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Network Competition: Workhorse Resurrection

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Abstract

I generalize the workhorse model of network competition (Armstrong, 1998; Laffont, Rey and Tirole, 1998a,b) to include income effects in call demand. Income effects imply that call demand depends also on the subscription fee, not only on the call price. In the standard case of differentiated networks, weak income effects are enough to deliver results in line with stylized facts: The networks have an incentive to agree on high mobile termination rates to soften competition. They charge a higher price for calls outside (off-net) than inside (on-net) the network. This vindicates the use of (a perturbation of) the workhorse model of network competition.

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1 Introduction

Authorities remain sceptical to network competition despite recent years' market growth and the significant benefits telecommunications have brought to consumers and producers over the course of the years.[1] A main concern are the *termination rates* the operators charge for connecting calls from other networks. By agreeing on high reciprocal termination rates, the networks can jointly commit to high call prices because of higher marginal call costs. Interconnection agreements between networks are legally enforceable because network externalities render interconnection desirable. For the fear of market power in the termination markets, authorities routinely cap termination rates, even for small networks. A common requirement, at least in Europe, is that termination rates not exceed estimated long run incremental cost.

Based upon the seminal contributions on network competition (Armstrong, 1998; Laffont, Rey and Tirole, 1998a,b) one would conclude that regulatory concern about excessive termination rates is exaggerated. To wit, the workhorse model shows that the existence of termination profit creates an incentive to increase termination rates. However, with high termination rates it is also more profitable to slash the subscription fee and attract more customers: A higher market share means that the network can save on call costs because a larger share of outgoing calls then terminates inside the network. In the workhorse model, increased termination profit and lower subscription fees exactly cancel out, leaving network profit independent of the termination rate (Laffont, Rey and Tirole, 1998a). There is no incentive to collude on the termination rate if it does not affect profit. In particular, the networks should not oppose to lowering their termination rates whenever the regulator calls for it. However, this is *not* how networks normally respond to tighter regulation. On the contrary, they vigorously oppose any reduction in termination rates. The observation that networks strongly resist termination regulation while the model predicts them to be indifferent, constitutes a *profit neutrality puzzle*.

The basic model assumes that the networks charge uniform prices for all calls. When all calls cost the same, consumers do not care about the size of the network they belong to. Size becomes important for the choice of network whenever networks price discriminate between calls inside the own network (on-net) and calls to other networks (off-net). If on-net calls are cheaper that off-net calls, as is usually the case, consumers minimize call expenditures by subscribing to the largest network - even if both networks charge the same price for calls and subscriptions. The larger is the network, the more advantageous it is to belong to it. Lowering the subscription fee becomes extra profitable to the individual network in this case of tariff-mediated network externalities (Laffont, Rey and Tirole, 1998b) because increased network size attracts additional customers. This network multiplier effect reinforces competition for subscribers and drives down equilibrium subscription profit. However, the network multiplier effect is weaker the cheaper are off-net calls in relation to on-net calls. The networks therefore soften competition by lowering termination rates, even below marginal termination cost (Gans and King, 2001). Based on these results, one would not expect colluding networks to oppose to cost-based price caps because the

[1] In a sample of 21 OECD countries, Röller and Waverman (2001) attribute one third of economic growth over the period 1970-90 to telecommunications infrastructure investments.

caps would not be binding to them. Rather, the welfare problem seems to be inferior termination rates. Collusion should also imply off-net prices below on-net prices, reflecting comparatively lower marginal call costs off-net than on-net. Yet the networks do complain about regulation, and off-net prices typically are higher than on-net prices. The discrepancy between predicted prices and observed prices under call price discrimination constitutes a *off-net price puzzle*.

I examine the robustness of the profit neutrality and off-net price puzzles by generalizing the workhorse - A-LRT - model of network competition to allow for income effects in call demand. In the presence of income effects, call demand depends also on the subscription fee and not only on call prices. Income effects open a channel through which high termination rates *soften* competition for subscribers, namely by lowering the marginal utility of income. Subscription demand depends on the consumer net surplus each network offers its customers. Consumer net surplus includes call utility, the cost of calls and the cost of the subscription. The lower is the marginal utility of income, the less important is the size of the subscription fee for the choice of network and the softer is competition for subscribers. An increase in the termination rate raises the marginal off-net call cost which is passed on to consumers through the off-net price. The more expensive are calls, the lower is the marginal utility of income. Thus, a higher termination rate means a lower marginal utility of income and by implication softer competition. Profit neutrality is a knife-edge result. Even the slightest income effect tips the scales in favour of high termination rates. Under uniform call prices, the networks generally collude by setting excessive termination rates, except in the special case of zero income effects when they are indifferent to the choice of termination rate.

The network externalities that arise under call price discrimination complicate the analysis because subscription demand is not necessarily uniquely defined anymore. Most of the papers on network competition correct for this problem by considering differentiated networks. When networks are differentiated, the price difference between off-net and on-net calls plays little role for the choice of network: The network multiplier effect is near insignificant. Then, even weak income effects are enough to turn the standard result around. The networks now jointly profit from setting a termination rate above the marginal cost of termination. In equilibrium, off-net prices are higher than on-net prices.

To summarize: In the standard case of differentiated networks, there is a model "ε-income effects" away from the A-LRT model which does not lead to counter-factual predictions of the termination rates and call prices. The puzzling profit neutrality and off-net price results of the workhorse model are non-robust to the inclusion of income effects in call demand. This vindicates the use of (a perturbation of) the workhorse model of network competition.

The puzzling predictions of the basic model have stimulated extensions of the workhorse model in many directions. Most recently, Hurkens and López (2010) analyze the importance of consumer expectations. They show that networks jointly profit from agreeing on excessive termination rates if consumers have passive expectations. Passive expectations means that consumers neglect the network multiplier effect when they choose network, which softens competition for consumers. All that matters for termination rates is to maximize termination profit.[2] Fully re-

[2]Passive expectations are related to the notion of competing in utilities instead of prices. When networks

sponsive consumers (as in A-LRT) and completely passive consumers (as in Hurkens and López, 2010) represent two extreme representations of consumer expectations. An intermediate stand is to assume that every subscriber only takes the actions of some other customers into account - consumers belong to so called "calling clubs". The smaller is the calling club, the weaker is competition for subscribers and the higher is the termination rate (Hoernig et al., 2009).[3]

Jullien et al. (2010) assume that a proportion of subscribers are "light" users. Light users hold subscriptions only because they value incoming calls and do not make outgoing calls. Remember that the networks can save on call costs by cutting the subscription fee and have a larger share of costs terminated inside the network. This incentive is weaker if a proportion of the network's subscribers do not make any outgoing calls. Thus, termination rates are higher when a fraction of the subscribers are light users.[4]

Armstrong and Wright (2009) consider network competition when there is a fixed-line network with locked-in subscribers in addition to mobile operators competing for mobile subscribers. As in the workhorse model, the mobile operators would like to set low termination rates between themselves to soften competition for subscribers, but charge a high termination rate from the fixed-line operator to exercise vertical market power. Upholding higher rates for fixed-line than mobile termination is impossible if the fixed-line operator can bypass termination by relaying calls via the competitor's mobile network. If fixed-line termination profit is sufficiently important and arbitrage possibilities prevent price discrimination, even mobile call termination is priced above marginal cost.[5]

The above papers represent substantial departures from the workhorse model by changing the assumptions of how consumers form expectations, considering heterogenous calling patterns, introducing fixed-line networks, and so forth. The present paper complements the existing literature by generalizing the workhorse model to include income effects in call demand. A small perturbation in this direction is all it takes to overturn the puzzling results of the workhorse model.

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compete in utilities (or consumer net surplus) they guarantee their subscribers a certain surplus independently of the number of subscribers. Then, network size does not matter to consumers even in the case of call price discrimination. Passive expectations and competition in utilities are not equivalent. In the latter case, the network has to adjust the pricing plan to account for changes in market share and keep surplus constant (Calzada and Valletti, 2008)

[3]This result rests on the assumption that the members of a calling club do *not* coordinate the choice of network. In case of coordination, calling clubs have no effect on the optimal termination rate (Calzada and Valletti, 2008; Gabrielsen and Vagstad, 2008).

[4]A heterogenous calling pattern is by itself not enough to overturn profit neutrality; see Dessein (2003) and Hahn (2004). Also, one can include call externalities and still maintain profit neutrality (Jeon et al., 2004; Berger, 2005). When the total market size is growing, the networks generally profit from a termination rate below termination cost (Dessein, 2003; Armstrong and Wright, 2009).

[5]There are a number of other circumstances under which profit neutrality fails; see Armstrong (2002) for an elaborate discussion. The termination rate affects profit if the networks are asymmetric (De Bijl and Peitz, 2002; Carter and Wright, 2003; Armstrong and Wright, 2009). In fact, asymmetric networks may fail altogether in reaching an agreement. Also, if networks compete in dimensions other than price, for example quality, they might benefit from a high termination rate in order to curb investments (Valletti and Cambini, 2005).

2 Uniform Call Prices: The Profit Neutrality Puzzle

The Model I generalize the workhorse model by Armstrong (1998) and Laffont, Rey and Tirole (1998a and b), henceforth A-*LRT*, to allow for income effects in call demand. A continuum of consumers with unit measure are uniformly distributed on the unit interval. Each consumer subscribes to one of two networks located at each end of the interval. I assume in this section that all calls have the same price, whereas the next section allows networks to price discriminate between calls inside (on-net) and outside (off-net) one's own network. The call pattern is balanced: Every subscriber to network $i = 1, 2$ places q_i calls at the price $p_i \geq 0$ per call to every other subscriber to maximize utility $U(q_i) + Z(y_i)$, subject to the budget constraint $p_i q_i + y_i + t_i \leq I$. Call utility features constant elasticity: $U(q) = (1 - 1/\eta)q^{1-1/\eta}$, with $\eta > 1$. Consumption y of the numeraire good renders utility $Z(y) = y - \varepsilon y^2/2$, where $\varepsilon \geq 0$. The workhorse, A-LRT model, features quasi-linear utility: $\varepsilon = 0$. Denote by t_i the subscription fee, and let I be exogenous income.

Utility maximization yields call demand $D_i = D(p_i, t_i)$, demand $Y_i = Y(p_i, t_i)$ for the numeraire good and a shadow price of the budget constraint $\Lambda_i = \Lambda(p_i, t_i)$. A difference between this model and A-LRT is that call demand now decreases in the subscription fee t_i and not only in the call price p_i; see the Appendix for the details. The consumer net surplus in network i is

$$v_i = V(p_i, t_i) = U(D(p_i, t_i)) + Z(Y(p_i, t_i)) + \Lambda(p_i, t_i)(I - p_i D(p_i, t_i) - Y(p_i, t_i) - t_i). \quad (1)$$

The consumer located at $k \in [0, 1]$ derives utility $v_0 + v_1 - \tau k$ from subscribing to network 1 and utility $v_0 + v_2 - \tau|1 - k|$ of subscribing to network 2, where v_0 is the utility of holding a subscription, whereas τ is the virtual transportation cost and a measure of horizontal differentiation. The customer base of network i equals

$$S_i = \alpha(v_i, v_j) = \frac{1}{2} + \frac{v_i - v_j}{2\tau}, \; i \neq j = 1, 2,$$

when all consumers belong to one network or the other. The market is fully covered ($S_1 + S_2 = 1$), if the two networks offer similar tariffs ($v_i - v_j$ is small), or the networks are sufficiently differentiated (τ is large). I employ the standard assumption that τ is sufficiently high to render the market fully covered.

The profit of network i under uniform call prices equals

$$\Pi_i = S_i[(p_i - S_i c - (1 - S_i)(c_o + a))D_i + t_i - f] + S_i(1 - S_i)(a - c_t)D_j,$$

where c_t (c_o) is the marginal cost of call termination (origination), $c = c_t + c_o$, and $f \geq 0$ is the per-subscriber cost. The network derives its profits from three sources. The first term inside the brackets is the profit on outgoing calls, which is positive if the call price exceeds the perceived marginal call cost $S_i c + (1 - S_i)(c_o + a)$ - a weighted average of calls inside and outside the network. Second, the network earns a profit on subscriptions. The final term constitutes the termination profit, which is positive if the markup on termination is positive.

Analysis Increasing the call price p_i leads to higher profits for a given customer base and a given number of outgoing calls. This is the first term in marginal profit below. However, the price increase comes at the cost of fewer subscribers and less outgoing calls:

$$\begin{aligned} \frac{\partial \Pi_i}{\partial p_i} &= S_i D_i + \frac{\partial S_i}{\partial p_i}[(p_i - S_i c - (1-S_i)(c_o + a))D_i + t_i - f] \\ &+ S_i\left(p_i - S_i c - (1-S_i)(c_o+a)\right)\frac{\partial D_i}{\partial p_i} \\ &+ \frac{\partial S_i}{\partial p_i} S_i (a - c_t) D_i + \frac{\partial S_i}{\partial p_i}(S_j - S_i)(a - c_t) D_j \end{aligned} \tag{2}$$

The first term on the last line term constitutes a cost composition effect. As the number of subscribers goes down, more calls are terminated outside than in inside the network. The cost composition effect is negative whenever off-net calls are more costly than on-net calls ($a \geq c_t$). The final term is the marginal effect on termination profit. Fewer subscribers tends to reduce subscription profit, but is mitigated by the fact that the number of incoming calls goes up. The second effect dominates the first if the network is large and termination markup positive. Thus, termination profit tends to balance market shares. Increasing the subscription fee t_i has similar effects:

$$\begin{aligned} \frac{\partial \Pi_i}{\partial t_i} &= S_i + \frac{\partial S_i}{\partial t_i}[(p_i - S_i c - (1-S_i)(c_o + a))D_i + t_i - f] \\ &+ S_i\left(p_i - S_i c - (1-S_i)(c_o+a)\right)\frac{\partial D_i}{\partial t_i} \\ &+ \frac{\partial S_i}{\partial t_i} S_i (a - c_t) D_i + \frac{\partial S_i}{\partial t_i}(S_j - S_i)(a - c_t) D_j. \end{aligned} \tag{3}$$

The network optimally sets call prices equal to the perceived marginal call cost, $c + (1 - S_i)(a - c_t)$, so as to maximize the social surplus inside the network and then uses the subscription fee to balance the loss of subscribers against surplus extraction. Lemma 1 generalizes the existence and uniqueness results (Proposition 7) in Laffont, Rey and Tirole (1998a) to the case of income effects:

Lemma 1 *Assume that each network charges a uniform price for calls. When the utility of subscribing to a network (v_0) is not too small, the degree of substitutability ($1/2\tau$) between the two networks is not too high, and the income effect (ε) is not too strong, there exists a unique and symmetric equilibrium. The call price equals perceived marginal call cost, $c + (a - c_t)/2$, and the subscription fee T satisfies:*

$$\frac{T-f}{T} = \frac{1}{-\frac{\partial S_i}{\partial t_i} 2T} - \frac{1}{2}\frac{(a-c_t)}{T} D(c + (a - c_t)/2, T). \tag{4}$$

Proof: See the Appendix.

The subscription fee T satisfies a modified Ramsey rule. The equilibrium elasticity of subscription demand with respect to the subscription fee

$$-\frac{\partial S_i}{\partial t_i} 2T = \frac{\Lambda(c + (a - c_t)/2, T)T}{\tau}$$

is a measure of the intensity of competition for subscribers. The lower is the elasticity of subscription demand, the higher is the equilibrium subscription fee, all else equal. Obviously, subscription elasticity is lower the stronger is the degree of network differentiation (the higher is τ), because then prices matter less for the choice of network. Second, subscription elasticity is lower the lower is the marginal utility of income (Λ_i) because the subscription fee then is less important for consumer net surplus. The Ramsey rule is corrected by the cost composition effect. Setting a low subscription fee and gaining a high market share is more profitable if off-net calls are more expensive on-net calls because the network then can save on call costs.

The networks choose the reciprocal termination rate a to maximize industry profit, which under symmetry is equivalent to maximizing network profit

$$\pi(a) = \tfrac{1}{2}(T(a) - f) + \tfrac{1}{4}(a - c_t) D(c + (a - c_t)/2, T(a)),$$

which consists entirely of subscription profit and termination profit since outgoing calls are priced at perceived marginal call cost. By agreeing on a higher termination rate, the two networks affect termination profit as well as subscription profit:

$$\pi'(a) = \tfrac{1}{2}T'(a) + \tfrac{1}{4}[D_j + (a - c_t)(\tfrac{1}{2}\tfrac{\partial D_j}{\partial p_j} + \tfrac{\partial D_j}{\partial t_j}T'(a))].$$

Each network runs a termination *deficit* whenever the termination rate lies below the marginal termination cost ($a \leq c_t$). If the subscription fee is increasing in the termination rate ($T'(a) \geq 0$), raising the termination rate unequivocally lowers the termination deficit (because $\partial D_j/\partial p_j < 0$ and $\partial D_j/\partial t_j \leq 0$; see the Appendix) and simultaneously increases the subscription profit. Thus, setting a termination rate below marginal termination cost is profitable only if the subscription fee decreases sufficiently fast in the termination rate. Differentiation of the equilibrium subscription fee (4) yields

$$T'(a) = \frac{-\frac{1}{2}\frac{\partial}{\partial p_i}\left(-\frac{\partial S_i}{\partial t_i}2T\right) - \frac{1}{2}(D_j + \frac{1}{2}(a - c_t)\frac{\partial D_j}{\partial p_j})(\frac{\partial S_i}{\partial t_i}2T)^2}{(\frac{\partial S_i}{\partial t_i}2T)^2\left(1 + \frac{1}{2}(a - c_t)\frac{\partial D_j}{\partial t_j} + \frac{1}{4\tau}\frac{\partial \Lambda_i}{\partial t_i}(\frac{\partial S_i}{\partial t_i})^2\right)}.$$

Increasing the termination rate affects the subscription fee through two channels.[6] An anti-competitive effect pulls in the direction of a higher subscription fee. Increasing the termination rate softens competition for subscribers because the marginal utility of income goes down and

[6] Note that the denominator is strictly positive for all $a \leq c_t$ because $\partial D_j/\partial t_j \leq 0$ and $\partial \Lambda_i/\partial t_i \geq 0$ with equality if and only if $\varepsilon = 0$, see the Appendix.

thereby the subscription elasticity:

$$\frac{\partial}{\partial p_i}\left(-\frac{\partial S_i}{\partial t_i}2T\right)\frac{\partial p_i}{\partial a}=\frac{1}{2}\frac{\partial \Lambda_i}{\partial p_i}\frac{T}{\tau}=\frac{1}{2}\frac{\varepsilon(\eta-1)U''(D_i)\,D_i}{\varepsilon p_i^2-U''(D_i)}\frac{T}{\tau}\leq 0.$$

Second, a higher termination rate reinforces the cost composition effect and tends to lower the equilibrium subscription fee. The anti-competitive effect does not appear in the workhorse model because subscription elasticity there is independent of the termination rate (formally: $\Lambda_i = 1$ for $\varepsilon = 0$). The cost composition effect exactly offsets the effect on termination profit, which renders profit independent of the termination rate. In the more general case of non-zero income effects, the anti-competitive effect is just big enough to pull in favour of high termination rates:

Proposition 1 *Assume that the conditions of Lemma 1 hold, so that there exists a unique and symmetric equilibrium. Then, network profit is independent of the termination rate if and only if the income effect is zero ($\varepsilon = 0$). In the presence of income effects ($\varepsilon > 0$), any profit maximizing access price (if it exists) lies strictly above the marginal cost of termination.*

Proof: See the Appendix.

To gain additional insight into the mechanism driving profit neutrality, return to the A-LRT model, i.e. assume that there are no income effects. Let $v(a) = V(c + (a - c_t)/2, T(a))$ be consumer net surplus in symmetric equilibrium given the termination rate a. Define $\zeta(v(a)) \equiv (\partial\alpha/\partial v_i|_{v_1=v_2=v(a)})2v(a)$, the equilibrium subscription elasticity with respect to consumer net surplus. With quasi-linear preferences, the shadow price of the budget constraint equals unity ($\Lambda = 1$), and the equilibrium subscription fee solves:

$$T = f + \frac{v(a)}{\zeta(v(a))} - \frac{1}{2}(a - c_t)\,D(c + (a - c_t)/2).$$

Substituting the subscription fee above into consumer net surplus $v(a)$ and the profit function $\pi(a)$ yields after simplifications

$$v(a) = \frac{\zeta(v(a))}{1+\zeta(v(a))}W(a),\; 2\pi(a) = \frac{1}{1+\zeta(v(a))}W(a), \tag{5}$$

where

$$W(a) = U(D(c + (a - c_t)/2)) + I - cD(c + (a - c_t)/2) - f$$

is social surplus net of the utility of holding a subscription (v_0) and of the cost of horizontal differentiation ($\min\{k\tau; \tau(1-k)\}$). Social surplus is divided between the consumers and the industry in proportion to the subscription elasticity $\zeta(v(a))$. Most of the surplus goes to the consumers whenever subscription demand is elastic because of an intense competition for subscribers. Conversely, the networks extract most of the surplus under inelastic subscription demand because competition is weak in this case. The networks affect by their choice of access price both the size $W(a)$ of the social surplus to be divided and the share of that surplus the networks receive through the effect on competition $\zeta(v(a))$.[7] Under profit neutrality, the intensity of competition changes in exact proportion with social surplus. To see the fundamental property behind this result, divide $2\pi(a)$ by $v(a)$ in (5) and rewrite: $\pi(a) = v(a)/2\zeta(v(a))$. Obviously, profit neutrality holds if and only if equilibrium subscription elasticity is proportional to consumer net surplus, i.e. $\zeta(v(a)) = kv(a)$ for some $k > 0$. The Hotelling model features proportional subscription demand at symmetric prices: $\zeta(v) = v/\tau$ for all v, and therefore profit neutrality follows.[8]

Profit neutrality is a knife-edge result because it hinges on equilibrium subscription elasticity being exactly proportional to consumer net surplus. Introducing even very small income effects breaks the proportionality and therefore profit neutrality. With income effects, social surplus grows faster than the intensity of competition for low termination rates, and so the profit maximizing termination rate is above the marginal cost of termination.

3 Call Price Discrimination: The Off-Net Price Puzzle

The Model I now generalize the model in the previous section by allowing the networks to price discriminate between calls within the network (on-net) and calls outside the network (off-net). Price discrimination creates network externalities in the sense that the optimal choice of network now depends also on the size of the network and not only on prices. Every subscriber to network $i = 1, 2$ places q_i^{on} calls at the price p_i^{on} per call to every subscriber in the same network (on-net), and q_i^{off} calls at the price p_i^{off} per call to every subscriber in network $j \neq i$ to maximize utility $S_i U(q_i^{on}) + S_j U(q_i^{off}) + Z(y_i)$ and subject to the budget constraint $S_i p_i^{on} q_i^{on} + S_j p_i^{off} q_i^{off} + y_i + t_i \leq I$.

Utility maximization yields on-net demand $D_i^{on} = D^{on}(\mathbf{p}_i, t_i, S_i)$, off-net demand $D_i^{off} = D^{off}(\mathbf{p}_i, t_i, S_i)$, demand $Y_i = Y(\mathbf{p}_i, t_i, S_i)$ for the numeraire good and a shadow price of the subscription fee $\Lambda_i = \Lambda(\mathbf{p}_i, t_i, S_i)$ when all consumers have a subscription, $S_1 + S_2 = 1$, and $\mathbf{p}_i = (p_i^{on}, p_i^{off})$ is the call-price profile of network i. Because of the income effect, on-net and off-net calls are substitutes, call demand decreases in the subscription fee and is ambiguous with respect to changes in the customer base; see the Appendix. Define

$$u_i^{on} = u^{on}(\mathbf{p}_i, t_i, S_i) = U\left(D^{on}(\mathbf{p}_i, t_i, S_i)\right) - \Lambda(\mathbf{p}_i, t_i, S_i) p_i^{on} D^{on}(\mathbf{p}_i, t_i, S_i)$$

[7] The socially optimal choice of access charge is c_t when no income effects are present. Differentiate: $W'(a) = U'(D_i)\frac{1}{2}\frac{\partial D_i}{\partial p_i} - c\frac{1}{2}\frac{\partial D_i}{\partial p_i} = \frac{1}{4}(a - c_t)\frac{\partial D_i}{\partial p_i}$, where I have used $U'(D_i) = p_i = c + (a - c_t)/2$. Social surplus $W(a)$ is single-peaked in a and reaches its global maximum at c_t because $\partial D_i / \partial p_i < 0$ and $W'(c_t) = 0$.

[8] More generally, all models in which market share is determined by the difference in consumer net surplus, $S_i = g(v_i - v_j)$, feature proportional subscription demand: $\zeta(v) = 2g'(0)v$. The random utility model first used by Dessein (2003) for the duopoly case and extended by Calzada and Valletti (2008) to the general n network case belongs to this class of models: $S_i = (1 + \sum_{j \neq i} e^{-\frac{1}{\gamma}\{v_i - v_j\}})^{-1}$, and $\zeta(v) = \frac{1}{\gamma}\frac{n-1}{n}v$. However, profit neutrality does not imply that subscription demand is a function of the differences in consumer net surplus. For example, $S_i = g((v_i/v_j)^{v_j} - 1)$ does not have this property, but still is proportional: $\zeta(v) = 2g'(0)v$.

the indirect utility of reaching an on-net subscriber in network i, and let $u_i^{off} = u^{off}(\mathbf{p}_i, t_i, S_i)$ be the similarly defined indirect utility of reaching an off-net subscriber from network i. Consumer net surplus in network i is

$$\begin{aligned} v_i &= V(\mathbf{p}_i, t_i, S_i) = S_i u^{on}(\mathbf{p}_i, t_i, S_i) + (1 - S_i) u^{off}(\mathbf{p}_i, t_i, S_i) + Z(Y(\mathbf{p}_i, t_i, S_i)) \\ &\quad + \Lambda(\mathbf{p}_i, t_i, S_i)(I - t_i - Y(\mathbf{p}_i, t_i, S_i)), \end{aligned}$$

when all consumers belong to one network or the other. Under the standard assumption of differentiated networks,

$$S_i = \alpha(V(\mathbf{p}_i, t_i, S_i), V(\mathbf{p}_j, t_j, 1 - S_i)) \tag{6}$$

uniquely defines subscription demand S_i in rational expectations equilibrium as a function of call prices $(\mathbf{p}_i, \mathbf{p}_j)$ and subscription fees (t_i, t_j). The profit of network i equals

$$\Pi_i = S_i[S_i\,(p_i^{on} - c)\,D_i^{on} + S_j(p_i^{off} - a - c_o)D_i^{off} + t_i - f] + S_i S_j\,(a - c_t)\,D_j^{off}.$$

Owing to price discrimination, call profit can now be split into the profit of outgoing on-net calls and the profit on outgoing off-net calls. Termination profit and subscription profit adds to network profit, as under uniform pricing.

Analysis By increasing the on-net price, the network earns a higher revenue per on-net call, but at the cost of a smaller number of subscribers and less on-net calls per subscriber. These three effects constitute the three first terms below:

$$\begin{aligned} \frac{\partial \Pi_i}{\partial p_i^{on}} &= S_i^2 D_i^{on} + \frac{\partial S_i}{\partial p_i^{on}}[S_i\,(p_i^{on} - c)\,D_i^{on} + S_j(p_i^{off} - a - c_o)D_i^{off} + t_i - f] \\ &+ S_i^2\,(p_i^{on} - c)\,\frac{\partial D_i^{on}}{\partial p_i^{on}} + \frac{\partial S_i}{\partial p_i^{on}}[(p_i^{on} - c)\,D_i^{on} - (p_i^{off} - a - c_o)D_i^{off}] \\ &+ \frac{\partial S_i}{\partial p_i^{on}}(S_j - S_i)\,(a - c_t)\,D_j^{off} + S_i\left[S_i\,(p_i^{on} - c)\,\frac{\partial D_i^{on}}{\partial S_i}\frac{\partial S_i}{\partial p_i^{on}}\right. \\ &+ \left. S_j(p_i^{off} - a - c_o)\left(\frac{\partial D_i^{off}}{\partial p_i^{on}} + \frac{\partial D_i^{off}}{\partial S_i}\frac{\partial S_i}{\partial p_i^{on}}\right) - S_j\,(a - c_t)\,\frac{\partial D_j^{off}}{\partial S_j}\frac{\partial S_i}{\partial p_i^{on}}\right]. \end{aligned} \tag{7}$$

The second term on the second line is a composition effect, same as under uniform pricing: Fewer subscribers means that relatively more calls are terminated off-net. The composition effect could be positive or negative depending on the profitability of on-net calls relative to off-net calls. The first term on the third line is marginal termination profit. Increasing the on-net price generally affects demand for all types of calls through the budget constraint. The remaining terms characterize these income effects. Raising the off-net price p_i^{off} and the subscription fee

t_i have similar effects. For example:

$$\begin{aligned}
\frac{\partial \Pi_i}{\partial t_i} &= S_i + \frac{\partial S_i}{\partial t_i}[S_i\,(p_i^{on} - c)\,D_i^{on} + S_j(p_i^{off} - a - c_o)D_i^{off} + t_i - f] \\
&+ \frac{\partial S_i}{\partial t_i}[(p_i^{on} - c)\,D_i^{on} - (p_i^{off} - a - c_o)D_i^{off}] \\
&+ \frac{\partial S_i}{\partial t_i}(S_j - S_i)\,(a - c_t)\,D_j^{off} + S_i\left[S_i\,(p_i^{on} - c)\left(\frac{\partial D_i^{on}}{\partial t_i} + \frac{\partial D_i^{on}}{\partial S_i}\frac{\partial S_i}{\partial t_i}\right)\right. \\
&+ \left. S_j(p_i^{off} - a - c_o)\left(\frac{\partial D_i^{off}}{\partial t_i} + \frac{\partial D_i^{off}}{\partial S_i}\frac{\partial S_i}{\partial t_i}\right) - S_j\,(a - c_t)\,\frac{\partial D_j^{off}}{\partial S_j}\frac{\partial S_i}{\partial t_i}\right].
\end{aligned} \tag{8}$$

Lemma 2 generalizes the existence and uniqueness results (Proposition 5) in Laffont, Rey and Tirole (1998b) to the case of income effects:

Lemma 2 *Assume that both networks price discriminate between on-net and off-net calls. When the utility of subscribing to a network (v_0) is not too small, the degree of substitutability ($1/2\tau$) between the two networks is not too high, and the income effect (ε) is not too strong, there exists a unique and symmetric equilibrium. Call prices equal marginal call cost: $P^{on} = c$ and $P^{off} = a + c_o$. The subscription fee satisfies:*

$$\frac{T - f}{T} = \frac{1}{-\frac{\partial S_i}{\partial t_i}2T} + \frac{1}{4}\frac{(a - c_t)}{T}\frac{\partial D_j^{off}(c, a + c_o, T, 1/2)}{\partial S_j}. \tag{9}$$

Proof: See the Appendix.

The network optimally sets call prices at marginal call cost to maximize the social surplus inside the network and then uses the subscription fee to balance the loss of subscribers against surplus extraction. The optimal subscription fee satisfies a modified Ramsey rule. The composition effect vanishes compared to the subscription fee (4) under uniform pricing: The network does not care about a larger fraction of outgoing calls being terminated off-net when the markup on all outgoing calls is zero. Instead, an expression related to termination profit shows up. A higher market share of the other network affects demand for off-net (as well as on-net) calls in that network through the income effect.

Just as was the case under uniform pricing, the subscription fee and termination profit are the sole sources of network profit

$$\pi(a) = \tfrac{1}{2}(T(a) - f) + \tfrac{1}{4}\,(a - c_t)\,D^{off}(c, a + c_o, T(a), 1/2)$$

because outgoing calls are priced at marginal cost. The marginal effect on industry profit of increasing the reciprocal termination rate a thus equals:

$$\pi'(a) = \tfrac{1}{2}T'(a) + \tfrac{1}{4}[D_j^{off} + (a - c_t)\,(\tfrac{\partial D_j^{off}}{\partial p_j^{off}} + \tfrac{\partial D_j^{off}}{\partial t_j}T'(a))]. \tag{10}$$

Whether setting an termination rate below marginal termination cost is profitable depends on the sensitivity of the subscription fee to changes in the termination rate. If the subscription fee is non-decreasing in the termination rate ($T'(a) \geq 0$), it is profitable to increase the termination rate from any point below marginal termination cost ($a \leq c_t$) because then termination deficit falls and subscription profit increases. Only if the subscription fee falls sufficiently in the termination rate can it be profitable to set a termination rate below the marginal cost of termination. The key to understanding termination rate collusion under call price discrimination therefore lies in exploring the sensitivity of the subscription fee to changes in the termination rate.

The equilibrium elasticity of subscription demand with respect to the subscription fee equals

$$-\frac{\partial S_i}{\partial t_i}2T = \frac{\Lambda_i T}{\tau - (u_i^{on} - u_i^{off})} \tag{11}$$

under call price discrimination. As under uniform pricing, subscription elasticity is lower the stronger is the degree of network differentiation (the higher is τ) and the lower is marginal utility of income (Λ_i). Under call price discrimination, an additional network multiplier effect intensifies competition. A lower subscription fee means a higher market share, all else equal. A larger market share implies in turn that a larger fraction of every subscribers' calls are terminated on-net. If it is more valuable to connect with someone in the same network compared to someone in the other network ($u_i^{on} > u_i^{off}$) a higher market share further accentuates the benefit of belonging to that network. In the presence of network effects, there is a lot to gain in terms of extra subscribers by lowering the subscription fee because the flow of consumers multiplies itself. This process is faster the larger is the net benefit of on-net calls compared to off-net calls (measured by $u_i^{on} - u_i^{off}$).

Importantly, the networks affect competition for subscribers through the choice of termination rate because a higher off-net price lowers the marginal value of income ($\partial\Lambda_i/\partial p_i^{off} \leq 0$) and strengthens the network effect ($\partial(u_i^{on} - u_i^{off})/\partial p_i^{off} > 0$). The net effect is ambiguous in general and depends on the magnitude of the income effect and the degree of network differentiation:

$$\frac{\partial}{\partial p_i^{off}}\left(-\frac{\partial S_i}{\partial t_i}2T\right) = \frac{T\Lambda_i}{p_i^{off}(\tau - (u_i^{on} - u_i^{off}))}\left(\underbrace{\frac{\partial(u_i^{on} - u_i^{off})}{\partial p_i^{off}}\frac{p_i^{off}}{\tau - (u_i^{on} - u_i^{off})}}_{+} + \underbrace{\frac{\partial\Lambda_i}{\partial p_i^{off}}\frac{p_i^{off}}{\Lambda_i}}_{-/0}\right).$$

The elasticity of the network effect is weak in the standard case of differentiated networks (τ is high). Nonetheless, the networks soften competition by setting a termination rate below the marginal cost of termination in the workhorse (A-LRT) model (see e.g. Gans and King, 2001) because marginal utility of income then is constant ($\Lambda_i = 1$). Even small income effects are enough to overturn this result, and render it profitable for the networks to agree on a termination rate above the marginal cost of termination:

Proposition 2 *Assume that the conditions of Lemma 2 hold, so that there exists a unique and* *symmetric equilibrium under call price discrimination. The profit maximizing access price lies below the marginal cost of termination if the income effect is zero. Then, the off-net price is lower than the on-net price. In the presence of income effects and if the networks are differentiated, the profit maximizing access price instead lies above the marginal cost of termination. In this second case, the off-net price is higher than the on-net price (If* $\varepsilon = 0$*, then* $P^{on} - P^{off} = c_t - a > 0$*. If* $\varepsilon > 0$*, but small, and* $\tau\varepsilon > 2/(\eta - 1)$*, then* $P^{off} - P^{on} = a - c_t > 0$*).*

Proof: See the Appendix.

The above results on access price collusion under uniform prices (Proposition 1) and under call price differentiation (Proposition 2) are derived under standard assumptions. The underlying assumption of differentiated networks is quite common in the literature because network differentiation allows a high degree of freedom in the choice of termination rates, while preserving concavity of the profit function and uniqueness of subscription demand. Propositions 1 and 2 demonstrate that it then only takes a minor departure from the workhorse, A-LRT, model to reverse the puzzling results and instead deliver results in line with regulatory concern and the pricing policies the networks actually use.

4 Conclusion

I generalize the workhorse model of network competition (Armstrong, 1998; Laffont, Rey and Tirole, 1998,a,b) to allow for income effects in call demand. In the standard case of differentiated networks, weak income effects are enough to deliver results in line with stylized facts: The networks have an incentive to agree on high mobile termination rates to soften competition, and not the other way around. The networks set off-net prices that are higher than on-net prices, and not the other way around. This vindicates the use of (a perturbation of) the workhorse model of network competition.

With income effects, call demand is sensitive to changes in disposable income, for example through a reduction in the subscription fee. The existence of income effects in call demand, and therefore the relevance of the model, is testable. I leave empirical examination of the model and the assumptions underlying it for future research.

Appendix

Call demand

Uniform call prices Construct the Lagrangian $\mathcal{L}_i = U(q_i) + Z(y_i) + \lambda_i(I - t_i - p_i q_i - y_i)$, where λ_i is the Lagrangian multiplier associated with the budget constraint. Total differentiation of the optimality conditions $U'(D_i) - \Lambda_i p_i = 0$, $Z'(Y_i) - \Lambda_i = 0$, $\Lambda_i(I - t_i - p_i D_i - Y_i) = 0$ and

$\Lambda_i \geq 0$ yield:

$$\begin{bmatrix} U''(D_i) & 0 & -p_i \\ 0 & Z''(Y_i) & -1 \\ -\Lambda_i p_i & -\Lambda_i & 0 \end{bmatrix} \begin{bmatrix} dD_i \\ dY_i \\ d\Lambda_i \end{bmatrix} = \begin{bmatrix} \Lambda_i dp_i \\ 0 \\ \Lambda_i (D_i dp_i + dt_i) \end{bmatrix}$$

under the assumption of a fully covered market, $S_1 + S_2 = 1$. Apply Cramer's rule to the optimality conditions:

$$\frac{\partial D_i}{\partial p_i} = \tfrac{\Lambda_i - Z''(Y_i)p_i D_i}{U''(D_i)+Z''(Y_i)p_i^2} < 0, \qquad \frac{\partial D_i}{\partial t_i} = \tfrac{-Z''(Y_i)p_i}{U''(D_i)+Z''(Y_i)p_i^2} \leq 0,$$
$$\frac{\partial \Lambda_i}{\partial p_i} = \tfrac{Z''(Y_i)U''(D_i)(\eta-1)D_i}{U''(D_i)+Z''(Y_i)p_i^2} \leq 0, \quad \frac{\partial \Lambda_i}{\partial t_i} = \tfrac{-Z''(Y_i)U''(D_i)}{U''(D_i)+Z''(Y_i)p_i^2} \geq 0.$$

Call Price Discrimination Construct the Lagrangian

$$\mathcal{L}_i = S_i U(q_i^{on}) + S_j U(q_i^{off}) + Z(y_i) + \lambda_i (I - t_i - S_i p_i^{on} q_i^{on} - S_j p_i^{off} q_i^{off} - y_i),$$

where $\lambda_i \geq 0$ is the Lagrangian multiplier associated with the budget constraint.

The three first-order conditions $U'(D_i^{on}) - \Lambda_i p_i^{on} = 0$, $U'(D_i^{off}) - \Lambda_i p_i^{off} = 0$, $Z'(Y_i) - \Lambda_i = 0$, along with the the complementary slackness condition $\Lambda_i (I - t_i - S_i p_i^{on} D_i^{on} - S_j p_i^{off} D_i^{off} - Y_i) = 0$ define four non-linear equations in the four unknowns $(D_i^{on}, D_i^{off}, Y_i, \Lambda_i)$. By total differentiation of the optimality conditions:

$$\begin{bmatrix} U''(D_i^{on}) & 0 & 0 & -p_i^{on} \\ 0 & U''(D_i^{off}) & 0 & -p_i^{off} \\ 0 & 0 & Z''(Y_i) & -1 \\ -\Lambda_i S_i p_i^{on} & -\Lambda_i S_j p_i^{off} & -\Lambda_i & 0 \end{bmatrix} \begin{bmatrix} dD_i^{on} \\ dD_i^{off} \\ dY_i \\ d\Lambda_i \end{bmatrix} = \begin{bmatrix} \Lambda_i dp_i^{on} \\ \Lambda_i dp_i^{off} \\ 0 \\ \Lambda_i (S_i D_i^{on} dp_i^{on} + S_j D_i^{off} dp_i^{off} \\ + dt_i + (p_i^{on} D_i^{on} - p_i^{off} D_i^{off}) dS_i) \end{bmatrix},$$

under the assumption of a fully covered market, $S_1 + S_2 = 1$. The determinant of the bordered Hessian is $-\Lambda_i U''(D_i^{on}) U''(D_i^{off}) H_i$, where

$$H_i = 1 + \frac{Z''(Y_i) S_i (p_i^{on})^2}{U''(D_i^{on})} + \frac{Z''(Y_i)(1 - S_i)(p_i^{off})^2}{U''(D_i^{off})} \geq 1.$$

Define the total call elasticity

$$\eta_i^{on} = -\frac{dD_i^{on}}{d(\Lambda_i p_i^{on})} \frac{\Lambda_i p_i^{on}}{D_i^{on}} = -\frac{U'(D_i^{on})}{U''(D_i^{on}) D_i^{on}},$$

and let η_i^{off} be similarly defined. In Laffont, Rey and Tirole (1998a,b), $U(q) = (1-\eta^{-1})^{-1} q^{1-\eta^{-1}}$, which implies a constant elasticity $\eta_i^{on} = \eta_i^{off} = \eta > 1$.

By repeated application of Cramer's rule, the following comparative statics results are straightforward:

$$\frac{\partial D_i^{on}}{\partial p_i^{on}} U''(D_i^{on}) H_i = \Lambda_i - Z''(Y_i)(S_i p_i^{on} D_i^{on} + \eta_i^{off} S_j p_i^{off} D_i^{off}),$$
$$\frac{\partial D_i^{on}}{\partial p_i^{on}} U''(D_i^{on}) H_i = \Lambda_i - Z''(Y_i)(S_i p_i^{on} D_i^{on} + \eta_i^{off} S_j p_i^{off} D_i^{off}),$$
$$\frac{\partial D_i^{off}}{\partial p_i^{on}} \frac{U''(D_i^{off})}{p_i^{off}} H_i = \frac{\partial \Lambda_i}{\partial p_i^{on}} H_i = Z''(Y_i)(\eta_i^{on} - 1) S_i D_i^{on},$$
$$\frac{\partial D_i^{on}}{\partial p_i^{off}} \frac{U''(D_i^{on}) H_i}{p_i^{on}} = \frac{\partial \Lambda_i}{\partial p_i^{off}} H_i = Z''(Y_i)(\eta_i^{off} - 1) S_j D_i^{off},$$
$$\frac{\partial D_i^{off}}{\partial p_i^{off}} U''(D_i^{off}) H_i = \Lambda_i - Z''(Y_i)(\eta_i^{on} S_i p_i^{on} D_i^{on} + S_j p_i^{off} D_i^{off}),$$
$$\frac{\partial D_i^{on}}{\partial t_i} \frac{U''(D_i^{on}) H_i}{p_i^{on}} = \frac{\partial D_i^{off}}{\partial t_i} \frac{U''(D_i^{off}) H_i}{p_i^{off}} = \frac{\partial \Lambda_i}{\partial t_i} H_i = -Z''(Y_i),$$
$$\frac{\partial D_i^{on}}{\partial S_i} \frac{U''(D_i^{on}) H_i}{p_i^{on}} = \frac{\partial D_i^{off}}{\partial S_i} \frac{U''(D_i^{off}) H_i}{p_i^{off}} = \frac{\partial \Lambda_i}{\partial S_i} H_i = Z''(Y_i)(p_i^{off} D_i^{off} - p_i^{on} D_i^{on}).$$

Proof of Lemma 1

Subtract $(\partial \Pi_i / \partial t_i) D_i$ from $\partial \Pi_i / \partial p_i$, using $\partial v_i / \partial p_i = (\partial v_i / \partial t_i) D_i = -\Lambda_i D_i$, to get

$$\frac{\partial \Pi_i}{\partial p_i} - \frac{\partial \Pi_i}{\partial t_i} D_i = S_i (p_i - S_i c - (1 - S_i)(c_o + a)) \left(\frac{\partial D_i}{\partial p_i} - D_i \frac{\partial D_i}{\partial t_i} \right).$$

Assuming that $S_i > 0$, the right-hand side of this expression is positive for all $p_i < S_i c + (1 - S_i)(c_o + a)$ and negative for all $p_i > S_i c + (1 - S_i)(c_o + a)$ because

$$\frac{\partial D_i}{\partial p_i} - \frac{\partial D_i}{\partial t_i} D_i = \frac{\Lambda_i}{U''(D_i) + Z_i'' p_i^2} < 0.$$

Therefore, the first-order conditions $\partial \Pi_i / \partial p_i = 0$ and $\partial \Pi_i / \partial t_i = 0$ are satisfied at $S_i > 0$ only if $P_i = S_i c + (1 - S_i)(c_o + a)$. At an interior optimum, therefore, outgoing calls are priced at weighted marginal call cost. In symmetric equilibrium, $S_i = 1/2$, so $P_1 = P_2 = c + (a - c_t)/2$.

Existence of a unique and symmetric equilibrium At $P_i = c + (1 - S_i)(a - c_t)$ and $\varepsilon = 0$, the profit function Π_i is strictly quasi-concave in t_i, the subscription fees are strategic complements and the reaction functions have a slope which is positive, but below unity; see Laffont, Rey and Tirole (1998a). By continuity, these properties extend also to the case with non-zero but weak income effects ($\varepsilon \gtrsim 0$). Hence, there exists a unique and symmetric equilibrium, provided v_0 is large, τ is large and ε is small. Given $P(a) = c + (a - c_t)/2$, the symmetric subscription fee solves the first-order condition

$$\frac{\partial \Pi_i}{\partial t_i} = 0 \Leftrightarrow T = f + \frac{1}{-2\frac{\partial S_i}{\partial t_i}} - (a - c_t)\, D(c + (a - c_t)/2, T)/2,$$

which can be rewritten on the Ramsey form (4). ■

Proof of Proposition 1

If v_0 is large, τ is large and ε is small, but positive, the equilibrium subscription fee is given by (4). In the Hotelling model $-2\partial S_i/\partial t_i = \Lambda(p_i, t_i)/\tau$, hence the symmetric subscription fee in this case solves:

$$T = f + \tau/\Lambda(c + (a - c_t)/2, T) - (a - c_t)\, D(c + (a - c_t)/2, T)/2.$$

By implicit differentiation:

$$T'(a) = -\frac{\left(\frac{\tau}{\Lambda^2}\frac{\partial \Lambda_i}{\partial p_i} + D_i + \frac{1}{2}(a - c_t)\frac{\partial D_i}{\partial p_i}\right)}{2 + \frac{2\tau}{\Lambda_i^2}\frac{\partial \Lambda_i}{\partial t_i} + (a - c_t)\frac{\partial D_i}{\partial t_i}},$$

which is of ambiguous sign. Plugging the expression for $T(a)$ into the equilibrium profit function, industry profit simplifies to $2\pi(a) = \tau/\Lambda(c + (a - c_t)/2, T(a))$. By substituting in the above expression for $T'(a)$:

$$2\pi'(a) = -\frac{\tau}{\Lambda_i^2}\left(\frac{1}{2}\frac{\partial \Lambda_i}{\partial p_i} + \frac{\partial \Lambda_i}{\partial t_i}T'(a)\right) = \frac{\tau}{\Lambda_i^2}\frac{\frac{\partial \Lambda_i}{\partial t_i}D_i - \frac{\partial \Lambda_i}{\partial p_i} + \frac{1}{2}(a - c_t)\left(\frac{\partial D_i}{\partial p_i}\frac{\partial \Lambda_i}{\partial t_i} - \frac{\partial Di}{\partial t_i}\frac{\partial \Lambda_i}{\partial p_i}\right)}{2 + \frac{2\tau}{\Lambda_i^2}\frac{\partial \Lambda_i}{\partial t_i} + (a - c_t)\frac{\partial D_i}{\partial t_i}}.$$

Recall, $\partial \Lambda_i/\partial t_i \geq 0$ and $\partial D_i/\partial t_i \leq 0$, so the denominator is strictly positive for all $a \leq c_t$. All terms in the numerator are zero whenever $\varepsilon = 0$ because then $\partial \Lambda_i/\partial t_i = \partial \Lambda_i/\partial p_i = \partial D_i/\partial t_i = 0$. For $\varepsilon > 0$, the first two terms in the numerator are strictly positive because then $\partial \Lambda_i/\partial t_i > 0$ and $\partial \Lambda_i/\partial p_i < 0$. The second term in the numerator is non-negative for all $a \leq c_t$ because $\partial \Lambda_i/\partial t_i \geq 0$, $\partial D_i/\partial p_i < 0$, $\partial \Lambda_i/\partial p_i \leq 0$ and $\partial D_i/\partial t_i \leq 0$. Thus, $\pi'(a) > 0$ for all $a \leq c_t$ and $\varepsilon > 0$. ■

Proof of Lemma 2

Marginal cost pricing of outgoing calls By total differentiation of (6):

$$dS_i = -\frac{\frac{\partial \alpha}{\partial v_i}\Lambda_i(S_i D_i^{on} dp_i^{on} + S_j D_i^{off} dp_i^{off} + dt_i)}{1 - \frac{\partial \alpha}{\partial v_i}(u_i^{on} - u_i^{off}) + \frac{\partial \alpha}{\partial v_j}(u_j^{on} - u_j^{off})}. \tag{12}$$

Take advantage of the fact that $\partial S_i/\partial p_i^{on} = (\partial S_i/\partial t_i)S_iD_i^{on}$ and $\partial S_i/\partial p_i^{off} = (\partial S_i/\partial p_i^{off})S_jD_i^{off}$, subtract (8) from (7) and $\partial \Pi_i/\partial t_i$ from $\partial \Pi_i/\partial p_i^{off}$ to get:

$$\begin{aligned}\frac{\partial \Pi_i}{\partial p_i^{on}} - S_iD_i^{on}\frac{\partial \Pi_i}{\partial t_i} &= S_i^2\left(p_i^{on}-c\right)\left(\frac{\partial D_i^{on}}{\partial p_i^{on}} - S_iD_i^{on}\frac{\partial D_i^{on}}{\partial t_i}\right)\\ &+ S_iS_j(p_i^{off}-a-c_o)\left(\frac{\partial D_i^{off}}{\partial p_i^{on}} - S_iD_i^{on}\frac{\partial D_i^{off}}{\partial t_i}\right)\end{aligned} \tag{13}$$

$$\begin{aligned}\frac{\partial \Pi_i}{\partial p_i^{off}} - S_jD_i^{off}\frac{\partial \Pi_i}{\partial t_i} &= S_i^2\left(p_i^{on}-c\right)\left(\frac{\partial D_i^{on}}{\partial p_i^{off}} - S_jD_i^{off}\frac{\partial D_i^{on}}{\partial t_i}\right)\\ &+ S_iS_j(p_i^{off}-a-c_o)\left(\frac{\partial D_i^{off}}{\partial p_i^{off}} - S_jD_i^{off}\frac{\partial D_i^{off}}{\partial t_i}\right)\end{aligned} \tag{14}$$

Under the assumption of $S_i > 0$, the right-hand side of (13) is strictly negative if $p_i^{on} > c$ and $p_i^{off} \leq a + c_o$ and strictly positive if $p_i^{on} < c$ and $p_i^{off} \geq a + c_o$ because

$$\begin{aligned}\frac{\partial D_i^{on}}{\partial p_i^{on}} - S_iD_i^{on}\frac{\partial D_i^{on}}{\partial t_i} &= \frac{\Lambda_i - Z''(Y_i)\eta_i^{off}S_jp_i^{off}D_i^{off}}{U''\left(D_i^{on}\right)H_i} < 0\\ \frac{\partial D_i^{off}}{\partial p_i^{on}} - S_iD_i^{on}\frac{\partial D_i^{off}}{\partial t_i} &= \frac{Z''(Y_i)\eta_i^{on}S_ip_i^{off}D_i^{on}}{U''(D_i^{off})H_i} \geq 0.\end{aligned}$$

At optimum $\partial \Pi_i/\partial p_i^{on} = \partial \Pi_i/\partial t_i = 0$, so $P_i^{on} \neq c$ is part of a profit maximizing two-part tariff only if $sgn\{P_i^{on} - c\} = sgn\{P_i^{off} - a - c_o\}$.

Add (13) and (14):

$$\begin{aligned}\frac{\partial \Pi_i}{\partial p_i^{on}} + \frac{\partial \Pi_i}{\partial p_i^{off}} - (S_iD_i^{on} + S_jD_i^{off})\frac{\partial \Pi_i}{\partial t_i} &= S_i^2\left(P_i^{on}-c\right)\left(\frac{\partial D_i^{on}}{\partial p_i^{on}} + \frac{\partial D_i^{on}}{\partial p_i^{off}} - (S_iD_i^{on} + S_jD_i^{off})\frac{\partial D_i^{on}}{\partial t_i}\right)\\ &+ S_iS_j(P_i^{off}-a-c_o)\left(\frac{\partial D_i^{off}}{\partial p_i^{on}} + \frac{\partial D_i^{off}}{\partial p_i^{off}} - (S_iD_i^{on} + S_jD_i^{off})\frac{\partial D_i^{off}}{\partial t_i}\right) = 0\end{aligned}$$

at optimum. After some algebraic manipulations:

$$\frac{\partial D_i^{on}}{\partial p_i^{on}} + \frac{\partial D_i^{on}}{\partial p_i^{off}} - (S_iD_i^{on} + S_jD_i^{off})\frac{\partial D_i^{on}}{\partial t_i} = \frac{\Lambda_i(1 + \varepsilon S_jP_i^{off}(P_i^{on} - P_i^{off})/U''(D_i^{off}))}{U''\left(D_i^{on}\right)H_i}. \tag{15}$$

As shown by Laffont, Rey and Tirole (1998b), profit maximization implies $P_i^{on} = c$, $P_i^{off} = a + c_o$ for $\varepsilon = 0$. By continuity, $\lim_{\varepsilon\to 0}\varepsilon S_jP_i^{off}(P_i^{on} - P_i^{off})/U''(D_i^{off}) = \lim_{\varepsilon\to 0}\varepsilon S_j(a + c_o)(c_t - a)/U''\left(U'^{-1}(a+c_o)\right) = 0$. Thus, for ε small, but positive, profit maximization implies (15) negative. By an analogous argument, even $\partial D_i^{off}/\partial p_i^{on} + \partial D_i^{off}/\partial p_i^{off} - (S_iD_i^{on} + S_jD_i^{off})(\partial D_i^{off}/\partial t_i)$ is negative for ε small. Thus, any equilibrium satisfying $P_i^{on} \neq c$ implies $sgn\{P_i^{on} - c\} = -sgn\{P_i^{off} - a - c_o\}$ for ε small but positive, which contradicts the necessary condition $sgn\{P_i^{off} - a - c_o\} = sgn\{P_i^{on} - c\}$, previously established. Thus, for ε small, but positive: $P_i^{on} = c$ and by implication also $P_i^{off} = a + c_o$.

Existence of a unique and symmetric equilibrium At $P_i^{on} = c$, $P_i^{off} = a + c_o$ and $\varepsilon = 0$, the profit function Π_i is strictly quasi-concave in t_i, the subscription fees are strategic complements and the reaction functions have a slope which is positive, but below unity; see Laffont, Rey and Tirole (1998b). By continuity, these properties extend also to the case with non-zero, but weak income effects. Hence, there exists a unique and symmetric equilibrium, provided v_0 is large, τ is large and ε is small. Outgoing calls are priced at effective marginal cost, $P_i^{on} = c$ and $P_i^{off} = a + c_o$. The subscription fee solves the first-order condition

$$\frac{\partial \Pi_i}{\partial t_i} = 0 \Leftrightarrow T = f + \frac{1}{-2\frac{\partial S_i}{\partial t_i}} + \frac{(a - c_t)}{4}\frac{\partial D_j^{on}}{\partial S_j},$$

which can be rewritten on the Ramsey form (9).

Proof of Proposition 2

First, some preliminaries. In the Hotelling model $\partial S_i / \partial t_i = -\Lambda_i/(2\tau + u_i^{off} - u_i^{on} + u_j^{off} - u_j^{on})$; see (12). Hence, the symmetric subscription fee solves:

$$T = f + \frac{\tau + u^{off}(c, a + c_o, T, 1/2) - u^{on}(c, a + c_o, T, 1/2)}{\Lambda(c, a + c_o, T, 1/2)} + \frac{1}{4}(a - c_t)\frac{\partial D_j^{off}(c, a + c_o, T, 1/2)}{\partial S_j}.$$

By total differentiation of the subscription fee:

$$T'(a) = \frac{-\left(\tau + U(D_i^{off}) - U\left(D_i^{on}\right)\right)\frac{\partial \Lambda_i}{\partial p_i^{off}} - \Lambda_i^2 D_i^{off} + \frac{\Lambda_i^2}{4}\left(\frac{\partial D_j^{off}}{\partial S_j} + (a - c_t)\frac{\partial^2 D_j^{off}}{\partial S_j \partial p_j^{off}}\right)}{\left(\tau + U(D_i^{off}) - U\left(D_i^{on}\right)\right)\frac{\partial \Lambda_i}{\partial t_i} + \Lambda_i^2\left(1 - \frac{1}{4}(a - c_t)\frac{\partial^2 D_j^{off}}{\partial S_j \partial t_j}\right)}.$$

Zero income effects ($\varepsilon = 0$) Now $\Lambda_i = 1$, $\partial \Lambda_i / \partial p_i^{off} = \partial \Lambda_i / \partial t_i = 0$, $\partial D_j^{off} / \partial S_j = 0$ and $\partial^2 D_j^{off} / \partial S_j \partial p_j^{off} = \partial^2 D_j^{off} / \partial S_j \partial t_j = 0$, so $T'(a) = -D_i^{off}$, which implies

$$2\pi'(a) = \tfrac{1}{2}[D_j^{off} + (a - c_t)\frac{\partial D_j^{off}}{\partial p_j^{off}}] + T'(a) = \tfrac{1}{2}[(a - c_t)\frac{\partial D_j^{off}}{\partial p_j^{off}} - D_j^{off}] < 0 \text{ for all } a \geq c_t,$$

where I have used symmetry, $D_i^{off} = D_j^{off}$, and $\partial D_j^{off} / \partial p_j^{off} < 0$.

Non-zero income effects ($\varepsilon > 0$) Since $\partial D_j^{off} / \partial p_j^{off} < 0$ and $\partial D_j^{off} / \partial t_j \leq 0$, it is sufficient that $T'(a) \geq 0$ for all $a \leq c_t$ to render $2\pi'(a) > 0$ for all $a \leq c_t$, see (10). I need to show that $T'(a) > 0$ for ε sufficiently low and τ sufficiently large. Let $\tau = \overline{\tau}\varepsilon^{-1}$, where $\overline{\tau}(\eta - 1) > 2$.

Recall, $\partial\Lambda_i/\partial p_i^{off} = -\varepsilon(\eta-1)S_jD_i^{off}/H_i$ and $\partial\Lambda_i/\partial t_i = \varepsilon/H_i$. Plug into $T'(a)$ above to get

$$T'(a) = \frac{\left(\overline{\tau}+\varepsilon U(D_i^{off})-\varepsilon U\left(D_i^{on}\right)\right)(\eta-1)D_i^{off}-2H_i\Lambda_i^2\left(D_i^{off}-\frac{1}{4}\frac{\partial D_j^{off}}{\partial S_j}-\frac{1}{4}\left(a-c_t\right)\frac{\partial^2 D_j^{off}}{\partial S_j\partial p_j^{off}}\right)}{2\left(\overline{\tau}+\varepsilon U(D_i^{off})-\varepsilon U\left(D_i^{on}\right)\right)+2H_i\Lambda_i^2\left(1-\frac{1}{4}\left(a-c_t\right)\frac{\partial^2 D_j^{off}}{\partial S_j\partial t_j}\right)}.$$

By inspection of the comparative statics in this appendix, $\partial D_j^{off}/\partial S_j \to 0$, $\partial^2 D_j^{off}/\partial S_j\partial p_j^{off} \to 0$ and $\partial^2 D_j^{off}/\partial S_j\partial t_j \to 0$, as $\varepsilon \to 0$. Moreover, $H_i \to 1$, $\Lambda_i \to 1$ and $D_i^{off} \to U'^{-1}(a+c_o)$ as $\varepsilon \to 0$. Thus, $\lim_{\varepsilon\to 0} T'(a) = (\overline{\tau}(\eta-1)-2)U'^{-1}(a+c_o)/2(\overline{\tau}+1) > 0$ in this case. ■

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