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A ROUTE TO THE ANCIENT DISCOVERY OF NON-UNIFORM PLANETARY MOTION

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The models of Greek kinematic astronomy, and the medieval traditions that descended from it, were constructed from "uniform circular motions". I do not know of any rigorous definition of this kind of motion in a classical text. In the simpler models it is easy to see what constitutes uniform motion:

(i) In an eccentric model with a stationary or moving apsidal line, such as Ptolemy's solar model (*Almagest* Book 3), the heavenly body travels with constant speed along its eccentre as measured from the apogee of the eccentre, or equivalently it travels with constant angular velocity as seen from the centre of the eccentre relative to the apsidal line.

(ii) In a simple epicyclic model, such as Ptolemy's first model for the Moon (*Almagest* 4), the body travels with constant speed along its epicycle as measured from the apogee of the epicycle, while the centre of the epicycle moves with constant speed along its deferent as measured from any fixed point on the deferent.

Ptolemy stretched the interpretation of uniform circular motion in his more complex models on several occasions:

(a) In the revised lunar model of *Almagest* 5.2, the centre of the Moon's epicycle travels along an eccentric deferent with a moving apsidal line, in such a way that the epicycle centre has constant angular velocity as seen from the centre of the cosmos, not from the centre of the eccentre.

(b) In the same model (as demonstrated in *Almagest* 5.5), the apogee of the epicycle, relative to which the Moon's own motion is uniform, is defined as lying on a radius through the centre of the epicycle and through a moving point diametrically opposite the centre of the eccentre on the circular path of the centre of the eccentre.[1]

(c) In the model for Mercury (*Almagest* 9), the centre of the epicycle travels along a revolving eccentre with uniform angular velocity as seen not from the centre of the eccentre but from a point whose position relative to the centre of the cosmos is sidereally stationary.

(d) In the models for the remaining four planets (*Almagest* 10–11), the centre of the epicycle travels along a sidereally fixed eccentre with uniform angular velocity as seen not from the centre of the eccentre but from a point twice as far from the centre of the cosmos along the apsidal line as the centre of the eccentre.

Ptolemy chooses to regard all these motions as uniform, though obviously not in the intuitive sense. Most notably, (a), (c), and (d) result in making the point that is

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moving "uniformly" vary in actual speed in its revolution with respect to the reference point.

Ptolemy states (*Almagest* 5.1) that he discovered the phenomena motivating (a) and (b) by examining elongations of the Moon from the Sun observed by Hipparchus and by himself, such as the ones that he adduces in *Almagest* 5.3 and 5.5 to demonstrate the characteristics and parameters of the revised model. This seems to be a credible assertion. In *Almagest* 9.9 Ptolemy determines the location of the centre of uniform motion of Mercury's model (c) through an analysis of observed greatest elongations of Mercury, and while the demonstration of Mercury's model in the *Almagest* manifestly does not reproduce the heuristic route by which Ptolemy arrived at that model, it is nevertheless plausible that the original derivation relied on greatest elongations.[2]

The special centre of uniform motion in (d), the so-called 'equant' point, presents the most historically interesting problem. Like (a), and unlike (b) and (c), the equant reproduces a real property of planetary motion rather than an artifact of imperfect observations or analysis. Moreover, Ptolemy demonstrates the distinct existence and location of the equant point, such that the centre of the eccentric deferent bisects the line from the centre of the Earth to the equant point, only for Venus. The basis of his demonstration is a variety of observation that is not possible for the three outer planets: greatest elongations observed when Venus's epicycle was calculated to be roughly halfway between the apogee and perigee of the deferent (*Almagest* 10.3). It is very difficult to believe that Ptolemy arrived at the equant in this way, and for this planet. Venus's eccentricity is rather small, so that while Ptolemy could perhaps have detected that Venus's centre of uniform motion seemed to be further from the centre of the cosmos than the centre of its eccentre, it would not be so easy to find the 2 to 1 ratio of distances (which is in fact more or less optimal for this type of model) or to justify its application to the remaining three planets. That Ptolemy demonstrates the equant for Venus is probably due to the circumstance that Venus is the first planet, in the sequence from innermost to outermost that Ptolemy chooses to follow, that he believes calls for this model type. He may also have wished to provide a 'proof' of the model precisely for the planet with the smallest eccentricity to ward off any doubt that it applies to all the planets except Mercury.

In an unpublished paper cited by Toomer in his translation of the *Almagest*, Swerdlow proposed a different route to the equant, in which the central role would have been played by the retrogradations of the outer planets, and in particular Mars.[3] A similar (but by no means identical) hypothesis was independently developed by Evans.[4] The starting point for both is a remark that Ptolemy makes in *Almagest* 10.6 when he is making the transition from his treatment of the inner planets to the outer ones, and that they see as an indication of how Ptolemy first arrived at the equant and the bisection of the eccentricity:

> In the case of the remaining three, i.e. Mars, Jupiter, and Saturn, we find a single model for their motion, similar to the one that was determined for Venus, namely

> the model according to which the eccentric circle, on which the centre of the epicycle always travels, is drawn around a centre that bisects the line between the centres of the ecliptic [i.e. the centre of the cosmos] and of the circle that brings about the uniform revolution of the epicycle [i.e. the 'equant circle' centred on the equant point]. For in the case of each of these planets, speaking in terms of a rather rough method, the eccentricity that is found by means of the greatest difference caused by the zodiacal anomaly proves to be approximately double the eccentricity derived from the magnitude of the retrogradations around the greatest and least distances of the epicycle.

What Ptolemy means by the eccentricity found "by means of the greatest difference caused by the zodiacal anomaly" is clear enough. The difference in question is that between the longitude at which the planet would have appeared if there were no zodiacal anomaly and the longitude at which it actually appears. Assuming a preliminary model involving an epicycle (for the synodic anomaly) and eccentric deferent (for the zodiacal anomaly) but no equant, Ptolemy measures this eccentricity for the three outer planets in the first parts of *Almagest* 10.7, 11.1, and 11.5 by analysis of a set of three observations of mean oppositions, analogous to the analysis by which he measured the anomalies of the Sun and Moon in *Almagest* 3.4 and 4.6.

Evans and Swerdlow interpret the "eccentricity derived from the magnitude of the retrogradations" as meaning the eccentricity in an epicycle-and-eccentre model that would produce the observed range of variation in the planet's retrograde arcs.[5] There is some divergence in the particulars, since in Evans's reconstruction of Ptolemy's reasoning, Ptolemy is speaking in the first instance of the eccentricity in an *equantless* model that would generate, roughly speaking, the observed maximum and minimum retrograde arcs, whereas Swerdlow takes Ptolemy's statement as applicable to an equant model in which the eccentricity of the centre of the eccentric is found, independently of the eccentricity of the equant, from the variation of the retrograde arcs. I will not attempt to summarize the courses of deduction that Evans and Swerdlow propose, merely remarking that both presume, with good reason, that Mars furnished the evidence, since it is the only one of the outer planets whose eccentricity is large enough to produce a conspicuous variation in retrograde arcs.[6] For the convenience of the reader, however, I will briefly review the behaviour of the retrograde arcs of the outer planets in relation to eccentre-and-epicycle models.

Figures 1–3 show the actual retrograde arcs of Mars, Jupiter, and Saturn for the first few synodic periods starting from A.D. 100, plotted as a function of the tropical longitude of the midpoint of the retrogradation. Also plotted on each graph are the maximum and minimum retrograde arcs predicted by (a) an equantless epicycle-and-eccentre model using the bisected eccentricity and apogee of Ptolemy's final equant model, (b) an equantless model using the eccentricity of the equant in the same model, and (c) Ptolemy's equant model.[7] As one would expect, Mars shows the most pronounced differences from model to model. What is most striking is that the equantless models predict the largest retrograde arcs near perigee, whereas the

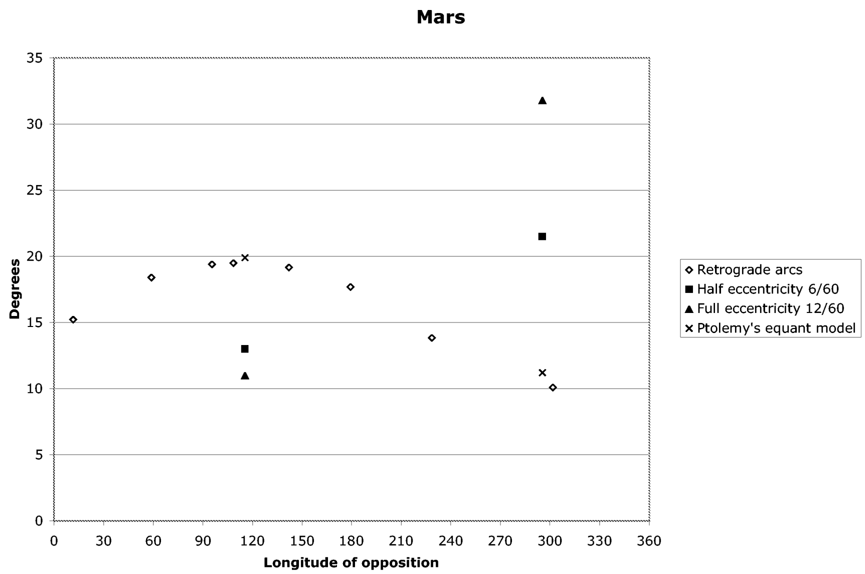

FIG. 1. Retrograde arcs of Mars *c.* A.D. 100.

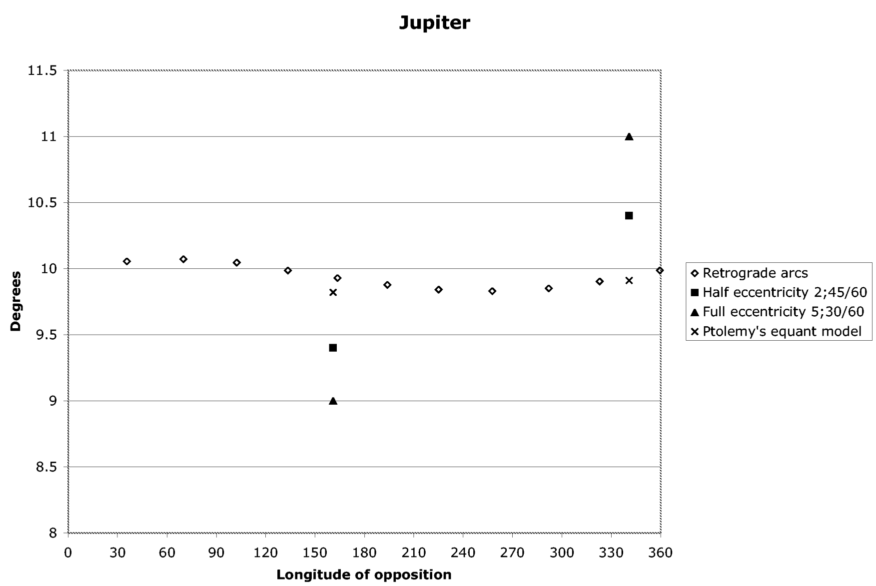

FIG. 2. Retrograde arcs of Jupiter *c.* A.D. 100.

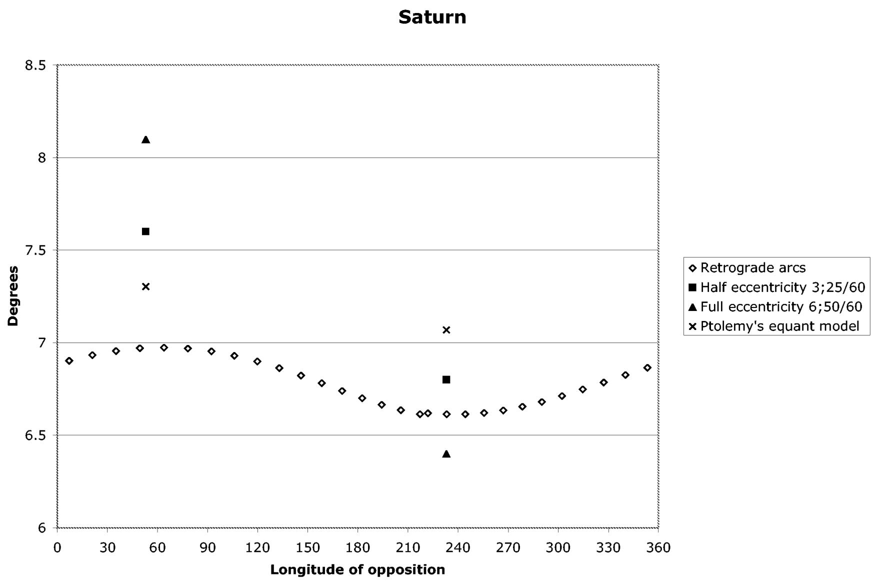

FIG. 3. Retrograde arcs of Saturn *c.* A.D. 100.

maximum for Mars is actually at apogee. Bisecting the eccentricity derived from oppositions yields a more reasonable maximum — at some cost to the minimum — but it is still situated in diametrically the wrong direction (hence Evans posits that an intermediate model with the apsidal line displaced 180° played a part in Ptolemy's route to the equant). For Jupiter even the equantless model with the half-eccentricity predicts too large a variation in the retrograde arc; the actual variation is only about a quarter of a degree, scarcely detectable for an ancient observer, and interestingly the maximum and minimum are significantly offset from the apsidal line.[8] Only for Saturn does the bisected eccentricity produce a perceptible improvement on the full eccentricity in reproducing the amplitude *and* phase of the retrograde arcs, though here Ptolemy's too large epicycle radius means that the predicted values are noticeably too large. Ptolemy's equant models turn out not to generate retrogradations that behave like the ones resulting from the equantless models with bisected eccentricity, and on the whole they fit the phenomena more closely — spectacularly so for Mars.

While I have no fault to find with Swerdlow's or Evans's hypothetical routes to the equant model, I do not think we need to feel bound by the passage from *Almagest* 10.6 quoted above to believe that historically the equant was discovered from retrograde arcs. In that passage, Ptolemy is not writing an historical or autobiographical account, but giving an ostensibly empirical justification for a modelling assumption, which might of course be related to the way Ptolemy arrived at the model, but for that matter might not. There are a number of comparable passages in the *Almagest* where

Ptolemy justifies a decision he makes about the structure of a model by appealing to a phenomenon that can be succinctly expressed and easily understood with minimal theoretical presuppositions.[9] Hence it seems justifiable to look for other observable planetary behaviours that could have suggested the bisection of the eccentricity. We would still expect such a behaviour to be manifested around the time of the retrogradations, since that is when the planet is nearest the Earth so that the deficiencies of a simple eccentre-and-epicycle model are most pronounced. In the following, I will describe how one can be led directly to Ptolemy's model by a kind of observation that was readily available and that was of obvious utility for kinematic modelling.

Neugebauer has drawn attention to several ancient Greco-Roman astronomical and astrological texts that mention a special stage in the synodic cycle of Mars when it is at an elongation at or near 90° from the Sun; two such situations, called by Pliny *nonagenarii* ("nintieths"), occur in each cycle, one roughly three months before opposition, the other roughly three months after opposition.[10] Neugebauer plausibly interpreted these as references to the points when Mars passes in direct motion through the longitude of its opposition. Such significance as the ancient sources assign to the *nonagenarii* is purely astrological, although Neugebauer suspected that they might also have been regarded for computational convenience as the limiting points of the more or less linear interval of the planet's direct motion. Since the term *nonagenarius* is strictly appropriate only for Mars, I will use the expression "triple passage" for the general situation of a planet passing three consecutive times through its point of opposition.

So far as I am aware, no actual reports of Mars passing direct through its recent or imminent point of opposition survive among the few planetary observation reports in classical sources (most are in the *Almagest* itself). However, the observation report of Jupiter passing close by δ Cnc on 241 B.C. Sept. 4 that Ptolemy uses to correct Jupiter's periodicities of mean motion in *Almagest* 11.3 catches the planet in just this situation. Ptolemy himself does not allude to the fact, or make any use of it, but it was likely the reason why this observation was preserved in whatever source he had, since a recently discovered papyrus fragment of an astronomical treatise from the generation before Ptolemy cites an observation report of Jupiter passing the very same star on 241 B.C. Dec. 31 at its opposition.[11] Hence it is reasonable to ask whether triple passages might have had a theoretical application in the work of Greek astronomers before Ptolemy.

In the following I designate the point of opposition of an outer planet Θ, and the points where a planet has the same longitude before and after opposition Θ_1 and Θ_2. Figure 4 shows how the triple passage would be accounted for by a simple epicyclic model of the kind that might have been current before Hipparchus's time. At opposition the planet lies along the radius from C, the centre of the deferent, to E_0, the centre of the epicycle. During the time T_1 between Θ_1 and Θ, the radius from the epicycle's centre to the planet revolves through the angle $\Delta\bar{\alpha}_1$, and the radius from C to the epicycle's centre through $\Delta\bar{\lambda}_1$, such that $r/R = \sin \Delta\bar{\lambda}_1 / \sin (\Delta\bar{\lambda}_1 + \Delta\bar{\alpha}_1)$,

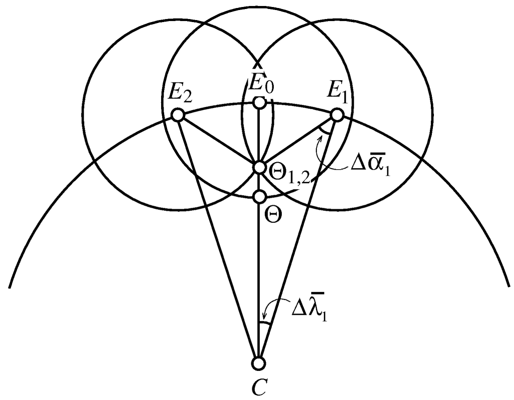

FIG. 4. Triple passage of a planet assuming an epicycle model.

where r and R are the radii of epicycle and deferent respectively. $\Delta\bar{\lambda}_1$ and $\Delta\bar{\alpha}_1$ can be calculated from the periodicities of the planet and the observed time T_1. Hence we readily obtain the size of the epicycle. (The same radius, of course, should follow from using T_2, the time between Θ and Θ_2.)

It is worth remarking in passing that little loss of accuracy need result if the observations of Θ_1 and Θ_2 are not made when the planet is at the exact longitude of opposition, but instead at a nearby longitude, say with reference to a conveniently situated star. To allow for this, one may calculate with $T_{avg} = \frac{1}{2}(T_1 + T_2)$ in place of T_1 or T_2; the error when Θ_1 and Θ_2 are equally displaced is small because the rate of change of the apparent velocity (as seen from C) near these moments is small.

If we hypothetically displace the observer from the centre of the deferent, we obtain an eccentre-and-epicycle model, the variety that (it is plausibly supposed) was commonly employed for the planets between Hipparchus and Ptolemy.[12] In this model the observed times T_1 and T_2 between Θ_1, Θ, and Θ_2 do not in general lead to an exact derivation of r/R, because the change of viewpoint from which Θ_1, Θ, and Θ_2 are seen as the same longitude destroys the relation between $\Delta\bar{\lambda}$, $\Delta\bar{\alpha}$, r, and R. However, this relation still holds when the opposition falls along the apsidal line, since then the apparent positions are the same as if one observed from C. If, therefore, one knows the approximate location of the apsidal line (say from consideration of the symmetrical distribution of planetary phenomena with respect to this line), one can look for oppositions occurring at or close to either the apogee or the perigee, and derive r/R as before. Obviously one would expect the observed T_1 and T_2 to be equal and to be the same duration at both ends of the apsidal line.[13]

But what would observations show? To find out, we select triple passages of the three outer planets in the sequence of years starting with A.D. 100 that fell close to each planet's apsidal line. We also include the analogous data for Venus's inferior conjunctions, which can be treated just like the oppositions of the outer planets except

TABLE 1.

Date of Opposition	Θ	T_1	T_2	T_{avg}
101 January 9	109° (near apogee)	83d	86d	84.5d
103 February 13	142°	83	85	84
105 March 22	180°	77	83	80
107 May 13	229°	68	71	69.5
109 July 27	302° (near perigee)	60	58	59
111 October 6	11°	73	68	70.5
113 November 21	58°	81	79	80

that the longitudes and dates of the conjunction must of course be calculated. The method is probably not applicable to Mercury because of the difficulty of observing this planet in the necessary configurations.

Thus for Mars a fifteen-year period of observations might yield the seven oppositions and associated time intervals shown in Table 1. The variation in T_{avg} is pronounced, and clearly dependent on longitude such that the maximum is at apogee and the minimum at perigee. Supposing the radial distance R to be a constant 60 units, and taking the extreme times as pertaining approximately to the situations where the opposition occurs *precisely* at apogee and perigee, we find:

Mars	T_{avg}	r/R
Apogee	84.5	42.2/60
Perigee	59	36.3/60

It seems reasonable, however, to posit that r remains constant, in which case the centre of uniform revolution must lie nearer the apogee than the perigee — that is, we require an equant. The displacement of the equant C from the centre of the deferent D (*cf.* Figure 5) is easy to determine: the angles ACE_A, PCE_P, $CE_A\Theta_{1A}$, and $CE_P\Theta_{1P}$ are known from the mean motions and the observed T_{1A} and T_{1P}, and hence the ratios of CD and of the epicycle radius to the radius of the deferent can be calculated trigonometrically. From the above data for Mars, we find that CD is approximately 5.7 units and r is approximately 39.1 units such that the radius is 60 units. These results compare very well with Ptolemy's determination that CD is 6 units and r 39.5 units.

It will suffice to summarize our simulated "observational" data for the remaining planets:

Jupiter	T_{avg}	r/R
Apogee	121	12.0/60
Perigee	116	11.0/60

$CD = 2.6$, $r = 11.5$ (Ptolemy: $CD = 2.75$, $r = 11.5$).

Saturn	T_{avg}	r/R
Apogee	136.5	6.7/60
Perigee	130	5.9/60

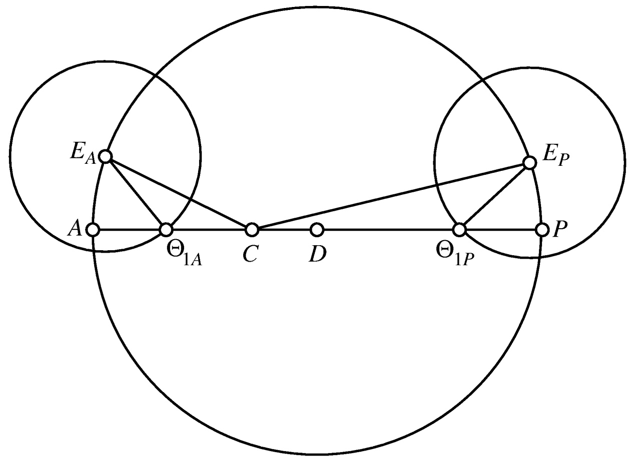

FIG. 5. Triple passages observed at apogee and perigee.

CD = 4.1, *r* = 6.3 (Ptolemy: *CD* =3.42, *r* = 6.5).

Venus	T_{avg}	r/R
Apogee	44.5	42.2/60
Perigee	41.5	36.3/60

CD = 1.0, *r* = 43.4 (Ptolemy: *CD* =1.25, *r* = 43.17).

The variation in T_{avg}, which motivates the equant, should have been detectable for all three planets from careful observations of stellar passages.[14]

The location of the observer along the apsidal line can now be estimated from the observed intervals in time and longitude between a pair of consecutive oppositions (or, for Venus, conjunctions) near the apogee or perigee. Figure 6 shows the configuration for a pair of oppositions, one of which (Θ_A) is at the apogee; as before, *C* is the centre of uniform motion, *D* is the deferent's centre, and now *T* is the observer. Angle *CTE* is the observed difference in true longitude between the two oppositions, and angle *ACE* is the difference in mean longitude, calculated from the observed difference in time. Since *CD* has already been determined, *CE* and hence *CT* can be found trigonometrically. In a case where the two oppositions are equally situated on either side of the apsidal line, one can of course carry out the same computations using half the observed difference in true longitude as *CTE* and half the calculated difference in mean longitude as *ACE*.

For example, Mars's opposition of 109 July 27 fell close to the perigee, as is evident from the near symmetry of T_{avg} in the tabulation above. This opposition is "observed" as occurring 801 days and 69.4° after its predecessor, and 806 days and 72.4° before its successor. From the first pair, we find *CT* = 11.2, and from the second pair, *CT* = 11.4 such that *R* = 60. Again, the oppositions of 101 January 9 and 103 February

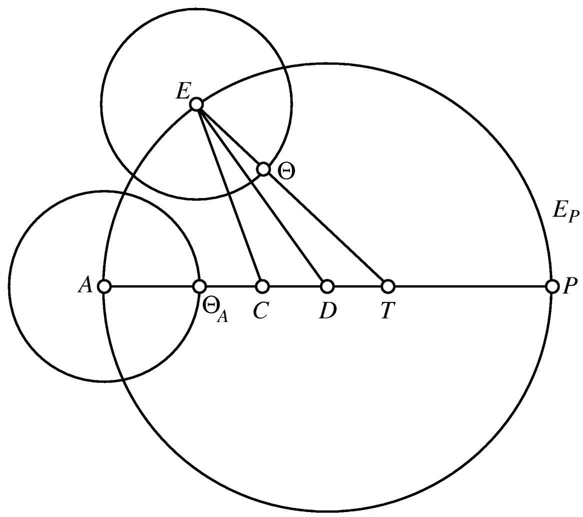

FIG. 6. Two consecutive oppositions.

13 were "observed" to be 765 days and 33.7° apart, being roughly equidistant from the apogee; from this pair, *CT* = 11.8.

Again, Jupiter's opposition of 107 March 6 was "observed" at longitude 163.9°, close to the apogee, and its predecessor and successor were both separated from it by 396 days and 30.2°; both pairs yield *CT* = 5.9. The oppositions of 100 August 18 and 101 September 24 were "observed" at longitudes 322.9° and 359.6°, roughly equidistant from the perigee. The intervals of 402 days and 36.7° yield *CT* = 5.7.

Data from four pairs oppositions of Saturn, two near apogee and two near perigee, lead to values of *CT* ranging from 6.2 to 7.0, with an average *CT* = 6.7. For Venus alone the method proves too sensitive to errors in the data to provide a meaningful estimate of *CT*.

It is clear that an analysis of oppositions along these lines could easily lead one to the recognition that *CT* is approximately twice *CD*, i.e. to the bisection of the eccentricity that Ptolemy assumes for all planets except Mercury. The observational basis that justifies placing the equant and the observer equidistant from the centre of the deferent is glaringly obvious in the case of Mars, but certainly noticeable for Jupiter, and perhaps also detectable for Saturn and Venus. The deduction of the parameters of each model could only be regarded as approximate, because of the necessity of calculating as if oppositions falling near the apsidal line lie exactly on it; however, the errors resulting from these simplifications are not large, as can be seen by comparing the eccentricities found above with those obtained by Ptolemy's ostensibly rigorous procedures.

Acknowledgements

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REFERENCES

1. In the description of the Moon's model in his later *Planetary hypotheses*, Ptolemy apparently repudiates this, now defining the apogee of the epicycle as lying on the radius from the centre of the cosmos through the epicycle's centre (J. L. Heiberg, *Claudii Ptolemaei Opera quae exstant omnia*, ii: *Opera astronomica minora* (Leipzig, 1907), 82–84).
2. N. M. Swerdlow, "Ptolemy's theory of the inferior planets", *Journal for the history of astronomy*, xx (1981), 29–60.
3. "The origin of Ptolemaic planetary theory", cited by G. J. Toomer, *Ptolemy's Almagest* (London, 1984), 480 n. 24. A revised version of Swerdlow's paper recently appeared as "The empirical foundations of Ptolemy's planetary theory", *Journal for the history of astronomy*, xxxv (2004), 249–71.
4. J. Evans, "Fonction et origine probable du point équant de Ptolémée", *Revue d'histoire des sciences*, xxxvii (1984), 193–213, and "On the function and the probable origin of Ptolemy's equant", *American journal of physics*, lii (1984), 1080–9.
5. Presumably influenced by this hypothesis, Toomer actually translates Ptolemy's word *proegeseis* as "retrograde arcs", although the Greek word is not that specific. (The literal meaning is "advancings", reflecting the notion that a planet is advancing when it is travelling in the same direction relative to the stars as the stars travel in their revolutions relative to the horizon.)
6. If Ptolemy meant his description to apply to an equantless model, it is in fact accurate only for Mars, and even then requires a reversal of the apogee and perigee. These facts are integral to Evans's reconstruction.
7. I use Ptolemy's final eccentricities for illustrative purposes only. As Swerdlow remarks, the eccentricity derived from a set of three oppositions assuming an equantless model is not constant, varying in the case of Mars by roughly ±10%.
8. This offset is due in part to the large angle between the apsidal lines of the Earth's and Jupiter's orbits. If it could have been detected in Antiquity (which I doubt), it would have revealed a defect in Ptolemy's equant model, which could be corrected by giving an eccentricity and equant to the epicycle as well as to the deferent. The phenomenon is noted by A. Aaboe, "A Late-Babylonian procedure text for Mars, and some remarks on retrograde arcs", in *From deferent to equant: A volume of studies in the history of science in the ancient and medieval Near East in honor of E.S. Kennedy* (*Annals of the New York Academy of Sciences*, d (1987)), 1–14, pp. 11–13.
9. To take a single example, in *Almagest* 3.4 Ptolemy justifies the use of an eccentre model for the Sun by the ostensible phenomenon that the time the Sun takes in passing from least to mean speed is always greater than the time from mean speed to greatest. In fact the alternative model that Ptolemy rejects, an epicycle with the Sun revolving in the same direction around the epicycle as the epicycle revolves around the Earth, differs from Ptolemy's model in true daily motion by a maximum of about 13″, with a maximum accumulated difference in longitude (at the octants) less than 6′. Hence it is highly questionable whether anyone in Antiquity could have verified Ptolemy's assertion from observations, and no one would suggest that this is how the eccentre model was arrived at historically. The introduction of the second lunar model in *Almagest* 5.1 is an exceptional instance where Ptolemy writes, using the first person, that he discovered a phenomenon in a particular stated manner.
10. O. Neugebauer, *A history of ancient mathematical astronomy* (3 vols, Berlin, 1975), 792.
11. See A. Jones, "A likely source of an observation report in Ptolemy's *Almagest*", *Archive for history of exact sciences*, liv (1999), 255–8, discussing *P. Oxy.* 61.4133. Others among the handful of

early planetary observations preserved in the *Almagest* catch a planet near a significant stage of its synodic cycle; thus the ancient observation of Mercury used in *Almagest* 9.10 has been identified by Toomer (note *ad loc.*) as a station, and the Babylonian observation of Saturn used in 11.7 very nearly coincided with opposition. Again Ptolemy does not make use of these facts, so that they probably tell us something about the nature of the channels by which Ptolemy got access to third-century B.C. planetary observations.

12. Ptolemy tells us in *Almagest* 9.2 (Toomer, *op. cit.* (ref. 3), 421–2) that such models had been employed by his predecessors or contemporaries. Pliny the Elder, *Naturalis historia* 2.63–64, has an obscure and muddled discussion of planetary apsidal lines that evidently refers to eccentre-and-epicycle models.
13. Although the procedure outlined here presumes only an approximate knowledge of the apsidal line, it deserves to be remarked how accurate most of the apsidal lines in Ptolemy's models are. About A.D. 140 the tropical longitudes of the apparent apogees of Saturn, Jupiter, Mars, and Venus were respectively (to the nearest half-degree) 236.5°, 160.5°, 116.5°, and 57.5°. (The apparent geocentric apsidal line is the line through the centres of the Earth's and the planet's orbit, which I have computed from the elements in J. Meeus, *Astronomical algorithms* (Richmond, 1991), 197–201; for discussion see A Aaboe, *Episodes from the early history of astronomy* (New York, 2001), 160–8.) Ptolemy's, in his tropical frame of reference by which longitudes are systematically about 1° too low for his time, are 233°, 161°, 115.5°, and 55°. Ptolemy's apogee for Mercury, at 190°, is notoriously distant from the actual apparent apogee (222.5°), undoubtedly as a consequence of the patchy observational record for this planet.
14. Babylonian observations of planets passing 'Normal Stars' (bright stars in the zodiacal belt) give a lower bound of what naked eye observation could achieve; see A. Jones, "A study of Babylonian observations of planets near Normal Stars", *Archive for history of exact sciences* (forthcoming, 2004) for general discussion. I know of 71 Babylonian reports of Jupiter, 106 reports of Mars, and 83 reports of Venus passing stars within two degrees of the ecliptic (I exclude reports that had, or may once have had, a statement that the planet was some distance ahead of or behind the star). The median absolute difference in longitude for the reported dates in the Jupiter observations is approximately 0.20°, i.e. a little more than two days of mean motion; the median for the Mars observations is approximately 0.46°, i.e. less than one day of mean motion; and the median for the Venus observations is approximately 0.56°, i.e. again less than one day of mean motion. Observations made carefully for theoretical applications could certainly have improved on these standards.