Product Scope and Entry Deterrence in Technology Markets

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We model an oligopolistic technology market in which firms endogenously choose product scope,

fixed costs are affected by exogenous technological progress, and there may be threat of entry. Our

analysis shows that equilibrium outcomes involve substantial overinvestment in product scope,

which benefit consumers and hurt firms, relative to the social optimum. Technological progress

generally increases consumer surplus and lowers firm profits. If entry is threatened bilaterally

across two converging markets, both either accommodate entrants from the rival market, or both

deter entry; continuous progress in technology can cause equilibria shifts, leading to discontinuous

and radical redistribution of surplus across markets.

JEL Classification: D43, L11, L13, L49

1 Overview and motivation

This paper presents a model of a technology market with horizontally differentiated products, in which firms make costly endogenous choices of product scope, the costs of which are influenced by an exogenous state of technological progress. We analyze two market structures – symmetric oligopoly with and without the threat of entry. We then apply these results to a model in which there is a bilateral threat of entry between two converging technology markets. In the process, we generalize existing models of horizontally differentiated competition, by providing a richer treatment of endogenous product scope, and by incorporating entry deterrence and inter-industry interaction within their framework.

Our context is a market for information technology-based products (henceforth referred to as a technology market), examples of which are software, handheld computers, cellular telephones, networking equipment and home electronics devices. Often, the level of functionality or scope of any individual product is chosen endogenously by its seller, from a wide range of feasible levels, and by expending a fixed cost which is increasing in the level of scope chosen. In the software industry, this would be the fixed cost of creating the software code. Once the source code has been created, per-unit variable costs are negligible. Analogously, in markets for semiconductor-based electronic devices, the fixed cost of scope manifests as a product design cost, which increases rapidly as higher scope drives up the complexity of the design process¹.

The fixed cost function is also influenced by the stage of advancement of technology, sometimes within the technology market, but more commonly in an upstream supplier industry, or in a downstream industry for a complementary product (Economides and Salop [1992]). This has been recognized in models of general-purpose technologies (see, for instance, Breshanan and Trajtenberg [1995]). For instance, the fixed cost of scope for electronic devices and computers depends on the state-of-the-art of the semiconductor technology that will eventually implement the product. As

1 As devices of this kind move from a 'hardwired' architecture to one in which the functionality of a product

¹As devices of this kind move from a 'hardwired' architecture to one in which the functionality of a product is implemented using software running on general-purpose hardware, the costs of increasing product scope become increasingly 'fixed', and have a lower influence on unit variable costs. However, even under the current paradigm, the variable costs of computers and other semiconductor-based electronics devices are commoditized. Branded computer and electronics products are rarely manufactured by the firm whose name is on the device – an increasing volume of this kind of manufacturing is done by contractors such as Flextronics and Solectron.

Moore's law ensures continuous rapid progress in this technology, the constraints that drive product design costs are relaxed, allowing firms to symmetrically achieve increases in scope at a lower fixed cost. The same reasoning applies to software developers, whose design and architecture costs are driven by how powerful the machines their software will run on are. Technological progress does lower fixed costs in other industries as well, but the rate at which this occurs in technology markets is far more substantial, as is its impact on firm strategy.

Consequently, the extent of competition in technology markets is determined more actively by the incumbent firms, and is more variable than standard models of product differentiation would suggest. The level of product differentiation is a function of product scope, which is controlled actively and varied over time by sellers. Moreover, product scope determines the fixed costs of each firm. This implies that in technology markets, two key inputs to computing zero-profit equilibria, and optimal concentration – fixed costs and transportation costs – are no longer exogenous. In addition, the fixed cost function itself changes rapidly over time, as technology progresses. These issues motivate the model of oligopoly with differentiated products that we describe in Section 2, and analyze in Section 3.

Another crucial determinant of pricing and product strategy in technology markets is the threat of entry. This issue has received considerable attention in the context of monopoly, especially in light of the recent Microsoft antitrust case. For instance, Fudenberg and Tirole [2000] describe how entry threats and the deterring value of an installed base can lower pricing in a technology market with network externalities (such as the operating systems market). Schmalensee [1999] describes how Microsoft's pricing is substantially lower than what optimal monopoly pricing would be, and that this reduction is largely driven by entry considerations; the magnitude of reduction in price and its effects on surplus have been estimated by Hall and Hall [2000], and by Schinkel [2002], among others. A related aspect of Microsoft's strategy that has received somewhat less attention is that they have increased their product scope to the point where the fixed costs of entry are substantial (upto \$9 billion, according to Hall and Hall). This highlights the potential role that costly and endogenously chosen product scope can play in deterring entry.

Microsoft notwithstanding, a majority of technology markets are oligopolies. Moreover, as noted by Gilbert and Vives [1986], sustained entry-deterrence by a single firm in a market that is not a natural monopoly is rare; a more common entry deterring situation being control by a few firms and higher industry concentration levels than can be explained by technological considerations alone. This is of particular interest when the fixed costs of scope that are chosen by incumbents can be used to deter entry, when the technological considerations are changing rapidly over time, and when these changes may alter whether or not a market is a natural monopoly (or more generally, what its socially optimal concentration is). Many of these issues are addressed in our analysis of oligopolistic entry deterrence, presented in Section 4.

Furthermore, a number of pairs of technology markets display a specific and unique kind of bilateral entry threat. For instance, progress in networking and communications technology, along with the ability to digitize and transmit voice over IP data networks has made it feasible for the providers of cable television to enter the residential voice telephony market. Simultaneously, this technology also makes it feasible for telephone companies to become providers of digital video entertainment over their telephone lines, which will eventually enable them to threaten entry into the core business of cable television providers. The viability of this kind of bilateral entry threat increases with technological progress that leads to higher-bandwidth data networks, as well as more sophisticated compression algorithms. Similarly, a move towards operating system and general-purpose hardware based product architectures in both the cellular telephony and handheld computing markets has led to analogous bilateral entry of firms into each others' markets. Progress in semiconductor technology makes the underlying hardware more powerful, leading to a corresponding increase in the viability of this kind of bilateral entry.

This kind of technology convergence across markets opens up a unique set of strategic considerations, wherein firms have to decide whether to enter a rival market while simultaneously considering the effects of their actions on entry deterrence and oligopoly profits within their own market². As discussed, progress in technology makes this kind of entry more technologically viable – however, it is not clear that it is strategically sound. In fact, as technological progress lowers the fixed costs of scope, the optimal strategic response may be to recede into one's core market, and focus on sustaining margins that are being eroded by decreased product differentiation. In Section 5, we analyze a model that captures these considerations.

The rest of the paper is organized as follows. We outline the elements of our basic model in

2A somewhat similar set of issues is addressed in Cooper (1989), with two firms and overlapping spatially differentiated markets. Our model is substantially different from his.

Section 2. The equilibrium of the *n*-firm oligopoly is derived in Section 3, where we also analyze effects of changes in technology, market size and concentration on price and product scope. In section 4, we derive the *n*-firm entry deterring equilibrium, and contrast its equilibrium scope, profits and surplus with both those obtained in the absence of an entry threat, as well as the socially optimal levels. In section 5, we apply the results of sections 3 and 4 to analyze a model of bilateral entry across technology markets, derive its unique equilibrium outcome, and describe how technological progress can lead to either increased accommodation, or to increased deterrence. We discuss our results in section 6, and conclude in section 7.

2 Basic model

This section presents our model of a market with endogenous and costly choice of product scope, which generalizes the models of both von Ungern-Sternberg [1988] and Hendel and Figueiredo [1997], and also forms the basis for our subsequent analyses of symmetric entry-deterring oligopoly, and bilateral oligopolistic entry.

2.1 Firms and products

Following Salop (1979), each potential product is represented by a point on the unit circle. There are n firms, each of which produces exactly one product, and shares identical production technology. Each firm j makes a costly choice $s_j \in (0, \infty)$ of product scope, and a choice of price p_j . For analytical simplicity, we assume a constant unit variable cost of production c. The fixed cost of scope depends on an exogenous state of technology τ , as well as the level of scope s. We make the following assumptions about the fixed cost function $F(s,\tau)$.

- $F(s,\tau) > 0, F_1(s,\tau) > 0, F_{11}(s,\tau) > 0$: Fixed costs are positive, increasing and convex in scope.
- $F_2(s,\tau) < 0, F_{22}(s,\tau) > 0$: The cost of providing a fixed level of scope is decreasing and convex in the state of technology τ .
- F₁₂(s,τ) < 0: The fixed cost of every unit increase in scope is lower at higher states of technology.

Numbered subscripts of functions represent partial derivatives with respect to the corresponding variable. This notation is preserved throughout the paper. The convexity of fixed costs in scope is widely prevalent for information technology products. For example, the fixed costs of developing software are convex in the number of lines of code, and in the number of function points, both of which increase with increased software functionality. Alternately, if the number of lines of code is constrained by design guidelines based on average end-user memory or processor constraints, then adding each new functionality requires a increasing level of investment in careful software architecture and optimization. Similarly, design costs for electronic devices increase at an increasing rate if engineers have to incorporate increasing functionality onto a circuit board of limited dimensions, with constraints on total battery needs and heat emission.

The properties we ascribe to $F(s,\tau)$ with respect to τ are consistent with commonly observed cost characteristics of technology products. For instance, the fixed costs of delivering a specified level of functionality in a cellular handset or a home electronics device decrease continuously as the raw power of semiconductor technology increases. This decrease in fixed costs is not due to a drop in the price of chips³, but because with more powerful microprocessors, DSP chips and memory chips, and higher feasible levels of miniaturization, it takes a lower investment in design or software to deliver the same level of functionality in a device. Analogously, when selling to a market in which users have personal computers with faster CPUs and more RAM, software manufacturers can more easily increase the size of their code base as well as the minimum memory/processor requirements. Consequently, their product scope can be increased with far less careful software architecture and optimization than was necessary at the lower level of technology (i.e., when their target customers had slower PC's).

2.2 Consumers

There is a mass of customers of total size m distributed uniformly around the circle. A customer located at distance x_j from firm j's product that has scope s_j receives utility

$$U(x_j, s_j) = v - x_j t(s_j)$$

³Note that there may also be a decrease in the variable cost of production due to a decrease in the cost of chips. This effect is likely to strengthen our results; however, in this paper, we focus on changes in fixed costs.

from this product, where t(s) is the misfit cost function that relates product scope to misfit or transportation costs. This function is assumed to have the following properties:

- $t(s) > 0, t_1(s) < 0$: Unit cost of misfit is positive and decreasing in scope
- $t_{11}(s) > 0$, $\frac{\partial}{\partial s}(\frac{-t_1(s)}{(t(s))^2}) \le 0$: The unit cost of misfit is sufficiently convex in scope

The shape of the function t(s) reflects the fact that as scope increases, the products become more general-purpose, and consequently, the disutility of misfit faced by consumers decreases. However, the benefits from decreasing misfit costs are generally diminishing as scope increases. The final (strong) assumption on the convexity of t(s) is necessary to ensure well-behaved best-response functions⁴.

Each consumer purchases exactly one product, and chooses the one that maximizes their surplus $U(x_j, s_j) - p_j$. As is customary, we assume that v is high enough so that all consumers get non-zero surplus from at least one product in equilibrium.

2.3 Sequence of events

Following the standard literature, we assume an exogenously specified symmetric location of firms around the unit circle. In the first stage, the firms simultaneously choose their levels of scope and prices – that is, each firm j chooses a pair (s_j, p_j) , given their relative locations. In the second stage, there may be potential entry. In the final stage of the game, consumers make their purchases. given the prices, scope and (symmetric) location of the firms.

3 Symmetric oligopoly

In this section, we solve the *n*-firm symmetric oligopoly model in the absence of any threat of entry. The model yields some independently interesting results about the relationship between product scope, price and profits, as well as the effects of progress in technology on equilibrium scope, profits and surplus. In addition, it serves as a basis for the model of bilateral entry analyzed in section 5.

The condition is equivalent to assuming that $2(t_1(s))^2 - t(s)t_{11}(s) \le 0$. It is satisfied, for instance, for any mixture of polynomials $\sum_{i=1}^n a_i s^{-b_i}$ in which $a_i \ge 0, b_i \ge 1$.

3.1 Consumer choice and demand

Each consumer's choice is governed by the price and scope of their two closest candidate products. A consumer located at a distance x from one of these firms (labeled firm i) will be located at a distance $\frac{1}{n} - x$ from its other adjacent firm (labeled firm i - 1). Refer to Figure 1. Suppose firm i has price and scope (p_i, s_i) , and firm i - 1 has price and scope (p_{i-1}, s_{i-1}) . The consumer chooses firm i if

$$v - p_i - xt(s_i) \ge v - p_{i-1} - (\frac{1}{n} - x)t(s_{i-1}). \tag{1}$$

The consumer indifferent between firm i and firm i-1 is therefore located at $x_{i,i-1}^*$, where:

$$v - p_i - x_{i,i-1}^* t(s_i) = v - p_j - (\frac{1}{n} - x_{i,i-1}^*) t(s_{i-1}), \tag{2}$$

or

$$x_{i,i-1}^* = \frac{p_{i-1} - p_i + \frac{1}{n}t(s_{i-1})}{t(s_i) + t(s_{i-1})},$$
(3)

and the demand received by firm i from the segment between i and i-1 is $mx_{i,i-1}^*$. Consequently, it follows that if firm i chooses (p_i, s_i) , and its two adjacent neighbors firm i-1 and firm i+1 choose (p_{i-1}, s_{i-1}) , and (p_{i+1}, s_{i+1}) , then the demand for firm i's product is:

$$q(p_i, s_i | p_{i-1}, s_{i-1}, p_{i+1}, s_{i+1}) = m \left(\frac{p_{i-1} - p_i + \frac{1}{n} t(s_{i-1})}{t(s_i) + t(s_{i-1})} + \frac{p_{i+1} - p_i + \frac{1}{n} t(s_{i+1})}{t(s_i) + t(s_{i+1})} \right). \tag{4}$$

3.2 Equilibrium

Suppose all firms except firm i choose price and scope (p, s). If firm i chooses (p_i, s_i) , then based on equation (4), its demand will be:

$$q(p_i, s_i | p, s, p, s) = 2m \left(\frac{p - p_i + \frac{1}{n} t(s)}{t(s_i) + t(s)} \right), \tag{5}$$

and consequently, its payoff function from the choice (p_i, s_i) will be:

$$\pi(p_i, s_i | p, s) = 2m(p_i - c) \left(\frac{p - p_i + \frac{1}{n} t(s)}{t(s_i) + t(s)} \right) - F(s_i, \tau).$$
 (6)

The symmetric Nash equilibrium can now be derived, and is presented in Proposition 1. All proofs are in Appendix A.

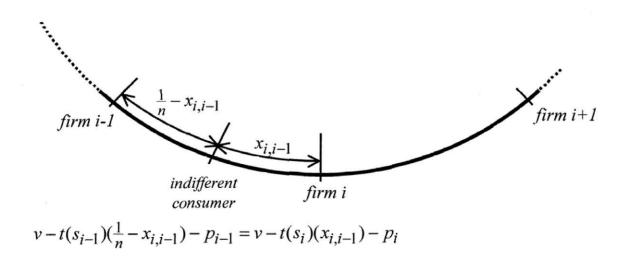


Figure 1: Illustration of the position of the consumer indifferent between firm i and firm i-l, for a market with n firms. Firm i receives demand $x_{i,i-l}$ from the segment between i-l and i, and a similar demand $x_{i,i+l}$ (not illustrated) from the segment between i and i+l.

Proposition 1 For an n-firm oligopoly, the symmetric Nash equilibrium choice $p_A^*(n), s_A^*(n)$ of price and scope is unique, and is specified by:

$$F_1(s_A^*(n), \tau) = \frac{-mt_1(s_A^*(n))}{2n^2};$$
 (7)

$$p_A^*(n) = c + \frac{t(s_A^*(n))}{n}.$$
 (8)

The choice of the subscript (and subsequently, the superscript) A for the functions describing the equilibrium outcomes is because these functions are later used to derive payoffs and welfare under entry accommodation, in contrast to entry deterrence. The expression (7) for price is similar to what is obtained in the standard model with exogenous misfit costs.

Under the symmetric equilibrium, each firm's equilibrium profits are:

$$\Pi^{A}(s_{A}^{*}(n), n) = \frac{mt(s_{A}^{*}(n))}{n^{2}} - F(s_{A}^{*}(n), \tau). \tag{9}$$

In addition, equilibrium consumer surplus is given by:

$$C^{A}(s_{A}^{*}(n), n) = m \left[2n \int_{0}^{\frac{1}{2n}} \left(v - xt(s_{A}^{*}(n)) - \left(c + \frac{t(s_{A}^{*}(n))}{n} \right) \right) dx \right], \tag{10}$$

which reduces to

$$C^{A}(s_{A}^{*}(n), n) = m(v - c) - \frac{5mt(s_{A}^{*}(n))}{4n},$$
(11)

and total surplus is:

$$T^{A}(s_{A}^{*}(n), n) = n\Pi(s_{A}^{*}(n), \tau) + C(s_{A}^{*}(n), \tau)$$

$$= m(v - c) - \frac{mt(s_{A}^{*}(n))}{4n} - nF(s_{A}^{*}(n), \tau).$$
(12)

3.3 Comparative statics

These comparative statics assume an exogenously specified number⁵ of firms n. Proposition 2 examines the effects of changing the number of firms n on the equilibrium choices of scope and price:

Proposition 2 (a) As n increases, the equilibrium level of scope $s_A^*(n)$ decreases, and the equilibrium level of price $p_A^*(n)$ also decreases:

$$\frac{ds_A^*(n)}{dn} = \frac{2mt_1(s_A^*(n))}{n[2n^2F_{11}(s_A^*(n), \tau) + mt_{11}(s_A^*(n))]} < 0,$$
(13)

⁵Rather than a zero-profit monopolistically competitive equilibrium number of firms.

and

$$\frac{dp_A^*(n)}{dn} = \frac{t_1(s_A^*(n))}{n} \frac{ds_A^*(n)}{dn} - \frac{t(s_A^*(n))}{n^2} < 0 \tag{14}$$

Part (a) of Proposition 2 indicates that as the oligopoly becomes more concentrated, firms will respond by lowering their choices of scope, so as to increase the level of differentiation between their products and those of their nearest competitors. This reduces the gross value (before adjusting for price changes) of each of the products for almost all consumers.

As a consequence of this adjustment in scope, the effects driving changes in price are two-fold. The second term on the RHS of equation (14) is the direct effect of increasing concentration on price, which is the only effect in the usual model with exogenous scope, and is always negative. On the other hand, the first term on the RHS of equation (14) represents the positive price adjustment by the firm in response to the equilibrium decrease in scope. Proposition 2(b) tells us that under the model's convexity assumptions about t(s), the negative effect always dominates the positive one, and therefore, increased industry concentration does lead to lower prices.

Next, we explore the impact of progress in technology, represented by an increase in the state of technology τ . Intuitively, since $F_2(s,\tau) < 0$ and an increase in τ shifts the fixed cost curve down for all choices of scope, one would expect scope to increase, and correspondingly, for prices to fall. This is confirmed in the next result:

Proposition 3 As technology progresses and τ increases, $s_A^*(n)$ increases, and $p_A^*(n)$ decreases:

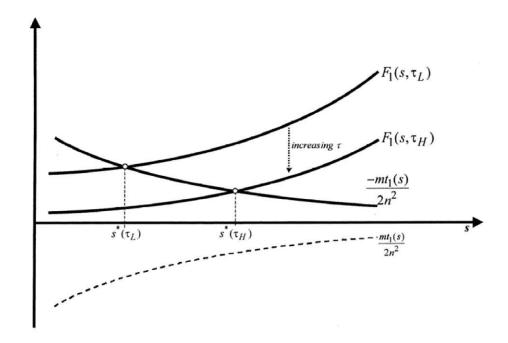
$$\frac{ds_A^*(n)}{d\tau} = \frac{-F_{12}(s_A^*(n), \tau)}{F_{11}(s_A^*(n), \tau) + \frac{mt_{11}(s_A^*(n))}{2n^2}} > 0,$$
(15)

and

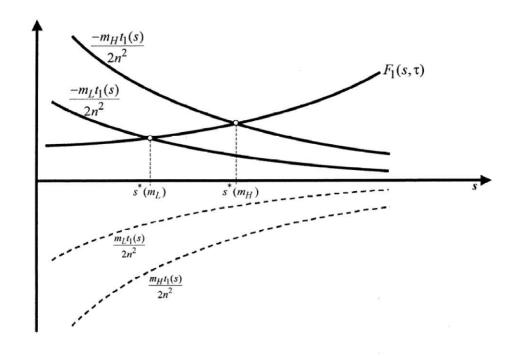
$$\frac{dp_A^*(n)}{d\tau} = \frac{t_1(s_A^*(n))}{n} \frac{ds_A^*(n)}{d\tau} < 0.$$
 (16)

The choice of scope is illustrated in Figure 2(a) for two different values of τ . The equilibrium choice $s_A^*(n)$ occurs where equation (7) is satisfied. Apart from the convexity of $F(s,\tau)$ and t(s), the key regularity assumption driving this result is that $F_{12}(s,\tau) < 0$, which ensures that the 'marginal fixed cost' curve $F_1(s,\tau)$ is lower everywhere as τ increases. In addition, the figure illustrates that the uniqueness of the equilibrium is a consequence of the opposite signs of the slopes of the functions $F_1(s,\tau)$ and $\frac{-mt(s)}{2n^2}$.

Variation in the m, which models the size of the market served, yields similar results:



(a) Changes in equilibrium scope as τ increases



(b) Changes in equilibrium scope as market size \boldsymbol{m} increases

Figure 2: Changes in the equilibrium level of scope $s_A(n)$ as technology progresses, and as market size increases.

Proposition 4 As the size of the market m increases, $s_A^*(n)$ increases, and $p_A^*(n)$ decreases:

$$\frac{ds_A^*(n)}{dm} = \frac{-t_1(s_A^*(n))}{2n^2 F_{11}(s_A^*(n), \tau) + m t_{11}(s_A^*(n))} > 0,$$
(17)

and

$$\frac{dp_A^*(n)}{dm} = \frac{t_1(s_A^*(n))}{n} \frac{ds_A^*(n)}{dm} < 0.$$
 (18)

Proposition 4, illustrated in Figure 2(b), indicates that enabling incumbent firms to expand sales of their products to a new market (for instance, by including consumers in a different geographical location) results in an increase in product scope and a reduction in price for all consumers, including those in the existing market. This may have interesting policy implications for standard setting in technology markets, as discussed in Section 6.

An immediate corollary of Propositions 2 and 3 is:

Corollary 1 An increase in τ or m always results in a strict increase in consumer surplus.

A strict increase in consumer surplus is expected for any change that causes a net increase in equilibrium scope, since the increase in scope both reduces the prices charged, and reduces the disutility from the cost of misfit. In general, the details of the shape of the fixed cost function are crucial in predicting an improvement in profitability or total welfare, though the latter always occurs at an earlier value of τ than the former (based on Corollary 1). Our analysis indicates that total surplus is generally increasing in τ for a wide range of polynomial cost functions, so long as t(s) has the convexity property described in section 2.2. However, individual firm profits are often decreasing in τ . This is explored further in an example in section 4.4.

3.4 Socially optimal scope and concentration

A social planner constrained to offering n symmetrically-located products, all with the same level of scope, would maximize total welfare:

$$T^{S}(s,n) = n \left(2m \int_{0}^{1/2n} (v - c - xt(s)) dx \right) - nF(s,\tau)$$
 (19)

$$= m(v - c) - \frac{mt(s)}{4n} - nF(s, \tau).$$
 (20)

First order conditions yield:

$$nF_1(s^*(n), \tau) = \frac{-mt_1(s^*(n))}{4m}. (21)$$

It is easily verified that $T_{11}(s,n) < 0$, and hence the first order conditions are sufficient. Since $F_1(s,\tau)$ is positive and strictly increasing in s, and $-t_1(s)$ is positive and strictly decreasing in s, the value $s^*(n)$ specified by (21) is unique.

Comparing (21) with (7) indicates that $s_A^*(n) > s^*(n)$, since $F_1(s^*(n), \tau) < \frac{-mt_1(s^*(n))}{2n^2}$. This indicates that the oligopolists always overinvest in product scope. A similar result is obtained by von-Ungern Sternberg [1988]. This issue is discussed further and contrasted with the equilibrium entry-deterring scope, in section 4.4.

Finally, we allow n to be endogenous, and briefly investigate what the socially optimal level of concentration in the industry is. As indicated by Proposition 5, the answer may not be unique:

Proposition 5 The level of industry concentration n_o^* which maximizes total surplus (in conjunction with the corresponding socially optimal choices of scope by each of the n_o^* firms), is given by

$$n_o^* = \frac{1}{2} \sqrt{\frac{mt(s_o^*)}{F(s_o^*)}},\tag{22}$$

where s_o^* is a critical point of the function $F(s,\tau)t(s)$.

4 Oligopoly with the threat of entry

In this section, we solve a model of entry deterrence by incumbent oligopolists. The sequence of events we analyze proceeds as follows. There are n incumbents, symmetrically located around the unit circle. In stage 1, the incumbents choose price and scope, with the restriction that the choices of scope by the incumbents are identical (which also ensure symmetric choices of price in equilibrium). In stage 2, a potential entrant evaluates entry, enters if it is profitable to do so at a level of scope identical to that of the incumbents, and chooses the optimal price for this level of scope, given the prices of the incumbents. In stage 3, based on the product offerings and prices, consumers make their purchase decisions, and the firms receive their payoffs.

Non-cooperative entry deterrence models similar to ours have been analyzed before, most commonly in the context of oligopoly with Cournot competition (for instance, Gilbert and Vives [1986]). In models of this kind, there is typically a commonly shared fixed cost of entry, and the solution is a non-cooperative quantity equilibrium that deters entry. Our model is more involved than this kind of model, in that the competing firms control both the level of fixed costs that influence the

entry decision (through their choice of scope), as well as the equilibrium strategic variable (price, in our case). Restricting the oligopolistic firms to choosing only identical levels of scope is analogous to this choice of fixed cost of entry being an 'industry-wide' entry deterring decision⁶ (as in Spence [1977]).

The restriction we place on the entrant – that she be constrained to choosing the same value of product scope as the incumbent – is consistent with our assumption of symmetric incumbent scope. While we consider the case of a single potential entrant, it is easily shown that if a single entrant is deterred, then so are multiple entrants.

4.1 The entrant's problem

If the incumbents choose symmetric prices and scope (p_D, s_D) , it can be shown that the entrant will choose to locate at a point equidistant from two incumbent firms. As a consequence, upon entry, the demand received by the entrant is identical to that received by a firm in an oligopoly with a total of 2n firms. Proceeding as in section 3.1 and 3.2, though with a fixed choice of scope, if the potential entrant chooses price p and scope s_D , then the entrant's payoff is:

$$\pi(p|p_D, s_D) = 2m(p - c) \left(\frac{p_D - p + \frac{1}{2n}t(s_D)}{2t(s_D)} \right) - F(s_D, \tau). \tag{23}$$

The entrant chooses p to maximize this, which leads to the following result:

Lemma 1 If the symmetric choices of the incumbents are (p_D, s_D) , then the equilibrium entrant payoff upon entry is:

$$\pi_E^*(p_D, s_D) = \frac{m}{4t(s_D)} \left(p_D - c + \frac{t(s_D)}{2n} \right)^2 - F(s_D, \tau). \tag{24}$$

4.2 Equilibrium with entry deterrence

Under symmetric choices of scope, entry is deterred only if each of the incumbents choose (p_D, s_D) such that $\pi_E^*(p_D, s_D) \leq 0$. Given the restrictions we have imposed on scope, we look for an equilibrium of a specific kind. Specifically, we look for a price-scope pair which satisfies the following condition:

⁶Clearly, a model in which the choices of scope form part of a non-cooperative Nash equilibrium would be more general – however, tractability reasons preclude that analysis in this paper.

- 1. Given symmetric choices of scope s_D by all firms, p_D is a symmetric Nash equilibrium choice of price, and
- 2. There is no other value of scope s'_D , which, when chosen symmetrically by all firms, along with its corresponding Nash equilibrium price p'_D , yields a higher payoff to any of the firms than the payoffs under the symmetric choice s_D , while simultaneously deterring entry

The second condition ensures that the industry-wide choice of scope that deters entry is collectively the best one for incumbents; the first condition simply states that given this choice, the price outcomes are the usual symmetric Nash equilibrium for horizontally differentiated products.

Under the assumption that entry is not blockaded (that is, that the symmetric n-firm equilibrium choices $p_A^*(n), s_A^*(n)$ do not naturally deter entry), the symmetric entry-deterring choices of price and scope are as follows:

Proposition 6 The symmetric n-firm entry-deterring choices $p_D^*(n)$, $s_D^*(n)$ of price and scope are specified by:

$$F(s_D^*(n), \tau) = \frac{9mt(s_D^*(n))}{16n^2}; \tag{25}$$

$$p_D^*(n) = c + \frac{t(s_D^*(n))}{n}.$$
 (26)

Note that the expression for price continues to be similar to what is obtained in the standard model with exogenous misfit costs. This is not surprising, since firms choose prices with the assumption of symmetric scope. Equilibrium firm profits are:

$$\Pi^{D}(s_{D}^{*}(n), n) = \frac{mt(s_{D}^{*}(n))}{n^{2}} - \frac{9mt(s_{D}^{*}(n))}{16n^{2}} = \frac{7mt(s_{D}^{*}(n))}{16n^{2}},\tag{27}$$

consumer surplus is:

$$C^{D}(s_{D}^{*}(n), n) = m(v - c) - \frac{5mt(s_{D}^{*}(n))}{4n},$$
(28)

and total surplus is:

$$T^{D}(s_{D}^{*}(n), n) = m(v - c) - \frac{13mt(s_{D}^{*}(n))}{16n}.$$
(29)

4.3 Comparative statics

We continue to treat industry concentration n as an exogenous parameter. As n increases, the optimal entry-deterring level of scope decreases:

Proposition 7 (a) As n increases, the efficient level of scope that deters entry $s_D^*(n)$ decreases:

$$\frac{ds_D^*(n)}{dn} = \frac{-18mt(s_D^*(n))}{n[16n^2F_1(s_D^*(n),\tau) - 9mt_1(s_D^*(n))]} < 0.$$

(b) As n increases, the equilibrium price $p_D^*(n)$ decreases if $F_1(s_D^*(n), \tau) \ge \frac{-9mt_1(s_D^*(n))}{16n^2}$.

Proposition 7(a) makes sense intuitively, because as the concentration of firms in the incumbent's industry increases, this reduces the revenue potential from entry for any potential entrant. Consequently, a lower equilibrium level of scope is required to deter entry.

It is likely that prices also decreases as a consequence, though this is not certain. As discussed in the proof of Proposition 6, $s_D^*(n) > s_A^*(n)$, which means that:

$$F_1(s_D^*(n), \tau) > \frac{-mt_1(s_D^*(n))}{2n^2} = \frac{-8mt_1(s_D^*(n))}{16n^2},$$

and consequently, it is probable that for many specific examples, the condition in Proposition 7(b) will be met. However, this is not unambiguously clear for all cost functions.

The sensitivity of scope $s_D^*(n)$ to changes in technology τ , and to changes in market size m are directionally the same as in the case of oligopoly with no threat of entry:

Proposition 8 (a) As τ increases, $s_D^*(n)$ increases:

$$\frac{ds_D^*(n)}{d\tau} = \frac{-F_2(s_D^*(n), \tau)}{F_1(s_D^*(n), \tau) - \frac{9mt_1(s_D^*(n))}{16n^2}} > 0.$$
(30)

(b) As m increases, $s_D^*(n)$ increases:

$$\frac{ds_D^*(n)}{dm} = \frac{9t(s_D^*(n))}{16n^2 F_1(s_D^*(n), \tau) - 9mt_1(s_D^*(n))} > 0.$$
(31)

In addition, as the equilibrium scope increases, prices are lower (as in Section 3). An intuitive explanation for Proposition 8(a) is that as technology progresses and τ increases, fixed costs fall for all levels of scope (because $F_2(s,\tau) < 0$). As a consequence, a higher level of scope is necessary in order to raise fixed costs to the level where entry is deterred. Similarly, as the market size m increases, so does the revenue opportunity of entering, warranting an equilibrium increase in scope.

4.4 Comparison with socially optimal product scope

Recall that a social planner constrained to offering n symmetric products maximizes:

$$T^{S}(s,n) = m(v-c) - \frac{mt(s)}{r} - nF(s,\tau),$$
 (32)

and chooses scope that satisfies:

$$nF_1(s^*(n), \tau) = \frac{-mt_1(s^*(n))}{4n}.$$
(33)

Also recall that $s_A^*(n) > s^*(n)$. Under the threat of entry, and absent entry-blockading, this over-investment in scope is accentuated further, since $s_D^*(n) > s_A^*(n)$, as discussed in the proof of Proposition 6. Now, if a social planner were to mandate the socially optimal level of scope, and allow firms to compete on price at this exogenously mandated level $s^*(n)$, then it is easily shown that the equilibrium price would be the familiar:

$$p^*(n) = c + \frac{t(s^*(n))}{n}. (34)$$

The corresponding firm profits and consumer surplus would be:

$$\Pi^{S}(s^{*}(n), n) = \frac{mt(s^{*}(n))}{n^{2}} - F(s^{*}(n), \tau)$$
(35)

and

$$C^{S}(s^{*}(n), n) = m(v - c) - \frac{5mt(s^{*}(n))}{4n}$$
(36)

This leads to the following result:

Proposition 9 Under mandated provision of the socially optimal level of scope $s^*(n)$ in an oligopoly with n firms, if entry is not blockaded at the free market equilibrium level of scope,

(a) Firm profits are higher than the corresponding equilibrium oligopoly profits, which are in turn higher than the entry-deterring equilibrium profits:

$$\Pi^{S}(s^{*}(n), n) > \Pi^{A}(s_{A}^{*}(n), n) > \Pi^{D}(s_{D}^{*}(n), n). \tag{37}$$

(b) Consumer surplus is lower than the corresponding consumer surplus under the oligopoly equilibrium, which is in turn lower than the consumer surplus under the entry-deterring equilibrium:

$$C^{S}(s^{*}(n), n) < C^{A}(s_{A}^{*}(n), n) < C^{D}(s_{D}^{*}(n), n).$$
 (38)

(c) Total surplus is higher than the corresponding total surplus under the oligopoly equilibrium, which is in turn higher than the total surplus under the entry-deterring equilibrium:

$$T^{S}(s^{*}(n), n) > T^{A}(s_{A}^{*}(n), n) > T^{D}(s_{D}^{*}(n), n).$$
 (39)

Proposition 9 shows that when firms choose product scope endogenously in an oligopoly, while the choices of the oligopolists are socially inefficient, consumers are better off, and firm profits are lower, than they would be under a regime where the socially optimal level of scope was mandated. Furthermore, all these effects – an overinvestment in product scope, a reduction in profits, yet a net increase in consumer surplus – are intensified when there is a threat of entry that is successfully deterred. In technology markets, where regulatory policy is often ostensibly aimed at making consumers better off, these results have important implications, which are discussed further in section 6.

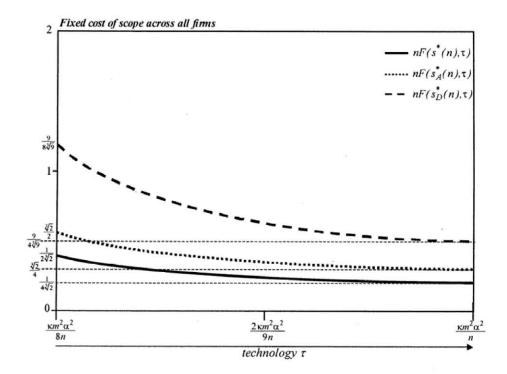
Technological progress (that is, an increase in τ) does not change the direction of the results of Proposition 9. However, it will change the relative magnitude of the differences between profits and surplus under the three scenarios. Figure 3 illustrates the variation in the relative magnitudes of fixed costs, profits, consumer surplus and total surplus for a specific example in which $F(s,\tau) = \frac{\kappa s^2}{2\tau}$, and $t(s) = \frac{\alpha}{s}$, for a representative range of values of τ . In general, it appears that technological progress mitigates the magnitude of overinvestment in scope, and the corresponding distortions away from the social optimum. Also, as illustrated in Figure 3(d), in this particular example, the symmetric oligopoly outcome yields values of total surplus that, while lower, are fairly close to the socially optimal level.

5 Converging technology markets

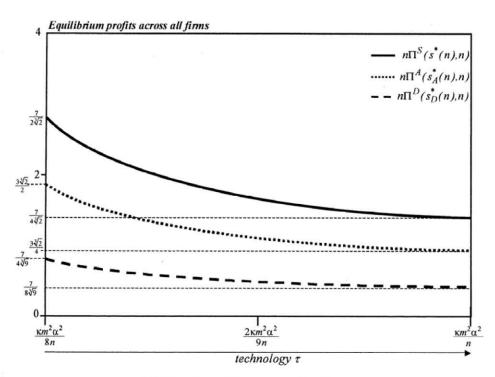
In this section, we use the results of section 3 and 4 to analyze a model of converging technology markets. As discussed in Section 1, this model is motivated by the observation that in many information technology industries, the primary threat of entry is from existing firms in related industries (rather than new start-up firms), and is often triggered by strategic responses to technological progress that makes mobility across industry boundaries feasible. Besides, in technology markets, this movement is often bilateral – that is, firms in a pair of industries threaten entry into each others' core markets.

5.1 Sequence and timing of events

There are two industries 1 and 2, each of which consists of n incumbent firms, and each of which has the demand and cost structure described in section 2. The consumers in each market are assumed



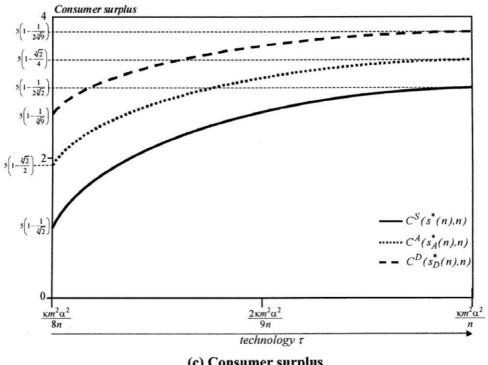
(a) Total fixed costs of scope across all n firms



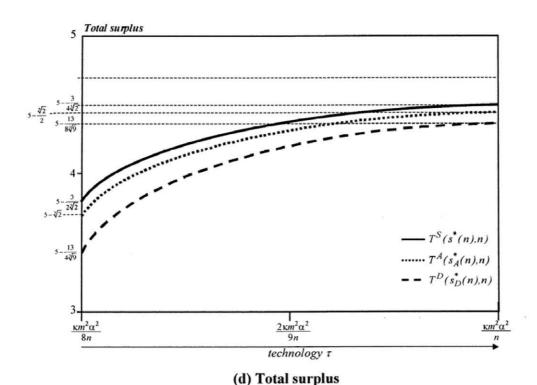
(b) Total profits across all n firms

$$F(s,\tau) = \frac{\kappa s^2}{2\tau} \qquad t(s) = \frac{\alpha}{s}$$

Figure 3: Changes in the relative magnitude of industry-wide fixed costs and profits as technology progresses, for a specific set of cost functions. The figures compare values under symmetric oligopoly, and under oligopoly with entry deterrence, with a scenario under which the socially entired level of scena is mandated. As π increases while scope increases, equilibrium fixed costs fall. More as τ increases. However, profits also fall under all three



(c) Consumer surplus



 $F(s,\tau) = \frac{\kappa s^2}{2\tau} \qquad t(s) = \frac{\alpha}{s}$

Figure 3: Changes in the relative magnitude of consumer surplus and total surplus as technology progresses, for a specific set of cost functions. The figures compare values under symmetric oligopoly, and under oligopoly with entry deterrence, with a scenario under which the socially entired level of scene is mandated. As a increases consumer surplus increases rapidly. However, the magn

Total surplus rises more gradually under all scenarios. V than the socially optimal level, the difference is minimal

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to be distinct. Apart from the firms in these two industries, we assume that there are no other potential entrants. The sequence of events we analyze is as follows. In the first stage, firms in each industry choose prices and levels of scope for the products in their own markets. In the second stage, based on the (known) price and scope choices of the firms in the other market, the firms decide whether to enter the other market. If entry occurs, the entrants choose optimal prices and scope for their products in their new market, under the same symmetric scope restriction imposed in Section 4. In the final stage, based on the prices and levels of scope in each market, consumers in each market make their purchase decisions, and firms receive their payoffs.

5.2 Firm decisions and payoffs

Each firm makes two sets of decisions. In the first stage, a firm chooses price and scope in their own industry, towards either trying to deter entry (D) in the later stage, or towards trying to accommodate entry (A) in the later stage. In the second stage, contingent on the decisions made by the firms in the other market, and their own first stage decisions, each firm decides whether to stay out (S) of the other industry, or whether to enter (E) the other industry.

We assume that in each of the industries, the decisions made by each firm are symmetric.

That is, each firm chooses the same level of price and scope, and also chooses the same deter/accommodate and enter/stay out decision. However, the symmetric decision can be different across the two industries. As in section 4, we assume that entry is not blockaded in either industry.

Under these assumptions, the choices of price and scope are governed by the equilibria described in sections 3 and 4. Therefore, the payoffs to each sequence of decisions can be derived according to the equilibrium profit functions Π^A and Π^D , adjusting the total number of firms in each industry based on whether firms from the other industry have entered or not. This leads to the following components of the final payoffs:

(a) If firms in an industry choose price and scope to deter entry (D), and entry is successfully deterred (S), each incumbent firm gets a payoff of $\Pi^D(s_D^*(n), n)$ from that industry⁷. This is the payoff from the entry-deterring equilibrium price and scope of the n-firm oligopoly. Note that D is the choice in the firm's own industry, and S is the choice of firms in the other industry.

⁷Since incumbent firms do not readjust their locations upon entry, the level of scope required to deter entry of a single firm s_D^* is identical to that required to deter entry of n firms.

	Industry 1 actions	Industry 2 actions	Industry 1 payoff
DSDS	Deter, Stay out	Deter, Stay out	$\Pi^D(s_D^*(n),n)$
DSDE	Deter, Stay out	Deter, Enter	0
DEDS	Deter, Enter	Deter, Stay out	$\Pi^D(s_D^*(n),n)$
DEDE	Deter, Enter	Deter, Enter	0
ASAS	Accommodate, Stay out	Accommodate, Stay out	$\Pi^A(s_A^*(2n),n)$
ASAE	Accommodate, Stay out	Accommodate, Enter	$\Pi^A(s_A^*(2n), 2n)$
AEAS	Accommodate, Enter	Accommodate, Stay out	$\Pi^A(s_A^*(2n), n) + \Pi^A(s_A^*(2n), 2n)$
AEAE	Accommodate, Enter	Accommodate, Enter	$2\Pi^A(s_A^*(2n),2n)$
DSAS	Deter, Stay out	Accommodate, Stay out	$\Pi^D(s_D^*(n),n)$
DSAE	Deter, Stay out	Accommodate, Enter	0
DEAS	Deter, Enter	Accommodate, Stay out	$\Pi^D(s_D^*(n), n) + \Pi^A(s_A^*(2n), 2n)$
DEAE	Deter, Enter	Accommodate, Enter	$\Pi^A(s_A^*(2n),2n)$
ASDS	Accommodate, Stay out	Deter, Stay out	$\Pi^A(s_A^*(2n),n)$
ASDE	Accommodate, Stay out	Deter, Enter	$\Pi^A(s_A^*(2n),2n)$
AEDS	Accommodate, Enter	Deter, Stay out	$\Pi^A(s_A^*(2n),n)$
AEDE	Accommodate, Enter	Deter, Enter	$\Pi^A(s_A^*(2n),2n)$

Table 1: Summary of payoffs under each combination of actions

- (b) If firms in an industry choose price and scope to deter entry, but entry occurs (E), then each incumbent and entrant firm gets a payoff of zero from that industry. This is the payoff from the entry-deterring equilibrium price and scope of the n-firm oligopoly, when symmetric entry by upto n additional firms occurs.
- (c) If firms in an industry choose price and scope to accommodate entry (A) and entry occurs (E), then each incumbent and entrant firm gets a payoff of $\Pi^A(s_A^*(2n), 2n)$, the equilibrium payoff from the 2n-firm oligopoly, from that industry.
- (d) If firms in an industry choose price and scope to accommodate entry (A) but entry does not occur (S), then each incumbent firm gets a payoff of $\Pi^A(s_A^*(2n), n)$ from their industry. This is the equilibrium payoff from the n-firm oligopoly, when scope is $s_A^*(2n)$, or at the level chosen for

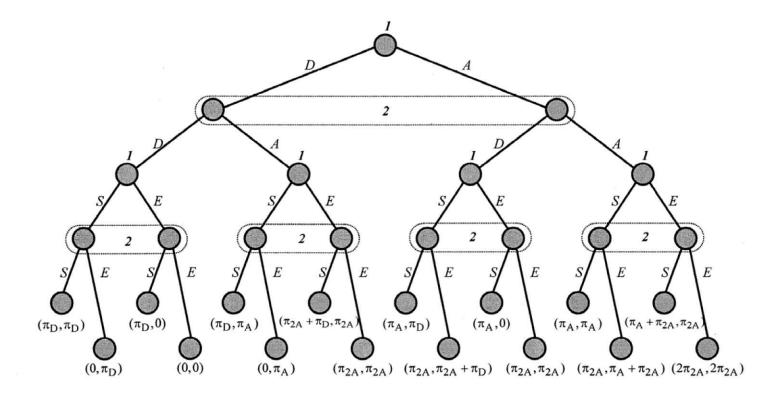
equilibrium with 2n firms⁸.

The payoffs from each combination of decisions, from the point of view of firms in one of the two industries, are summarized in Table 1. A comprehensive explanation of each of these payoffs is cumbersome, so we describe just a few illustrative cases. Under the first combination of actions DSDS, there are n firms in industry 1, each of whom have chosen entry deterrence, and each of whom have chosen not to enter industry 2. As a consequence, each firm gets the equilibrium n-firm entry-deterring oligopoly payoff $\Pi^D(s_D^*(n), n)$ from industry 1, and gets no payoff from industry 2, for a total payoff of $\Pi^D(s_D^*(n), n)$. Under the second combination of actions DSDE, on the other hand, while firms in industry 1 have chosen to stay out of industry 2 and to deter entry in industry 1, the firms in industry 2 have chosen to enter industry 1. Consequently, the firms in industry 1 get zero payoff from their own industry (since entry has occurred by n firms, all firms get zero payoff), and zero payoff from industry 2 (since they have chosen to stay out).

Similarly, under the combination DEAS, firms in industry 1 successfully deter entry from their own industry (payoff of $\Pi^D(s_D^*(n), n)$), and enter industry 2, where they are accommodated (payoff of $\Pi^A(s_A^*(2n), 2n)$). Under the combination ASDE, the firms accommodate entry in their own industry (payoff of $\Pi^A(s_A^*(2n), 2n)$), and stay out of industry 2 (payoff of zero), for a total payoff of $\Pi^A(s_A^*(2n), 2n)$. Similar reasoning yields the payoffs for all the other combinations listed in Table 1.

5.3 Equilibrium

Having specified the payoffs to the firms under each set of actions, we now solve for the subgame perfect Nash equilibria of the bilateral entry game. Its extensive form and payoffs are shown in Figure 4, for a representative player from each industry. While choosing equilibria in the subgames, we assume that if a player is indifferent between E and S (that is, the payoffs from entering and $\frac{8}{3}$ One could argue that under a set of actions in which firms choose to accommodate in an industry, but entry does not occur, the equilibrium payoff should be $\Pi^A(s_A^*(n), n)$ – simply the n-firm oligopoly payoff, rather than the higher value $\Pi^A(s_A^*(2n), n)$, since $\Pi^A(s_A^*(2n), n)$ is not a Nash equilibrium payoff. However, this would be inconsistent with the firm making their price and scope choices prior to knowing whether entry has occured. As it turns out, this does not affect the results – this outcome is never on the subgame perfect equilibrium path, and under either assumption (or any convex combination thereof), the actual equilibrium remains unchanged.



$$\begin{split} \pi_D &= \Pi^D(s_D^*(n), n) \\ \pi_A &= \Pi^A(s_A^*(2n), n) \\ \pi_{2A} &= \Pi^A(s_A^*(2n), 2n) \end{split}$$

Figure 4: Extensive form of the bilateral entry game described in Section 5, with payoffs based on the equilibria derived in Sections 3 and 4.

		Industry 2	
		Deter	Accommodate
Industry 1	Deter	$\Pi^{D}(s_{D}^{*}(n),n),\Pi^{D}(s_{D}^{*}(n),n)$	$\Pi^{D}(s_{D}^{*}(n), n) + \Pi^{A}(s_{A}^{*}(2n), 2n),$
			$\Pi^A(s_A^*(2n), 2n)$
	Accommodate	$\Pi^A(s_A^*(2n),2n),$	$2\Pi^{A}(s_{A}^{*}(2n),2n), 2\Pi^{A}(s_{A}^{*}(2n),2n)$
		$\Pi^D(s_D^*(n), n) + \Pi^A(s_A^*(2n), 2n)$	

Table 2: Nash payoffs from second-stage subgames, for each of the first stage actions from staying out are equal), then the player chooses S (to stay out)⁹.

Proposition 10 The bilateral entry game has a unique subgame perfect Nash equilibrium.

- (a) If $\Pi^D(s_D^*(n), n) \ge \Pi^A(s_A^*(2n), 2n)$, then the equilibrium strategies of all firms are DS (deter, stay out), the equilibrium choice of scope is $s_D^*(n)$, the equilibrium prices are $p_D^*(n)$, and the equilibrium payoffs to each firm are $\Pi^D(s_D^*(n), n)$.
- (b) If $\Pi^D(s_D^*(n), n) < \Pi^A(s_A^*(2n), 2n)$, then the equilibrium strategies of all firms are AE (accommodate, enter), the equilibrium choice of scope is $s_A^*(2n)$, the equilibrium prices are $p_A^*(2n)$, and the equilibrium payoffs to each firm are $2\Pi^A(s_A^*(2n), 2n)$.

The derived payoff matrix for the first stage of the game (that is, after solving for the Nash equilibrium outcomes of the second stage subgames¹⁰) is summarized in Table 2. When $\Pi^D(s_D^*(n), n) < \Pi^A(s_A^*(2n), 2n)$, the equilibrium outcome of the game is Pareto-efficient, since the payoffs to the firms are higher at the equilibrium outcome than in any other feasible outcome. On the other hand, if $\Pi^A(s_A^*(2n), 2n) \leq \Pi^D(s_D^*(n), n) < 2\Pi^A(s_A^*(2n), 2n)$, the game becomes similar to a one-shot prisoners dilemma. Both firms would be better off under the Accomodate-Accomodate outcome, but since Deter is a dominant strategy for both players, they end up at the inefficient entry-deterring outcome.

⁹One could interpret this as implicitly assuming a small cost of mobility across industries, which would imply that unless profits from entering are strictly higher, the player does not enter. We do not explicitly specify such a cost, however, since it would then affect the optimal choice of entry-deterring scope, thereby complicating the analysis substantially.

¹⁰As shown in the proof of Proposition 10, the outcomes of the second-stage subgames are independent of the relative values of $\Pi^A(s_A^*(2n), 2n)$ and $\Pi^D(s_D^*(n), n)$. In a subgame perfect equilibrium, the payoff matrix for the first stage is therefore always as illustrated in Table 2.

5.4 Technological progress and equilibrium changes

Proposition 10 shows that the relative magnitudes of the *n*-firm entry-deterring equilibrium profits $\Pi^D(s_D^*(n), n)$ and the 2*n*-firm standard oligopoly profits $\Pi^A(s_A^*(2n), 2n)$ play the crucial role in determining the equilibrium outcome of the bilateral entry game. As τ increases, both these profit functions tend to decrease¹¹. If one decreases more rapidly than the other, this can cause a shift from one equilibrium outcome to another, resulting in a significant change in industry concentration and investment in product scope, and a redistribution of surplus across firms and consumers. In this section, we discuss two possible cases where this occurs.

The first case is illustrated in Figure 5(a), and represents a situation in which technological progress has a higher impact on the entry-deterring profits. Both $\Pi^D(s_D^*(n), n)$ and $\Pi^A(s_A^*(2n), 2n)$ are decreasing as τ increases. While $\Pi^D(s_D^*(n), n)$ starts out higher (indicating the optimality of entry deterrence at lower levels of technology τ), it decreases more rapidly than $\Pi^A(s_A^*(2n), 2n)$. At a critical point τ^* , the profit functions cross, after which accommodation of entry is optimal, and $\Pi^A(s_A^*(2n), 2n) > \Pi^D(s_D^*(n), n)$. While the progress in τ and the changes in these two functions were gradual, the changes in firm profits are substantial and discontinuous, since the equilibrium outcome now shifts to AEAE, resulting in a doubling of firm profits and industry concentration in both industries. Moreover, there is an accompanying substantial drop in product scope, and a corresponding drop in fixed costs. In addition, since the equilibrium shifts from an entry-deterring one, to a standard 2n firm oligopoly, consumer surplus is likely to drop substantially.

The second case, illustrated in Figure 5(b), is where technological progress has a greater impact on the 2n-firm oligopoly profits. Again, both $\Pi^D(s_D^*(n), n)$ and $\Pi^A(s_A^*(2n), 2n)$ are decreasing as τ increases, but in this case, $\Pi^A(s_A^*(2n), 2n)$ starts out higher, and decreases more rapidly. Consequently, bilateral entry is accommodated initially, until the curves cross, at which point entry deterrence becomes the equilibrium strategy. At this point, profits fall substantially, as firms recede into their core industries, and raise their investments in product scope, so as to deter entry. However, consumer surplus rises sharply, as the value of individual products increases, with a possible drop in prices. While it is technologically feasible (and bilaterally profit improving) to $\overline{}^{11}$ We know that $\Pi^D(s_D^*(n), n)$ is always strictly decreasing in τ . A comparable result has not been established for

 $\Pi^A(s_A^*(2n), 2n)$; however, it appears to decrease over a variety of candidate fixed cost and misfit cost functions, so

long as the convexity assumptions in Section 2 are satisfied.

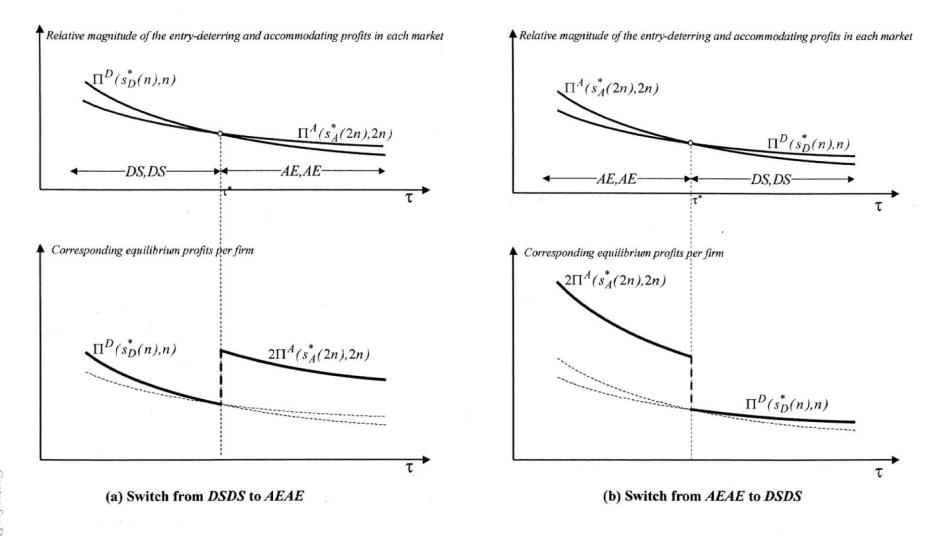


Figure 5: Shift in equilibrium outcomes as technology progresses. In 5(a), n-firm entry deterring profits decrease more rapidly than 2n firm entry accommodating profits. As a consequence, at a critical technology level τ^* , the curves cross, and the equilibrium configuration shifts from *Deter-Stay* by both firms, to *Accommodate-Enter* by both firms. This results in a substantial and discontinuous jump in profits. Figure 5(b) illustrates the opposite case – when the 2n-firm entry accommodating profits decrease more rapidly, resulting in a discontinuous fall in profits at τ^* . These situations arise only when the curves cross – if one is always higher than the other, there will be no equilibrium shift or jump/fall in profits or surplus.

continue competing in both markets, it is no longer strategically viable to do so.

The interesting aspect of both these cases is that while technological progress leads to 'innovation' of sorts in both scenarios, the outcomes for firms and consumers are starkly different. In the first case, when it is accompanied by an expansion by firms into new markets, and increase in the number of firms in both industries, consumers paradoxically suffer on account of technological progress. in the latter case, where there is focused and high individual investment by each firm in their core markets, albeit at a level that is socially inefficient, consumers nevertheless benefit substantially.

6 Discussion and conclusions

Our analysis of oligopoly with endogenous scope and the threat of entry has yielded a number of new results. We have shown that when firms in technology markets are able to respond to changes in industry concentration by adjusting both prices and product scope, equilibrium reductions in scope mitigate the price reductions that would otherwise be warranted. This adjustment leads to a lower increase in consumer surplus, as well as a possible increase in each firm's investment in scope (which is already at an inefficiently high level, as Proposition 9 has shown). This suggests that regulation that aims to improve total welfare by opening up technology markets to new competitors must proceed with care. This is even more crucial when dealing with a market in which firm strategies are driven by entry deterrence. In this case, an increase in concentration amplifies the scope reductions described above, because there is also a response to the reduced threat of entry. As a consequence, prices may actually rise, and consumer surplus may fall.

We have also shown that the equilibrium outcome in an oligopoly with or without the threat of entry is to choose levels of scope that are socially inefficient. As technology progresses, the response by incumbent firms is to further increase product scope, thereby often reducing their own profits, and continuing an inefficient transfer of surplus to consumers. This is consistent with observed long-term trends of hedonic price reductions in technology markets. In this context, encouraging entry-deterring behavior under the argument that it benefits consumers is unlikely to be good long-term policy. However, if a policy maker were to attempt to rectify this inefficiency (by mandating a level of scope, for instance, while still letting firms compete on price, or by influencing industry concentration), this is bound to reduce consumer surplus. The key observation here is that these

inefficiently high investments in scope always benefit consumers. As a consequence, regulatory action that is economically optimal is unlikely to be politically viable, and vice versa.

An increase in the total market size for the product results in an increase in product scope and a reduction in price for all consumers, including those in the existing market. This is consistent with firms being able to spread their fixed costs of scope over a higher number of consumers—consequently, they increase scope, and reduce prices in response. For instance, if wireless technology developed for a national market were compatible with the standards in other national markets, this would translate into gains not just for the manufacturers of wireless handsets and communications equipment in the first market, but also for consumers in this market, since they would benefit from significantly better products in their own market, at a lower price. Consequently, government regulatory policy that encourages (or mandates) shared standards, even at the cost of mandating that firms invest more in product design and software so as to cater to a multinational audience, will lead to substantial consumer benefits, and will do so in a manner that improves firm profits. This may be instructive for markets like the United States, which has chosen a purely industry-driven approach to standards setting for cellular telephony.

Technological progress often leads firms to compete in each others' previously distinct markets. This has been highlighted recently by digital convergence and the sudden increase in products and services that span traditionally distinct industry boundaries, but is not a new occurrence in technology markets. For instance, Breshanan and Greenstein (1999) talk about the 'competitive crash' in the computer industry in the early 1990's, when, as described in their paper, "...seller rents were dramatically reallocated across market segments. Firms that had previously supplied different segments now competed for the same consumers." This 'competitive crash' was not preceded by a technological shock, and for the most part, neither has the current trend towards product convergence.

Our results suggest that over time, gradual progress in technology may lead to cycles in which there are periods of gradual price and profit declines, followed by sudden changes as firms cross industry boundaries. The sudden changes occur when the equilibrium shifts to one of entry accommodation. Immediately following this shift, if entry is blockaded, there is a period of relative 'calm', after which technology progresses to the point where it becomes necessary to deter entry in one's markets again. The change at this point, and following it, are still gradual, — until

technology progresses to the point where accommodation becomes optimal again, thereby causing another dramatic industry realignment.

Formalizing these technology cycles in a dynamic model is one goal of our current work. In addition, we are extending the model of section 5 to support asymmetric concentration n and market size m, so as to enable the analysis of mobility decisions when one firm has more to gain from entry, and the other has more to lose from entry. We are also investigating the effects of heterogeneity in response to changes in τ , with possible information asymmetry. We hope to address these issues in the near future.

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A Appendix: Proofs

Proof of Proposition 1

In a symmetric equilibrium, the choice (p_i, s_i) for firm i should be a best-response to the choice (p, s) by all other firms. The first-order conditions for maximizing $\pi(p_i, s_i|p, s)$ with respect to p_i and s_i yield:

$$\frac{2m(p-2p_i+c+\frac{t(s)}{n})}{t(s)+t(s_i)} = 0; (40)$$

$$-t_1(s_i)\frac{2m(p-p_i+\frac{t(s)}{n})(p_i-c)}{(t(s)+t(s_i))^2} - F_1(s_i,\tau) = 0.$$
(41)

Solving (40) for p_i yields:

$$p_i = \frac{1}{2}(p + c + \frac{t(s)}{n}),\tag{42}$$

and substituting (42) into (41) yields the following equation for s_i :

$$m\left(p - c + \frac{t(s)}{n}\right)^2 \left(\frac{-t_1(s_i)}{(t(s) + t(s_i))^2}\right) = F_1(s_i, \tau). \tag{43}$$

Since $F(s_i, \tau)$ is increasing and strictly convex in s_i , the RHS of (43) is positive and strictly increasing. Also

$$\frac{\partial}{\partial s_i} \left(\frac{-t_1(s_i)}{(t(s) + t(s_i))^2} \right) = \frac{2(t_1(s_i))^2 - t_{11}(s_i)(t(s) + t(s_i))}{(t(s) + t(s_i))^3}.$$

Since $t_1(s_i) < 0$, and under our convexity assumptions, $2(t_1(s_i))^2 - t_{11}(s_i)t(s_i) \le 0$, the LHS of (43) is strictly decreasing in s_i . As a consequence, equation (43) is satisfied for a unique s_i , and the best-response of firm i is unique.

To determine the symmetric equilibrium, first substitute $p_i = p = p_A^*(n)$ in equation (42) to get:

$$p_A^*(n) = c + \frac{t(s_A^*(n))}{n},$$
 (44)

and then $s_i = s = s_A^*(n)$ along with equation (44) to get:

$$F_1(s_A^*(n), \tau) = -\frac{mt_1(s_A^*(n))}{2n^2}. (45)$$

From equation (44) and the fact that $t_1(s) < 0$, it is clear that for a given $s_A^*(n)$, the choice of $p_A^*(n)$ is unique. Since $F(s,\tau)$ is strictly convex, $F_1(s,\tau)$ is positive and strictly increasing in s. Also, since t(s) is strictly decreasing and strictly convex, $-t_1(s)$ is positive and strictly decreasing in s. As a consequence, equation (45) specifies a unique value of $s_A^*(n)$, which implies that the symmetric equilibrium is unique, and completes the proof.

Proof of Proposition 2

(a) Totally differentiating both sides of equation (45) with respect to n yields

$$F_{11}(s_A^*(n), \tau) \left(\frac{ds_A^*}{dn}\right) = -\frac{mt_{11}(s_A^*(n))}{2n^2} \left(\frac{ds_A^*}{dn}\right) + \frac{mt_1(s_A^*(n))}{n^3},\tag{46}$$

which rearranges to

$$\frac{ds_A^*(n)}{dn} = \frac{2mt_1(s_A^*(n))}{2n^3F_{11}(s_A^*(n), \tau) + mnt_{11}(s_A^*(n))}. (47)$$

Since $t_1(s) < 0$, $F_{11}(s, \tau) > 0$, and $t_{11}(s) > 0$, it follows that $\frac{ds_A^*(n)}{dn} < 0$.

(b) Totally differentiating both sides of equation (44) with respect to n yields

$$\frac{dp_A^*(n)}{dn} = \frac{t_1(s_A^*(n))}{n} \frac{ds_A^*(n)}{dn} - \frac{t(s_A^*(n))}{n^2}.$$
 (48)

Substituting equation (47) into (48) yields:

$$\frac{dp_A^*(n)}{dn} = \frac{2m(t_1(s_A^*(n)))^2}{2n^4F_{11}(s_A^*(n), \tau) + mn^2t_{11}(s_A^*(n))} - \frac{t(s_A^*(n))}{n^2},\tag{49}$$

which rearranges to:

$$\frac{dp_A^*(n)}{dn} = \frac{1}{n^2} \frac{m[2(t_1(s_A^*(n)))^2 - t_{11}(s_A^*(n))t(s_A^*(n))] - 2n^2 F_{11}(s_A^*(n), \tau)t(s_A^*(n))}{2n^2 F_{11}(s_A^*(n), \tau) + mt_{11}(s_A^*(n))}.$$
 (50)

Under the convexity assumptions imposed on t(s), we know that $2(t_1(s))^2 - t_{11}(s)t(s) \le 0$. Since $F_{11}(s,\tau) > 0$, this implies that the numerator of the RHS of equation (48) is strictly negative. The result follows.

Proof of Proposition 3

Totally differentiating both sides of equation (45) with respect to τ yields:

$$F_{11}(s_A^*(n), \tau) \left(\frac{ds_A^*(n)}{d\tau} \right) + F_{12}(s_A^*(n), \tau) = -\frac{mt_{11}(s_A^*(n))}{2n^2} \left(\frac{ds_A^*(n)}{d\tau} \right), \tag{51}$$

which rearranges to:

$$\frac{ds_A^*(n)}{d\tau} = \frac{-F_{12}(s_A^*(n), \tau)}{F_{11}(s_A^*(n), \tau) + \frac{mt_{11}(s_A^*(n))}{2n^2}}.$$
 (52)

Since $F_{12}(s,\tau) < 0$, $F_{11}(s,\tau) > 0$, and $t_{11}(s) > 0$, it follows that $\frac{ds_A^*(n)}{d\tau} > 0$. Next, totally differentiating both sides of equation (44) with respect to τ yields:

$$\frac{dp_A^*(n)}{d\tau} = \frac{t_1(s_A^*(n))}{n} \frac{ds_A^*(n)}{d\tau},\tag{53}$$

and since $t_1(s) < 0$ and $\frac{ds_A^*(n)}{d\tau} > 0$, it follows that $\frac{dp_A^*(n)}{d\tau} < 0$.

Proof of Proposition 4

Totally differentiating both sides of equation (45) with respect to m yields:

$$F_{11}(s_A^*(n), \tau) \left(\frac{ds_A^*(n)}{dm} \right) = -\frac{t_1(s_A^*(n))}{2n^2} - \frac{mt_{11}(s_A^*(n))}{2n^2} \left(\frac{ds_A^*(n)}{d\tau} \right), \tag{54}$$

which yields:

$$\frac{ds_A^*(n)}{dm} = \frac{-t_1(s_A^*(n))}{2n^2 F_{11}(s_A^*(n), \tau) + mt_1(s_A^*(n))}.$$
 (55)

Since $t_1(s) < 0$, $F_{11}(s,\tau) > 0$, and $t_{11}(s) > 0$, it follows that $\frac{ds_A^*(n)}{dm} > 0$. Totally differentiating both sides of equation (44) with respect to m yields:

$$\frac{dp_A^*(n)}{dm} = \frac{t_1(s_A^*(n))}{n} \frac{ds_A^*(n)}{dm},\tag{56}$$

and the result follows.

Proof of Proposition 5

The social planner solves:

$$\max_{s,n} T^{S}(s,n) = m(v-c) - \frac{mt(s)}{4n} - nF(s,\tau).$$
 (57)

First order conditions with respect to s and n yield:

$$n_o^* F_1(s_o^*, \tau) = \frac{-mt_1(s_o^*)}{4n_o^*};$$
 (58)

$$F(s_o^*, \tau) = \frac{mt(s_o^*)}{4(n_o^*)^2}. (59)$$

Solving (58) and (59) for s_o^* yields:

$$F_1(s_o^*, \tau)t(s_o^*) + F(s_o^*, \tau)t_1(s_o^*) = 0.$$
(60)

This implies that s_o^* solves:

$$\frac{\partial}{\partial s}[F(s,\tau)t(s)] = 0.$$
 (61)

The result follows.

Proof of Lemma 1

Assuming entry, the entrant chooses the value of p that maximizes $\pi(p|p_D, s_D)$. From equation (23), first-order conditions for the equilibrium choice of p result in:

$$p_E^* = \frac{1}{2}(p_D + c + \frac{t(s_D)}{2n}),\tag{62}$$

which when substituted back into (23) yields:

$$\pi_E(p_D, s_D) = \frac{m}{4t(s_D)} \left(p_D - c + \frac{t(s_D)}{2n} \right)^2 - F(s_D, \tau). \tag{63}$$

Also, differentiating equation (23) twice with respect to p, one gets:

$$\pi_{11}(p|p_D, s_D) = -\frac{2m}{t(s_D)},$$
(64)

which verifies that the entrant's payoff function is strictly concave in p for any positive value of s_D . The result follows.

Proof of Proposition 6

Suppose $p_D^*(n)$, s is a candidate symmetric equilibrium choice in stage 1 which deters entry. If entry is deterred, the demand that each firm receives in the final stage is governed by the same equations as in the n-firm oligopoly. Under symmetric scope, in order for $p_D^*(n)$ to be part of a symmetric Nash equilibrium, it must satisfy the first-order condition analogous to equation (40), or:

$$p_D^*(n) = \frac{1}{2}(p_D^*(n) + c + \frac{t(s)}{n}),\tag{65}$$

which solves to:

$$p_D^*(n) = c + \frac{t(s)}{n}.$$
 (66)

Given a price $p_D^*(n)$, using Lemma 1, any value of scope s that deters entry must satisfy:

$$\frac{m}{4t(s)} \left(p_D^*(n) - c + \frac{t(s)}{2n} \right)^2 \le F(s, \tau).$$
 (67)

Substituting equation (66) into equation (67) yields:

$$\frac{m}{4t(s)} \left(\frac{3t(s)}{2n}\right)^2 \le F(s,\tau),\tag{68}$$

or

$$\frac{9mt(s)}{16n^2} \le F(s,\tau). \tag{69}$$

Now, the profits to each firm from a choice $p_D^*(n)$, s are:

$$\pi = \frac{m}{n}(p_D^*(n) - c) - F(s, \tau), \tag{70}$$

which when combined with equation (66) yields:

$$\pi = \frac{mt(s)}{n^2} - F(s, \tau). \tag{71}$$

Equation (71) indicates that π is strictly decreasing in s. In addition, since entry is not blockaded, it follows that at the equilibrium oligopoly choices of price and scope $(p_A^*(n), s_A^*(n))$, the entrant finds it profitable to enter:

$$\frac{m}{4t(s_A^*(n))} \left(p_A^*(n) - c + \frac{t(s_A^*(n))}{2n} \right)^2 - F(s_A^*(n), \tau) > 0.$$
 (72)

Substituting the expression for $p_A^*(n)$ from equation (44) into equation (72), one gets:

$$\frac{9mt(s_A^*(n))}{16n^2} > F(s_A^*(n), \tau). \tag{73}$$

Moreover, the RHS of equation (69) is strictly increasing in s, and the LHS is strictly decreasing in s. This implies that if s satisfies (69), $s > s_A^*(n)$. Consequently, the equilibrium value of s, denoted $s_D^*(n)$ satisfies (69) as a strict equality, or:

$$\frac{9mt(s_D^*(n))}{16n^2} = F(s_D^*(n), \tau),\tag{74}$$

which completes the proof.

Proof of Proposition 7

(a) Totally differentiating both sides of equation (25) with respect to n:

$$F_1(s_D^*(n), \tau) \frac{ds_D^*(n)}{dn} = \frac{9mt_1(s_D^*(n))}{16n^2} \frac{ds_D^*(n)}{dn} - \frac{9mt(s_D^*(n))}{8n^3},\tag{75}$$

which can be rearranged to yield:

$$\frac{ds_D^*(n)}{dn} = \frac{-18mt(s_D^*(n))}{n[16n^2F_1(s_D^*(n), \tau) - 9mt_1(s_D^*(n))]}.$$
 (76)

Since t(s) > 0, $t_1(s) < 0$, and $F_1(s, \tau) > 0$, the result follows.

(b) Totally differentiating both sides of equation (??) with respect to n:

$$\frac{dp_D^*(n)}{dn} = \frac{t_1(s_D^*(n))}{n} \frac{ds_D^*(n)}{dn} - \frac{t(s_D^*(n))}{n^2}.$$
 (77)

Substituting equation (76) into (77) and rearranging yields:

$$\frac{dp_D^*(n)}{dn} = \frac{-t(s_D^*(n))}{n^2} \left(\frac{16n^2 F_1(s_D^*(n), \tau) + 9mt_1(s_D^*(n))}{16n^2 F_1(s_D^*(n), \tau) - 9mt_1(s_D^*(n))} \right). \tag{78}$$

The result follows.

Proof of Proposition 8

(a) Totally differentiating both sides of equation (25) with respect to τ :

$$F_1(s_D^*(n), \tau) \frac{ds_D^*(n)}{d\tau} + F_2(s_D^*(n), \tau) = \frac{9mt_1(s_D^*(n))}{16n^2} \frac{ds_D^*(n)}{d\tau}, \tag{79}$$

which rearranges to

$$\frac{ds_D^*(n)}{d\tau} = \frac{-F_2(s_D^*(n), \tau)}{F_1(s_D^*(n), \tau) - \frac{9mt_1(s_D^*(n))}{9mt_1(s_D^*(n))}}.$$
(80)

Since $F_2(s,\tau) < 0$, $F_1(s,\tau) > 0$, and $t_1(s) < 0$ for all s, the result follows.

(b) Totally differentiating both sides of equation (25) with respect to m:

$$F_1(s_D^*(n), \tau) \frac{ds_D^*(n)}{dm} = \frac{9mt_1(s_D^*(n))}{16n^2} \frac{ds_D^*(n)}{dm} + \frac{9t(s_D^*(n))}{16n^2}, \tag{81}$$

which rearranges to

$$\frac{ds_D^*(n)}{dm} = \frac{9t(s_D^*(n))}{16n^2 F_1(s_D^*(n), \tau) - 9mt_1(s_D^*(n))}.$$
(82)

Since t(s) > 0, $F_1(s, \tau) > 0$, and $t_1(s) < 0$ for all s, the result follows.

Proof of Proposition 9

(a) In each of the three cases, given the appropriate (i.e. either equilibrium or mandated) level of scope s, the expression for profits is

$$\Pi^{i}(s,n) = \frac{mt(s)}{n^2} - F(s,\tau). \tag{83}$$

Since $t_1(s) < 0$ and $F_1(s,\tau) > 0$, the expression above is strictly decreasing in s. Consequently, using $s^*(n) < s_A^*(n) < s_D^*(n)$, the result follows.

(b) In each of the three cases, given the appropriate (i.e. either equilibrium or mandated) level of scope s, the expression for consumer surplus is

$$\Pi^{i}(s,n) = m(v-c) - \frac{5mt(s)}{4n}.$$
 (84)

Since $t_1(s) < 0$, the expression above is strictly increasing in s; we know that $s^*(n) < s^*_A(n) < s^*_D(n)$, and therefore, the result follows.

(c) From (32), at a given level of scope s, the expression for total surplus is

$$T(s,n) = m(v-c) - \frac{mt(s)}{4n} - nF(s,\tau).$$
(85)

Differentiating both sides of (85) twice with respect to s:

$$T_{11}(s,n) = \frac{-mt_{11}(s)}{4n} - nF_{11}(s,\tau), \tag{86}$$

and since $t_{11}(s) > 0$ and $F_{11}(s,\tau) > 0$ for all s, this establishes that T(s,n) is strictly concave in s. Now, by definition, $s^*(n)$ is the global maximizer of T(s,n), which, when combined with the strict concavity of T(s,n), implies that T(s,n) is strictly decreasing in s for $s > s^*(n)$. Since $s^*(n) < s^*_A(n) < s^*_D(n)$, the result follows.

Proof of Proposition 10

First, consider the payoff matrix for the second-stage subgame that follows a choice of Deter by firms in both industries (the DD subgame):

		Industry 2	
		Stay out (S)	Enter(E)
Industry 1	Stay out (S)	$\mathbf{\Pi}^D(\mathbf{s}_D^*(\mathbf{n}),\mathbf{n}),\ \mathbf{\Pi}^D(\mathbf{s}_D^*(\mathbf{n}),\mathbf{n})$	$0,\;\Pi^D(s_D^*(n),n)$
	Enter (E)	$\Pi^D(s_D^*(n), n), \ 0$	0, 0

Subgame DD

Clearly, S is a weakly dominant strategy for both players. Since we choose S over E when they yield the same payoffs, the Nash equilibrium of this subgame is SS.

Next, consider the payoff matrix for the DA subgame:

		Industry 2	
		$Stay\ out\ (S)$	Enter(E)
Industry 1	Stay out (S)	$\Pi^{D}(s_{D}^{*}(n), n), \ \Pi^{A}(s_{A}^{*}(2n), n)$	$0, \Pi^A(s_A^*(2n), n)$
	Enter (E)	$\mathbf{\Pi}^D(\mathbf{s}_D^*(\mathbf{n}),\mathbf{n}) + \mathbf{\Pi}^A(\mathbf{s}_A^*(2\mathbf{n}),2\mathbf{n}),$	$\Pi^A(s_A^*(2n), 2n), \ \Pi^A(s_A^*(2n), 2n)$
		$\mathbf{\Pi}^A(\mathbf{s}_A^*(\mathbf{2n}),\mathbf{2n})$	

Subgame DA

S is a weakly dominant strategy for firms in industry 2, and E is a dominant strategy for firms in industry 1. Consequently, the Nash equilibrium is ES.

Next, consider the payoff matrix for the AD subgame:

		Industry 2	
	1 -	Stay out (S)	$Enter\ (E)$
Industry 1	Stay out (S)	$\Pi^{A}(s_{A}^{*}(2n), n), \ \Pi^{D}(s_{D}^{*}(n), n)$	$\Pi^A(\mathbf{s}_A^*(\mathbf{2n}),\mathbf{2n}),$
			$\boldsymbol{\Pi}^D(\mathbf{s}_D^*(\mathbf{n}),\mathbf{n}) + \boldsymbol{\Pi}^A(\mathbf{s}_A^*(\mathbf{2n}),\mathbf{2n})$
	Enter (E)	$\Pi^A(s_A^*(2n), n), 0$	$\Pi^{A}(s_{A}^{*}(2n), 2n), \ \Pi^{A}(s_{A}^{*}(2n), 2n)$

Subgame AD

This is simply the DA payoff matrix transposed, with payoffs exchanged. In this case, S is a

weakly dominant strategy for firms in industry 1, and E is a dominant strategy for firms in industry 2, which leads to the Nash equilibrium SE.

Finally, the payoff matrix for the AA subgame is:

		Industry 2	
		Stay out (S)	Enter(E)
Industry 1	Stay out (S)	$\Pi^A(s_A^*(2n), n), \ \Pi^A(s_A^*(2n), n)$	$\Pi^{A}(s_{A}^{*}(2n), 2n),$ $\Pi^{A}(s_{A}^{*}(2n), n) + \Pi^{A}(s_{A}^{*}(2n), 2n)$
***************************************	Enter (E)	$\Pi^A(s_A^*(2n), n) + \Pi^A(s_A^*(2n), 2n),$	$2\Pi^A(\mathbf{s}_A^*(\mathbf{2n}),\mathbf{2n}),\; 2\Pi^A(\mathbf{s}_A^*(\mathbf{2n}),\mathbf{2n})$
		$\Pi^A(s_A^*(2n), 2n)$	

Subgame AA

Clearly, E is a dominant strategy for both players, leading to the Nash equilibrium EE.

Therefore, under subgame perfection, the payoffs as seen by the players when making their stage 1 decisions are as follows:

		Industry 2	g 2 0 0 0 0
		Deter	Accommodate
Industry 1	Deter	$\Pi^D(s_D^*(n),n),\Pi^D(s_D^*(n),n)$	$\Pi^{D}(s_{D}^{*}(n), n) + \Pi^{A}(s_{A}^{*}(2n), 2n),$ $\Pi^{A}(s_{A}^{*}(2n), 2n)$
	Accommodate	$\Pi^A(s_A^*(2n), 2n),$	$2\Pi^{A}(s_{A}^{*}(2n),2n),2\Pi^{A}(s_{A}^{*}(2n),2n)$
		$\Pi^{D}(s_{D}^{*}(n), n) + \Pi^{A}(s_{A}^{*}(2n), 2n)$	

First stage payoffs, given Nash outcomes in the second stage

If $\Pi^D(s_D^*(n), n) > \Pi^A(s_A^*(2n), 2n)$, then Deter (D) is a dominant strategy for both players. On the other hand, if $\Pi^D(s_D^*(n), n) < \Pi^A(s_A^*(2n), 2n)$, then Accommodate (A) is a dominant strategy for both players. If $\Pi^D(s_D^*(n), n) = \Pi^A(s_A^*(2n), 2n)$, then any combination of actions is a Nash equilibrium. Consistent with our earlier assumption of firms choosing to stay out rather than enter, we choose DD as the outcome in this case (it is a knife's edge case and has no bearing on the subsequent discussion). The result follows.