To Surcharge or Not To Surcharge?  
A Two-Sided Market Perspective of the No-Surcharge Rule

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To Surcharge or Not To Surcharge?
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Abstract

In Electronic Payment Networks (EPNs) the No-Surcharge Rule (NSR) requires that merchants charge the same final good price regardless of the means of payment chosen by the customer. In this paper, we analyze a three-party model (consumers, merchants, and proprietary EPNs) to assess the impact of a NSR on the electronic payments system, in particular, on competition among EPNs, network pricing to merchants and consumers, EPNs’ profits, and social welfare.

We show that imposing a NSR has a number of effects. First, it softens competition among EPNs and rebalances the fee structure in favor of cardholders and to the detriment of merchants. Second, we show that the NSR is a profitable strategy for EPNs if and only if the network effect from merchants to cardholders is sufficiently weak. Third, the NSR is socially (un)desirable if the network externalities from merchants to cardholders are sufficiently weak (strong) and the merchants’ market power in the goods market is sufficiently high (low). Our policy advice is that regulators should decide on whether the NSR is appropriate on a market-by-market basis instead of imposing a uniform regulation for all markets.

Keywords: Electronic Payment System, Market Power, Network Externalities, No-Surcharge Rule, Regulation, Two-sided Markets, MasterCard, Visa, American Express, Discover.

JEL: L13, L42, L80.

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1 Introduction

The No-Surcharge Rule (NSR). The NSR means that a merchant charges at most the same amount for a payment card transaction as for cash. If the merchant decides to apply a discount for payments in cash that discount cannot be extended to any specific card brand. If a merchant wants to offer a discount to a given card brand, then he must extend it to all the other comparable card brands. Economides (2009) compares the NSR as if “Coca-Cola were to impose the requirement that a can of Pepsi be sold at the same price as a can of Coke”, which would enhance the incentives for collusive behavior among companies.

Payment cards have been experiencing fast growth which has drawn attention to some of the contentious features of this industry, namely the NSR. In several countries, the NSR has been under examination by regulatory and competition authorities, central banks and courts. For example, in the U.S., on October 5th, 2010, Visa and MasterCard reached a settlement with the U.S. DOJ that allows merchants to reward consumers for paying with credit or debit cards that charge the merchant lower fees, while American Express Co. (AmEx) vowed to fight a government antitrust lawsuit. In early 2010, the Portuguese Government decided to make the NSR mandatory by law claiming consumer protection and that the use of electronic payments is more efficient than cash and thus should be protected. In other countries, such as Australia, the Netherlands, Sweden, and the United Kingdom, the NSR has been abolished (Prager et al., 2009). Critics of the NSR have claimed that it inefficiently encourages the use of more costly forms of payment (credit cards) over the less costly (cash), as well as more costly credit cards compared to less costly credit cards, leading to a “Gresham’s Law of Payments.”

Description of the paper: goals and results. In this paper, we investigate the impact of the NSR on competition and pricing among proprietary Electronic Payment Networks (EPNs), on EPNs’ profits and on social welfare. We also provide insights on the desirability of laws and contractual rules about surcharging payment cards. We base

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1 The payment cards industry includes credit, debit, and prepaid cards. For the purposes of this paper, we do not distinguish among types of payment cards.

2 Although infrequent, there have been cases where card payments were discounted relatively to cash, e.g. in Germany during the transition to the euro. Also in Argentina and Colombia since 2003 Governments have been providing VAT discounts to transactions processed with debit or credit cards.

3 Transactions done on electronic payment networks in the U.S. exceeded $1.7 trillion in 2002 (Schwartz & Vincent, 2004). In 2006, payment cards were used in 47 billion transactions for a total of $3.1 trillion (Shy & Wang, 2010). In 2008, debit and prepaid card purchases topped $3.285 trillion (almost a quarter of U.S. GDP).

our analysis on a three-party model, as described in figure 1, with consumers, merchants and profit-maximizing EPNs setting fees to both merchants and cardholders.

Figure 1: A three-party card network. See Economides (2009) for further details.

We start by deriving a series of results with elasticities characterizing the platform optimal pricing conditions with surcharge, and then examine the NSR’s impact on the elasticities. We show that: (i) the platform’s profit maximization problem can be decomposed in two parts: setting a total fee, i.e., the sum of the merchant fee with the cardholder average fee per transaction, and setting relative fees, i.e., which fraction of the total fee is paid by each type of end-user; (ii) consumers demand for electronic payment services become less elastic with respect to merchant fees under the NSR; and (iii) the total fee is inflated under the NSR.

In our second set of results, we derive and compare the EPN pricing and profit in the market equilibrium under surcharge versus under the NSR. We show that the NSR implementation (i) rebalances the fee structure in favor of cardholders and (ii) increases EPNs’ profits if and only if the network effect exerted by the number of merchants on cardholders’ utility is sufficiently weak. In a nutshell, the reasoning for these findings is as follows. The NSR makes cardholders less sensitive to merchant fees since merchant fee differences among EPNs cannot be translated into purchase price differences. Hence EPNs competition is softer in the merchant side of the market under the NSR. As a result, merchant fees rise and the number of merchants accepting electronic transactions declines. Since cardholders are sensitive to the number of merchants accepting card payments (the network effect), EPNs will then reduce membership fees. Moreover, if the network effect is sufficiently strong, cardholders’ demand will sharply decline with the reduction on the number of merchants and EPNs’ profits will decline.

5 In this example, AmEx chooses to charge the merchant a $3 fee for the $100 transaction, while cardholders do not pay any per transaction fee but may have to pay a membership fee. Note that the merchant receives a net value of $97 (= $100 – $3), that is, the purchase value discounted from the merchant fee.

6 Our analysis primarily addresses a closed network, but it may also characterize a four-party network if acquirers (issuers) are identical and perfectly competitive, while issuers (acquirers) are identical and collude when setting the fees to cardholders (merchants). One advantage of a three-party model is that we do not need to be concerned with the interchange fee (IF), which, in a four-party setup compensates the issuing bank each time cardholders use the card in a purchase.
In the welfare analysis, we analyze the NSR’s impact on the number of end-users of the electronic payment system and on the purchase price. We show that the NSR reduces payment card acceptance by merchants, expands the number of cardholders and raises the equilibrium price in the goods market. We close the set of welfare results with the surplus variations that the NSR implies to each group of end-users and to society as a whole. We show under which conditions the society is better off with the NSR implementation. The NSR will be socially (un)desirable if network effects from merchants to cardholders are sufficiently weak (strong) and merchants market power in the goods market is sufficiently high (low). In our framework, the NSR implementation raises the merchant fee and consequently reduces card acceptance. Thus, on the one hand, if network effects on cardholders’ utility are strong, the NSR destroys value in the cardholder side of the market. This is the case provided that the network size of card acceptance matters to cardholders and, according to previous results, under the NSR fewer stores accept payment cards. On the other hand, if merchants’ market power in the goods market is sufficiently high (e.g., under a monopoly) the price in the goods market is essentially defined by consumer willingness-to-pay. Thus, the increase in merchant fees (marginal cost of selling the good) due to the NSR is not passed-through to the purchase price of the good, but cardholders benefit because of a discount (or reward) on the cardholder fees. In this case, the NSR accomplishes implicitly the task of partially correcting the merchants’ market power distortion in the goods market.

We conclude the paper by discussing policy considerations and possible interventions on the electronic payment system with regard to the NSR imposition. We consider the pros and cons of forbidding the NSR versus no regulatory intervention emphasizing that one size policy does not fit all markets, since, in general, there are significant market power differences across goods and geographic markets within the same country. According to our welfare results, regulators should take into account the merchants’ market power in the goods and geographic markets and the extent of network effects and decide on the NSR on a market-by-market basis instead of imposing a rule common to all markets.

Background. Formal economic analysis of electronic payment systems was initiated by Baxter (1983) with an analysis of the NaBanCo litigation. The theoretical payment card literature has been growing, especially during the last decade, by addressing the issue of how costs of payment cards are and might be divided among EPNs, merchants and cardholders. The models considered in this literature point out that EPNs may charge fees significantly in excess of their costs to merchants and provide incentives to cardholders to increase card adoption and usage. To a great extent, this literature has not distinguished prepaid cards from debit or credit cards. Usually these models (e.g., Rochet & Tirole

\textsuperscript{7}See Frankel & Shampine (2006) for a summary on the NaBanCo case (National Bancard Corporation vs. Visa U.S. Inc.).
(R&T) (2002, 2003), Cabral (2006), Wright (2010)) focus on the adoption and usage of payment cards versus all other payment instruments and have showed that competition levels among merchants and among EPNs, along with consumer and merchant demand elasticities, are relevant factors in determining model outcomes.

EPNs are a type of two-sided markets. The two-sided markets literature has been employed to investigate the structure of fees paid by cardholders and merchants. This strand of literature combines the network economics literature, which studies how agents' utility changes with participation of other agents in the network, and the multiproduct firm literature, which investigates how firms choose prices when offering more than one product.

The seminal articles in two-sided markets by R&T (2003, 2006) and Armstrong (2006) investigate the determinants of the price balance between two groups of end-users (e.g., consumers and merchants) when each group exerts a network effect on the other, and both are intermediated by a platform (e.g., an EPN). Some of the discussed determinants of the price balance are: the possibility of multi-homing (access to more than one platform), platform differentiation, presence of same-side externalities, platform compatibility, per-transaction or lump-sum pricing and relative size of cross-group externalities. However, as far as we know, the two-sided markets literature has been silent about the NSR implications on platform fees, profits and welfare, since it assumes that end-users are not allowed to negotiate prices of platform services.

Chakravorti & Roson (2006) compare the welfare level when two networks operate as competitors and as a cartel. One of their findings corroborates the conclusion of R&T (2003) that network competition does not imply, from a social standpoint, a better or worse balance of fees between consumers and merchants. Chakravorti & Roson show that, in general, the welfare gain of a drop in the total network fee more than compensates the deterioration in the efficiency of the fee balance. Moreover, network competition unambiguously increases consumer and merchant surpluses.

Gans and King (2003) show that, under a general four-party model of a payment system, abolishing the NSR is one sufficient condition to reach the neutrality of the IF, i.e., variations in the IF do not lead to changes in consumers’ decisions on purchases, consumers’ and merchants’ adoption decisions and issuers’, acquirers’ or merchants’ profits. However, Gans and King did not do a welfare analysis.

Wright (2003) undertakes the welfare analysis of the NSR under two-merchant competition extremes: monopoly and perfect competition. The author shows that (i) the NSR is socially desirable when merchants operating in a monopoly EPN engage in price discrimination based on payment instruments, and (ii) under Bertrand competition among

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8See Chakravorti (2010) for an excellent review of the growing payment card literature and discussion of the impact of regulatory interventions on card adoption, usage, and welfare.
merchants, the social surplus does not change regardless the existence of the NSR. Wright explains that if merchants are monopolists, the imposition of the NSR prevents them from surcharging excessively. Therefore, the NSR increases social surplus. If merchants compete à la Bertrand, they pass to consumers the full benefits and costs associated with the means of payment used to complete the transaction. Hence, under the NSR, competitive merchants only accept cash or only accept card payments, and prices in the goods market are equal to the respective marginal cost net of benefits. Under surcharging, competitive merchants accept both types of payment and price discriminate. However, Wright did not consider competition among EPNs or intermediate cases of merchant market power.

Schwartz and Vincent (2006) investigates the NSR welfare distribution effects among cash users and card users when merchants are local monopolists. Although the authors allow for elastic demand in the goods market, they assume that consumers are exogenously divided between cash or card users. They conclude that the NSR harms cash users and merchants and is profitable to EPNs.

Our model differs from the existing literature in several aspects. As far as we know, we are the first to introduce the NSR analysis in a two-sided market environment. Articles studying the NSR have not considered network effects in the analysis, while in our results network effects play an important role. Also, we do not assume a specific market structure in the goods market. In fact, all market structures from perfect competition to monopoly can be used in our model. While in past literature the price for goods is derived assuming a given market structure, we assume that the price follows a reduced form that depends on the degree of competition among merchants. This approach allows us to test explicitly the impact of small market power changes in the goods market on the social desirability of the NSR.

2 The Three-party Card Payment Model

Consider a model of payment card network competition with three agent types: proprietary EPNs, consumers and merchants. There are three payment instruments: cash, as the default payment instrument accessible to all consumers and merchants at no cost, and two EPNs, EPN 1 and 2. Without loss of generality, cash payments are set to generate zero surplus both to payees and payors, whilst EPNs offer a service that may yield positive benefits for consumers and merchants. The elements of our model are as follows:

(i) Electronic Payment Networks, 1 and 2, are profit-maximizing and compete simultaneously and non-cooperatively in a two-sided market, charging membership fees $f_1$ and $f_2$ to the cardholders and transaction fees $m_1$ and $m_2$, to merchants, respectively. Card payments require the payee (merchant) and the payor (cardholder) to have a common
payment platform.

(ii) Consumers choose at most one of the two possible EPNs. Those who decide to make payments using an EPN are the cardholders, the remaining consumers pay all transactions in cash. Each consumer makes one transaction with each existing merchant, and therefore, the number of transactions in the economy is fixed regardless of the payment instruments used. Consumers are heterogeneous in their preferences towards EPNs. For example, AmEx and Discover may differ in terms of lines of credit or billing cycles, and consumers have idiosyncratic preferences concerning those characteristics.

Consumer $h$’s surplus of using EPN $i = 1, 2$ is private information (this prevents merchants from price discriminating among consumers) and defined by

$$ U^h_i = U^h_i (f_i, p_i, D^m_i), \quad (1) $$

where $D^m_i$ is the number of merchants on platform $i$, $p_i$ is the purchase price of a unit good when payment is processed through EPN $i$, and $f_i$ is the membership fee paid by consumers using platform $i$.

We assume that consumer surplus in (1) satisfies the following properties

(i) $\frac{\partial U^h_i}{\partial f_i} < 0$, (ii) $\frac{\partial U^h_i}{\partial p_i} < 0$, and (iii) $\frac{\partial U^h_i}{\partial D^m_i} > 0$. \quad (2)

Properties (i) and (ii) in (2) imply that consumers prefer lower membership fees and lower prices for goods. Intuitively, we can think of the unspent funds on goods and membership fee as reverting to some other activity (where payment cards cannot be used) generating surplus to cardholders. Property (iii) implies that cardholders prefer EPNs with larger acceptance. When choosing a payment instrument, a consumer equates his idiosyncratic surplus of using a card from EPN 1 against the surplus of using a card from EPN 2, and checks whether the highest of the two is indeed positive; otherwise a consumer chooses cash as his payment instrument and gets zero surplus.\[9\]

We assume that consumers know all prices before their card membership decisions and that cardholders make all payments by card to the extent that this is feasible. In other words, cardholders will only pay cash if the merchant does not accept payment through the EPN to which the consumer subscribes. This may be due to liquidity constraints or other properties coupled with card usage such as theft-insurance for goods purchased with the card, or even dispute-resolution protection by EPNs.

\[9\] As an example of a functional form for $U^h_i$ take

$$ U^h_i = (h^c_i - (p_i - p_0)) D^m_i - f_i $$

where $p_0$ denotes the price of the good in a cash transaction, and $h^c_i$ is the idiosyncratic preference of consumer $h$ for EPN $i$. It satisfies all properties in (2) for $h^c_i > p_i - p_0$. \[7\]
Each consumer buys one unit of the good per merchant. The price per unit of the good is \( p_i \) if the transaction is processed by EPN \( i \), or \( p_0 \) if it is a cash transaction. Regardless of how the transaction is effected, consumers are willing to pay \( v \) for the unit good.

Each cardholder subscribes to one EPN, that is, cardholders “single-home,” and pay a fixed (e.g., annual membership) fee, \( f_i \), if they use EPN \( i \), allowing them to make an unlimited number of transactions at zero fee per transaction. The single-homing hypothesis for cardholders is supported by Rysman (2007) that found empirical evidence in which, although cardholders in U.S. may hold payment cards from more than one EPN, most of them have a *top of the wallet* card, i.e., they prefer to use mainly one card. In our model, this is equivalent to assuming that the benefit of a second card is always lower than its membership fee \(^{[10]}\).

(iii) *Merchants* can multi-home, i.e., besides cash, they have the option to accept payments through both EPNs paying a per transaction charge according to the EPN employed on each transaction. We disregard possible steering strategies in which a merchant might decide to refuse a network not because its net benefit is negative but so as to induce consumers to choose another payment network in which the merchant has a higher net benefit. Without loss of generality, the marginal cost of producing the goods demanded by consumers is normalized to zero. Merchants bear the merchant fee as a supply cost for cashless transactions, while not facing costs for cash transactions.

Merchants are heterogeneous in their gross surpluses \( b \) for cashless transactions. However, for the sake of simplicity, we assume that the gross surplus \( b \) is independent of the EPNs. Merchant’s surplus \( b \) from a cashless transaction may arise from cash-handling costs’ reduction or from increased security. The *additional* surplus of a merchant, indexed by \( b \), who accepts electronic payments is given by,

\[
S^b = \sum_{i=1}^{2} \max \left\{ (p_i - p_0 + b - m_i) D^c_i; 0 \right\},
\]

where \( D^c_i \) denotes the number of cardholders on platform \( i \). Note that a merchant will accept to run transactions under an EPN as long \( p_i - p_0 + b - m_i \geq 0 \).

(iv) Under surcharge, the *equilibrium price in the goods market*, \( p_i \), is given by a weighted average\(^{[11]}\) of the net cost of selling the good, \( m_i - b \), and consumer’s willingness-to-pay, \( v \). For simplicity, we assume that the marginal cost of producing the good is zero and, \( m_i - b \) is the merchant cost net of benefit \( b \) of accepting the payment via EPN \( i \). Mathematically we have \( p_i = \beta v + (1 - \beta) (m_i - b) \). The price of the same transaction in cash is \( p_0 = \beta v \). Lemma 1 highlights the appropriateness of this simplification that will

\(^{[10]}\) The additional benefit of an EPN \( j \) card for an EPN \( i \) cardholder is based only on the number of merchants that accept EPN \( j \) and do not accept EPN \( i \).

\(^{[11]}\) The weight \( \beta \) measures merchants’ market power toward consumers in the goods market.
be particularly relevant in the welfare analysis (section 5). When surcharging is allowed, prices in the goods market may differ according to the EPN employed to complete the payment, i.e., \( p_1 \) may differ from \( p_2 \).

We use a set of general assumptions regarding the end-users demands (assumption 1 on consumers and assumption 2 on merchants below) and the equilibrium price in the goods market (reduced form solution in Lemma 1 below). The detailed description of the different agents and price determination in the goods market follows.

**Consumers.** Formally, consumers, (superscript \( c \) hereafter) demand function for EPN \( i \)'s services arises from the mass of consumers satisfying the following two inequalities: \( U^h_i \geq U^h_j \) and \( U^h_i \geq 0 \). Let consumers demand be represented by

\[
D^n_i \equiv D^n_i \left( f_i - \alpha f_j, S (p_i - \alpha p_j), D^m_i - \alpha D^m_j \right), \ i, j = 1, 2 \text{ and } i \neq j,
\]

where \( S \) is a dummy variable that takes value 1 if surcharge is allowed or 0 under the NSR and, \( 0 < \alpha < 1 \) measures to what extent EPNs 1, 2 are substitute payment instruments from the consumers’ standpoint. Equation (4) satisfies assumption 1 below.

**Assumption 1 (Consumers demand):** Consumers demand for EPN \( i \)'s services in (4) is a twice differentiable function decreasing in \( \Delta f_i \) and \( \Delta p_i \), and increasing in \( \Delta D^m_i \), where

\[
\begin{align*}
\Delta f_i & \equiv f_i - \alpha f_j, \\
\Delta p_i & \equiv S (p_i - \alpha p_j), \\
\Delta D^m_i & \equiv D^m_i - \alpha D^m_j.
\end{align*}
\]

Intuitively, assumption 1 means that when choosing an EPN, consumers compare the fee and price of purchasing goods using EPN \( i \), \( f_i \) and \( p_i \), against similar values of purchasing using EPN \( j \), \( f_j \) and \( p_j \). According to (i) and (ii) in (2), consumers demand for EPN \( i \) should decrease in \( \Delta f_i \) and \( \Delta p_i \) since \( U^h_i \geq U^h_j \) is harder to satisfy. For any fixed \( (f_j, p_j) \), \( U^h_i \geq 0 \) is harder to satisfy as \( f_i \) or \( p_i \) increase. If \( \alpha \) increases, the degree of substitution between EPNs from the consumers’ perspective will be higher (see Singh and Vives (1984)). Similar rationale applies to the goods price when \( S = 1 \), and to merchant acceptance coverage variations.

Cardholders take into account the cross-group externality captured by \( \Delta D^m_i \), that is, they care about the extent of merchant acceptance offered by each network. The larger the merchant acceptance by EPN \( i \) relatively to network \( j \), the larger will be the demand

\[12\text{Under the NSR, } \Delta p_i = 0, \text{ consumers only equate membership fees and the number of merchants accepting each EPN in their payment instrument decisions.}\]
for payment services of network $i$, *ceteris paribus*, since $U_i^h \geq U_j^h$ and $U_i^h \geq 0$ are then easier to satisfy given the definition of consumers utility in (1).

**The goods market equilibrium.** For simplicity we treat the equilibrium prices in the goods market as a reduced form solution $(p_0^*, p_1^*, p_2^*)$. When surcharging is allowed, the prices are given by

$$\begin{align*}
p_0^* &= \beta v, \\
p_1^* &= \beta v + (1 - \beta) (m_1 - b), \\
p_2^* &= \beta v + (1 - \beta) (m_2 - b),
\end{align*}$$

where $\beta \in [0, 1]$ denotes merchant market power in the goods market. When the NSR is imposed, the goods market price is the same regardless of the payment instrument chosen by the consumer, i.e., $p_0^{NSR} = p_1^{NSR} = \lambda_2 p_0^* + (1 - \lambda) p_1^* = \beta v + (1 - \beta) (1 - \lambda) (m_i - b)$ in the symmetric equilibrium, $\lambda \in [0, 1].$

The assumed reduced form solutions (5) to (7) are general in the sense that they can mimic, with an appropriate $\beta$, the price equilibria of standard Micro and IO models of firm competition. For example, if $\beta = 0$, then $(p_0^*, p_1^*, p_2^*) = (0, m_1 - b, m_2 - b)$, corresponding to the perfectly competitive market outcome in which prices equal the net marginal costs. If $\beta = 1$, then $(p_0^*, p_1^*, p_2^*) = (v, v, v)$ merchants have maximum market power and set prices equal to consumer’s maximum willingness-to-pay. In the case of duopolistic competition à la Hotelling, prices correspond to the sum of the net marginal cost, $m_i - b$, plus a transportation cost $t$. The analog to our reduced form pricing can be re-written as $p_i = \beta (v - (m_i - b)) + m_i - b$, $\beta$ set equal to $\beta = \frac{t}{v - (m_i - b)}$. In the Cournot oligopoly with $N$ firms, constant marginal costs $m_i - b$, and linear demand $P = v - bQ$, the equilibrium price is $P^* = \frac{1}{N+1} v + \frac{N}{N+1} (m_i - b)$, which corresponds to setting $\beta = \frac{1}{N+1}$ in the reduced form solution. Lemma 1 generalizes the application of the reduced form solution in the goods market equilibrium.

\[13\] If both EPNs increase their membership fees by one dollar, the total impact in the number of cardholders in the economy will be negative. From the definition in (4),

$$\frac{\partial D_C^*}{\partial f_i} + \frac{\partial D_C^*}{\partial f_j} + \frac{\partial D_C^*}{\partial f_j} = \left( \frac{\partial D_C^*}{\partial f_i} + \frac{\partial D_C^*}{\partial f_j} \right) (1 - \alpha) = 2 (1 - \alpha) \frac{\partial D_C^*}{\partial f_i} < 0,$$

since $\frac{\partial D_C^*}{\partial f_i} < 0$ by assumption 1 and $0 < \alpha < 1$.

\[14\]This is equivalent to saying that $p_1^*$ and $p_2^*$ are bounded, $p_i^* \in [m_i - b, v], \ i = 1, 2$. We assume that merchants do not face costs for setting multiple prices for the same product, i.e., there is no cost of surcharging.

\[15\]We assume that under the NSR the equilibrium price falls between $p_0^*$ and $p_1^*$. This reflects either the NSR, or even when merchants are allowed to surcharge, that they choose to not do so. Empirically we find that merchants do not usually set differential prices depending on the payment mean. Frankel (1998) calls this phenomenon *price coherence.*
Lemma 1 (Goods Market Reduced Form Solution): Consider market \( k \) characterized by (i) constant net marginal cost \( k \) of providing the good, (ii) consumer willingness-to-pay \( v \), and (iii) \( v > k \). For any level of competition among firms in the market there exists a unique \( \beta \in [0, 1] \) such that the equilibrium price \( p_k^* \) can be written as \( p_k^*(\beta) = \beta v + (1 - \beta) k \).

Proof: All proofs are in an appendix.

Merchants. Given the merchant fees \((m_1, m_2)\) and goods market prices \((p_0, p_1, p_2)\), merchants choose whether to request access either to EPN 1 or 2, multi-home by accepting both EPNs or accept cash only. Formally, merchants (superscript \( m \) hereafter) demand function for EPN \( i \) services corresponds to the mass of merchants that satisfies the non-negativity of the first argument in (3), i.e., \( p_i - p_0 + b - m_i \geq 0 \Leftrightarrow b \geq m_i - (p_i - p_0) \). Thus, merchants’ demand is

\[
D_i^m(m_i) \equiv \Pr (b \geq m_i - (p_i - p_0)) , \quad i = 1, 2 \tag{8}
\]

where \( b \) follows a distribution with support \([0, b]\). Equation (8) satisfies assumption 2 below.

Assumption 2 (Merchants demand): Merchants demand for EPN \( i \)'s services is a twice differentiable function such that \( D_i^m(m_i)' < 0 \).

Remark 1 (Merchants Demand): Merchants demand is defined by (8). Under the NSR, \( p_1^{NSR} = p_2^{NSR} = p_0^{NSR} \), condition (8) becomes \( \Pr (b \geq m_i^{*}\vert_{NSR}) \). Under surcharging \( p_i^* - p_0^* = (1 - \beta) (m_i^{*} - b) \). Therefore, \( \Pr (b \geq m_i^{*} - (p_i^* - p_0^*)) = \Pr (b \geq m_i^{*} - (1 - \beta) (m_i^{*} - b)) = \Pr (b \geq m_i^{*}) \), which is identical to the condition that defines merchants demand under the NSR. Hence, the merchants demand functional form is the same regardless of whether the NSR is imposed or not. Murphy and Ott (1977) suggest that cash customers impose more costs than card users on merchants’ profits. In fact, currently there are businesses that are no longer accepting cash.

Despite the fact that merchant demand functions are independent of the number of cardholders in each network, we still have cross-group network effects because the surplus of a merchant depends on the number of cardholders as defined in (3). In this aspect, our approach is similar to R&T (2003), where the total surplus of a merchant accepting EPN \( i \), with gross per transaction surplus \( b \) is \( (b - m_i) D_i^c \) depends on the number of cardholders \( D_i^c \).


\[17\] However, R&T (2003) assumed that the No-Surcharge Rule is always imposed.
Platforms. EPN $i$ chooses simultaneously and non-cooperatively the end-user fees, $f_i$ per cardholder and $m_i$ per transaction. Without loss of generality, assume that platforms have costs normalized to zero or alternatively interpret $f_i$ and $m_i$ as price-to-cost margins. Each merchant completes one transaction with each one of the cardholders, resulting in a total number of transactions processed by EPN $i$ of $D_i^c D_i^m$. Platform $i$ solves the following maximization problem.

\[
\max_{f_i, m_i} \Pi_i = f_i D_i^c + m_i D_i^m, \quad i = 1, 2 \quad \text{and} \quad i \neq j
\]  
subject to
\[
D_i^c = D_i^c (f_i - \alpha f_i, S (p_i (m_i) - \alpha p_j (m_j)), D_i^m - \alpha D_j^m) \quad \text{from (4)}
\]
\[
D_i^m = D_i^m (m_i) \quad \text{from (8)}
\]

A summary of the model’s notation is shown in table 1.

<table>
<thead>
<tr>
<th>Notation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p_i$</td>
<td>price of a unit of a good with payment processed under EPN $i$</td>
</tr>
<tr>
<td>$p_0$</td>
<td>price of a unit of a good when cash is used for payment</td>
</tr>
<tr>
<td>$f_i$</td>
<td>cardholder membership (annual) fee at EPN $i$</td>
</tr>
<tr>
<td>$m_i$</td>
<td>merchant fee per transaction processed under EPN $i$</td>
</tr>
<tr>
<td>$D_i^c$</td>
<td>number of cardholders on EPN $i$</td>
</tr>
<tr>
<td>$D_i^m$</td>
<td>number (mass) of merchants on EPN $i$</td>
</tr>
<tr>
<td>$S$</td>
<td>indicator variable taking value 1 if surcharge is allowed, 0 otherwise</td>
</tr>
<tr>
<td>$v$</td>
<td>consumer’s willingness-to-pay for a unit of a good</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>substitution degree among EPNs</td>
</tr>
<tr>
<td>$b$</td>
<td>merchant benefit of a cashless transaction relatively to cash</td>
</tr>
<tr>
<td>$\bar{b}$</td>
<td>highest value of $b$</td>
</tr>
<tr>
<td>$\beta$</td>
<td>merchant market power in the goods market</td>
</tr>
</tbody>
</table>

3 The Market Equilibrium

The market equilibrium concept used is the Nash equilibrium defined below, where $(f, m) \equiv (f_1, f_2; m_1, m_2)$.

Definition (Market Equilibrium): A market equilibrium is a pair of pairs $(f_i^*, m_i^*)$, $i = 1, 2$, such that $\forall i$, $(f_i^*, m_i^*)$ solves $\max \Pi_i$, defined in (9), subject to end-user demands $(D_i^c (f, m), D_i^m (m_i))$, (4) and (8), taking as given the fee choices $(f_j, m_j)_{j \neq i}$ of EPN $j$.
A main goal of the paper is to understand the impact of the NSR on platforms competition, pricing structure, and profits. The roadmap for this section is as follows. First we derive a series of results with elasticities characterizing the platforms optimal pricing conditions. Then, we verify the NSR’s impact on those elasticities. Our results show that (i) the EPNs’ profit maximization problem consists of two parts: setting the average total fee level per transaction, and setting the relative fees, (ii) under the NSR, consumers demand for EPNs becomes less elastic with respect to merchant fees, and (iii) under the NSR, the total fee is higher.

Second, once the optimal fee mechanism for networks is disassembled, we derive the market equilibrium pricing and profits. We compare market equilibrium fees and profits under surcharging versus under the NSR. We show that the NSR (i) rebalances the pricing structure in favor of cardholders, and (ii) increases platforms’ profits if and only if the network externality exerted by merchants over cardholders is sufficiently weak.

### 3.1 The Elasticity Rule

#### 3.1.1 The platforms’ optimal private solution

Lemma 2 shows the rule that profit maximizing platforms follow when choosing the pricing structure.

**Lemma 2 (Platforms’ Optimal Private Solution):** Profit maximizing platforms set fees according to the following rule,

\[
s_i + m_i = \frac{s_i}{\varepsilon^c} = \frac{m_i}{\varepsilon^m + \varepsilon^{c,m}}
\]

where

\[
s_i = \frac{f_i}{D_i^m}, \quad \varepsilon^c = -\frac{dD_i^c}{ds_i} \frac{s_i}{D_i^c}, \quad \varepsilon^{c,m} = -\frac{dD_i^c}{dm_i} \frac{m_i}{D_i^m},
\]

This result is reminiscent of a finding by R&T (2003), in the sense that it shows that the network’s maximization problem of choosing the optimal fees can be decomposed in two parts: (i) setting the (average) total fee level \( s_i + m_i \) and (ii) setting the relative fees ratio \( s_i/m_i \),

\[
\frac{s_i}{\varepsilon^c} = \frac{m_i}{\varepsilon^m + \varepsilon^{c,m}} \iff \frac{s_i}{m_i} = \frac{\varepsilon^c}{\varepsilon^m + \varepsilon^{c,m}}.
\]

The novelty on this result is the introduction of the effect of a variation in network size. The term \( \varepsilon^{c,m} \) in the EPN optimal pricing rule arises because the cardholders are sensitive to the number of merchants in each EPN and to their fees, as these are reflected
in the final goods prices. In R&T (2003) such interaction does not exist and therefore $\varepsilon^{c,m} = 0$.

### 3.1.2 Elasticity decomposition and the NSR impact

The introduction of the cross elasticity term $\varepsilon^{c,m}$ measures consumers demand variation with respect to changes in merchant fees. Therefore, the cross elasticity term plays an important role since it captures, among other effects, how consumers change their demand for network services in the presence of the NSR. To investigate the NSR impact on consumers’ demand, it is convenient to decompose the cross elasticity $\varepsilon^{c,m}$ in order to separate the NSR (pricing) effect from the remaining effects. Lemma 3 presents this decomposition.

**Lemma 3 (Elasticity decomposition):** The profit-maximizing rule (10) can be re-written as

$$ s_i + m_i = \frac{s_i}{\varepsilon^m} = \frac{m_i}{\varepsilon^m (1 + \varepsilon^{D_r,D^m}) + \varepsilon^{D_r,\mathbf{p} \in \mathbf{p},m}}, $$

with the following notation,

$$ \varepsilon^m = -\frac{dD^m_i}{dm_i} D^m_i, \quad \varepsilon^{D_r,\mathbf{p}_i \in \mathbf{p},m} = -\frac{\partial D^c_i}{\partial (\mathbf{p}_i)} D^c_i, \quad \varepsilon^{\mathbf{p},m} = \frac{\partial \mathbf{p}_i}{\partial m_i} \mathbf{p}_i, \quad \varepsilon^{D_r,D^m} = \frac{\partial D^c_i}{\partial D^m_i} \frac{D^m_i}{D^c_i}. $$

We can show that

$$ \varepsilon^{c,m} = \varepsilon^{D_r,\mathbf{p}_i \in \mathbf{p},m} + \varepsilon^{D_r,D^m} \varepsilon^m, \quad (12) $$

where $\varepsilon^{y,x}$ refers to the percentage impact on $y$ of 1% change in variable $x$. From (12) we observe that the impact of merchant fees on cardholders demand for EPNs can be decomposed in two effects.

(i) The *goods market price effect*, $\varepsilon^{D_r,\mathbf{p}_i \in \mathbf{p},m}$, due to merchant fee differences that enhance goods market price differences when EPN-based prices are allowed. Note that the NSR influences the cross elasticity $\varepsilon^{c,m}$ through the goods market price effect. Specifically, when the NSR is binding, from the consumers’ standpoint the goods market price does not vary irrespective of the EPN chosen to process payments. In our model, the NSR is equivalent to imposing $S = 0 \Rightarrow \mathbf{p}_i = 0$ which by its turn implies $\varepsilon^{D_r,\mathbf{p}_i \in \mathbf{p},m} = 0$.

(ii) The *cross-group externality effect*, $\varepsilon^{D_r,D^m} \varepsilon^m$, is due to the assumption that cardholders prefer EPNs with larger merchant acceptance. Since the number of merchants

18 Under the NSR, the goods market price effect is

$$ \varepsilon^{D_r,\mathbf{p}_i \in \mathbf{p},m} = -\frac{\partial D^c_i}{\partial (\mathbf{p}_i)} D^c_i \frac{\partial \mathbf{p}_i}{\partial m_i} \mathbf{p}_i = 0, $$

since $\partial (\mathbf{p}_i) / \partial m_i = 0$ due to the fact that $\mathbf{p}_i = 0$ as a result of the NSR.
in each EPN is influenced by merchant fees, cardholders behavior will also be indirectly influenced by those fees.

Given the elasticity decomposition shown in Lemma 3, the following result regarding the NSR arises.

**Proposition 1** *(No-Surcharge Rule impact):* Relatively to the market equilibrium without restrictions, under the NSR regime, entailing \( \triangle p_i = 0 \), (i) the cross elasticity of consumers demand to merchant fees becomes less elastic and, (ii) EPNs increase the average total fee level per transaction.

### 3.2 Pricing Structure and Profits at the Market Equilibrium

This section presents and compares the market equilibrium pricing structure and profits under surcharging and under the NSR. We show first that the NSR biases the pricing structure in favor of cardholders. Second, we show that the NSR will increase platforms’ profits if and only if the network externality exerted by merchants over cardholders is sufficiently weak.

#### 3.2.1 Market equilibrium under surcharging

The optimal conditions from profit maximization in (9) are

\[
\begin{align*}
\frac{\partial \Pi_i}{\partial f_i} &= D^c_i + f_i \frac{\partial D^c_i}{\partial f_i} + m_i \left( \frac{\partial D^m_i}{\partial f_i} D^c_i + \frac{\partial D^c_i}{\partial f_i} D^m_i \right) = 0, \\
\frac{\partial \Pi_i}{\partial m_i} &= f_i \frac{dD^c_i}{dm_i} + \left( D^m_i D^c_i + m_i \left( \frac{dD^m_i}{dm_i} D^c_i + D^m_i \frac{dD^c_i}{dm_i} \right) \right) = 0,
\end{align*}
\]

where \( \frac{\partial D^m_i}{\partial f_i} = 0 \) since \( D^m_i \) in (8) does not depend on cardholder fees \( f_i \), and

\[
\frac{dD^c_i}{dm_i} \equiv \frac{\partial D^c_i}{\partial m_i} + \frac{\partial D^c_i}{\partial (\triangle p_i)} \frac{\partial \triangle p_i}{\partial m_i},
\]

as a matter of terminology simplification.

The optimality conditions can be re-written as

\[
\begin{align*}
f_i &= \frac{D^c_i + m_i \frac{\partial D^c_i}{\partial f_i} D^m_i}{\frac{dD^c_i}{dm_i}}, \\
m_i &= -\frac{f_i \frac{dD^c_i}{dm_i} + D^m_i D^c_i}{\frac{dD^c_i}{dm_i}}.
\end{align*}
\]

To guarantee that the pricing solution from system (13) is indeed a maximizer of platform’s profit we introduce assumption 3.
**Assumption 3** *(Demand functions linearity):* Functions \( D_c^i(\cdot) \) and \( D_m^i(m_i) \) are linear in all the respective arguments.

Assumption 3 implies that \( D_c^i(\cdot) \) and \( D_m^i(m_i) \) second-order partial derivatives are zero. In the appendix, we show that assumption 3 is sufficient to guarantee that the system \([13]\) defines a profit maximizing solution for EPN \( i \). Graphically, the system is represented by two downward-sloping curves intersecting at the equilibrium point.\(^{19}\)

![Figure 2: Illustration of optimal pricing conditions and the equilibrium point.](image)

Solving the system of simultaneous equations \([13]\) for \((f_i, m_i)\) we have:

**Proposition 2** *(Market Equilibrium under surcharging):* The market equilibrium with surcharging is characterized by (i) merchant per transaction fees \( m_i^* = \frac{\partial D_m^i}{\partial m_i} \frac{\partial \Pi_i}{\partial f_i} \frac{\partial D_m^i}{\partial f_i} \), (ii) cardholder membership fees \( f_i^* = -\frac{\partial D_m^i}{\partial m_i} \frac{\partial D_c^i}{\partial f_i} + \frac{\partial D_m^i}{\partial m_i} \left( \frac{\partial D_c^i}{\partial f_i} \frac{\partial D_m^i}{\partial f_i} \right) \), and (iii) platform’s profit \( \Pi_i^* = \left( \frac{\partial D_c^i}{\partial f_i} \right)^2 \), for \( i = 1, 2 \).

### 3.2.2 Market equilibrium under the NSR

Under the NSR, the goods market is ruled by a single price irrespective the EPN used to complete transactions. The following Proposition shows that the NSR rebalances the pricing structure in favor of the cardholders and to the detriment of merchants.

**Proposition 3** *(Changes in pricing structure under the NSR):* Relatively to the market equilibrium with surcharging, the EPN pricing structure under the NSR decreases cardholder membership fees and increases merchant per transaction charges.

We have seen from Proposition 1 that under the NSR consumers’ demand for EPNs is less elastic to variations of merchant fees. This is because under the NSR merchant \(^{19}\)This can be shown by applying the implicit function theorem to the FOCs that define the equilibrium.
fee differences cannot be reflected in purchase price differences. Since, under the NSR, consumers are less responsive to merchant fees, EPNs charge higher fees to merchants. However, higher merchant fees will cause the mass of merchants accepting electronic payments to decline. This in turn reduces cardholder valuation of the EPN. Thus, in accordance with the EPN’s best-response function, membership fees should be lower, otherwise the EPNs would lose cardholders and profits would decline.

We now study the NSR profitability, that is, the conditions under which the NSR results in an increase of EPNs’ profits. Proposition 4 below shows that the NSR will be a profitable strategy for networks if and only if the externality that merchants exert over cardholders, \( \frac{\partial D^e_i}{\partial m_i} = \frac{\partial D^e_i}{\partial (D^m_i)} \frac{dD^m_i}{dm_i} \), is weak enough.

The intuition for the result is as follows. Suppose that merchants exert a large positive externality, that is, consumers are willing to pay a much higher membership fee if EPN \( i \) has a larger merchant acceptance. Hence, if EPNs implement the NSR, by Proposition 3, merchant per transaction charges will increase, and by Assumption 2 the number of merchants on the network will decrease. But then cardholders demand will suffer a sharp cutback that could only be compensated by a sufficiently large discount on the membership fee. However, such a large discount would be unprofitable for EPNs.

Also note that if consumers demand strongly varies with \( f_i \), that is, if consumers are strongly responsive to membership fees, then the NSR will be a profitable strategy since, by Proposition 3, it induces a membership fee reduction and thus invigorates cardholder demand. Proposition 4 presents the formal condition that assures profitability of the NSR for an EPN. As corollary, if the profitability condition holds for one firm, then, under symmetry, it will hold for both.

**Proposition 4 (The NSR profitability):** The NSR is a profitable strategy for an EPN if and only if network externalities exerted by merchants over cardholders are sufficiently weak, i.e., iff \( \left| \frac{\partial D^e_i}{\partial m_i} \right| < \frac{\partial D^e_i}{\partial f_i} D^m_i \).

**Corollary to Proposition 4:** Under symmetry of end-user demands, if the NSR is a profitable strategy for an EPN, then it will be a dominant strategy for both EPNs.

For the rest of the exposition, we assume that if the NSR is implemented by an EPN it is profitable, and condition \( \frac{\partial D^e_i}{\partial m_i} > D^m_i \frac{\partial D^e_i}{\partial f_i} \) in Proposition 4 holds.

### 4 Welfare Analysis

In this section, we first check the NSR impact on the number of end-users in the electronic payment system and on the goods market price. We show that the NSR reduces

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\(^{20}\)Note that \( \partial D^e_i/\partial m_i \) measures cardholder demand variation to merchant fee. Therefore, the larger the derivative (in absolute value), the larger will be the cross-group externality that merchants exert.
the number of merchants accepting card payments, increases the number of cardholders, and raises the equilibrium goods market price paid by cardholders.

We then investigate the surplus variations that the NSR implies on each group of agents. We also discuss the total welfare variation and show conditions under which society is better off under the NSR equilibrium. We highlight that (i) merchant market power, $\beta$, in the goods market and (ii) the network externality from merchants to cardholders are two relevant determinants of whether the NSR is socially desirable.

According to Proposition 3, merchants per transaction charges are higher under the NSR. Therefore, by assumption 2, the number of merchants accepting payment cards will unambiguously decrease. Regarding cardholders, the analysis is more complex in the sense that we find two opposite effects on cardholders demand: the decrease on cardholder fee and the increase on merchant fees that diminishes the number of merchants accepting cards. However, assuming that the network effects exerted by merchants over cardholders are sufficiently weak (condition from Proposition 4), the former effect dominates the latter and the NSR net effect on cardholders demand will be positive.

The effect on the goods market price paid by cardholders is straightforward by Lemma 1. Since merchants per transaction charges are part of their marginal cost, the equilibrium price paid by cardholders increases. Proposition 5 formalizes these intuitions.

**Proposition 5 (NSR impact on the number of end-users and goods market price):** Relatively to the surcharging case, the NSR leads to (i) a reduction on the number of merchants accepting card payments, (ii) an increase on the number of cardholders’ and (iii) an increase on the equilibrium goods market price paid by cardholders.

**Remark 2.** Despite the fact that cardholder fees are lower under the NSR, cardholders face additional expenditure related with the price adjustment in the goods market due to the merchant fees increase. Note that, as merchants market power increases, the price increase in the goods market, due to the NSR, is smaller. In the limit, if the goods market has a monopolistic structure $\beta = 1$, then $\Delta p_i = 0$. When merchant market power in the goods market is high, prices follow closely consumer willingness-to-pay. In other words, merchants do not pass-through the marginal cost of card usage to cardholders. If merchant market power is high ($\beta \approx 1$) and the NSR is introduced, then cardholders will keep paying (approximately) the same price in the goods market but membership fees will be lower. Proposition 6 below shows the merchants’ market power relevance for the goods market as one determinant of the NSR social desirability.

We discuss the variations on merchant and cardholder payment surpluses due to the

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21 The effect of the NSR on the number of transactions on platform $i$, $D_m^p D_c$, is unclear. As we have seen from Proposition 5 the number of merchants decreases but there is an increase on the number of cardholders. Therefore, it is not clear which will be the dominant effect.
NSR implementation. Merchant surplus on EPN \(i\) is defined as

\[
Surplus_i^m = \left( \int_{0}^{b} D_i^m(x_i) \, dx_i \right) D_i^c,
\]

where \(b\) corresponds to merchant highest willingness-to-pay per cashless transaction, defined by \(D_i^m(b) = 0, \ i = 1, 2; \int_{0}^{b} D_i^m(x_i) \, dx_i\) is the merchant surplus per transaction and \(D_i^c\) is the number of transactions that each merchant will process through EPN \(i\).

For the sake of simplicity, cardholders’ total surplus on network \(i\) is denoted by

\[
V_i^c \equiv V_i^c(f_i, p_i(m_i), D_i^m),
\]

satisfying similar properties as in (2) for an individual cardholder.

\[
\frac{\partial V_i^c}{\partial f_i} < 0, \frac{\partial V_i^c}{\partial p_i} < 0 \text{ and } \frac{\partial V_i^c}{\partial D_i^m} > 0.
\]

We highlight that cardholder surplus decreases with expenditure \(f_i\) and \(p_i(m_i)\), since marginal cardholders will stop using the EPN when total expenditure increases and those who remain at the EPN will see their individual surplus to decrease. Additionally, the derivative with respect to the number of merchants captures two effects: the change on the number of transactions under EPN \(i\) and, the impact on cardholders willingness-to-pay for \(i\)’s payment card.

Lemma 4.1 summarizes the value functions variations, introduced by the NSR, on both agent types. Lemma 4.2 shows the expression for social surplus variation.

**Lemma 4.1 (Variations of EPNs’ profit and end-users’ total surpluses):** Let

\[
\begin{align*}
V_i^m & \equiv \int_{0}^{b} D_i^m(x_i) \, dx_i \text{ and} \\
V_i^c & \equiv V_i^c(f_i, p_i(m_i), D_i^m) \text{ denote cardholders’ total surplus at EPN } i,
\end{align*}
\]

then the approximated variation, due to the NSR,

(i) on EPN \(i\)’s profit is \((1 - \alpha) 2D_i^c \left( \frac{\partial D_i^c}{\partial m_i} - \frac{\partial D_i^c}{\partial f_i} D_i^m \right) \frac{\partial V_i^c}{\partial f_i} \frac{\partial V_i^c}{\partial p_i} \frac{\partial V_i^c}{\partial D_i^m} \frac{\partial V_i^c}{\partial D_i^c} \frac{\partial V_i^c}{\partial m_i},
\]

(ii) on merchants’ total surplus is \(2 \sum_{i=1}^{2} \left[ V_i^m \left( \frac{\partial D_i^m}{\partial m_i} - D_i^m \frac{\partial D_i^m}{\partial f_i} \right) - D_i^m (m_i) D_i^c \right] \frac{\partial V_i^c}{\partial f_i} \frac{\partial V_i^c}{\partial p_i} \frac{\partial V_i^c}{\partial D_i^m} \frac{\partial V_i^c}{\partial D_i^c} \frac{\partial V_i^c}{\partial m_i},
\]

and

(iii) on cardholders’ total surplus is \(2 \sum_{i=1}^{2} \left[ \frac{\partial V_i^c}{\partial p_i} \frac{\partial V_i^c}{\partial m_i} - \frac{\partial V_i^c}{\partial f_i} \frac{\partial V_i^c}{\partial m_i} + \frac{\partial V_i^c}{\partial D_i^m} \frac{\partial V_i^c}{\partial D_i^c} \right] \frac{\partial V_i^c}{\partial f_i} \frac{\partial V_i^c}{\partial p_i} \frac{\partial V_i^c}{\partial D_i^m} \frac{\partial V_i^c}{\partial D_i^c} \frac{\partial V_i^c}{\partial m_i}.
\]

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Note that EPN’s profit variation is positive iff \( \frac{\partial D_c^e}{\partial m_i} > \frac{\partial D_c^e}{\partial f_i} D_i^m \) as derived and interpreted in Proposition 4. Regarding merchants, since \( \frac{\partial D_c^e}{\partial m_i} dD_{m_i} > 0 \) and \( \frac{\partial D_c^e}{\partial m_i} - \frac{dD_c^e}{dm_i} > 0 \), the sign of their surplus variation depends positively on

\[
sign \left\{ V_i^m \left( \frac{\partial D_c^e}{\partial m_i} - D_i^m \frac{\partial D_c^e}{\partial f_i} \right) - D_i^m D_i^e \right\} = sign \left\{ \frac{\partial D_c^e}{\partial m_i} - D_i^m \frac{\partial D_c^e}{\partial f_i} - \frac{D_i^m D_i^e}{V_i^m} \right\}
\]

which is undetermined without further assumptions.

Given \( \frac{\partial D_c^e}{\partial f_i} < 0 \) by assumption 1, the sign of the cardholder surplus variation depends negatively on

\[
sign \left\{ \frac{\partial V_c^e (1 - \beta)}{\partial p_i} \frac{dD_{m_i}^c}{dm_i} - \frac{\partial V_c^e}{\partial f_i} \frac{dD_{m_i}^c}{dm_i} + \frac{\partial V_c^e}{\partial D_i^m} \right\}.
\]

The previous expression depends on the merchant market power \( \beta \). Hence, if \( \beta \) is sufficiently high such that (14) is negative, consumers will benefit from the NSR. The reason why cardholder surplus variation (due to the NSR) depends positively on merchant market power is similar to that of Remark 2.

Finally, social surplus variation is as follows:

**Lemma 4.2 (Total Welfare Variation):** The social welfare variation due to the NSR is approximately given by

\[
\Delta W \approx \sum_{i=1}^{2} \left[ \left( V_i^m \frac{\partial D_c^e}{\partial f_i} - 2 (1 - \alpha) D_i^e \right) \left( \frac{\partial D_c^e}{\partial m_i} - D_i^m \frac{\partial D_c^e}{\partial f_i} \right) \right] + \frac{\partial D_c^e}{\partial m_i} \frac{dD_c^e}{dm_i} - \frac{dD_c^e}{dm_i}
\]

Since

\[
\frac{\partial D_c^e}{\partial m_i} \frac{dD_c^e}{dm_i} - \left( \frac{\partial D_c^e}{\partial f_i} \right)^2 \frac{dD_c^e}{dm_i} < 0,
\]

the relevant term that determines the social welfare variation is,

\[
\frac{\partial V_c^e}{\partial m_i} \frac{dD_c^e}{dm_i} > \frac{\partial V_c^e}{\partial f_i} D_i^m,
\]

hence, for \( \beta = 1 \), expression in (14) is negative.

Recall that, by assumption, cash payments do not generate value to both payee and payor. Thus, cash payments are discarded from the welfare analysis.
in which
\[
\left( V_m^{i} \frac{\partial D_i^c}{\partial f_i} - 2 \left( 1 - \alpha \right) D_i^c \right) \left( \frac{\partial D_i^c}{\partial m_i} - \frac{\partial D_i^c}{\partial f_i} D_i^m \right) < 0 \text{ holding the condition from Proposition 4.}
\]

The term with undetermined sign is
\[
\frac{\partial V_i^c}{\partial p_i} (1 - \beta) - \frac{\partial V_i^c}{\partial f_i} D_i^m + \frac{\partial V_i^c}{\partial D_i^m} \frac{dD_i^m}{dm_i} - D_i^c D_i^m,
\]
depending on merchants market power in the goods market. Proposition 6 shows under what conditions the NSR is socially (un)desirable.

**Proposition 6** (The NSR impact on total welfare): The NSR will be socially (un)desirable if the network externality exerted by merchants on cardholders is sufficiently weak (strong) and merchants market power in the goods market is sufficiently high (low), i.e., if

\[
\left| \frac{\partial D_i^c}{\partial m_i} \right| < \left( > \right) \left| \frac{\partial D_i^c}{\partial f_i} \right| D_i^m \text{ and } \beta > \left( < \right) 1 - \frac{\frac{\partial V_i^c}{\partial D_i^m}}{\frac{\partial V_i^c}{\partial f_i}} D_i^m.
\]

Proposition 6’s main message is that the network externality condition that assures the NSR profitability to EPNs may be insufficient to guarantee a better social outcome. In order to assure social desirability, the NSR has to be applied in markets whose merchants have sufficiently high market power, i.e., define prices according to consumers willingness-to-pay and do not fully pass-through the marginal cost of sales (including the card usage) to cardholders (recall Remark 2). Hence, under the NSR, in a market whose merchants have sufficiently high market power, cardholders do not pay much more for their purchases while benefit from a discount on the membership fee. In these cases, the NSR acts as a pricing distortion (see Proposition 3) that partially corrects the opposite price distortion in the goods market due to merchants market power. Recalling the expression *fight fire with fire*, a way to combat a distortion is with another distortion.

On the other hand, if the market for goods is highly competitive, i.e., market price is close to cost, then the NSR will implicitly generate distortions by inflating merchant costs when serving cardholders. In that case, the NSR will introduce a distortion in a market which had no distortions, making society worse off.
Therefore, although merchant market power is irrelevant to EPNs when deciding on the implementation of the NSR, it is fundamental in determining the NSR desirability from the social perspective. In fact, the higher the merchant market power $\beta$, the bigger the likelihood of the NSR being socially desirable.

The network effect exerted by merchants on cardholders also affects total surplus. If the network effect is sufficiently strong, then the NSR will reduce cardholder surplus due to the decrease in the number of merchants accepting card payments. Hence, the social perspective suggests the existence of a relationship between merchants market power $\beta$ and network effect exerted by merchants on cardholders, $\partial D^c_i / \partial m_i$. Proposition 7 presents the social indifference equation which is the set of allocations with coordinates $(\beta, NE) \in [0, 1] \times \mathbb{R}_0^+$ where society is indifferent to whether the NSR is implemented. Let $NE$ denote the network effect that the number of merchants accepting card payments has on cardholders demand, i.e., $-\partial D^c_i / \partial m_i$.

**Proposition 7 (The Social Indifference Equation):** The set of allocations with coordinates $(\beta, NE) \in [0, 1] \times \mathbb{R}_0^+$ such that society is indifferent to the NSR implementation, i.e., $\Delta W = 0$, is characterized by

$$NE = \left( \frac{\partial V^c}{\partial p_i} (1 - \beta) - \frac{\partial V^c}{\partial f_i} D^m_i + \frac{\partial V^c}{\partial f_i} \frac{\partial D^c_i}{\partial f_i} - D^c_i D^m_i \right) \frac{\partial D^c_i}{\partial f_i} \frac{\partial D^c_i}{\partial f_i} - 2 (1 - \alpha) D^c_i - D^m_i \frac{\partial D^c_i}{\partial f_i},$$

where $NE \equiv -\partial D^c_i / \partial m_i$.

Proposition 7 highlights the existence of a relationship between the network externality that merchants exert on cardholders and merchant market power in the goods market. Under the NSR, $\partial (NE) / \partial \beta = \frac{\partial V^c}{\partial p_i} \frac{\partial D^c_i}{\partial f_i} / \left( 2 (1 - \alpha) D^c_i - V^m_i \frac{\partial D^c_i}{\partial f_i} \right) > 0$, hence even if the $NE$ is significant it might be the case that the NSR is socially desirable when the merchant market power is sufficiently high.

From Proposition 4, we can write the EPN $i$’s indifference equation between implementing the NSR or not as

$$\partial D^c_i / \partial m_i - \partial D^c_i / \partial f_i D^m_i = 0 \Leftrightarrow NE = -\partial D^c_i / \partial f_i D^m_i.$$

Therefore, at the point of indifference, an increase of $NE$ (cross-group Network Effects) will make the NSR an unprofitable strategy.

Figures 3 and 4 depict the set of points where the NSR is a profitable strategy for EPN $i$ (areas $A$ and $B$) and compares it to the set of points where the NSR is socially desirable (areas $B$ and $C$).

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24 By introducing a cost of surcharging the merchant market power threshold, from which the NSR is socially desirable, should decrease.
The NSR is a profitable strategy as long as the network effect from merchants on cardholders is below the threshold \textit{EPN indifference line}, regardless of $\beta$. However, from the social perspective, for low levels of $\beta$, even if $NE$ is below the EPN’s indifference curve, the NSR might not increase social welfare (the area $A$). In area $A$ there is a misalignment of social and network interests. On the one hand, the NSR increases networks’ profits, but, on the other, its social cost (price distortions due to the increase on merchant fee) reduces the social benefit (namely, lower cardholder membership fee).

As we redo the cost-benefit analysis for higher levels of merchant market power in the goods market, \textit{i.e.}, higher $\beta$ (keeping the $NE$ fixed at some positive level and below the EPN indifference curve) the social benefit from the NSR increases since it contributes to correct market power distortions. We get then into area $B$ where both network and social interests are aligned in favor of the NSR implementation.

Area $C$ corresponds to the situation when the NSR social benefit, amending the high merchant market power, is sufficiently high that compensates the social cost, under strong network externalities (\textit{i.e.}, above the EPN indifference line). Area $C$ is characterized by the divergence of network and social interests. Hence, in the absence of regulation or transfers, despite the fact that the NSR is socially desirable, EPNs will choose not to implement it. Although area $C$ may not exist, area $B$ where network and social interests are aligned on NSR implementation will necessarily exist (see the proof of Proposition 8).

\footnote{See figure 4 for an illustration where area $C$ does not exist.}
Proposition 8 shows that area $B$ always exists. In other words, there exists a set of allocations where the NSR is socially desirable and simultaneously profitable for EPNs.

**Proposition 8** (Existence of allocations where the NSR is socially desirable and profitable): For $\partial V_i^c / \partial f_i$ sufficiently negative, there exists a set of allocations with coordinates $\beta, NE \in [0, 1] \times \mathbb{R}_0^+$ such that the NSR is simultaneously socially desirable and profitable for EPNs.

In the following section we discuss possible policy interventions taking into consideration the results previously derived.

### 5 Policy Interventions: one size does not fit all

In this section, we discuss policy considerations and possible interventions in the payment card industry with regard to the NSR. We start by considering the pros and cons of abolishing the NSR versus no regulatory intervention, i.e., letting EPNs decide on the NSR implementation. We conclude that one policy does not fit all markets. In general, there are significant differences from market to market. We claim that regulators should take into account those market specificities, namely the merchants market power, deciding about the NSR on a market-by-market basis instead of uniformly regulating all markets.

#### 5.1 Eliminating restrictions on differential pricing

During the last decade courts and policymakers have investigated the business practices of payment networks. In most countries, card networks impose the NSR preventing
merchants from setting different prices across EPNs. However, abolishing restrictions on differential pricing may be an attractive policy option for society as a whole. For example, the Reserve Bank of Australia has decided to remove the NSR.

Some authors claim that abolishing the NSR would remove a restraint of trade. Nonetheless, this economic justification is questionable. For instance, according to our analysis (see footnote 20) it is unclear that the number of card transactions will increase or decrease without the NSR. Proposition 5 shows that without the NSR on the one hand the number of merchants accepting card payments will increase, but on the other hand fewer consumers will use payment cards. Hence, the net effect on the number of transactions is not clear \textit{a priori}.

Here, we highlight some of the pros and cons of this policy. The NSR exclusion has the advantage of being a transparent policy, easy to implement and enforce; it does not require the regulator to have information about costs and benefits of any of the agents involved in a transaction. Its applicability and effect does not depend on the card type (debit, prepaid or credit), the network organizational structure (three-party or four-party systems), or its pricing strategy. Moreover, it may allow for goods market price changes so to reflect the real costs and benefits of card transactions to merchants. Hence, consumers may internalize the externalities tied to the use of payment cards, which would promote more efficient payment card use from the social perspective. However, this argument is valid only as long as merchants’ behavior is sufficiently competitive (see Remark 2). In order for differential pricing to correctly internalize externalities, these price differences must accurately reflect social costs and benefits. If merchants have market power, they might obstruct the NSR suppression policy from encouraging the efficient utilization of card payments by distorting prices and fees away from the social costs and benefits. Another disadvantage of this policy is that it may generate confusion and uncertainty among consumers, if merchants set a different price for each payment means. Also, merchants would bear extra costs of setting and managing a system with several prices for each product. In particular, we should expect increased (menu) costs to merchants of updating price lists, pamphlets, and shelf prices.

5.2 \textit{Laissez faire, laissez aller, laissez passer}

An alternative policy for antitrust authorities regarding the payment card system is simply not to intervene. We discuss here some pros and cons of the \textit{laissez faire} policy.

First, it is not clear \textit{ex-ante} that the market outcome is less efficient than what would result with intervention. For example, in Proposition 8 we show that there exists a set of allocations (area $B$) where EPNs choices regarding the NSR adoption are compatible with the socially desirable choices. Nonetheless, while it is ambiguous that the market
outcome is inefficient, it is also unclear that it is efficient. For example, consider area $A$ in Figure 3. In that set, the NSR implementation is optimal from the network perspective but undesirable from the social standpoint.

Second, policy interventions may generate unforeseen and unintended adverse consequences for the payment card system. However, private or government legal actions based on antitrust laws are important to provide effective means to deal with competition issues on the payment card industry. Furthermore, litigation implies substantial costs and without regulation it would significantly increase uncertainty with regard to the outcome of possible negotiations or of a verdict in court. Regulatory indecision may also delay the introduction of innovation in the electronic payment system.

Third, entry and innovation have occurred in the payment industry (e.g. PayPal) reflecting the free market performance to tackle merchant concerns about high merchant fees for payment card transactions. However, because of network effects and consumer inertia, the establishment of new payment networks is hard. Hence, the extent to which these entrants will serve as effective competitors for the established networks is unclear, particularly when faced with well-established incumbent networks.

5.3 One size does not fit all

Different policy choices have been made by policymakers regarding the payment card industry over the last two decades. For example, in Australia, the Netherlands, Sweden, and the United Kingdom, the NSR has been abolished, while in many countries the NSR still prevail. In the U.S. this rule has been abolished by MasterCard and Visa but not by American Express, which opposes the DOJ on this issue. The policy dichotomy does not imply that only one group of nations has made an accurate analysis. In fact, reality may fit Figure 3 with countries that abolished the NSR lying on area $A$, while countries that protect the NSR by law, or simply allow networks to impose the NSR on contracts, lying on area $B$. According to our model, when deciding the NSR adoption or refusal, policymakers should take into account (i) the degree of competition among merchants ($\beta$ of the model) that characterizes the economy, and, (ii) the weight of the network externality that merchants exert over cardholders relatively to consumer sensitiveness towards membership fees. Different nations likely have different estimates of the two determinants for the NSR refusal or adoption. Hence, our model is compatible with the dichotomy on policy choice. In general, to set a uniform payment card policy worldwide would not serve the social interest of each nation or region.

Both the elimination of restrictions on differential pricing and laissez faire policies have advantages and drawbacks; after arguing in favor of policy segmentation by country, we further argue in favor of policy customization by market. That is, policymakers
should take into account merchants’ market power, choosing the best policy on a market-by-market basis instead of uniformly regulating all markets. More specifically, when the network effect of merchants on cardholders is sufficiently weak (condition of NSR profitability for networks) then policymakers should concentrate their efforts on implementing the NSR only on less competitive markets where merchants do not pass-through the marginal cost of card usage to cardholders (see Remark 2). Just like different countries adopt different policies, we propose the extension of this rationale to the industry level. When one policy does not fit all markets, then virtue lies in choosing the right policy that best suits each individual industry.

6 Conclusions

In this paper, we built a three-party model with consumers, merchants and electronic payment networks. We extend the literature on electronic payment networks that sheds light on the effects of the No-Surcharge Rule on networks’ pricing, profits and social welfare. We debate some possible policy interventions and claim that card payments should be regulated on a market-by-market basis. For the sake of simplicity, our theoretical model does not distinguish among different types of payment card (debit, credit, prepaid) and may fail to capture important real-world features such as the role of credit that would probably influence the model’s results.

Our first set of results relates to the seminal work of R&T (2003) extending its analysis to include the effect of a variation on network size. We show that the existence of network effects adds a specific cross elasticity term to the formula for optimal EPN pricing. We derive a series of results based on elasticities showing that (i) the platform’s profit maximization problem can be decomposed in two steps: (1) setting the total fee level, and (2) the relative fees, (ii) consumers demand for payment services becomes less elastic with respect to merchant fee under the NSR, and (iii) the absence of surcharge variations amongst EPNs holds back network competition resulting in higher total fee levels.

In a second set of results, we show first that the NSR rebalances the relative fees in favor to cardholders and against the merchants. We also investigate under which circumstances the NSR is a profitable strategy for EPNs. We find that the NSR increases EPNs’ profits if and only if the cross-group externality exerted by merchants on cardholders is sufficiently weak. The NSR inflates merchant fees decreasing the merchant demand for EPNs, therefore if the cross-group network effect is strong, consumer demand and, by implication, EPNs’ profits will both sharply decrease.

In the welfare analysis, we show that the NSR reduces the number of merchants accepting card payments, increases the number of cardholders and raises the equilibrium goods market price paid by cardholders. We investigate the surplus variations that the
NSR implies to each group of agents and to society as a whole. We show that (i) merchants’ market power $\beta$ in the goods market and the (ii) network effect exerted by merchants on cardholders are two relevant determinants of whether the NSR is socially desirable or not.

We conclude from the welfare analysis that the answer to the question in the title, “to surcharge or not to surcharge?”, has a bifurcation: a private answer to EPNs and a social answer to policymakers. Regarding the EPNs decision making process all that matters for the NSR implementation is the network effect exerted by merchants on cardholders: it must be sufficiently weak; otherwise EPNs would lose end-users on both sides of the market. The social preference concerning the NSR is in general different because society is concerned not only with the network effect, but also with the merchants market power in the goods market. For example, suppose that network effects are strong to the point that EPNs are unwilling to implement the NSR. Even in this case, it is still possible for the NSR to be a socially desirable policy in final goods markets characterized by high market power. To take another example, suppose the network externality is weak leading networks to find the NSR implementation optimal, but if the goods market is very competitive, society as a whole may prefer to abolish the NSR.

7 References


Bedre, Özlem & Emilio Calvano (December 2009), Pricing Payment Cards, ECB Working Paper Series.


## 8 Appendix

### 8.1 Proofs

**Proof of Lemma 1:** First, note that the equilibrium price in market $k$ satisfies $k \leq p^*_k \leq v$. On the one hand, if $p^*_k > v$ then no consumer will buy the good and the market shuts down for that range of prices. On the other hand, if $p^*_k < k$, no merchant will produce the good since the price does not cover the net marginal cost $k$ of supplying the good and the market shuts down.

Second, the extreme values of $p^*_k$ are covered by function $p_k (\beta)$ when $\beta = 0$ and $\beta = 1$, $p_k (\beta = 0) = k$ and $p_k (\beta = 1) = v$.

Third, note that the function $p_k (\beta) : [0, 1] \rightarrow [k, v]$ is continuous in $\beta$. Therefore regarding the intermediate values of $p^*_k$, $p_k (0) = k < p^*_k < v = p_k (1)$, we can guarantee by the Intermediate Value Theorem that there exists at least one $\beta \in [0, 1]$, such that $p_k (\beta) = p^*_k$.

Fourth, since $\frac{dp_k (\beta)}{d\beta} = v - k > 0$ by assumption (iii), $p_k (\beta)$ is strictly increasing in $\beta$ and we can assure the uniqueness of $\beta \in [0, 1]$ satisfying $p^*_k = \beta v + (1 - \beta) k$, specifically $\beta = \frac{p^*_k - k}{v - k}$. □

**Proof of Lemma 2:** Substituting $f_i$ by $D^m_i s_i$ in (9) and taking the log we get\(^{26}\)

\[
\max_{s_i, m_i} \ln \Pi_i = \ln (s_i + m_i) + \ln D^m_i + \ln D^c_i, \quad i = 1, 2.
\]

\(^{26}\)We assume log-concavity of the profit function (9). This assumption is sufficient to guarantee that the FOCs define a profit-maximizer.
By the first-order conditions of the problem in (15) we have

\[
\begin{align*}
\frac{\partial \ln \Pi_i(s_i,m_i)}{\partial s_i} &= \frac{1}{s_i + m_i} + \frac{dD_i^c}{D_i^c} = 0 \\
\frac{\partial \ln \Pi_i(s_i,m_i)}{\partial m_i} &= \frac{1}{s_i + m_i} + \frac{dD_i^m}{D_i^m} + \frac{dD_i^c}{D_i^c} = 0
\end{align*}
\]

\[\Leftrightarrow \begin{cases} 
 s_i + m_i = \frac{s_i}{e^i} \\
 s_i + m_i = \frac{m_i}{e^{m_i + e^{c,m}}}.
\end{cases}\]

Proof of Lemma 3:

\[
\begin{align*}
\varepsilon^{c,m} &= -\frac{dD_i^c m_i}{dm_i D_i^c} \\
&= -\left( \frac{\partial D_i^c}{\partial (\uparrow p_i)} \frac{\partial (\uparrow p_i)}{\partial m_i} + \frac{\partial D_i^m}{\partial D_i^m} \frac{dD_i^m}{dm_i} \right) m_i D_i^c \\
&= -\frac{\partial D_i^c}{\partial (\uparrow p_i)} \frac{\partial (\uparrow p_i)}{\partial m_i} m_i - \frac{\partial D_i^m}{\partial D_i^m} \frac{dD_i^m}{dm_i} m_i D_i^c \\
&= \varepsilon^{D^c,\uparrow p} \varepsilon^{p,m} + \varepsilon^{D^m,\varepsilon^{c,m}}.
\end{align*}
\]

Therefore,

\[
\begin{align*}
\varepsilon^{m} + \varepsilon^{c,m} &= \varepsilon^{m} + \varepsilon^{D^c,\uparrow p} \varepsilon^{p,m} + \varepsilon^{D^m,\varepsilon^{c,m}} \\
&= \varepsilon^{m} \left(1 + \varepsilon^{D^c,\varepsilon^{c,m}}\right) + \varepsilon^{D^c,\uparrow p} \varepsilon^{p,m}.
\end{align*}
\]

Plugging the result into the system of optimal equations from Lemma 2, we reach the result

\[
\begin{cases} 
 s_i + m_i = \frac{s_i}{e^i} \\
 s_i + m_i = \frac{m_i}{e^{(m_i + e^{c,m})}}.
\end{cases}\]

Proof of Proposition 1: (i) From (12) we have

\[
\varepsilon^{c,m} = \varepsilon^{D^c,\uparrow p} \varepsilon^{p,m} + \varepsilon^{D^m,\varepsilon^{c,m}},
\]

where under surcharging \((S = 1)\),

\[
\varepsilon^{D^c,\uparrow p} \varepsilon^{p,m} = -\frac{\partial D_i^c}{\partial (\uparrow p_i)} \frac{m_i \partial (\uparrow p_i)}{\partial m_i} D_i^c > 0,
\]

since \(\frac{\partial D_i^c}{\partial (\uparrow p_i)} < 0\) by assumption 1 and \(\frac{\partial (\uparrow p_i)}{\partial m_i} = \frac{\partial p_i}{\partial m_i} > 0\) by Lemma 1. Under the NSR \(\varepsilon^{D^c,\uparrow p} \varepsilon^{p,m} = 0\) since prices must be equal regardless of the payment instrument. There-
fore, the cross elasticity of consumers demand under the NSR, \( \varepsilon_{NSR}^{c,m} \), satisfies

\[
\varepsilon_{NSR}^{c,m} = \varepsilon^{Dc,Dm} < \varepsilon^{c,m}.
\]

(ii) From (10), and the fact that \( \varepsilon_{NSR}^{c,m} < \varepsilon^{c,m} \), we can establish the following inequality,

\[
s_i^{NSR} + m_i^{NSR} = \frac{m_i}{\varepsilon^m + \varepsilon_{NSR}^{c,m}} > \frac{m_i}{\varepsilon^m + \varepsilon^{c,m}} = s_i + m_i,
\]

where \( s_i^{NSR} + m_i^{NSR} \) is the average total fee level per transaction under the NSR. □

**Proof of Proposition 2**: Part (i) and (ii) come straight from solving the system of simultaneous equations (13) with respect to \( (f_i, m_i) \). Provided that \( i = f_i D_i^c + m_i D_i^m D_i^c \) by definition, and substituting \( m_i^* \) and \( f_i^* \) by the optimal expressions from (i) and (ii), respectively, we reach

\[
\Pi_i^* = -\frac{\partial D_i^m}{\partial m_i} D_i^c + D_i^m \left( \frac{\partial D_i^m}{\partial m_i} - \frac{\partial D_i^c}{\partial f_i} D_i^m \right) D_i^c + \frac{\partial D_i^m}{\partial m_i} \frac{\partial D_i^m}{\partial f_i} D_i^m D_i^c = \left( \frac{D_i^c}{\partial f_i} \right)^2. \quad \Box
\]

**Proof of Proposition 3**: The NSR constraint \( S = 0 \) implies \( \Delta p_i = 0 \) and \( \frac{\partial D_i^c}{\partial m_i} \bigg|_{NSR} = \frac{\partial D_i^c}{\partial m_i} \). Introducing \( \frac{\partial D_i^c}{\partial m_i} \bigg|_{NSR} = \frac{\partial D_i^c}{\partial f_i} \) in the system of simultaneous equations (13) and solving w.r.t. \( (f_i, m_i) \), we get

\[
m_i^* \bigg|_{NSR} = \frac{\partial D_i^m}{\partial m_i} \left( \frac{\partial D_i^m}{\partial f_i} - \frac{\partial D_i^c}{\partial f_i} D_i^m \right),
\]

\[
f_i^* \bigg|_{NSR} = -\frac{\partial D_i^m}{\partial m_i} D_i^c + D_i^m \left( \frac{\partial D_i^m}{\partial m_i} - \frac{\partial D_i^c}{\partial f_i} D_i^m \right). \quad \Box
\]

Since \( \frac{\partial D_i^c}{\partial m_i} > \frac{\partial D_i^c}{\partial m_i} \), then \( m_i^* \bigg|_{NSR} > m_i^* \) and \( f_i^* \bigg|_{NSR} < f_i^* \). □

**Proof of Proposition 4**: From Proposition 2, (iii) we have \( \Pi_i^* = \left( \frac{D_i^c}{\partial f_i} \right)^2 \). Taking \( \frac{\partial D_i^c}{\partial f_i} \) as constant by Assumption 3, it is clear that EPN’s equilibrium profit will only increase if more consumers access the network. Therefore, the NSR will be a profitable strategy if the number of consumers on the EPN increases. The variation on the number of consumers on EPN \( i \), \( \Delta D_i^c \equiv D_i^c \bigg|_{NSR} - D_i^c \), is approximately given by

\[
\Delta D_i^c \approx \frac{\partial D_i^c}{\partial m_i} \Delta m_i + \frac{\partial D_i^c}{\partial f_i} \Delta f_i.
\]
Hence, and the NSR will be a profitable strategy if
\[
\frac{\partial D^e_i}{\partial m_i} \Delta m_i + \frac{\partial D^e_i}{\partial f_i} \Delta f_i > 0 \iff \frac{\Delta f_i}{\Delta m_i} < -\frac{\partial D^e_i}{\partial f_i}.
\]

Computing \( \Delta f_i \) and \( \Delta m_i \),
\[
\Delta f_i \equiv f^*_i|_{NSR} - f^*_i = -\frac{dD^m_i}{dD_i} D^e_i + D^m_i \left( \frac{\partial D^m_i}{\partial m_i} - \frac{\partial D^m_i}{\partial f_i} D^m_i \right) - \left( -\frac{dD^m_i}{dD_i} D^e_i + D^m_i \left( \frac{\partial D^m_i}{\partial m_i} - \frac{\partial D^m_i}{\partial f_i} D^m_i \right) \right) = \frac{D^m_i \left( \frac{dD^e_i}{dD_i} - \frac{\partial D^e_i}{\partial f_i} \right)}{\frac{dD^m_i}{dD_i} - \frac{\partial D^m_i}{\partial f_i}},
\]
\[
\Delta m_i \equiv m^*_i|_{NSR} - m^*_i = \frac{\partial D^e_i}{\partial m_i} m^*_i - \frac{\partial D^e_i}{\partial f_i} m^*_i - \frac{dD^m_i}{dD_i} D^e_i + D^m_i \left( \frac{\partial D^m_i}{\partial m_i} - \frac{\partial D^m_i}{\partial f_i} D^m_i \right) = \frac{\partial D^e_i \partial D^m_i}{\partial f_i},
\]
therefore,
\[
\Delta f_i \Delta m_i = -\frac{D^m_i \left( \frac{dD^e_i}{dD_i} - \frac{\partial D^e_i}{\partial f_i} \right)}{\frac{dD^m_i}{dD_i} - \frac{\partial D^m_i}{\partial f_i}} = -D^m_i,
\]
and the NSR will be a profitable strategy if
\[
\frac{\Delta f_i}{\Delta m_i} < -\frac{\partial D^e_i}{\partial f_i} \iff -D^m_i < \frac{\partial D^e_i}{\partial m_i} \iff \frac{\partial D^e_i}{\partial m_i} > D^m_i \frac{\partial D^e_i}{\partial f_i}
\]
which is equivalent to
\[
\left| \frac{\partial D^e_i}{\partial m_i} \right| < \left| \frac{\partial D^e_i}{\partial f_i} \right| D^m_i. \ \Box
\]

**Proof of Corollary to Proposition 4:** If both EPNs engage in the NSR strategy, the variation on the number of consumers on EPN \( i \), \( \Delta D^e_i \equiv D^e_i|_{NSR} - D^e_i \), will be approximately given by
\[
\Delta D^e_i \approx \frac{\partial D^e_i}{\partial m_i} \Delta m_i + \frac{\partial D^e_i}{\partial f_i} \Delta f_i + \frac{\partial D^e_i}{\partial m_j} \Delta m_j + \frac{\partial D^e_i}{\partial f_j} \Delta f_j.
\]
Under symmetry
\[
\Delta m_i = \Delta m_j \text{ and } \Delta f_i = \Delta f_j.
\]
Hence,
\[
\Delta D^e_i \approx \left( \frac{\partial D^e_i}{\partial m_i} + \frac{\partial D^e_i}{\partial m_j} \right) \Delta m_i + \left( \frac{\partial D^e_i}{\partial f_i} + \frac{\partial D^e_i}{\partial f_j} \right) \Delta f_i.
\]
By the definition in (4) and $\Delta f_i \equiv f_i - \alpha f_j$ we can write that

$$\frac{\partial D_i^c}{\partial (\Delta f_i)} = \frac{\partial D_i^c}{\partial f_i}$$

thus,

$$\frac{\partial D_i^c}{\partial f_j} \equiv \frac{\partial D_i^c}{\partial (\Delta f_i)} \frac{\partial (\Delta f_i)}{\partial f_j} = -\alpha \frac{\partial D_i^c}{\partial f_i},$$

and similarly,

$$\frac{\partial D_i^c}{\partial m_j} = -\alpha \frac{\partial D_i^c}{\partial m_i}.$$

Hence,

$$\Delta D_i^c \approx (1 - \alpha) \left[ \frac{\partial D_i^c}{\partial m_i} \Delta m_i + \frac{\partial D_i^c}{\partial f_i} \Delta f_i \right]$$

where $1 - \alpha > 0$ and the NSR will be a profitable strategy for both EPNs if

$$\frac{\partial D_i^c}{\partial m_i} \Delta m_i + \frac{\partial D_i^c}{\partial f_i} \Delta f_i > 0,$$

which corresponds to the NSR profitability condition for an EPN (see proof of Proposition 4). $\square$

**Proof of Proposition 5:** (i)

$$\Delta D_i^m \approx (1 - \alpha) \frac{dD_i^m}{dm_i} \Delta m_i = (1 - \alpha) \frac{\partial D_i^c}{\partial m_i} \frac{dD_i^m}{dm_i} < 0.$$

(ii)

$$\Delta D_i^e \approx (1 - \alpha) \left( \frac{\partial D_i^e}{\partial m_i} \Delta m_i + \frac{\partial D_i^e}{\partial f_i} \Delta f_i \right) =$$

$$= (1 - \alpha) \left( \frac{\partial D_i^e}{\partial m_i} \frac{dD_i^m}{dm_i} - \frac{\partial D_i^c}{\partial f_i} \frac{dD_i^e}{df_i} \right) \left( \frac{\partial D_i^c}{\partial m_i} - \frac{dD_i^m}{dm_i} \right),$$

where $\frac{\partial D_i^e}{\partial f_i} \frac{dD_i^m}{dm_i} > 0$, $\frac{\partial D_i^e}{\partial m_i} - \frac{dD_i^e}{dm_i} > 0$, and $\frac{\partial D_i^c}{\partial m_i} - \frac{dD_i^m}{dm_i} > 0$ by Proposition 4 (EPN profitability).

(iii)

$$\Delta p_i = (1 - \beta) \Delta m_i = (1 - \beta) \frac{\partial D_i^c}{\partial m_i} \frac{dD_i^m}{dm_i} \frac{\partial D_i^c}{\partial m_i} \geq 0. \square$$

**Proof of Lemma 4.1:** (i)
\[ \Delta \Pi_i^* \approx (1 - \alpha) \left( \frac{\partial \Pi_i^*}{\partial m_i} \Delta m_i + \frac{\partial \Pi_i^*}{\partial f_i} \Delta f_i \right) = (1 - \alpha) \left( 2D_i^c \frac{\partial D_i^c}{\partial m_i} \Delta m_i + 2D_i^c \frac{\partial D_i^c}{\partial f_i} \Delta f_i \right) \]

\[ = (1 - \alpha) \left( 2D_i^c \frac{\partial D_i^c}{\partial m_i} \frac{\partial^2 D_i^c}{\partial m_i \partial f_i} - \frac{\partial D_i^c}{\partial f_i} \frac{\partial D_i^c}{\partial m_i} \right) - 2D_i^c \left( \frac{\partial^2 D_i^c}{\partial m_i \partial f_i} \right) \]

\[ = (1 - \alpha) 2D_i^c \left( \frac{\partial D_i^c}{\partial m_i} - \frac{\partial D_i^c}{\partial f_i} D_i^m \right) \frac{\partial D_i^c}{\partial m_i} \frac{\partial D_i^c}{\partial f_i} \frac{D_i^m}{D_i} \]

(ii)

\[ \Delta W^m = \sum_{i=1}^{2} \left[ \left( \int_{m_{i|NSR}}^{0} D_i^m (x_i) \, dx_i \right) D_i^c \bigg|_{NSR} - \left( \int_{m_i}^{0} D_i^m (x_i) \, dx_i \right) D_i^c \right], \]

\[ \Delta W^m \approx \sum_{i=1}^{2} \left[ \left( \frac{d V_i^m}{d m_i} D_i^c + V_i^m \frac{d D_i^c}{d m_i} \right) \Delta m_i + V_i^m \frac{\partial D_i^c}{\partial f_i} \Delta f_i \right], \]

with \( V_i^m \equiv \int_{m_i}^{0} D_i^m (x_i) \, dx_i \) and \( \frac{d V_i^m}{d m_i} = -D_i^m (m_i) \). Hence,

\[ \Delta W^m \approx \sum_{i=1}^{2} \left[ \left( -D_i^m D_i^c + V_i^m \frac{\partial D_i^c}{\partial m_i} \right) \frac{d D_i^m}{d m_i} \frac{d D_i^c}{d m_i} + V_i^m \frac{\partial D_i^c}{\partial f_i} D_i^m \left( \frac{d D_i^c}{d m_i} - \frac{d D_i^c}{d f_i} \right) \right] = \]

\[ = \sum_{i=1}^{2} \left[ V_i^m \left( \frac{\partial D_i^c}{\partial m_i} - D_i^m \frac{\partial D_i^c}{\partial f_i} \right) - D_i^m D_i^c \right] \frac{d D_i^m}{d m_i} \frac{d D_i^c}{d f_i} \frac{d D_i^c}{d m_i} \frac{d D_i^c}{d f_i} \frac{d D_i^c}{d m_i} \frac{d D_i^c}{d f_i}. \]

(iii)
\[ \Delta W^c = \sum_{i=1}^{2} \left[ V_i^c (f_i |_{NSR}, p_i (m_i |_{NSR}), D_i^m |_{NSR}) - V_i^c (f_i (p_i (m_i ), D_i^m)) \right] \]

\[ \approx \sum_{i=1}^{2} \left[ \frac{\partial V_i^c}{\partial f_i} \Delta f_i + \frac{\partial V_i^c}{\partial p_i} \Delta p_i + \frac{\partial V_i^c}{\partial D_i^m} \Delta D_i^m \right] \]

\[ = \sum_{i=1}^{2} \left[ \frac{\partial V_i^c}{\partial f_i} D_i^m \left( \frac{\partial D_i^m}{\partial m_i} - \frac{\partial D_i^m}{\partial f_i} \right) + \frac{\partial V_i^c}{\partial p_i} (1 - \beta) \left( \frac{\partial D_i^m}{\partial m_i} - \frac{\partial D_i^m}{\partial f_i} \right) + \frac{\partial V_i^c}{\partial D_i^m} \left( \frac{\partial D_i^m}{\partial m_i} - \frac{\partial D_i^m}{\partial f_i} \right) \right] \]

\[ \Delta W^c \approx \sum_{i=1}^{2} \left[ \frac{\partial V_i^c (1 - \beta)}{\partial p_i} \frac{\partial D_i^m}{\partial m_i} - \frac{\partial V_i^c D_i^m}{\partial f_i} + \frac{\partial V_i^c}{\partial D_i^m} \right] \frac{\partial D_i^m}{\partial m_i} - \frac{\partial D_i^m}{\partial f_i}.
\]

Note that \( \Delta p_i \equiv p_i^{NSR} - p_i^* \). Hence, \( \Delta p_i = (1 - \beta) [(1 - \lambda) (m_i^{NSR} - b) - (m_i - b)] = (1 - \beta) [\Delta m_i - \lambda (m_i^{NSR} - b)] \). Since the indifferent merchant has benefit \( b = m_i^{NSR} \), plugging this in \( \Delta p_i \) arises \( \Delta p_i = (1 - \beta) \left( \frac{\partial D_i^m}{\partial m_i} - \frac{\partial D_i^m}{\partial f_i} \right) \big/ \left( \frac{\partial D_i^m}{\partial m_i} - \frac{\partial D_i^m}{\partial f_i} \right) \). \( \square \)

**Proof of Lemma 4.2:**

\[ \Delta W \equiv \Delta W^c + \Delta W^m + \sum_{i=1}^{2} \Delta \Pi_i \]

\[ \Delta W^c + \Delta W^m + \sum_{i=1}^{2} \Delta \Pi_i \approx \sum_{i=1}^{2} \left[ \frac{\partial V_i^c (1 - \beta)}{\partial p_i} \frac{\partial D_i^m}{\partial m_i} - \frac{\partial V_i^c D_i^m}{\partial f_i} + \frac{\partial V_i^c}{\partial D_i^m} \left( \frac{\partial D_i^m}{\partial m_i} - \frac{\partial D_i^m}{\partial f_i} \right) \right] \frac{\partial D_i^m}{\partial m_i} - \frac{\partial D_i^m}{\partial f_i} + \]

\[ + \sum_{i=1}^{2} \left[ V_i^m \left( \frac{\partial D_i^m}{\partial m_i} - \frac{\partial D_i^m}{\partial f_i} \right) - D_i^m \frac{\partial D_i^m}{\partial f_i} \right] \frac{\partial D_i^m}{\partial m_i} - \frac{\partial D_i^m}{\partial f_i} + (1 - \alpha) \sum_{i=1}^{2} \left[ 2 \frac{\partial D_i^m}{\partial m_i} \left( \frac{\partial D_i^m}{\partial m_i} - \frac{\partial D_i^m}{\partial f_i} \right) \right] \frac{\partial D_i^m}{\partial m_i} - \frac{\partial D_i^m}{\partial f_i}^2 \]

\[ \sum_{i=1}^{2} \left[ \left( \frac{\partial V_i^c}{\partial p_i} (1 - \beta) - \frac{\partial V_i^c}{\partial f_i} \right) D_i^m + \frac{\partial V_i^c D_i^m}{\partial m_i} - D_i^m \frac{\partial D_i^m}{\partial f_i} \right] \frac{\partial D_i^m}{\partial m_i} - \frac{\partial D_i^m}{\partial f_i} + \left( \frac{\partial V_i^c D_i^m}{\partial m_i} - D_i^m \frac{\partial D_i^m}{\partial f_i} \right) \frac{\partial D_i^m}{\partial m_i} - \frac{\partial D_i^m}{\partial f_i} \frac{\partial D_i^m}{\partial f_i}^2. \square \]

**Proof of Proposition 6:** Condition \( \left| \frac{\partial D_i^m}{\partial m_i} \right| > \left( \frac{\partial D_i^m}{\partial f_i} \right) \) implies

\[ \left( V_i^m \frac{\partial D_i^m}{\partial f_i} - 2 (1 - \alpha) D_i^m \right) \left( \frac{\partial D_i^m}{\partial m_i} - \frac{\partial D_i^m}{\partial f_i} D_i^m \right) < 0 \]
and $\beta > (<) 1 - \frac{D_i^C D_i^{m_1} - \frac{\partial V_c}{\partial f_i} dD_i^{m_1} + \frac{\partial V_c}{\partial f_i} dD_i^{m_1}}{\partial V_c}$ implies

$$\frac{\partial V_c}{\partial p_i} (1 - \beta) - \frac{\partial V_c}{\partial f_i} D_i^{m_1} + \frac{\partial V_c}{\partial D_i^{m_1}} \frac{dD_i^{m_1}}{dm_i} - D_i^c D_i^{m_1} > (<) 0.$$ 

Therefore,

$$\left( \frac{\partial V_c}{\partial p_i} (1 - \beta) - \frac{\partial V_c}{\partial f_i} D_i^{m_1} + \frac{\partial V_c}{\partial D_i^{m_1}} \frac{dD_i^{m_1}}{dm_i} - D_i^c D_i^{m_1} \right) \frac{\partial D_i^c}{\partial f_i} + \left( \frac{V_i \frac{\partial D_i^c}{\partial f_i}}{V_i \frac{dD_i^{m_1}}{dm_i} - 2 (1 - \alpha) D_i^c} \right) \left( \frac{\partial D_i^c}{\partial m_i} - \frac{dD_i^{m_1}}{dm_i} \right) < (>) 0,$$

implying $\Delta W > (<) 0$. □

**Proof of Proposition 7:** The result comes straightforward by the expression from Lemma 4.2, equating it to zero and solve it while defining $\text{NE} \equiv -\frac{\partial D_i^c}{\partial m_i}$. □

**Proof of Proposition 8:** Substituting $\beta = 1$ in the expression for the social indifference line (Proposition 7) we get

$$\left( \frac{\partial V_c}{\partial f_i} (1 - \beta) - \frac{\partial V_c}{\partial f_i} D_i^{m_1} + \frac{\partial V_c}{\partial D_i^{m_1}} \frac{dD_i^{m_1}}{dm_i} - D_i^c D_i^{m_1} \right) \frac{\partial D_i^c}{\partial f_i} + \left( \frac{V_i \frac{\partial D_i^c}{\partial f_i}}{V_i \frac{dD_i^{m_1}}{dm_i} - 2 (1 - \alpha) D_i^c} \right) \left( \frac{\partial D_i^c}{\partial m_i} - \frac{dD_i^{m_1}}{dm_i} \right) \equiv \omega^{\text{Social}}.$$

For $\frac{\partial V_c}{\partial f_i}$ sufficiently negative, i.e.,

$$\frac{\partial V_c}{\partial f_i} < \frac{\partial V_c}{\partial D_i^{m_1}} \frac{dD_i^{m_1}}{dm_i} - D_i^c,$$

this implies $\omega^{\text{Social}} \geq 0$. Therefore, when $\beta = 1$, the social indifference equation has

$$\text{NE} \geq 0,$$

which guarantees

$$\left\{ \left( \beta, \text{NE} \right) \in [0, 1] \times \mathbb{R}_0^+ : \quad \text{NE} \leq \left( \frac{\partial V_c}{\partial p_i} (1 - \beta) - \frac{\partial V_c}{\partial f_i} D_i^{m_1} + \frac{\partial V_c}{\partial D_i^{m_1}} \frac{dD_i^{m_1}}{dm_i} - D_i^c D_i^{m_1} \right) \frac{\partial D_i^c}{\partial f_i} + \left( \frac{V_i \frac{\partial D_i^c}{\partial f_i}}{V_i \frac{dD_i^{m_1}}{dm_i} - 2 (1 - \alpha) D_i^c} \right) \left( \frac{\partial D_i^c}{\partial m_i} - \frac{dD_i^{m_1}}{dm_i} \right) \right\} \neq \emptyset,$$

that is, assures the existence of a non-empty set of allocations in $(\beta, \text{NE}) \in [0, 1] \times \mathbb{R}_0^+$ where the NSR is socially desirable. In order to show the existence of a non-empty subset of those allocations that is also profitable to EPNs, consider the analysis for $\beta = 1$. Hence, (i) society is better off under the NSR iff

$$0 \leq \text{NE} \leq \omega^{\text{Social}} \quad \text{and} \quad (16)$$

(ii) EPNs increase profits iff

$$0 \leq \text{NE} \leq -\frac{\partial D_i^c}{\partial f_i} D_i^{m_1} \equiv \omega^{\text{Network}}, \quad (17)$$

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where $\omega^{\text{Network}} \geq 0$. The intersection of the sets defined by inequalities (16) and (17) at $\beta = 1$ is defined by

$$0 \leq NE \leq \min \{ \omega^{\text{Network}}, \omega^{\text{Social}} \},$$

where $\min \{ \omega^{\text{Network}}, \omega^{\text{Social}} \} \geq 0$. Hence, the intersection is a non-empty subset of $(\beta, NE) \in [0,1] \times \mathbb{R}_0^+$. □

### 8.2 Platform’s maximization problem

Taking the FOC of the program in (9) we have

$$\left\{ \begin{array}{l}
D_i^c + f_i \frac{\partial D_i^c}{\partial f_i} + m_i \frac{\partial D_i^c}{\partial m_i} D_i^m = 0 \\
f_i \frac{\partial D_i^c}{\partial m_i} + \left( D_i^m D_i^c + m_i \left( \frac{\partial D_i^m}{\partial m_i} D_i^c + D_i^m \frac{\partial D_i^c}{\partial m_i} \right) \right) = 0.
\end{array} \right.$$

Computing the Hessian matrix,

$$H = \begin{bmatrix}
\frac{\partial^2 \Pi}{\partial f_i^2} & \frac{\partial^2 \Pi}{\partial f_i \partial m_i} \\
\frac{\partial^2 \Pi}{\partial m_i^2}
\end{bmatrix},$$

$$\frac{\partial^2 \Pi}{\partial f_i^2} = \frac{\partial D_i^c}{\partial f_i} + \frac{\partial D_i^c}{\partial m_i} \frac{\partial^2 D_i^c}{\partial f_i^2} + f_i \frac{\partial^2 D_i^c}{\partial f_i^2} + m_i \frac{\partial^2 D_i^c}{\partial m_i^2} D_i^m = 2 \frac{\partial D_i^c}{\partial f_i} + \frac{\partial^2 D_i^c}{\partial f_i^2} \left( f_i + m_i D_i^m \right)$$

$$\frac{\partial^2 \Pi}{\partial m_i^2} = f_i \frac{\partial D_i^c}{\partial m_i} + \left( \frac{\partial D_i^m}{\partial m_i} D_i^c + D_i^m \frac{\partial D_i^c}{\partial m_i} \right) = \frac{\partial^2 \Pi}{\partial m_i^2}.$$

Note that $\frac{\partial^2 \Pi}{\partial f_i \partial m_i} = \frac{\partial^2 \Pi}{\partial m_i \partial f_i}$ by Schwarz & Young’s theorem of symmetry of cross-partial derivatives. By Assumption 3 the demand functions $D_i^c (\cdot)$ and $D_i^m (\cdot)$ are linear in their arguments and, by Lemma 1, $p_i$ is linear in $m_i$, therefore

$$\frac{\partial^2 D_i^c}{\partial f_i^2} = \frac{\partial D_i^c}{\partial m_i} = \frac{\partial^2 D_i^c}{\partial f_i \partial m_i} = \frac{\partial^2 D_i^m}{\partial m_i^2} = 0. \quad (18)$$

Using (18) and replacing $m_i$ by the equilibrium expression $m_i^* = \frac{\partial D_i^c}{\partial m_i} \frac{\partial D_i^m}{\partial m_i}$ in the second-order derivatives, we get

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\[ \frac{\partial^2 \Pi_i}{\partial f_i^2} = 2 \frac{\partial D_i^c}{\partial f_i} < 0 \]
\[ \frac{\partial^2 \Pi_i}{\partial f_i \partial m_i} = \frac{dD_i^c}{dm_i} + \frac{\partial D_i^c}{\partial f_i} \left( D_i^m + m_i^* \frac{dD_i^m}{dm_i} \right) = 2 \frac{dD_i^c}{dm_i} < 0 \]
\[ \frac{\partial^2 \Pi_i}{\partial m_i^2} = 2 \left( \frac{dD_i^m}{dm_i} D_i^c + \frac{dD_i^c}{dm_i} D_i^m + m_i^* \frac{dD_i^m}{dm_i} D_i^c \right) = 2 \frac{\partial D_i^c}{\partial f_i} \frac{dD_i^m}{dm_i} + \left( \frac{dD_i^c}{dm_i} \right)^2 < 0. \]

Thus, it arises that

\[ |H_1| = \frac{\partial^2 \Pi_i}{\partial f_i^2} < 0 \]
\[ |H_2| = \frac{\partial^2 \Pi_i \partial^2 \Pi_i}{\partial f_i^2 \partial m_i^2} - \frac{\partial^2 \Pi_i \partial^2 \Pi_i}{\partial f_i \partial m_i \partial m_i \partial f_i} = 4 \left( \frac{\partial D_i^c}{\partial f_i} \frac{dD_i^m}{dm_i} D_i^c + \left( \frac{dD_i^c}{dm_i} \right)^2 \right) - 4 \left( \frac{dD_i^c}{dm_i} \right)^2 \]
\[ = 4 \frac{\partial D_i^c}{\partial f_i} \frac{dD_i^m}{dm_i} D_i^c > 0, \]

assuring that \( H \) is definite negative. Hence \( (m_i^*, f_i^*) \) is a profit maximizer.