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Interest Rate Targeting and the Dynamics of Short-Term Rates

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Abstract: We explore the link between the overnight fed funds rate, which is actively targeted by the Federal Reserve, and longer-maturity term fed funds rates. We develop a term-structure model which explicitly accounts for interest rate targeting and for the predictability of future target changes. The model is able to replicate some qualitative features of the dynamic behavior of deviations of short-term rates from the target.

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1 Introduction

This paper studies the link between the targeting activity of the overnight fed funds rate on the part of the Federal Reserve, and the time-series properties of short-term interbank rates. While the relation between the overnight fed funds rate and bank reserves or other monetary aggregates have been extensively investigated [see, for example, Bernanke and Blinder (1992)], the effects of interest rate targeting on the behavior of short-term rates seem somewhat neglected. Important exceptions include Cook and Hahn (1989), who look at the reaction of government-bond rates to target changes, and Simon (1990), who concentrates on tests of the expectations-hypothesis relation between overnight fed funds rates and three-month Treasury-bill rates.

One crucial piece of information to our analysis is the daily series of overnight fed funds rate targets set by the Federal Reserve or, more precisely, the "indications of the fed funds rate expected to be consistent with the degree of reserve pressure specified by the Federal Open Market Committee (Federal Reserve Bank of New York)." This series is from the Federal Reserve Bank of New York, and was used in the related paper by Balduzzi, Bertola, and Foresi (1993) to investigate the implications of targeting procedures for the performance of the expectations hypothesis.

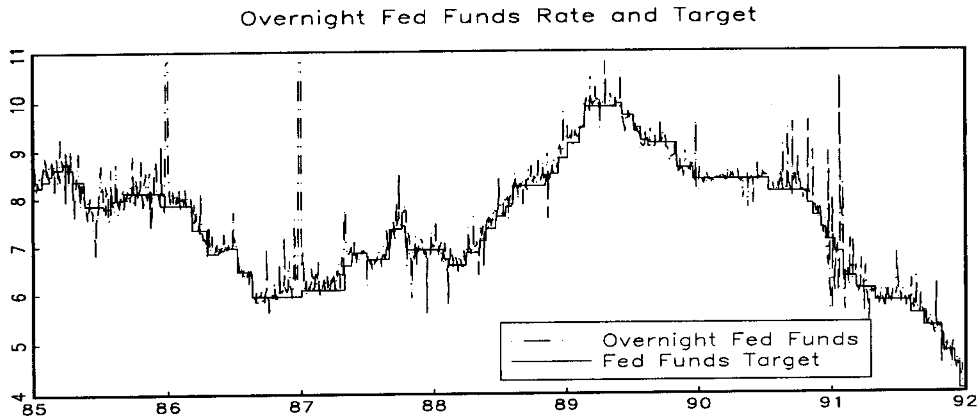
The top panel of Figure 1 displays the overnight fed funds rate and the target, while the bottom panel shows the one-, two-, and three-month term fed funds rates. The sample period goes from January 3, 1985, to December 31, 1991. (Details on the data are presented in Appendix A.)

It is apparent from the figure that the overnight rate quickly reverts to a target which, in turn, is highly persistent. Hence, we may model the overnight rate as the sum of two components: a random-walk-like target, and a mean-reverting deviation from the target.[1]

In turn, if we take an expectations-hypothesis view of the term structure, longer-maturity rates should be averages of expected future overnight rates. As the maturity of the contract increases, the more time is given (in expectation) to the overnight rate to revert to the expected future target (which equals the current one). As a result, short-term rates should be "close" to the target, the more so, the longer the maturity.

Consider, however, the autocovariance functions of spreads of overnight-, one-, two-, and three-month rates from the target which are shown in Figure 2. Overnight spreads are quite volatile about the target, with a standard deviation of 29 basis points, but short-lived. Surprisingly, though, one-, two-, and three-month spreads are also quite volatile, with standard deviations of about 30 basis points, and quite long-lived. Moreover, both volatility and persistence *increase* with maturity (except for an inversion of such ordering between the

[1]The notion that interest rate targeting induces a random-walk-like behavior in short-term rates has been discussed, for example, by Mankiw and Miron (1986). This is also an assumption used in popular "abitrage-free" models of the term structure, such as Ho and Lee (1986).



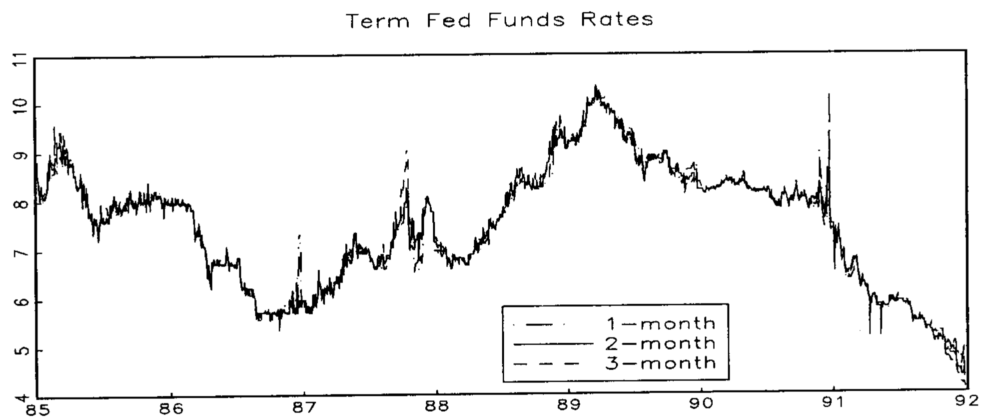


Figure 1: **Fed fund rates**

one- and the two-month term fed funds rates, at very short lags).

This paper develops a simple expectations-hypothesis model of the term structure that accounts for the stylized facts above. We formalize the notion that *i)* the Federal Reserve *targets* the overnight rate, *ii)* target changes are *predictable.* As a result, two factors drive short-term rate spreads: the current deviation of the overnight rate from the target, and expectations of future target changes. The first factor is quickly mean reverting, and its relevance decreases with the maturity of the instrument. The second factor, on the other hand, is persistent, and its relevance *increases* with maturity, thus making longer-maturity spreads more volatile and persistent.

The paper is organized as follows: Section (2) presents the term-structure model and studies its implications for the time-series properties of spreads of different maturities; in Section (3) we amend the model to account for the biweekly pattern in overnight rates, and we produce theoretical autocovariance functions for spreads of different maturity.

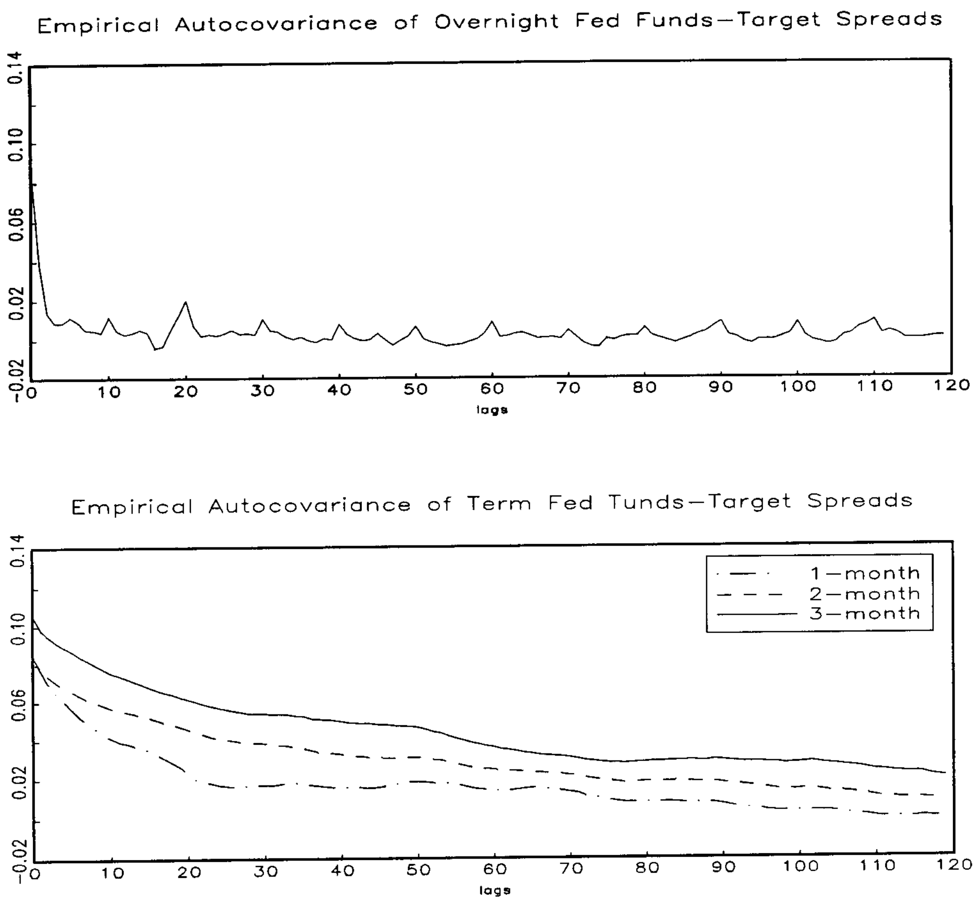


Figure 2: **Empirical autocovariance of fed fund rates**

2 Interest rate targeting and the term structure

This section develops a model of the term structure of interest rates which explicitly incorporates persistent expectations of future changes in the target.

2.1 A term-structure model

Our analysis draws on the term-structure model of Balduzzi, Bertola, and Foresi (1993) which emphasizes the role of interest rate targeting. We model the fluctuations of the overnight rate around the target as reverting towards a zero mean

$$r_t - \bar{r}_t = (1-k)(r_{t-1} - \bar{r}_{t-1}) + \epsilon_t, \tag{1}$$

where ϵ_t is a white-noise error with standard deviation σ_ϵ, and $0 < k < 1$ is a given constant. The overnight rate is more tightly targeted the higher the mean-reversion parameter k and the smaller σ_ϵ.

As to the behavior of the target, we argue the following: Target changes are not influenced by the temporary deviations of the overnight rate from the target. Moreover, the Federal Reserve typically revises the target by small increments, so as not to "whipsaw" the market; when a sizable change in the target is required, several small changes in the same direction are implemented. This gives rise to positive serial correlation in target changes that we model as follows:

$$\Delta \bar{r}_{N_t} = \rho \; \Delta \bar{r}_{N_t - 1} + \xi_{N_t}, \tag{2}$$

where N_t denotes the number of target changes between time zero and time t, $0 < \rho < 1$ regulates the serial correlation in target changes, and ξ_{N_t} is a mean-zero, serially uncorrelated error independent from ϵ_t. The absence of an intercept term in (2) implies the unconditional average of target changes to have mean zero. Note that equation (2) holds only as an approximation, since target changes take place by *discrete* amounts, typically a multiple of 12.5 basis points.

We denote with z_t the time-t market expectation of the *next* target change, formed at the *end* of day t. *Between* target changes, the expectation of the next target change is revised according to the daily flow of new information. *On the day* of a target change z_t relates to the realized target change through the parameter ρ. We formalize this behavior as follows

$$z_t = \begin{cases} z_{t-1} + \zeta_t, & \text{if there is no target change at } t, \\ \rho \Delta \bar{r}_t + \zeta_t, & \text{if there is a target change at } t, \end{cases}$$

where ζ_t is a mean-zero serially uncorrelated error, independent from ϵ_t, with standard deviation σ_ζ.

On the day of a target change the market already accumulates information on the following target change. This assumption is consistent with the timing of the data used in the following Section (3) which are 5p.m. closing quotes: on the day of the target change we observe interest rates which incorporate the target-change event, as well as additional information concerning the *next* target change.

The probability ν of a target change taking place on any given day is assumed to be constant. Hence, the probability distribution of the number of target changes, n, over s periods is given by

$$\binom{s}{n} \nu^n (1 - \nu)^{s-n}.$$

Note that our assumptions on the behavior of $\Delta \bar{r}_t$ imply that $\bar{r}_t$ is nonstationary. Such nonstationarity should not be taken literarily, but can be viewed as an approximation of the behavior of the overnight target over short periods of time.

Finally, we assume the interest rate of maturity τ, $R(\tau)$, to satisfy

$$R_t(\tau) = \sum_{s=0}^{\tau-1} \frac{E_t\{r_{t+s}\}}{\tau},$$

a pure expectations-hypothesis model of the term structure of interest rates.

Under these hypotheses, interest rates are linear in the two factors z_t and $(r_t - \bar{r}_t)$ (see Appendix B for details)

$$R_t(\tau) = \bar{r}_t + L_{r_t-\bar{r}_t}(\tau)\,(r_t - \bar{r}_t) + L_{z_t}(\tau) z_t, \tag{3}$$

with factor loadings

$$L_{r_t-\bar{r}_t}(\tau) \equiv \frac{1-(1-k)^\tau}{k\tau}, \tag{4}$$

and

$$L_{z_t}(\tau) \equiv \frac{1}{\tau}\sum_{s=1}^{\tau-1}\left[\sum_{n=1}^{s}\binom{s}{n}\nu^n(1-\nu)^{s-n}\frac{1-\rho^n}{1-\rho}\right]. \tag{5}$$

The factor loadings (5) and (4) depend on the features of interest rate targeting. An increase in targeting intensity affects negatively $L_{r_t-\bar{r}_t}(\tau)$: as k increases, the overnight rate reverts more quickly to the target, making current spreads less relevant for future overnight rates, and hence for current short-term rates as well. More important for our main point, the longer the maturity, the *smaller* the factor loading $L_{r_t-\bar{r}_t}(\tau)$.

The factor loading $L_{z_t}(\tau)$ increases when target changes become more frequent (higher ν), and hence future target changes are more likely. More persistence in target changes (higher ρ) increases $L_{z_t}(\tau)$: current expectations of the next target change have higher "information content" as to the subsequent ones. Unlike $L_{r_t-\bar{r}_t}(\tau)$, $L_{z_t}(\tau)$ is *increasing in* τ*:* the longer the maturity, the higher the chance that the target will change before the maturity of the instrument, and the more relevant the current expectation z_t in determining average interest rates before maturity. This feature is crucial for our model to replicate the ranking of autocovariance functions observed in Figure 2.

2.2 Time-series properties of spreads

We now investigate the time-series properties of the spreads of interest rates of different maturity from the target.

If contemporaneous and lagged overnight spreads do not influence the Federal Reserve's decision to revise the target, they should also be irrelevant to market participants' revision of z_t. Thus, the assumed orthogonality of the two processes $\Delta\bar{r}_t$ and $r_t - \bar{r}_t$ implies that z_t is orthogonal to $r_t - \bar{r}_t$. Based on equation (3) we can then write the *autocovariance* function of the maturity-τ spread, $\text{Cov}\left[R_{t+s}(\tau) - \bar{r}_{t+s}, R_t(\tau) - \bar{r}_t\right]$, which represents the main object of interest of this paper. We have

$$\text{Cov}\left[R_{t+s}(\tau) - \bar{r}_{t+s}, R_t(\tau) - \bar{r}_t\right] = L_{r_t - \bar{r}_t}(\tau)^2 \text{ Cov}(r_{t+s} - \bar{r}_{t+s}, r_t - \bar{r}_t) + L_{z_t}(\tau)^2 \text{ Cov}(z_{t+s}, z_t). \quad (6)$$

As we discussed above, longer-maturity spreads attach a larger weight to the expected-target-change factor z. Hence, they inherit the time series properties of the former to a larger extent than shorter-maturity rates.

For given factor loadings, $L_{z_t}(\tau)$ and $L_{r_t - \bar{r}_t}(\tau)$, the autocovariance functions in (6) depend on the autocovariances of z_t and $r_t - \bar{r}_t$ which we calculate in the following.

Autocovariance function of $r_t - \bar{r}_t$

The autocovariance of $r_t - \bar{r}_t$ is simply that of an AR(1) process,

$$\text{Cov}(r_{t+s} - \bar{r}_{t+s}, r_t - \bar{r}_t) = \frac{(1-k)^s \sigma_\epsilon^2}{1-(1-k)^2}, \quad (7)$$

which decreases with the targeting intensity k at any number of lags.

Autocovariance function of z_t

For simplicity, consider first the case of $\rho = 0$, that is no autocorrelation in target changes:

$$z_{t+s} = \begin{cases} z_t + \sum_{j=1}^{s} \zeta_{t+j}, & \text{if there is no target change at } t, \\ \sum_{j=0}^{s} \zeta_{t+j}, & \text{if there is a target change at } t, \end{cases}$$

where ζ_t denotes a serially independent mean-zero error, with constant variance σ_ζ^2. The unconditional autocovariance of z_t is, by the law of total probabilities, $\text{Cov}(z_{t+s}, z_t) = (1-\nu)^s \text{Var}(z_t)$. To calculate $\text{Var}(z_t)$, note that the innovations ζ_t are serially independent and cumulate only from the time of the *last* target change. Conditional on the last target change having occurred at time t^*, $\text{Var}(z_t|\text{last target change at } t^*) = (t - t^* + 1)\sigma_\zeta^2$, and therefore the unconditional variance is $\text{Var}(z_t) = \sum_{s=0}^{\infty} \nu(1-\nu)^s (s+1)\sigma_\zeta^2 = (1/\nu)\sigma_\zeta^2$.

When target changes are autocorrelated, the expectation series $\{z_t\}$ takes such autocorrelation into account:

$$z_{t+s} = \begin{cases} z_t + \sum_{j=1}^{s} \zeta_{t+j}, & \text{if there is no target change at } t, \\ \rho(z_{t-1} + \eta_t) + \sum_{j=0}^{s} \zeta_{t+j}, & \text{if there is a target change at } t, \end{cases}$$

where η_t is an expectational error defined as $\eta_t \equiv \Delta \bar{r}_t - z_{t-1}$ which is realized only when a target change is implemented. In Appendix C we show that the autocovariance function of

z_t equals

$$\mathrm{Cov}(z_{t+s}, z_t) = \sum_{n=0}^{s} \binom{s}{n} \nu^n (1-\nu)^{s-n} \rho^n \frac{\rho^2 \sigma_\eta^2 + \sigma_\zeta^2/\nu}{1-\rho^2}, \tag{8}$$

where σ_η^2 denotes the variance of η_t.

Note that a higher correlation in target changes (ρ) increases the variance and persistence of z_t. On the other hand, when target changes become more frequent (higher ν) the variability and average persistence of z_t *decreases*: a higher number of target changes means that z_t is more often reset close to $\Delta\bar{r}_t$, which in turn reverts to zero across target-change events [see equation (2)].

Moreover, the parameters characterizing the information-acquisition process also affect the time-series properties of z_t. A higher variance of ζ_t increases the variance and the persistence of z_t. As the market's expectations on the next target change become less accurate (higher σ_η), the variability and persistence of z_t also increases.

Substituting (8) and (7) into (6) yields an explicit expression for the theoretical autocovariance function of spreads from the target.

3 Empirical analysis

This section extends the model illustrated above to account for the marked biweekly pattern in the overnight fed funds rate, and produces theoretical autocovariance functions for spreads of different maturities.

3.1 A term-structure model with biweekly effects

The autocovariance functions of Figure 2 show a marked biweekly pattern in overnight spreads and, to a lesser extent, in one-month spreads as well. Hence, we proceed to incorporate such biweekly effects into our term-structure model.[2]

We modify equation (1) along the lines in Balduzzi, Bertola, and Foresi (1993) and assume

$$r_t - \bar{r}_t = d_t + (1-k_t)(r_{t-1} - \bar{r}_{t-1} - d_{t-1}) + \epsilon_t, \tag{1'}$$

where $d_t = d_{t+10}$ is a time-varying intercept which captures changes in the central tendency of r_t over the maintenance period. Similarly, we allow for a time-varying mean-reversion parameter $k_t = k_{t+10}$ and a time-varying standard deviation $\sigma_{\epsilon t+10} = \sigma_{\epsilon t}$.

[2]The relevance of periodicities in the overnight fed funds rate over the maintenance period is well known; see for example, Campbell (1987), and Barret, Slovin, and Sushka (1988).

Some care is needed in the treatment of weekends. There are no market quotes for overnight rates during the weekend: the probability of a target change on Saturday or Sunday is nil and thus expectations of target levels prevailing for interest rates quoted on Friday are appropriate also for the (shadow) interest rate of Saturday and Sunday. For other holidays in the sample we proceed "as if" a target change were possible during any non-weekend day.

Interest rates are then given by (see Appendix B for details)

$$R_t(\tau) = \bar{r}_t + L_{z_t}(\tau)z_t + L_{r_t-\bar{r}_t}(\tau)\,(r_t - \bar{r}_t - d_t) + L_{d_t}(\tau), \quad \tau = 30, 60, 91, \qquad (3')$$

where, since Friday-overnight rates regulate contracts which mature on Monday, $L_{z_t}(\tau)$ now equals

$$L_{z_t}(\tau) \equiv \frac{1}{\tau}\sum_{s=1}^{\hat{\tau}-1}(1+2F_t)\left[\sum_{n=1}^{s}\binom{s}{n}\nu^n(1-\nu)^{s-n}\frac{1-\rho^n}{1-\rho}\right], \qquad (5')$$

with $\hat{\tau} = \tau - \text{floor}(\tau 2/7)$ and F_t a dummy variable which equals one on Fridays and zero otherwise; $L_{r_t-\bar{r}_t}(\tau)$, because of the biweekly effect in the mean reversion, is given by

$$L_{r_t-\bar{r}_t}(\tau) \equiv \frac{1}{\tau}\left[1 + \sum_{s=t+1}^{t+\tau-1}\prod_{j=1}^{s-t}(1-k_{t+j})\right], \qquad (4')$$

and finally

$$L_{d_t}(\tau) \equiv \sum_{i=0}^{\tau-1} d_{t+i}/\tau.$$

3.2 Autocovariance functions

We now compare the implications of our model with the data in the "metric" of the autocovariance functions. The theoretical autocovariance functions calculated in Section (2.2) are also modified to account for weekends and for the biweekly patterns in the overnight rate (see Appendix D for details).

In order to implement the model we need to estimate several parameters characterizing the overnight rate process, as well as the target-change process, and its understanding on the part of market participants.

The time-varying intercept for the i-th day of the maintenance period, d_i, is set equal to the sample average of $r_t - \bar{r}_t$ for that day. Let $i = 1$ denote the second Monday of the biweekly-maintenance period (which ends on Wednesday, $i = 3$); we have

$$d_i = .08,\ .016,\ .19,\ .15,\ .064,\ .12,\ .092,\ .042,\ .050,\ -.015, \quad \text{for } i = 1, \ldots, 10.$$

These figures suggest that the overnight rate has been fairly "close" to the target in our sample: the average spread amounts to less than eight basis points. Also, the biweekly pattern is evident, with the largest average spread from the target on the Wednesday ending the maintenance period.

Similarly, the time-varying mean-reversion parameter for the i-th day of the maintenance period, k_i, is chosen to match the first-order sample autocovariance of $r_t - \bar{r}_t - d_t$ for that day: $k_i = \text{Cov}((r_i - \bar{r}_i - d_i), (r_{i-1} - \bar{r}_{i-1} - d_{i-1}))/\text{Var}(r_i - \bar{r}_i - d_i)$. We have

$$k_i = .21,\ .64,\ -.21,\ .74,\ .56,\ .42,\ .30,\ .64,\ .35,\ .47,\ \text{ for } i = 1,\ \ldots, 10.$$

These figures confirm that interest rate targeting during our sample was quite effective. The average k_i equals .41, which means that almost half of the spread net of d_i on any given day was "reabsorbed" by the next one. The biweekly pattern is also apparent, and in fact on the Wednesday closing the maintenance period the previous-day's spread is exacerbated rather than reduced.

Finally the time-varying standard deviation for the i-th day of the maintenance period, $\sigma_{\epsilon i}$, is chosen to match the sample variance of $r_t - \bar{r}_t$ for that day:

$$\sigma_{\epsilon i} = \sqrt{\text{Var}(r_i - \bar{r}) - k_i^2 \text{Var}(r_{i-1} - \bar{r})}.$$

We have

$$\sigma_{\epsilon i} = .26,\ .24,\ .49,\ .20,\ .15,\ .20,\ .16,\ .17,\ .16,\ .17,\quad \text{for } i = 1,\ \ldots, 10.$$

Again, we have an indication of the exceptional volatility of the overnight fed funds rate at the end of the maintenance period.

The parameter ν is set equal to the observed frequency of target changes in our sample, .036, while $\rho = .62$ is obtained from an OLS regression of $\Delta\bar{r}_{N_t}$ on $\Delta\bar{r}_{N_{t-1}}$.

The remaining parameters σ_ζ and σ_η characterize the behavior of the *unobservable* market expectation z_t. While the two parameters cannot be recovered directly from the data, we can make inference on them based on our model. More precisely, we chose σ_ζ and σ_η as to minimize the sum of squared deviations of the theoretical autocovariance functions $\text{Cov}\left[R_{t+s}(\tau) - \bar{r}_{t+s}, R_t(\tau) - \bar{r}_t\right]$, from their empirical counterparts, for $\tau = 30,\ 60,\ 91$, and lags $s = 0,\ 30,\ 60,\ 120$. Note, from (8), that σ_η and σ_ζ cannot be separately identified, and our algorithm optimizes with respect to $\text{Var}(z_t) = (\rho^2\sigma_\eta^2 + \sigma_\zeta^2/\nu)/(1-\rho^2)$ (see Appendix C for this last equality). The least squares algorithm yields $\text{Var}(z_t) = .14$, which is somewhat large, given that the typical size of a target change in our sample is 12.5 or 25 basis points. Still, this result is understandable given the restrictive assumptions of our expectations-hypothesis model: we probably attribute to z_t some of the variability in term fed funds rates which is due to changes in unmodeled liquity and term premia.

Figure 3 presents the theoretical autocovariance functions for overnight and short-term spreads. Comparison of Figures 2 and 3 shows how our model successfully replicates the ranking of autocovariance functions.

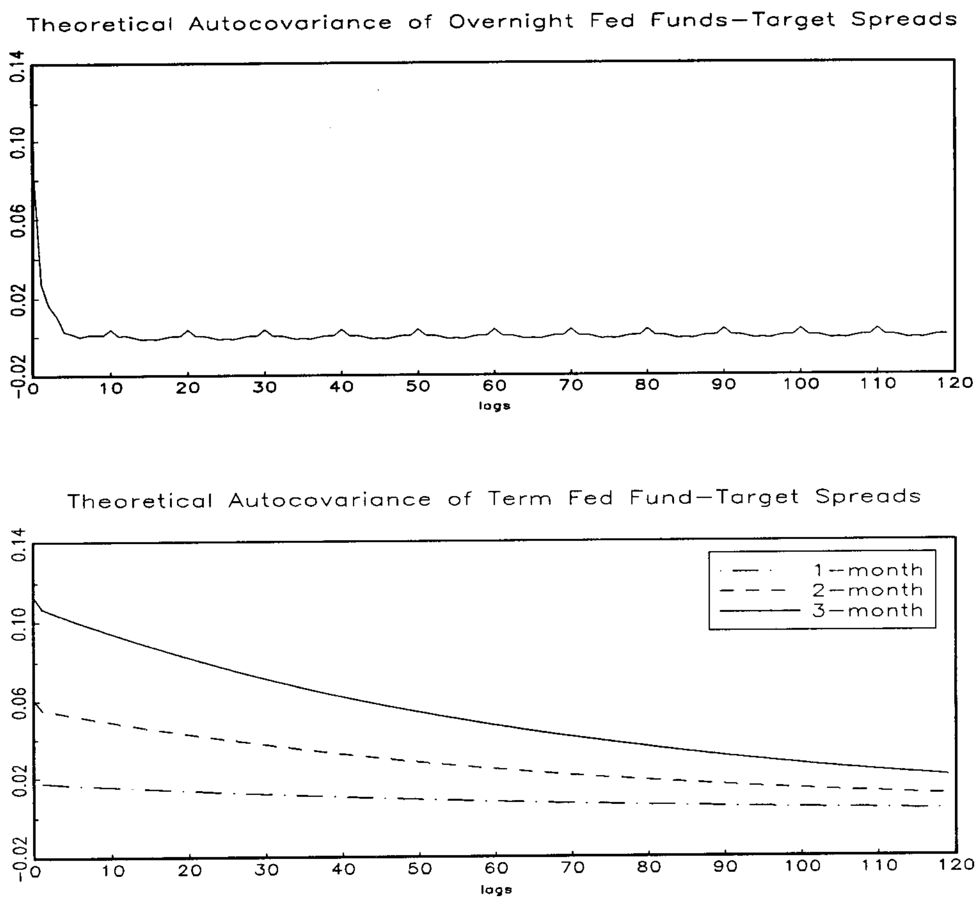


Figure 3: **Theoretical autocovariance of fed fund rates**

When the Fed targets the overnight rate, *two* factors drive longer-maturity spreads: short-lived deviations of the overnight rate from the target, and persistent expectations of the next target change. It is the *second* factor which makes spreads of all maturities variable and persistent, and allows us to reproduce the observed ranking of autocovariance functions.

Our model replicates some of the biweekly pattern in the autocovariance function of overnight spreads, while such periodicity is absent from longer-term spreads. The intuition for this is that the averaging of future expected overnight fed funds rates, which takes place through the expectations hypothesis, "gets rid" of seasonalities. Hence, the biweekly pattern which we do observe in actual one-month spreads is probably due to the behavior of liquidity and term premia which are absent form our framework.

Appendix

A Data

When a target *range* for the overnight fed funds rate was indicated, the midpoint of that range was used as the target value.

Daily closing-quote series for interest rates on overnight, one-, two-, and three-month fed funds of comparable liquidity and risk characteristics are from the Board of Governors of the Federal Reserve.

Market holidays are filled with observations from the last day the market was open, and all interest rates are converted to a continuously compounded basis

Three observations for the fed funds rate are treated as outliers (12-31-1985, 12-30-1986, and 12-31-1986) and replaced with the value of the target on that same day.

B Factor loadings

For any $t + s$ we have the definitional relation

$$E_t(r_{t+s}) \equiv \bar{r}_t + E_t\left[\sum_{j=1}^{N_{t+s}-N_t} \Delta\bar{r}_{N_t+j} + (r_{t+s} - \bar{r}_{t+s})\right] \tag{9}$$

where N_{t+s} is the number of target changes between time zero and time $t + s$ and $\Delta\bar{r}_{N_t+j}$ is the j-th target change after time t. Since target changes occur with fixed daily probability ν, and their timing is independent of z_t, we can condition on their total number N_{t+s} and implement the known (binomial) form of the distribution of N_{t+s}:

$$E_t\left(\sum_{j=1}^{N_{t+s}-N_t} \Delta\bar{r}_{N_t+j}\right) = \sum_{n=1}^{s}\binom{s}{n}\nu^n(1-\nu)^{s-n}\, E_t\left(\sum_{j=1}^{n}\Delta\bar{r}_{N_t+j}\middle| N_{t+s} = N_t + n\right).$$

Using the law of iterated expectations on equation (2), the expected size of the $N_t + j$-th target-change realization is

$$E_t(\Delta\bar{r}_{N_t+j}|n \geq j \geq 1) = \rho^{j-1}z_t,$$

hence

$$\begin{aligned} E_t\left(\sum_{j=1}^{N_{t+s}-N_t} \Delta\bar{r}_{N_t+j}\middle| N_{t+s} = N_t + n\right) &= \sum_{n=1}^{s}\binom{s}{n}\nu^n(1-\nu)^{s-n}\sum_{j=1}^{n}\rho^{j-1}z_t \\ &= z_t\sum_{n=1}^{s}\binom{s}{n}\nu^n(1-\nu)^{s-n}\frac{1-\rho^n}{1-\rho}. \end{aligned} \tag{10}$$

Substituting (10) in (9), we find that

$$E_t(\bar{r}_{t+s}) - \bar{r}_t = z_t \left[\sum_{n=1}^{s} \left(\begin{array}{c} s \\ n \end{array} \right) \nu^n (1-\nu)^{s-n} \frac{1-\rho^n}{1-\rho} \right].$$

Averaging this expression over the horizon relevant to an instrument of maturity τ yields the factor loading on the next target-change expectation z_t in equation (3):

$$L_{z_t}(\tau) \equiv \frac{1}{\tau} \sum_{s=1}^{\tau-1} \left[\sum_{n=1}^{s} \left(\begin{array}{c} s \\ n \end{array} \right) \nu^n (1-\nu)^{s-n} \frac{1-\rho^n}{1-\rho} \right].$$

Moreover, it is easy to obtain

$$E_t(r_{t+s} - \bar{r}_{t+s}) = (r_t - \bar{r}_t)(1-k)^s,$$

and the loading on $r_t - \bar{r}_t$ in the term-structure model (3) is

$$L_{r_t - \bar{r}_t}(\tau) = \frac{1-(1-k)^\tau}{k\tau}. \tag{11}$$

C Autocovariance function of z_t

Iterating the law of motion of z_t backwards and using the law of total probabilities, we obtain the unconditional autocovariance of z_t:

$$\text{Cov}(z_{t+s}, z_t) = \sum_{n=0}^{s} \left(\begin{array}{c} s \\ n \end{array} \right) \nu^n (1-\nu)^{s-n} \rho^n \text{Var}(z_t),$$

where $\text{Var}(z_t)$ is given by

$$\begin{aligned} \text{Var}(z_t) &= \rho^2 \text{Var}(z_t) + \rho^2 \text{Var}(\eta_t) + \text{Var}\left(\sum_{n=0}^{s} \zeta_{t+n} \right) \\ &= \rho^2 \text{Var}(z_t) + \rho^2 \sigma_\eta^2 + \sum_{s=0}^{\infty} \nu(1-\nu)^s (s+1) \sigma_\zeta^2 = \frac{\rho^2 \sigma_\eta^2 + \sigma_\zeta^2/\nu}{1-\rho^2}, \end{aligned}$$

and thus

$$\text{Cov}(z_{t+s}, z_t) = \sum_{n=0}^{s} \left(\begin{array}{c} s \\ n \end{array} \right) \nu^n (1-\nu)^{s-n} \rho^n \frac{\rho^2 \sigma_\eta^2 + \sigma_\zeta^2/\nu}{1-\rho^2}.$$

D Autocovariances and biweekly effects

When biweekly effects are explicitly accounted for, some modifications to our discussion of the time-series properties of interest rates are needed. While the autocovariance function of z_t does not change, that of $r_t - \bar{r}_t$ is modified as follows: We can write

$$\text{Cov}\left(r_{i+s} - \bar{r}_{i+s}, r_i - \bar{r}_i\right) = \text{Cov}\left(r_{i+s} - \bar{r}_{i+s} - d_{i+s}, r_i - \bar{r}_i - d_i\right) + \text{Cov}\left(d_{i+s}, d_i\right),$$

where i denotes the day of the biweekly period. We have

$$\text{Cov}\,(r_{i+s} - \bar{r}_{i+s} - d_{i+s}, r_i - \bar{r}_i - d_i) = \prod_{j=1}^{s}(1 - k_{i+j})\text{Var}(r_i - \bar{r}_i - d_i), \; i = 1, 2, \ldots 10,$$

The i-th variance $\text{Var}(r_i - \bar{r}_i - d_i)$ is found by solving the system

$$\text{Var}(r_i - \bar{r}_i - d_i) = (1 - k_i)^2\text{Var}(r_{i-1} - \bar{r}_{i-1} - d_{i-1}) + \sigma^2_{\epsilon i}, \qquad i = 1, 2, \ldots 10,$$

to yield

$$\text{Var}(r_i - \bar{r}_i - d_i) = \frac{[(1 - k_i)^2(1 - k_{i-2})^2 \ldots (1 - k_{i-8})^2]\,\sigma^2_{\epsilon i-9} + \ldots + (1 - k_i)^2\sigma^2_{\epsilon i-1} + \sigma^2_{\epsilon i}}{1 - [(1 - k_i)^2(1 - k_{i-2})^2 \ldots (1 - k_{i-9})^2]},$$

for $i = 1, 2, \ldots 10$. The second component of the autocovariance function of $r_t - \bar{r}_t$ can be easily calculated as follows:

$$\text{Cov}\,(d_{i+s}, d_i) = (d_{i+s} - \bar{d})(d_i - \bar{d}),$$

where $\bar{d}$ is the average of d_i over the ten days of the biweekly maintenance period.

Moreover, since the quantity $L_{d_t}(\tau)$ also follows a biweekly pattern, we have

$$\text{Cov}\left[L_{d_{i+s}}(\tau), L_{d_t}(\tau)\right] = \left[L_{d_{i+s}}(\tau) - \bar{L}_{d_t}(\tau)\right]\left[L_{d_i}(\tau) - \bar{L}_d(\tau)\right],$$

where $\bar{L}_{d_t}(\tau)$ is the average $\bar{L}_d(\tau)$ over the ten days of the biweekly maintenance period.

We obtain the autocovariance function of any yield in excess of the overnight target as the *average* of the ten autocovariance functions:

$$L_{z_{i+s}}(\tau)L_{z_i}(\tau)\text{Cov}(z_{t+s}, z_t) + L_{d_{i+s}}(\tau)L_{d_i}(\tau)\text{Cov}\,(r_{i+s} - \bar{r}_{i+s} - d_{i+s}, r_i - \bar{r}_i - d_i) + \text{Cov}\left[L_{d_{i+s}}(\tau), L_{d_t}(\tau)\right].$$

As shown in equations (5') and (4') also $L_{z_t}(\tau)$ and $L_{r_t - \bar{r}_t}(\tau)$ are time varying and follow a biweekly pattern. This does not complicate the analysis any further though, since the levels of z_t and $r_t - \bar{r}_t - d_t$ are *free* of biweekly periodicity.

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