THE DECLINING PRICE PARADOX OF NEW TECHNOLOGIES

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May 1987

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Working Paper Series

STERN #IS-87-35

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ABSTRACT

The declining prices of new technology products often results in a tendency for many decision makers to wait for lower prices, and to postpone a capital investment.

This paper makes a distinction between the prices of technology elements and the prices of components and systems. There are many cases where the price reduction over time applies only to some elements of the system, while the total price of the improved system remains almost the same. For those cases, a DECLINING PRICE PARADOX is spelled out. The Paradox suggests that the more the price of the investment is subject to future reduction, the more urgent it is to invest in this technology. The paper incorporates learning considerations in the investment decision making, and states the conditions where the paradoxox applies.

1. Introduction

Assume that a CEO faces the decision whether to invest in a new computerized system. The Vice President for Operations, supported by the VP of Marketing, emphasizes the essential nature of the investment to the improvement of the manufacturing facilities that will enable the company to stay ahead of its competitors and keep its competitive edge. However, the controller and some of senior executives point out the declining prices of equipment of this kind. In spite of the favorable contribution to productivity, they suggest waiting and procuring the system at better prices. What should the CEO do? What tools should he use in making his decision?

Recent research has shown that traditional financial methods for investment evaluation fall short in measuring the real merit of investing in new technology. Difficulties in quantifying indirect variables such as learning, increased manufacturing flexibility, commitment to quality and the necessity to safeguard the firm's competitive position may cause the understatement of the benefit of new technologies (see Kaplan, [1983], [1984b], [1986]), Gold, [1983] Suresh and Meredith, [1985]). Other researchers have pointed out that the traditional approach falls short in evaluating the strategic economics of new technologies (see, for example, Skinner, [1984]). For further research done in this field the reader may refer to Meredith, [1986], and Kaplan, [1984a].

However, recent research has not focused on the evidence of declining prices which must be taken into consideration in the justification of new technology. Thus, in many cases, the "real" manager's dilemma is not whether to buy the new technology-- often this decision has already been made -- but when to buy it. The declining prices of new technologies results in a tendency of many decision makers to wait for better prices, and to postpone the needed investment. This paper shows that in most cases this intuitive passive behavior does not fit the need to maximize the Net Present Value of the investment or the need for survival in a competitive environment.

There is no doubt that in today's marketplace a delay in investment may yield better prices in the future. In order to show that this passive approach is not always correct, this paper presents a step by step analysis, which will provide insights for decision makers.

The models developed hereafter approach a finite set of problems in which there is no technological risk in investing today in the new technology. The following assumptions are made:

- The new technology has already proved itself and is not considered "premature" or risky; the quality of the new products is broadly recognized as better than that of the old technology.
- There is no evidence of an alternative new revolutionary technology that is going to replace this technology in a short time, and only incremental evolutionary improvements are expected.

For example, consider the Personal Computer (PC) technology today. It is well known to small businesses as well as large organizations that this technology is a proven mature one. It is also known that by now there is no alternative revolutionary technology that might shortly replace it. Thus, many managers know that an investment in PC technology does not involve technological risks. The only question they often ask is whether to buy this technology today or maybe wait a couple of months to obtain better prices or improved performance.

In section 2, the traditional approach to investment decisions is presented. Section 3 introduces us to the importance of the learning process in the implementation and adoption of new technologies. The model is thus modified to handle the important benefits of the user's learning. The learning curve models, together with the model presented in section 2, may advocate different decisions.

Section 4 presents the DECLINING PRICE PARADOX. In real life there are many cases where the price reduction over time is for unit of performance only, while the total price of the improved system remains almost the same; the DECLINING PRICE PARADOX is spelled out for such cases. The Paradox suggests that the more the price of the investment is subject to future reduction, the more learning we are going to face in the future, since the extra performance features are growing at about the same rate. Thus, in order to compete and survive, it is worthwhile to buy the technology as soon as possible. For example, assume that a manager has decided to purchase a Personal Computer system for financial modeling and graphics presentations. Though he expects a price decline within a couple of months for the same system, or an improved system for the same price, the paradox suggests he should purchase it now.

Section 5 draws conclusions and calls for further research.

2. Model No. 1: The Traditional DCF Approach

The simple classical approach examines the myopic dilemma whether to buy the system today or tomorrow. If we buy the system today, the price is A; if we buy it "tomorrow" the discounted price will be B, and the natural assumption is that $B < A$.

On the other hand, there are benefits gained from applying the new technology. If we apply the system today, the Net Present Value (NPV) of the benefits will be X; tomorrow, Y, assuming $X>Y$. Figure 2.1 summarizes the simple model.

Insert Figure 2.1 about here

The decision criteria according to the Discounted Cash Flow (DCF) evaluation is as follows:

If $X - A > Y - B$, 0, buy today.

If $0 > X - A > Y - B$, do not invest.

If $X - A < Y - B < 0$, do not invest.

If $X - A < Y - B > 0$, buy tomorrow.

This DCF approach, like other financial performance measures, gives us an illusion of objectivity and precision. However, it ignores both the impact of the time needed to learn any new technology and the behavior of the competitors. Current research has shown that temporary noteworthy declines in productivity often accompany the introduction of new process technology (Kaplan, [1986]). Thus, if we apply the new technology too late we may lose our competitive edge.

In the next sections we develop an alternative approach which copes both with the learning effects and strategic competition considerations.

3. Learning Curve Considerations

We will now incorporate learning curve considerations in the decision making process. As noted, some investments in new process technology have important learning characteristics. Thus, even if calculations of the net present value suggest a postponement the investment, investing today can still be valuable by permitting managers and workers to gain experience with the technology. The problem may therefore be presented as having the following alternatives:

- o To wait for the price of the new technology to decline

or

- o Invest now, at today's prices, and gain learning experience.

To model this approach, we will use the learning curve models, which are well known in the literature (see, for example, Chase and Aquilano, [1985]): having an experience in some process or technology, the process time and the cost decrease.[1]

Most of the literature deals with the learning benefits of the producer of a product or a service. In this paper we focus on the user's learning process.

It is common to assume a logarithmic relationship (Yelle, [1979]) between the production cost (or time) of a unit and the number of units produced (or time of use of the new system) according to

(3.1) $Y_x = KX^n$

where

x = Unit number

Y_x = Number of hours (cost) required to produce the x^{th} unit

K = Number of direct labor hours (cost) required to produce the first unit

n = Log b/log 2

b = Learning percentage

Figure 3.1 shows the learning curve of the two options stated above.

Insert Figure 3.1 about here

Assume that Firm I invests "today" (at time t_0) while Firm II decides to wait for better prices and invests in time t_1. Curves

[1] The learning curve considerations have been introduced to the accounting literature. Brenneck, [1959] used them for better judgement in conjunction with traditional breakeven analysis. Morse, [1972] developed a model for reallocating the production cost along time, according to the learning curve phenomenon.

I and II show the behavior of the two firms. This model assumes stationary learning slopes, i.e., that it will take the same amount of time to master the new equipment whenever purchased. However, at a critical time, t*, when both firms will have to face the market, Firm I has a definite advantage over Firm II. These considerations can be incorporated into the traditional DCF model.

Let us look at Figure 3.1 and modify model No. 1. We will refer to the modified model as Model No. 2.

The experience advantage of Firm I is represented by the shaded area -- designated $L(t^*, t_2)$ -- may be calculated as follows:

$$(3.2)\quad L(t^*, t_2) = K^*(t_2 - t_0) + \int_{t_2}^{t^*} K^*(t-t_2)^n dt - \int_{t_0}^{t^*} K^* t^n dt$$

$$= K^*(t_2 - t_0) + \{K/(n+1)\}^*\{(t^* - t_2)^{n+1} - t^{*n+1} + t_0^{n+1}\}$$

Let us assume that the prices are declining according to

$$(3.3)\quad B = A^*d^{(t_2-t0)}$$

where

d is the decline prices per period, $0 < d < 1$

Incorporating (3.2) and (3.3) into the traditional DCF evaluation modifies our decision rules:

Invest today if

$$(3.4)\quad X - A + L(t^*, t_2) > Y - A^*d^{(t2-t_0)}\ ,\ 0$$

Buy tomorrow if

$$(3.5)\quad X - A + L(t^*, t_2) < Y - A^*d^{(t2-t0)} > 0$$

Do not invest if

(3.6) $X - A + L(t^*, t_2), Y - A^*d^{(t2-t0)} < 0$

Thus, we can use the modified DCF evaluation for better decision making.

Moreover, if (3.4) or (3.5) is positive, and given a certain t*, dictated by the market, we can maximize the NPV. Optimization yields the best timing (t_2) for the investment:

(3.7) $\underset{t_2}{\text{MAX}} \{L(t^*, t_2) - (A - A^*d^{(t2-t0)}\}$

Figure 3.2 is a plot of the cost reduction vs gain in learning. There may be an optimal time T for investment.

The evaluating of (3.7) results in some important conclusions for the decision makers:

- o At some point of time, t*, we measure the benefits. As t* is smaller, the learning benefits are higher and may dominate the decision. In our ever changing markets, where t* becomes shorter from year to year, the firm has much incentive to apply the new technology early.
- o The optimal timing for the investment T is affected by the expected price reduction coefficient d.

<u>Model No. 3</u>, introduced hereafter, deals with a problem which often arises in many companies: If the company invests in the current technology now, they will still have to buy the latest technology later. Would it be worthwhile to wait for the newer technology, or buy the technology now and thus "pay twice" when the newer technology will becomes available?

For example, let us consider two businessmen operating at the same environment:

o Mr. I bought a system for his business some time ago, and paid the high price (A) of the new technology.

o Mr. II is going to buy a system today. This system is a better one, and has a better performance. Mr. I is willing to upgrade his system, or to buy a new one (at an incremental cost of A1) in order to keep track with the business. No doubt that the time required for Mr. I to close the technology gap will be much shorter than that for Mr. II. The learning experience plays an essential part in those decisions.[2]

In order to quantify these learning benefits, the basic learning curve model is modified to asses the value of introducing new technologies (see Yelle, 1979 and Globerson, 1980).

Figure 3.3 illustrates the model.

Insert Figure 3.3 about here

At time 0, both people hold the same production time and performance H. At time t_0 Mr. I buys the new technology. He has a temporary decline in productivity and produces the first product at cost K. Naturally, Mr. II has a temporary advantage over him till time t_1. The long term considerations of Mr. I

[2] Kaplan (1986) has argued that these learning effects have characteristics similar to buying options in financial markets. By investing today you buy an option over tomorrow's complex technology.

have shown to be worthwhile. By time t_2, Mr. II decides to invest in the modern technology. Mr. I, willing to keep his competitive edge in the market upgraded his system, and apparently has paid "twice" for the system. Mr II's cost has increased by HG, reflecting the adoption of a complex technology. Note that HG > HK reflecting the fact that the new system is more complex than the old one. However, Mr. I conversion cost is only EI < GH. The fact that the new system is more productive is demonstrated by the steeper slopes of the new learning curves.[3] As a matter of fact, Mr. I has apparently paid "twice" for his technology (at points t_0 and t_2). However, he may gain two different benefits from his acts:

o An operational payoff, resulting from the low cost of his services or products the whole time.

o A strategic benefit by selling his product at any time $t > t_1$ at a lower price than his competition. This may mean survival for his firm.

We will now modify Model No. 2 to comply with the "double payment" effect. We assume that the new technology learning curve slope ($_m$) will be the same for the two persons, while Mr. I starts from a lower point.

To find the operational benefit, we can calculate the area between the two learning curves, which represents the extra

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[3] The improved productivity of the new system because of the advanced technology and also some externalities.

benefits to Mr. I. Thus, for any $t^*>t_2$ the gain in learning (L) is

$$(3.8)\quad L(t^*, t_2) = H^*(t_2 - t_0) + \int_{t_2}^{t^*} G^*(t - t_2)^m dt - \int_{t_0}^{t_2} K^*(t-t_0)^n dt - \int_{t_2}^{t^*} I_*(t - t_2)^m dt =$$

$$= H^*(t_2-t_0) + G^*(t^*-t_2)^{m+1}/(m+1) - K^*(t_2-t_0)^{n+1}/(n+1) - I^*(t^*-t_2)^{m+1}/(m+1)$$

$$= H^*(t_2-t_0) + (G-I)^*(t^*-t_2)^{m+1}/(m+1) - K^*(t_2-t_0)^{n+1}/(n+1)$$

Modifying (3.7) to find the optimal time to invest in the new technology, yields

$$(3.9)\quad \max_{t_2} \{L(t^*, t_2) - [A+A_1(t) - (A^*d(t_2-t_0)]\}$$

where $A_1(t)$ is the upgrading cost at time t.

4. The Declining Price Paradox of New Technologies

The Declining Price Paradox states:

Under the Paradox conditions, the more the price of the investment is subject to future reduction, the more urgent it is to invest in this technology.

This apparent paradox is explained by the following two considerations:

o The greater the price reduction expected, the more complex future systems are going to be. This will result in more learning time. The mechanism is explained in this section.

o High price reduction means that more competitors will be able to afford this technology, and for strategic survival it is necessary to buy the technology as soon as possible.

In a systematic approach to this paradox, we use the following terms:

o The total system to be implemented will be called the system.

o The system consists of several components.

o The components are built of elements.

Let us refer to the PC example:

The total PC system is the system. It consists of a PC (monitor, keyboard and CPU unit), a printer and a modem, each of them being a component. Each component consists of elements, e.g., the PC component elements are: memory chips, floppy or hard disk etc.

The Declining Price Paradox is valid under the following conditions:

1. Prices of the technological elements are in decline. Thus, the prices of components having the same elements are in decline. If you buy the same system some time in the future, you will pay much less than today.

2. Tomorrow's systems will be more complicated than today's systems. They will include more elements and components, and the learning time will be longer. For example, IBM PC, introduced in 1981, had two floppy disk drives. In the next generation one of the floppy disks was replaced by a hard disk, an element much more complex to learn.

3. The price of the system remains almost the same, but performance is improved.

Under these three conditions, the higher the price reduction (d) expected, the greater will be the future learning costs (HG, see Figure 3.2). This will bias the decision toward t_2 ----> t_0. The competition thus forces companies and individuals to buy the complex state of the art technology. The decline of prices of the elements does not necessarily mean a decline in the prices of components or systems. In many cases, the component price remains the same, while yielding much more options and better performance: the systems have more components than before, and more complex functions. The result is that the learning time increases.

Insert Figure 4.1 about here

Figure 4.1, presents the expected reduction in element cost (condition No.1). The system cost remains constant (condition No.3). Thus (condition No.2) the cost of learning may increase to master the additional components added to the system. The shape of the extra learning curve would apparently be convex since more functions are added to the system. However, the trend in software and hardware development toward more friendly software may offset the convex function toward a linear one.[4] Therefore the more the element prices are subject to future decline (the right hand side of the horizontal axis), it is suggested to purchase the system earlier.

Figure 4.2 shows the mechanism that affect the Declining Technology Price Paradox:

Insert Figure 4.2 about here

The technology and other external forces result in element price decline. This price decline has two implications:

o Prices of the existing systems decline.

o Introduction of improved systems.

These implications result in increased competition, and a company that uses the old technology is exposed to a strategic threat. Sooner or later the firm will invest in the new system, and will spend more time on learning.

5. Conclusions

This paper has discussed the problem of investing in and justifying new technologies from a learning point of view.

Some points are to be emphasized:

o The learning time should always be taken into consideration

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[4] Some empirical indications may support this argument: Lotus 123 by Lotus Development has the same friendly interface in both release 1A and 2.01. The package list price for the two releases remains the same ($495) while the number of options has almost doubled, resulting in more learning time. The same trend is identified in other PC software and hardware (e.g., Disk Operating System, dBASE package etc.)

whenever examining an investment in some new technology.

o Buying today's technology may provide an advantage whenever an organization goes to a new generation of technology, by reducing learning time and being able to use the new technology earlier.

o The price reduction affects competitors as well, and the organization can be exposed to both operational competition (cost reduction) and strategic threat (lose of market share). The benefits gained by absorbing the new technology sooner may be considered much more than the tangible price reduction.

o The technology price reduction is sometimes a mirage for both tactical and strategic considerations. The price reduction affects <u>elements</u>, and not <u>systems</u>. As the price of elements decline, systems tend to become more complex and their price do not tend to decrease.

o Strategic implications should be considered for the optimal timing for buying the technology.

Further research has to be done:

o Empirical studies should be carried out to assess the prices of elements, components and systems. These studies should determine the technological areas where the methods described here are best applied.

o A Decision Support System (DSS) approach to this problem can supply managers with a powerful analytical tool for their decisions.

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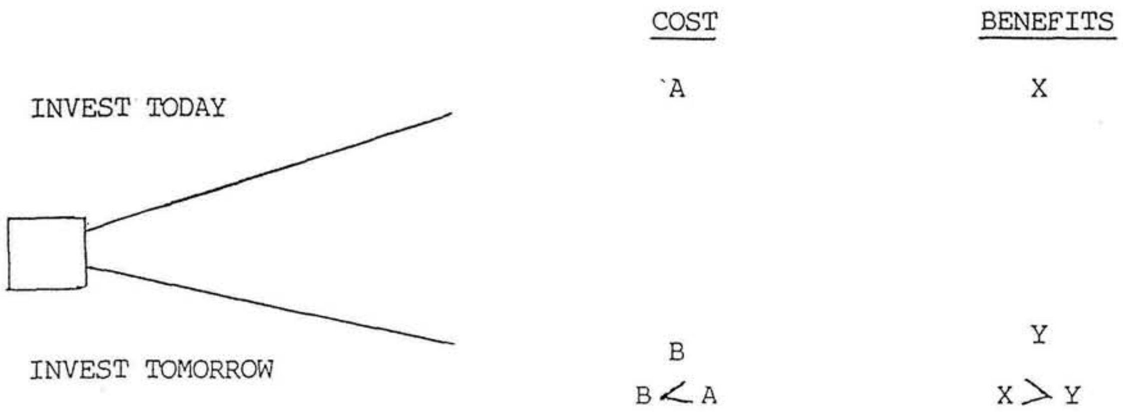
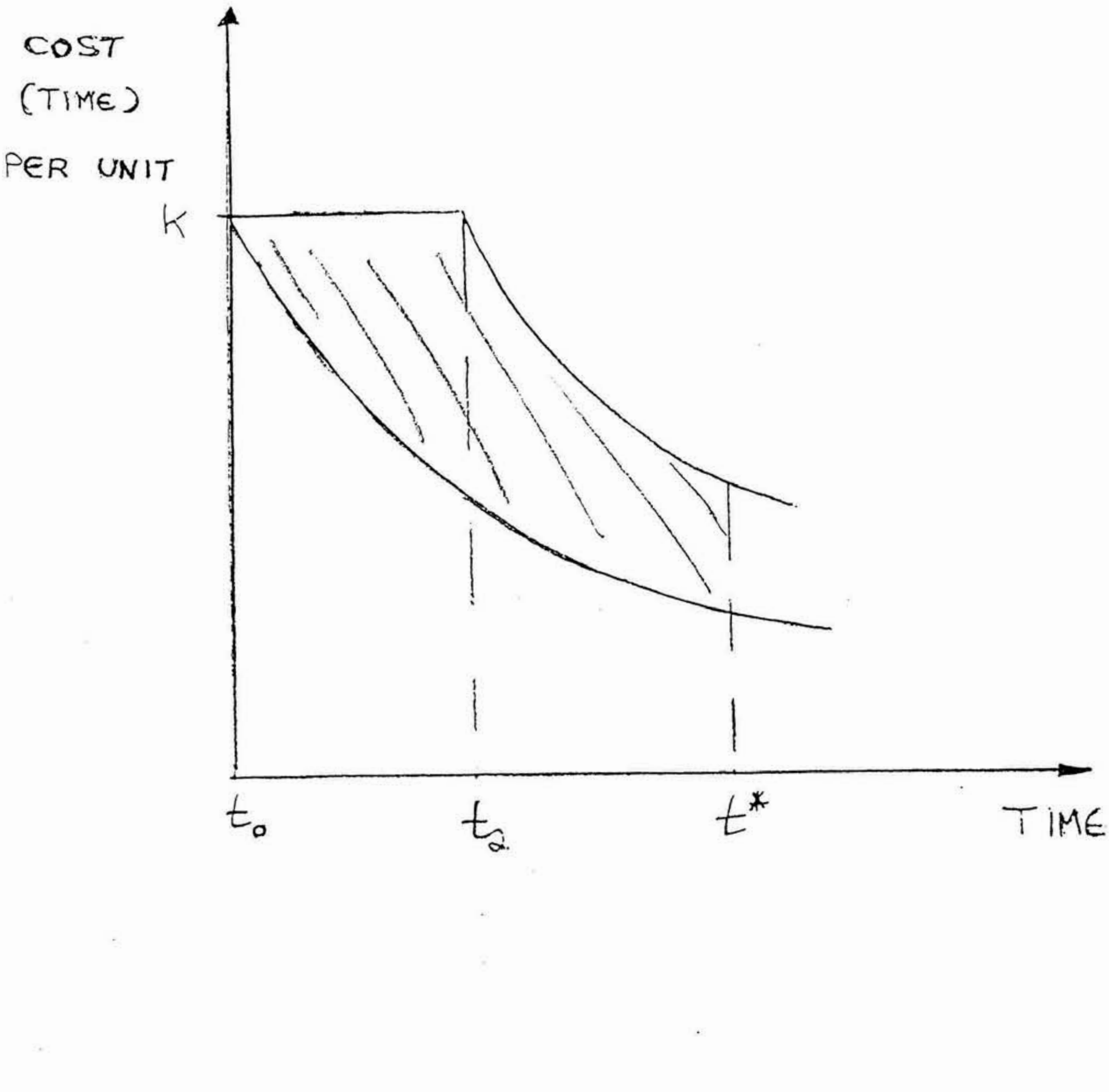
FIGURE 2.1: THE DCF APPROACH



'IGURE 3.1: LEARNING CURVE CONSIDERATIONS



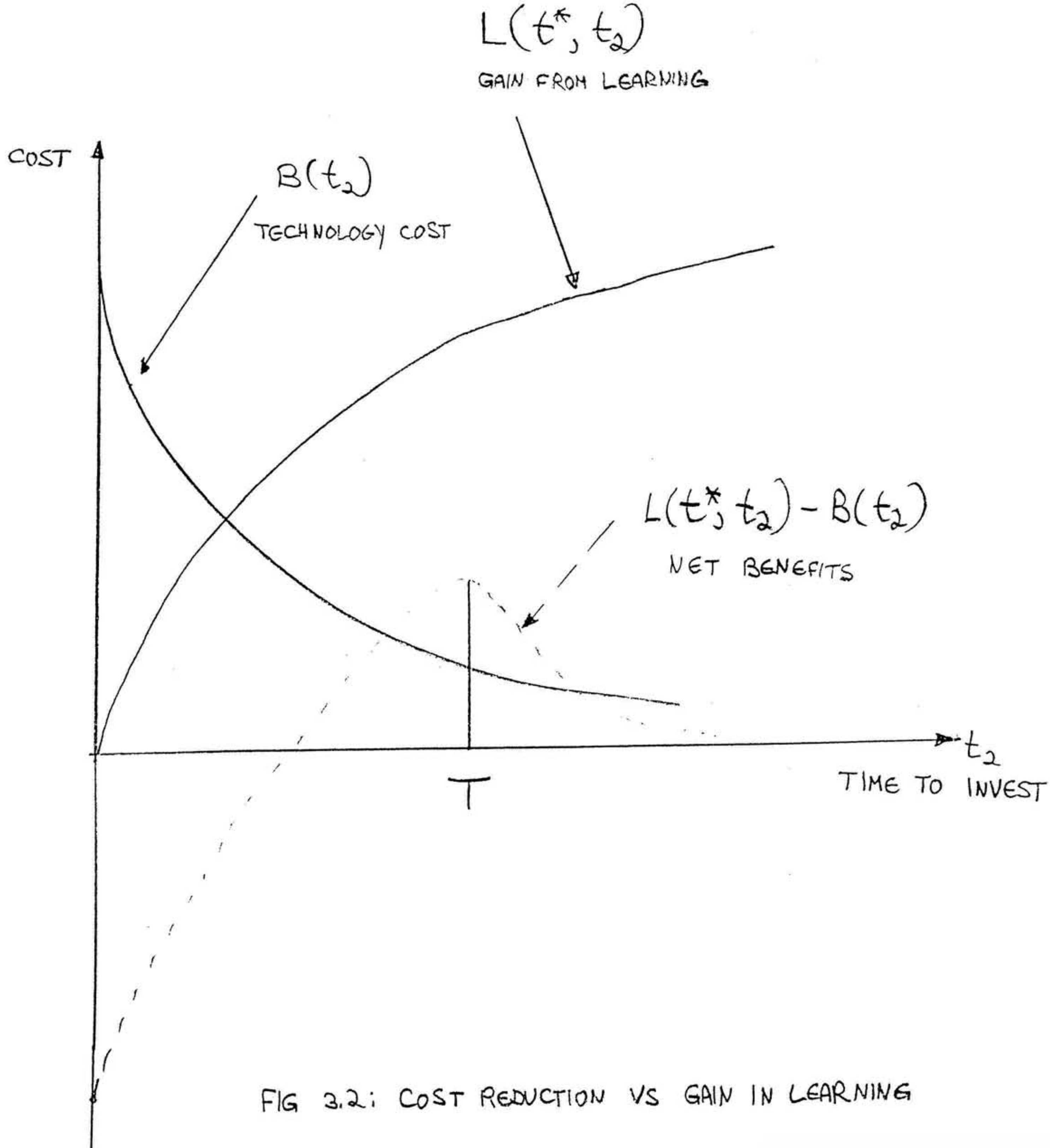


FIG 3.2: COST REDUCTION VS GAIN IN LEARNING

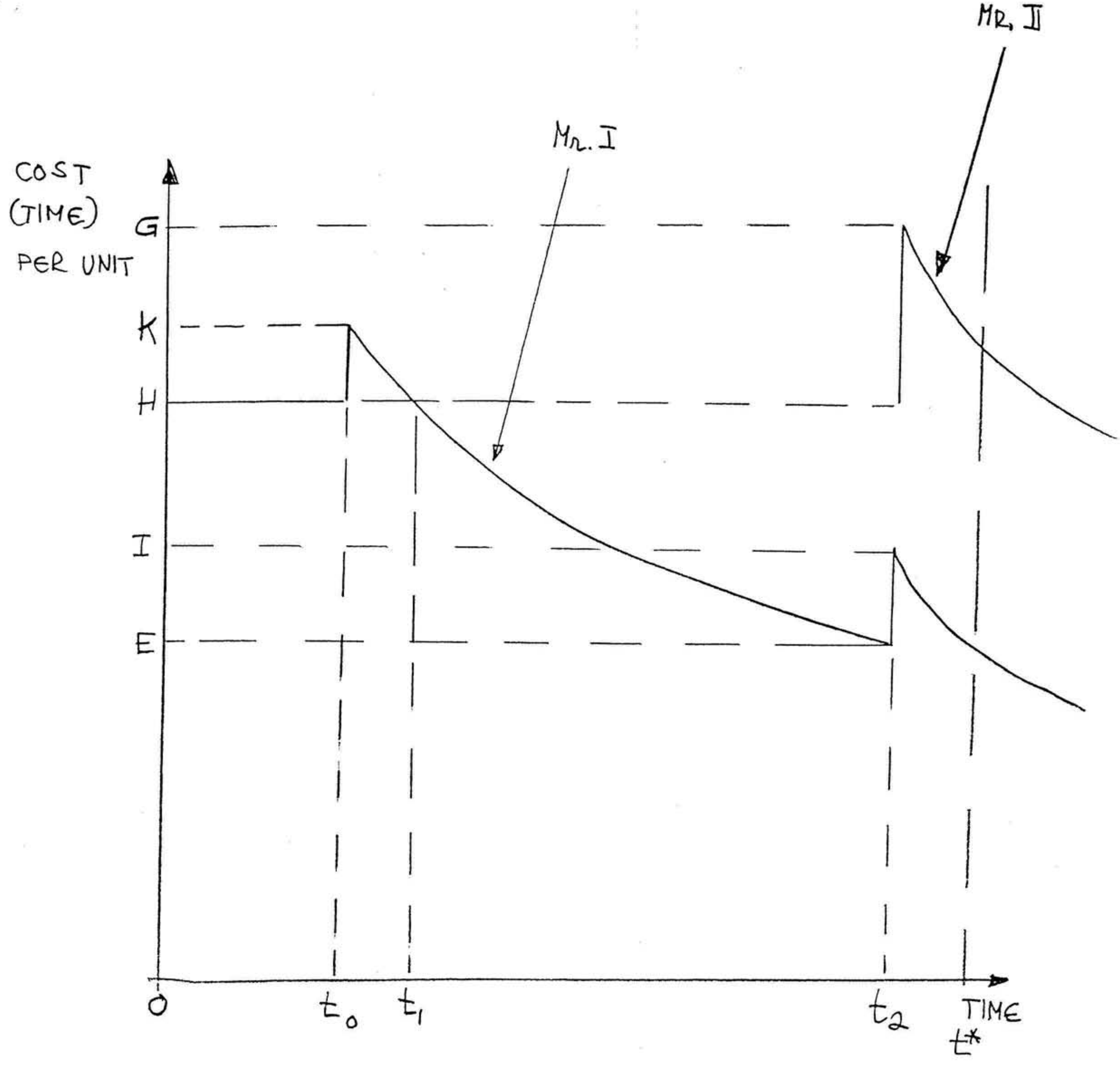


FIG 3.3: THE "PAY TWICE" MODEL

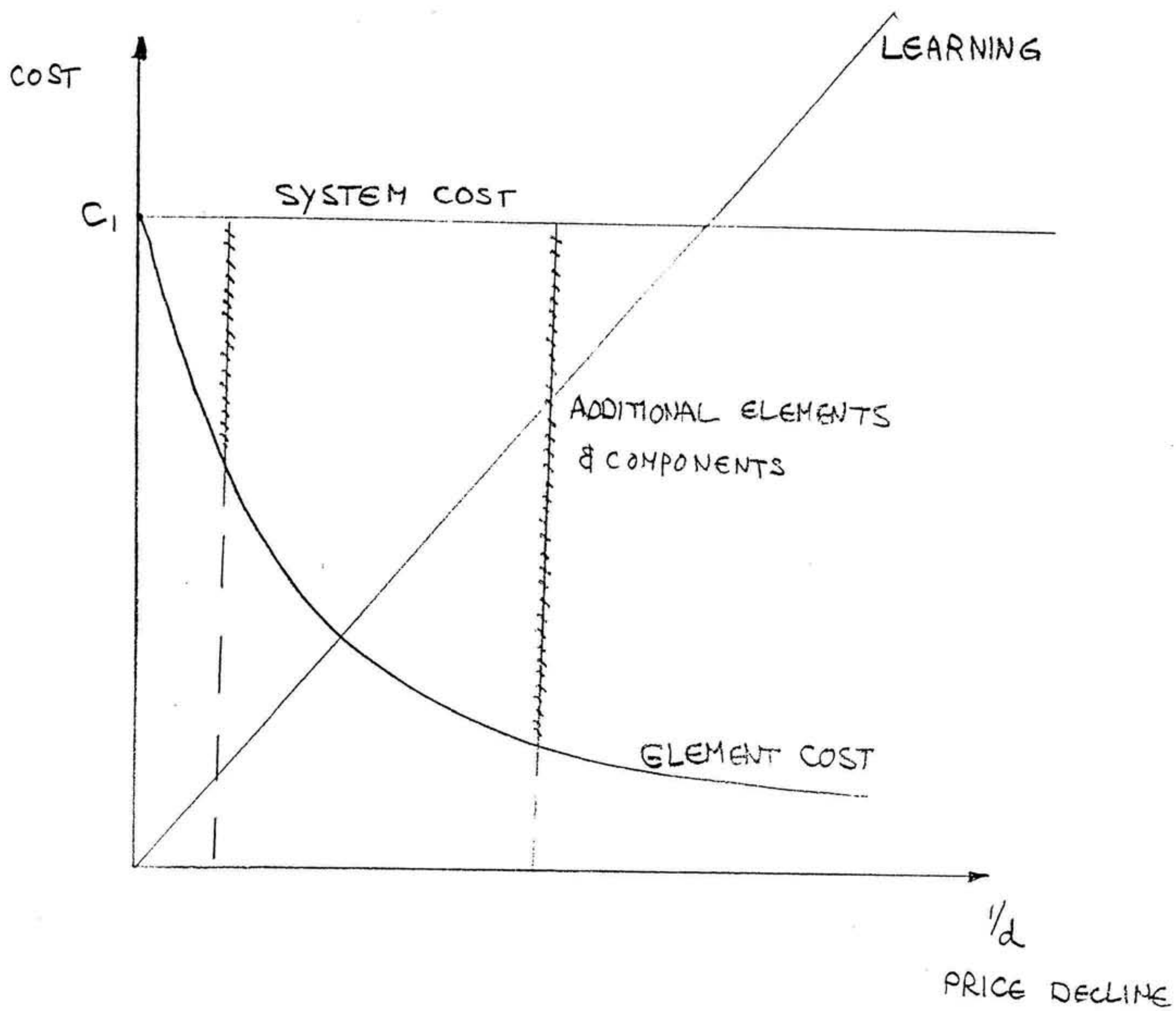
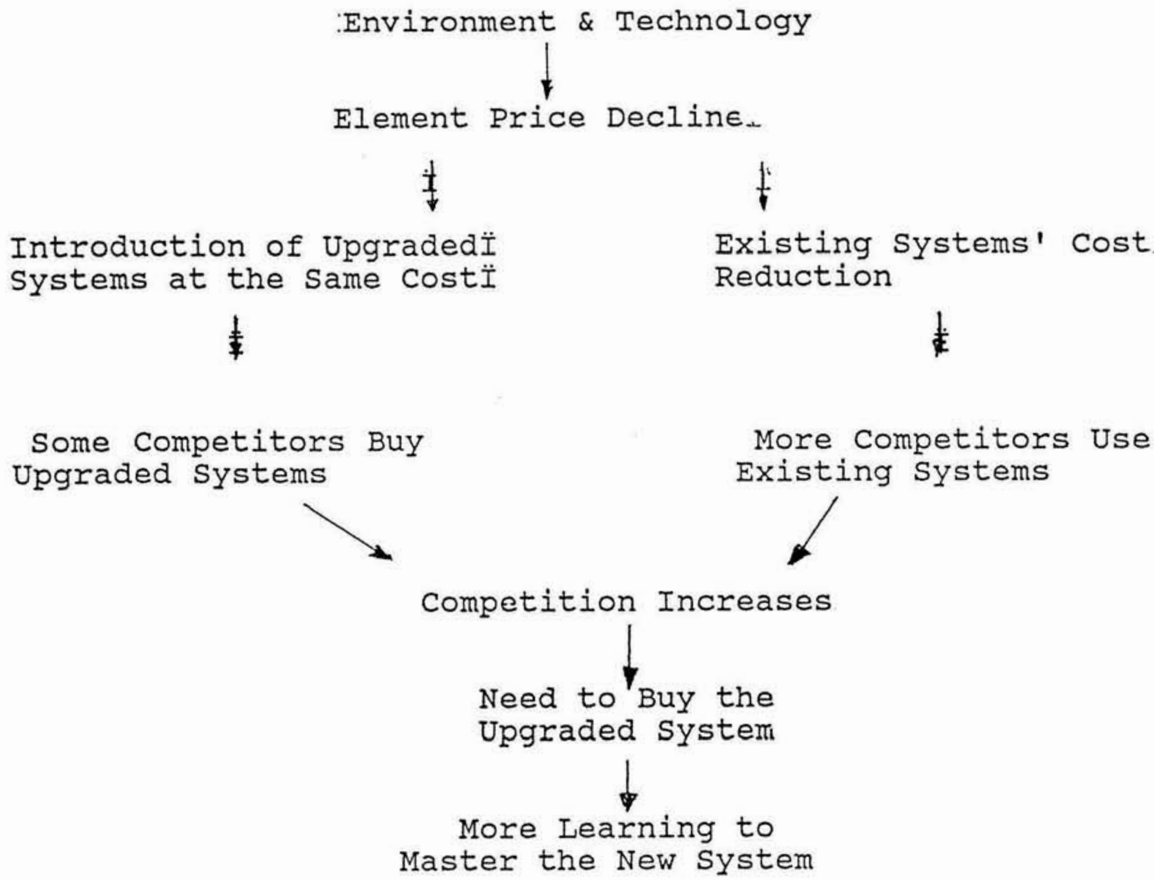


FIG 4.1: THE DECLINING ELEMENT COST AND THE INCREASING LEARNING COST

Figure 4.2: The Declining Price Paradox



pragmatics" [Carnap 47], Morris felt the need to expand upon this definition. In [Morris 46] he says that pragmatics is "that portion of semiotic which deals with the origins, uses, and effects of signs within the behavior in which they occur."

Montague's conception of pragmatics [Montague 68], [Montague 70a], and [Montague 73], based upon Bar-Hillel's [Bar-Hillel 54] discussion of indexical expressions, represents a departure from the traditional view. Hamblin [Hamblin 73] felt that Montague's incorporation of a pragmatic component directly in the syntax and semantics was unconventional, and felt the need "to defend pragmatics from this weakened interpretation...Pragmatics is the study of the *use* (not just reference) of language of all kinds; or, if it is not, we need a new name for the study that complements syntax and semantics. Montague's 'pragmatics' would be better classed as a special part of semantics." Dowty [Dowty 78], while admitting that "the linguist's use of the term pragmatics is far from standardized," adopts the view that it should encompass direct and indirect speech acts, presuppositions, and implicatures, and explicitly rejects Montague's use of the term to encompass a treatment of indexical expressions.

What we propose in our theory of questions is that the proper place for considering the answer(s) to a question is in a separate theory of pragmatics for the language. We do not propose a completely general theory of pragmatics. But we believe that incorporating into our fragment a formal pragmatic component that treats the notion of a response to a question is defensible as at least one component of a **theory of language use**. In the first place, Montague notwithstanding, it falls within the confines of pragmatics as that term is generally understood. For whether one speaks of "the use and effects" of language [Morris 46], the "relation of signs to their interpreters" [Morris 38], the notion of "speech acts" [Dowty 78], or the "linguistic means for effecting literal purposes" [Kasher 77], it is clear that the notion of responding to a question is encompassed. Our attempt to formalize a pragmatic component to QE-III accords well with what Stalnaker [Stalnaker 72] sees as the goals of "a formal semiotics no less rigorous than present day logical syntax and semantics." Those goals, he goes on to say, include an analysis of such linguistic acts as "assertions, commands, ..., requests ... to find necessary and sufficient conditions for the successful (or perhaps in some cases normal) completion of the act."

A second argument in favor of this approach comes from looking at the way that linguists have described the concept of a question. Linguists have traditionally classified sentences into four distinct types: declarative, interrogative, imperative, and exclamatory. A glance through some standard text or reference books on English grammar reveals two separate approaches to the rationale behind this scheme. According to one school, as in [Roberts 54] it is based upon the "different kinds of meaning" a sentence may have. The other school, as in [QuirkGreenbaum 74] considers that the distinction is based upon such criteria as word order in the sentence, presence or absence of a subject, the presence of an interjection, etc. Clearly the disagreement is over whether to consider this a syntactic or a semantic distinction. Perhaps in some sense it is mainly a pragmatic one, reflecting both the use and effects of the utterance.

Finally this approach in its technical details is both simple and elegant. It removes from semantics the burden of providing an account of the response to a question, and allows it to do what semantics theories have always done best, account for **reference**. Then, just as the semantics of a language is based upon its syntax, the pragmatics is based upon both the syntactic and semantic analyses (or, in Hamblin's phrase, it "complements syntax and semantics."). The simplicity with which we can state the pragmatic rules for our fragment, which take into account the notion of the answer to a question, is based upon this ability to have both the syntax and the semantics at hand upon which to build a theory of pragmatics. An example should make this clear.

In QE-III, questions denote (a semantic concept) just as declarative sentences do. Thus QE-III gives the following semantic analyses for "Who manages whom?" in the syntactic category WH-Question, and for "He manages him" in the category declarative sentence:

who manages whom? $\rightarrow \exists x[x(i) = u_2 \wedge EMP_*'(i)(u_1) \wedge AS\text{-}1(u_1,x)]$

he manages him $\rightarrow \exists x[x(i) = u_2 \wedge EMP_*'(i)(u_1) \wedge AS\text{-}1(u_1,x)]$

Both are treated as *denoting* the same object with respect to an index, a variable assignment, and a model. But they are *interpreted* differently in the pragmatics. Pragmatics in QE-III is a function that, given a derivation for an expression of QE-III together with its syntactic category and its (semantic) denotation, returns a (possibly) new object in the same model as its pragmatic interpretation. Thus, although we view pragmatics as a separate component of a language theory, it is closely allied to the semantics – both provide interpretations of linguistic expressions within the context of the same logical model. The formal definition of the pragmatic component in the next chapter will effect that these two sentences, interpreted pragmatically, denote what the following expressions of IL_s denote:

who manages whom? $\rightarrow \lambda u_2 \lambda u_1 \exists x[x(i) = u_2 \wedge EMP_*'(i)(u_1) \wedge MGR'(i)(x) \wedge AS\text{-}1(u_1,x)]$

he manages him $\rightarrow \exists x[x(i) = u_2 \wedge EMP_*'(i)(u_1) \wedge MGR'(i)(x) \wedge AS\text{-}1(u_1,x)]$

The pragmatic interpretation of the question is the set of n-tuples that answer it, while of the declarative sentence is the same as its denotation.

3.5.6. The Pragmatics of QE-III

The pragmatics which we give here for QE-III is a simple theory of the effects of producing an expression in that language within the assumed context of a question-answering environment. That is, we assume that a user of QE-III is using the language to produce some effect within this context, and it is this effect which we formalize as the pragmatic component of the language definition. We could, of course, have defined the pragmatics in the same manner as the semantics was defined, i.e. inductively over the syntax. However in doing so we would have seemed to be giving some status or importance to the pragmatic interpretation of expressions in every category of QE-III. Because we had no real intuition about what the **pragmatic interpretation** of, say, the expression "in 1978" represented, we decided upon a different form of the definition. Accordingly our definition provides a pragmatic interpretation for expressions in any of the several sentential categories of the language, namely T-YNQ, T-WHQ, WHENQ, and T-t. (Chapter VI contains a discussion both of some of the issues involved in our decision to present a separate pragmatic component to the formal theory of QE-III, as well as some of the considerations for the present form of this theory.)

The following preliminary definitions are needed before stating the pragmatic rules.

1. By $/\alpha\backslash$ is meant a derivation tree for the meaningful expression α of QE-III, as informally understood from our discussion of the syntax. We further assume that nodes of derivation trees are labelled with ordered triples <A,B,C> such that A is the meaningful expression derived at that node, B is its syntactic category, and C is the rule of syntax applied at that step in the derivation. For simplicity, we shall refer to component A of the root of $/\alpha\backslash$ as α, and to the component B as CAT($/\alpha\backslash$).

2. The translation rules guarantee that corresponding to any derivation tree $/\alpha\backslash$ for $\alpha \in ME_{QE\text{-}III}$ there is a *unique* translation into IL_s. By T($/\alpha\backslash$) shall be understood this unique translation, and by $[/\alpha\backslash]_M$ the denotation of $/\alpha\backslash$ (provided indirectly via T($/\alpha\backslash$)) with respect to the model M.

3. There are two standard ways of defining a (Tarskian) model-theoretic semantics. One is to define the notion of denotation with respect to a model M only, in which case formulas, e.g., denote the set of their satisfying variable assignments. The other, and more usual procedure is to define the denotation with respect to a model M and a variable assignment g, in which case a formula denotes either True or False. The two notions are, for all practical purposes, equivalent. Since for the purposes of pragmatics we shall want to consider that open formulas denote the set of their satisfying variable assignments, we shall in this section refer to the notion of denotation with respect to a model M only.

4. If $[/\alpha\backslash]_M$ is a function whose domain is As(M), the set of all possible variable assignments over M, and if further $V=\{v_1, ..., v_k\}$ is a set of variables of IL_s, then by $\Pi_V([/\alpha\backslash]_M)$ is understood the restriction of $[/\alpha\backslash]_M$ to the domain V. Note that if $V = \emptyset$, then $\Pi_V([/\alpha\backslash]_M$ is defined to be just $[/\alpha\backslash]_M$.

5. If f is any function with domain As(M), then **now**(f) is the restriction of f to the domain $As_{now}(M)$, where $As_{now}(M) = \{g \mid g \in As(M) \text{ and } g(i) = F(\mathbf{now})\}$, that is, that subset of the possible variable assignments for M for which the distinguished time variable *i* is interpreted as denoting that state denoted by the constant **now**.

6. By $FV(/\alpha\backslash)$ we shall understand the set $\{i_1, i_2, ..., i_n\}$ of indices of the "variables" (expressions of the form [it-CASE-i]) occurring free in α. This notion will not be defined rigorously here, but would be defined inductively over the structure of $/\alpha\backslash$ in the usual manner, with particular attention paid to which rules bind occurrences of variables (all of the PTQ substitution rules) and which rules leave them free (e.g., the rules that introduce WH-Terms.) This definition would be analogous to the definition of the set FV_e of variables of type e occurring free in a logical expression, in particular in the expression $T(/\alpha\backslash)$. It is clear that if $FV(/\alpha\backslash) = \{i_1, ..., i_n\}$ then $FV(T(/\alpha\backslash)) = \{u_{i_1}, ..., u_{i_n}\}$. However we emphasize that $FV(/\alpha\backslash)$ is defined over the derivation tree of α (i.e., over the syntax of QE-III) and makes no reference to the (intermediate) translation of this tree into IL_s.

7. Finally, if β is a meaningful expression of IL_s, and if the free variables of type e in β, $FV_e(\beta) = \{u_{i_1}, u_{i_2}, ..., u_{i_n}\}$, are such that $u_{i_1}, u_{i_2}, ..., u_{i_n}$ are in alphabetical order, then $LC_{FV_e}(\beta)$ is the unique expression: $\lambda u_{i_n} ... \lambda u_1 \beta$ formed by first prefixing β with λu_{i_1}, then prefixing λu_{i_2} to the result, and so on.

In order to understand the form of some of the following definitions we state the following fact (the proof follows directly from the translation rules of QE-III): If β is the translation of any meaningful expression α of QE-III, then the free variables of α are all of type e, except for the possible exception of the distinguished variable i of type s.

The rules of pragmatics which we now state constitute a definition of the pragmatic function, in a manner analogous to the way in which the translation rules constitute a translation relation. In particular they constitute a definition of the function P: $P : /QE\text{-}III\backslash \rightarrow M \cup \{ ERROR \}$ which assigns to any derivation tree of a meaningful expression α of QE-III, either an object in the model M or the distinguished symbol "ERROR" as its **pragmatic interpretation**.

P1. If $CAT(/\alpha\backslash) \notin$ {WHENQ, T-WHQ, T-t, T-YNQ} then $P(/\alpha\backslash) =$ ERROR.

P2. If $CAT(/\alpha\backslash) \in$ {WHENQ, T-WHQ, T-t, T-YNQ} then $P(/\alpha\backslash) = \Pi_{FV_e}(\mathbf{now}([/\alpha\backslash]_M))$

Rule P1 ensures that only **sentences** have a pragmatic interpretation. Rule P2 ensures that all sentences are interpreted with respect to the "current" state index, and that in the case of questions, the infinite sequences of variables that the question denotes is projected down to include only the questioned variables.

It is clear that the set of sequences given by Π_{FV_e} (**now** $([/\alpha\backslash]_M)$) is equivalently represented by the denotation of the expression LC_{FV_e} (λiT($/\alpha\backslash$)(**now**)) of IL_s with respect to M and g. P2 is therefore alternatively defined as: $P(/\alpha\backslash) = [LC_{FV_e}(\lambda i T(/\alpha\backslash)(\mathbf{now}))]_{M,g}$.

What this alternative definition allows us to do is to utilize the semantic notion of denotation to define the pragmatic interpretation of sentences in QE-III. For it allows us to take a translation T($/\alpha\backslash$) of any sentence α and determine its pragmatic interpretation as the denotation of the expression: LC_{FV_e} (λiT($/\alpha\backslash$)(**now**)) and thus evaluate the pragmatic interpretation of α in terms of the semantics of IL_s by means of this simple syntactic transformation on T($/\alpha\backslash$).

3.5.7. Conclusions

The QE-III theory defines the denotation of a question in exactly the same way as the denotation of the corresponding declarative sentence that has pronouns in place of the interrogatives, but defines its pragmatic interpretation as the set of n-tuples that "answer" it. We have discussed our initial attempts to accomplish this result directly, by having WH-Terms denote functions from sets of properties to sets of individuals that had those properties. Technically, we discovered that to accomplish this directly required a considerable complication of the semantics throughout the structure of our fragment. And we discovered, as we shall see, that others with similar goals had also been forced to introduce more complexity into their logical model in order to accomplish these goals in the semantic component of their theory. Finally we realized that by eliminating as a goal of the semantics the capturing of the answer(s) of questions, we could leave the basic semantic theory of PTQ intact, and moreover we could easily accomplish this goal in the pragmatics.

This concludes our informal discussion of the syntax, semantics, and pragmatics of QE-III. We now proceed to discuss the theory in relation to some of the other work in the field of Montague Semantics that has attempted to extend the PTQ fragment to include a theoretical account of the syntax and semantics of questions.

3.6. Related Work

3.6.1. Introduction

Two common threads run through much of the recent work on formalizing a theory of questions. The first is the idea that all questions should be defined so as to denote objects of the same type. Generally this has meant propositions or sets of propositions, but it seems that even before the choice of just what questions denote was made, this "single semantics" viewpoint had been adopted. The other, as we have already pointed out, is that some account of the answer(s) to a question should be included at least as a component of its semantics. When combined with other factors these two biases have led to somewhat different results. Thus [Hamblin 73] suggests that a question denotes the set of all "propositions that count as answers to it;" [Karttunen 77] "the set of propositions expressed by[its] true answers;" Bennett [Bennett 77], [Bennett 79] (and also [Belnap 82], who worked with Bennett on the theory) "sets of open propositions: functions from sequences of individuals to propositions."

3.6.2. Karttunen

As we have said [Karttunen 77] presents an analysis of the semantics of questions that falls within the single-semantics tradition. ([Hamblin 73] earlier proposed a treatment similar to Karttunen's, but his theory was not worked out in as much detail.) In Karttunen's theory, for example, the question "Who manages John?" would roughly be translated as: $\lambda p \exists x[p(i) \wedge p = \lambda i\ [\text{manage'}(i)(x)(i),\text{John}])$. Semantically this expression, when interpreted with respect to a model and a state, denotes the set which contains, for each person x that manages John, the proposition that x manages John.

Such a treatment of the semantics of questions seems inappropriate to us for two related reasons. First it seems to confuse propositions with the sentence s that express them. Whatever a proposition might be in our informal use of the term, it is in the formal system defined by $\mathbb{IL}_s$ a function from indices to truth values, or equivalently a set of indices. In order to see why this seems an inappropriate choice for the semantic object denoted by a question, consider a model in which the constants manage' and love' are interpreted as follows:

```
             | 1978 → {<Mary.John>. <Susan.John>} |
F(manage') = | 1979 → {<Mary.John>. <Bill.John> } |
             | 1980 → {<Bill.John> }              |
```

```
            | 1978  → {<Bill, John>, <Susan, John> } |
F(love') =  | 1979  → {<Bill, John> }                 |
            | 1980  →   ϕ                             |
```

Now consider the two queries ***Q1***: "Who manages John in 1978?", which translates to $\lambda p \exists x[\text{manage}'(1978)(x,\text{John}) \wedge p = \lambda i \text{manage}'(i)(x,\text{John})]$, and ***Q1***: "Who loves John 1n 1978?", which translates to $\lambda p \exists x[\text{love}'(1978)(x,\text{John}) \wedge p = \lambda i \text{love}'(i)(x,\text{John})]$. Given these translations, the interpretation of these two queries in this model, [Q1] and [Q2], is:

[Q1] = { {1978,1979} /* ***Mary manages John*** */
{1978} } /* ***Susan manages John*** */

[Q2] = { {1978,1979} /* ***Bill loves John*** */
{1978} } /* ***Susan loves John*** */

Under this interpretation both queries, which are obviously quite distinct, denote exactly the same set of propositions in the model, the set containing the proposition {1978,1979} and the proposition {1978}. Thus under this interpretation we cannot distinguish between these two questions -- they are semantically equivalent in the database under this theory.

The second and related objection is that under this interpretation all direct mention of the entities (Mary, Susan, John, ...) involved disappears. Instead the theory claims that the question denotes a set that contains sets of states (years). What this implies is that there is no obvious way of going backwards from these objects in the model (the sets of propositions) to some *useful* expression in a language (English) that names them. Since in this theory the denotation of questions "loses" the people involved, we have no simple way to recover their names and report them to the questioner. The theory neglects considering the *use* and *effects* of the question. Moreover there seems to be no way even to add a pragmatic component to such a theory in order to account for a question's answer(s), for on the one hand the syntax has no mention of the *names* of the individuals involved, nor on the other hand does the denotation involve any *individuals* at all. In the pragmatics of our theory the two queries would instead be interpreted, with respect to a given database, as follows (where [Qi] now means the pragmatic interpretation of Qi): [Q1] = {Mary,Susan}, and [Q2] = {Bill,Susan}.

With these interpretations we have not lost the people involved, and there is an obvious relationship between these objects and English expressions for them ("Mary and Susan" and "Bill and Susan") as well as the relations in the database that express the same information:[5]

Q1
Mary
Susan

Q2
Bill
Susan

As noted earlier, Bennett discussed the issue of the logic of questions in two separate papers, and collaborated with Belnap in the development. Their theory is presented cumulatively in [Bennett 77], [Bennett 79] and in [Belnap 82]. Motivated again by the goal of a single semantics, and even more strongly by a desire to account for the individuals that answer the question, Belnap and Bennett develop a theory that incorporates sequences of individuals into the model theory. Thus a question like "Whom does John love?" is treated as denoting a set of functions from sequences of individuals to propositions. Essentially all and only those sequences that close the open proposition "John loves [it-ACC-0]" and make it True are included in this denotation. What this is tantamount to is incorporating the standard (Tarskian) notion of a variable assignment into the model theory, instead of leaving it in the meta-theory of the logic. For technical reasons the entire system must be altered to include these sequences, so that even sentences are no longer translated into formulas, but rather into expressions denoting sets of such sequences. This rippling effect of of the complications to the semantics is extraordinarily reminiscent of the problems we had in formulating a theory with inductive WH-Terms!

In order to accomplish this result, the set of types of the IL is expanded to include as a basic type n, expressions of type n denoting a natural number. Thus the natural numbers must be included as objects in the model, as well as functions constructed from them. Of particular interest in their theory are the functions from N to individuals, i.e. sequences. The ripple effect necessitates that "all expressions of English [denote] functions from sequences of individuals to their usual extensions" [Bennett 79]. Even sentences are no longer translated into formulas, but rather into expressions of type $<<n,e>,t>$ that denote sets of sequences. Unfortunately the results of this complication to the logic and the English translations do not seem to justify the cost. Certainly this theory represents a step closer to the goal of capturing explicitly in the denotation the individuals that answer the question, so it is an improvement over the proposition proposal. But these individuals are hidden somewhere inside *infinite* sequences of individuals, with no indication of their position within those sequences.

[5] [Tichy 78] makes many of the same points that we make here regarding the proposition idea.

An example should clarify this point. In order to understand it, we provide the following table of the types of the variables used:

Variable symbols	Type of variable symbol
P	<e,t> : sets of individuals
r, s	<n,e> : sequences of individuals
O	<<n,e>,<s,t>> : open propositions

In the Bennett/Belnap theory, an "open" sentence like "John loves him" is translated as: $[\lambda s[\text{love}'([\hat{}\lambda PP(s(1))])(\text{John}')]$, which denotes (ignoring intensions) the set of sequences such that John loves the first member of each of them. The corresponding question "Whom does John love?"[6] would be: $[\lambda O[O = [\lambda s[\hat{}\text{love}'([\hat{}\lambda PP(s(1))])(\text{John}')]] \wedge \exists r[{}^{\vee}O(r)]]]$. which denotes a set of open propositions. But these again involve infinite sequences of individuals, and there is no indication of which projection of these sequences represents the individuals that John loves.

This problem, of having the individuals that constitute the answer embedded in infinite sequences without knowing how to project them out is the same one that we have in our semantic theory. For our semantics translates questions into open formulas, which denote the set of variable assignments that satisfy the formula. Our relegating to pragmatics the task of projecting these variable assignments could also be used to solve this problem here. But if this is the case, then what is gained by paying the price for the complication to the model theory and the translation rules? This use of sequences in effect duplicates the variable assignment of their Tarskian meta-theory (albeit restricted to the domain D_e) in the object language and in the logical model with no noticeable advantages.

3.6.3. Hausser and Zaefferer

The proposal of Hausser and Zaefferer ([HausserZaefferer 78] and hereafter H-Z) is quite different from the other theories we have discussed, and makes a number of interesting points. The theory is motivated early in the paper by a discussion of the range of answers that are possible to any given question, and a classification of these possibilities as ranging from "minimal" to "redundant." For example, in answer to the question "Who dates Mary?" the following list of possibilities is cited:

[6] actually their syntax does not cover direct questions, and so this is really their treatment of "John loves him" in the category of Basic Question; it seems clear, however, that they intend the semantics of the corresponding direct question to be the same

(a) Bill.
(b) Bill does.
(c) Bill does so.
(d) Bill dates her.
(e) Bill dates Mary.

Answer (e), of course, is just what the propositional approach would say that the question denotes (assuming Mary is "going steady" with Bill.) H-Z goes on to say, however, that "the truth value of the answer expression will depend on the question in the context of which it is uttered, except for [the completely redundant answer]. This shows that redundant answers are not very interesting from a semantical point of view since their semantic representation is identical to that of ordinary declarative sentences. Since both redundant and non-redundant answers are possible, and since non-redundant answers are generally much more natural, we hold that no serious theory of questions and answers should restrict itself to a treatment of redundant answers alone, and that it should be able to handle both."

H-Z then proceeds to develop a theory to account for all of these possible answers, by extending the PTQ grammar and the logic IL. This theory replaces the model theory of IL by what they call a "context-model." In essence this model is an IL-model expanded to include as model-theoretic objects the entire language of IL itself. Minimal answers are then translated into expressions that denote formulas when interpreted within the context of a preceding question. This is accomplished technically by including in the logic a set of context variables, and by including an abstraction over a context variable in the translation of the non-redundant answers. A context variable denotes an expression of IL, viz. the question that has set up the context. This idea of a context allows H-Z to define a semantics not just for questions like "Who dates Mary?" but also for each of the answers (a) through (e) in such a way that each of them is equivalent in extension.

H-Z's concern with the semantics of the answers to the questions, which at first sight seems to be our concern, is in fact another issue. For our theory, while it takes the answers of questions into account, is essentially not a theory of answers but a theory of questions. Of course, in the context of a more complete and user-friendly question-answering system, the ability to keep track formally of the context of the discourse and to express the answer in a number of different ways, is very attractive. Such a system would need the ability to go "backwards" from expressions in the logic to expressions in English with the same interpretation; [Friedman 81] discusses this issue from the point of view of the PTQ fragment. But the development in H-Z of the semantics of the questions themselves, although motivated from this different concern with the equivalence of redundant and non-redundant answers, does also lead them to an analysis of question-semantics outside of the single-semantics framework. Their analysis "lets questions denote different types of sets according to the type of that expression which is the critical one in any kind of answer." In other words, their semantic analysis of answers is quite similar to our pragmatic analysis of questions! The following table comparing the types assigned to various kinds of questions by their semantics and our pragmatics might help to make this analogy clearer:

Question class	Our typing	H-Z typing
yes-no	t	<<s,<<s,t>,t>>,t>
1 individual	<e,t>	<<s,f(T)>,t>
2 individual	<e,<e,t>>	<<s,f(T)>,<<s,f(T)>,t>>

3.6.4. Scha and Gunji

The work of Scha on the PHLIQA1 project [Scha 83] and Gunji [Gunji 81], both being developed concurrently with the development of QE-III [Clifford 82b], are remarkably similar in spirit, though not in detail, to the present work. The close parallels in the motivation of these three works indicate a trend among many researchers toward developing a formal foundation for computer systems that do natural language processing.

The major theoretical difference between QE-III and that of the PHLIQA1 project of Scha are that we make a distinction between the semantics and pragmatics of sentences in QE-III, so that the pragmatic interpretation of questions in QE-III is closely analogous to Scha's semantics for the same question. We continue to believe that this separation between the *denotation* of a sentence (given by the semantic component of the language) and its *interpretation* (given by the pragmatic component) is a simpler and more easily extendible approach to the problem of providing a formal account of "meaning."

Much of the motivation for the work reported in [Gunji 81], namely to provide a formal pragmatics to a language specification by means of the computational application of a "super-interpreter" after the completion of the syntactic and semantic interpretation, is the same as ours. Gunji's "super-interpreter," in fact, is quite clearly the computational realization of what we have termed our pragmatic interpretation. The major difference between these two projects is in the scope of their languages, which reflect Gunji's focus on conversation implicatures and ours on querying historical databases. Whereas Gunji's work covers declarative and imperative sentences, and True-False questions, whereas QE-III resulted from a concentration on WH-questions and an explicit treatment of time-denoting expressions.

This concludes our informal discussion of the goals and philosophy behind the definition of the fragment QE-III, and its relation to other recent work in the area of formal question semantics. The next section provides an overview of QE-III through a series of examples and discussions illustrating the major features of the language. (The complete definition of the syntax, semantics, and pragmatics of QE-III can be found in [Clifford 87].)

4. Examples from the QE-III Fragment

4.1. Introduction

This section presents and discusses examples of the syntactic and translation rules of the QE-III fragment. As we pointed out in 3, the PTQ fragment stands essentially intact as the core of QE-III. There are, however, certain changes to this core. One major change is our use of the logic IL_s as the intermediate translation language; this logic is a modification to Montague's IL, and makes explicit the "hidden" abstraction over indices that is a part of the evaluation process in Montague's PTQ analysis. With respect to IL, the major change is that in IL_s, we evaluate any expression α with respect to a state s by by forming the expression: $[\lambda i\alpha](s)$.

The other major aspect of QE-III is the inclusion of a formal pragmatic component to the language, on an equal standing with the syntactic and semantic components. The formalized pragmatic component of QE-III was primarily motivated by the desire to simplify the provision of an interpretation for **questions** in a formal way. As we showed in Section , the pragmatic interpretation of any sentential expression was essentially the denotation of the expression formed by λabstracting over all of the free individual variables and also evaluating with respect to **now**.

In addition to these changes in the underlying logic and method of evaluation, the following additional modifications have been made to the rules of the PTQ fragment:

1. rule S4 has been modified to perform the single function of combining a Term with an IV to form a sort of proto-sentence. It no longer performs the verb inflection for 3rd person singular present tense. The entire treatment of tense and time adverbials is now performed more systematically by rules S101 through S106. (The tensing functions of S17 have therefore been totally eliminated.)

2. Montague's use of the variables he_0 and him_0 amounted to a simple technique of case marking in order to choose the appropriate personal pronoun upon substitution of a Term. We have expanded this technique somewhat, using variables of the form [it-CASE-i] where CASE

ranges over {NOM,DAT,ACC} and i over the natural numbers.

3. rule S9 for combining a sentence adverbial ("Necessarily") with a sentence, has been eliminated. This is because the only sentence adverbials in QE-III are Time Adverbials which are brought in together with the tense marker in rules S104 - S106.

4. it is well known that there are problems with the PTQ treatment of conjunction and disjunction of Terms and IVs (see discussion in [Friedman 79] and [Bennett 74]). While Friedman's bracketing solution is ultimately more acceptable (both by virtue of its generality and, of particular interest, its natural correspondence to a LISP implementation), we have for simplicity of presentation adopted Bennett's simple solution of marking all Basic Verbs with a # marker which is removed when the verb is ultimately tensed. (We choose this solution because the points we wish to make have only to do with the verbs, and are easily understood with this technique.)

For ease of understanding the translations to follow, the following table shows the types of the variables used:

Variable symbols	Type of variable symbol
x , y , z , x_0 , x_1 , ...	<s,e> : individual concepts (ICs)
P , Q , Q_1, Q_2, ...	<s,<<s,e>,t>> : properties of ICs
p , q , q_1, q_2, ...	<s,t> : propositions
i	s : distinguished state variable wrt which all expressions are evaluated
i_1 , i_2 ,...	s : states
W	<s,<<s,<<s,e>,t>>,t>> :properties of properties of ICs

4.2. PTQ-like Examples from the QE-III Fragment

Before illustrating some of the added features of the QE-III database query fragment, we present a simple example within the syntactic range of the PTQ fragment (up to vocabulary differences) in order to contrast the way these two fragments derive and translate it. For example, under one analysis

(4-1) John manages Mary

would have the following derivation tree in QE-III:

```
         John manages Mary   S104
                 |
                 |
         John #manage Mary   S4
            /   \
           /     \
          /       \
         John   #manage Mary   S5
                     /\
                    /  \
                   /    \
             #manage    Mary
```

The syntactic and translation rules illustrated in this example are **S4** to form an Untensed Clause from a SUBJect and PREDicate, **S5** to form a Verb Phrase from a TransVerb and a Direct Object, and **S104** to form a Present Tense Clause.

Several points arise with this example. First we note that this analysis tree presents the derivational history of non-basic expressions in the language in the obvious way. Each node is labelled with a meaningful expression in QE-III; in case the expression is non-basic, it is further labelled by the syntactic rule by which it was constructed, and is given children labelled with the expressions from which it was obtained. [Montague 70b] provides a more formal definition of analysis trees; it should be sufficient to point out that the language is defined in such a way that to each analysis tree (though not necessarily to each meaningful expression) there corresponds a unique translation into the intermediate logical language.

This analysis of Example 4-1 illustrates several departures from the corresponding PTQ analysis. First we note that the basic verb is prefixed with #, and this prefix remains even after S4 is applied to combine the Term "John" with the Intransitive Verb Phrase "#manage Mary." Second the rule S104 is new. It takes an untensed sentence as input and gives a (present) tensed sentence as output. Thus we have characterized tense as a property not of verbs but of clauses, although this property in English is realized by the inflection of the main verb of the clause. The importance of this characterization will be made clearer when we consider the interaction of tense with interrogative sentences.

This method of introducing tenses into a sentence obviates the need for undoing the English verb inflections that would be required by a method (such as in PTQ or in [Dowty 79] that *always* introduced present tense first, subject to possible subsequent modifications. [Dowty 79] (fn.5, Ch.7) makes a similar point -- though still in terms of introducing the tense via a SUBJ + PRED rule -- but does not incorporate the idea into the fragment presented there.

In a number of the PTQ rules Montague makes use of the auxiliary notions of the gender of a CN or a T, and the third person singular form of a verb. These notions are never defined with the same rigor which Montague demanded of other characteristics of his logic and grammar, presumably because he felt they were obvious and uninteresting. As in [Bennett 74] we make use of a number of similar auxiliary notions in our rules. This example points out two such notions, viz. that of the tense of a clause and the case of a variable. In our fragment a clause is either untensed or tensed, and belongs to a different category (though of the same logical type) in either case. A variable introduced into a sentence is either uncased, or one of NOM, ACC or DAT.

The translation of Example 4-1 corresponding to the above analysis tree is as follows:

Mary $\Longrightarrow \lambda P \exists x[P(i)(x) \wedge x(i) = \text{Mary}]$

#manage $\Longrightarrow \lambda W \lambda x[W(i)(\lambda i \lambda y[\text{AS-1}(y(i),x) \wedge \text{EMP}_*'(i)(y(i)) \wedge \text{MGR}'(i)(x)])]$

#manage Mary $\Longrightarrow \lambda W \lambda x[W(i)(\lambda i \lambda y[\text{AS-1}(y(i),x) \wedge \text{EMP}_*'(i)(y(i)) \wedge \text{MGR}'(i)(x)])](\lambda i \lambda P \exists x[P(i)(x) \wedge x(i) = \text{Mary}])$

$\rightarrow \lambda x \exists z[\text{AS-1}(z(i),x) \wedge \text{EMP}_*'(i)(z(i)) \wedge \text{MGR}'(i)(x) \wedge z(i) = \text{Mary}]$

John $\Longrightarrow \lambda P \exists y[P(i)(y) \wedge y(i) = \text{John}]$

John #manage Mary $\Longrightarrow \lambda P \exists y[P(i)(y) \wedge y(i) = \text{John}]\ (\lambda i \lambda x \exists z[\text{AS-1}(z(i),x) \wedge \text{EMP}_*'(i)(z(i)) \wedge \text{MGR}'(i)(x) \wedge z(i) = \text{Mary}])$

$\rightarrow \exists y[\text{AS-1}(\text{Mary},y) \wedge \text{EMP}_*'(i)(\text{Mary}) \wedge \text{MGR}'(i)(y) \wedge y(i) = \text{John}]$

John manages Mary $\Longrightarrow \exists y[\text{EMP}_*'(i)(\text{Mary}) \wedge \text{MGR}'(i)(y) \wedge y(i) = \text{John} \wedge \text{AS-1}(\text{Mary},y)]$

The pragmatic interpretation is represented by: $\exists y\ [\text{EMP}_*'\ (\text{now})(\text{Mary}) \wedge \text{MGR}'\ (\text{now})(y) \wedge y(\text{now}) = \text{John} \wedge \text{AS-1}(\text{Mary},y)]$.

Our treatment of Proper Terms is slightly different from the PTQ treatment, in that the translations include an individual-concept variable whose extension at the state *i* is asserted to be the indicated individual. This is done because in HRDM all individuals of interest must be playing a role in the database, and roles can only be filled by individual concepts. Further, as we discussed in Section 3, verbs are treated as objects of the same type as in PTQ, but they are analyzed in terms of the database schema.

4.3. Temporal Reference in QE-III

In addition to its indication by means of the tense system, temporal reference in English is also indicated by certain time adverbials (today, last year, ...) and also by prepositional phrases (in 1978, on Monday...). Care must be taken in order to analyze properly the semantics of sentences which involve an interaction between tenses and these other temporal indicators. They cannot be applied sequentially as operators to a clause, or the semantics will be incorrect. (David Dowty [Dowty 79] makes the same observation.) The following derivation for

(4-2) Peter earned 25K in 1978.

illustrates this aspect of QE-III:

```
        Peter earned 25K in 1978    S108
           /              \
          /                \
         /                  \
  in 1978  S113    Peter #earn 25K  S4
     /  \          (derived as in example 4-1)
    /    \
  in    1978
```

This example illustrates rule **S108** which simultaneously adds a tense (past) and a time adverbial, and

S113 which forms a temporal prepositional phrase. The pragmatic interpretation correctly indicates that there is some state in the past that is also in the set of states 1978 at which the present tense sentence Peter earns 25K is true: $\exists i_1 \exists y[1978'(i_1) \wedge [i_1 < \mathbf{now}] \wedge EMP_*'(i_1)(Peter) \wedge SAL'(i_1)(y) \wedge y(i_1) = 25K \wedge AS\text{-}1(Peter,y)]$.

If we had introduced the two temporal indicators (the tense and "in 1978") separately, in either order, the resulting interpretations would be incorrect:

```
          Peter earned 25K in 1978
            /              \
           /                \
       (PAST)      Peter #earn 25K in 1978
                        /        \
                       /          \
                   in 1978     Peter #earn 25K
```

interpreted as: $\exists i_2 \exists i_1 \exists y[[i_2 < \mathbf{now}] \wedge 1978'(i_1) \wedge EMP_*'(i_1)(Peter) \wedge SAL'(i_1)(y) \wedge y(i_1) = 25K \wedge AS\text{-}1(Peter,y)]$. This places the three times i_1, i_2 and **now** on the time line as follows:

```
-------|--------|------------------
       i2      now
```

with i_1 *anywhere* on the time line in 1978.

The reverse order of sequential introduction is also incorrect:

```
        Peter earned 25K in 1978
          /             \
         /               \
     in 1978     Peter earned 25K
                    /          \
                   /            \
               (PAST)      Peter #earn 25K
```

since it is interpreted as: $\exists i_2 \exists i_1 \exists y[1978'(i_2) \wedge [i_1 < i_2] \wedge EMP_*'(i_1)(Peter) \wedge SAL'(i_1)(y) \wedge y(i_1) = 25K \wedge AS\text{-}1(Peter,y)$.

Here the two times are located as follows:

```
-------|------------|----
       i1           i2, in 1978
```

The properties of Peter are asserted to be true in state i_1, but i_1 may or may not be in 1978, and may or may not be in the past (with respect to **now**.) Only the **simultaneous** introduction of these temporal operators provides the correct interpretation.

Example 4-3 illustrates how tense is treated as a property of clauses in compound sentences, and how these tenses are independent of one another. It also illustrates how relative clauses are maintained in the QE-III fragment:

(4-3) Peter manages an employee such that he earned 30K.

Under the most likely analysis, this sentence is interpreted in QE-III as: $\exists w \exists x \exists y \exists i_1 [\mathrm{EMP}_*'(\mathbf{now})(x(\mathbf{now})) \wedge \mathrm{MGR}'(\mathbf{now})(w) \wedge w(\mathbf{now}) = \mathrm{Peter} \wedge \text{AS-1}(x(\mathbf{now}),w) \wedge \mathrm{EMP}_*'(i_1)(x(\mathbf{now})) \wedge \mathrm{SAL}'(i_1)(y) \wedge y(i_1) = 30\mathrm{K} \wedge 1978'(i_1) \wedge [i_1 < \mathbf{now}] \wedge \text{AS-1}(x(\mathbf{now}),w)]$.

Example 4-4 illustrates how propositions can be treated in almost the same way as time constants for denoting sets of states.

(4-4) John worked before Mary worked.

This sentence is analyzed as asserting that there was some state S1 before **now** at which John worked, and that S1 was also before some other state S2 before **now** at which Mary worked:
$\exists i_1 [[i_1 << (\lambda i_2 \mathrm{EMP}_*'(i_2)(\mathrm{Mary}))] \wedge [i_2 < \mathbf{now}] \wedge [i_1 < \mathbf{now}] \wedge \mathrm{EMP}_*'(i_1)(\mathrm{John})]$.

Similarly we can combine simple time expressions with prepositions to form temporal adverbials, as in Example 4-5:

(4-5) Rachel worked before yesterday.

which is analyzed as: $\exists i_1 [[i_1 << \mathrm{yesterday}'(\mathbf{now})] \wedge [i_1 < \mathbf{now}] \wedge \mathrm{EMP}_*'(i_1)(\mathrm{Rachel})]$. Notice that this translation places two restrictions upon when the state i_1 can occur in time:

1. $[i_1 << \mathrm{yesterday}'(\mathbf{now})]$ because of "before yesterday," and

2. $[i_1 < \mathbf{now}]$ because of the past tense. Since a time before yesterday must be before **now** (by the meaning of "yesterday"), a Meaning Postulate for words such as "yesterday" might well be in order here to remove this redundancy and reduce the final translation to: $\exists i_1 [[i_1 << \mathrm{yesterday}'(\mathbf{now})] \wedge \mathrm{EMP}_*'(i_1)(\mathrm{Rachel})]$.

We now proceed to discuss the other additional rules of the QE-III fragment. These rules either form expressions that have particular relevance to the database realm (possessives, role specifications, etc.) or form interrogative sentences. We will look first at the questions; some of the considerations involved in the framing of these rules for database querying purposes was given in Section 3.

4.4. Questions in QE-III

Consider the following query:

(4-6) Who managed Rachel?

translated as: $\exists i_1 \exists y [[i_1 < i] \wedge \mathrm{EMP}_*'(i_1)(\mathrm{Rachel}) \wedge \mathrm{MGR}'(i_1)(y) \wedge y(i_1) = u \wedge \text{AS-1}(\mathrm{Rachel},y)]$. Recall that the pragmatics provides a representation for the answer to questions, and that the pragmatic interpretation of this query is denoted by the expression: $\lambda u\ \exists i_1 \exists y [[i_1 < \mathbf{now}] \wedge \mathrm{EMP}_*'(i_1)(\mathrm{Rachel}) \wedge \mathrm{MGR}'(i_1)(y) \wedge y(i_1) = u \wedge \text{AS-1}(\mathrm{Rachel},y)]$ formed by binding all free occurrences of the variable i to the constant **now**, and

λ-abstracting over all of the free individual variables.

This example illustrates why the tense must be considered a property of the entire clause, rather than just of the verb phrase, if the semantics of the question is to come out right. For suppose instead that we derived 4-6 as follows:

```
        Who managed Rachel?
         /          \
        /            \
     who     [it-NOM-0] managed Rachel
```

The translation would then be: $\exists y \exists i_1 [y(i) = u \wedge [i_1 < i] \wedge EMP_*'(i_1)(Rachel) \wedge MGR'(i_1)(y) \wedge AS\text{-}1(Rachel,y)]$. The problem with this translation is that the manager-IC y is not "tensed" properly. When evaluated, this query will return the set of individuals u who are the extension of Rachel's manager-IC, not at some time in the past, but **now**. Because "who" has wider scope in this derivation, the past tense operator could not capture the free i of the translation of "who." The question, under our treatment, is correctly analyzed as Who (past) managed (past) Rachel?} rather than as **Who (*now*) managed (past) Rachel?** In order to get this reading, tenses (and tenses + TmADVerbials) must be brought in last over all clause, including interrogative sentences.

Interrogative Terms (WHT's) can also be derived from common nouns and the interrogative determiners such as "which," as seen in Example 4-7:

(4-7) Who manages which employees?

which is interpreted as: $\lambda u_1 \lambda u_2 \exists y [EMP_*'(now)(u_2) \wedge MGR'(now)(y) \wedge y(now) = u_1 \wedge AS\text{-}1(u_2,y)]$.

Example 4-8 illustrates a 3-Term interrogative, using the three-place verb "#supply" and a rule (a simple extension of the two-place case, essentially taken from [Dowty 79]) for combining such a verb with an indirect object to form a two-place verb:

(4-8) What does who supply to whom?

The interpretation[7] is: $\lambda u_3 \lambda u_2 \lambda u_1 [COMP_*'(now)(u_3) \wedge DEPT_*'(now)(u_2) \wedge ITEM_*'(now)(u_1) \wedge REC\text{-}3(u_3,u_2,u_1)]$.

Example 4-9 illustrates a more complicated question that requires, in terms of the database representation, a "join" of two relations:

(4-9) Who works for a department such that it sells shoes?

It is interpreted as: $\lambda u \exists x [EMP_*'(now)(u) \wedge DEPT_*'(now)\ (x(now)) \wedge AS\text{-}1(u,x) \wedge ITEM_*'(now)(Shoes) \wedge$

---

[7] REL-3 indicates that there is a 3-ary relationship among the indicated three individuals.

REC-2(x(now),Shoes)].

Yes-No questions can take two forms in the fragment:

(4-10) Is it the case that Peter earns 30K?

and

(4-11) Does Peter earn 30K?

Both of these questions receive the same interpretation: $\exists x[EMP_*'(i)(Peter) \wedge SAL'(i)(x) \wedge x(i) = 30K \wedge AS\text{-}1(Peter,x)]$.

"When" questions, very important in an historical database context, are illustrated by the following example:

(4-12) When did Peter earn 25K?

interpreted as: $\lambda i_1 \exists y[[i_1 < now] \wedge EMP_*'(i_1)(Peter) \wedge SAL'(i_1)(y) \wedge y(i_1) = 25K \wedge AS\text{-}1(Peter,y)]$.

Finally, the next two examples illustrate the interaction of "when" and an already-formed Term question, and the interaction of "when" with time phrases.

(4-13) When did who manage whom?

interpreted as: $\lambda u_2 \lambda u_1 \lambda i_1 \exists x[\,[i_1 < now] \wedge EMP_*'(i_1)(u_1) \wedge MGR'(i_1)(x) \wedge x(i_1) = u_2 \wedge AS\text{-}1(u_1,x)]$ and

(4-14) When and to whom did company A sell item B yesterday?

interpreted as: $\lambda i_1 \lambda u_2 \lambda u_1\ \exists x[[i_1 < now] \wedge yesterday'(i_1) \wedge DEPT_*'(i_1)(u_1) \wedge x(i_1) = u_1 \wedge COMP_*'(i_1)(A) \wedge ITEM_*'(i_1)(B) \wedge REL\text{-}3(A,B,u_1)]$.

This concludes the examples of the kinds of queries expressible in the language QE-III, and the semantics and pragmatics that the fragment provides for them. We now present some of the other additions we have made to the PTQ fragment in order to express certain other common query constructions.

4.5. Miscellaneous Features of QE-III

The use of possessives is very common in database queries, and is easily incorporated into the fragment as in:

(4-15) Who is Peter's manager?

which is interpreted as: $\lambda u \exists x[MGR'(now)(x) \wedge x(now) = u \wedge AS\text{-}1(Peter,x)]$. An alternative way of phrasing the same question uses "of" instead of the possessive marker: and ultimately receives the same

(4-16) Who is a manager of Peter?

interpretation. Finally, specification of the "role" played by an individual in the database can also be accomplished by means of the word "as":

(4-17) Who has Peter as manager?

interpreted as: $\lambda u \exists z[MGR'(now)(z) \wedge z(now) = Peter \wedge AS\text{-}1(u,z)]$, or by the simple concatenation of the role and a Term:

(4-18) Who sells item 37?

interpreted as: $\lambda u[DEPT_{*}'(now)(u) \wedge ITEM_{*}'(now)(37) \wedge REC\text{-}2(u,37)]$.

5. Conclusion

The problem of modelling the semantics of time is one which is beginning to be explored by researchers in a number of different areas of Computer Science. We believe that formal logic can make an important contribution to our understanding and specification of the properties of time that we with to incorporate into our models and systems. Using the logic IL_s and the framework of MS, we have presented in this paper an overview of the HRDM, which is a formalization of the concept of an historical database. HRDM provides for the modelling of historical information in a DBMS, for the specification of constraints on the way that information can change over time, and for a query language for accessing that information with specific reference to its temporal dimension.

To complement the relational query language of HRDM ([CliffordCroker 87]), we have in this paper described a formal English database query language, QE-III, which is defined in a MS framework. QE-III incorporates a formal syntax, semantics and pragmatics to account for an interpretation of question that accords with the interpretation of HRDM, including an account of multiple-WH questions, an a semantics and pragmatics of time, and a grammar that is conducive to a computer implementation. In addition to its formal syntax and parallel semantics, QE-III is provided with a formal pragmatics which provides a representation for the answer(s) to a question as a function of its syntax and semantics. We believe that this approach, and the whole area of **formal** pragmatics as a component of language theory, is a fertile area for further research.

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