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# *A Model of Target Changes and the Term Structure of Interest Rates*

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# A Model of Target Changes and the Term Structure of Interest Rates

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**Abstract:** We explore the effects of overnight-rate targeting on nominal interest rates of longer maturities. In a realistic model of noisy targeting and infrequent target changes, expectations of future policy actions introduce persistent spreads between interest rates of different maturities. Some empirical features of U.S. money-market daily interest rate data are broadly consistent with our theoretical assumptions and results. Not surprisingly, however, the data reject the expectations-hypothesis (EH) relation that we take as a working assumptions. A newly available series of historical interest-rate targets and simple tests based on our theoretical insights suggest that the EH rejection may be due to erroneous market expectations of the policy-induced component of fed funds dynamics. We briefly discuss how the size and volatility of such expectations may be interpreted from the perspective of our theoretical framework.

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## 1 Introduction

In many models of the nominal term structure, monetary authorities affect interest rates mainly through inflationary expectations, and perhaps through short-term liquidity effects as well. In practice, market participants attach particular significance to the stance of monetary policy when assessing the outlook for short-term interest rates. The present paper takes this perspective seriously. At the theoretical level, we formalize the idea that short-term interest rates are mainly determined by the current and expected future pattern of monetary policy. We also relate empirical interest-rate dynamics to a newly available historical series of interest-rate targets.

We view the overnight rate of interest on fed funds as the prime instrument of monetary policy, and we model it as the sum of two components: the "target," which is changed infrequently by the monetary authority in a way that imparts martingale-like behavior to interest rates; and the deviations from the target, which are the outcome of continuous market equilibrium and exhibit mean reversion toward zero. Our approach is closely related to that of recent and less recent work on "peso problems" and interest-rate differentials in the exchange rate literature.[1] Like most contributions to that literature, we focus on expectational relations between nominal interest rates of different maturities, and do not explicitly address more substantial policy issues. Still, our theoretical approach does offer insights on many issues of academic and non-academic interest, especially in light of renovated emphasis on interest-rate targeting and discount rate changes. Both our model and U.S. data feature long-memory spreads between overnight and longer-term interest rates when the former are the immediate instrument, and the latter the intermediate target, of monetary policy. This suggests that the style of official intervention may only loosely control even short-term interest rates, and allows us to extract from interest-rate and target data an estimate of the market's expected size and direction of target changes.

Section 2 proposes a stylized model to illustrate the basic ideas we put forward: when the infrequent character of real-life target changes at a daily time scale is explicitly accounted for, expectations of future policy actions introduce persistent spreads between interest rates of different maturities. Longer-term yields are then driven by three factors: the current target, the fluctuations of overnight rates about the target, and the expectations of future target changes.

[1]The references most relevant to our work are Bertola and Svensson (1992), Rose and Svensson (1991), and Lindberg, Svensson, and Söderlind (1991), where the notion of stochastic "devaluation risk" is introduced and empirically implemented on exchange- and interest-rate data.

Section 3 confronts the model with recent U.S. money-market interest rates and with historical target data which were made available to us by the Federal Reserve Bank of New York. Several features of the data are qualitatively consistent with our modeling approach, but the expectations-hypothesis relation that we take as a working assumption is rejected. Our modeling approach and data provide an interpretation for this common finding: when overnight rates are viewed as the sum of the official target process and deviations from it, the former component appears responsible for much of the expectational bias in the data, suggesting that it is the policy-induced component of fed funds dynamics to be erroneously anticipated by the market.

In Section 4 we consider specific models of expectation formation, and extract a series of expected target changes. The estimated series has statistical properties which are consistent with modeling assumptions and predicts well the direction, if not the size, of realized changes. The data do not contain sufficient information to precisely identify whether the expectational bias is mostly due to erroneous anticipation of target changes' size or of their timing. Section 5 summarizes what is learned from our theoretical and empirical work, and concludes outlining directions for further research.

## 2 A simple model of interest-rate targeting

It is well understood that the process for short-term interest rates has displayed very long memory since the Federal Reserve System ("the Fed") was established in the 1920s. Mankiw and Miron (1986) and others show that previously important interest-rate seasonals have disappeared after the Fed's inception, and that short-term nominal interest rates have approximately followed a martingale process. The long memory in interest rates is viewed as the result of direct or indirect intervention of the Fed with the objective of stabilizing the economy. Goodfriend (1990) relates this view to institutional information on interest-rate-targeting practices. Historically, the Fed has indeed used direct targeting of overnight rates to stabilize longer-term interest rates. Goodfriend notes that targets are changed infrequently in practice, and surveys empirical evidence showing the Fed's influence on interest rates.

Accordingly, we model the process followed by the overnight fed funds rate, $r_t$, as the sum of a target-rate process $\bar{r}_t$, and deviations from it. The resulting perspective on the term structure of interest rates is quite different from that afforded by the customary decomposition of nominal rates in (expected) inflation and real interest rates. The two views are complementary, however, and the one we choose in this paper has a number of novel implications. First, it reverses the usual view of the relation between inflation and interest rates: it is the stabilization policy, implemented by interest-rate targeting, which induces long memory in nominal interest rates and *thus* inflation [ Goodfriend (1990) ]. Second, it allows us to do without inflation data, whose quality and frequency fail to match those of interest-rate data. Third, it points to the core of the effects of monetary intervention on interest rates at the short end of the term structure: from this perspective, explaining interest rates requires understanding the character of interest-rate targeting on the part of the Fed.

The model outlined in the rest of this section illustrates our basic theoretical perspective in the simplest possible way. The specific modeling assumptions formalize ideas which feature prominently in the relevant theoretical and empirical literature and, while not meant to be fully realistic, the resulting model is representative of a wider class of model with qualitatively similar implications (see Section 4 below for possible extensions).

We model the limited and mean-reverting nature of fed funds-rate fluctuations around the target by the first-order stochastic difference equation

$$r_t - \bar{r}_t = (1-k)(r_{t-1} - \bar{r}_{t-1}) + \epsilon_t, \tag{1}$$

where $\epsilon_t$ is a white-noise error, and $0 < k < 1$ is a given constant. The $r_t$ process is more tightly targeted the smaller the standard deviation $\sigma$ of $\epsilon_t$, and the higher the mean-reversion parameter $k$.

As to the behavior of the target $\bar{r}_t$, we do not fully specify the way in which the Fed decides target changes. We model the process of target changes in minimal fashion taking target changes to be independent of the process (1), and to be *infrequent,* with $\nu < 1$ the known probability of a target change on any day $t$. We also assume in this section that, when a target change occurs, all adjustment "pressure" is released, and only the accrual of new information leads the Fed to contemplate a new target change in either direction. This rather extreme assumption incorporates Mankiw and Miron's (1986) idea that Fed targeting is responsible for martingale-like behavior of short-term rates.

In the following, we focus on *market expectations* of future target changes, which we model in an equally minimal fashion. We denote with $z_t = E_t(\Delta\bar{r}_{\hat{t}})$ the market's expectation, as of $t$, of the size of the *next* target-change realization, occurring at time $\hat{t} > t$. In reality, of course, such expectations presumably depend in complex ways on a variety of detailed policy-relevant information. By definition, however, the expectation revisions represented by changes in $z_t$ are unpredictable: only new information should induce the market to revise its expectation of future target changes occurring at time $\hat{t}$. Together with the "pressure release" assumption above, such unpredictability implies a simple univariate representation for the $\{z_t\}$ process:

$$z_t = \begin{cases} z_{t-1} + \text{error}_t, & \text{when } t \neq \hat{t}; \\ \text{error}_t, & \text{when } t = \hat{t}, \end{cases} \tag{2}$$

where the error is unpredictable on the basis of the market's past information (which obviously includes $z_{t-1}$). The expectational nature of $z_t$ implies that it should follow a martingale when target changes are *not* realized and, by assumption, $z_t$ is reset to zero (or, more generally, to a value drawn from an independent, mean-zero distribution) at every time $\hat{t}$ when a target change is realized.

Before proceeding, we note that the process (2) is meant to describe expectations *as formed by the market.* Systematic discrepancies between the $\{z_t\}$ process and the process governing actual target changes may arise for at least one reason: the Fed may not follow a stable policy or may hide the one it follows, so as to enhance the effectiveness of policy actions by surprising the market. As a result, the changing array of parameters characterizing the process of target changes is not known to the market, requiring potentially endless learning on their part.

To link the dynamics of overnight rates (and targets) to longer-maturity yields, we suppose that arbitrage keeps the interest rates on longer-term maturities in line with market expectations of future overnight fed funds rates. A linear expectations-hypothesis term structure is then a fair description of the equilibrium relation between short-term and overnight fed funds rates:

$$R_t = \sum_{s=0}^{\tau-1} \frac{E_t(r_{t+s})}{\tau}, \tag{3}$$

i.e., the yield $R_t$ on a $\tau$ days-maturity loan at time $t$ is the average of the future overnight rates $\{r_{t+s}\}$ expected to prevail during the life of the credit instrument.[2]

In the Appendix we calculate each term of the summation in (3) using our assumptions, to obtain

$$R_t = \bar{r}_t + \left[1 - \frac{1-(1-\nu)^\tau}{\nu\tau}\right] z_t + \left[\frac{1-(1-k)^\tau}{k\tau}\right] (r_t - \bar{r}_t). \tag{4}$$

The resulting term structure features three factors: the target $\bar{r}_t$, which (on the basis of market information) follows a generalized random walk on random time steps; the expectation of the next target change's size $z_t$, which is stationary but has local martingale dynamics

[2]See Cook and Hahn (1990) and Campbell and Shiller (1991) for recent surveys of theoretical and empirical issues relevant to (3).

between target changes; and the deviation $r_t - \bar{r}_t$ of overnight rates from current targets, which is stationary and reverts linearly towards zero.

As the time to maturity shortens, the yield converges to the overnight fed funds rate, $R_t(1) = r_t$: thus, day-by-day fluctuations in the very short end of the term structure reflect mainly expected movements around the current target $\bar{r}_t$. At the other extreme, $R_t(\infty) = \bar{r}_t + z_t$ if $\nu > 0$. The further into the future one looks, the less important is the current deviation from target ($k$ is the relevant parameter for the fading relevance of this factor), and the more relevant the possibility of a target change (by $z_t$, given current information). This "expected target change" factor is novel and peculiar to our interest-rate-targeting setup.

The stylized model of this section is quite close in spirit to Mankiw and Miron's (1986) framework of analysis but, as is appropriate at the very fine time scale we wish to consider, it accounts for *infrequency* of target changes. Only if $\nu = 1$ would target changes occur every day, to imply that the overnight rate itself would behave as a martingale (plus a mean reverting process). Expectations of future target changes play a separate and very important role in our model, and we shall see in Section 4 that our perspective leads quite naturally to a relaxation of the resetting assumptions above.

## 3 Taking the model to the data

We now turn to confront the insights afforded by the model outlined above with real-world data which may be generated by a similar mechanism.

New daily historical fed funds targets were made available to us by the Federal Reserve Bank of New York (FRBNY). The target data are those on which the trading desk's open market operations were based on any given day, or "indications of the fed funds rate expected to be consistent with the degree of reserve pressure specified by the Federal open market committee (where a trading range was indicated, the midpoint of that range is provided)." While professional "fed watchers" are usually able to infer current targets from a variety of economic and policy variables, the historical data we use were not previously disclosed. In line with evidence of renewed emphasis on fed funds rates, however, from 1991 the FRBNY publishes target data in the Spring issue of its Quarterly Review, resuming a practice interrupted in 1983.

As to overnight and longer-term interest rate data, we obtained from the Board of Governors of the Fed daily closing-quote series for overnight interest rates on fed funds and for three-month "term fed funds" of comparable liquidity and risk characteristics.[3]

Figure 1

Figure 1 plots the target fed funds rate, the overnight fed funds rate, and the three-month fed funds rate from January, 1985 through December, 1991. All three series are translated on a continuously compounded basis,[4] and market holidays are filled with observations from the last day the market was open.

We see in the Figure that target changes are indeed infrequent on a daily time scale. There are 66 target changes in the seven-year span of Figure 1, hence the target is changed (on average) only every five weeks or so. The data also tell us that targets are most often specified in quarter points (before continuous compounding), so that realized target changes are usually 25 or 50 basis points in absolute value. In principle, this is not a problem for the simple model specified above: as long as the market attaches positive probability to different quarter-point increments, the *expected* size $z_t$ of target changes can be a continuous random variable even when target-change realizations (hence expectational errors) have discrete distributions. Indeed, such lumpiness in target changes may be naturally associated to their infrequency. The Fed might conceivably intervene using a finer single-tick mesh, but usually decides to specify targets in quarter-point (eight ticks) increments. Specifying "round" targets might well facilitate communications between policy makers and the New York desk and, to the extent that the Fed intends to clearly signal its policy moves, between the desk and the market as well. The timing of target changes would then be determined by rounding to the closest quarter-point of an underlying, continuously updated "shadow" target process.

The overnight rate is quite volatile around the current target, reflecting the inherent difficulty of controlling a price target by quantity intervention in a turbulent market: in general, the change in reserve assets implemented by the desk's open-market operations is not exactly consistent with the specified overnight-rate target. Further, desk operations are implemented shortly after 11 AM in New York, and no coincident market-rate series

[3]While similar theoretical and empirical work could be performed on T-bill or commercial-paper yields, the liquidity, tax, and default characteristics of such securities are quite different from those of interbank instruments, and the time-varying effects introduced by such features [ for which see, e.g., Simon (1990) and Bernanke (1990) ] would spoil our analysis of expectational factors.

[4]Let $r^q$ be the quoted overnight fed funds rate, the corresponding continuously compounded rate $r$ is given by $r = 100[\ln(1 + r^q/360000)365]$. The same formula applies to quoted fed-funds-rate targets. Similarly, quoted three-month fed funds rates $R^q$ are converted to their continuously-compounded counterpart according to $R = 100[\ln(1 + R^q\ 91/360000)365/91]$.

is available to us. Our interest-rate data are 5 PM closing rates, and time lag introduces additional noise in the spread between measured overnight rates and targets.

Fed funds rates also display pronounced spikes in the last two days of "reserve maintenance periods," when banks must meet reserve requirements calculated over two-week "computation periods" ending on Monday. Every other Tuesday and Wednesday, the banking sector as a whole may be trying to increase net reserve positions or to unload excess reserves. The resulting market tensions impart wide fluctuations to the overnight fed funds rate: our empirical work below takes such seasonal effects into account.[5]

Official documents suggest that the interest-rate targeting perspective of our model may not be equally applicable to all available data. Chairman Volcker's anti-inflationary policies officially focused on monetary-aggregate rather than interest-rate targets. Quantity targets were de-emphasized starting in 1982, and gradually replaced by the semi-official interest-rate targets plotted by the step function of Figure 1. It is apparent in the Figure that targets were still not strictly implemented in the first part of the available sample. Indeed, only in 1987 did the Fed stop declaring targets for M1, a clear indication of the mounting difficulty of controlling aggregates rather than interest rates. Recent official Fed documents reflect a prevalence of price over quantity targets which was also typical of monetary policy in the 1970s [ e.g., Hetzel (1981) ].[6] As it is apparent from Figure 1, however, tight targeting was *de facto* abandoned in the aftermath of the 1987 stock market crash (no formal fed funds-target rate was indicated from October 19, 1987 through November 3, 1987) and during the highly volatile Gulf War period (from August 1990 through February 1991).

This institutional information confirms that the *modus operandi* of monetary policy is far from constant over time. We choose to work on data from November 4, 1987 through August 1, 1990, when the character of monetary policy appears relatively homogeneous: the period roughly coincides with the tenure of Alan Greenspan as Chairman of the Fed, but excludes

[5] Biweekly clearing of the market for reserve assets might in principle invalidate our assumption that innovations to the overnight rate/target spread are independent of those in target-change expectations: market-clearing overnight rates might conceivably themselves reflect anticipations of target changes if banks distribute reserve requirements towards times of expected lower cost within the maintenance period. However, the evidence in Campbell (1987) indicates such anticipation effects are not apparent in overnight-rate data. They would only operate at biweekly horizons anyway, and we abstract from them in our theoretical models and empirical work on 3-month term fed funds rates.

[6] In their section on policy implementation, recent issues of the Quarterly Review of the FRBNY (Spring 1990; Spring 1991, pp.66–71; Spring 1992, p.84) lament instability of the relation between fed funds rates and the amount of borrowing. In 1990, for example, the trading desk of the FRBNY was prompted to signal policy moves so clearly as to minimize possibility of misunderstanding, because it was considered paramount to fix the right rate rather than adjust the reserves.

obvious sources of instability such as the 1987 stock market crash period and the highly volatile period following the invasion of Kuwait. It will become apparent below, however, that even during this sub-period interest rates need not have been based on a stable expectational structure, and appear to incorporate the likelihood of future and ongoing "monetary regime" shifts.

Figure 2

For this period, Figure 2 displays the autocorrelation functions of overnight and three-month interest rates (both decay very slowly, as is also apparent from the long cycles of these series in Figure 1), as well as autocorrelations of their spread from contemporaneous targets.

Autocorrelations of the spread between overnight rates and targets display biweekly seasonality corresponding to the maintenance period, but otherwise decay very quickly, consistently with the mean-reverting dynamics postulated in equation (1). Conversely, the spread between three-month rates and targets is just as persistent as the level of the three-month interest rate. Such long-lived spreads are qualitatively consistent with the theoretical model outlined above, where deviations of longer-term interest rates from the overnight rates and targets are driven by target-change expectations $z_t$ as well as by mean-reverting dynamics around the current target. With $z_t$ a martingale between target changes and relatively infrequent target changes, the spread between short-term and overnight interest rates should indeed display long memory to the extent that intervals between target changes are long relative to the life of the credit instrument.[7]

### 3.1 Seasonality in the overnight rate process

We now estimate the law of motion of overnight interest rates around the target. To account for the biweekly seasonal effects which are apparent in Figures 1 and 2, we use ten level dummies (one for each working day of the maintenance period) and ten mean-reversion dummies. We estimate the model

$$r_t - \bar{r}_t = d_t + (1 - k_t)(r_{t-1} - \bar{r}_{t-1} - d_{t-1}) + \epsilon_t, \tag{5}$$

[7]These findings are not specific to the limited period we consider. Indeed, sizable and persistent spreads between 3-month fed funds rates and contemporaneous targets are quite apparent in Figure 1, indicating that "expected target change" term-structure factors are quantitatively important for all recent U.S. monetary policy.

where $t$ indexes business days;[8] $r_t$ is the overnight fed funds rate; $\bar{r}_t$ is the target; $d_t = d_{t+10}$ is a time-varying intercept with biweekly periodicity, which captures seasonality in the conditional mean of $r_t$; $k_t = k_{t+10}$ is a time-varying mean-reversion parameter, also with biweekly periodicity, meant to capture variations in targeting intensity. As not only the level, but also the volatility of the fed funds rate is affected by the maintenance-period cycle, we allow the standard deviation $\{\sigma_t\}$ of the serially uncorrelated error $\epsilon_t$ to vary over time, and impose a seasonal pattern with $\sigma_{t+10} = \sigma_t$.[9] We estimate the process in (5) by maximum likelihood, assuming $\epsilon_t$ to be normally distributed. The results are reported in Table 1.

Table 1

The model fits the data well, and the residuals have satisfactory statistical properties. Biweekly patterns are statistically significant for levels ($\{d_t\}$), serial correlations ($\{k_t\}$), and volatilities ($\{\sigma_t\}$). These parameters are precisely estimated, and offer insights into the nature of targeting and seasonal patterns during the period under scrutiny.[10] The intercept parameters $\{d_t\}$ are all close to zero, indicating that systematic seasonal effect on the level of the fed funds rate are economically (if not statistically) insignificant. Conversely, the intensity of mean-reversion changes quite dramatically over the biweekly period: we find *diversion* from the target on day 3 of the biweekly cycle, the Wednesday closing the maintenance period, while strong reversion towards the target is induced on the following Thursday and Friday. The variability of innovations in the fed funds rate also follows a marked seasonal pattern. The standard deviations $\{\sigma_t\}$ are higher on average during the first week of the period, especially between Tuesday and Thursday, with a peak on Wednesday.

## 3.2 The performance of the expectations hypothesis

We next seek evidence on the basic expectational relation (3) by estimating the equation

$$\sum_{s=0}^{\tau-1} \frac{r_{t+s}}{\tau} - r_t = \alpha \left[ R_t - r_t \right] - \phi_t + \eta_{t+\tau-1}, \tag{6}$$

[8] Annualized Friday rates are those applicable to positions spanning the weekend; missing data for market holidays other than weekends are filled with the previous trading day's rates.

[9] The stochastic seasonality exhibited by the fed funds rate could alternatively be captured by an autoregressive process with ten lags. However, the coefficients of our simple AR(1) process with seasonal variation are much easier to interpret.

[10] See Campbell (1987) for similar empirical evidence on weekly seasonals in fed funds rates in 1980-83, when reserve requirements were lagged over weekly maintenance periods.

where $R_t$ is the interest rate on a loan of maturity $\tau$, $\phi_t$ is a time-varying intercept which may capture liquidity and term effects, and $\eta_t$ is an expectational error. Under the premium-augmented expectations hypothesis,

$$R_t = \sum_{s=0}^{\tau-1} \frac{E_t(r_{t+s})}{\tau} + \phi_t, \qquad (3')$$

and the coefficient $\alpha$ of the regression should equal one if expectations are unbiased.

Table 2

In Table 2 we present results from estimation of (6) on $R_t$, the three-month fed funds rate. We allow for an intercept $\phi_t$ with biweekly seasonal periodicity, i.e., we impose $\phi_t = \phi_{t+10}$ for every $t$. In spite of the very pronounced "maintenance period" seasonal pattern, the hypothesis that $\phi_t = 0$ for all $t$ is not rejected: thus, the data are consistent with at least an unconditional version of the premium-free expectational relation (3).

Conditionally, however, the expectations hypothesis does not receive support from the data. While the spread between three-month and overnight rates has substantial predictive power for future behavior of the latter, the slope coefficient $\alpha$ is significantly lower than the unitary value implied by (3). Unexplained time-variation in unobservable term premia could of course be responsible for excess volatility of longer-term interest rates, which would bias $\alpha$ towards zero in the way suggested by Mankiw and Miron's (1986) work on quarterly data of three- and six-month maturity. Interestingly, however, the problem appears much less severe in our sample of recent daily observations than for the longer series of quarterly three- and six-month rate data considered by Mankiw and Miron. We choose to take the absence of term premia as a maintained identifying assumption, and we proceed to examine the origin of the bias in tests of (3′) in terms of expectational errors.

As in Section 2, we suppose that longer-term interest rates correctly embody the market's expectations of future overnight rates. However, we relax the standard requirement that the data-generating process is completely known to economic agents, and reinterpret the performance of tests of the expectations hypothesis from the vantage point of our simple theoretical framework and newly-available target data. As argued above, imperfect knowledge of policy rules is indeed likely to be important in real-life settings, and market expectations of future fed funds rates may well be biased, as documented by Simon (1990) with quarterly data from the Goldsmith-Nagan Survey for various recent periods. While Simon does not attribute expectational errors to specific features of interest-rate dynamics, our target data and estimated fed funds process help us identify the source of expectational biases.

Under the expectations hypothesis (3′), the regressor of equation (6) can be decomposed as follows:

$$\begin{aligned} R_t - r_t &= E_t\left(\frac{\sum_{s=0}^{\tau-1}\bar{r}_{t+s}}{\tau} - \bar{r}_t\right) + E_t\left[\frac{\sum_{s=0}^{\tau-1}(r_{t+s}-\bar{r}_{t+s})}{\tau} - (r_t - \bar{r}_t)\right] \\ &\equiv E_t\left(\Delta_{\bar{r}_t}\right) + E_t\left(\Delta_{r_t-\bar{r}_t}\right). \end{aligned} \tag{7}$$

Thus, $E_t\left(\Delta_{\bar{r}_t}\right)$ denotes the market's forecast of relevant future target-change variations,

$$E_t\left(\Delta_{\bar{r}_t}\right) = R_t - \bar{r}_t - E_t\left[\frac{\sum_{s=0}^{\tau-1}(\bar{r}_{t+s} - \bar{r}_{t+s})}{\tau}\right],$$

and $E_t\left(\Delta_{r_t-\bar{r}_t}\right)$ denotes the forecast of the relevant variations around future targets.

In the context of our model, the latter expectational component can be identified if the relatively stable process driving overnight fed funds rates around current targets is well understood by market participants, who know its form and the parameters $\{d_t, k_t, \sigma_t\}$. Projecting (5) forward yields $E_t(r_{t+s} - \bar{r}_{t+s}) = d_{t+s} + (r_t - \bar{r}_t - d_t)\prod_{i=1}^{s}(1 - k_{t+i})$, which can be used in the relevant calendar-day summations to obtain from (7):

$$E_t\left(\Delta_{\bar{r}_t}\right) = R_t - \bar{r}_t - L_{r_t-\bar{r}_t}(r_t - \bar{r}_t - d_t) - L_{d_t}, \tag{8}$$

where

$$L_{d_t} \equiv \frac{1}{\tau}\sum_{s=0}^{\hat{\tau}-1}(1 + 2F_{t+s})d_{t+s}$$

is a time-varying (but empirically rather small) intercept induced by seasonal effects, and

$$L_{r_t-\bar{r}_t} \equiv \frac{1}{\tau}\left[1 + \sum_{s=1}^{\hat{\tau}-1}(1 + 2F_{t+s})\prod_{i=1}^{s}(1 - k_{t+i})\right]$$

is the term-structure loading on seasonally adjusted deviations from target. In these expressions, $\hat{\tau}$ is the number of business days that fall within the maturity $\tau$ of the instrument, and $F_{t+s}$ is a dummy variable which equals one on Fridays, and zero otherwise: this accounts for the fact that overnight rate quoted on Friday regulates contracts which expire on the following Monday, and hence has an effective maturity of three days.

Sample counterpart of $E_t\left(\Delta_{r_t-\bar{r}_t}\right)$ can be computed from (8) inserting estimates of $\{k_t, d_t\}$ in the factor-loading expressions $L_{r_t-\bar{r}_t}$ and $L_{d_t}$. The resulting expectational series can then be compared to the realized target-change series. For such comparisons to be statistically meaningful, we need to account for the fact that the factor-loading expressions are (highly nonlinear) functions of the parameters given in Table 2. We therefore implement a simple Monte Carlo procedure based on the estimated asymptotic covariance matrix of the estimates (see the Appendix), and report empirical standard errors for each of the point-estimate comparisons below.

When we regress $\Delta_{\bar{r}_t}$ on the sample counterpart of $E_t(\Delta_{\bar{r}_t})$ and a constant, we obtain a slope coefficient $\alpha' = 0.503$, with negligible empirical standard error. Recalling that we had $\alpha = 0.576$ in Table 2, where the test was run on raw data, we find that the expectations-hypothesis test uncovers an even *more pronounced* bias when focused on the target component of fed funds dynamics. To interpret this result, consider that the theoretical value of $\alpha'$ (to which the estimated parameter should converge in probability) is given by

$$\text{plim } \alpha' = \frac{\text{cov}^* \left[\Delta_{\bar{r}_t}, E_t\left(\Delta_{\bar{r}_t}\right)\right]}{\text{var}^* \left[E_t\left(\Delta_{\bar{r}_t}\right)\right]},$$

where an asterisk (*) denotes moments calculated with respect to the *true* target-change process. If the rational-expectations restriction $E_t(\Delta_{\bar{r}_t}) = E_t^*(\Delta_{\bar{r}_t})$ is violated, the $\alpha'$ regression coefficient is lower than one in probability limit if

$$\text{cov}^* \left[\Delta_{\bar{r}_t}, E_t\left(\Delta_{\bar{r}_t}\right)\right] < \text{var}^* \left[E_t\left(\Delta_{\bar{r}_t}\right)\right], \tag{9}$$

as is the case in our data. As the dynamics of fed funds deviations from target are independent of the targeting process, theoretical value of the regression coefficient $\alpha$ is

$$\text{plim } \alpha = \frac{\text{cov}^* \left[\Delta_{\bar{r}_t}, E_t\left(\Delta_{\bar{r}_t}\right)\right] + \text{var}^* \left[E_t\left(\Delta_{r_t - \bar{r}_t}\right)\right]}{\text{var}^* \left[E_t\left(\Delta_{\bar{r}_t}\right)\right] + \text{var}^* \left[E_t\left(\Delta_{r_t - \bar{r}_t}\right)\right]} > \text{plim } \alpha',$$

the inequality following from $\text{var}^* \left[E_t\left(\Delta_{r_t - \bar{r}_t}\right)\right] > 0$ and (9). Hence, and quite intuitively, the empirically more pronounced bias of the expectations-hypothesis test for the target component of fed funds is consistent with our modeling framework if the remaining component of fed-funds dynamics, namely their fluctuations around the target, is well understood and correctly anticipated by market participants.

Of course, time-varying term premia could also induce the finding that $\alpha' < \alpha < 1$ under standard rational-expectations assumptions. It is instructive, however, to consider which moment conditions would need to be satisfied by such exogenous premia to rationalize the result. The Appendix derives probability limits of $\alpha$ and $\alpha'$ allowing for a general premium process and maintaining rational expectations. Those expressions suggest that time-varying premia might counterfactually imply that $\alpha$ is estimated to be *smaller* than $\alpha'$. In fact, stationary term premia need not covary strongly with the highly persistent expected-target-change component $E_t^*(\Delta_{\bar{r}_t})$ which our data and modeling approach make it possible to identify. Inasmuch as premia covary little with expectations of policy changes, it would be hard for them to strongly bias the estimated coefficient $\alpha'$ below one. Conversely, term premia (especially if liquidity-motivated) might well correlate positively with the overnight rate's spread from its currently targeted level, or with $E_t(\Delta_{r_t-\bar{r}_t}) = E_t^*(\Delta_{r_t-\bar{r}_t})$. Such correlation could easily imply that the estimated $\alpha$ coefficient differs from one by a more substantial quantity than $\alpha'$ does.

## 4 Expected target changes

The empirical procedures and results of the previous section did not need to be specific as to which time-series properties may be attributed to the target-changes process by market participants, or which ones might be appropriate to describe its actual behavior. Recall that in Section 2 the timing of target changes was governed by a constant-probability binomial distribution; the expected size $z_t$ of the next realized target change was modeled as a martingale between target changes (reflecting its expectational nature), and assumed to be reset to zero, on average, upon realization of a target change. When applied to the market's expectations of future fed-fund behavior, these assumptions made it possible to derive a conveniently simple factor-loading expression for $z_t$.

In light of Section 3's evidence, the behavior of the realized and anticipated target-changes series may be quite loosely related to each other. In this section, however, we briefly discuss to what extent the behavior of the targeting process and/or of its market-expectations counterpart may be captured by the model in Section 2, or by more complex and realistic versions of it.

Consider the timing assumptions first. It would not be difficult to specify and solve a model where the probability of a target change is time-varying (and at least partially reset upon target-change realizations). To see whether such extension is called for by our data, note that a constant daily probability $\nu$ of a target change would imply that the length of constant-target spells is binomially distributed with parameter $\nu$, and unrelated to other observable data.

Figure 3

Figure 3 plots the sample spells' empirical distribution function, along with their theoretical counterpart based on the empirical frequency of target-change events. The two panels of the Figure measure time in calendar days and in business days (as in our theoretical model), respectively. In both cases, the empirical and theoretical distributions are quite close in shape and position: formally, the Kolmogorov-Smirnov goodness-of-fit statistic [ see, e.g., De Groot (1986), p.556 ] fails to reject the null hypothesis at conventional significance levels ($p$-values are 29.01% and 24.05% for the distributions of Figure 3.a and 3.b, respectively). Consistently with the empirical evidence of Figure 3, the data reveal no pattern of serial correlation in the length of constant-target spells, indicating that the daily target-change probability $\nu$ is well approximated by a constant in this sample. We have also informally tested for correlation between the target changes' size and timing. Target-change sizes do not vary much around their quarter-point mode, and are essentially unrelated to the length of the previous no-change spell length. We conclude that the data provide little information about such potentially interesting features, and no evidence against the model's timing assumptions.

Conversely, the "resetting" assumption of Section 2 is patently at odds with the data. This assumption was motivated above by Mankiw and Miron's (1986) martingale-policy ideas, but turns out to be too stringent for the period and time scale we consider. In our sample, target changes were quite clearly *correlated* over time (see Figure 1). The thirteen positive target changes occurring in 1988 and early 1989 are followed by nine negative ones, and such sign runs are of course extremely unlikely in a small sample of target changes.

It is quite straightforward to incorporate such serial correlation in the market's expectation-formation mechanisms, along the lines of Section 2. For example, consider maintaining the assumption that the expectational $\{z_t\}$ process has martingale dynamics between target changes, but writing

$$\Delta\bar{r}_{N_t} = \rho\Delta\bar{r}_{N_t-1} + \text{error}_{N_t}, \tag{10}$$

where $N_t$ denotes the number of target changes up to time $t$, the constant $\rho$ denotes the correlation of target changes over the random time steps of the target change's process, and the error has mean zero from market participants' point of view.[11]

If the law of motion (10) characterizes the market's expectations, the univariate repre-

[11] As noted above, the fact that realized target changes most often have 25- or 50-point size simply induces a discrete probability distribution on the error term in (10), which is inconsequential to our modeling of expectations as a continuous process.

sentation of the next target change's expected size $z_t$ is:

$$z_t = \begin{cases} z_{t-1} + \text{error}_t, & \text{when } t \neq \hat{t}; \\ \rho \Delta \bar{r}_t + \text{error}_t, & \text{when } t = \hat{t}. \end{cases} \tag{2'}$$

Target-change expectations are indeed reset at $\hat{t}$ if $\rho = 0$, but persist across target-change realizations, as the data suggest, if $\rho > 0$.

As in the derivation of (4), we proceed under the assumption (3) that longer-term rates reflect the market's expectations of future overnight rates to obtain:

$$R_t = \bar{r}_t + L_{r_t - \bar{r}_t}(r_t - \bar{r}_t - d_t) + L_{z_t} z_t + L_{d_t}, \tag{11}$$

where $L_{r_t - \bar{r}_t}$ and $L_{d_t}$ are defined above after equation (8), and the loading on the next target-change expectation $z_t$ (derived in the Appendix) is

$$L_{z_t} \equiv \frac{1}{\tau} \sum_{s=1}^{\hat{\tau}-1} (1 + F_{t+s}) \left[ \sum_{i=1}^{s} \binom{s}{i} \nu^i (1-\nu)^{s-i} \frac{1-\rho^i}{1-\rho} \right], \tag{12}$$

a rather formidable, but easily programmed function of $\nu$ and of the newly introduced parameter $\rho$.

Given estimates of the parameters determining the factor loadings in (11), it is possible to extract target-change expectations from observable interest rate and target series:

$$z_t = \frac{R_t - \bar{r}_t - (r_t - \bar{r}_t - d_t)\, L_{r_t - \bar{r}_t} - L_{d_t}}{L_{z_t}}. \tag{13}$$

This expression can also be used, in conjunction with the estimated parameters' asymptotic covariance matrix, to compute associated standard errors from the Monte Carlo experiments discussed in the Appendix.

In keeping with the perspective of Section 3, the parameters of the process followed by the fed fund's deviations from target may be taken to characterize both actual realizations and the market's expectations. In what follows we explore the possibility of similarly inferring estimates of $\nu$ and $\rho$ from actual data. The daily probability $\nu$ of a target change may be estimated by the empirical frequency of target changes. The target changes 24 times in our sample of 716 business days. Since the term-structure model above presumes a constant probability of target changes on every calendar day in the relevant forecasting horizon, we use the estimate $\nu = 24/716 = 0.034$ and appropriate asymptotic standard errors (see the

Appendix).[12] A benchmark value of $\rho$ and associated standard errors are also straightforwardly estimated from the sample of realized target changes (see Appendix for details).

Using the estimates of Table 2 to compute the deviations factor loading $L_{r_t-\bar{r}_t}$ and the time-varying intercept $L_{d_t}$, the term-structure relationship (13) yields point estimates of the target-change expectation series $\{z_t\}$.

Figure 4

Figure 4 plots the point estimate of the $z_t$ series and empirical two-standard-error bands. While the process of target changes is responsible for the EH failure in our modeling framework, a calibration exercise like the one we perform can offer insights into the character of the market's misunderstanding of official targeting. The simple model featuring autocorrelated target changes captures the idea that more than one target change is expected after the next one is implemented and, while unmodelled time-variation in $\rho$ and/or $\nu$ could potentially account for part of the extracted $z_t$ series' variability, the model does appear to capture some aspects of the market's expectation-formation process.[13]

The sign of future policy actions is most often correctly predicted in Figure 4, but their size is substantially overestimated at times, consistently with the evidence from the expectations-hypothesis test. Most strikingly, the three-month fed funds rate surged much higher than overnight market rates and targets at the end of 1987. In our framework, this indicates that the market was expecting sharply higher interest rates in the very near future, yet the events of late 1987 lie outside of the error bands implied by our estimated model's parameters. Was the market "irrational" in entertaining such expectations, i.e., were arbitrage opportunities open to "smart" investors during that period? The question is of course very difficult to answer with a single string of data, as it is well understood in the related literature on learning and "peso problems" [ see e.g. Lewis (1991) ]. No surge in overnight rates was

[12] In reality, of course, target changes could only be observed on days when the money market was open. Implicitly, we are allowing the "shadow" target process triggering target changes to be continuously updated during weekends and holidays, at the same speed as on business days. The evidence of Figure 3 indicated that this does not do violence to our data: more realistic assumptions would have only minor effects on our results.

[13] In the spirit of the calibration exercise, it may be interesting to compare the $z_t$ series extracted form *data* with those the model assumes as to the market's *expectations* in (2'). Regressing $z_t - z_{t-1}$ on a constant and $z_{t-1}$ only *between* target changes, we find that the intercept and slope coefficients (which should both be zero under the null) are 0.000 and −0.019 with small empirical standard errors. A regression of $z_i$ on a constant and $\Delta\bar{r}_i$ across realignment days yields an intercept of −0.086 and a slope coefficient of 0.637 (with empirical standard errors of 0.020 and 0.146, respectively), when the $\rho$ estimate used in the expectational factor-loading of expression (12) is 0.737.

realized *ex post,* as the Fed successfully injected liquidity to control the market turbulence induced by the stock market crash (which, despite our *a priori* sample selection, does affect our data). Yet, a replay of the events which led to the Great Depression in 1929 was certainly possible in 1987: on the basis of what is essentially a single observation, no objective statistical procedure can ascertain whether the probability attached to the event by the market was in any sense irrational. The expectational overshooting of mid-1989, though not as sharp, can be similarly interpreted in terms of the market's misperceptions of the extent to which the Fed would be prepared to ease monetary policy after the interest-rate peak of early 1989.

## 5 Concluding comments

This paper proposes a formal framework of analysis for the effects of interest-rate targeting on the term structure of interest rates. At a general level, our perspective and data offer insights into the nature of the bias found by tests of the expectations hypothesis: given that the variation in the fed funds rate is generated mainly by changes in targets rather than by fluctuations about the target, and since the latter fluctuations are easily modeled and should be well understood by the market, we gather evidence indicating that the bias pertains to the policy-induced dynamics of the fed funds rate.

At a more practical level, we propose stylized models of expectations formation, and use their parametric structure to infer market expectations from interest-rate data. These models offer interesting theoretical insights as to the number and time-series properties of the factors driving the term structure of nominal interest rates in realistic settings. More complex parameterizations of the type of model we consider are possible, and might better fit historical experience. Inasmuch as unstructured tests of the expectations hypothesis indicate that the process of target changes is *not* well anticipated by the market, however, specifying sophisticated mechanisms of expectation formation and data generation may never eliminate expectation biases, but only yield insights on their source.

Our theoretical and empirical work does suggest several directions for further research. First, we may want to ask whether the expected target changes we measure are consistent with the Fed's *desiderata.* In its effort to anticipate future policy, the public accumulates information, and this translates into highly persistent spreads between overnight targets and longer-term rates. Such slack between the instruments and objectives of monetary policy may or may not be desirable from the authorities' point of view. The variability of the innovations in $z_t$ is an indicator of how frantic is the information-acquisition process, and of how successful is the Fed in keeping its intentions secret and preserving a discretionary role for policy. At the same time, however, a higher variability of the innovations in $z_t$ means looser control on longer-term money market rates. A trade-off could arise between secrecy and interest-rate control, of which the authorities should be (and probably are) aware. In the same spirit, it would be important to evaluate different monetary regimes in terms of the features of the fed funds rate process and, through the lenses of our model, compare the market's understanding of monetary policy across different periods.

Finally, we have shown that realistic specifications of interest-rate targeting processes have distinctive implications for the joint behavior and serial correlation properties of money market rates of different maturities. For example, deviations of shorter-term rates from the target mainly reflect the short-lived variability of the fed funds rate about the target, and should exhibit short memory; deviations of longer-term rates from the target are mainly driven by expectations of proximate target changes, and should be long-lived. These and other implications deserve to be formally tested in further work on interest rates of different maturities, and such testing may provide additional measures of our framework's descriptive validity.

## Appendix

## A The simple term structure model

Each of the expectations on the right-hand side of (3) can be conditioned on whether or not a target change occurs in the relevant forecast horizon:

$$\begin{aligned} E_t(r_{t+s}) &= \Pr(\hat{t} > t+s)E_t(r_{t+s}|\hat{t} > t+s) + \Pr(\hat{t} \le t+s)E_t(r_{t+s}|\hat{t} \le t+s) \\ &= E_t(r_{t+s} - \bar{r}_{t+s}) + \Pr(\hat{t} > t+s)E_t(\bar{r}_{t+s}|\hat{t} > t+s) \\ &\quad + \Pr(\hat{t} \le t+s)E_t(\bar{r}_{t+s}|\hat{t} \le t+s) \\ &= (r_t - \bar{r}_t)(1-k)^s + \Pr(\hat{t} > t+s)E_t(\bar{r}_{t+s}|\hat{t} > t+s) \\ &\quad + \Pr(\hat{t} \le t+s)E_t(\bar{r}_{t+s}|\hat{t} \le t+s), \end{aligned} \tag{A.1}$$

where we make use of the assumption that deviations of the fed funds rate at $t+s$ are independent of target change dynamics to obtain the second equality, while we calculate $E_t(r_{t+s} - \bar{r}_{t+s})$ from (1) to obtain the last equality.

Our assumptions yield simple expressions for the expectations of future targets appearing on the right-hand side of (A.1). If no target changes have occurred as of time $t+s$, we obviously have

$$E_t(\bar{r}_{t+s}|\hat{t} > t+s) = \bar{r}_t. \tag{A.2}$$

As to the terms where $s > \hat{t} - t$ is the conditioning event, we have

$$\begin{aligned} E_t(\bar{r}_{\hat{t}+s}) &= \bar{r}_t + E_t(\bar{r}_{\hat{t}} - \bar{r}_t) \\ &= \bar{r}_t + E_t(z_{\hat{t}-1}) \\ &= \bar{r}_t + z_t, \qquad \text{for all } s \ge 0; \end{aligned} \tag{A.3}$$

the first equality follows from the assumption that the target is expected to remain constant after $\hat{t}$ [ formally, $E_t(z_{\hat{t}}) = 0$ and $z_t$ follows a martingale after $\hat{t}$ ], and the second equality is implied by $z_t$'s martingale behavior between $t$ and $\hat{t}$.

Using (A.2) and (A.3) in (A.1), and noting that $\Pr(\hat{t} > t+s) = (1-\nu)^s$, we obtain:

$$E_t(r_{t+s}) = (r_t - \bar{r}_t)(1-k)^s + (1-\nu)^s\bar{r}_t + [1-(1-\nu)^s](\bar{r}_t + z_t). \tag{A.4}$$

The nominal yield on instruments of any maturity is now straightforward to calculate using (A.4) in the summations on the right-hand side of (3):

$$R_t = \bar{r}_t + \left[1 - \frac{1-(1-\nu)^\tau}{\nu\tau}\right] z_t + \left[\frac{1-(1-k)^\tau}{k\tau}\right](r_t - \bar{r}_t). \tag{4}$$

## B Empirical standard errors

All statistics which depend on estimates of the parameters $\{k_t, d_t\}$, $\nu$, and $\rho$ are subject to randomness due to variability of the estimates around the true values. We account for this by drawing 100 samples from the estimated joint asymptotic distribution of the estimates, and compute the empirical standard errors of the quantities of interest. By standard results, the asymptotic distribution of the $\{k_t, d_t, \sigma_t\}$ estimates in Table 2 is multivariate normal and, by our independence assumption, is independent of $\nu$ and $\rho$ estimates. The sample frequency $\hat{\nu}$ is bounded between zero and one and has asymptotic Beta distribution. To simplify programming, we draw Monte Carlo samples from the asymptotic distribution of the monotone log-odd ratio transformation $\ln(\hat{\nu}/(1-\hat{\nu}))$. Maximum likelihood estimation of the log-odds ratio yields the sampling frequency as the $\hat{\nu}$ point estimate, and a normal asymptotic distribution. (The log-odds ratio point estimate is $-3.362$, with standard error 0.212.)

We use a similar approach to account for sampling variability in estimation of the target-change correlation parameter $\rho$ introduced in Section 4. We estimate the model

$$\Delta\bar{r}_{N_t} = \frac{e^{\xi}}{1+e^{\xi}}\Delta\bar{r}_{N_t-1} + \text{error}_{N_t},$$

by nonlinear least squares, thus ensuring that the estimate of $\rho = e^{\xi}/(1+e^{\xi})$ lies between zero and one. We obtain an estimate $\xi = 1.030$ (or $\rho = 0.737$). The standard error of the (asymptotically normal) $\xi$ estimate is 0.742.

## C Performance of the expectations hypothesis

We consider an alternative framework where:

$$E_t\left(\Delta_{\bar{r}_t}\right) = E_t^*\left(\Delta_{\bar{r}_t}\right).$$

We further allow for a time-varying premium $\phi_t$ which we assume to covariate positively with $E_t\left(\Delta_{\bar{r}_t}\right)$ and $E_t\left(\Delta_{r_t-\bar{r}_t}\right)$. Under these additional assumptions, when we regress the realized $\Delta_{\bar{r}_t}$ on our model's estimate of $E_t\left(\Delta_{\bar{r}_t}\right)$, we obtain a slope coefficient, $\alpha'$, whose theoretical value is

$$\text{plim } \alpha' = \frac{\text{var}^*\left[E_t\left(\Delta_{\bar{r}_t}\right)\right] + \text{cov}^*\left[\phi_t, E_t\left(\Delta_{\bar{r}_t}\right)\right]}{\text{var}^*\left[E_t\left(\Delta_{\bar{r}_t}\right)\right] + \text{var}^*\left(\phi_t\right) + 2\text{cov}^*\left[\phi_t, E_t\left(\Delta_{\bar{r}_t}\right)\right]}.$$

Similarly, the theoretical coefficient of the expectations-hypothesis regression, $\alpha$, becomes

$$\text{plim } \alpha = \frac{\text{var}^*\left[E_t\left(\Delta_{\bar{r}_t}\right)\right] + \text{var}^*\left[E_t\left(\Delta_{r_t-\bar{r}_t}\right)\right] + \text{cov}^*\left[\phi_t, E_t\left(\Delta_{\bar{r}_t}\right)\right] + \text{cov}^*\left[\phi_t, E_t\left(\Delta_{r_t-\bar{r}_t}\right)\right]}{\text{var}^*\left[E_t\left(\Delta_{\bar{r}_t}\right)\right] + \text{var}^*\left[E_t\left(\Delta_{r_t-\bar{r}_t}\right)\right] + \text{var}^*\left(\phi_t\right) + 2\text{cov}^*\left[\phi_t, E_t\left(\Delta_{\bar{r}_t}\right)\right] + 2\text{cov}^*\left[\phi_t, E_t\left(\Delta_{r_t-\bar{r}_t}\right)\right]}.$$

Straightforward algebra shows that the theoretical value of $\alpha'$ is *greater* than the theoretical value of $\alpha$ if

$$\begin{aligned}\text{cov}^*\left[\phi_t, E_t\left(\Delta_{r_t-\bar{r}_t}\right)\right] \; & \left(\text{var}^*\left[E_t\left(\Delta_{\bar{r}_t}\right)\right] - \text{var}^*\left(\phi_t\right)\right) \\ & > \text{var}^*\left[E_t\left(\Delta_{r_t-\bar{r}_t}\right)\right]\left(\text{var}^*\left(\phi_t\right) + \text{cov}^*\left[\phi_t, E_t\left(\Delta_{\bar{r}_t}\right)\right]\right).\end{aligned}$$

This inequality is satisfied, for example, when the variability of $E_t\left(\Delta_{\bar{r}_t}\right)$ is substantially greater than that of $\phi_t$ and $E_t\left(\Delta_{r_t-\bar{r}_t}\right)$, while $\phi_t$ covariates strongly with $E_t\left(\Delta_{r_t-\bar{r}_t}\right)$ and weakly with $E_t\left(\Delta_{\bar{r}_t}\right)$.

## D Serially correlated target changes

For any $s > 0$ we have the definitional relation

$$E_t(r_{t+s}) \equiv E_t\left[(r_{t+s} - \bar{r}_{t+s}) + \bar{r}_t + \sum_{j=1}^{N_{t+s}-N_t} \Delta\bar{r}_{N_t+j}\right], \tag{D.1}$$

where $N_{t+s}$ is the number of target changes between time zero and time $t+s$ and $\Delta\bar{r}_{N_t+j}$ is the $j$-th target change *after* time $t$. Since changes occur with fixed daily probability $\nu$, and their timing is independent of $z_t$, we can condition on their total number $N_{t+s}$ and implement the known (binomial) form of the distribution of $N_{t+s}$:

$$E_t\left(\sum_{j=1}^{N_{t+s}-N_t} \Delta\bar{r}_{N_t+j}\right) = \sum_{i=1}^{s}\binom{s}{i}\nu^i(1-\nu)^{s-i}\, E_t\left(\sum_{j=1}^{i}\Delta\bar{r}_{N_t+j}\,\middle|\, N_{t+s} = N_t + i\right). \tag{D.2}$$

Using the law of iterated expectations on equation (2′), the expected size of the $j$th target-change realization is

$$E_t(\Delta\bar{r}_{N_t+j} | i \geq j \geq 1) = \rho^{j-1} z_t, \tag{D.3}$$

hence

$$\begin{aligned} E_t\left(\sum_{j=1}^{N_{t+s}-N_t} \Delta\bar{r}_{N_t+j}\,\middle|\, N_{t+s} = N_t + i\right) &= \sum_{i=1}^{s}\binom{s}{i}\nu^i(1-\nu)^{s-i}\sum_{j=1}^{i}\rho^{j-1}z_t \\ &= z_t\left[\sum_{i=1}^{s}\binom{s}{i}\nu^i(1-\nu)^{s-i}\frac{1-\rho^i}{1-\rho}\right]. \end{aligned} \tag{D.4}$$

Substituting (D.4) in (D.1), we find that

$$E_t(\bar{r}_{t+s}) = \bar{r}_t + z_t\left[\sum_{i=1}^{s}\binom{s}{i}\nu^i(1-\nu)^{s-i}\frac{1-\rho^i}{1-\rho}\right]. \tag{D.5}$$

Averaging this expression over the horizon relevant to an instrument of maturity $\tau$ yields equation (12) in the main text.

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**Table 1. The fed funds-rate process: maximum likelihood estimates.**

We estimate the model

$$r_t - \bar{r}_t - d_t = (1 - k_t)(r_{t-1} - \bar{r}_{t-1} - d_{t-1}) + \epsilon_t,$$

where $r_t$ is the overnight fed funds rate; $\bar{r}_t$ is the target; $d_t$ is a time-varying intercept parameter with biweekly periodicity; $k_t$ is a time-varying parameter which regulates mean reversion towards the target, also with biweekly periodicity. The white-noise error term $\epsilon_t$ is allowed to display a biweekly seasonal heteroskedasticity pattern. The model is estimated by maximum likelihood under the assumption of normality, using daily data for the period 1987:11:05–1990:08:01. We report the Ljung-Box portmanteau statistic, $Q$, distributed chi-square (degrees of freedom in parenthesis). For each of the three sets of parameters $\{d_t\}$, $\{k_t\}$, and $\{\sigma_t\}$, we report likelihood-ratio test statistics $\lambda_{LR}$ for the hypothesis of no biweekly seasonality. These statistics have chi-square distributions (degrees of freedom in parenthesis).

**Statistics**

| | |
|---|---|
| $\bar{R}^2$ | 0.308 |
| D-W | 1.979 |
| $Q(80)$ | 160.3 |

**Parameter estimates**

| | Coefficient (s.e.) | | Coefficient (s.e.) | | Coefficient (s.e.) |
|---|---|---|---|---|---|
| $k_1$ | 0.274 | $d_1$ | 0.053 | $\sigma_1$ | 0.130 |
| | (0.134) | | ( 0.011 ) | | ( 0.019) |
| $k_2$ | 0.476 | $d_2$ | -0.009 | $\sigma_2$ | 0.138 |
| | (0.107) | | ( 0.012 ) | | ( 0.019) |
| $k_3$ | -0.213 | $d_3$ | 0.081 | $\sigma_3$ | 0.398 |
| | (0.295) | | ( 0.033 ) | | ( 0.052) |
| $k_4$ | 0.759 | $d_4$ | 0.104 | $\sigma_4$ | 0.190 |
| | (0.051) | | ( 0.016 ) | | ( 0.025) |
| $k_5$ | 0.763 | $d_5$ | 0.043 | $\sigma_5$ | 0.115 |
| | (0.062) | | ( 0.010 ) | | ( 0.014) |
| $k_6$ | 0.120 | $d_6$ | 0.084 | $\sigma_6$ | 0.094 |
| | (0.079) | | ( 0.007 ) | | ( 0.016) |
| $k_7$ | 0.297 | $d_7$ | 0.070 | $\sigma_7$ | 0.115 |
| | (0.096) | | ( 0.009 ) | | ( 0.017) |
| $k_8$ | 0.349 | $d_8$ | 0.019 | $\sigma_8$ | 0.073 |
| | (0.051) | | ( 0.005 ) | | ( 0.013) |
| $k_9$ | 0.577 | $d_9$ | 0.026 | $\sigma_9$ | 0.094 |
| | (0.101) | | ( 0.007 ) | | ( 0.014) |
| $k_{10}$ | 0.103 | $d_{10}$ | 0.003 | $\sigma_{10}$ | 0.062 |
| | (0.070) | | ( 0.005 ) | | ( 0.014) |
| $\lambda_{LR}(9)$ | 131.9 | $\lambda_{LR}^2(9)$ | 62.8 | $\lambda_{LR}(9)$ | 459.9 |

## Table 2. A test of the expectations hypothesis

We estimate the model

$$\sum_{s=t}^{t+90} \frac{r_s}{91} - r_t = \alpha\,[R_t - r_t] - \phi_t + \eta_{t+90},$$

where $R_t$ is the interest rate on a loan of maturity 91 days, $\phi_t$ is a time-varying intercept with biweekly seasonal periodicity which captures liquidity and term effects, and $\eta_{t+90}$ is an expectational error. We use daily overlapping observations, and the standard errors (in parentheses) are adjusted for heteroskedasticity and serial correlation of moving-average form in the residuals $\eta_{t+90}$ [ see Hansen (1982) ]. We report the Wald test statistic $\lambda_W$ for the null hypothesis that $\phi_t = 0 \; \forall \; t$. The statistic is based on an adjusted estimated covariance matrix [ Newey and West (1987) ] and is distributed *chi*-square (degrees of freedom in parenthesis). The sample period is 1987:11:04–1990:05:04.

**Statistics**

| | |
|---|---|
| $\bar{R}^2$ | 0.371 |

**Parameter estimates**

| | Coefficient (s.e.) |
|---|---|
| $\alpha$ | 0.576 (0.182) |
| $\phi_1$ | -0.003 (0.068) |
| $\phi_2$ | 0.035 (0.068) |
| $\phi_3$ | -0.004 (0.083) |
| $\phi_4$ | -0.019 (0.067) |
| $\phi_5$ | 0.007 (0.063) |
| $\phi_6$ | -0.009 (0.070) |
| $\phi_7$ | -0.002 (0.070) |
| $\phi_8$ | 0.018 (0.068) |
| $\phi_9$ | 0.018 (0.068) |
| $\phi_{10}$ | 0.024 (0.057) |
| $\lambda_W(10)$ | 16.38 |

Figure 1. Overnight and Three-month Fed Funds Rates and Target

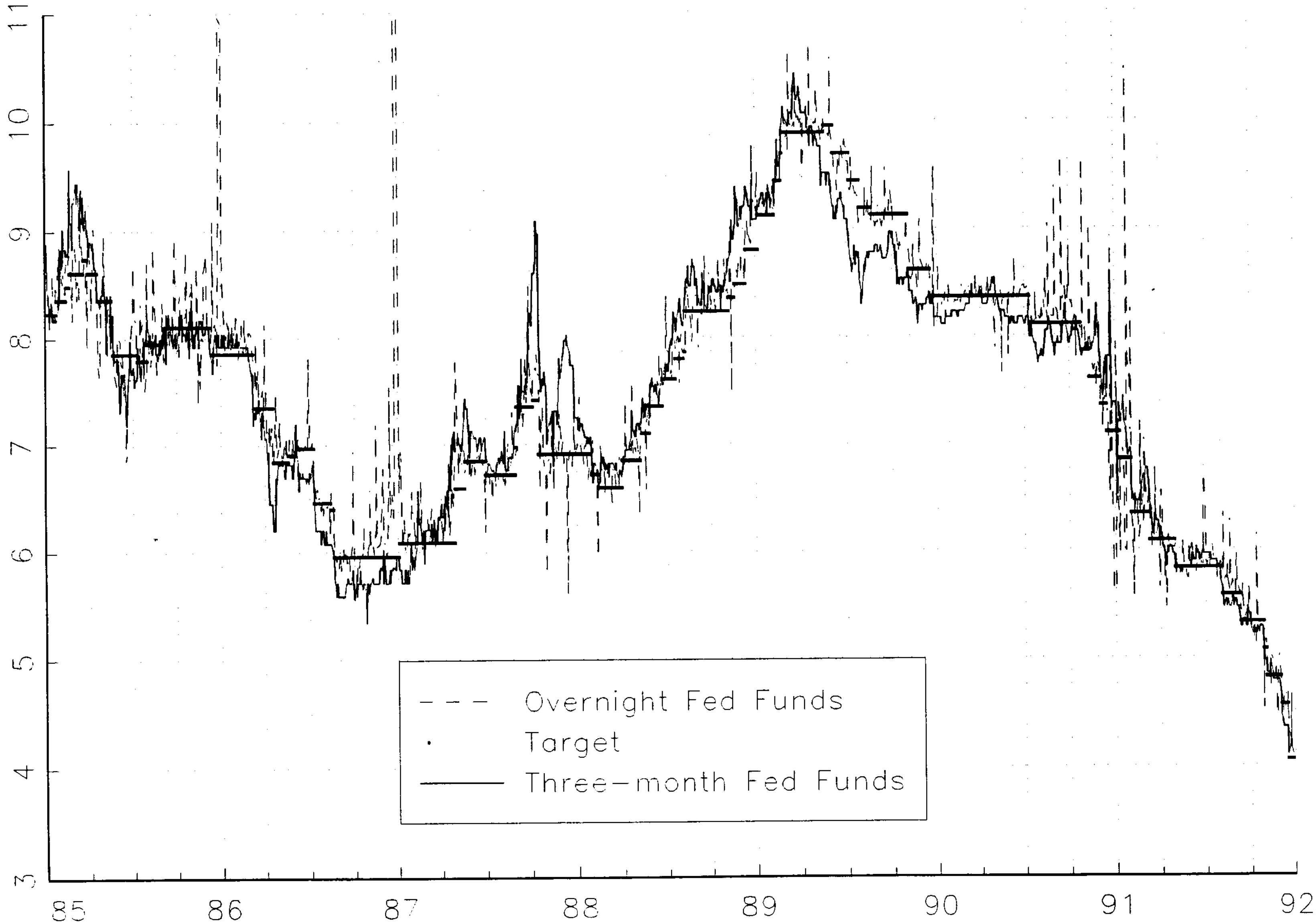


Figure 2. Autocorrelation Functions

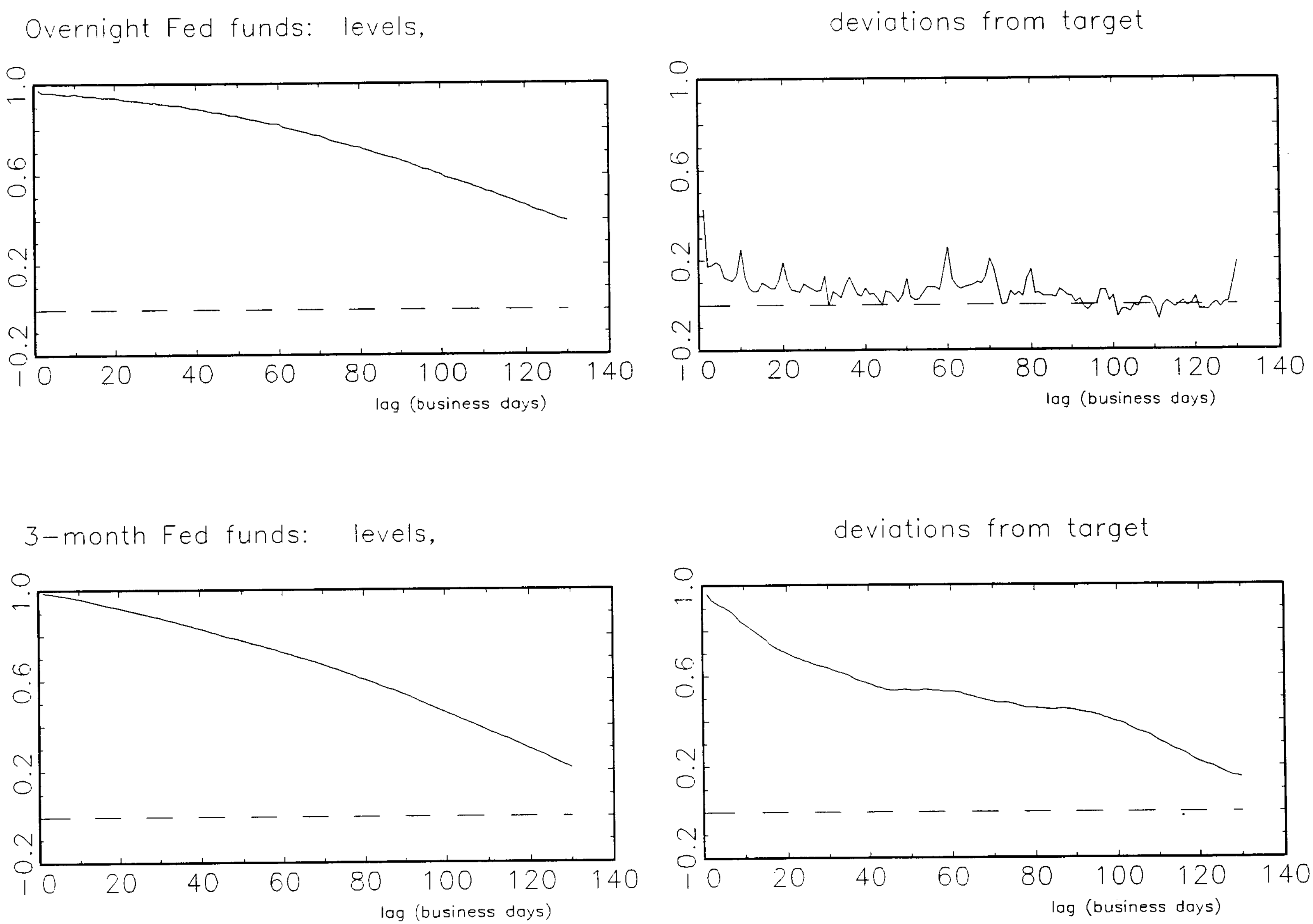
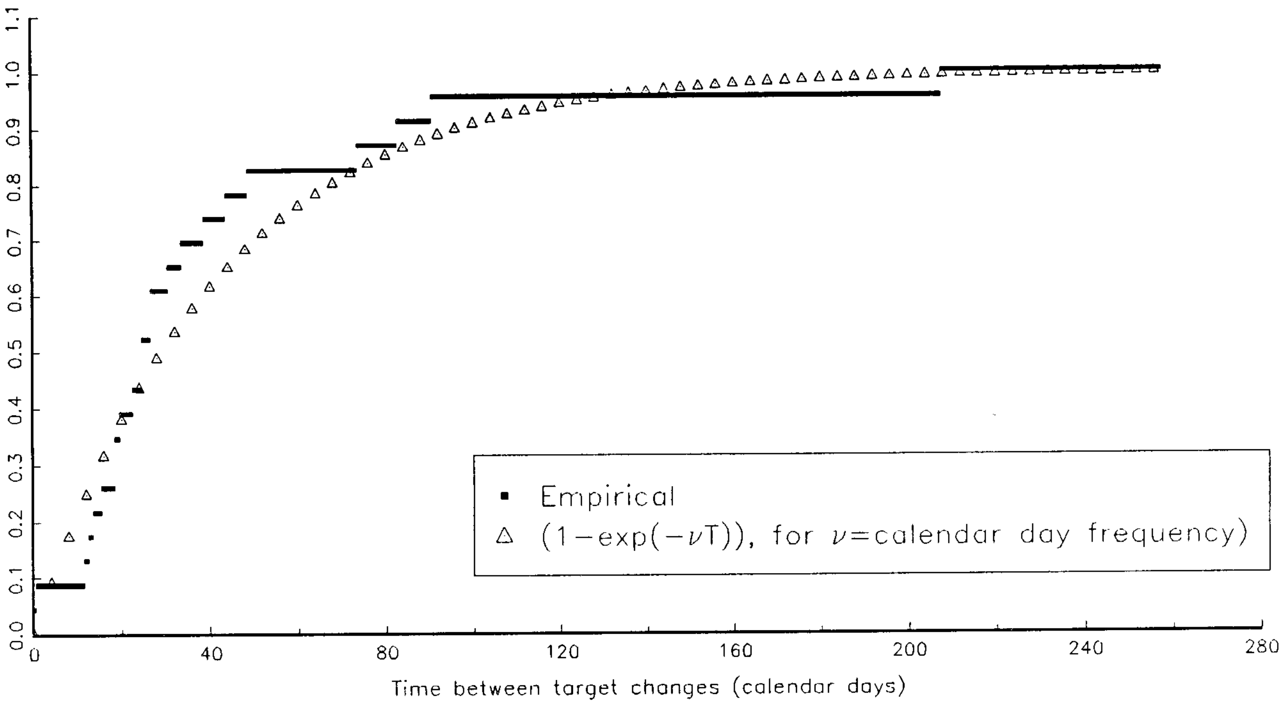


Figure 3: Empirical vs theoretical CDF of no-change spells

3.a: Calendar time

Kolmogorov-Smirnov statistic=0.69, p-value=27.85%




3.b: Business time

Kolmogorov-Smirnov statistic=0.70, p-value=28.92%

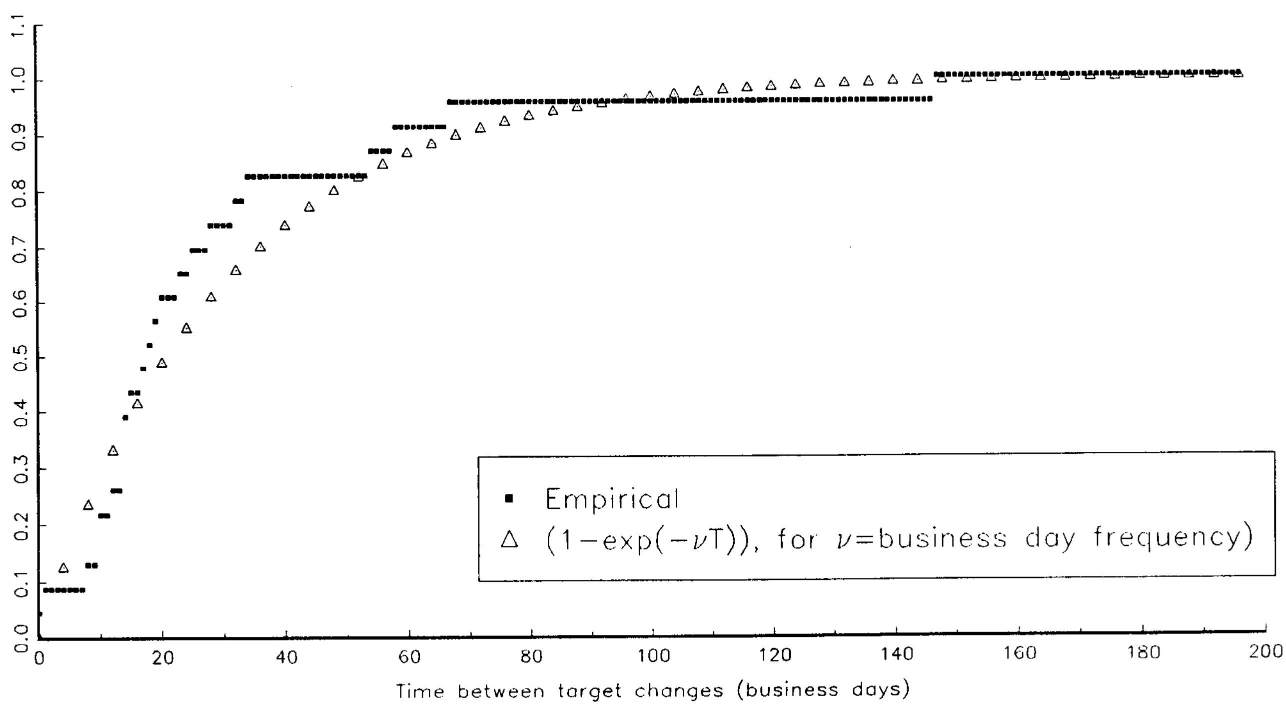


Figure 4. Target Change Expectations