Figure 1. *A Babylonian tablet (B.M. 37236) listing undated phases of Mars according to the System A scheme. By permission of the Trustees of the British Museum.*

The Adaptation of Babylonian Methods in Greek Numerical Astronomy

*By Alexander Jones**

THE DISTINCTION CUSTOMARILY MADE between the two chief astronomical traditions of antiquity is that Greek astronomy was geometrical, whereas Babylonian astronomy was arithmetical. That is to say, the Babylonian astronomers of the last five centuries B.C. devised elaborate combinations of arithmetical sequences to predict the apparent motions of the heavenly bodies, while the Greeks persistently tried to explain the same phenomena by hypothesizing kinematic models compounded out of circular motions. This description is substantially correct so far as it goes, but it conceals a great difference on the Greek side between the methods of, say, Eudoxus in the fourth century B.C. and those of Ptolemy in the second century of our era. Both tried to account for the observed behavior of the stars, sun, moon, and planets by means of combinations of circular motions. But Eudoxus seems to have studied the properties of his models purely through the resources of geometry. The only numerical parameters associated with his concentric spheres in our ancient sources are crude periods of synodic and longitudinal revolution, that is to say, data imposed on the models rather than deduced from them.[1] By contrast, Ptolemy's approach in the *Almagest* is thoroughly numerical.[2] In its theoretical aspect, Ptolemy's astronomy set out to describe the apparent motions of the sun, moon, and planets according to models composed of parts, the dimensions and speeds of which were all calculated from quantitative observations. Its practical aspect expressed the behavior of the models in the form of numerical tables by which the apparent positions and other observable phenomena of the heavenly bodies could be predicted for a specified time.

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The research for this article was carried out under a Canada Research Fellowship (Social Sciences and Humanities Research Council of Canada).

[1] Simplicius, *Commentary on Aristotle's De Caelo,* in *Commentaria in Aristotelem Graeca,* Vol. VII: *Simplicii in Aristotelis De caelo Commentaria,* ed. J. L. Heiberg (Berlin, 1894), pp. 493–507. I choose Eudoxus for this comparison because we are relatively well informed by Simplicius about the arrangement of his models, the astronomical phenomena they were intended to explain, and the grounds on which they were first criticized. The same conditions, as I intend to argue elsewhere, probably still applied to the astronomy of Apollonius (ca. 200 B.C.).

[2] G. J. Toomer, *Ptolemy's "Almagest"* (London: Duckworth; New York: Springer, 1984).

The earliest Greek astronomer known to have extensively applied numerical methods to geometrical models is Hipparchus (active ca. 150–125 B.C.).[3] In the *Almagest* Ptolemy passes on much information about how Hipparchus determined the parameters of his solar and lunar models, both when he agrees with Hipparchus's results (as in the measurement of the solar eccentricity and apsidal line) and when he finds fault with them (as in finding the dimensions and periods of the lunar model).[4] Not only do these passages prove the numerical character of Hipparchus's work; but the fact that Ptolemy nowhere discusses comparable measurements by anyone other than Hipparchus is a weighty argument that Hipparchus was the founder of this methodology. This conclusion also finds support in the lack of evidence that any Greek before Hipparchus's time possessed three of the foundations of Greek numerical astronomy: sexagesimal arithmetic, degree measure, and a table of chords for trigonometric computations.[5]

The Mesopotamian origin of the first two of these three resources points to another respect in which Hipparchus differed from his predecessors: his extensive use of elements of Babylonian astronomy. This is not to deny that Babylonian astronomical concepts and conventions, for example the ecliptic and its division into twelve equal zodiacal signs, had entered Greek astronomy from the time of Eudoxus on. But Hipparchus had access to Babylonian data, both observational and theoretical, on a much larger scale and of a much more technical nature than anything that we find in earlier Greek sources. In 1900 F. X. Kugler showed from the recently deciphered astronomical cuneiform texts that the fundamental period relations of Hipparchus's lunar theory, namely his value for the mean synodic month and its ratio to the anomalistic and draconitic months, were parameters taken from the heart of the complex Babylonian System B lunar theory.[6] Significant as it was, this discovery was only the beginning: Babylonian elements have since been recognized in numerous aspects of Hipparchus's astronomical work.[7] At the same time, the study of astrological texts, Greco-Egyptian papyri of the Roman period, and Indian treatises descended from Greek originals has revealed extensive traces of Babylonian predictive schemes, to the degree that it is now evident that many of the most advanced methods of Babylonian

[3] A good general survey of Hipparchus's work is G. J. Toomer, "Hipparchus," in *Dictionary of Scientific Biography,* ed. C. C. Gillispie, 16 vols. (New York: Scribners, 1970–1980) (hereafter *DSB*) Vol. XV, pp. 207–224. For details of the topics discussed here see also section 4 of A. Jones, "Models and Tables of Ancient Astronomy, 200 B.C. to A.D. 300," Part 37.4 of *Aufstieg und Niedergang der Römischen Welt,* ser. II, *Principat* (Berlin/New York: de Gruyter, in press).

[4] Ptolemy, *Almagest* 3.4 (solar model), 4.2, 7, and 9 (lunar periods), and 4.11 (lunar eccentricity).

[5] Degrees and sexagesimal arithmetic first appear in Greek in the *Anaphorikos* of Hypsicles, who wrote a generation or so later than Apollonius, i.e., roughly contemporary with Hipparchus if not a decade or two earlier. See V. De Falco, M. Krause, and O. Neugebauer, *Hypsikles: Die Aufgangszeiten der Gestirne* (Abhandlungen der Akademie der Wissenschaften in Göttingen, Philol.-hist. Kl., 3rd Ser., 62) (Göttingen, 1966); and for the date, G. L. Huxley, "Studies in Greek Astronomers," *Greek, Roman, and Byzantine Studies,* 1963, *4*:83–105. The chord table was probably Hipparchus's invention, as argued by G. J. Toomer, "The Chord Table of Hipparchus and the Early History of Greek Trigonometry," *Centaurus,* 1973, *18*:6–28.

[6] F. X. Kugler, *Die babylonische Mondrechnung* (Freiburg, 1900), pp. 20–40.

[7] Most of the evidence is collected and discussed in G. J. Toomer, "Hipparchus and Babylonian Astronomy," in *A Scientific Humanist: Studies in Memory of Abraham Sachs,* ed. E. Leichty, M. deJ. Ellis, and P. Gerardi (Occasional Publications of the Samuel Noah Kramer Fund, 9) (Philadelphia, 1988), pp. 353–362.

planetary and lunar theory were substantially transmitted into Greek and practiced widely in the Hellenistic world.[8]

How the Greeks came by this knowledge remains controversial, and indeed no single channel of transmission seems adequate to explain the whole range of Babylonian astronomical lore attested in Greek sources. Hipparchus (and through him, Ptolemy) had access to reports of lunar eclipses from Babylon ranging from 747 B.C. to the early fourth century B.C., if not later, as well as Babylonian planetary observations from the second half of the third century B.C. The source of these reports was almost certainly the astronomical archive of Babylon. Significant numbers of observational texts from this archive are extant and are in the course of publication.[9] We can now see that the reports in Ptolemy's *Almagest* represent only a selection from a mass of information so enormous that a translation into Greek of the whole, or even of a large fraction of it, is inconceivable. The only plausible alternative seems to be that some Greek, having considerable technical competence and a specific idea of what sort of observations he wanted, extracted reports from the archive with the collaboration of the astronomers of Babylon. Perhaps this happened more than once, for the Greeks had a long-standing interest in eclipse observations, and the conventions according to which the Babylonian reports in the *Almagest* were translated and dated vary. G. J. Toomer plausibly suggests that Hipparchus himself was responsible for fetching some of the Babylonian observations.[10] That he organized the reports in the published form in which Ptolemy found them is practically certain.[11]

Direct contact with Babylonian astronomers could also explain the Babylonian theoretical parameters in Hipparchus's works. It does not suffice, however, to account for the fact, coming to be fully appreciated only in recent years, that the Babylonian predictive schemes were themselves practiced in Ptolemy's day in substantially unaltered form. The most striking proof of this is a second-century papyrus published by Otto Neugebauer in 1988 that, for all that it uses Greek numerals, is effectively a fragment of a Babylonian System B lunar "ephemeris" in which visibility or eclipse phenomena were computed for a succession of syzygies.[12] This document was a surprise to historians of ancient astronomy, although the shock was perhaps greater than it need have been; for there was already evidence that the rather less complicated Babylonian schemes for predicting planetary phases were used in Roman Egypt.[13]

By what means, then, did the Babylonian predictive schemes pass into the hands of the astrologers of Ptolemy's day? It is at the very least superfluous to

[8] Jones, "Models and Tables" (cit. n. 3), sect. 3.

[9] A. J. Sachs and H. Hunger, *Astronomical Diaries and Related Texts from Babylonia,* 2 vols. to date (Österreichische Akademie der Wissenschaften, Phil.-hist. Kl., 195 and 210) (Vienna, 1988–1989). For the contents of the observational texts see also A. J. Sachs, "A Classification of the Babylonian Astronomical Tablets of the Seleucid Period," *Journal of Cuneiform Studies,* 1948, *2*:271–290; and Sachs, "Babylonian Observational Astronomy," *Philosophical Transactions of the Royal Society of London,* 1974, Series A, *276*:43–50.

[10] Toomer, "Hipparchus and Babylonian Astronomy" (cit. n. 7), pp. 357–360.

[11] Ptolemy, *Almagest* 9.2, refers to an "edition" by Hipparchus of older planetary observations.

[12] O. Neugebauer, "A Babylonian Lunar Ephemeris from Roman Egypt," in Leichty *et al., Scientific Humanist* (cit. n. 7), pp. 301–304. The papyrus is in a private collection.

[13] The evidence is most compelling for Mars and Venus. See B. L. van der Waerden, "Aegyptische Planetenrechnung," *Centaurus,* 1972, *16*:65–91; and A. Jones, "A Second-Century Greek Ephemeris for Venus," *Archives Internationales d'Histoire des Sciences,* in press.

insist that they had to be published in treatises by Greek astronomers before reentering the domain of practical use. One need only assume that scribes trained in the Babylonian astronomical centers occasionally carried their skills elsewhere, and that the methods of astronomical computation that we find in Roman Egypt and still later in India belong to a continuous tradition in which a change of language was an event of little practical significance.[14] Hipparchus and later Greek astronomers could have obtained details of Babylonian theory either in Babylon or from "Chaldeans" residing elsewhere.

Thus two parallel streams of mathematical astronomy emerged in the Hellenistic world during the second century B.C.: an arithmetical astronomy originating in Babylonia and carried on by astrologers, and a numerical-geometrical astronomy founded by Hipparchus and continued by "scientific" astronomers such as Ptolemy. Until quite recently one could have maintained that the two streams maintained separate courses until at last, in late antiquity, the arithmetical approach dried up. Hipparchus, it is true, had assigned to the revolutions in his models certain period relations taken from the Babylonian schemes, but then periodicity was almost the only theoretical assumption shared by the Babylonian schemes and the Greek kinematic models.[15] On a more general level, the very inspiration for Hipparchus's quantification of the models might be traced to his knowledge of what the Babylonians had accomplished.[16] But surely the two traditions, based as they were on fundamentally divergent principles, had little else of substance to offer each other!

However, the assumption that the Greek arithmetical and geometrical traditions developed independently has less foundation in actual documents than in a tacit assumption that the two astronomies had the same scope and the same methodological homogeneity in Hipparchus's time as the original Babylonian systems on the one hand and Ptolemy's system on the other. The harmonious pairing of a theoretical process from observations to quantitative models and a practical process back from the models to predictive schemes or tables is such a prominent feature in the *Almagest* and its modern successors that one is tempted to take it as a pattern for the astronomies of earlier times. It does apply to the period when the Babylonian schemes evolved, probably the fifth and fourth centuries B.C., through the discovery of mathematical models that reproduced the observed distribution of astronomical phenomena.[17] Here theory was the servant of prediction, and we have little reason to believe that the Babylonian astronomers made any important theoretical advances after their computational schemes had attained the more or less canonical forms exhibited in the cuneiform texts of the last three centuries B.C. Once the predictions had attained an accuracy at least as good as that of the observations, there was little motive for deeper investigation. The astronomy that the Hellenistic Greeks received from the hands of the Babylonians was by then more a skill than a science: the quality of the predictions was

[14] In Egyptian astronomical papyri, Greek and Demotic Egyptian are used indiscriminately.

[15] See O. Neugebauer, "Problems and Methods in Babylonian Mathematical Astronomy, Henry Norris Russell Lecture, 1967," *Astronomical Journal,* 1967, *72*:964–972, on p. 972.

[16] Toomer, "Hipparchus and Babylonian Astronomy" (cit. n. 7), p. 361.

[17] For the principles involved see A. Aaboe, "Observation and Theory in Babylonian Astronomy," *Centaurus,* 1980, *24*:14–35.

proverbial, but in all likelihood the practitioners knew little or nothing of the origins of their schemes in theory and observations.

Meanwhile, Greek geometrical astronomy had developed as a "philosophical" science devoted to reconciling eternally valid theoretical models with continually verified observational facts. The date of a particular appearance of Venus as morning star, the magnitude of a particular eclipse, the longitude of Jupiter on a particular date were events the prediction of which possessed no scientific interest. This state of affairs changed with Hipparchus, but less than one might expect.

Hipparchus's use of small numbers of carefully selected observations to determine the parameters of his models had no precedent, so far as we know, in either Greek or Babylonian astronomy. But the resemblance of these procedures to those of Ptolemy in the *Almagest* should not seduce us into thinking that Hipparchus and Ptolemy had the same goals. Ptolemy carefully designed the *Almagest* to give the deduction of his models an apparently ineluctable logical sequence. The solar theory is established first, and purely from observations of solstices and equinoxes; he next works out the lunar theory using the solar theory, then the star catalogue and theory of precession using the solar and lunar theories, and finally the planetary theory using all the preceding work. The plan of each part is itself determined by a chain of cumulative dependence. Ptolemy's treatment of the solar and preliminary lunar models in *Almagest* 3–4 is characteristic. He must first get at least an approximation of the periods of uniform circular motion in his model from a selection of observations over long intervals. Second, he has to measure the ratio of radii of the circular components by applying these mean motions to a selection of observations over short intervals. Third, he determines the instantaneous configuration of the model for an epoch date, from which all subsequent configurations can be extrapolated using the mean motions. Last, he computes tables by which the apparent position of the body can be predicted for a given date. Each step depends on the steps that preceded. If any subsequent corrections are made to the parameters (as happens with the lunar mean motions), Ptolemy takes care to show that his initial assumption of less accurate data introduced no significant errors.

The order of deduction in the *Almagest* is logical but not necessarily chronological, and Ptolemy makes no claim that he actually worked out the stages of his system in exactly the order that he presents. In fact, the notorious solstices and equinoxes that he adduces to confirm the length of the tropical year and the long-term constancy of the lengths of the seasons in *Almagest* 3.1 were allegedly observed in A.D. 139/140, so that the observations on which his solar theory (and therefore the whole system) rests are among the very latest he recorded.[18] What is essential is that the subordinate theories could not be *published* until Ptolemy had empirically justified all the data assumed in their derivation.

Although Hipparchus solved many of the individual problems of establishing parameters of the solar and lunar models in essentially the same way as Ptolemy,

[18] Ptolemy records his own observations from A.D. 127 to 141. For my argument the genuineness of Ptolemy's solstice and equinox observations is irrelevant; this vexed topic is expertly investigated by J. P. Britton, "On the Quality of Solar and Lunar Observations and Parameters in Ptolemy's *Almagest*" (Ph.D. diss., Yale Univ., 1967).

Table 1. Hipparchus's writings on solar and lunar theory, and their counterparts in the *Almagest*

	Almagest	Hipparchus
Solar theory		
Length of the year	3.1	"On the Shifting of the Solsticial and Equinoctial Points," "On the Length of the Year"
Eccentricity and apogee	3.4	Determination of eccentricity and apogee
Epoch and tables	3.2, 6–8	No known work
Lunar theory		
Periods of mean motion	4.2–3	Confirmation of Babylonian System B period relations
Size of epicycle	4.6	Measurement using eccentric model Measurement using epicyclic model
Epoch and tables	4.4, 8, 10	No known work

his researches manifestly had no such broad plan. In the first place, Hipparchus did not write a single comprehensive treatise on astronomy; his contributions to the theory of the sun's and moon's motions were made in several monographs, each devoted to a specific topic. Table 1 correlates the relevant chapters of Books 3 and 4 of the *Almagest* with Hipparchus's writings, identified either by title (where this is known) or by content. The chronology of these monographs is not certain, but it is possible to deduce an approximate sequence from their relation to each other and to the Hipparchian observations reported by Ptolemy.[19] The work on the solar model, corresponding to *Almagest* 3.4, was published probably not long after the vernal equinox of 146 B.C. but before the first measurement of the lunar anomaly, and the second measurement of the lunar anomaly was made before 128 B.C. The works on the length of the year and precession, however, were written later than the vernal equinox of 128 B.C. This means that Hipparchus did not establish the primary element of the solar theory, the period of the sun's mean motion, until near the end of his career, and hence his published researches on the lunar theory cannot have relied on a completely worked out solar model.

Another glaring difference between the Hipparchian and Ptolemaic programs is the apparent absence of predictive tables in Hipparchus's work. Only one author, the astrologer Vettius Valens (late second century), speaks of Hipparchian tables for the sun, and lunar tables published by Hipparchus are nowhere mentioned.[20] If the solar tables that Vettius Valens knew were truly Hipparchus's, and not a later fabrication based perhaps on Hipparchus's solar theory, the point still remains that Hipparchus could scarcely have composed them when he wrote his treatises on the lunar theory; for at this stage he had no confirmed value for any of the fundamental periods (the sidereal, tropical, and anomalistic years) of the solar model.

[19] For the details see Jones, "Models and Tables," sect. 4; and Toomer, "Hipparchus" (both cit. n. 3).

[20] Vettius Valens, *Vettii Valentis Antiocheni Anthologiarum Libri Novem,* ed. D. Pingree (Leipzig: Teubner, 1986), p. 339. In the context it seems unlikely that Valens would have omitted a reference to Hipparchian lunar tables if he had known of any.

But if Hipparchus did not make tables, how did he find the computed positions of the sun and moon that he needed for his theoretical investigations? Considering how generous Ptolemy is with information about other aspects of Hipparchus's theoretical work, he is oddly taciturn whenever reference has to be made to Hipparchus's methods of calculating these positions. I suspect, indeed, that Hipparchus often merely stated the results without discussing how he got them. Only through recent analyses of certain passages of the *Almagest* has the surprising fact emerged that Hipparchus used Babylonian arithmetical schemes to compute both solar and lunar longitudes.[21]

The evidence for Hipparchus's method of computing solar positions comes from Ptolemy's account of Hipparchus's measurements of the ratio of radii between the circles in his lunar model, in which the initial data were the intervals of time and longitude separating three observed lunar eclipses. The intervals of time could be deduced from the observation reports, but the longitudes had to be calculated because parallax interferes with the observed positions. When Ptolemy adopts Hipparchus's procedure in *Almagest* 4.6, he uses his already established solar theory and tables to compute the sun's longitude for the three times of mid-eclipse; the moon's longitude is of course exactly 180° from the sun's. Ptolemy elsewhere (*Almagest* 4.11) quotes the intervals of longitude that Hipparchus assumed in his two measurements of the lunar radial ratio, and it is easy to confirm that these numbers cannot have resulted from a correct trigonometric calculation based on Hipparchus's solar model, with or without tables. The Babylonian System A lunar scheme (with some slight modifications to be discussed presently) furnished him with a much easier approach. In this scheme the synodic arc, that is, the progress in longitude from one opposition (or conjunction) to the next, is determined solely by the sun's longitude at the former date: the ecliptic is divided into two zones, and a constant synodic arc is associated with each zone. Given an initial solar longitude for the first eclipse (possibly estimated from proximity to a solstice or equinox), Hipparchus was able to find the longitudes for the other two eclipses by simple arithmetic.

For Hipparchus's lunar computations, we must turn to the measurements of positions of fixed stars that he is known to have made at various stages of his career. Two conditions determined how he had to find stellar positions: he could easily observe a star's location relative to the moon when the moon passes nearby, but unfortunately the lunar theory was still too insecure to permit an accurate direct computation of the moon's own position at the time of the observation. In exceptional instances, Hipparchus was able to sight the relative positions of the moon and stars during lunar eclipses. For such observations the solar longitude, and hence also the true lunar longitude, could be estimated from the interval of time between the eclipse and the nearest equinox or solstice; the lunar position would then be corrected for parallax. But Ptolemy describes in *Almagest* 7.2 a more general procedure that could be used on any night. During the day before or after the stellar observation, one observes the apparent elongation be-

[21] A. Jones, "Hipparchus's Computations of Solar Longitudes," *Journal for the History of Astronomy,* 1991, *22*:101–125; and Jones, "The Development and Transmission of 248-Day Schemes for Lunar Motion in Ancient Astronomy," *Archive for History of Exact Sciences,* 1983, *29*:1–36, on pp. 23–27.

tween the sun and moon. The solar longitude itself is either calculated or measured using an instrument such as Ptolemy's armillary sphere. The apparent lunar longitude resulting from adding the observed elongation to the sun's longitude must then be corrected for parallax to obtain the moon's true longitude at the time of the diurnal sighting. To this one adds (or subtracts) the product of the moon's current daily motion in longitude by the fraction of a day separating the diurnal and nocturnal sightings to get the true longitude at the time of nocturnal sighting, which must be corrected again for parallax to obtain the apparent position. Since only a few degrees in this final result are contributed by the lunar theory, the error due to the imperfections of this theory can be counted on to be small.

Although Ptolemy does not tell us so, it is very probable that this procedure too goes back to Hipparchus. Ptolemy actually quotes (*Almagest* 4.3 and 5) three diurnal observations of the elongation between the sun and moon that Hipparchus made in 128/127 B.C. Because Ptolemy uses these observations to measure the second anomaly of the moon, modern historians have suggested that Hipparchus too suspected the existence of a second anomaly and was attempting to investigate it.[22] However, it is doubtful whether Hipparchus had actually refined his lunar model to the point where he could have detected the second anomaly. What confirms that these three sightings were associated with *stellar* observations is the fact that Ptolemy has preserved (unnecessarily, for his own purposes) Hipparchus's citation of the moon's daily motion for one of the sightings.[23] The way that this is expressed is critical. Hipparchus writes that the moon's "course" (*δρόμος*) was the 241st, an expression that long eluded explanation until Toomer recognized that it referred to the index number of the day in question in a table of daily motion over a 248-day lunar anomalistic period.[24] This table originally formed part of a Babylonian arithmetical scheme for predicting lunar longitudes for a sequence of consecutive days, associated with the lunar System B.[25] The exact pattern of the Babylonian daily motion table (a linear zigzag function) is described by Geminus in the first century, while adaptations of it are well attested in the Roman period. Without further information it seems safest to assume that Hipparchus used the unmodified Babylonian table.

The debt of Hipparchus's theoretical researches to Babylonian astronomy thus extends beyond his use of Babylonian observations and period relations, justifiable from Ptolemy's point of view, to a reliance on predictive schemes that were not derived from his assumed theoretical models. For Ptolemy such practices would have been utterly irreconcilable with the apodictic and didactic purpose of the *Almagest*. Hipparchus apparently was satisfied to have recourse to the already existing schemes on the grounds that they yielded sufficiently accurate

[22] E.g., O. Neugebauer, *A History of Ancient Mathematical Astronomy* (Berlin: Springer, 1975), p. 309; and Toomer, *Ptolemy's "Almagest"* (cit. n. 2), p. 217 n. 2.

[23] The only dated Hipparchian stellar observation in the *Almagest* is one of Regulus in 128 B.C. (*Almagest* 7.2), and this is also the year that Ptolemy uses for the epoch of Hipparchus's stellar observations in general. G. Grasshoff, *The History of Ptolemy's Star Catalogue* (New York: Springer, 1990), p. 154, concludes on statistical grounds that the bulk of Hipparchus's stellar observations were made about ten years earlier, in the early 130s.

[24] Toomer, *Ptolemy's "Almagest,"* p. 224 n. 14.

[25] Jones, "248-Day Schemes" (cit. n. 21), pp. 2–11.

predictions (or so he believed). For his immediate needs, correct numbers were all that mattered.

Hipparchus put the Babylonian schemes to use in still another way. In the absence of direct observations, he could resort to the schemes to argue for the existence or nonexistence of various astronomical phenomena. It would not be surprising if the prominence of the zodiacal anomaly in the Babylonian planetary schemes provided Hipparchus with ammunition in his polemic against the Greek planetary models hypothesized in his time, since by Ptolemy's account (*Almagest* 9.2) these models generated a constant synodic anomaly. The eclipse schemes were still more useful because they analyzed the solar and lunar motions into factors that could be interpreted in the light of geometrical models. We have one elegant instance of this principle in Hipparchus's extant *Commentary on Aratus,* where he uses it to refute Eudoxus's hypothesis of a solar motion in latitude.[26] Hipparchus argues that if the sun exhibited a noticeable latitudinal deviation from the ecliptic, then the earth's shadow would also deviate from the ecliptic. Yet, he goes on, the eclipse predictions of the "astrologers" (*ἀστρολόγοι*) assume no such deviation, and lead to calculated eclipse magnitudes in error by never more than two digits, which is insignificant. At the date he was writing, the only methods of prediction that could fit this description were the Babylonian schemes.

If the arithmetical methods offered themselves as ready-made substitutes for missing components in the new Hipparchian astronomy, the geometrical theory might in turn be the source of improvements in schemes that had been cut off from their empirical origins. Probably no Greek astronomer had the faintest notion of why the particular structures that occurred in the Babylonian schemes had been chosen, but this was no obstacle to tinkering superficially with the numerical parameters associated with these structures. The phenomenon can already be seen in Hipparchus's calculations of the solar longitudes for the eclipses used in his two measurements of the lunar radial ratio.[27] When making his first measurement he was not content to take over the rules of the System A lunar scheme as he found it. Without altering the constant synodic arcs associated with the two zones of the ecliptic, he changed the definition of the zones so that they were exactly equal and centered on the apsidal line that he had determined theoretically for his solar model. Making the zones equal was a mistake, because it introduced a systematic error in the predicted longitudes that was large enough to throw his first measurement of the lunar model seriously off. For his second measurement, Hipparchus also changed the synodic arcs, probably to conform with the maximum and minimum apparent velocities resulting from the eccentricity in his solar model. This correction brought about a considerable improvement in the accuracy of the ensuing calculations. If Hipparchus did eventually publish solar tables for the prediction of the sun's longitude on given dates, they could have used a similar zone structure with the constant increments scaled to correspond to days instead of synodic months.[28]

[26] Hipparchus, *Hipparchi in Arati et Eudoxi Phenomena Commentarium,* ed. K. Manitius (Leipzig: Teubner, 1894), pp. 88–90.

[27] Jones, "Hipparchus's Computations" (cit. n. 21).

[28] A pre-Ptolemaic solar table has now turned up in an unpublished papyrus, *P. Oxy. Inv.* 32 4B.1/E(1-2) a. A zone scheme much more refined than Hipparchus's two-zone scheme apparently underlies the table, but the associated speeds are derived from Hipparchus's model.

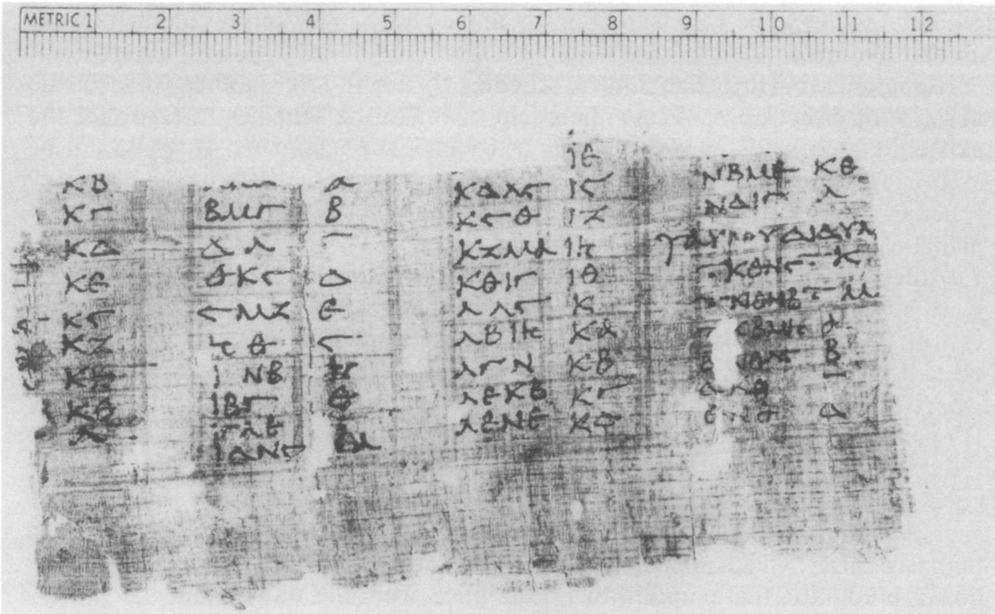

Figure 2. *Part of an auxiliary table (*P. Mich. *151) belonging to a Greek adaptation of the System A Mars scheme. By permission of the Department of Rare Books and Special Collections, University of Michigan Library.*

A papyrus dating from the third century of our era, *P. Heid. Inv.* 4144 + *P. Mich.* 151 (cf. Figure 2), testifies to the existence of a scheme for Mars related to the Babylonian System A scheme in much the same way as Hipparchus's solar schemes were related to their Babylonian ancestor.[29] Like other schemes from Greco-Egyptian papyri that will be discussed below, this scheme cannot be dated precisely or attributed to any particular astronomer, although we are obviously dealing with developments after Hipparchus but uninfluenced by Ptolemy. Mars's pronounced orbital eccentricity results in a very uneven distribution of occurrences of the planet's phases over the ecliptic. The Babylonian scheme reproduced this distribution very successfully by dividing the ecliptic into six equal zones, with associated synodic arcs varying from a minimum of 30° to an ideal maximum of 90°. Through B. L. van der Waerden's analysis of the data for Mars in Egyptian astronomical "Almanacs," we know that the original Babylonian scheme, employing these synodic arcs, was known in Roman Egypt.[30] In this papyrus, however, a new set of values for the synodic arcs are associated with the six Babylonian zones, exhibiting a much smaller range of variation, from $40\frac{3}{4}°$ to 57°. The modified scheme generates such poor predictions of the longitudes of Mars's phases that it clearly cannot have been based on, or even confronted with, systematic observations of the phases. On the other hand, the numbers agree closely with the synodic arcs that one would get from a geometrical

[29] A. Jones, "Babylonian and Greek Astronomy in a Papyrus Concerning Mars," *Centaurus*, 1991, *34* (in press). Earlier discussions: O. Neugebauer, "A New Greek Astronomical Table (P. Heid. Inv. 4144 + P. Mich. 151)," *Historisk-filosofiske Meddelelser: Danske Videnskabernes Selskab*, 1960, *39*(1); and Neugebauer, *History of Ancient Mathematical Astronomy* (cit. n. 22), pp. 946–948.

[30] Van der Waerden, "Aegyptische Planetenrechnung" (cit. n. 13), pp. 77–87.

model in which Mars travels on an epicycle revolving around an eccentric deferent, with eccentricity of $\frac{1}{10}$ or less. The astronomer who devised the papyrus scheme, I conclude, had settled on this excessively small eccentricity for Mars's orbit, used it to calculate synodic arcs for six initial longitudes at 60° intervals, and imposed these values on the System A zones.

The Babylonian schemes were directed primarily at the prediction of planetary phases, eclipses, and other lunar phenomena connected with syzygies, but even before their transmission into Greek they had been extended to address the problems of predicting day-to-day positions and the dates when a planet entered a new zodiacal sign. The growth of Hellenistic horoscopic astrology undoubtedly encouraged the development of more handy, if not always more accurate, methods. Daily motion in Babylonian astronomy was characteristically handled by interpolation between phases. The simplest hypothesis, linear motion, gave rise to "velocity schemes" that broke up a planet's synodic period into a sequence of intervals of constant speed; the patterns varied through the ecliptic in order to account for the effects of zodiacal anomaly. There is little evidence for velocity schemes in the extant Greek documents, but they do turn up in early Indian texts that depend on Greek sources.[31] In most cases the Indian velocity schemes are intended to apply throughout the ecliptic; thus all attempt to describe the zodiacal anomaly is abandoned in the interest of simplicity. In other respects these schemes seem to have been modeled on Babylonian patterns without interference from Greek kinematic theory; a possible exception is an elaborate (and fragmentarily preserved) velocity scheme for Venus that, as van der Waerden has shown, gives a good approximation of epicyclic motion.[32]

A more refined treatment of daily motion attested in cuneiform texts is to interpolate between phases using higher-order difference sequences, that is, to assume that the daily motion, or even the daily *change* in motion, increases or decreases by constant differences.[33] Bridging the interval in time and longitude between two given phases with a second- or third-order sequence of terminating sexagesimal numerals is not a trivial problem, and the two surviving examples, for Jupiter and Mercury, may be scribal exercises rather than specimens of normal Babylonian astronomical practice. Nevertheless we have two second-century papyri, *P. Dem. Carlsberg* 32 and *P.S.I.* 1492, that show similar methods applied to the daily motion of Mercury and Saturn.[34] Algebraic difficulties are eliminated in these "templates" by tabulating a single standard mean synodic cycle composed of stretches of second-order and linear motion. The template was used in conjunction with a list of epoch longitudes corresponding to the beginning of each synodic period; one obtained the longitude for the day in question by reading off from the template the progress in longitude corresponding to the number of days

[31] For summaries and references see D. Pingree, "History of Mathematical Astronomy in India," in *DSB*, Vol. XV, pp. 533–633, on pp. 540–542.

[32] B. L. van der Waerden, "The Motion of Venus in Greek, Egyptian and Indian Texts," *Centaurus*, 1988, *31*:105–113.

[33] P. Huber, "Zur täglichen Bewegung des Jupiter nach babylonischen Texten," *Zeitschrift für Assyriologie*, 1957, N.S., *18*:265–303.

[34] O. Neugebauer and R. Parker, *Egyptian Astronomical Texts*, 3 vols. (Providence, R.I.: Brown Univ. Press, 1969), Vol. III, pp. 240–241 and pl. 79B; and A. Jones, "A Greek Saturn Table," *Centaurus*, 1984, *27*:311–317.

since epoch, and adding it to the epoch longitude. There may have been a separate correction for zodiacal anomaly, but we have no evidence for it. The numerical patterns of the two preserved templates seem to be uninfluenced by geometrical models.

New theoretical considerations did find their way into a similar template scheme for lunar motion.[35] This lunar scheme seems to have been very popular during the first three centuries of our era, and enough relevant texts survive to permit an almost complete reconstruction. The template is a descendant of the Babylonian table of lunar daily motion over a 248-day anomalistic period, which we have already met in connection with Hipparchus. In both the Greek scheme and its Babylonian ancestor the lunar daily motion is described as a linear zigzag function, alternately increasing and decreasing by constant differences between fixed maxima and minima. The assumed periods of longitude and anomaly are more accurate in the template scheme than in the Babylonian version, and the difference between maximum and minimum daily motion has been reduced to about $\frac{5}{7}$ of the Babylonian value. The last change results in a pattern of longitudinal motion that closely fits a simple epicyclic model with a radial ratio of about $5\frac{1}{3}$:60, which is very near Ptolemy's ratio of the epicycle radius, $5\frac{1}{4}$:60. This suggests that the Babylonian function was adapted to bring it into accord with a post-Hipparchian model correctly describing the moon's longitudes at syzygies (when the second anomaly vanishes), that is, a model based on eclipse observations.

Entirely without precedent in the Babylonian sources is the treatment of lunar latitude in the Greek template scheme. The epoch list and the template tabulate not only the moon's progress in longitude but also its argument of latitude, that is, its elongation from the northern limit of its orbit. The coordinated variation in the argument of latitude and the daily motion shows that the inventors of the template scheme were fully aware of the interdependence of lunar anomaly and latitude, an effect that has no counterpart in the Babylonian lunar schemes but follows directly from consideration of a kinematic model. From an obscure passage in Pliny's *Natural History* (2.68–76) it appears that planetary template schemes existed in the first century that described latitudinal motion in a similar way; the extant papyrus templates concern only longitudinal motion.[36]

The pre-Ptolemaic astronomy that emerges from the study of the fragmentary contemporary documents is a strange symbiosis of arithmetical and geometrical methods that will be sure to disconcert anyone who expects to find a record of steady scientific progress from the Babylonians through Hipparchus to Ptolemy. Theoretical work operated with geometrical hypotheses but did not disdain to employ initial data derived from arithmetical schemes; the predictive schemes, for their part, readily absorbed new parameters from geometrical theory without casting off their arithmetical structures. For Hipparchus the availability of the Babylonian predictive methods was, on the whole, a boon. It must be remembered that, unlike Ptolemy, Hipparchus could not use his Greek predecessors' work as a first approximation in determining the parameters of his models; without the arithmetical schemes he would have had to invent new ways of dealing

[35] Jones, "248-Day Schemes" (cit. n. 21), pp. 14–23.
[36] A. Jones, "Pliny on the Planetary Cycles," *Phoenix*, 1991, *45* (in press).

with subsidiary problems that were only incidental to the topics of his researches. On the other side, the efforts of Hellenistic astronomers to accommodate Babylonian schemes to their models at first gave valuable experience in astronomical table making; the lunar template scheme is an especially successful fusion of Babylonian and Greek concepts that well deserved its wide reception in antiquity. The attempts to tamper with the planetary schemes were less happy and mark a step backward, both in accuracy of prediction and in theoretical sophistication, from the Babylonian originals.

During this same period, methods of astronomical prediction were developed that actually described, rather than merely being fitted to, geometrical models, using mean motions and equation tables calculated by the correct trigonometrical formulas. These methods are known to us primarily through the Indian astronomical traditions, which are practically silent about the empirical basis of the parameters of their models. To date, comparable tables have not turned up in any papyrus; but an astronomical inscription discovered at Keskinto on Rhodes, probably dating from about 100 B.C., evinces the use of huge compound periods of planetary anomaly characteristic of the Indian systems.[37] Perhaps the "aeon-tables" mentioned in various ancient sources belonged to this tradition.[38] The Indian tables already approach Ptolemy's ideal of "exhibiting the uniform circular motion,"[39] but Ptolemy seems to have been the first to understand the importance of working out the entire system of the heavenly bodies according to a logical deductive plan and a consistent methodology. The *Almagest* was so successful in this respect that it is only with much difficulty that we are coming to appreciate its originality.

[37] Neugebauer, *History of Ancient Mathematical Astronomy* (cit. n. 22), pp. 698–705.

[38] B. L. van der Waerden, "Ewige Tafeln," in *Arithmos-Arrythmos: Skizzen aus der Wissenschaftsgeschichte: Festschrift für Joachim Fleckenstein zum 65. Geburtstag,* ed. K. Figala and E. H. Berninger (Munich: Minerva, 1979), pp. 285–293; and Toomer, *Ptolemy's "Almagest"* (cit. n. 2), p. 422 n. 12.

[39] The phrase is quoted from *Almagest* 9.2; Ptolemy describes the concept as one of his goals in tabular presentation near the end of 3.1.