Entrepreneurial Innovations in Network Industries*

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Abstract

We contribute to the literature network effects by allowing entrepreneurs to sell their innovations to incumbents in addition to entering the industry. We identify three new effects. Stronger network effects make selling innovations attractive, as incumbents bid up the sales price in fear of letting a rival obtain the innovation. This improves innovation incentives. Increased compatibility, however, reduces innovation incentives by reducing the relative advantage the owner of the innovation gets, in turn resulting in a lower sales price. Finally, bidding competition for innovations is crucial. Innovation waves can occur in network industries as bidding competition is fierce in young industries with several players competing for the top spot, but weak in mature industries with a clear leader.

Keywords: Entrepreneurship; entry; compatibility; innovation; network effects; standardization.

JEL classification: D40, L10

1 Introduction

An entrepreneur with an innovation has two main ways in which to commercialize it: entry into the market or a sale/licensing to an incumbent firm. So far, the literature has analyzed how network effects affect innovation incentives allowing only for entry. But as an exit route for venture capital backed startups, sale seems to be a more important commercialization route than entry (IPO), as measured by total deal value. This is true even during the IT boom in 1999-2001, when IPOs were frequent (see figure 1). In this paper, we ask two questions: "How does network effects and compatibility affect the entrepreneurs decision to sell or enter the industry?", and "What is the effect of network effects..."
and compatibility on innovation incentives allowing for both sale of the innovations and entry into the market?"

Allowing for the sale of innovations leads to a model that can explain why most of the recent large tech acquisitions are acquisitions of products with strong network effects (see table 1). In network industries, a small advantage over a rival is amplified by network effects. Entrepreneurial innovations that help incumbents compete are thus very valuable. This leads to strong bidding competition between incumbents, resulting in a high sales price and strong innovation incentives. Thus, we should expect a rapid pace of innovation and frequent sales of innovations at high prices in industries with strong network effects.

This effect, however, is moderated by the degree of compatibility between each firm’s network. With more compatible networks, an advantage over rivals is less important. Valuations for innovations by entrepreneurs are thus lowered, leading to lower prices paid for innovations, more entry instead of sale, and reduced innovation incentives. This suggests caution should be used when imposing standards, and it calls for empirical studies to measure the effect of standardization on prices paid for entrepreneurial innovations.

A further aspect that arises with allowing sale of innovations in addition to entry is bidding competition between incumbents for innovations. In network industries, the market is often scattered when the industry is young, as several players compete for the lead position in the industry. During this initial stage, bidding competition between incumbents for innovations is likely to be strong, as a small lead can tip the industry in favor of the owner of the innovation. Once time passes, however,
Table 1: Recent large technology acquisitions. Many are characterized by strong network effects.

<table>
<thead>
<tr>
<th>Company</th>
<th>Acquired by</th>
<th>Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>DoubleClick</td>
<td>Google</td>
<td>$3.1 billion</td>
</tr>
<tr>
<td>Skype</td>
<td>Ebay</td>
<td>$2.6 billion</td>
</tr>
<tr>
<td>YouTube</td>
<td>Google</td>
<td>$1.65 billion</td>
</tr>
<tr>
<td>AOL (5% stake)</td>
<td>Google</td>
<td>$1 billion</td>
</tr>
<tr>
<td>MySpace</td>
<td>News Corp</td>
<td>$580 million</td>
</tr>
<tr>
<td>Facebook (1.6% Stake)</td>
<td>Microsoft</td>
<td>$240 million</td>
</tr>
<tr>
<td>dMarc Broadcasting</td>
<td>Google</td>
<td>$102 million</td>
</tr>
<tr>
<td>Feedburner</td>
<td>Google</td>
<td>$100 million</td>
</tr>
<tr>
<td>Grouper</td>
<td>Sony</td>
<td>$65 million</td>
</tr>
<tr>
<td>Flickr</td>
<td>Yahoo</td>
<td>$35 million</td>
</tr>
<tr>
<td>del.icio.us</td>
<td>Yahoo</td>
<td>$35 million</td>
</tr>
</tbody>
</table>

and the industry stabilizes with one large leader, rivals might give up trying to overtake the leader. This can result in reduced bidding competition for innovations and reduced innovation incentives. This argument suggest that "innovation waves" could occur in network industries: initially innovation incentives are strong, but over time as the industry matures, innovation incentives are reduced due to the absence of bidding competition for entrepreneurial innovations.

Related Literature. Our paper contributes to the literature on entry in network industries and to the literature on the commercialization pattern of entrepreneurial innovations. There is a sizeable literature on entry in network industries.\(^1\) To the best of our knowledge, we are the first to allow for the sale of innovations to incumbents in addition to entry.\(^2\) The existing literature on the pattern of commercialization for entry and sale (licensing), shows that commercialization by sale (licensing) is more likely when entry costs are high, the entrepreneurial firm lacks complementary assets, brokers facilitating trade are available, and the expropriation problem associated with asset transfers is low.\(^3\) Abstracting from asymmetric information problems, we add to this literature by showing that when the innovation is commercialized in an oligopolistic market, it is more likely that the innovation is commercialized through a sale to an incumbent the stronger the network effects and the weaker compatibility between products is.

## 2 The Model

Consider an entrepreneur that exerts effort to discover an innovation.\(^4\) If successful, the entrepreneur chooses between entering the market or selling the innovation to an incumbent. The game has the following structure: In stage 1, the entrepreneur decides how much effort to invest in increasing the

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\(^2\)We also add to the literature on R&D and standardization in network industries (e.g. Katz and Shapiro (1985a), Farrell and Saloner (1985), Farrell and Saloner (1996), Church and Gandal (1993), Choi (1996a,b), Niswan and Minehart (2007) and Cabral and Salant (2009)) by allowing for the sale of innovations and by considering outside entrepreneur’s R&D instead of incumbents R&D.

\(^3\)See, for instance, Anton and Yao (1994), Gans and Stern (2000, 2003), and Gans et al. (2002).

\(^4\)The theoretical model is based on a combination of Katz and Shapiro (1985a) and Norback and Persson (2007).
1. **Innovation:**
Entrepreneur $e$ chooses effort to invent $\rho e$.
(where $\rho e$ increases the probability of discovering an invention of quality $k$.)

2. **Entry/Sale:**
Acquisition/entry and exit game

3. **Product market interaction:**
Oligopoly

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**Figure 2:** The timing of the game.

Probability of discovering a new innovation. In stage 2, the entrepreneur decides on either entering the market or selling the innovation to an incumbent in an auction. In stage 3, there is product market competition between the firms in the industry. See figure 2 for an illustration. The game is solved in a standard fashion though backwards induction.

2.1 **Stage 3: Product Market Competition**

There are $n$ symmetric firms competing in an oligopoly industry with network effects. The strength of the network effects is given by $z \in [0, \hat{z}]$, where $z = 0$ corresponds to no network effects and $z = \hat{z}$ is the maximum strength of network effects so that at least two firms can compete ($n \geq 2$). The degree of compatibility between the firms products is measured by $c \in [0, z]$, where $c = 0$ implies that each firm’s product only benefits from its own network (incompatibility) and $c = z$ implies that each firm’s product benefits equally from the networks of all products sold (full compatibility).

One of the firms in the industry owns an innovation of quality $k \in R^+$. This innovation is developed by the entrepreneur, and the owner is thus either the entrepreneur that has entered or one of the incumbents that have purchased the innovation from the entrepreneur.

Firm $j$ chooses an action $x_j$ to maximize its direct product market profits $\pi_j(x_j, x_{-j}, l, k, z, c)$, which depend on its own action, its rivals’ actions, the owner of the innovation, $l$, the quality of the
innovation, $k$, the strength of network effects, $z$, and the degree of compatibility between products, $c$. We make the assumption that given the expectations of network sizes, there exists a unique Nash equilibrium in actions $x^*_j(l, k, z, c)$. Then, we define the reduced form profit function for firm $j$ as $\pi_j(l, k, z, c)$.

There can be three types of firms, the entering entrepreneur ($E$), the acquiring incumbent ($A$) and non-acquiring incumbents ($N$). There are two types of ownership: entry into the market ($l = e$) and sale to an incumbent ($l = i$). For expositional purposes, we will suppress the arguments $k$, $z$, and $c$, referring to the reduced form profit functions as $\pi_A(l)$, $\pi_E(l)$ or $\pi_N(l)$.

The inherent quality of the innovation will typically vary and to capture this, we define the quality of the innovation by the effect on reduced form profits: $\frac{d\pi_A(l)}{dk} > 0$, $\frac{d\pi_E(l)}{dk} > 0$, and $\frac{d\pi_N(l)}{dk} < 0$. Consequently, the reduced form profit for the possessor of the innovation is strictly increasing in the quality of the innovation, whereas increased quality strictly decreases the rivals’ profits.

The main assumption of our model is on how network effects and compatibility affect reduced form profit functions, given the quality of the innovation:

**Assumption 1** Network effects amplify the advantage of owning an innovation: $\frac{d\pi_A(l)}{dz} > 0$, $\frac{d\pi_E(l)}{dz} > 0$, and $\frac{d\pi_N(l)}{dz} < 0$. Compatibility reduces the advantage of owning an innovation: $\frac{d\pi_A(l)}{dc} < 0$, $\frac{d\pi_E(l)}{dc} < 0$, and $\frac{d\pi_N(l)}{dc} > 0$.

The first part states that the reduced-form profit for the possessor of an innovation is strictly increasing in the strength of the network effects, while increased network effects strictly decrease the profits of the rivals. Network effects amplify the advantage of owning an innovation if expectations of network sizes track quality (Farrell and Katz (1998)), or if consumer have rational expectations and the innovation gives the acquirer a higher quality product.

The second part is based on that increasing the degree of compatibility makes products more similar, as less emphasis is put on network size. This term "leveling" in the literature. Leveling reduces the profits of the firm with the largest network and increases profits of rivals, particularly if price competition is intensified (see e.g. Farrell and Saloner (1992)). We show in the Appendix that this assumption valid for a large parameter set in the Linear Cournot Network (LCN) Model, in which the innovation reduces marginal costs and expectations are rational.

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5 A large part of the literature on network effects focuses on coordination problems, multiple equilibria, tipping and path dependence. With this assumption, we are able to sidestep many complications and obtain a cleaner analysis. We, however, acknowledge that it comes at the cost of sidestepping many important issues in markets with network effects.

6 Empirical evidence on this effect from 19 markets with network effects is given in Tellis, Yin and Niraj (2008), who show that the presence of network effects enhance the effect of quality on market share. Liebowitz and Margolis (2001) also argue that quality largely explains success in software markets, which suggests that higher quality attracts more users and that the effect on market share and profits is then larger the stronger the network effects are.


8 Some empirical evidence that compatibility affects rivals of the owner of the innovation differently than the strength of network effects is given in Liu, Kremerer and Smith (2007). They document empirically that increasing compatibility in the flash card market reduces the effect of installed bases on price premiums while larger installed bases increases price premiums. I.e. there appears to be network effects in this market but the price premiums they allow are reduced when the degree of compatibility increases.
2.2 Stage 2: Sale or Entry

The entry or sale process is depicted as an auction where \( I \) incumbents simultaneously post bids, and the entrepreneur either accepts or rejects these bids. If all bids are rejected, the entrepreneur enters the market. Each incumbent announces a bid, \( b_i \), for the innovation: \( b = (b_1, \ldots, b_i, \ldots, b_I) \in R^I \) is the vector of these bids. Following the announcement of \( b \), the innovation may be sold to one of the incumbents at the bid price, or remain in the ownership of the entrepreneur, \( e \). If more than one bid is accepted, the bidder with the highest bid obtains the innovation. If there is more than one incumbent with such a bid, each such incumbent obtains the innovation with equal probability. The acquisition is solved for Nash equilibria in undominated pure strategies. There is a smallest amount, \( \varepsilon \), chosen such that all inequalities are preserved if \( \varepsilon \) is added or subtracted.

In case all bids are rejected and the entrepreneur enters, the entrepreneur face a reduced form entry cost equal to \( F(z, c) \). The cost of entry depends on the strength of the network effects in the industry and the degree of compatibility. Installed bases can create incumbent advantages that are costly to overcome. Stronger network effects lead to larger fixed entry costs, but increased compatibility reduces them.

Assumption 2 Network effects increases entry barriers, compatibility reduces them: \( \frac{dF(z,c)}{dc} > 0 \) and \( \frac{dF(z,c)}{dz} < 0 \).

Stage 3 can thus be interpreted as the long-run stable industry configuration, while stage 2 (entry or sale) depicts the short run in which incumbent advantages must be overcome and fixed entry costs paid.

To further simplify the entry stage, we assume that the market structure is entry-neutral. This implies that \( \pi_A(i, k, z, c) = \pi_E(e, k, z, c) \) and that in case of entry, each incumbent remains in the industry with probability \( \frac{n-1}{n} \) (one thus exits). The auction is an auction with externalities, implying that the bidders care about what happens if they do not win the auction. Specifically, the players have the following valuations for obtaining or retaining the innovation:

- \( v_{ii} \) is the value for an incumbent of obtaining the innovation when it would otherwise be obtained by a rival incumbent:
  \[
  v_{ii} = \pi_A(i) - \pi_N(i).
  \]

The first term shows the profit when possessing the innovation; the second term the profit if a rival incumbent obtains the innovation.

- \( v_{ie} \) is the value for an incumbent of obtaining the innovation, when the entrepreneur would otherwise keep it and enter the market:
  \[
  v_{ie} = \pi_A(i) - \left( \frac{n-1}{n} \right) \pi_N(e).
  \]

9 It can be shown in the Linear Cournot Model we outline in the appendix that this assumption holds for \( k \in [k(e), k(i)] \), i.e. \( k \) not too large or too small.
The first term is the profit of the incumbent when obtaining the innovation. The second term is the profit of the incumbent when the entrepreneur enters, incorporating the probability \( \frac{n-1}{n} \) that the incumbent remains in the market after entry. The profit for an incumbent of not obtaining the innovation is different in this case, due to the change of identity of the firm that would otherwise possess the innovation.

\[ v_e = \pi_E(e) - F(z, c). \]  

We can now proceed to solve for the equilibrium ownership structure given these valuations. Since incumbents are symmetric, the valuations \( v_{ii}, v_{ie} \) and \( v_e \) can in general be ordered in six different ways, as shown in table 2. These inequalities are useful for solving the model and illustrating the results. We can state the following lemma.

**Lemma 1** The equilibrium ownership structure and the acquisition price are described in table 2.

<table>
<thead>
<tr>
<th>Ineq.:</th>
<th>Definition:</th>
<th>Ownership ( l^* ):</th>
<th>Acquisition price, ( S^* ):</th>
<th>Reward, ( R_E )</th>
</tr>
</thead>
<tbody>
<tr>
<td>I1:</td>
<td>( v_{ii} &gt; v_{ie} &gt; v_e )</td>
<td>( i )</td>
<td>( v_{ii} )</td>
<td>( v_{ii} )</td>
</tr>
<tr>
<td>I2:</td>
<td>( v_{ii} &gt; v_e &gt; v_{ie} )</td>
<td>( i ) or ( e )</td>
<td>( v_{ii} )</td>
<td>( v_{ii} ) or ( v_e )</td>
</tr>
<tr>
<td>I3:</td>
<td>( v_{ie} &gt; v_{ii} &gt; v_e )</td>
<td>( i )</td>
<td>( v_{ii} )</td>
<td>( v_{ii} )</td>
</tr>
<tr>
<td>I4:</td>
<td>( v_{ie} &gt; v_e &gt; v_{ii} )</td>
<td>( i )</td>
<td>( v_e )</td>
<td>( v_e )</td>
</tr>
<tr>
<td>I5:</td>
<td>( v_e &gt; v_{ii} &gt; v_{ie} )</td>
<td>( e )</td>
<td>.</td>
<td>( v_e )</td>
</tr>
<tr>
<td>I6:</td>
<td>( v_e &gt; v_{ie} &gt; v_{ii} )</td>
<td>( e )</td>
<td>.</td>
<td>( v_e )</td>
</tr>
</tbody>
</table>

Table 2: Equilibrium ownership structure, acquisition price and entrepreneurial reward.

**Proof.** See appendix A. ■

Lemma 1 shows that when one of the inequalities \( I1, I3, \) or \( I4 \) holds, the innovation is obtained by one of the incumbents. Under \( I1 \) and \( I3 \), the acquiring incumbent pays the acquisition price \( S = v_{ii} \) as incumbents bid up the price to preempt other incumbents from acquiring the innovation (preemptive acquisition). Under \( I4 \), one of the incumbents obtains the innovation to prevent the entrepreneur from entering the market (entry deterring acquisition). When \( I5 \) or \( I6 \) holds, the entrepreneur keeps the innovation and enters the market. When \( I2 \) holds, there exist multiple equilibria. In one type of equilibrium, the incumbents expect the entrepreneur to sell the innovation and they try to preempt rivals from obtaining the innovation by bidding up the price. In the other equilibrium, the incumbents do not expect the entrepreneur to sell the innovation, and nobody has an incentive to bid a price above the entry valuation of the incumbent.

In our setting, \( v_{ie} > v_{ii} \) holds so the relevant inequalities are \( I3, I4 \) and \( I6 \). Figure 3 then illustrates Lemma 1. The net value of entry deterrence \( (v_{ie} - v_e) \) and preemption \( (v_{ii} - v_e) \) is increasing in \( k \) and hence slopes upwards Figure 3 (i). For low values of \( k \), inequality \( I6 \) holds and the entrepreneur keeps the innovation and enters the market. As \( k \) increases, inequality \( I4 \) holds and it is optimal for an incumbent to obtain the innovation to prevent the entrepreneur from entering the market. Finally, if \( k \) increases further, \( I3 \) holds and the innovation becomes important enough so that incumbents...
compete to preemptively acquire it. As Figure 3 (iii) shows, the entrepreneurial reward is increasing in \( k \) as the innovation becomes more valuable. Once a pre-emptive acquisition occurs, however, the entrepreneurial reward moves from \( v_e \) to \( v_{ii} \). This is the extra effect on innovation incentives arising from the possibility of selling the innovation to incumbents.

### 2.3 Stage 1: Research Intensity by the Entrepreneur

Given the entrepreneurial reward \( (R_E) \) from Lemma 1, the entrepreneur undertakes effort, \( \rho \), to discover the innovation. Let the innovation costs \( y(\rho) \) be an increasing convex function in effort, i.e. \( y'(\rho) > 0 \), and \( y''(\rho) > 0 \). Let the probability of success be \( \omega \) and the probability of failure \( 1 - \omega \), where \( \omega \in [0, 1] \) and probability \( \omega \) is an increasing concave function of effort, i.e. \( \omega'(\rho) > 0 \) and \( \omega''(\rho) < 0 \).

\[
\Pi_e = \omega(\rho) R_E - y(\rho)
\]

is then the expected net profit of undertaking effort. Maximizing this expression, the entrepreneurs optimal effort \( \rho^*(R_e) \) is implicitly given from:

\[
\frac{d\Pi_e(\rho, v, c)}{d\rho} = \omega'(\rho) R_E - y'(\rho) = 0,
\]

with the associated (satisfied) second-order condition \( \frac{d^2\Pi_e}{d\rho^2} = R_E \omega'' - y'' < 0 \). We can also state the following lemma.

**Lemma 2** The entrepreneur’s effort is increasing in the entrepreneurial reward: \( \frac{d\rho^*(R_e)}{dR_e} = -\frac{\omega'(\rho^*)}{R_E \omega''(\rho^*) - y''(\rho^*)} > 0. \)

Any factor increasing \( R_E \in \{v_{ii}, v_e\} \) thus increases entrepreneurs effort and hence also the probability of discovering the innovation.

### 3 Network Effects, Compatibility and the Decision to Enter or Sell

Having set up a model allowing the entrepreneur to both enter the market and sell the innovation, we can now study how network effects and compatibility affect the decision to enter the industry or sell the innovation. From table 2, a sale takes place if \( v_{il} > v_e \). The following proposition then holds.

**Proposition 1** (i) Stronger network effects \( (z) \) increases the likelihood of the innovation being sold to an incumbent. (ii) A greater degree of compatibility \( (c) \) decreases the likelihood of the innovation being sold to an incumbent.

To see this, note that the condition for when a preemptive acquisition takes place is \( v_{ii} > v_e \) and for when an entry-deterring acquisition of the innovation takes place is \( v_{ie} > v_e \). Differentiating \( v_{ii} - v_e \)
Figure 3: The equilibrium ownership structure. By varying the quality of the innovation, we vary who obtains it in equilibrium. For low $k$, the entrepreneur keeps the innovation. For intermediate values of $k$, an entry deterring acquisition occurs. For high values, a pre-emptive acquisition occurs resulting in the entrepreneurial reward $v_{ii}$ instead of $v_e$. 

Net value of Entry-deterrence:

$$v_{ie} - v_e = \pi_A(i) - \pi_E(e) - (\frac{\pi_N}{\pi_i})\pi_N(e) + F(z,c)$$

Net value of Preemption:

$$v_{ii} - v_e = \pi_A(i) - \pi_E(e) - \pi_N(i) + F(z,c)$$
and \( v_{ie} - v_e \) in \( z \) gives

\[
\begin{align*}
\frac{d(v_{ii} - v_e)}{dz} &= \frac{d(\pi_A(i) - \pi_N(i) - \pi_E(e) + F(z,c))}{dz} \\
&= -\frac{d\pi_N(i)}{dz} + \frac{dF(z,c)}{dz} > 0 \\
\frac{d(v_{ie} - v_e)}{dz} &= \frac{d(\pi_A(i) - \frac{n-1}{n}\pi_N(e) - \pi_E(e) + F(z,c))}{dz} \\
&= -\left(\frac{n-1}{n}\right)\frac{d\pi_N(e)}{dz} + \frac{dF(z,c)}{dz} > 0
\end{align*}
\]

(6) (7)

Hence, as the strength of the network effects in the industry increases, a sale becomes more likely. The intuition is the following. An increase in the strength of the network effects increases the first term in \( v_e = \pi_E(e) - F(z,c) \) by assumption 1. Once the entrepreneur has entered the industry, stronger network effects work to his or her benefit. The first term in \( v_{ii} = \pi_A(i) - \pi_N(i) \) and \( v_{ie} = \pi_A(i) - \frac{n-1}{n}\pi_N(e) \) increase by the same amount as the first term in \( v_e \). But stronger network effects also hurt non-acquirers (the second term in \( v_{ii} \) and \( v_{ie} \)). This leads to an additional increase in the valuation of an incumbent when network effects increase. As the Economist (1999) writes:

"Companies like Cisco, Intel and Microsoft recognize the threat posed by nimble young firms getting technologies to market at unimaginable speeds," says Red Herring’s Brian Taptich. "And they are willing to pay extremely high premiums to protect their franchises."

On top of this, stronger network effects decrease the value of entering since fixed entry costs increase (the second term in \( v_e \)). In sum, stronger network effects increases the likelihood of the entrepreneur selling the innovation. Both increasing the sales price and increasing the difficulty of entry promotes a sale of the innovation. In figure 3 (i), an increase in network effects shift the curves \( (v_{ie} - v_e) \) and \( (v_{ii} - v_e) \) upwards, resulting in an increased likelihood of a sale.

Regarding compatibility, differentiating \( v_{ii} - v_e \) and \( v_{ie} - v_e \) in \( c \) gives

\[
\begin{align*}
\frac{d(v_{ii} - v_e)}{dc} &= \frac{d(\pi_A(i) - \pi_N(i) - \pi_E(e) + F(z,c))}{dc} \\
&= -\frac{d\pi_N(i)}{dc} + \frac{dF(z,c)}{dc} < 0 \\
\frac{d(v_{ie} - v_e)}{dc} &= \frac{d(\pi_A(i) - \gamma(e)\pi_N(e) - \pi_E(e) + F(z,c))}{dc} \\
&= -\left(\frac{n-1}{n}\right)\frac{d\pi_N(e)}{dc} + \frac{dF(z,c)}{dc} < 0
\end{align*}
\]

(8) (9)

Hence, as the degree of compatibility in the industry increases, a sale becomes less likely. Increased compatibility between products hurts the owner of the innovation and reduces the first terms in \( v_e \), \( v_{ie} \) and \( v_{ii} \) by equal amounts. Non-acquiring incumbents, however, benefit from increased compatibility.
leading to a further decrease in the valuation of incumbents (the second terms in $v_{ie}$ and $v_{ii}$). Since increased compatibility between products also decreases the costs of entry into the industry, there is an additional increase in the value of entering the industry for the entrepreneur (the second term in $v_e$). Both effects promote entry. In figure 3 (i), an increase in compatibility shifts the curves $(v_{ie} - v_e)$ and $(v_{ii} - v_e)$ downwards, resulting in an decreased likelihood of a sale.

4 Network Effects, Compatibility and Innovation Incentives

Given the entrepreneur’s decision to sell the innovation or enter the industry, how does network effects and compatibility affect innovation incentives? Recall that the entrepreneurial reward ($R_E$) is $v_{ii}$ in case of a preemptive acquisition occurs and $v_e$ in case of entry or an entry deterring acquisition. We obtain the following proposition.

Proposition 2 (i) When the innovation is sold, network effects ($z$) unambiguously increases innovation incentives. Under entry, network effects may increase or decrease innovation incentives. (ii) When the innovation is sold, increased compatibility ($c$) unambiguously decreases innovation incentives. Under entry, compatibility may increase or decrease innovation incentives.

To see the first part relating to network effects, suppose the entrepreneur sells the innovation. Then $R_E = v_{ii}$ and

$$\frac{d\rho^*(v_{ii})}{dz} = \frac{d\rho^*(v_{ii})}{dv_{ii}} \frac{dv_{ii}}{dz}$$

$$= \frac{d\rho^*(v_{ii})}{dv_{ii}} \left( \frac{d\pi_A(i)}{dz} - \frac{d\pi_N(i)}{dz} \right) > 0.$$  

Network effects increase the sales price, thereby increasing innovation incentives. If the entrepreneur enters the market, then $R_E = v_e$ and

$$\frac{d\rho^*(v_e)}{dz} = \frac{d\rho^*(v_e)}{dv_e} \frac{dv_e}{dz}$$

$$= \frac{d\rho^*(v_e)}{dv_e} \left( \frac{d\pi_A(i)}{dz} - \frac{d\pi_N(i)}{dz} - \frac{dF(z,c)}{dz} \right).$$

The sign depends on how the strength of network effects affects the value of entering the industry. This is the standard trade-off often analyzed in the literature (e.g. Segal and Whinston (2007)). Stronger network effects increase innovation incentives (everyone wants to become the next Microsoft). However, stronger network effects also make entry much harder. What we add to the discussion is the positive effect on innovation incentives from the ability to sell the innovation to competing incumbents. The effect on innovation incentives can be illustrated in figure 3 (iii), with a shift of $v_{ii}$ upwards, and a shift of $v_e$ upwards or downwards.
To see the second part relating to compatibility, suppose selling the innovation is optimal. Then, $R_E = v_{ii}$ and

$$\frac{d\rho^*(v_{ii})}{dc} = \frac{d\rho^*(v_{ii})}{dv_{ii}} \frac{dv_{ii}}{dc} = \frac{d\rho^*(v_{ii})}{dv_{ii}} \left( \frac{d\pi_A(i)}{dc} - \frac{d\pi_N(i)}{dc} \right) < 0,$$

which is negative as the profits from having the innovation are reduced and the profits from not having the innovation are increased. Compatibility can thus reduce innovation incentives by reducing the sales price of innovations. If the entrepreneur enters the market, then, $R_E = v_e$ and

$$\frac{d\rho^*(v_e)}{dc} = \frac{d\rho^*(v_e)}{dv_e} \frac{dv_e}{dc} = \frac{d\rho^*(v_e)}{dv_e} \left( \frac{d\pi_E(e)}{dc} - \frac{d\pi_N(e)}{dc} - \frac{dF(z,c)}{dc} \right).$$

Research intensity is either increasing or decreasing in the degree of compatibility. This can be illustrated in figure 3 (iii), with a shift of $v_{ii}$ downwards, and a shift of $v_e$ upwards or downwards.

5 The Importance of Bidding Competition

In network industries, the market is often scattered when the industry is young. Once the industry matures, a dominant firm often emerges. How does such dominance affect innovation incentives and the entrepreneur’s decision to sell or enter the industry?

Consider an industry with only one dominant firm, arising, for example, after the entrepreneur has entered the industry or sold the innovation and all non-acquiring incumbents have been forced to exit the industry. A new entrepreneur considers undertaking effort to come up with an innovation. The setup is as above, with an innovation stage, and entry stage and a product market stage.

In the product market stage, there can be two configurations. Either the entrepreneur enters and the incumbent becomes a non-acquirer, or the incumbent acquires the innovation and becomes an acquirer without any rivals. The reduced form profit functions are $\pi_A(i), \pi_E(e)$ or $\pi_N(e)$. In the entry or sale stage, there are now only two valuations:

- $v_{ie} = \pi_A(i) - \pi_N(e)$ is the value for the incumbent of obtaining the innovation, when the entrepreneur would otherwise keep it and enter the market.

- $v_e = \pi_E(e) - F(z,c)$ is the value for the entrepreneur of retaining the innovation and entering the market.

With only two valuations, the entrepreneur sells the innovation if $v_{ie} > v_e$. But in case of a sale
there is no bidding competition, so the incumbent has no incentives to bid higher than slightly above $v_e$.

**Proposition 3** Bidding competition increases innovation incentives. In the absence of bidding competition, the entrepreneur sells the innovation if $v_e > v_i$. Otherwise, the entrepreneur enters the industry. The entrepreneurial reward is $R_E = v_e$ in both cases. With bidding competition, the entrepreneurial reward is $R_E = \max\{v_e, v_{ii}\}$, with $v_{ii} > v_e$ under a sale.

This proposition can be illustrated in figure 3 by removing region $I3$. Either entry or an entry deterring acquisition takes place, and the entrepreneurial reward is simply $v_e$ in both cases. As before, 

$$
\frac{d(v_{ie} - v_e)}{dz} = -\frac{d\pi_N(e)}{dz} + \frac{dF(z,c)}{dz} > 0,
$$

so network effects increases the likelihood of sale, while compatibility increases the likelihood of entry, 

$$
\frac{d(v_{ie} - v_e)}{dc} = -\frac{d\pi_N(e)}{dc} + \frac{dF(z,c)}{dc} < 0.
$$

Regarding innovation incentives, since the reward is $R_E = v_e$, network effects and compatibility may increase or decrease innovation incentives.

This suggests a dynamic effect; bidding competition—and thus innovation incentives—may vary over time in network industries. In network industries, the market is often scattered when the industry is young, as several players compete for the lead position in the industry. During this initial stage, bidding competition between incumbents for innovations is likely to be strong, as a small lead can tip the industry in favor of the winner. However, once time passes and the industry shifts in favor of one dominant incumbent, bidding competition between incumbents may weaken and in the extreme disappear completely. Reduced innovation incentives follow. This suggests that "innovation waves" could occur in network industries. Initially, innovation incentives are strong, but over time as the industry matures, bidding competition for new innovations disappear and innovation incentives weaken.

## 6 Extensions

### 6.1 Entry Leads to More Firms in the Market

The assumption of market structure neutral entry implies that the number of firms in the industry is the same before and after entry. If the number of firms are allowed to vary, our results may be affected. Suppose entry leads to a less concentrated market structure, i.e. if a sale occurs there are $n$ active firms in the market, whereas if entry occurs there are $n + 1$ active firms in the market. To this end, replace the assumption of market structure neutral entry with the following assumption that entry happens without the exit of incumbents. This implies that: (i) $\pi_A(i)$ may differ from $\pi_E(e)$, and (ii) the probability of remaining in the market as a non-acquirer when the entrepreneur enters is $\gamma(e)$ and the probability of remaining in the industry when an incumbent acquires the innovation is $\gamma(i)$.

Since all incumbents remain on the market, the difference in value for sale and the value of entering becomes:

$$
v_{ii} - v_e = [\pi_A(i) - \pi_E(e) + F(z,c)] - [\gamma(l)\pi_N(l)], \ l = \{e, i\},
$$

(14)
whereas the effect of changing the strength of the network effects becomes:

\[ v'_{i,e,z} - v'_{e,z} = \left[ \frac{d\pi_A(i)}{dz} - \frac{d\pi_E(e)}{dz} \right] - \gamma(l) \frac{d\pi_N(l)}{dz} \geq 0 \quad l = \{e, i\}. \tag{15} \]

Thus, the effects on the entrant and the acquirer of an increase in the strength of the network effect may differ, i.e. \( \frac{d\pi_A(i)}{dz} \neq \frac{d\pi_E(e)}{dz} \). We cannot determine the sign of the effect of an increase in the strength of network effects on the net value of an acquisition. As long as \( \frac{d\pi_A(i)}{dz} \) is not sufficiently lower than \( \frac{d\pi_E(e)}{dz} \), however, an increased network effect will increase the likelihood of a sale since \( \frac{d\pi_N(l)}{dz} \) is still negative. Innovation incentives will then also increase.

### 6.2 Entry Cost and Incumbent Installed Bases

In our model, we have incorporated all effects from incumbent installed bases in \( F(z,c) \). The intended interpretation is to consider product market competition in stage 3 as a long-run outcome, in which incumbent advantages are no longer important. Another way of potentially accounting for installed bases is to introduce a stage 0, in which incumbents sign-up customers that constitute the installed base in stage 3. This would imply that our model is more short term overall, as incumbent advantages would essentially persist forever. This is not very likely to be the case. Furthermore, accounting for installed bases in this way—instead of assuming any benefits for incumbents to be captured in the fixed entry cost—would not affect our results, unless installed bases change now network effects and compatibility affect reduced form profit functions (assumption 1).

### 6.3 Other Selling Mechanisms and Licensing

In our setup, a sale takes place through a sealed-bid first price auction with externalities. The motivation for this is that we believe that it captures the most essential features of bidding competition between incumbents in situations where acquisitions are used to gain access to new innovations. This simple setup, however, implies that the potential rents from using a more sophisticated mechanism are foregone. Jehiel, Moldovanu and Stacchettis (1999) have shown that sophisticated mechanisms might be needed to maximize the revenues in auctions with externalities. It might be that all firms in the industry need to provide some transfers to the seller. It is likely, however, that more complicated mechanisms require the seller of the innovation to have an unrealistically strong commitment power (see Jehiel and Moldovanu(2000)) and that more sophisticated auctions would potentially allow the entrepreneur to extract a larger price for its innovation.

Our model should also be fairly robust to incorporating licensing of the innovation. First, if the entrepreneur licences the innovation to only one incumbent, then, for all purposes, licensing equals sale in our model and our results go through unchanged. Such a setting is natural when the innovation consists of an indivisible asset, in terms of capital or human capital. Second, if the entrepreneur licences the innovation to a large number of incumbents or licences the innovation and simultaneously enters the industry, our results may weaken. In this kind of situation, the seller must determine how many licences to sell. Such an issue is studied in the literature on patent licensing. For example, Katz and Shapiro (1986b) allow a seller to commit to the number of licences to sell and show that there
exists an equilibrium where some potential buyers are left without a licence. Hence, licensing to one incumbent only is a possible outcome in situations where the entrepreneur can sell multiple licences. Our model is thus valid also in those situations where multiple licences can be sold.

6.4 Asymmetric Incumbents

In our model, we have focused on the case of completely symmetric incumbents before the sale of the innovation. However, in network industries, asymmetries in terms of network size between firms, for example, are common. Further, the innovation may be of different use to different incumbents so that they may vary in their valuation of the innovation. The crucial part relating to asymmetries between incumbents is the outcome of the auction in case the entrepreneur sells the innovation. In that situation, incumbents will have different valuations of the innovation, and the auction game will, in general, be very tedious to solve since one needs to keep track of many possible orders of valuations. A sufficient condition for a sale, however, is then that there exists an incumbent firm $d \in I$ for which the net value of a take-over acquisition is positive $v^d_{ic} - v_e > 0$, where:

$$v^d_{ic} - v_e = \left[ \pi^d_A(i) - \pi_E(e) + F(z, c) \right] - \gamma^d(e)\pi^d_N(l). \quad (16)$$

The main difference from the above is that the effects on the entrant’s profit $\pi_E(e)$ and firm $d$’s profit $\pi^d_A(i)$ of an increase in the strength of network effects might now differ also when exit occurs, since the entrant and firm $d$ might now use the innovation differently. As long as $\frac{d\pi^d_A(i)}{dz}$ is not sufficiently lower than $\frac{d\pi_E(e)}{dz}$, however, an increase in the strength of the network effects will be conducive to innovation for sale, since $\frac{d\pi^d_N(l)}{dz}$ is still negative.

Moreover, if firms are asymmetric as non-acquirers, the exit game will look different. In particular, the most inefficient firm(s) would know that it (they) would exit if they did not acquire the innovation. This would then imply that sale would always be the equilibrium outcome if entry were to trigger exit(s), i.e. $\gamma^d(e) = 0$. An explicit acquisition model with asymmetric firms is needed to analyze these issues in detail, but due to space limitations, this is left to future research.

7 Concluding Remarks

In this paper, we set out to answer the following questions: "How does network effects and compatibility affect the entrepreneur’s decision to sell or enter the industry?", and "What is the effect of network effects and compatibility on innovation incentives allowing for both sale of the innovations and entry into the market?". We developed a model of entry into network industries, allowing also for the possibility of selling the innovation to incumbents. We showed that increasing network effects tend to increase the likelihood of a sale of an innovation, while compatibility decreases it. The stronger the network effects, the stronger the innovation incentives as incumbents compete to acquire the innovation from the entrepreneur. Compatibility, however, can decrease innovation incentives by reducing incumbents valuations of obtaining the innovation. We also underscored that bidding competition for innovations is important for innovation incentives, and that if markets become concentrated over time,
innovation waves can occur as bidding competition for new innovations is strong when the industry is young, but weakens as it matures and a clear leader emerges.

Policy implications. Our results have several policy implications. First, policy makers should put more emphasis on the sale of innovations when evaluating entry barriers to an industry. A sale allows the entrepreneur to avoid entry barriers, and can also substantially increase innovation incentives if there is intense bidding competition between incumbents for the innovation. Second, it is common that economists support public policy in favor of increasing compatibility, see e.g. Farrell and Klemperer (2007), because competition between incompatible networks is usually found to be more profitable. As we point out, however, innovation incentives may be reduced if compatibility reduces incumbents willingness to pay for entrepreneurial innovations. Thus, we suggest that a careful analysis on the likely effects on innovation incentives from the sale of innovations should be undertaken before implementing policies aiming to increase compatibility in an industry with network effects. Third, policies promoting bidding competition between incumbents for entrepreneurial innovations may be a good way to increase innovation incentives in network industries.

Empirical implications. Our model gives rise to several empirically testable predictions: (i) the ratio of sale of innovations to entry in network industries should be higher the larger the network effects are, (ii) the implementation of policies increasing compatibility should decrease the ratio of sale of innovations to entry and reduce the pace of innovation, and (iii) total innovation output (e.g. patents) by entrepreneurs should be higher when network effects are strong. Testing these predictions is a good avenue for further research.

Appendix

A Proof of Lemma 1

First, note that \(b_i \geq \max v_{il}, l = \{e, i\}\) is a weakly dominated strategy since no incumbent will post a bid equal to or above its maximum valuation of obtaining the innovation and that firm \(e\) will accept a bid, iff \(b_i > v_e\).

A.1 Inequality I1

Consider equilibrium candidate \(b^* = (b^*_1, b^*_2, ..., yes)\). Let us assume that incumbent \(w \neq e\) is the incumbent that has posted the highest bid and obtains the innovation and firm \(s \neq d\) is the incumbent with the second highest bid.

Then, \(b^*_w \geq v_{ii}\) is a weakly dominated strategy. \(b^*_w < v_{ii} - \varepsilon\) is not an equilibrium, since firm \(j \neq w, e\) then benefits from deviating to \(b_j = b^*_w + \varepsilon\), since it will then obtain the innovation and pay a price lower than its valuation of obtaining it. If \(b^*_w = v_{ii} - \varepsilon\), and \(b^*_s \in [v_{ii} - \varepsilon, v_{ii} - 2\varepsilon]\), then no incumbent has an incentive to deviate. By deviating to \(no\), the entrepreneur’s payoff decreases since it foregoes a selling price exceeding its valuation, \(v_e\). Accordingly, the entrepreneur has no incentive to deviate and thus, \(b^*\) is a Nash equilibrium.

Let \(b = (b_1, ..., b_m, no)\) be a Nash equilibrium. Let incumbent \(h\) be the incumbent with the highest
bid. The entrepreneur will then say no iff \( b_h \leq v_e \). But incumbent \( j \neq e \) will have the incentive to deviate to \( b' = v_e + \varepsilon \) in stage 1, since \( v_{ie} > v_e \). This contradicts the assumption that \( b \) is a Nash equilibrium.

### A.2 Inequality I2

Consider equilibrium candidate \( b^* = (b^*_1, b^*_2, ..., y) \). Then, \( b^*_w \geq v_{ij} \) is a weakly dominated strategy. \( b^*_w < v_{ij} - \varepsilon \) is not an equilibrium since firm \( j \neq w, e \) then benefits from deviating to \( b_j = b^*_w + \varepsilon \), since it will then obtain the innovation and pay a price lower than its valuation of obtaining it. If \( b^*_w = v_{ii} - \varepsilon \), and \( b^*_e \in [v_{ii} - \varepsilon, v_{ii} - 2\varepsilon] \), no incumbent has an incentive to deviate. By deviating to no, the entrepreneur’s payoff decreases since it foregoes a selling price exceeding its valuation, \( v_e \). Accordingly, the entrepreneur has no incentive to deviate and thus, \( b^* \) is a Nash equilibrium.

### A.3 Inequality I3

Consider equilibrium candidate \( b^* = (b^*_1, b^*_2, ..., yes) \). Then, \( b^*_w \geq v_{ii} \) is a weakly dominated strategy. \( b^*_w < v_{ii} - \varepsilon \) is not an equilibrium since firm \( j \neq w, e \) then benefits from deviating to \( b_j = b^*_w + \varepsilon \), since it will then obtain the innovation and pay a price lower than its valuation of obtaining it. If \( b^*_w = v_{ii} - \varepsilon \), and \( b^*_e \in [v_{ii} - \varepsilon, v_{ii} - 2\varepsilon] \), then no incumbent has an incentive to deviate. By deviating to no, the entrepreneur’s payoff decreases, since it foregoes a selling price exceeding its valuation, \( v_e \). Accordingly, the entrepreneur has no incentive to deviate and thus, \( b^* \) is a Nash equilibrium.

Let \( b = (b_1, ..., b_m, no) \) be a Nash equilibrium. The entrepreneur will then say no iff \( b_h \leq v_e \). But incumbent \( j \neq e \) will then have the incentive to deviate to \( b' = v_e + \varepsilon \) in stage 1, since \( v_{ie} > v_e \). This contradicts the assumption that \( b \) is a Nash equilibrium.

### A.4 Inequality I4

Consider equilibrium candidate \( b^* = (b^*_1, b^*_2, ..., yes) \). Then, \( b^*_w > v_e \) is not an equilibrium since firm \( w \) would then benefit from deviating to \( b_w = v_e \). \( b^*_w < v_e \) is not an equilibrium, since the entrepreneur would then not accept any bid. If \( b^*_w = v_e - \varepsilon \), then firm \( w \) has no incentive to deviate. By deviating to \( b'_j \leq b^*_w \), firm \( j \)’s, \( j \neq w, e \), payoff does not change. By deviating to \( b'_j > b^*_w \), firm \( j \)’s payoff decreases since it must pay a price above its willingness to pay \( v_{ij} \). Accordingly, firm \( j \) has no incentive to deviate. By deviating to no, the entrepreneur’s payoff decreases since it foregoes a selling price above its valuation, \( v_e \). Accordingly, the entrepreneur has no incentive to deviate and thus, \( b^* \) is a Nash equilibrium.

Let \( b = (b_1, ..., b_m, yes) \) be a Nash equilibrium. If \( b_w \geq v_{ii} \), then firm \( w \) will have the incentive to deviate to \( b' = b_w - \varepsilon \). If \( b_w < v_{ii} \), the entrepreneur will have the incentive to deviate to no, which contradicts the assumption that \( b \) is a Nash equilibrium.
Let \( b = (b_1, ..., b_m, no) \) be a Nash equilibrium. The entrepreneur will then say \( no \) iff \( b_h \leq v_e \). But incumbent \( j \neq d \) will have the incentive to deviate to \( b' = v_e + \varepsilon \) in stage 1 since \( v_{ie} > v_e \), which contradicts the assumption that \( b \) is a Nash equilibrium.

### A.5 Inequalities I5 or I6

Consider equilibrium candidate \( b^* = (b^*_1, b^*_2, ..., no) \), where \( b^*_j < v_e \) \( \forall j \in J \). It then directly follows that no firm has an incentive to deviate and thus, \( b^* \) is a Nash equilibrium.

Then, note that the entrepreneur will accept a bid iff \( b_j \geq v_e \). But \( b_j \geq v_e \) is a weakly dominating bid in these intervals, since \( v_e > \max\{v_{ii}, v_{ie}\} \).

### B Linear Cournot Network (LCN) Model

Here we present a simple linear Cournot model with network effects and compatibility that illustrates product market competition in stage 3. The model is essentially Katz and Shapiro (1985a) with a linear network effect, differences in marginal costs brought about through innovation and a degree of compatibility between products. In particular, it is shown that assumption 1 is valid for a large parameter set.

As above, there are \( n \geq 2 \) firms competing in an oligopoly industry with network effects. The strength of network effects in the industry is given by \( z \in [0, 1] \), where \( z = 0 \) corresponds to a standard industry with no network effects. The degree of compatibility between networks is measured by \( c \in [0, z] \), where \( c = 0 \) implies that each firm’s product only benefits from its own network (incompatibility) and \( c = z \) implies that each firm’s product equally benefits from the networks of all products sold. One of the firms, firm \( A \), possesses an innovation, which reduces its marginal costs by \( k \in [0, b] \) from the common marginal cost \( b \).

The goods sold are homogeneous, but consumers are heterogeneous in their willingness to pay for the good, such that consumer \( r \) chooses to buy from the firm \( i \) that offers the highest utility given by

\[
r + zq^e_i + cq^e_{-i} - p_i.
\]

(17)

In this expression, \( r \) is uniformly distributed between \(-\infty, A\) and \( q^e_i \) is the expected number of consumers purchasing the product of firm \( i \) and \( q^e_{-i} \) denotes the sum of the expected number of consumers purchasing rivals’ products. We assume the timing to be such that the consumers first form expectations on network sizes and then firms set quantities taking the expectations as given. Since the goods are homogeneous, it must be the case that firms’ generalized prices given by \( p^g_i = p_i - vq^e_i - cq^e_{-i} \) are equal. Since \( r \) is uniform on \([-\infty, A]\), the total number of consumers buying at this generalized price is \( Q = A - p^g \). As firms compete in quantities, the market clears at the generalized price \( p^g = A - Q \).

In the following, we only need to keep track of differences between firm \( A \) with marginal cost \( b - k \) and the other symmetric \((n - 1)\) firms, subscript \( N \), with marginal costs \( b \). Individual prices are then
The second-order conditions, given by

\[ p_A = A - Q + vq_A^* + c((n - 1)q_N^*) \]
\[ p_N = A - Q + vq_N^* + c((n - 2)q_N^* + q_A^*) \]

with \( Q = q_A + (n - 1)q_N \). The resulting profits are

\[ \pi_A = (A - (n - 1)q_N - q_A + vq_A^* + c((n - 1)q_N^*) - b + k)q_A \]  
\[ \pi_N = (A - (n - 1)q_N - q_A + vq_N^* + c((n - 2)q_N^* + q_A^*) - b)q_N \]

Firms set quantities taking rivals’ quantities and the expectation of network sizes as given. This results in optimal quantities of

\[ q_A = \frac{A - b + kn - c(n - 1)(q_A^* - 2q_N^*) + (n(q_A^* - q_N^*) + q_N^*)z}{1 + n} \]  
\[ q_N = \frac{A - b - k + 2cq_A^* - 3cq_N^* + c(q_N^* - q_A^*)z + 2q_N^*z}{1 + n} \]

The second-order conditions, \(-2 < 0\) and \((1 - n) < 0\), are satisfied. Assuming that the expectations are correct, \(q_A^* = q_A\) and \(q_N^* = q_N\) must hold. This gives equilibrium quantities equal to

\[ q_A^* = \frac{k(n - 1)}{n(1 + c - z)} + \frac{k + An - bn}{n(1 + c + n - cn - z)} \]
\[ q_N^* = \frac{k - ck + b(1 + c - z) + A(z - c - 1)}{(1 + c - z)(z - n - 1 + c(n - 1))} \]

and profits

\[ \pi_A^*(k, z, c) = \frac{(b(1 + c - z) + A(z - c - 1) + k(c(n - 2) - n + z))^2}{(1 + c - z)^2(z - n - 1 + c(n - 1))^2} \]
\[ \pi_N^*(k, z, c) = \frac{(k - ck + b(1 + c - z) + A(z - c - 1))^2}{(1 + c - z)^2(z - n - 1 + c(n - 1))^2} \]

We need to restrict our parameter values such that these quantities and profits are positive. In particular, note that we need \(k < k^{\text{max}} = \frac{(A - b)(1 + c - z)}{1 - c}\) to ensure that \(q_N^* > 0\) and \(\pi_N^*(k, z, c) > 0\).

We can now see that

\[ \frac{d\pi_A(k, z, c)}{dk} = \frac{(c(n - 2) - n + z)((b - A)(1 + c - z) + k(c(n - 2) - n + z))}{(1 + c - z)^2(-1 + c(n - 1) - n + z)^2} \]
\[ \frac{d\pi_N(k, z, c)}{dk} = \frac{(c - 1)((c - 1)k + A(1 + c - z) + b(-1 - c + z))}{(1 + c - z)^2(-1 + c(n - 1) - n + z)^2} \]

The first expression is positive and the second is negative if

\[ k > \frac{(A - b)(1 + c - z)}{c(n - 2) - n + z} < 0 \]
\[ k < k^{\text{max}} \]
The first condition always holds since \( n \geq 2 \), \( z < 1 \) and \( c < z \) imply that the right-hand side is negative. The second condition says that \( k \) should be sufficiently small to ensure that \( q_N^* > 0 \) and \( \pi_N^*(k,z,c) > 0 \).

With regards to network effects, we get

\[
\frac{d\pi_A(k,z,c)}{dz} = \frac{X((A-b)(1+c-z)^2 + kY)}{(1+c-z)^3(-1 + c(-1+n) - n + z)^3}
\]

\[
\frac{d\pi_N(k,z,c)}{dz} = \frac{Z((b-A)(1+c-z)^2 + (-1+c)k(-2+c(-2+n) - n + 2z))}{(1+c-z)^3(-1 + c(-1+n) - n + z)^3}
\]

with \( X = 2((b-A)(1+c-z) + k(c(-2+n) - n + z)) \), \( Y = (-1 + n + c^2(3 + (-3+n)n) + (n - z)^2 + 2c(1 + n - n^2 + (-2+n)z)) \) and \( Z = 2((-1+c)k + (A-b)(1+c-z)) \). The first expression is positive if

\[
k < \frac{(A-b)(1+c-z)}{c(n-2)-n+z} < 0
\]

\[
k > \frac{(A-b)(1+c-z)^2}{1-n-c^2(3 + (n-3)n) - (n - z)^2 - 2c(1 + n - n^2 + (n-2)z)} < 0.
\]

The first equation never holds as the right side is negative and \( k \) should be larger than zero. It is thus irrelevant. The second equation is also negative, since the denominator is negative. Hence, stronger network effects always benefit the owner of the innovation.

The second expression, \( \frac{d\pi_N(k,z,c)}{dz} < 0 \), holds for \( k \) that satisfy

\[
k \in [k_{\text{min}}, k_{\text{max}}],
\]

with \( k_{\text{min}} = \frac{(A-b)(1+c-z)^2}{(c-1)(-2+c(-2+n)-n+2z)} \). The lower bound is positive since \(-2+c(-2+n)-n+2z = -2(1+c-z) + n(c-1) < 0 \). This implies that for stronger network effects to hurt the non-owners of the innovation, \( k \) must be sufficiently large.

With regards to compatibility we obtain

\[
\frac{d\pi_A(k,z,c)}{dc} = -\frac{2(n-1)((b-A)(1+c-z) + k(c(n-2) - n + z)J}{(1+c-z)^3(-1 + c(-1+n) - n + z)^3}
\]

\[
\frac{d\pi_N(k,z,c)}{dc} = -\frac{2((c-1)k + A(1+c-z) + b(z-c-1))H}{(1+c-z)^3(-1 + c(-1+n) - n + z)^3}
\]

with \( J = -A(1+c-z)^2 + b(1+c-z)^2 + (c-1)k(-2+c(n-2) - n + 2z) \) and \( H = A(n-1)(1+c-z)^2 - b(n-1)(1+c-z)^2 + k(3 + (c-2)c(n-1) + n - 4z + z^2) \). The first expression is negative for \( k \) such that

\[
k < \frac{(A-b)(1+c-z)}{c(n-2)-n+z} < 0
\]

\[
k > k_{\text{min}},
\]

The first condition is negative and thus irrelevant. The second is positive and is the same as the lower bound for \( \frac{d\pi_N(k,z,c)}{dz} < 0 \).
For the second expression, the nominator is positive for

\[
k \in \left[ \frac{(A - b)(1 - n)(1 + c - z)^2}{3 + (c - 2)c(n - 1) + n - 4z + z^2}, k_{\text{max}} \right].
\]

(31)

The lower bound is always negative since \(3 + (c - 2)c(n - 1) + n - 4z + z^2 > 0\). Hence, greater compatibility always benefits rivals to the owner of the innovation.

References


