Software Innovation and the Open Source threat

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Abstract

In this paper I study how innovation investment in a software duopoly is affected by the fact that one of the firms is, or might become Open Source. Firms can either be proprietary source (PS) or open source (OS) and have different initial technological levels. An OS firm is a for profit organization whose basic software is OS and it is distributed for free. The OS firm, however, is able to make profits from selling complementary software and, on the cost side, it receives development help from a community of users. I first compare a duopoly composed by two PS firms with a mixed duopoly of a PS and OS firm and I find that a PS duopoly might generate more innovation than a mixed duopoly if the initial technological gap between firms is small. However if this gap is large, a PS duopoly generates less innovation than a mixed duopoly. I then extend the setting to allow PS firms to switch to OS or to remain PS. A PS firm wants to become OS if it gets behind enough in the technological race against a competitor. I find that the outside option to become OS might soften competition on innovation since the technological leader prefers to reduce his innovation investment to avoid the OS switch of the follower. Therefore, although the switch to OS could generate higher investment levels ex-post it might generate lower investment ex-ante. In this context I find that a government subsidy to OS firms could be potentially harmful for innovation.

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1 Introduction

An Integrated Development Environment or IDE is a software application that computer programmers use along with a programming language to develop new software. It consists of a set of tools such as a source code editor, a compiler and a debugger for which the IDE provides a single and unified user interface[1]. In 2001 an important event occurred in the IDE software market: IBM donated a $40 million worth code to found the Eclipse Open Source project[i]. The origin of this code was VisualAge[ii], a Proprietary Source IDE developed by IBM that failed to retain market share against its main competitor, Microsoft Visual Studio[iii]. The Open Source project was quite successful in gaining adoption and creating a community that contributed to the development and improvement of the software. Although Eclipse is distributed for free, IBM is still able to make profits from selling complementary goods such as IBM Rational software: a set of development tools to "extend the Eclipse Platform"[iv,v,vi]. Due to the project success, Eclipse was considered "the first IDE to seriously challenge Microsoft's popular Visual Studio"[vii,viii]. Moreover, the fact that the software is free, forced Microsoft to provide Visual Studio Shell: a basic stripped-down free version of his IDE product[ix]. The VisualAge/Eclipse story shows how "dangerous" a dying Proprietary Source project could be for a market leader if it turns into a successful Open Source project.

The IBM VisualAge/Eclipse story is an example of the increasing involvement of for profit firms in Open Source (OS) as opposed to the community initiated and managed projects[2]. Like in our example the "sponsor" firm starts the OS project by releasing valuable internally developed code and inviting a community of users to join and collaborate with the project by, for example, solving code "bugs" or helping to develop new features. The fact that the code is open usually implies that the "sponsor" firm looses the ability to make profits by charging a price for the software license. However the firm can sell complementary software or services such as additional tools, support and customization[3]. Some of these "sponsored" projects, like MySQL and JBoss, are born from scratch as OS. Others, like Eclipse or OpenOffice, were

[1] see http://en.wikipedia.org/wiki/Integrated_development_environment

[2] see West and O'Mahony (2005)

[3] Dahlander (2004) presents a detailed case study of how firms generate returns by selling a variety of products and services related to OS software.

[i] http://news.cnet.com/IBM-makes-40-million-open-source-offer/2100-1001_3-275388.html
[ii] http://en.wikipedia.org/wiki/Eclipse_(software) [iii] http://www.informationweek.com/news/global-cio/show Article.jhtml?articleID=23902341 [iv] http://www-306.ibm.com/software/rational/eclipse/
[v] http://www.technical-insight.com/my_samples/Eclipse the Competition with IBM Rational Tools.htm
[vi] http://www.builderau.com.au/news/soa/IBM-gets-Rational-with-open-source/0,339028227,320277506,00.htm
[vii] http://www.cmswire.com/cms/industry-news/the-coming-eclipse-of-visual-studio-000964.php
[viii] http://www.devsource.com/c/a/Languages/Eclipse-Behind-the-Name/ [ix] http://www.theserverside.net/news/thread.tss?thread_id=45690

originally Proprietary Source (PS) software that became OS usually after they failed to retain a significant market share against a competitor. A PS software getting behind in an innovation race against a competitor may find catching up very costly and the help provided by a OS community might reduce this cost.

Another interesting example comes from the Enterprise Resource Planning (ERP) software market. The ERP "is an integrated information system that serves all departments within an enterprise"[4]. Although some OS alternatives already exists, the market is mainly dominated by two firms offering PS software: SAP AG and Oracle[x,xi,xii]. SAP AG is the market leader, with a share that is almost three times the share of Oracle. In a 2007 interview[xiii], SAP's chief executive dismissed any Open Source threat to his company star product, despite the fact that his main competitor had been moving aggressively into OS in other software areas[xiv,xv] and could potentially decide to do the same in the ERP market. He argues that programming ERP software is unappealing and boring for OS developers since it implies, for example, dealing with legal and accounting issues:"I have never seen anyone who likes doing that. That is not fun, there is no choice. The boring bits are a strength of SAP's". In other words, the market leader thinks his product is protected against the possibility of OS competition. He believes that, as an OS firm, his competitor will not get significant development help from the community of users and therefore becoming OS will not be a profitable decision. It is clear from this example that the market leaders are aware of, and do take into consideration, their rivals trade-offs from becoming OS. A natural question that arises is what would be the leader's reaction if confronting a OS competitor becomes more likely.

The key question that emerges from both examples and that I address in this paper is how the leader's behavior is affected by the possibility of facing an OS rather than a PS competitor. The leader's incentives to innovate, for example, will probably be affected if the follower becomes OS. For instance, if the OS project succeeds to develop a community that helps to improve the software, then the leader will face a rival with a development cost advantage. Moreover, the leader's revenue will get hurt since the OS software is usually distributed for free and this will probably force the leader to lower it's own price. Then it is plausible to think that the market leader might prefer to modify his actions (i.e. slow down the technological progress of his software) in order to avoid the OS switch of the follower. In

[4] see http://encyclopedia2.thefreedictionary.com/erp

[x] http://en.wikipedia.org/wiki/List_of_ERP_vendors [xi] http://www.destinationcrm.com/Articles/ReadArticle.aspx?ArticleID=47784 [xii] http://www.destinationcrm.com/Articles/CRM-News/Daily-News/The-Dynamic-Duo-SAP-and-Oracle-Still-Lead,-But-Oracle-Offers-More-42857.aspx
[xiii] http://www.cbronline.com/article_news.asp?guid=5630C807-7CF1-4FDB-98FA-5063A87A4D33
[xiv] http://www.cbronline.com/article_news.asp?guid=5630C807-7CF1-4FDB-98FA-5063A87A4D33
[xv] http://www.erpsoftware-news.com/2006/02/oracle_vs_sap_o.html

other words, the OS threat might soften competition in the market.

In this paper I study how innovation investment in a software duopoly is affected by the fact that one of the firms is, or might become, OS. In particular I focus on how the investment of a PS market leader changes with the possibility of an OS switch by a PS follower. To this end, I build a two stage duopoly model where in the first stage firms invest in innovation and in the second stage they compete in prices. At the beginning of the game, firms are endowed with different initial levels of technology. There are two kind of consumers in the market. One group just uses a basic software and the other group also needs some extra good (additional tools, complementary software, support, etc.). In this context, an OS firm is a for profit organization whose basic software is OS. The OS firm earns no income from selling basic software licenses but is still able to have profits on the extra good. On the cost side, the OS firm, receives development help from a community of users which reduces the firm's innovation costs. I fist consider a basic setup where I compare the investment in a duopoly composed by two PS firms versus one with a PS and an OS firm. Then I extend the basic setup to allow a PS follower in the first stage to choose between becoming OS or remaining PS. In this context I analyze how the leader's investment incentives are affected by this potential switch.

With this framework I get three main results. First, if the initial technological gap between firms is small, a duopoly with two PS firms generates *more* innovation than a mixed duopoly of an PS and OS firm. However if the initial technological gap is large then a duopoly with two PS firms generate *less* innovation than a mixed duopoly. Second, in a context of a duopoly with two PS firms, the outside option for the market follower to become OS might soften competition on innovation. Therefore, although the switch to OS could generate higher investment levels ex-post it might generate lower investment incentives ex-ante. Finally, in this simplified context, I analyze the effect of a government subsidy to OS firms and I find that it might be potentially harmful for innovation.

The paper is organized as follows. Section 2 presents the model. Section 3 solves the basic setup and compares a two PS firm duopoly with a mixed duopoly. Section 3 solves the extended setup where the follower can choose between being OS or PS and analyses the effect of the OS "threat". Section 4 relates this paper with previous literature on this subject. Finally, Section 5 concludes.

2 The model

The following model adapts the framework of Sorenson (1995). Consider a horizontally differentiated duopoly à la Hotelling where each firm sells two goods: a basic software s and a complementary good t. Good t could be thought, for instance, as set of extra tools or support and customization of the basic software s. I assume maximal differentiation on the Hotelling line: firm l, the leader, is located at 0 and firm f, the follower is located at 1. There is a unit mass of consumers uniformly distributed on the line. Consumers are divided in two types:

the first group, of size $\gamma \in (0,1)$, just needs the basic software s and does not require the complementary good t. The second group, of size $(1-\gamma)$, needs to consume both software s and good t together. Both types of consumers are evenly distributed in the line and only buy one unit of the software s or one unit of the bundle $s+t$. To distinguish both groups of consumers I speak of the s and $s+t$ markets.

The net utility of a consumer who only needs good s and is located at $x \in [0,1]$ is given by

$$u_x = \begin{cases} s_l^1 - x - p_l^s & \text{if buys } s \text{ from leader,} \\ s_f^1 - (1-x) - p_f^s & \text{if buys } s \text{ from follower,} \\ 0 & \text{if does not buy.} \end{cases}$$

where s_l^1 and s_f^1 are the gross utilities derived from using the basic software and p_l^s and p_j^s are their prices. Net utility also depends linearly on the disutility consumers suffer from using a software different from their ideal variant. Therefore x (or $1-x$) represents the "distance" between the consumers ideal variant and the one the leader (or follower) is offering. Notice I assume that the disutility per unit of distance is 1.

The net utility of a consumer who needs both s and t and is located at $x \in [0,1]$ is given by

$$u_x = \begin{cases} s_l^1 - x - p_l^{s+t} & \text{if buys } s+t \text{ from leader,} \\ s_f^1 - (1-x) - p_f^{s+t} & \text{if buys } s+t \text{ from follower,} \\ 0 & \text{if does not buy.} \end{cases}$$

Again s_l^1 and s_f^1 represent gross utilities and p_l^{s+t} and p_f^{s+t} are the prices for the set of goods $s+t$. For simplicity I assume that good t does not bring any extra gross utility and does not increase the disutility per unit of distance[5].

I consider a basic and an extended setup. The basic setup consists in a two stage game where, in the first stage, firms invest in product innovation and in the second stage they simultaneously choose prices and then users choose one of the product. At the beginning of the game firms are endowed with initial levels of s_l^0 and s_f^0 defined as the pre "innovation" gross utilities each firm can provide to consumers. I define $g_0 = s_l^0 - s_f^0$ as the pre-innovation technological gap. I assume $g_0 > 0$; there exists an initial technological *advantage* for the leader and an initial technological *disadvantage* for the follower. This assumption is a shortcut to the idea that firms were previously competing in the market with uneven development success. Following this interpretation, firms invest in the first stage of the game to increase their demand from a new inflow of customers arriving in the second stage.

Investment in innovation increases the gross utility that future users will derive from buying their software. The post-innovation technological gap $g_1 = s_l^1 - s_f^1$ could be higher or lower

[5] If we assume that the good t increases the disutility per unit of distance then it increases the horizontal differentiation and the price each firm can charge for the bundle $s+t$. This however will not change the main results of the paper.

than the initial g_0 depending on firms relative investment expenditures. I assume a Stackelberg framework for the first stage: firm l, the leader, chooses investment first. Following our examples, I consider that the leader is always a PS firm while the follower can be either PS or OS. A "PS duopoly" refers to a situation where both firms are PS and a "mixed duopoly" refers to a situation in which the leader is PS and the follower is OS.

Investment costs differ depending on whether the firm is OS or PS. If the firm is PS (leader or follower), to increase s_i^0 to $s_i^1 = s_i^0 + I_i$ costs

$$C(I_i) = \frac{(I_i)^2}{2}. \tag{1}$$

On the other hand, as we have seen in the Eclipse example, the OS project can receive development help from a community of user-developers. This development help is captured in the model by assuming that it reduces innovation costs for the OS firm. Therefore, if the follower is OS, to increase s_f^0 to $s_f^1 = s_f^0 + I_f$ costs

$$C(I_f) = \begin{cases} 0 & \text{if } I_f \leq I_H, \\ \frac{(I_f - I_H)^2}{2} & I_f > I_H. \end{cases} \tag{2}$$

where I_H is a positive constant that represents the development help provided by a community of user-developers.

This cost formulation features the idea that community help reduces the marginal cost of innovation for low values of I_f, however this help is limited so the reduction in marginal cost converges to 0 as I_f increases. As we mentioned before, if firms were previously competing in the market, this development help can be thought as coming from consumers that are using preceding versions of the software. If the software code is open and distributed for free, users of past versions of the software, along with future new customers, benefit from the software progress. The fact that I_H is constant simplifies the mathematics of the problem. An possible alternative formulation is to assume that the level of help depends on the initial technological gap g_0. This is a shortcut to the idea that help is proportional to past levels of demand and therefore on the past amount of users of the software. This alternative assumption does not change our main results.

Since investing $I_f < I_H$ is costless, the OS follower invests $I_f > I_H$ as long as it is profitable to do so. Otherwise the minimum level of investment for the OS project is I_H. Behind this assumption is the idea that, if the OS follower exits the market and abandons the software development, the community of user-developers invest I_H anyway and the project will at least progress I_H[6]. We could think that users of old versions always find it profitable to improve the software for their own use, even if the OS follower exits the market and new customers will only buy from the market leader.

[6] This assumption is not essential for the main results of the paper but simplifies some computations. It will only affect the profit of the leader when it becomes a constrained monopolist. We could alternatively assume that if the OS follower exits the market, all development of the project is abandoned.

In the second stage the two firms choose their prices simultaneously. At this point we find a second difference between an OS and a PS firm. The OS firm is a for profit organization whose basic software s is OS. The fact that the source code is open usually reduces the ability to have revenue by directly charging a positive price for the software licence. Therefore, if the firms is OS I assume that $p_f^s = 0$. However, like in the VisualAge-Eclipse example, OS firms can still earn profit on complementary PS software and services. I capture this by assuming that the OS follower can still choose a positive p_f^{s+t} in the second stage[7]. Throughout the analysis I assume that s_l^1 and s_f^1 are large enough that the market is covered.

While in the basic setup the OS or PS status of the follower is not decided by the firm, in the extended setup this choice is allowed: in the first stage, after observing the leaders investment decision, a PS follower first decides whether to remain PS or become OS and then it chooses the level of investment I_f. If the PS follower chooses to become OS it forgoes income in the basic software market ($p_f^s = 0$) but gains development help I_H from the user-developers that reduces its innovation costs.

3 Analysis of the Basic Setup

In this section I analyze and compare the basic setup for both, a duopoly composed by two PS firms and a mixed duopoly composed by a PS leader and a OS follower. I solve the game by backward induction, obtaining first the equilibrium prices of the second stage and then solving the investment levels of the first stage.

3.1 Stage two: market game

At stage two, the post-innovation technological gap $g_1 = s_l^1 - s_f^1$ is given. For simplicity I assume that the marginal cost of producing both s or t is 0.

3.1.1 PS duopoly

Since marginal costs are zero, when both firms are PS they face the following revenue maximization problems:

$$\max_{p_l^s, p_l^{s+t}} \pi_l = p_l^s \left[\gamma \frac{1 + p_f^s - p_l^s + g_1}{2} \right] + p_l^{s+t} \left[(1-\gamma) \frac{1 + p_f^{s+t} - p_l^{s+t} + g_1}{2} \right]. \tag{3}$$

$$\max_{p_f^s, p_f^{s+t}} \pi_f = p_f^s \left[\gamma \frac{1 + p_l^s - p_f^s - g_1}{2} \right] + p_f^{s+t} \left[(1-\gamma) \frac{1 + p_l^{s+t} - p_f^{s+t} - g_1}{2} \right]. \tag{4}$$

[7] In order to keep things simple we assume that the fact that the code is open has not brought any additional competition to the OS firm for the good t. A possible way to account for the potential competition is to introduce limit pricing. If we assume that $\widetilde{p}_{f\prime}^t > 0$ is the price at which a firm $f\prime$ located at 1 enters the t market, then the firm f must charge a price $P_f^t \leq \widetilde{P}_{f\prime}^t$ to avoid entry.

The expressions in square brackets in (3) and (4) are the usual demands functions obtained by assuming that the market is covered and locating the consumer who is indifferent between buying from the leader or the follower. These demands are expressed in terms of the post-innovation gap $g_1 = s_l^1 - s_f^1$.

Computing the reaction functions and solving the system we get the following equilibrium prices for the two PS firms case:

$$\widehat{p}_l^s = \widehat{p}_l^{s+t} = 1 + \frac{g_1}{3}, \quad \widehat{p}_f^s = \widehat{p}_f^{s+t} = 1 - \frac{g_1}{3}.$$

The prices of the basic software s and of the bundle $s+t$ are the same since the complement good t does not bring any extra product differentiation and is produced at zero marginal cost. We could think that in this market $s+t$ bundles are sold to everyone and some users just do not use the t tools. A positive level of post-innovation gap g_1 (the leader provides a higher gross utility) implies that the leader is able to charge a higher price than the follower. Substituting the equilibrium prices in the revenue function we obtain, for $g_1 \leq 3$

$$\overline{\pi}_l^{ps} = \gamma \frac{\left(1 + \frac{g_1}{3}\right)^2}{2} + (1-\gamma) \frac{\left(1 + \frac{g_1}{3}\right)^2}{2} = \frac{1}{2}\left(1 + \frac{g_1}{3}\right)^2. \tag{5}$$

$$\overline{\pi}_f^{ps} = \gamma \frac{\left(1 - \frac{g_1}{3}\right)^2}{2} + (1-\gamma) \frac{\left(1 - \frac{g_1}{3}\right)^2}{2} = \frac{1}{2}\left(1 - \frac{g_1}{3}\right)^2. \tag{6}$$

Revenues are increasing in g_1 for the leader and decreasing for the follower.

For $g_1 > 3$ the follower exists and the leader becomes a constrained monopolist for both the s and the $s+t$ markets. In this case the Nash equilibrium prices are

$$\widehat{p}_f^s = \widehat{p}_f^{s+t} = 0, \quad \widehat{p}_l^s = \widehat{p}_l^{s+t} = g_1 - 1.$$

Notice that the equilibrium requires that the leader serves all the market. With these prices the leader obtains the following revenue

$$\overline{\pi}_l^{cm} = g_1 - 1. \tag{7}$$

3.1.2 Mixed duopoly

When the leader is PS and the follower is OS, firms face the following maximization problems:

$$\max_{p_l^s, p_l^{s+t}} \pi_l = p_l^s \left[\gamma \frac{1 - p_l^s + g_1}{2}\right] + p_l^{s+t} \left[(1-\gamma) \frac{1 + p_f^{s+t} - p_l^{s+t} + g_1}{2}\right], \tag{8}$$

$$\max_{p_f^{s+t}} \pi_f = p_f^{s+t} \left[(1-\gamma) \frac{1 + p_l^{s+t} - p_f^{s+t} - g_1}{2}\right]. \tag{9}$$

Notice that the revenue functions in (8) and (9) are the same as (3) and (4) in the PS duopoly but assuming that $p_f^s = 0$. The fact that the rival can not charge a positive price for the basic software s has reduced the leader's demand for any p_l^s. Computing the reaction functions and solving the system we get the following equilibrium prices for the mixed duopoly case:

$$\widehat{p_l^s} = \frac{1}{2} + \frac{g_1}{2}, \ \ \widehat{p_l^{s+t}} = 1 + \frac{g_1}{3}, \ \ \widehat{p_f^t} = 1 - \frac{g_1}{3}.$$

The leader no longer charges the same price for the software s and the bundle $s+t$: since prices are strategic complements and $p_f^s = 0$, it charges a lower price for the basic software. Substituting the equilibrium prices in the revenue function we obtain

$$\overline{\pi}_l^{mix} = \gamma \frac{\left(\frac{1}{2} + \frac{g_1}{2}\right)^2}{2} + (1-\gamma) \frac{\left(1 + \frac{g_1}{3}\right)^2}{2}, \tag{10}$$

$$\overline{\pi}_f^{mix} = (1-\gamma) \frac{\left(1 - \frac{g_1}{3}\right)^2}{2}. \tag{11}$$

Not only the follower's revenue has decreased but also the leader's revenue from the s market is smaller. However the revenue difference with the two PS case decreases as g_1 approaches to 3. Therefore *an OS follower is particularly harmful for industry revenue when the post-innovation gap is small* (when qualities offered by both firms are similar). Notice also that the fact that the follower is OS has not affected the revenue coming from the $s+t$ market for both firms.

Like in the two PS case the equilibrium prices are valid as long as $g_1 \leq 3$. For $g_1 > 3$ the follower exists and the leader becomes a constrained monopolist for both the s and the $s+t$ markets. The Nash equilibrium prices are

$$\widehat{p_f^s} = \widehat{p_f^{s+t}} = 0, \ \widehat{p_l^s} = \widehat{p_l^{s+t}} = g_1 - 1.$$

and the leader obtains the following revenue

$$\overline{\pi}_l^{cm} = g_1 - 1.$$

3.2 Stage one: Innovation

At stage one, firms invest to increase the gross utility users derive when buying each software. Given that $s_i^1 = s_i^0 + I_i$ for $i = \{l, f\}$ then the post innovation gap is

$$\begin{aligned} g_1 &= s_l^0 + I_l - s_f^0 - I_f, \\ &= g_0 + I_l - I_f. \end{aligned}$$

Since I assume a Stackelberg framework to solve the model we first compute the follower's reaction function and then solve the leader's level of investment.

3.2.1 PS duopoly

When the PS follower chooses his level of investment, the leader has already decided on I_l. From (6) and (1) the PS follower chooses I_f to maximize profit

$$\max_{I_f} \Pi_f = \begin{cases} \frac{1}{2}\left(1 - \frac{g_0+I_l-I_f}{3}\right)^2 - \frac{(I_f)^2}{2} & \text{if } g_0 + I_l - 3 \leq I_f, \\ \\ -\frac{(I_f)^2}{2} & \text{if } g_0 + I_l - 3 > I_f. \end{cases} \tag{12}$$

Solving (12) we derive the reaction function of the PS follower for a given g_0 and I_l.

$$R^{ps}(g_0 + I_l) = \begin{cases} \frac{3}{8} - \frac{1}{8}(g_0 + I_l) & \text{if } g_0 + I_l \leq 3, \\ \\ 0 & \text{if } g_0 + I_l > 3. \end{cases} \tag{13}$$

The followers innovation investment is decreasing both in the pre-innovation technological gap $g_0 = s_l^0 - s_f^0$ and in the rival's investment level I_l (I_f and I_l are strategic substitutes). We can see from (12) that both a higher g_0 (initial technological disadvantage) and a higher I_l decrease the marginal revenue from innovation.

If $g_0 + I_l > 3$ the PS follower finds investment non profitable so $R^{ps} = 0$. Since $g_1 = g_0 + I_l > 3$ the firm will have no demand in stage two so it exits the market and the leader becomes a constrained monopolist.

Given the reaction function (13), the leader chooses its investment level by maximizing profits. The profit function depends on whether the leader is a constrained monopolist or not. The leader becomes a constrained monopolist if $I_l > 3 - g_0$. From (5), (7) and (1) the leader's problem is given by:

$$\max_{I_l} \Pi_l = \begin{cases} \frac{1}{2}\left(1 + \frac{g_0+I_l-R^{ps}}{3}\right)^2 - \frac{(I_l)^2}{2} & \text{if } I_l \leq 3 - g_0, \\ \\ g_0 + I_l - 1 - \frac{(I_l)^2}{2} & \text{if } I_l > 3 - g_0. \end{cases} \tag{14}$$

Solving (14) we find the leader's optimal level of investment when both firms are PS, $\widehat{I}_l^{ps}(g_0)$, which is a function of the initial technological gap g_0

$$\widehat{I}_l^{ps}(g_0) = \begin{cases} \frac{9}{55}g_0 + \frac{21}{55} & \text{if } g_0 \leq \overline{g}_0, \\ \\ 1 & \text{if } g_0 > \overline{g}_0. \end{cases} \tag{15}$$

where

$$\overline{g}_0 = 2.1297.$$

The value $\overline{g}_0$ is the threshold point at which the leader decides to become a constrained monopolist. It is obtained by equalizing the leader's maximum profit as a duopolist with the profit as a constrained monopolist subject to the restriction that $I_l > 3 - g_0$.

The leader investment is increasing in the pre-innovation gap, since a higher g_0 increases the marginal revenue of innovation. At $\overline{g}_0$ the investment function $\widehat{I}_l^{ps}(g_0)$ presents a discontinuity since $\widehat{I}_l^{ps}$ "jumps" from 0.73 to 1 to induce the follower's exit from the market.

Substituting $\widehat{I}_l^{ps}$ into the reaction function of the follower (13), we obtain the follower's equilibrium investment when both firms are PS, $\widehat{I}_f^{ps}(g_0)$:

$$\widehat{I}_f^{ps}(g_0) = \begin{cases} \frac{18}{55} - \frac{8}{55}g_0 & \text{if } g_0 \leq \overline{g}_0, \\ \\ 0 & \text{if } g_0 > \overline{g}_0. \end{cases}$$

Notice that $g_1 = \frac{72}{55}g_0 + \frac{3}{55} > g_0$, so the gap increases after the innovation period and the difference $g_1 - g_0$ increases with g_0: the leader has a stronger incentive to innovate than the follower and this incentive is increasing in g_0. This finding is in line with Sorenson (1995) who uses an horizontally differentiated duopoly à la hotelling to examine the persistence of market leadership in a duopoly. Sorenson (1995) assumes, however, a Cournot game in the innovation stage. Here the fact that the leader is first mover only increases the difference $g_1 - g_0$ compared to a Cournot game.

Although $\widehat{I}_f^{ps}$ is decreasing in g_0 and $\widehat{I}_l^{ps}$ is increasing in g_0, we have that total investment in innovation

$$\widehat{I}^{ps} = \widehat{I}_f^{ps} + \widehat{I}_l^{ps} = \frac{1}{55}g_0 + \frac{39}{55}. \tag{16}$$

increases with g_0.

The following proposition summarizes the results so far

Proposition 1 *The PS leader invests more than the PS follower which increases the technological gap between firms (Sorenson 1995). There is a threshold value of pre-innovation technological gap $\overline{g}_0$ such that for $g_0 > \overline{g}_0$ the leader induces the exit of the follower. Total innovation investment, $\widehat{I}^{ps}$, is an increasing function of g_0.*

3.2.2 Mixed duopoly

The OS follower chooses its investment level by maximizing profits for a given level of g_0, I_l and help I_H. From (11) and (2) the OS follower problem is

$$\max_{I_f}\Pi_f = \begin{cases} (1-\gamma)\frac{\left(1-\frac{g_0+I_l-I_f}{3}\right)^2}{2} - \frac{(I_f-I_H)^2}{2} & \text{if } g_0+I_l-3 \leq I_f, \\ \\ -\frac{(I_f-I_H)^2}{2} & \text{if } g_0+I_l-3 > I_f. \end{cases} \quad (17)$$

Solving (17) we can derive the reaction function:

$$R^{mix}(g_0+I_l) = \begin{cases} \frac{9I_H-3\gamma+3}{\gamma+8} - \frac{1-\gamma}{\gamma+8}(g_0+I_l) & \text{if } g_0+I_l \leq 3+I_H, \\ \\ I_H & \text{if } g_0+I_l > 3+I_H. \end{cases} \quad (18)$$

The follower's innovation investment is increasing in the help provided by the community and decreasing both in the pre-innovation technological gap $g_0$and in the leader investment level I_l (I_f and I_l are strategic substitutes).

At $g_0+I_l = 3+I_H$ the OS follower finds investing non profitable and abandons software development, leaving the leader as a constrained monopolist. Notice that the OS follower exits the market for a larger value of g_0+I_l compared to the PS follower. As I mentioned in section (2) the minimum level of investment for the OS project is I_H since I assume that even if the OS follower exits the market the OS community still invests I_H to develop the software for their own use.

Comparing the reaction functions in a mixed duopoly and in a PS duopoly we obtain the following proposition

Proposition 2 *If the help provided by the user-developers, I_H, is higher than a positive threshold $\overline{I_H}$, then the level of investment of a PS follower is smaller that the level of investment of a OS follower for all positive values of pre-innovation technological gap g_0 and leader's investment I_l such that the firm is present in the market. If the help, I_H, is lower than a threshold $\overline{I_H}$, then for small values of g_0+I_l, the PS follower invests more than the OS follower and for large values of g_0+I_l the opposite is true.*
Proof. *see Appendix* ■

Two opposing effects generate this result. First compared to the PS firm, the OS follower has no income on the s market. This means that for the same (g_0+I_l), the OS firm has a lower marginal revenue from innovation, which reduces the incentive to invest in innovation. Second, the help from the community reduces the marginal cost of innovation for the OS follower which increases the incentive to invest in innovation. If the help provided by the community is not too large (i.e.: $I_H \leq \overline{I_H} = \frac{3}{8}\gamma$), the first negative effect prevails for small values of (g_0+I_l). Since income from the s market is decreasing in (g_0+I_l), the first effect fades away so the second effect prevails for higher values of (g_0+I_l).

Figure 1 illustrates proposition 2 for $\gamma = \frac{1}{2}$.The three curves represent the follower's reaction functions: the solid line corresponds to a PS follower, the dashed line corresponds to an

OS follower with $I_H = \frac{1}{4} > \overline{I_H}$ and the dotted line corresponds to an OS follower with $I_H = \frac{1}{10} < \overline{I_H}$. In the latter case we can see that for low values of $g_0 + I_l$ the PS follower invests more than a OS follower.

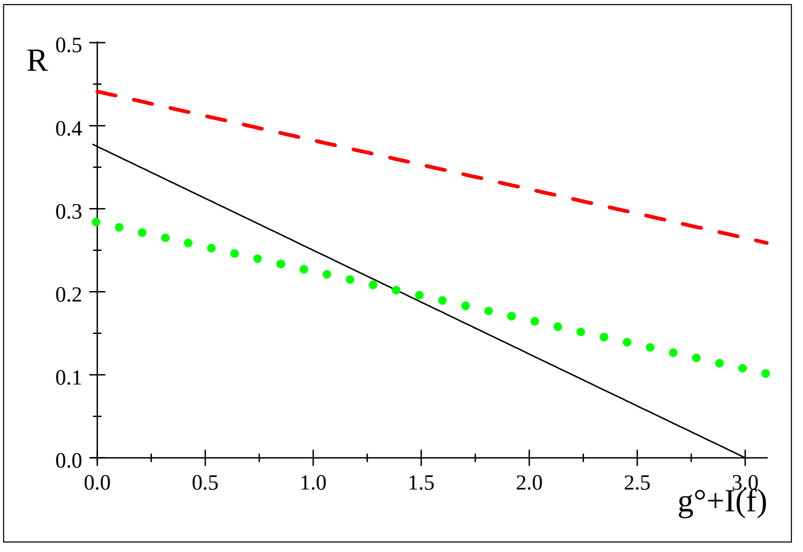

Figure 1: Follower's reaction functions as OS and PS firm.

Given the reaction function (18), the leader computes its optimal level of innovation investment. The profit function depends on whether the leader is a constrained monopolist or not. The leader becomes a constrained monopolist if $I_l > 3 + I_H - g_0$. From (10), (7) and (1) the leader's problem is given by:

$$\max_{I_l} \Pi_l = \begin{cases} \frac{\gamma\left(\frac{1}{2}+\frac{g_0+I_l-R^{mix}}{2}\right)^2}{2} + \frac{(1-\gamma)\left(1+\frac{g_0+I_l-R^{mix}}{3}\right)^2}{2} - \frac{(I_l)^2}{2} & \text{if } I_l \leq I_H + 3 - g_0, \\ g_0 + I_l - I_H - 1 - \frac{(I_l)^2}{2} & \text{if } I_l > I_H + 3 - g_0. \end{cases} \tag{19}$$

Comparing the second expression of the leader's profits in (14) and in (19) we can see that the assumption of a minimum level of investment for the OS project only reduces the constrained monopolist profits by I_H, which in turn will affect the threshold level of g_0 where the leader decides to become a constrained monopolist.

Solving (19) we find the optimal level of investment of the leader

$$\widehat{I_l}^{mix}(g_0) = \begin{cases} \frac{45\gamma+36}{4\gamma^2+19\gamma+220}(g_0 - I_H) + \frac{12\gamma^2-15\gamma+84}{4\gamma^2+19\gamma+220} & \text{if } g_0 \leq \overline{\overline{g}}_0, \\ 1 & \text{if } g_0 > \overline{\overline{g}}_0. \end{cases} \tag{20}$$

where

$$\overline{\overline{g}}_0 = I_H + \frac{34\gamma - 8\gamma^2 + 136 + +\frac{804\gamma - 880 + 60\gamma^2 + 16\gamma^3}{\sqrt{19\gamma + 4\gamma^2 + 220}}}{45\gamma + 36}.$$

The value $\overline{\overline{g}}_0$ is the threshold point at which the leader decides to become a constrained monopolist. An interesting issue is whether the leader in a mixed duopoly is willing to induce the exit of the follower for larger or smaller value of g_0 as opposed to a PS duopoly. Comparing $\overline{\overline{g}}_0$ with the threshold value in a PS duopoly, $\overline{g}_0$, we have $\overline{\overline{g}}_0 < \overline{g}_0$ only if

$$I_H < \underline{I_H} = 2.1297 - \frac{34\gamma - 8\gamma^2 + 136 + \frac{804\gamma - 880 + 60\gamma^2 + 16\gamma^3}{\sqrt{19\gamma + 4\gamma^2 + 220}}}{45\gamma + 36}. \tag{21}$$

and $\overline{\overline{g}}_0 \geq \overline{g}_0$ otherwise.

Since $\underline{I_H}$ is increasing in γ, a sufficient condition to have $\overline{\overline{g}}_0 > \overline{g}_0$ for all $\gamma \in (0,1)$ is that $I_H > 0.1297$. There are two opposing effects for this result. On the one hand the leader's income in the s market is smaller in a mixed duopoly. This makes the income of a constrained monopolist more tempting so the leader should be willing to induce exit for a lower g_0. On the other hand, the help I_H the follower obtains in the mixed duopoly implies that, in order to become a constrained monopolist, the leader requires a higher investment effort. Moreover since I assume that the OS community invests I_H even if the OS follower exits, the income of the constrained monopolist is reduced[8]. Then, because of I_H the leader should be willing to induce exit for a higher g_0. Therefore if help is sufficiently small (i.e.: $I_H < 0.1297$) the leader in a mixed duopoly decides to become a constrained monopolist for lower g_0.

The equilibrium investment of the leader is decreasing in the help provided by the OS community to the follower. Then, for the equilibrium to make sense, I need to impose a condition on I_H. Since $\widehat{I}_l^{mix}$ is increasing in g_0 I assume I_H is such that $\widehat{I}_l^{mix}|_{g_0=0} \geq 0$ for all $\gamma \in (0,1)$. A sufficient condition is that:

Condition 1 $I_H \leq 1$.

Substituting $\widehat{I}_l^{mix}$ into the reaction function of the follower we obtain the equilibrium value

$$\widehat{I}_f^{mix} = \begin{cases} \frac{4\gamma^2+28\gamma-32}{4\gamma^2+19\gamma+220} g_0 + \frac{(252-9\gamma)I_H+72-72\gamma}{4\gamma^2+19\gamma+220} & \text{if } g_0 \leq \overline{\overline{g}}_0, \\ \\ I_H & \text{if } g_0 > \overline{\overline{g}}_0. \end{cases}$$

The equilibrium investment of the follower is such that all the help provided by the community is used since $\widehat{I}_f^{mix}$ is higher than I_H for all $0 < g_0 < \overline{\overline{g}}_0$ and $\gamma \in (0,1)$.

[8] All the main results are not affected by $\overline{\overline{g}}_0 \lesseqgtr \overline{g}_0$. If we assume alternatively that if the OS follower exits the market, all development of the project is abandoned.then the income of the constrained monopolist is not reduced and we have $\overline{\overline{g}}_0 < \overline{g}_0$ for all I_H.

The comparison of the leader's optimal level of investment with that obtained in a PS duopoly yields the following proposition.

Proposition 3 *If the help provided by the user-developers, I_H, is higher than a positive threshold $\overline{\overline{I_H}}$, then the leader's investment in a PS duopoly is higher than in a mixed duopoly for all positive values of pre-innovation technological gap $g_0 \in [0, \overline{g}_0]$. If help is lower than the threshold $\overline{\overline{I_H}}$ then the leader's investment in a PS duopoly is higher than in a mixed duopoly for low values of g_0 while the opposite is true for large values of g_0.*

Proof. see Appendix ■

Three effects generate this behavior. First when the follower is open source the leader's income from the s market is reduced which lowers its incentive to invest. This effect fades away as g_0 approaches to the threshold value where the leader becomes a constrained monopolist. Second, Proposition 2 tells us that the OS follower invests more than a PS follower for large values of g_0 which in turn reduces the leader incentive to invest since I_f and I_l are strategic substitutes. This effect increases with the community help I_H provided to the follower. Finally, the follower's price in the s market is fixed at zero and does not react to changes in the leaders investment, which increases the marginal revenue from innovation for the leader. This effect exists as long as the follower is present in the market. While the first and second effects, both reduce the incentive to invest of the leader in mixed duopoly, the third one increases this incentive. When the help provided to the follower is sufficiently high $I_H > \overline{\overline{I_H}}$ the first and second effect offsets the third one for all positive values of g_0, otherwise there is a range of g_0 in which leader's investment in a mixed duopoly is higher than in a PS duopoly.

Figure 2 illustrates Proposition 3 for $\gamma = \frac{1}{2}$. Each of the three curves represent the leader's investment as a function of the pre-innovation technological gap g_0: the solid line corresponds to the PS duopoly, the dashed line corresponds to the mixed duopoly with $I_H < \overline{\overline{I_H}}$ and the dotted line correspond to the mixed duopoly with $I_H > \overline{\overline{I_H}}$. The discontinuity of the solid line represents the point $\overline{g}_0$ at which the investment of the leader in a pure PS duopoly "jumps" to induce the follower's exit. We can see that if $I_H < \overline{\overline{I_H}}$ we have an interval $[\widehat{g}_0, \overline{g}_0]$ where the leader's investment in a mixed duopoly is higher than in a PS duopoly. This interval is represented by the point where the dashed line crosses the solid line and the discontinuity point the of the solid line. Notice that a higher I_H moves the curve to the southeast reducing the interval $[\widehat{g}_0, \overline{g}_0]$. For $I_H > \overline{\overline{I_H}}$ this interval disappears.

Proposition 2 tells us that for large values of community help, the follower investment is larger in a mixed duopoly compared to a PS duopoly. On the other hand, from proposition 3 we know that the opposite is true for the leader's investment. Therefore, the extra follower's investment might be compensated by a smaller leader's investment. Then, it is interesting to analyze what happens to total investment in innovation in a mixed duopoly, $\widehat{I}^{mix}$, compared to a PS duopoly, $\widehat{I}^{ps}$.

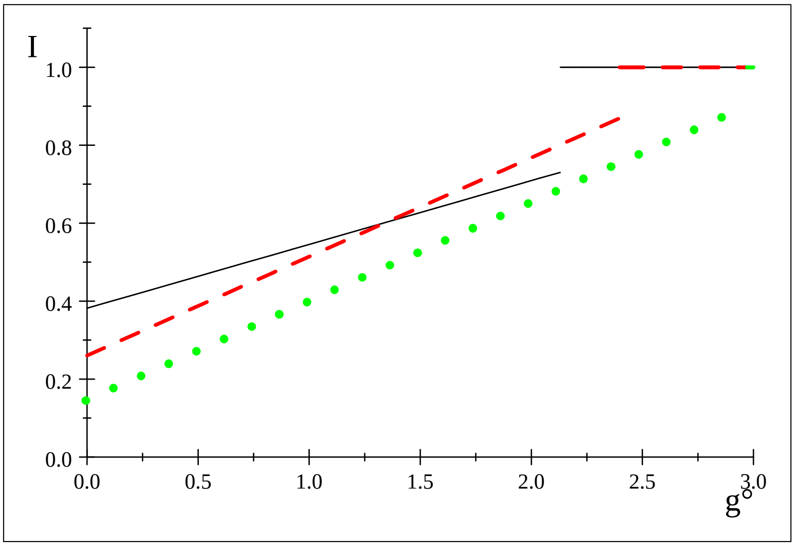

Figure 2: Leader's inverstment for mixed and PS duopoly cases.

Proposition 4 *If the help provided by the user-developers, I_H, is higher than a positive threshold $\widetilde{I_H}$, then a mixed duopoly generates higher investment than a PS duopoly for all positive values of pre-innovation technological gap $g_0 \in [0, \overline{g}_0]$. If help is lower than the threshold $\widetilde{I_H}$ then a PS duopoly generates more investment for low values of g_0 (i.e.:$g_0 \to 0$) and a mixed duopoly generates higher investment for large values of g_0.*

Proof. *see Appendix* ■

From Propositions 2 and 3 we know that there are several effects at play. The interesting thing to notice is that even if the help provided by the community is zero, for certain values of g_0 total investment in innovation is higher in a mixed duopoly. In this case profits are lower for both firms but the fact that the follower's price in the basic software market is fixed increases the leader's incentive to invest.

Figure 3 illustrates Proposition 4 for $\gamma = \frac{1}{2}$. Each of the three curves represents total investment as a function of the pre-innovation technological gap g_0: the solid line corresponds to the pure PS duopoly, the dotted line corresponds to the mixed duopoly with $I_H = 0 < \widetilde{I_H}$ and the dashed line correspond to the mixed duopoly with $I_H > \widetilde{I_H}$. We can see that if $I_H < \widetilde{I_H}$ for low values of g_0 total investment in a PS duopoly is higher and for large values of g_0 total investment is higher in a mixed duopoly even at $I_H = 0$. A higher I_H moves the curve for the mixed duopoly upward reducing the region where a PS duopoly generates more investment.

The main idea of Proposition 4 is that a mixed duopoly configuration is better than a PS duopoly in terms of investment in innovation if the help from the community is large or

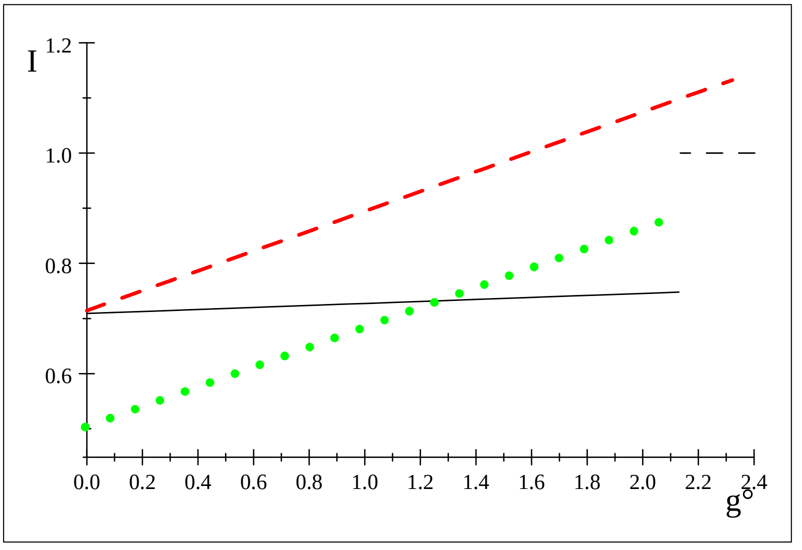

Figure 3: Total investment for the PS and mixed duopoly cases

if the technological gap between firms is large. If the government cares about investment in innovation, then Proposition 4 seems to suggest that there is room for policy intervention. In particular a policy that incentivize technological laggards that are PS to become OS (for example a conditional transfer or a tax cut for OS firms) might increase innovation investment in the market. However, as we will see in the extended setup, this kind of policy might have the opposite effect if they are not carefully designed to take into consideration the market leader's investment behavior.

4 Extended Setup: PS duopoly with OS threat.

In this section we solve the extended setup. Compared to the basic setup, now in the first stage the PS follower after observing the leaders investment decision, first decides whether to remain PS or become OS and then chooses its level of investment I_f.

From the computations of the basic setup we have all the ingredients to solve the extended game. In order to keep things simple and to reduce the number of cases to analyze I assume that $0 \leq g_0 < \overline{g}_0$. In other words I require g_0 to be in a range where the leader in a pure PS duopoly finds inducing the follower's exit non profitable.

Depending on the follower's OS/PS choice in the first stage the game continues according to the PS duopoly or the mixed duopoly of the basic setup: the follower invests according to R^{mix} or R^{ps} and in the second stage firms receive revenues $\overline{\pi}_f^{ps}$ and $\overline{\pi}_l^{ps}$ or $\overline{\pi}_f^{mix}$ and $\overline{\pi}_l^{mix}$. Therefore, to decide between OS and PS the follower compares, for a given $g_0 + I_l$, the maximum profit functions

$$\overline{\Pi}_f^{ps} = \frac{1}{2}\left(1 - \frac{g_0 + I_l - R^{ps}}{3}\right)^2 - \frac{(R^{ps})^2}{2}, \tag{22}$$

$$\overline{\Pi}_f^{mix} = (1-\gamma)\frac{\left(1 - \frac{g_0 + I_l - R^{mix}}{3}\right)^2}{2} - \frac{\left(R^{mix} - I_H\right)^2}{2}. \tag{23}$$

These profits are equal when

$$(g_0 + I_l)^* = \frac{1}{9\gamma}\left(27\gamma - 8I_H(1-\gamma) - 2\sqrt{2}I_H(8+\gamma)\sqrt{\frac{1-\gamma}{\gamma+8}}\right). \tag{24}$$

Therefore if $(g_0 + I_l) \leq (g_0 + I_l)^*$ the follower prefers to remain PS, and if $(g_0 + I_l) > (g_0 + I_l)^*$ it becomes OS.

Now we turn to the leader who must decide on I_l at the beginning of the game. The firm knows that for $g_0 + I_l \leq (g_0 + I_l)^*$ the follower remains PS, so using the leader's optimal investment function (15) we know that

$$\begin{aligned} g_0 + \widehat{I}_l^{ps}(g_0) &\leq (g_0 + I_l)^*, \\ g_0 + \frac{9}{55}g_0 + \frac{21}{55} &\leq (g_0 + I_l)^*, \\ g_0 &\leq g_0^*. \end{aligned}$$

where

$$g_0^* = \frac{55}{576\gamma}\left(27\gamma - 8I_H(1-\gamma) - 2\sqrt{2}I_H(8+\gamma)\sqrt{\frac{1-\gamma}{\gamma+8}}\right) - \frac{21}{64}. \tag{25}$$

The threshold value g_0^* is decreasing in the help provided by the community and increasing in the relative size of the $s+t$ market. In order to make the problem interesting I need g_0^* to be positive and smaller than the threshold value at which the leader decides to become a constrained monopolist. Then, to have $0 < g_0^* < \overline{g}_0$, I need to impose the following condition

Condition 2 $\frac{15\,840 I_H + 3025 I_H^2}{15\,840 I_H + 3025 I_H^2 + 23\,328} < \gamma < \frac{-1760 I_H + 495 I_H^2 + 256\sqrt{55} I_H}{-1760 I_H + 495 I_H^2 + 256\sqrt{55}(I_H - 2) + 3808}$ *for all* $I_H > 0$.

As we shall see later in Figure 4, this condition holds for a large set of I_H and γ.

If the value of g_0 is larger than the threshold point g_0^* and if the the leader invests according to 20 then the follower becomes OS. If this is the case the leader's profits jumps from $\overline{\Pi}_l^{ps}|_{g_0=g_0^*}$ to $\overline{\Pi}_l^{mix}|_{g_0=g_0^*}$, where $\overline{\Pi}_l^{ps}$ and $\overline{\Pi}_l^{mix}$ are, respectively, the leader's maximum profit functions in a PS and in a mixed duopoly. Under Conditions 1 and 2 we obtain the following proposition

Proposition 5 *If conditions 1 and 2 are verified then at the threshold level of pre-innovation technological gap g_0^* the leader's maximum profits are higher in a PS duopoly than in a mixed duopoly. Moreover there is an interval $[g_0^*, g_0^{**}]$ for which the leader finds profitable to avoid the PS follower to become OS by investing in such a way to maintain $(g_0 + I_l) = (g_0 + I_l)^*$.*

Figure 4 illustrates the first part Proposition 5. The area between the dashed lines represents the set of parameters for which $\overline{\Pi}_l^{ps}|_{g_0=g_0^*} > \overline{\Pi}_l^{mix}|_{g_0=g_0^*}$. The area inside the solid lines represents the combination of parameters defined by Conditions 1 and 2. For all the combinations of γ and I_H that verify Conditions 1 and 2 we have that $\overline{\Pi}_l^{ps}|_{g_0=g_0^*} > \overline{\Pi}_l^{mix}|_{g_0=g_0^*}$.

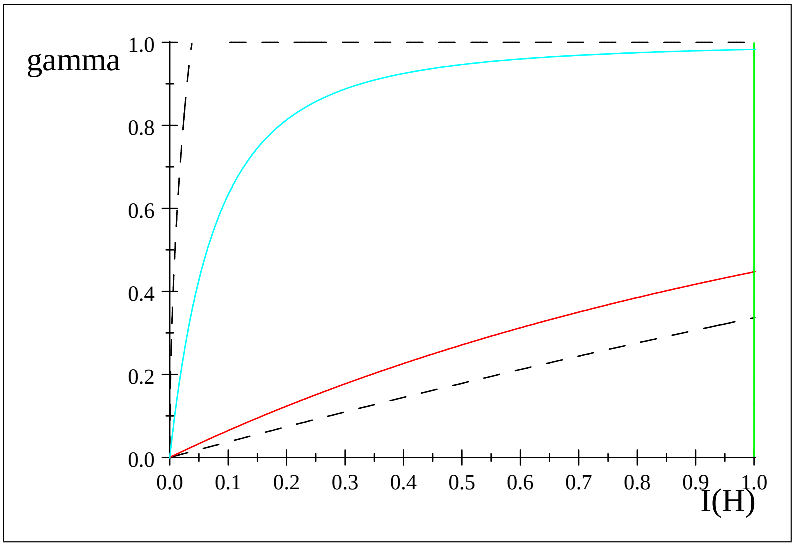

Figure 4: Profits and set of I_H and γ that verifies condition 1 & 2

Since $\overline{\Pi}_l^{ps}$ is strictly increasing in g_0, the leader's optimal investment function for the interval $[g_0^*, g_0^{**}]$ is

$$\widehat{I}_l = (g_0 + I_l)^* - g_0.$$

The function $\widehat{I}_l$ is such that the follower always observes $(g_0 + I_l) = (g_0 + I_l)^*$ and therefore it remains PS. Since $(g_0 + I_l)$ is constant along the interval $[g_0^*, g_0^{**}]$the leader gets a constant income equal to

$$\widetilde{\pi}_l^{ps}|_{g_0=g_0^*} = \frac{1}{2}\left(1 + \frac{-\frac{3}{8} + \frac{9}{8}(g_0 + I_l)^*}{3}\right)^2.$$

Then the maximum profit function for the interval $[g_0^*, g_0^{**}]$ is

$$\widetilde{\Pi}_l\left(g_0\right)=\widetilde{\pi}_l^{ps}|_{g_0=g_0^*}-\frac{\left(\left(g_0+I_l\right)^*-g_0\right)^2}{2}. \tag{26}$$

Notice that $\widetilde{\Pi}_l$ is tangent to $\overline{\Pi}_l^{ps}$ at $g_0=g_0^*$. Since $\widetilde{\pi}_l^{ps}|_{g_0=g_0^*}$ is constant and the cost is decreasing in g_0, the maximum profit function $\widetilde{\Pi}_l$ is concave in g_0 and increasing at the tangent point $g_0=g_0^*$. Moreover $\widetilde{\Pi}_l$ attains its maximum at

$$g_0=\left(g_0+I_l\right)^*\geq g_0^*. \tag{27}$$

where $\widehat{I}_l=0$.

Figure 5 illustrates the situation. The three curves represent the leader's maximum profits: the solid one corresponds to a PS duopoly ($\overline{\Pi}_l^{ps}$), the dotted one to a mixed duopoly ($\overline{\Pi}_l^{mix}$) and the dashed one ($\widetilde{\Pi}_l$) corresponds a situation where the leader sets I_l such that $\left(g_0+I_l\right)=\left(g_0+I_l\right)^*$. The function $\widetilde{\Pi}_l$ is concave and tangent to Π_l^{ps} at $g_0=\widetilde{g}$.

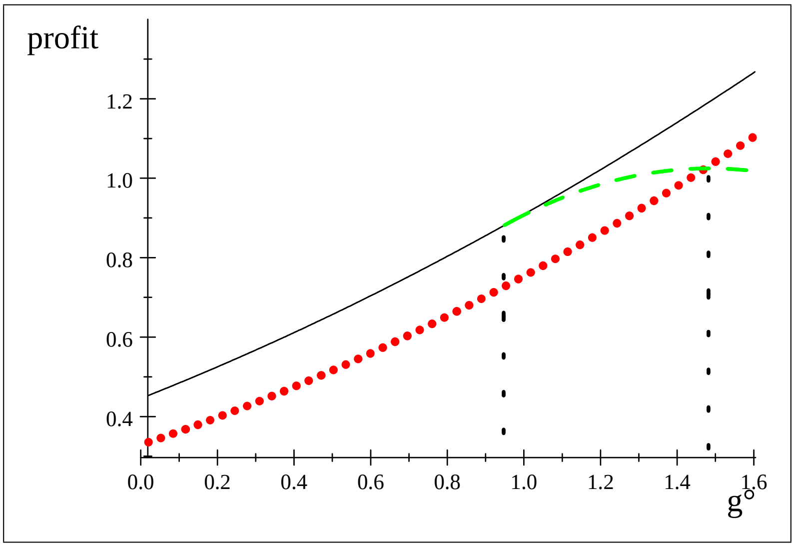

Figure 5: Profit functions

We still have to define the upper limit g_0^{**} of the interval$[g_0^*,g_0^{**}]$. A natural candidate is the value g_0 at which $\widetilde{\Pi}_l\left(g_0\right)=\overline{\Pi}_l^{mix}\left(g_0\right)$. This value, however, could imply $\widehat{I}_l<0$. I consider only non negative levels of investment therefore from (27), g_0^{**} is defined in the following way

$$g_0^{**}=\min_{g_0}\left\{g_0=\left(g_0+I_l\right)^*;\widetilde{\Pi}_l\left(g_0\right)=\overline{\Pi}_l^{mix}\left(g_0\right)\right\}. \tag{28}$$

The leader's equilibrium investment for the interval $0\leq g_0<\overline{g}_0$ is

$$\widehat{I_l} = \begin{cases} \frac{9}{55} g_0 + \frac{21}{55} & \text{if } g_0 \leq g_0^*, \\ \\ (g_0 + I_l)^* - g_0 & \text{if } g_0^* < g_0 \leq g_0^{**}, \\ \\ \frac{45\gamma+36}{4\gamma^2+19\gamma+220} (g_0 - I_H) + \frac{12\gamma^2-15\gamma+84}{4\gamma^2+19\gamma+220} & \text{if } g_0^{**} < g_0 < \overline{\overline{g}}_0 \leq \overline{g}_0, \\ \\ 1 & \text{if } g_0^{**} \leq \overline{\overline{g}}_0 < g_0 \leq \overline{g}_0. \end{cases}$$

Since $\widehat{I_l}$ is decreasing for $g_0 \in [g_0^*, g_0^{**}]$ and $\widehat{I_f}$ is constant we have that

Proposition 6 *The "threat" of OS switch from the follower softens competition between firms in the interval* $[g_0^*, g_0^{**}]$*. Total investment in innovation in this interval is decreasing in* g_0*. Moreover, total investment is lower compared to the PS and mixed duopoly cases from the basic setup.*
Proof. *see Appendix* ■

Considering what we have learned from propositions 4, 5 and 6 we can conclude that:

- If the past accumulated technological gap is small ($g_0 < g_0^*$) then we should expect a PS duopoly configuration. The PS firms may invest *more* or *less* than in a mixed duopoly depending on the level of g_0 and I_H (Proposition 4). In particular, if I_H is small then it is more likely that the PS duopoly generates *more* invest than a mixed duopoly.
- If the accumulated technological gap is big ($g_0 > g_0^{**}$) then we should expect the laggard to switch to OS and firms investing *more* than a PS duopoly (Proposition 4 and 5).
- If accumulated technological gap is intermediate, ($g_0 \in [g_0^*, g_0^{**}]$) then we should expect a PS duopoly configuration with firms doing *low* investment.(Proposition 5 and 6)

Although the OS switch can trigger high investment and lower prices, the threat of OS switch can trigger the opposite. As I have already discussed at the end of section 3 in the context of the basic setup, Proposition 4 suggests that a policy that incentivize technological laggards that are PS to become OS might increase innovation investment in the market. However from Proposition 5 we learn that we should take into consideration the incentives of the technological leader.

Consider, in this simplified setting, the effect of a subsidy consisting in a fixed transfer $\bar{t}$ to the OS firm (we could think alternatively on a tax cut). This transfer reduces both the lower and the upper limits of the interval $[g_0^*, g_0^{**}]$. Therefore the success of the policy depends on the initial level of g_0. If g_0 is such that with the policy we end up in the interval $[g_0^*, g_0^{**}]$ then the result is that there is no switch at all and less innovation. Therefore a policy to support OS could end up being potentially harmful to innovation.

According to our very stylized model, in order to be successful the transfer should take into consideration the incentive of the market leader to avoid the OS switch of the follower. Our framework suggests that instead of being a fixed amount, it should depend negatively on g_0 and I_H, and positively on γ. However it is very unlikely that the policymaker has accurate information on these variables.

A example should help us clarify these results.

Example 1 *Assume that* $\gamma = \frac{1}{5}$, *and* $I_H = \frac{1}{5}$. *From (24) we compute the level of* $(g_0 + I_l)$ *at which* $\Pi_f^{mix} = \Pi_f^{ps}$

$$(g_0 + I_l)^* = 1.484.$$

Using (25) we computes the threshold value

$$g_0^* = 0.947.$$

If $g_0 > g_0^*$ *and if the leader invest according to (15), the follower becomes OS. If this is the case the leader's profits jumps from* $\overline{\Pi}_l = 0.88$ *to* $\overline{\Pi}_l = 0.72$. *To avoid this the leader sets* I_l *such that*

$$\widehat{I}_l = 1.484 - g_0.$$

According to (28) the upper limit g_0^{**} *of the interval where the leader avoids the OS switch is given by:*

$$\begin{aligned} g_0^{**} &= \min_{g_0} \left\{ (g_0 + I_l)^* - g_0 = 0; \widetilde{\Pi_l}(g_0) - \overline{\Pi}_l^{mix}(g_0) = 0 \right\}, \\ &= \min_{g_0} \{1.484; 1.481\} = 1.481. \end{aligned}$$

The equilibrium investment I_l *would then be*

$$\widehat{I}_l = \begin{cases} \frac{9}{55} g_0 + \frac{21}{55} & \text{if } g_0 \leq 0.947\,, \\ 1.484 - g_0 & \text{if } 0.947 < g_0 \leq 1.481, \\ 0.201 g_0 + 0.323 & \text{if } 1.481 < g_0 \leq 2.2. \end{cases}$$

Figure 6 depicts total investment as a function of g_0. *For values of* $g_0 \in [0, 0.947]$ *we have a PS duopoly without OS threat. For values* $g_0 \in [0.947\,16, 1.4815)$ *we still have a PS duopoly but the leader invests in such a way to avoid the follower's OS switch. Along the interval total investment is decreasing in* g_0. *Finally for* $g_0 \in [1.481, 2.2]$ *the leader finds avoiding the OS switch to costly so we have a mixed duopoly and total investment jumps and becomes increasing again with respect to* g_0. *The investment in a mixed duopoly is also depicted for* $g_0 \in [0, 1.481]$ *with a dashed line. This shows that for all* g_0 *there is clear gain in terms of innovation investment if the PS follower can be induced to become OS. Lets now assume that*

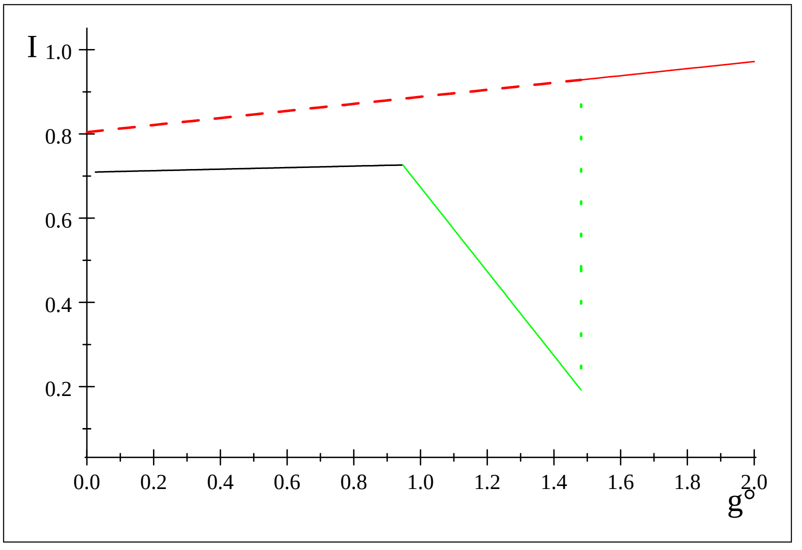

Figure 6: Total investment under OS "threat".

the government announces, at the beginning of the game, a transfer $\bar{t} = 0.01$ *to OS firms. Then the new level of* $(g_0 + I_l)$ *at which* $\Pi_f^{mix} = \Pi_f^{ps}$ *is:*

$$(g_0 + I_l)_t^* = 1.115 < 1.484.$$

and the threshold value g_0^* *is*

$$g_0^* = 0.630 < 0.947.$$

To avoid the follower becoming OS the leader sets I_l *such that*

$$\widehat{I_l} = 1.115 - g_0.$$

The upper limit g_0^{**} *of the interval where the leader avoids the OS switch is given by:*

$$\begin{aligned} g_0^{**} &= \min_{g_0} \{1.115; 1.16\}, \\ &= 1.115. \end{aligned}$$

As we can see, only if $g_0 \in (1.115, 1.481)$ *this transfer will induce the PS follower to become OS. If* $g_0 \geq 1.481$ *the PS follower will become OS anyway so the transfer is useless. If* $g_0 \leq 1.115$ *this policy does not induce the OS switch. Moreover if* $g_0 \in [0.630, 1.115]$ *the transfer reduces investment in innovation.*

5 Relation to the literature

In this paper I presented an horizontally differentiated duopoly framework, that links competition, incentives to innovate, exit and decisions of software firms to become OS.

Two papers are closely related to my framework: Sorenson (1995) and Schmidt and Schnitzer (2003). Sorenson (1995) uses an horizontally differentiated duopoly à la Hotelling to examine the persistence of market leadership in a duopoly. As in my model firms first invest in product-improving R&D prior to competing in prices and they differ in their initial level of product quality. He finds that the leader in a horizontally differentiated market always becomes increasingly dominant. His framework is similar to my PS duopoly case in the Basic Setup, with the exception that he assumes a Cournot game in the first stage and each firms sells only one good. My paper adapts his framework to account for the specificities of an OS firm. The OS firm generates no revenue from the basic software but has a positive income from selling a complementary good and it also receives free help from a community of developers which lowers the firm's innovation costs. Another difference is that I add an intermediate step in the game to allow the follower to choose between remaining PS or becoming OS. As in Sorenson (1995), in my Basic Setup the technological leader always becomes increasingly dominant however in the Extended setup this is no longer true: the PS leader may invest in such a way that the post-innovation technological gap is lower than the pre-innovation technological gap, in order to avoid the OS switch of the PS follower.

Schmidt and Schnitzer (2003) build a model to study the effect of government forcing some agencies to adopt OS and the effect this adoption has on welfare and incentives to improve quality on a PS developer. They assume that there are two software alternatives in the market, one PS and the other OS. Each firm is located at one extreme of the hotelling line. The PS software there is sold by a profit maximizing firm that sets the price and invest in software quality. In contrast, the OS software price is set at 0 and there is no investment in quality. Consumers are divided in three categories. The first group always buy the PS software, the second one always buy the OS software and the third group may buy either of the two. They model the preferences of the third group with a Hotelling model of horizontal product differentiation. They find that if the government reduces the amount of consumers of the third group to increase the number of the second group, this reduces welfare and investment incentives of the PS firm. The are two main differences with my model. First I assume a for profit firm behind the OS software that receives revenue from selling a complement good and invests in product quality improvement. Second I endogenize the decision of becoming OS. This two differences have an important affect on the incentives to invest of the PS firm.

Comino and Manetti (2005) also use a Hotelling model to analyze the impact on welfare of government policies supporting open source software. They assume that there are two software alternatives in the market, one PS and one OS. The PS firm chooses the price while OS software is free. Consumers are divided between those who are informed about the existence of the OS software and those who are not. Under this assumption some uninformed consumers, who are far from the PS firm may remain "inactive" by not using any software. Contrary to Schmidt and Schnitzer (2003) they find that mandated adoption may increase social welfare since it forces some "inactive" consumers to use OS software and therefore enjoy a positive benefit. They also find that a policy that informs uninformed users about the existence of the OS alternative also increases welfare. On the other hand a subsidy for consumers to use OS software always reduces welfare.

Gaudeul (2008) also analyzes the effects of competition between OS and PS software but in the context of a Vickrey-Salop model of spatial product differentiation. The author shows that a mixed industry with OS and PS projects may exhibit large OS projects cohabiting with more specialized PS projects and this configurations is better in terms of welfare compared to a industry with only PS firms.

Besides Schmidt and Schnitzer (2003), the issue of innovation investment in the context of OS has been studied in several papers.

Economides and Katsamakas (2005) build a model to compare investment incentives of platform and application developers when the platform can be OS or PS. The application developers and the PS platform decide their level of investment as profit maximizing firms while the OS platform investment depends user-developers that maximize the sum of consumer surplus and a reputation value. They find that the level of investment in applications is larger when the operating system is open source. The comparison for platforms depends, among others, on reputation effects and the number of developers.

Bitzer and Schröder (2005) study the effect on innovation of OS entry by comparing innovation incentives in a software Monopoly vs a software duopoly. Rather that competing in price or quantities, software producers compete in technology levels. They assume that the OS producer is a reputation maximizing agent, where reputation is a fixed monetary reward times the demand for the software. Compared to a PS firm the OS firm also has lower innovation costs. The authors find that entry of a low-cost OS producer increases the technology set of the for profit firm beyond the level that would have resulted from a for profit entry with higher costs. This result is similar to what I find in my framework where a mixed duopoly may generate more innovation than a PS duopoly. However the reasons for each result are different. While in my framework the key ingredient is that OS follower's price in the basic software market is fixed at zero, in Bitzer and Schröder (2005) is that the technology levels are strategic complements.

Verani (2006) presents a differentiated duopoly model to study investment software quality under a Proprietary or Open Source regime. Firms first invest in quality and then fix prices. Under a PS regime the quality achieved by each firm depends only on their own investment while under an OS regime there are spillover effects. The author finds that investment is increasing in the spillover rate when the goods are substitutes and independent but is decreasing in when the goods are close to perfect complements.

Darmon et. al. (2007) study the diffusion of both PS and OS software with a two-step game model where first the PS firm chooses price and quality and then users decide whether to use a PS software, OS software or neither. The authors find a winner-takes-all competition may arise between types of software, which may lead to the exit of one of them.

The trade off that a for profit firm faces to become OS or to release his code as OS has been analyzed before in the literature.

Lerner and Tirole (2004) suggest that behind the firm's code donation there is a strategy of "giving away the razor (the released code) to sell more razor blades" (complement goods and services). They also suggest that "the temptation to go open source is particularly strong when the product is lagging behind the leader and making few profits". In contrast Commino and Manetti (2007) assume that by releasing the code the firm obtains the collaboration of developers which increases the software quality. Baake and Wichmann (2004) suggest that the OS code releasing reduces the firm developing cost but also reduces the reservation price consumers are willing to pay for the commercial version. Similar to Lambardi (2008) the trade off in this paper is: less income on the software market in exchange of less development or innovation costs.

6 Conclusions

Open source has become a widespread phenomena in the software industry. While some open source projects are born as new alternatives to existing proprietary software some others originate from failed proprietary software experiences. Successful OS projects such as the Eclipse example analyzed here show that OS may be a viable outside option for those PS software that failed to retain market share against a competitor. Donald K. Rosenberg, author of Open Source: The Unauthorized White Papers, summarizes the later situation as follows: "The software world is filled with the casualties of Microsoft competition. The return of Open Source provides an opportunity for those of them still able to lift a hand".

It is also clear form the Eclipse example that the market leader can suffer significantly when the follower becomes OS so it might be profitable to avoid this switch. The SAP example shows us that market leaders have learn to take into consideration the followers temptations to become OS.

In this paper I to study how the behavior of the leader is affected by the fact that the follower is, or might become, OS. In particular I analyze how innovation investment in a software duopoly is affected by this OS switch.

I presented an horizontally differentiated duopoly setup, that links competition, incentives to innovate, exit and decisions of software firms to become OS. Although the model is very stylized I believe it gives some interesting insight and sheds some light on the OS phenomena. First I show that if the initial technological gap between firms is small, a duopoly with two PS firms might generate more innovation than a mixed duopoly of an PS and OS firm. However if the initial technological gap is big then a duopoly with two PS firms generates less innovation than a mixed duopoly. Second, in a context of a duopoly with two PS firms, the outside option for the technological laggard to become OS can soften competition on innovation. In fact I show that if the technological gap falls into a certain "intermediate" region (not too low or large) total investment in innovation could be substantially low, due to the fact that the technological leader may prefer to reduce his innovation investment in order to avoid the temptation of the follower to become Open Source. Since mixed duopoly can generate more innovation than a PS duopoly I analyzed the possibility of introducing a subsidy (fixed transfer) to incentivize technological laggards that are PS to become OS. Our framework suggests, however; that such policy could end up being potentially harmful to innovation.

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6.1 Proof of Proposition 2

We will compare the reaction functions of the follower as a PS firm, denoted $R^{ps}(g_0+I_l)$,and as an OS firm, $R^{mix}(g_0+I_l)$. If $g_0+I_l<3$ the PS follower will be present in the s market, so the difference between the reaction functions is given by:

$$\begin{aligned} R^{ps}-R^{mix} &= \frac{3}{8}-\frac{1}{8}(g_0+I_l) \\ &\quad -\left(\frac{(9I_H-3\gamma+3)}{\gamma+8}-\frac{1-\gamma}{\gamma+8}(g_0+I_l)\right) \end{aligned}$$

rearranging terms we get

$$R^{ps}-R^{mix}=-\frac{9}{8}\frac{\gamma}{\gamma+8}(g_0+I_l)+\frac{9(3\gamma-8I_H)}{8(\gamma+8)}$$

Since $(g_0+I_l)>0$, the first term is always negative for all values of $\gamma\in(0,1)$. The second term can be positive or negative. A sufficient condition for the second term to be non positive is:

$$I_H\geq\overline{I_H}=\frac{3}{8}\gamma$$

Then if $I_H\geq\overline{I_H}$, we have that $R^{ps}<R^{mix}$ for all values of $0\leq(g_0+I_l)<3$. On the other hand if $0<I_H<\overline{I_H}$ the second term is positive and for values of $(g_0+I_l)\to 0$ it exceeds the first term, therefore we have $R^{ps} > R^{mix}$. As $(g_0 + I_l)$ increases the first negative term exceeds the second one so we have that $R^{ps} < R^{mix}$. In particular it is always the case that when $(g_0 + I_l) \to 3$ we have that

$$R^{ps} - R^{mix} = -9\frac{I_H}{\gamma + 8}$$

which is always negative for $I_H > 0$ and $\gamma \in (0, 1)$.

6.2 Proof of Proposition 3

We will compare the leader's equilibrium investment in a PS duopoly, $\widehat{I}_l^{ps}$, and in a mixed duopoly, $\widehat{I}_l^{mix}$. From (15) and (20) we know that the expression to be compared are:

$$\widehat{I}_l^{ps} = \tfrac{9}{55}g_0 + \tfrac{21}{55} \quad \text{if } g_0 < \overline{g}_0$$

$$\widehat{I}_l^{mix} = \tfrac{45\gamma+36}{4\gamma^2+19\gamma+220}(g_0 - I_H) + \tfrac{12\gamma^2-15\gamma+84}{4\gamma^2+19\gamma+220} \quad \text{if } g_0 < \overline{\overline{g}}_0$$

Notice that the threshold values $\overline{g}_0$ and $\overline{\overline{g}}_0$ at which the leader decides to become a constrained monopolist are different. Therefore the interval values of g_0 for which the comparison between the PS and mixed duopoly is valid depends on $\overline{g}_0 \lesseqgtr \overline{\overline{g}}_0$. We know from (21) that if

$$I_H > \underline{I_H} = 2.1297 - \frac{34\gamma - 8\gamma^2 + 136 + \frac{804\gamma - 880 + 60\gamma^2 + 16\gamma^3}{\sqrt{19\gamma + 4\gamma^2 + 220}}}{45\gamma + 36}$$

then $\overline{g}_0 < \overline{\overline{g}}_0$ and $\overline{g}_0 > \overline{\overline{g}}_0$ otherwise.

The difference between $\widehat{I}_l^{ps}$ and $\widehat{I}_l^{mix}$ is given by

$$\begin{aligned}\widehat{I}_l^{ps} - \widehat{I}_l^{mix} &= \left(\frac{9}{55} - \frac{45\gamma + 36}{4\gamma^2 + 19\gamma + 220}\right) g_0 \\ &\quad + \left(\frac{21}{55} - \frac{12\gamma^2 - 15\gamma + 84}{4\gamma^2 + 19\gamma + 220}\right) + I_H \frac{45\gamma + 36}{4\gamma^2 + 19\gamma + 220}\end{aligned}$$

For all $\gamma \in (0, 1)$ the first term is negative and the second and third are always positive (given $g_0 \geq 0$ and $I_H > 0$). For low values of $g_0 \to 0$ the expression is positive implying $\widehat{I}_l^{ps} > \widehat{I}_l^{mix}$. As g_0 grows the expression becomes smaller. A sufficient condition for the expression to become negative is that at the end of the interval is negative. Therefore

- If $I_H > \underline{I_H}$ then $\overline{g}_0 < \overline{\overline{g}}_0$ so the sufficient condition for the expression to become negative for $g_0 \in [0, \overline{g}_0]$ is that

$$I_H < \overline{\overline{I_H}} = \frac{256\gamma - 4\gamma^2}{275\gamma + 220}\left(\frac{34}{9} - \frac{2}{9}\sqrt{55}\right) - \frac{136\gamma - 64\gamma^2}{275\gamma + 220}$$

- If $I_H < \underline{I_H}$ then $\overline{g}_0 > \overline{\overline{g}}_0$ so the sufficient condition for the expression to become negative for $g_0 \in \left[0, \overline{\overline{g}}_0\right]$ is that

$$I_H < \overline{\overline{I_H^b}} = \left(-\frac{8}{9}\frac{\gamma\left(2\gamma^2 - 130\gamma + 128\right)}{5\gamma + 4}\right)\sqrt{\frac{1}{4\gamma^2 + 19\gamma + 220}} + \frac{8}{9}\frac{\gamma\left(\gamma + 17\right)}{5\gamma + 4}$$

Notice that $\overline{\overline{I_H^b}} > \overline{\overline{I_H}} > \underline{I_H}$ for all $\gamma \in (0,1)$. From the analysis of both cases (i.e.: $I_H < \underline{I_H}$ and $I_H > \underline{I_H}$) we conclude that:

- We always have that for values of $g_0 \to 0$, $\widehat{I}_l^{ps} > \widehat{I}_l^{mix}$
- If $I_H < \underline{I_H}$ we always have that for $g_0 \to \overline{\overline{g}}_0$, $\widehat{I}_l^{ps} < \widehat{I}_l^{mix}$.
- If $\underline{I_H} < I_H < \overline{\overline{I_H}}$ we always have that for $g_0 \to \overline{g}_0$, $\widehat{I}_l^{ps} < \widehat{I}_l^{mix}$.
- If $I_H = \overline{\overline{I_H}}$, then $\widehat{I}_l^{ps} > \widehat{I}_l^{mix}$ except at $\overline{g}_0$ where $\widehat{I}_l^{ps} = \widehat{I}_l^{mix}$
- If $I_H > \overline{\overline{I}}$ and since $\overline{\overline{I_H}} > \underline{I_H}$ then $\widehat{I}_l^{ps} > \widehat{I}_l^{mix}$ for all $g_0 \in [0, \overline{g}_0]$

Given $I_H < \overline{\overline{I_H}}$, we can define $\widehat{g}_0$ as the level of g_0 such that $\widehat{I}_l^{ps} - \widehat{I}_l^{mix} = 0$. The value $\widehat{g}_0$ is given by

$$\widehat{g}_0 = \frac{275\gamma + 220}{256\gamma - 4\gamma^2} I_H + \frac{16\gamma - 34}{\gamma - 64}$$

and $\widehat{g}_0 < \overline{g}_0$ if $\underline{I_H} < I_H < \overline{\overline{I_H}}$ and $\widehat{g}_0 < \overline{g}_0$ if $I_H < \underline{I_H}$

6.3 Proof of Proposition 4

We are will compare the sum of the equilibrium investments of the leader and the follower in a PS duopoly, $\widehat{I}^{ps}$, and in a mixed duopoly, $\widehat{I}^{mix}$.

$$\widehat{I}^{ps} = \tfrac{1}{55} g_0 + \tfrac{39}{55} \quad \text{if } g_0 < \overline{g}_0$$

$$\widehat{I}^{mix} = \tfrac{4\gamma^2 + 73\gamma + 4}{4\gamma^2 + 19\gamma + 220} g_0 + \tfrac{216 - 54\gamma}{4\gamma^2 + 19\gamma + 220} I_H + \tfrac{12\gamma^2 - 87\gamma + 156}{4\gamma^2 + 19\gamma + 220} \quad \text{if } g_0 < \overline{\overline{g}}_0$$

The interval values of g_0 for which the comparison between the PS and mixed duopoly is valid depends on $\overline{g}_0 \lesseqgtr \overline{\overline{g}}_0$. We know form the proof of proposition 3 that if $I_H > \underline{I_H}$ then $\overline{g}_0 > \overline{\overline{g}}_0$ and if $I_H < \underline{I_H}$ then $\overline{g}_0 < \overline{\overline{g}}_0$.

The difference between $\widehat{I}^{mix}$ and $\widehat{I}^{ps}$ is given by

$$\begin{aligned} \widehat{I}^{mix} - \widehat{I}^{ps} \quad = \quad & \left(\frac{4\gamma^2 + 73\gamma + 4}{4\gamma^2 + 19\gamma + 220} - \frac{1}{55}\right) g_0 + \left(\frac{216 - 54\gamma}{4\gamma^2 + 19\gamma + 220}\right) I_H \\ & + \left(\frac{12\gamma^2 - 87\gamma + 156}{4\gamma^2 + 19\gamma + 220} - \frac{39}{55}\right) \end{aligned} \qquad (29)$$

For all $\gamma \in (0,1)$ and for positives values of I_H and g_0 the first and second terms are positive and the third one is negative.

- If $I_H > \underline{I_H}$ then $\overline{g}_0 < \overline{\overline{g}}_0$ and for high values of g_0 (i.e. $g_0 \to \overline{g}_0$) the expression (29) is positive for all $\gamma \in (0,1)$ even at $I_H = \underline{I_H}$.

- If $I_H < \underline{I_H}$ then $\overline{g}_0 > \overline{\overline{g}}_0$ and for high values of g_0 (i.e. $g_0 \to \overline{\overline{g}}_0$) the expression (29) is positive for all $\gamma \in (0,1)$ even at $I_H = 0$.

Then, in both cases, a sufficient condition for (29) to become negative is that at least at $g_0 = 0$ it is negative. This happens if

$$I_H < \widetilde{I_H} = \frac{307\gamma - 28\gamma^2}{660 - 165\gamma}$$

So provided $I_H < \widetilde{I_H}$ (i.e.: the help is not very large) for low values g_0 we have that $\widehat{I}^{ps} > \widehat{I}^{mix}$ and for high values of g_0 we have that $\widehat{I}^{mix} > \widehat{I}^{ps}$. On the other hand if $I_H > \widetilde{I_H}$ and since $\widetilde{I_H} > \underline{I_H}$ for all $\gamma \in (0,1)$ then $\widehat{I}^{mix} > \widehat{I}^{ps}$ for all $g_0 \in [0, \overline{g}_0]$.

Given $I_H < \widetilde{I_H}$, we can define $\widehat{\widehat{g}}_0$ as the level of g_0 such that $\widehat{I}^{ps} - \widehat{I}^{mix} = 0$. The value $\widehat{\widehat{g}}_0$ is given by

$$\widehat{\widehat{g}}_0 = \frac{307\gamma - 660 I_H - 28\gamma^2 + 165\gamma I_H}{222\gamma + 12\gamma^2}$$

and $\widehat{\widehat{g}}_0 < \overline{g}_0$ if $\underline{I_H} < I_H < \widetilde{I_H}$ and $\widehat{\widehat{g}}_0 < \overline{g}_0$ if $I_H < \underline{I_H}$

6.4 Proof of Proposition 6

We going to show that total investment in innovation in the interval $(g_0^*, g_0^{**}]$ is lower compared to the PS and mixed duopoly cases. From equation (16) we know that total investment in innovation $\widehat{I}^{ps}$ from the basic setup is increasing in g. In the extended setup total investment for the interval $(g_0^*, g_0^{**}]$ is

$$\widehat{I}^{pse} = \frac{3}{8} + \frac{7}{8}(g_0 + I_l)^* - g_0$$

which is decreasing in g_0 for a given $(g_0 + I_l)^*$. Since $\widehat{I}^{ps} = \widehat{I}^{pse}$ at $g_0 = g_0^*$, then $\widehat{I}^{ps} > \widehat{I}^{pse}$ for the interval $(g_0^*, g_0^{**}]$.

From the proof of Proposition 4 we know that if $g_0 > \widehat{\widehat{g}}_0$ then $\widehat{I}^{mix} > \widehat{I}^{ps}$. Since $\widehat{\widehat{g}}_0 < g_0^*$ for all $I_H > 0$ and $\gamma \in (0,1)$ then we have that $\widehat{I}^{mix} > \widehat{I}^{ps}$for $(g_0^*, g_0^{**}]$. We have shown above that for this interval $\widehat{I}^{ps} > \widehat{I}^{pse}$ then $\widehat{I}^{mix} > \widehat{I}^{pse}$.