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Introduction of Software Products and Services Through Public “Beta” Launches *

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Abstract

Public “Beta” launches have become a preferred route of entry into the markets for new software products and web site based services. While beta testing of novel products is nothing new, typically such tests were done by experts within firm boundaries. What makes public beta testing so attractive to firms? By introducing semi-completed products in the market, the firm can target the early adopter population, who can then build the potential market through the word of mouth effect by the time the actual version of the product is launched. In addition, the information gathered through the usage of the public beta gives significant insights into customer preferences and consequently helps in building a better product. We build these marketing and product development implications in an analytical model to compare the different product introduction strategies like “skimming” or “penetration pricing”with beta launches. This analysis is done for products of branded and unbranded Web 2.0 companies like Google and Flickr etc. We also examine the impact of different monetization models like direct pricing and advertising on the beta launch strategy.

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1 Introduction

It is very common to come across pubic beta launches of software products and web site based services. Examples of software products that have used betas are Internet Explorer and Microsoft Office (while introducing upgrades) and μtorrent (a bit torrent client). Examples of web site betas are Gmail and Flickr, the earlier of these still remains in beta. Why is this strategy of introducing software products and services becoming popular? To answer this we must first understand the possible reasons behind why firms may find this strategy attractive.

One reason is to do with product development. Beta testing phase has typically been the comprehensive testing phase in which software is stress-tested prior to market launch. This testing is done by technically adept testers who provide feedback on bugs and potential feature alterations. It has long been recognized that this way of testing software is costly and inefficient, and has become even more so with the increasing complexity present in the software products today. Previous academic literature points out the economic advantages of involving end-users in the product development process (Eric von Hippel 1998). With the advent of Internet and the penetration of broadband, it is becoming easier to involve users in the software testing process at low cost. This makes comprehensive testing feasible since the software is tested on combinations of software and hardware components (Computing Canada March 3, 2006). Further, it improves the identification of the features and feature alterations desired by end users and improves the efficiency of application development. Close involvement with end-users helps implement the tried and tested methodology of agile development (Computer Weekly October 10, 2006). Thus public beta launches may help improve the cost efficiency of the product development process in a significant way.

The other reason for adoption of public beta launches is to do better introductory marketing of the product. Marketing literature has long studied optimal product launch strategies. The focus has been on studying the sales curve of the product and relating it to firm's choice variables like pricing and advertising. Mahanjan, Muller, and Bass (1990) provide a comprehensive survey of these product diffusion models. The work of Krishnan, Bass, and Jain (1999) relates the optimal pricing strategy to sales growth patterns. For a durable product, several situations may influence the optimal price path. Jain, Muller, and Vicassim (1999) find that existence of customers with different valuations, usage levels and price sensitivities drive the choice of skimming (decreasing price path) or penetration pricing (increasing price path) strategies. Uncontrolled software piracy has also been cited as a reason for skimming pricing strategies (Nascimento and Vanhonacker (1988)). In a competitive situation, it has been suggested that an initial high price is appropriate for products with a unique advantage over others (Hultink, Hart, Robben, and Griffin (2000). However, if diffusion of the product is important, penetration pricing may be employed to hinder product launches by competitors (Choffray and lilien (1984), Kalish and Lilien (1990)). Besanko and Winston (1990) focus on the how expectations of declining prices by rational customers impact the firm's pricing policy. Other notable works that study pricing are by Raman and Chatterjee (1995) and Krishnan, Bass, and Jain (1999). Bayus (1992), Norton and Bass (1987) and Padmanabhan and Bass (1991) study an interesting variation of the pricing problem where they consider the presence of improved generations of a product. This situation is the closest to our work since one could consider the public beta product and the final product as successive product generations. In addition to pricing, the advertising of new products is important to raise product awareness. Horsky and Simon (1983) Kalish (1985), Krishnan and Jain (2006) and Krishnamoorthy, Prasad, and Sethi (2008) have analyzed optimal advertising spending levels for new products along the product life cycle. In addition to explicit advertising, word of mouth effect has also been long recognized to an important factor in increasing sales (Bass (1969)). To create a strong word of mouth effect, marketers must ensure that the product is adopted by sufficient numbers of consumers early on in the product life cycle. These early adopters then help in diffusing the product knowledge in the rest of the potential market, thus serving as enablers of the product adoption process. The customers who have a propensity to adopt the product early are called innovators while those who tend to follow them are called imitators in marketing parlance. Horsky (1990) reports that if word of mouth effects are weak, the monopolist prefers a price skimming strategy. In case of experience goods, the word of mouth effect may be negative. This aspect has been explored by Mahajan, Muller, and Kerin (1984). Other reasons impacting the demand pattern of a new product have been studied by Golder and Tellis (1997) who focus on the takeoff portion of the sales curve where the product gets established in the market whereas Gupta, Jain, and Sawhney (1999) study the sales for a product whose demand is effected by the sales of a complementary product (e.g. hardware and software). Some other papers that focus on interesting aspects of the new product adoption process are by Souza, Bayus, and Wagner (2004), Song (2003), Smith (1986), Golder and Tellis (1997), Bridges, Yim, and Briesch (1995), Narasimhan (1989) and Lee and Connor (2003).

Creating an initial adopter population is a huge challenge and there is a realization that beta launches may help acquire early adopters. The Wall Street Journal November 28, 2005 observes that there is nothing else in the marketing world where marketers knowingly introduce a flawed or an inadequate product, yet it helps grow the product's user base. Articles from New York Times July 22, 2004 and TechNewsWorld March 29, 2005 report that early adopters of products attach a lot of value to having a branded new product that few other people may have. There are several examples of this phenomenon. Accounts on Google's Gmail were only made available through invitations when Gmail was introduced. This created such a frenzy for acquiring a Gmail address that at one point Gmail invitations were auctioned off at Ebay for as high as $200. Another example is that of new versions of the popular World of Warcraft Computer game of Blizzard Entertainment. Test accounts of this game went for as much as $500 on Ebay (New York Times (July 22, 2004)). Thus exclusivity may be quite important for innovators in the context of products of big banners like Google etc and these firms may in response create scarcity for the beta product. This presents an interesting contrast with Web 2.0 products which are typically started by relatively unknown firms. The business models of these products are usually based on value addition to the product through inputs of other users. For example websites such as del.icio.us becomes a more valuable product for its users as people use it and add bookmarks satisfying a wide range on interests. Clearly, the product development of these products depends not only upon the R&D efforts of the firm but also upon the users the web site. Thus network externalities are extremely important for creating value for such products and therefore such firms may want to ease adoption by the innovators rather than create scarcity. Clearly, beta launches for increasing product adoption may proceed in dissimilar ways by branded and unbranded Web 2.0 firms. Our purpose in this paper is to focus on both these types of firms and do a comprehensive analysis of their beta launch policies accounting for the product development and marketing benefits of public betas. Thus this paper combines product development and marketing issues which addresses a significant gap in the literature. Exceptions are work by Ofek and Savary 2003 and Dogan, Ji, Mookerjee, and Radhakrishnan (2008). Ofek and Sarvary (2003) consider firms' choices of R&D and marketing variables such as advertising but do not consider the sales impact of word of mouth effects, which we do. Dogan, Ji, Mookerjee, and Radhakrishnan (2008) do focus on word of mouth effects but do not consider the demand side impact of exclusivity and network externalities, which is a central feature of our work.

We now discuss the specific questions that a firm needs to consider when considering a beta launch. In order to do so, we first define a beta product as one that is of a lesser quality than the final product. The first set of questions relate to the firm's decisions on the choice variables of product quality and price in the beta phase as well as the final launch. It is interesting to note that the so called beta versions could be of very high quality despite the perception that they must have inherent flaws due to their beta status. A New York Times July 22, 2004 article reports that, "Gmail feels like a final release...it might be much more of a marketing tool than an actual beta in the traditional sense." Essentially, a new product may be a final release, yet they may be called betas. There is also a general perception that firms do not charge for betas. While this may be more common, there are examples where firms have put a price to consumers using the beta versions. For example, Flickr charged $59.99 per year when it was still in beta phase (ZDNet News February 11, 2005) and Microsoft charged users $32 for testing out Windows 95 (Economist August 31, 1996). We want to analyze whether putting a price to betas is an optimal strategy. The second major question is to analyze the conditions under which a beta launch is optimal as compared to other possible strategies associated with new product introductions such as skimming or penetration pricing along with an upfront launch of the final product. In this work, we aim to provide answers to both these set of questions using an analytical model.

Many times, the revenue models of Internet based products (e.g. Gmail) employ an advertising model in place of a direct pricing model. We propose to study variations of the direct pricing models proposed earlier in this paper to analyze how beta launch decisions may be impacted due to change in the monetization approach. The rest of the paper is structured as follows. Section 2 presents the model with direct pricing for the branded product when exclusivity is important. Section 3 focuses on the branded product with monetization through advertising. Section 4 presents the model with direct pricing for the Web 2.0 based firms when network externalities rather than exclusivity are important and Section 5 extends this analysis to the advertising based revenue model. Section 6 offers concluding remarks.

2 Branded Products with Direct Pricing

2.1 The Model

We consider a two period model. In the first period, the firm takes the following two decisions in sequence:

1. Firm decides the quality (v) for the initial launch and develops it.

2. Firm decides the price (b) for the initial launch and introduces product.

This sequence reflects the fact that product quality is determined well in advance of product pricing. There are two types of customers: innovators (or experimentors) and imitators. When a new product is released, the innovators try out the product in the first period. The imitators get informed about the product details through innovators and enter the market in the second period. Let the number of imitators brought in by each innovator is x. In the second period, the firm addresses the expanded market consisting of both the innovators and imitators. In order to do so, it again takes two decisions sequentially:

1. Firm decides the additional quality (u) to be developed for the final launch, and,

2. Firm decides the price (p) for the final launch.

We take the maximum possible final product quality to be T. Thus $v \in [0, T]$ and $u \in [0, T - v]$. If the firm does not improve the quality of its product in the second period ($u = 0$), it implies that the firm entered the market with the final product right in the first period. On the other hand, if the firm improves the quality in the second period ($u > 0$), we take the first period product to be a beta launch.

Next, we define the utility functions for the innovators in periods 1 and 2 and for the imitators in period 2. In the first period, the value proposition for innovators arises from (i) using the product; this is dependent on the quality of the product, and, (ii) the value that they get because of having a "new" product which few others have. This kind of value is recognized in literature to be the value from exclusivity, or "snob effect" Pesendorfer (1995). We model the value from exclusivity to be increasing in the product quality in that period (v). If a fraction t_1 of the innovators buy the product, the simplest utility function that captures the above features is $v + v(1 - t_1)a$, where $a > 0$ captures the value of exclusivity associated with the particular brand. The more branded the firm launching the product, the higher the value of a will be. By the second period, the product is no longer new and so no longer provides exclusivity. Consequently, both the innovators and imitators, have the same utility function which presents the value from using the product, $v + u$, in this period.

We assume that all customers (imitators and innovators) are heterogeneous with respect to their value of outside opportunities which are uniformly distributed in $[0, 1]$ in each period. This reflects foregone utility from other options (individual rationality). Suppose the price of initial launch is b. Then the number of innovators who try the initial launch can be found from

$$v + v(1 - t_1)a - b \geq t_1, \tag{1}$$

which reflects that fact that all innovators who derived value greater than their outside option try the product.

In the second period, the total number of potential buyers for the final product are all the innovators and xt_1 imitators. Let the price of the final product be p. We assume that price discrimination between imitators and innovators is not possible. Further, those who bought the product in the first period can no longer use it in the second period. This may happen due to first period subscription for a web site expiring in the second period, or the functionality of a software being turned off after a pre-specified time, as is common with trial versions. The innovators who buy the final product can be found from

$$v + u - p \geq t_2, \tag{2}$$

where t_2 is the fraction of innovators who buy the product. Similarly, the fraction of imitators who try the final product is found from

$$v + u - p \geq t_3. \tag{3}$$

Thus the numbers of imitators who buy the final product are $t_3 x t_1$.

We now define the profit function of the firm. To do this, we first look at the revenues. In the second period, since there are t_2 innovators and $t_3 x t_1$ imitators buying the product, the revenue function is given by

$$R_2 = p(t_2 + t_3 x t_1). \tag{4}$$

Similarly, in the first period, since there are t_1 innovators buying the product, the revenue function is given by

$$R_1 = b t_1. \tag{5}$$

Next, we look at the fixed and marginal costs associated with the product. We assume that the marginal cost of a new copy of the software is 0. However, there is a cost involved in R&D to produce the initial release and the subsequent final release. We assume that these costs are linearly increasing in product quality. Thus product development costs in the first period are Fv. We further assume that the feedback obtained from innovators who use the initial release can be used as an additional input to improve the final product (if the initial product does not incorporate full functionality). We model this by assuming that the cost of the additional functionality incorporated in the final product is reduced from Fu to $(F - Jt_1)u$. This reflects that the larger the number of innovators who try out the beta, the better the feedback, and hence the higher the cost savings. Note that we assume the cost of product development to be linearly increasing in quality, unlike other literature where cost of product development is taken to be convex in quality. Our cost structure captures the essence of the nature of R&D costs and at the same time helps us to focus on the split of quality development between the beta release and the final product.

The profit functions for the firm in the second period can therefore be written as follows:

$$\pi_2 = pt_2 + p(t_3xt_1) - (Fu - Jt_1u). \tag{6}$$

The total profit for the firm in both periods is given by the sum of the profits for the two periods, i.e.

$$\pi_1 = (bt_1 - Fv) + \pi_2, \tag{7}$$

where the first term in paranthesis reflect the profit in the first period.

2.2 Analysis and Results

We are mainly interested in the following: first, what are the characteristics of a public beta launch (functionality, price); and second, when is launch of public beta optimal. Note that, instead of pre-committing to a full quality final launch upfront, the firm decides on the quality increment (if any) after the initial launch with a view to maximizing profits in final launch. We perform the analysis assuming that the maximum quality the firm can go to is T, and take v and u as the qualities for initial release and additional quality in final release, respectively. To solve the firms profit maximization problem, we will use the standard backward induction technique.

We first report on the fraction of the market captured by the firm in periods 1 and 2.

LEMMA 1. *The market fraction captured by the firm are given as follows:*

$$t_1 = \frac{-b + v(1+a)}{1+av} \tag{8a}$$

$$t_2 = -p + u + v \tag{8b}$$

$$t_3 = -p + u + v. \tag{8c}$$

In addition,

$$v - 1 \le b \le v(1+a) \tag{9a}$$

$$(u + v - 1) \le p \le (u + v). \tag{9b}$$

Our next result characterizes the possible equilibrium product introduction and pricing strategies for the firm. We introduce some simplifications in the model at this point. First, we restrict the maximum technically feasible quality of the product, $T = 1$. This restriction implies that the outside opportunity for some customer matches the direct utility (utility without the exclusivity value) from using the product. In other words, the product is not "too superior" to the other options available in the market. Second, we also restrict the number of imitators bought in by each innovator so that $x < 4(1+a)$. This ensures that the market remains uncovered in period 1 ($t_1 < 1$).

PROPOSITION 1. *If the firm decides to introduce a product, it has the following options:*

1. *It can introduce the product with the technically feasible quality right in the first period so that* $v = 1$, $b = \frac{1+a}{2} - \frac{x}{8}$, $u = 0$, $p = \frac{1}{2}$.
2. *It can introduce the product with a quality less than the technically feasible quality in the first period and then improve the product to the technical feasible quality in the second period so that:*

   A. *If* $J > \sqrt{1+a}$, $v = v_*$, $b = 0$, $u = 1 - v_*$ *and* $p = \frac{1}{2}$.

   B. *If* $1 < J < \sqrt{1+a}$

   - **i.** $x > \frac{4(1+a)(1+a-J^2)}{aJ}$, $v = v_*$, $b = 0$, $u = 1 - v_*$ *and* $p = \frac{1}{2}$.
   - **ii.** $x \le \frac{4(1+a)(1+a-J^2)}{aJ}$, $v = v_{th1}$, $b = 0$, $u = 1 - v_{th1}$ *and* $p = \frac{1}{2}$.

   C. *If* $J < 1$, $v = v_{th1}$, $b = 0$, $u = 1 - v_{th1}$ *and* $p = \frac{1}{2}$.

where

$$v_{th1} \triangleq \frac{4J + x}{4(1 + a + J)} \tag{10a}$$

$$v_* \triangleq \frac{-2J + \sqrt{4J^2 + 4aJ^2 + aJx}}{2aJ}. \tag{10b}$$

The above result tells us that one possible strategy is for the firm to introduce the product in the first period after developing it fully. In this case, the firm may price the product differently in the first and second periods. The next corollary reports an interesting finding regarding the relative pricing of the full product in the two periods.

COROLLARY 1. *If $x < 4a$, $b > p$, i.e. the firm follows a "skimming" strategy while if $4a < x < 4(1 + a)$, the firm follows a "penetration" pricing strategy.*

Thus we find that if the word of mouth is very strong, the firm prefers to price low in the first period and then increase its price. On the other hand, it prefers to go from a high price to a low price in the second period if the word of mouth effect is weak. The threshold on the word of mouth effect is dependent upon the value of exclusivity, a. Thus, increase in customer utility due to exclusivity increases the propensity to do skimming pricing.

The other possible strategy for the firm is to introduce the product in the first period after developing it only partially (beta strategy). The interesting finding here is that such partial quality launches must be done at a first period price of zero. This is very much in confirmation with the anecdotal evidence since we see most beta launches to be free. This also points out that some of the real world examples that are called beta but have a price tag attached may actually be closer to a high quality initial launch.

In choosing between the full quality launch and the beta launch, the firm considers the tradeoffs between savings accrued in the product development process (advantage of beta launch) and the revenue benefits of selling the product in the first period and the potentially larger market mobilization in the second period (advantage of full quality launch). We now provide some numerical example to illustrate the nature of these two equilibria.

Figure 2.2 illustrates firm's profits with the beta launch and full quality launch policies. The parameter values are $a = 1, F = 2, x = 7$ and J is varied from 1.1 to 1.414. The beta launch is optimal with $v = v_*$ in this parameter set. It can be seen that profits with beta launch dominate the full quality launch profits. Thus this example shows that beta launch can be the equilibrium policy under some situations.

Figure 2.2 illustrates the impact of increase in the value of exclusivity on the firm's profits with the beta launch and full quality launch policies. The parameter values are $F = 2, J =$

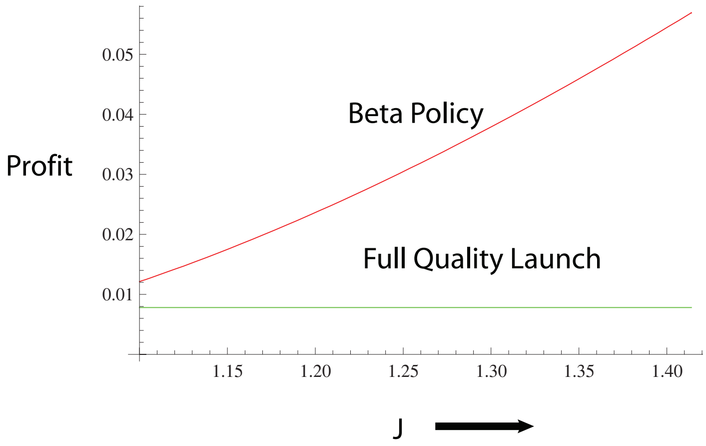

Figure 1: Beta launches can be an equilibrium policy

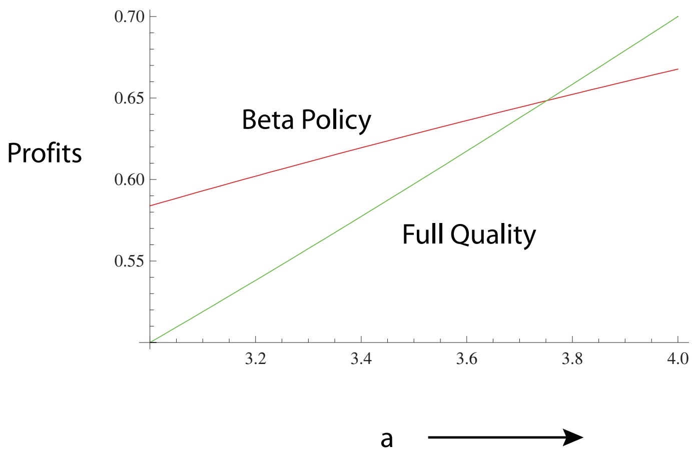

Figure 2: Impact of Exclusivity

$1.8, x = 8$ and a is varied from 3 to 4. The beta launch is optimal with $v = v_*$ in this parameter set. It can be seen that profits with the full quality launch increase beyond the profits with beta launch as a increases. Thus exclusivity can move the choice of the product launch strategy in favor of the full quality launch.

Figure 2.2 illustrates the impact of increase in the word of mouth effect on the firm's profits with the beta launch and full quality launch policies. The parameter values are $F = 2, J = 1.39, a = 3$ and x is varied from 10.25 to 10.29. The beta launch is optimal with $v = v_*$ in this parameter set. It can be seen that profits with the beta launch overtake the profits with full quality launch as x increases. Thus stronger word of mouth effects seem to favor beta

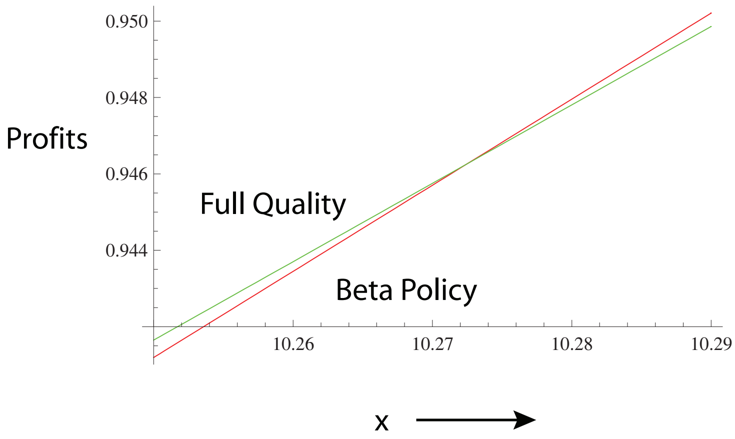

Figure 3: Impact of word of mouth effects

launches.

3 Branded Products with Monetization Through Advertising

In this section, we look at the firm's product launch options where the firm uses advertising for monetization rather than direct pricing. The model is similar to the one used in Section 2 except that the firm does not set prices for the launches. Instead the firm is realizes an exogenously given value g per customer that it acquires. Thus, g reflects the value of an eyeball in the online advertising industry. We are again interested in characterizing the firms optimal product launch and pricing strategies.

3.1 The Model

Since the firm doesn't charge the customers, (1)-(3) may be modified to

$$v + v(1 - t_1)a \geq t_1, \tag{11}$$

$$v + u \geq t_2, \tag{12}$$

and

$$v + u \geq t_3. \tag{13}$$

Note that the above equations reflect that the customers do not suffer a cost due to the advertising shown to them. The typical advertisements load very quickly with the availability of broadband and so we assume that this imposes only negligible delay costs on the consumers.

The firm takes decisions in the following sequence:

1. Firm decides the quality (v) for the initial launch and develops it.
2. Firm decides the additional quality (u) to be developed for the final launch and develops the final product.

These decisions are taken in the first and second period, respectively.

Next, given that the firm derives value g for every customer acquired, the profit functions in the two periods can be written as

$$\pi_2 = g[t_2 + t_3 x t_1] - (F - J t_1)u \tag{14}$$

and

$$\pi_1 = (g t_1 - F v) + \pi_2, \tag{15}$$

where the first two terms reflect the profit in the first period.

3.2 Analysis and Results

We first look at the equilibrium values of various market fractions.

Lemma 2. *The equilibrium values of various fractions are given as follows:*

$$t_1 = \frac{v(1+a)}{1+av} \tag{16a}$$

$$t_2 = u + v \tag{16b}$$

$$t_3 = u + v. \tag{16c}$$

To solve the firms profit maximization problem, we will use the standard backward induction technique. Our next result identifies the possible product launch strategies for the firm.

PROPOSITION 2. *In equilibrium, the possible options for the firm are as follows*

1. *If $J > \frac{g(1+x)}{1+a}$, the firm prefers a beta launch with $v = v_{opt}$ and $u = 1 - v$.*
2. *If $J \leq \frac{g(1+x)}{1+a}$, the firm does a full quality product launch.*

where

$$v_{opt} \triangleq \frac{-J + \sqrt{agJ + J^2 + aJ^2 + agJx}}{aJ}. \tag{17a}$$

Thus if the benefit from reduction of product development costs are large enough ($J > \frac{g(1+x)}{1+a}$), the firm prefers a beta launch. The threshold is increasing in word of mouth effect and decreasing in the value of exclusivity. Thus strong word of mouth effects make a beta strategy unlikely, whereas a strong exclusivity makes a beta launch strategy more likely.

Next, we are interested in exploring the differences between the impact of value of exclusivity (a) and word of mouth effect (x) on the product launch strategies with the advertising and the direct pricing model. We represent profit with beta launch as π_β^A and with full quality launch as π_{full}^A. We find that $\frac{\partial \pi_\beta^A}{\partial a} > 0$ and $\frac{\partial \pi_{full}^A}{\partial a} = 0$. Thus, while exclusivity increases the profits with beta launches, it has no impact with full quality product launch, unlike the direct pricing model. This result points to the increased importance the firm must attach to savings in product development from user feedback if it wants to take advantage of its brand image (and corresponding exclusivity). We also find that π_β^A and π_{full}^A are both increasing in x. This effect is thus similar to the situation with direct pricing.

We now want to explore how the relative qualities in beta launches differ with the direct pricing and advertising models. The following result answers this question.

PROPOSITION 3. *If the firm prefers the advertising model with a beta launch over the direct pricing model with beta launch, it must be that $v_{opt} > v_*$.*

Clearly, then, the firm must invest much more upfront to develop its beta product if it is utilizing the advertising model to monetize its web site. This happens because g must be large enough for the profits with advertising to be higher. At these value of g, the marginal revenue from first period quality is higher for the beta. Hence the firm prefers to have a larger value of v.

4 Web 2.0 Products with Direct Pricing

In this section, we look at the beta launch under an externalities based model. As discussed in the Introduction, this model captures the product launch incentives of Web 2.0 based web sites where a lot of product value is generated through user participation (e.g. Flickr). The details of the model are similar to the one used in Section 2 except for the following differences. First, in both periods, the value of a product depends positively on other users in the systems. This is a case of positive externality, as opposed to the negative externality experienced by innovators in the the first period in Section 2. Second, the extent of the positive externality depends on the variety of inputs provided by the users. For example, in the context of Flickr, the more the different types of pictures, say of marine life and mountain portraits, (as opposed to merely the number of pictures) that are uploaded and tagged, the more the value created for the users. This is in line with the long tail argument that users get value when a variety of their interests are served rather when they get more of the same.

We are now interested in the following questions. First, what are the characteristics of a beta launch, and second, is a public beta launch optimal.

4.1 The Model

As already discussed, we assume that the positive externalities depend on the variety of user interests (and not just numbers). The variety brought about by innovators is different from the variety brought about by imitators. This captures the realistic situation that the potential market of imitators is bigger and hence the variety that they bring to the table is greater. We normalize the value of variety from imitators to just the fraction of the imitator market that is covered (t_3). Correspondingly, the value of variety created by innovators is dt_1 and dt_2 in periods 1 and 2, where $d < 1$. In this case, (1)-(3) may be modified to

$$v + vdt_1a - b \geq t_1, \tag{18}$$

$$(v + u)[1 + (dt_2 + t_3)a] - p \geq t_2, \tag{19}$$

and

$$(v + u)[1 + (dt_2 + t_3)a] - p \geq t_3. \tag{20}$$

The profit functions remain unchanged from Section 2. Also note that hereon we use the parameter a to represent the strength of externalities rather than exclusivity (as in the earlier sections).

4.2 Analysis and Results

The firm takes the same sequence of decisions as in Section 2.

We first look at the equilibrium values of various fractions.

LEMMA 3. *The equilibrium values of various fractions are given as follows:*

$$t_1 = \frac{b - v}{-1 + adv} \tag{21a}$$

$$t_2 = -\frac{p + u + v}{-1 + a(1 + d)(u + v)} \tag{21b}$$

$$t_3 = -\frac{p + u + v}{-1 + a(1 + d)(u + v)}. \tag{21c}$$

In addition, when $b < v$ and $p < (u + v)$,

$$a * d * v < a(d + 1)(u + v) < 1 \tag{22}$$

Proof. The proof is along the same lines as Lemma 1. For marginal customers, (18)-(20) are satisfied with equalities. Solving the three simultaneous equations, we get (21). □

We next look into possible strategies for the firm. Though we do not completely characterize the firm strategies in terms of available parameters, we identify a significantly reduced set of strategies, which include public beta launches. Our main result is as follows

Proposition 4. *In equilibrium, the possible options for the firm are as follows.*

- *If $a > a_*$ then $b = 0$. In addition, the possible strategies are:*
  - $v = v_{th0},\ u = 0,\ p = \frac{v_{th0}}{2}$.
  - $v = T,\ u = 0,\ p = \frac{T}{2}$.
  - $v = v_{**},\ u = T - v_{**},\ p = \frac{T}{2}$,
- *If $a < a_*$ then $p = \frac{T}{2}$. In addition, the possible strategies are:*
  - $v = T,\ b = \frac{T}{8}(4 - \frac{Tx}{1-a(1+d)T}),\ u = 0$.
  - $v = v_{thT},\ b = 0,\ u = T - v_{thT}$.
  - $v = v_{**},\ b = 0,\ u = T - v_{**}$,

where

$$
\begin{aligned}
a_* &= \frac{1}{(1+d)}(\frac{1}{T} - \frac{x}{4}) \\
v_{th0} &= \frac{4}{4a(1+d)+x} \\
v_{thT} &= \frac{T(4J(-1+a(1+d)T) - Tx)}{4(1+J)(-1+a(1+d)T)} \\
v_{**} &= \frac{2J(-1+a(1+d)T) + \sqrt{J(-1+a(1+d)T)(-4J(-1+adT)(-1+a(1+d)T) + adT^2x)}}{2adJ(-1+a(1+d)T)}.
\end{aligned}
$$

Note that the strategy $v = v_{**},\ u = T - v$ corresponds to the traditional beta launch: a firm would release a product with full functionality at the end, with an optimal decision on the first period quality.

The following example demonstrates that beta launches may turn out to be the optimal policy under certain parameter conditions even when the utility function of customers is driven by positive externalities rather than exclusivity.

Example 1. *When $T = 1,\ F = 1,\ J = 10,\ x = 1,\ d = 0.1$, we get the following. When $a < a_* = 0.6818$ (Figure 4), $\pi_{uTb0v}(v = v_{**},\ b = 0,\ u = T - v_{**}, p = T/2)$ is irrelevant (the relevant profit function is convex), $\pi_{uTb0vth}(v = v_{thT},\ b = 0,\ u = T - v_{thT},\ p =$*

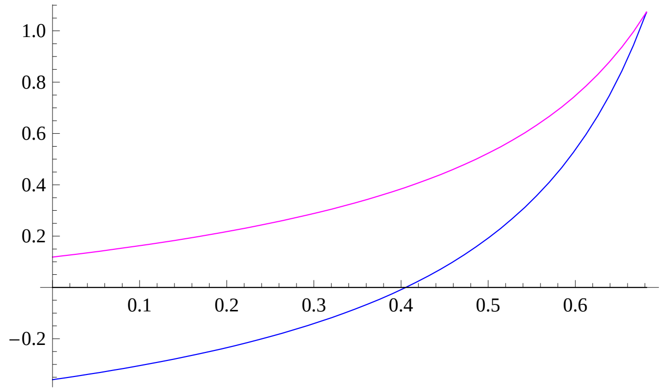

Figure 4: The plot of various profits versus a ($a < a_*$) in Example 1. The color coding is as follows. BLUE: $\pi_{u0bvT}(v = T,\ b > 0,\ u = 0,\ p = T/2)$; PINK: $\pi_{uTb0vth}(v = v_{thT},\ b = 0,\ u = T - v_{thT},\ p = T/2)$.

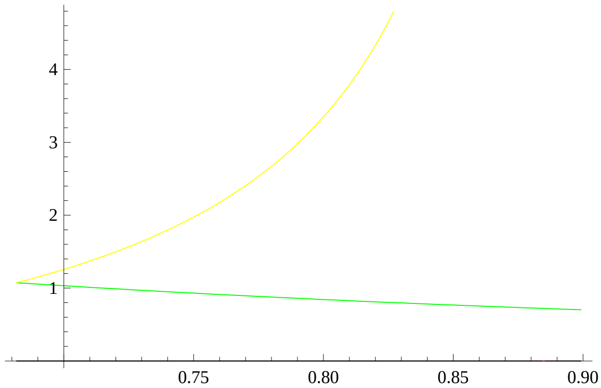

Figure 5: The plot of various v versus a ($a > a_*$) in Example 1. The color coding is as follows. GREEN: $\pi_{u0b0vth}(v = v_{th0},\ b = 0,\ u = 0,\ p = v_{th0}/2)$; YELLOW: $\pi_{u0b0vT}(v = T,\ b = 0,\ u = 0,\ p = T/2)$.

$T/2)$ is larger than $\pi_{u0bvT}(v = T,\ b > 0,\ u = 0,\ p = T/2)$, and the optimum policy is $v = v_{thT},\ b = 0,\ u = T - v_{thT},\ p = T/2$. When $a > a_*$ (Figure 5), again π_{uTb0v} is irrelevant (the relevant profit function is convex), $\pi_{u0b0vT}(v = T,\ b = 0,\ u = 0,\ p = T/2)$ is larger than $\pi_{u0b0vth}(v = v_{th0},\ b = 0,\ u = 0,\ p = v_{th0}/2)$, and the optimum policy is $v = T,\ b = 0,\ u = 0,\ p = T/2$.

5 Web 2.0 Products with Advertising

In this section, we look at the beta launch under an advertising model. This model is similar to the one used in Section 4 except that the firm does not set prices for the launches. Instead the firm is realizes an exogenously given value g per customer that it acquires. Then we are interested in the following questions. First, what are the characteristics of a beta launch, and second, when is a public beta launch optimal and how does it compare to the public beta launch under the direct pricing model.

5.1 The Model

Since the firm doesn't charge the customers, (18)-(20) may be modified to In this case, (18)-(20) may be modified to

$$v + vdt_1a \geq t_1, \tag{24}$$

$$(v+u)[1+(dt_2+t_3)a] \geq t_2, \tag{25}$$

and

$$(v+u)[1+(dt_2+t_3)a] \geq t_3. \tag{26}$$

The profit functions remain unchanged from Section 3.

5.2 Analysis and Results

The firm takes decisions in the following sequence:

1. Firm decides the quality (v) for the initial launch and develops it.
2. Firm decides the additional quality (u) to be developed for the final launch and develops the final product.

These decisions are taken in the first and second period, respectively. To solve the firms profit maximization problem, we will use the standard backward induction technique.

We first look at the equilibrium values of various fractions.

LEMMA 4. *The equilibrium values of various fractions are given as follows:*

$$t_1 = \frac{v}{1 - adv} \tag{27a}$$

$$t_2 = \frac{u+v}{1 - a(1+d)(u+v)} \tag{27b}$$

$$t_3 = \frac{u+v}{1 - a(1+d)(u+v)}. \tag{27c}$$

In addition,,

$$a * d * v < a(d+1)(u+v) < 1 \tag{28}$$

Proof. (27) follow from Lemma 2 by substituting $b = 0$ and $p = 0$. Further, (28) follows from the conditions $t_1 > 0$ and $t_2 > 0$ □

We next look into possible strategies for the firm. Though we do not completely characterize the firm strategies in terms of available parameters, we identify a significantly reduced set of strategies, which include public beta launches. Our main result is as follows

PROPOSITION 5. *In equilibrium, the possible options for the firm are as follows*

- $v = T,\ u = 0,$
- $v = v_{opte},\ u = T - v_{opte},$

where $v_{opte} \triangleq \frac{J(-1+a(1+d)T)+\sqrt{-J(-1+a(1+d)T)(J(1+a^2d(1+d)T^2-a(T+2dT))+adp(-1+a(1+d)T-Tx))}}{adJ(-1+a(1+d)T)}$.

Note that the strategy $v = v_{opt},\ u = T - v$ corresponds to the traditional beta launch: a firm would release a product with full functionality at the end, with an optimal decision on the first period quality.

We next look at an illustrative example to demonstrate that under some parameter conditions, the beta launch policies can be optimal under some parameter conditions.

EXAMPLE 2. *When* $T = 1,\ F = 1,\ J = 10,\ x = 1,\ d = 1,$ *we get we get the following. In the beginning* $v_{opte} < T$, $\pi_{uTv}(v = v_{opte},\ u = T - v_{opte})$ *is the maximum, and the optimum policy is* $v = v_{opte},\ u = T - v_{opte}$. *However, when* $v_{opte} > T$, $\pi_{u0vT}(v = T,\ u = 0)$ *is the maximum, and the optimum policy is* $v = T,\ u = 0$..

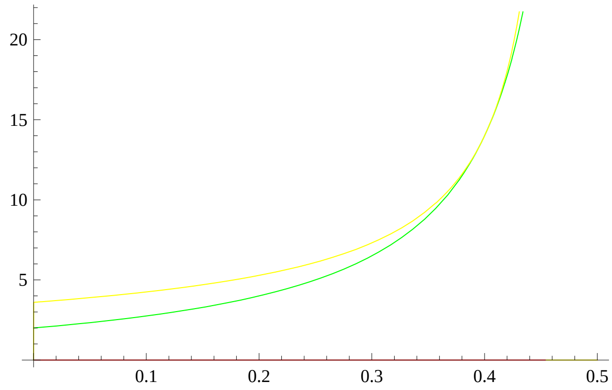

Figure 6: The plot of various π versus a in Example 2. The color coding is as follows. GREEN: $\pi_{u0vT}(v = T,\ u = 0)$; YELLOW: $\pi_{uTv}(v = v_{opte},\ u = T - v_{opte})$.

6 Concluding Remarks

The current analysis shows that with product development cost saving considerations and marketing implications, launching of public betas could be an equilibrium strategy, both for branded firms like Google who take advantage of the "snob" effect by restricting access when introducing a new product and for unbranded Web 2.0 firms who rely on network externalities to make their offering more valuable. Our analysis yields quite general insights in that it shows that beta launch strategies compete with the more common product introduction strategies through price skimming or penetration pricing. We show that beta launches must be characterized by a zero price irrespective of the type of firm. This says that most firms have got it right when they don't price their betas. Some of the rare exceptions which do attach a price to their betas may be doing it due to some specific nature of the product category.

In case of branded products, we also find that increases in the "snob" effect will increase the chances of a full quality product introduction with a skimming pricing strategy. Further, stronger word of mouth effects favor the beta launch policy over the full quality product introduction. Finally adopting a monetization model based on advertising causes the firm to increase the quality of the beta product. We are also able to show that beta launches can be optimal for Web 2.0 firms, both when they directly price their products and when they use monetization through advertising.

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Appendix

Proof of Lemma 1. The proof is straightforward. The equilibrium values will be defined by the marginal consumer: a consumer that is indifferent between using the product and his outside option. First we look at the indifferent innovator in the first period and solve $v+v(1-t_1)a-b=$

t_1 (1) to get $t_1 = \frac{-b+v(1+a)}{1+av}$. Next we look at the indifferent innovator in the second period and solve $v + u - p = t_2$ (2) to get $t_2 = -p + u + v$. Finally, we look at the indifferent innovator in the second period and solve $v + u - p = t_3$ (3) to get $t_3 = -p + u + v$.

Note that $t_1 \leq 1$ implies $b \geq v$ and $t_1 \geq 0$ implies $b \leq v + av$. Also, note that $t_2 \leq 1$ implies $p \geq u + v - 1$ and $t_2 \geq 0$ implies $p \leq u + v$. $\square$

Proof of Proposition 1. We first look at the pricing decision $p \in [0, \infty)$ in the second stage. Using the equilibrium values (8), using (6) the profit function of the second period may be written as

$$\begin{aligned}\pi_2 &= p(t_2 + t_3 x t_1) - (Fu - Jt_1 u) \\ &= \frac{-u(F + aFv + J(b - (1+a)v)) - p^2(1 - bx + vx + av(1+x))}{(1+av)} + \\ &\quad \frac{p(u+v)(1 - bx + vx + av(1+x))}{(1+av)}.\end{aligned} \tag{29}$$

Since $\frac{d^2\pi_2}{dp^2} = -\frac{2(1-bx+vx+av(1+x))}{(1+av)} < 0$ and $\frac{d\pi_2}{dp}(0) = u + v + (u+v)\frac{(-b+v+av)x}{1+av} > 0$, from (9) in Lemma 1 (i.e., $t_1 > 0 \Rightarrow av + v > b$), π_2 is concave in p and we have an interior solution from $\frac{d\pi_2}{dp} = 0$. That is,

$$p = \frac{(u+v)}{2}. \tag{30}$$

Next, we look at the quality decision $0 \leq u \leq T - v$ in the second stage. Substituting for p in (29), we get

$$\pi_{2u} = \frac{-4u(F + aFv + J(b - (1+a)v)) + (u+v)^2(1 - bx + v(a + x + ax))}{4(1+av)}. \tag{31}$$

Since $\frac{d^2\pi_{2u}}{du^2} = \frac{(1-bx+vx+av(1+x))}{2(1+av)} > 0$, from (9) in Lemma 1 (i.e., $t_1 > 0 \Rightarrow av + v > b$), π_{2u} is convex in u. Thus, in the second period, the firm will either go to the maximum possible quality (i.e., $u = T - v$) or for the minimum possible quality (i.e., $u = 0$). For now, we keep both the possibilities alive.

Next, we look at the price decision $b \in [0, \infty)$ in the first stage. Using $p = \frac{(u+v)}{2}$, the profit function in the first stage can be written as

$$\begin{aligned}\pi_1 &= (bt_1 - Fv) + \pi_2 \\ &= -Fv + \frac{b(-b+v+av)}{1+av} + \\ &\quad \frac{-4u(F+aFv+J(b-(1+a)v)) + (u+v)^2(1-bx+v(a+x+ax))}{4(1+av)}.\end{aligned} \tag{32}$$

Since $\frac{d^2\pi_1}{db^2} = -\frac{2}{(1+av)} < 0$, π_1 is concave in b. Assuming an interior solution, $\frac{d\pi_1}{db} = 0$ gives

$$b = \frac{-Ju+v+av}{2} - \frac{(u+v)^2 x}{8}. \tag{33}$$

The value of b is obviously dependent on the quality decision u in the second period. Note that firm cannot offer cash incentives for trying out beta (negative price) because it cannot discriminate between potential customers and "free-riders" who may get the beta just to get the cash that goes with it.

Before looking at the decision on the first stage quality, we derive conditions on v that cause the first period price to be zero. First consider the case when the decision is to have the maximum possible quality in the second period, i.e., $u+v=T$ and $v \leq T$. Then $b = \frac{(-JT+(1+a+J)v)}{2} - \frac{T^2x}{8}$, and $b > 0$ for $v > \frac{4JT+T^2x}{4(1+a+J)} = v_{th1}$. Next, consider the case with minimum possible quality in second period i.e. $u = 0$ and $v \leq T$. Then $b = -\frac{v(-4-4a+vx)}{8}$, and $b > 0$ for $v < \frac{4(1+a)}{x} = v_{th2}$.

We are now ready to look at the decision on the first stage quality v. We look at four cases, as follows. (Note that, for each of the these cases, the domain for v is different, to be consistent with our earlier analysis.) First consider the case when the b=0 constraint is not binding and the quality in the second period is the maximum possible, i.e., $u = T - v$ and $v_{th1} \leq v \leq T$. Substituting for b, and using $u = T - v$, the first period profit in (32) can be written as

$$\begin{aligned}\pi_{1v1} = &\frac{16(JT+v+av-Jv)^2 - 64F(T+aTv) + h^2T^4x^2}{64(1+av)} + \\ &\frac{8T^2(2+(JT+v-Jv)x+av(2+x))}{64(1+av)}.\end{aligned} \tag{34}$$

Since $\frac{d^2\pi_{1v1}}{dv^2} = \frac{(4(-1+J)+a(-4+4JT+T^2x))^2}{32(1+av)^3} > 0$, this profit function is convex in v. Hence the possible solutions are at the corners, i.e., at $v = v_{th1}$ or at at $v = T$. That is, the firm's optimal strategy is given by $v \in \{v_{th1}, T\},\ b \geq 0,\ u = T - v,\ p = \frac{T}{2}$.

Next consider the case when the $b = 0$ constraint is not binding and the quality in the second period is minimum possible, i.e., $u = 0$ and $0 \leq v \leq v_{th2}$. Substituting for b, and using $u = 0$, the first period profit in (32) can be written as

$$\pi_{1v2} = \frac{16v((1+a)^2v + (1+av)(-4F+v)) + 8(1+a)v^3x + h^2v^4x^2}{64(1+av)}. \tag{35}$$

Since $\frac{d^2\pi_{1v2}}{dv^2} = \frac{8a^3v^3(2+x)+8a(4+h^2v^3x^2+3v(2+x+vx))+2(8+3h^2v^2x^2+4h(2+3vx))+a^2(16+3h^2v^4x^2+8v^2(6+(3+v)x))}{32(1+av)^3} > 0$, this profit function is convex in v. Hence the possible solutions are at the corners, i.e., at $v = 0$ or at $v = v_{th2}$. That is, the firm's optimal strategy is given by $v \in \{0, v_{th2}\},\ b \geq 0,\ u = 0,\ p = \frac{v}{2}$.

Next consider the case when the b=0 constraint is binding and the quality in the second period is the maximum possible, i.e., $u = T - v$ and $0 \leq v \leq v_{th1}$. Substituting for $b = 0$ first, and using $u = T - v$, the first period profit in (32) can be written as

$$\pi_{1v3} = -FT + \frac{4(1+a)J(T-v)v + T^2(1+v(a+x+ax))}{4(1+av)}. \tag{36}$$

Since $\frac{d^2\pi_{1v3}}{dv^2} = -\frac{(1+a)(4J(1+aT)+aT^2x)}{2(1+av)^3} < 0$ and $\frac{d^2\pi_{1v3}}{dv^2}(0) = \frac{(1+a)T(4J+Tx)}{4} > 0$, this profit function is concave in v and has a possible non-zero solution given by $\frac{d\pi_{1v3}}{dv} = 0$. i.e., at

$$v = \frac{-2J + \sqrt{4J^2 + 4aJ^2T + aJT^2x}}{2aJ} \triangleq v_* \tag{37}$$

. However, depending on the relationship between v_* and v_{th1}, the optimum value is either at v_{th1} (when $v_* \geq v_{th1}$) or at v_* (when $v_* \leq v_{th1}$). That is, the firm's optimal strategy is given by $v \in \{v_*, v_{th1}\},\ b = 0,\ u = T - v,\ p = \frac{T}{2}$

Finally, consider the case when the b=0 constraint is binding and the quality in the second period is minimum possible, i.e., $u = 0$ and $v_{th2} \leq v \leq T$. Substituting for $b = 0$ first, and using $u = 0$, the first period profit in (32) can be written as

$$\pi_{1v4} = \frac{v[-4F + v(1 + \frac{(1+a)vx}{1+av})]}{4}. \tag{38}$$

Since $\frac{d^2\pi_{1v4}}{dv^2} = \frac{(1+3vx+a^3v^3(1+x)+3av(1+x+vx)+a^2v^2(3+(3+v)x))}{2(1+av)^3} > 0$, this profit function is convex in v. Hence the possible solutions are at the corners, i.e. at $v = v_{th2}$ or at $v = T$. That is, the firm's optimal strategy is given by $v \in \{v_{th2}, T\},\ b = 0,\ u = 0,\ p = \frac{v}{2}$.

The following analysis is now simplified taking $T = 1$. We find that $v_{th1} < 1$ requires $x < 4(1 + a)$. However, under this condition $v_{th2} > 1$. Further, $v_* < v_{th1}$ requires either $J > \sqrt{1 + a}$, or $1 < J < \sqrt{1 + a}$ and $x > \frac{4(1+a)(1+a-J^2)}{aJ}$. This leads us to the results in the statement of the proposition. □

Proof of Lemma 2. The proof follows from Lemma 1 by substituting $b = 0$ and $p = 0$. □

Proof of Proposition 2. We first look at the quality decision $0 \le u \le T-v$ in the second stage. Using the equilibrium values (16) as well as (14) the profit function of the second period may be written as

$$\begin{aligned}\pi_2 &= g(t_2 + t_3xt_1) - (Fu - Jt_1u) \\ &= \frac{(1+a)Juv - F(u+auv) + gh(u+v)(1+vx+av(1+x))}{1+av}.\end{aligned} \tag{39}$$

This is linear in u. Thus the equilibrium choice of u is either 0 or $1 - v$.

First, we look at the quality decision $u = 0$. From (16) and (15), the profit function in the first stage can be written as

$$\begin{aligned}\pi_{10} &= (gt_1 - Fv) + \pi_2 \\ &= \frac{v(g(2+a) - F(1+av) + gv(a+x+ax))}{1+av}.\end{aligned} \tag{40}$$

Since $\frac{d^2\pi_{10}}{dv^2} = -\frac{(2(1+a)g(a-x)}{1+av)^3}$, pi_{10} is convex in v if $a < x$ and concave otherwise. When $a < x$, π_{10} is maximized at the corners, i.e., either at $v = 0$ or at $v = T$. However, $v = 0$, $u = 0$ solution yields zero profits. Hence the only situation of interest is when $v = T$.

Next, when $F > F_1(v)$ and $a > x$, π_{10} is concave and the maximizer can be either at the corner or at an internal point. We need to examine the potential internal solution. Solving the first order condition $\frac{d\pi_{10}}{dv} = 0$, we get this as

$$v = -\frac{-aF + g(a + x + ax) \pm \sqrt{-(1+a)g(a-x)(gx + a(-F + g(1+x)))}}{a(gx + a(-F + g(1+x)))}.$$

Since $a > x$, we must have $(gx + a(-F + g(1+x))) < 0$. That is, $g < \frac{aF}{h(a+x+ax)} \triangleq g_i$. Then, this internal solution is given by

$$v_{fix} = -\frac{-aF + g(a + x + ax) + \sqrt{-(1+a)g(a-x)(gx + a(-F + g(1+x)))}}{a(gx + a(-F + g(1+x)))}. \quad (41)$$

With some algebraic manipulation, it is easy to see that $v_{fix} > 0$ requires $F > 2g + ag$ (at $T = 1$). However, the firm profits with v_{fix} are positive only when $F < 2g + ag$. Thus v_{fix} can never be an admissible strategy.

Next we explore the situation when $u = T - v$. From (16) and (15), the profit function in the first stage can be written as

$$\begin{aligned} \pi_{1T} &= (gt_1 - Fv) + \pi_2 \\ &= \frac{(1+a)J(T-v)v - FT(1+av) + g((1+a)v + T(1 + v(a + x + ax)))}{1 + av}. \end{aligned} \quad (42)$$

Since $\frac{d^2\pi_{1T}}{dv^2} = -\frac{(2(1+a)(J+a(g+JT+gTx))}{1+av)^3} < 0$, π_{1T} is concave in v and the maximizer can be either at the corner or at an internal point. We need to examine the potential internal solution. Solving the first order condition $\frac{d\pi_{1T}}{dv} = 0$, we get this as

$$v = \frac{-J \pm \sqrt{agJ + J^2 + aJ^2T + agJTx}}{aJ}.$$

Since the optimal solution should be positive, the only solution of interest is

$$v_{opt} = \frac{-J + \sqrt{agJ + J^2 + aJ^2T + agJTx}}{aJ} > 0. \quad (43)$$

That is, we need not look at $v = 0$ as an optimal solution in this case. Further, $v_{opt} < T$ if $J > \frac{g(1+x)}{1+a}$ (at $T = 1$).

Thus the firm will do a full quality launch when $J \leq \frac{g(1+x)}{1+a}$. When $J > \frac{g(1+x)}{1+a}$, the firm must choose between the beta launch and full quality launch strategies. Comparing the profits of the firm with these two strategies, we find that the one with beta launch dominates when $J > \frac{g(1+x)}{1+a}$. □

Proof of Proposition 3. Recall that the traditional beta launchs are made with first period quality v_{opt} and v_* under the advertising and direct pricing models, respectively. From (42), the profit under the advertising model is given by

$$\pi_{adv} = \frac{2J - 2\sqrt{J(J + a(g + JT + gTx))}}{a^2} + \frac{a(-aFT + J(2 + T + aT) + g(1 + a + aT + (1 + a)Tx) - 2\sqrt{J(J + a(g + JT + gTx))})}{a^2}.$$

Similarly, from (36), the profit under the direct pricing model is given by

$$\pi_{price} = \frac{4(1 + a)J(2 + aT) - 4\sqrt{J(4(J + aJT) + aT^2x)}}{4a^2} + \frac{a(T^2x + aT(-4F + T + Tx) - 4\sqrt{J(4(J + aJT) + aT^2x)})}{4a^2}.$$

Note that v_{opt} is increasing in g while v_* is independent of g. Further, $v_{opt} = v_*$ at $g = \frac{T^2x}{4(1+Tx)} \triangleq g_{th}$. It can be verified that $\pi_{price} - \pi_{adv} = \frac{T^2}{4+4Tx} > 0$ at $g = g_{th}$. Further, $\frac{d\pi_{adv}}{dg}(g = 0) = \frac{-(1+a)J(1+Tx)+\sqrt{J^2(1+aT)}(1+a+aT+(1+a)Tx)}{a\sqrt{J^2(1+aT)}} > 0$ and $\frac{d^2\pi_{adv}}{dg^2} = \frac{(1+a)(J+JTx)^2}{2(J(J+a(g+JT+gTx)))^{3/2}} > 0$. That is, π_{adv} is positive and is always increasing in g. Thus, if the advertising strategy were to be preferred over the direct pricing strategy (i.e., $\pi_{adv} > \pi_{price}$), it must be that $g > g_{th}$. This implies that if the advertising model is employed, it must be that $v_{opt} > v_*$. □

Proof of Proposition 4. We first look at the pricing decision $p \in [0, \infty)$ in the second stage. Using the equilibrium values (21), using (6) the profit function of the second period may be written as

$$\pi_2 = -Fu + \frac{Ju(b - v)}{-1 + adv} - \frac{p(-p + u + v)}{-1 + a(1 + d)(u + v)} - \frac{p(b - v)(-p + u + v)x}{(-1 + adv)(-1 + a(1 + d)(u + v))} \tag{44}$$

When $b < v$ and $p < u+v$, from (22), $\frac{d^2\pi_2}{dp^2} = (2(-1+adv+bx-vx))/((-1+adv)(-1+a(1+d)(u+v))) < 0$ and $\frac{d\pi_2}{dp}(0) = -\frac{((u+v)(-1+adv+bx-vx)}{(-1+adv)(-1+a(1+d)(u+v)))} > 0$. π_2 is concave in p and we have an interior solution from $\frac{d\pi_2}{dp} = 0$. That is,

$$p = \frac{(u+v)}{2}. \tag{45}$$

That is, the assumption $p < u+v$ is consistent.

Next, we look at the quality decision $0 \leq u \leq T-v$ in the second stage. Substituting for p in (44), we get

$$\pi_{2u} = -Fu + \frac{Ju(b-v)}{-1+adv} - \frac{(u+v)^2}{4(-1+a(1+d)(u+v))} - \frac{(b-v)(u+v)^2 x}{4(-1+adv)(-1+a(1+d)(u+v))}. \tag{46}$$

When $b < v$, from from (22), $\frac{d^2\pi_{2u}}{du^2} = \frac{1-adv-bx+vx}{2(-1+adv)(-1+a(1+d)(u+v))^3} > 0$. That is, π_{2u} is convex in u. Thus, in the second period, the firm will either go to the maximum possible quality (i.e., $u = T-v$) or for the minimum possible quality (i.e., $u = 0$). For now, we keep both the possibilities alive.

Next, we look at the price decision $b \in [0, \infty)$ in the first stage. Using $p = \frac{(u+v)}{2}$, the profit function in the first stage can be written as

$$\begin{aligned} \pi_1 &= (bt_1 - Fv) + \pi_2 \\ &= -Fu - Fv + \frac{b(b-v)}{-1+adv} + \frac{Ju(b-v)}{-1+adv} - \\ &\quad \frac{(u+v)^2}{4(-1+a(1+d)(u+v))} - \frac{(b-v)(u+v)^2 x}{4(-1+adv)(-1+a(1+d)(u+v))}. \end{aligned} \tag{47}$$

Since $\frac{d^2\pi_1}{db^2} = -\frac{2}{(1+adv)} < 0$, from (22), π_1 is concave in b. Assuming an interior solution, $\frac{d\pi_1}{db} = 0$ gives

$$b = \frac{-4Ju(-1+a(1+d)(u+v)) + u^2x + v^2(4a(1+d)+x) + 2v(-2+2a(1+d)u+ux)}{8(-1+a(1+d)(u+v))}. \tag{48}$$

The value of b is obviously dependent on the quality decision u in the second period. Note that firm cannot offer cash incentives for trying out beta (negative price) because it cannot discriminate between potential customers and "free-riders" who may get the beta just to get the cash that goes with it.

Before looking at the decision on the first stage quality, we derive conditions on v that cause the first period price to be zero. First consider the case when the decision is to have the maximum possible quality in the second period, i.e., $u + v = T$ and $v \leq T$. Then

$$b = \frac{-4J(-1+a(1+d)T)(T-v) + 4(-1+a(1+d)T)v + T^2x}{8(-1+a(1+d)T)} < v, \tag{49}$$

where the inequality follows from (22) (so $b < v$ is consistent), and $b > 0$ for $v > \frac{T(4J(-1+a(1+d)T)-Tx)}{4(1+J)(-1+a(1+d)T)} \triangleq v_{thT}$. Next, consider the case with minimum possible quality in second period i.e. $u = 0$ and $v \leq T$. Then

$$b = \frac{v(-4+4a(1+d)v+vx)}{8(-1+a(1+d)v)} < v, \tag{50}$$

where the inequality follows from (22) (so $b < v$ is consistent), and $b > 0$ for $v < \frac{4}{4a(1+d)+x} \triangleq v_{th0}$. Note that when $a < \frac{1}{(1+d)}(\frac{1}{T} - \frac{x}{4}) \triangleq a_*$ we get $v_{thT} < T$ as well as $v_{th0} < T$.

We are now ready to look at the decision on the first stage quality v. We first focus on the case when the quality in the second period is the minimum possible, i.e. $u = 0$. Then we have the following cases

- $a > a_*,\ 0 < v < v_{th0},\ b > 0.$
- $a > a_*,\ v_{th0} < v < T,\ b = 0.$
- $0 < a < a_*,\ 0 < v < T,\ b > 0.$

When $b > 0$, substituting for b (50) in (47), we get the first period profit as

$$\begin{aligned}\pi_{1u0b} = -v[&\frac{64F(-1+adv)(-1+a(1+d)v)^2}{64(-1+adv)(-1+a(1+d)v)^2)} \\ &+ \frac{v(32+v(16a^2(1+d)(1+2d)v+x(8+vx)-8a(6+vx+d(8+vx)))}{64(-1+adv)(-1+a(1+d)v)^2)}]\end{aligned} \tag{51}$$

From $b < v$ and (22) it can be verified that $\frac{d^2\pi_{1u0b}}{dv^2} > 0$. That is π_{1u0b} is convex in v and the optimum solution is given by $v \in \{0, v_{th0}\}$ when $a > a_*$ and by $v \in \{0, T\}$ when $a < a_*$.

We denote the eventual profits corresponding to $v \in \{0, v_{th0}, T\}$ as $\pi_{u0bv0},\ \pi_{u0bvth},\ \pi_{u0bvT}$, respectively. Similarly, when $b = 0$, substituting for b in (47), we get the first period profit as

$$\pi_{1u0b0} = \frac{v}{4}\frac{-4F + (v(1 - adv + vx)}{(-1 + adv)(-1 + a(1 + d)v))}. \tag{52}$$

From $b < v$ and (22) it can be verified that $\frac{d^2\pi_{1u0b0}}{dv^2} > 0$. That is π_{1u0b} is convex in v and the optimum solution is given by $v \in \{v_th0, T\}$ when $a > a_*$. We denote the eventual profits corresponding to $v \in \{v_{th0}, T\}$ as $\pi_{u0b0vth},\ \pi_{u0b0vT}$, respectively. It can then be easily verified that $\pi_{u0bvth} = \pi_{u0b0vth}$ (since $b = 0$ at $v = v_{th0}$), and we are left with four profits: $\pi_{u0bv0},\ \pi_{u0b0vth},\ \pi_{u0b0vT},\ \pi_{u0bvT}$.

We now look at the case when the quality in the second period is the maximum possible, i.e., $u = T - v$. Then we have the following cases

- $0 < a < a_*,\ 0 < v < v_{thT},\ b = 0.$
- $0 < a < a_*,\ v_{thT} < v < T,\ b > 0.$
- $a > a_*,\ 0 < v < T,\ b > 0.$

When $b > 0$, substituting for b (49) in (47), we get the first period profit as

$$\pi_{1uTb} = -\frac{\pi_{1uTbN}}{64(-1 + a(1 + d)T)^2(-1 + adv)}, \tag{53}$$

where $\pi_{1uTbN} = 16(-1+J)^2v^2+64FT(-1+a(1+d)T)^2(-1+adv)-32(-1+J)Tv(J+a(1+d)(-1+J)v)+T^4(-4a(1+d)J+x)^2+8T^2(2+2J^2(1+a(1+d)v(4+a(1+d)v))+v(-2ad+2a^2(1+d)^2v+x)-Jv(4a(1+d)(2+a(1+d)v)+x))-8T^3(2a^2(1+d)(-d-2(1+d)J+2(1+d)J^2)v-Jx+a(1+d)(2+4J^2+vx-Jvx))$. From $b < v$ and (22) it can be verified that $\frac{d^2\pi_{1uTb}}{dv^2} = -\frac{(-4+4J(1+a^2d(1+d)T^2-a(T+2dT))+aT(4+4d-dTx))^2}{32(-1+a(1+d)T)^2(-1+adv)^3} > 0$. That is π_{1u0b} is convex in v and the optimum solution is given by $v \in \{v_{thT}, T\}$ when $a < a_*$. We denote the eventual profits corresponding to $v \in \{v_{thT}, T\}$ as $\pi_{uTbvth},\ \pi_{uTbvT}$, respectively.

Similarly, when $b = 0$, substituting for b in (47), we get the first period profit as

$$\pi_{1uTb0} = -FT + \frac{-4Jv^2 + 4JTv(1 + a(1 + d)v) + T^2(1 - a(d + 4(1 + d)J)v + vx)}{4(-1 + a(1 + d)T)(-1 + adv)}. \tag{54}$$

Now, $\frac{d^2\pi_{1uTb0}}{dv^2} = \frac{-4J(-1+adT)(-1+a(1+d)T)+adT^2x}{2(-1+a(1+d)T)(-1+adv)^3}$, and it is possible that π_{1uTb0} either convex or concave. If convex, then the optimum solution is given by $v \in \{v_{thT}, T\}$ when $a < a_*$ and by $v \in \{0, T\}$ when $a > a_*$. If concave, then the optimum solution is given by $v \in \{0, v_{thT}, v_{**}\}$ when $a < a_*$ and by $v \in \{0, v_{**}, T\}$ when $a > a_*$, where

$$v_{**} \triangleq \frac{2J(-1 + a(1 + d)T) + \sqrt{J(-1 + a(1 + d)T)(-4J(-1 + adT)(-1 + a(1 + d)T) + adT^2x)}}{2adJ(-1 + a(1 + d)T)}$$

is the solution to $\frac{d\pi_{1uTb0}}{dv} = 0$. We denote the eventual profits corresponding to $v \in \{0, v_{thT}, v_{**}, T\}$ as $\pi_{uTb0v0},\ \pi_{uTb0vth},\ \pi_{uTb0v},\ \pi_{uTb0vT}$, respectively. It can then be easily verified that $\pi_{uTbvth} = \pi_{uTb0vth}$ (since $b = 0$ at $v = v_{thT}$), $\pi_{uTbvT} = \pi_{u0bvT}$ (when $a < a_*$), $\pi_{uTb0vT} = \pi_{u0b0vT}$ (when $a > a_*$), $\pi_{uTb0v0} \leq \pi_{u0bvT}$ and we are left with two additional profits: $\pi_{uTb0v},\ \pi_{uTbv0th}$.

Finally, it can be verified that $\pi_{u0bv0} = 0$. Overall, we need to compare three profits depending on the value of a: when $a > a_*$, we compare $\pi_{u0b0vth},\ \pi_{u0b0vT},\ \pi_{uTb0v}$; and when $a < a_*$, we compare $\pi_{u0bvT},\ \pi_{uTb0vth},\ \pi_{uTb0v}$. □

Proof of Proposition 5. We first look at the quality decision $0 \leq u \leq T-v$ in the second stage. Using the equilibrium values (27) as well as (14) the profit function of the second period may be written as

$$\begin{aligned} \pi_2 &= g(t_2 + t_3xt_1) - (Fu - Jt_1u) \\ &= -Fu - \frac{Juv(-1 + a(1 + d)(u + v)) + g(u + v)(-1 + adv - vx)}{(-1 + adv)(-1 + a(1 + d)(u + v))}. \end{aligned} \tag{55}$$

Since $\frac{d^2\pi_2}{du^2} = -\frac{2a(1+d)g(-1+adv-vx)}{(-1+adv)(-1+a(1+d)(u+v))^3} > 0$ (28), π_2 is convex in u and the optimal decision belongs to the set $u \in \{0, T - v\}$.

Next, we look at the quality decision $0 \leq v \leq T$ in the first stage. We first look at the case when the second period decision is $u = 0$. In this case the provider profit may be written as, using (27) and (55),

$$\begin{aligned}\pi_{1u0} &= gt1 + \pi 2 - Fv \\ &= v(-F + \frac{g(2 - a(1+2d)v + vx)}{(-1+adv)(-1+a(1+d)v))}\end{aligned} \tag{56}$$

Since $\frac{d^2\pi_1}{dv^2} = -\frac{(2g(-a(1+2d)+a^4 d(1+3d+4d^2+2d^3)v^3 - x + 3a^2 d(1+d)v(2+vx) - a^3 d(1+3d+2d^2)v^2(3+vx))}{1+a^2 d(1+d)v^2 - a(v+2dv))^3} > 0$ (28), π_{1u0} is convex in v, and the optimal decision is given by $v \in \{0, T\}$. We denote the eventual profits corresponding to $v \in \{0, T\}$ as $\pi_{u0v0},\ \pi_{u0vT}$, respectively.

Finally, we look at the case when $u = T - v$. In this case, the provider profits may be written as, using (27) and (55),

$$\begin{aligned}\pi_{1uT} &= gt1 + \pi 2 - Fv \\ &= \frac{\pi_{1uTN}}{(-1+a(1+d)T)(-1+adv)},\end{aligned} \tag{57}$$

where $\pi_{1uTN} = -J(-1+a(1+d)T)(T-v)v - FT(-1+a(1+d)T)(-1+adv) + g(v + T(1 - a(1+2d)v + vx))$. Now $\frac{d^2\pi_{1uT}}{dv^2} = -\frac{2(J+a^2 d(1+d)T(g+JT) - a(JT + d(g+2JT+gTx)))}{(-1+a(1+d)T)(-1+adv)^3}$. This could be positive as well as negative. That is, π_{1uT} may be convex or concave. If it is convex, the optimal decision is given by $v \in \{0, T\}$. However, if it is concave, the optimal decision may be given by the internal solution v_{opte}, which is the solution to $\frac{d\pi_{uT}}{dv} = \frac{J(a(1+d)T^2 + v(2-adv) + T(-1-2a(1+d)v + a^2 d(1+d)v^2)) + g(-1+a(1+d)T - Tx)}{(-1+a(1+d)T)(-1+adv)^2} = 0$. Then, we get $v_{opte} = \frac{J(-1+a(1+d)T) + \sqrt{-J(-1+a(1+d)T)(J+a^2 d(1+d)T(g+JT) - a(JT+d(g+2JT+gTx)))}}{adJ(-1+a(1+d)T)}$. We denote the eventual profits corresponding to $v \in \{0, v_{opte}, T\}$ as $\pi_{uTv0},\ \pi_{uTv},\ \pi_{uTvT}$, respectively.

Since the strategy $v = T,\ u = 0$ is the same as $v = T,\ u = T - v$, overall we need to compare four profits: $\pi_{u0v0},\ \pi_{u0vT},\ \pi_{uTv0},\ \pi_{uTv}$. This can be further reduced to three by noticing that $\pi_{u0vT} \geq \pi_{uTv0}$ always. $\square$