ORTHOGONAL INFORMATION STRUCTURES:
A MODEL TO EVALUATE THE INFORMATION PROVIDED BY A SECOND OPINION

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ABSTRACT

The paper discusses the value of information when a number of independent sources provide information related to a common set of states of nature.

The starting point is the Information Economic model of Information Structures. The model is augmented to represent independence of informational sources by means of orthogonality of the information structures.

A new mathematical operator, **orthogonal product**, is defined and its properties are probed. It is shown that this operator maintains some mathematical properties such as closure, association, unity element, null element, etc. It is demonstrated how the orthogonal product represents the notion of multi-source information.

The paper proves that an orthogonal product is generally more informative than its multipliers, namely, if cost is not considered a constraining factor, then there is a non-negative value to obtaining a second opinion.

The paper concludes with a numerical example and a discussion on the applicability of the model of orthogonality.

Keywords: Information Economics, Information Structures, Value of Information, Decision Models

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1. INTRODUCTION

The Information Economic approach to information evaluation is based on an Information Structure (IS) model developed by Marschak[9], McGuire and Radner[11], Demski[5], and others, and later expanded in a number of articles (e.g.,[1],[2],[3]). The model portrays an information system as a stochastic (Markov) matrix of probabilities which transform states of nature to signals. The decision maker (DM) has to select the optimal decision rule under given values of a priori probabilities for states of nature, and given values of payoffs. The Information Economic model proposes a partial rank ordering of Information Structures by using Blackwell Theorem[10].

The IS model basically deals with one source of information generating signals in a stochastic manner. In reality, however, there are many cases where the DM has to consider a number of signals based on the same set of states of nature but generated by 'independent' information systems. The IS model does not deal with such cases. Since the power of a model is in its correspondence to a real decision problem (Demski[5]), there is a need for a modification to the traditional model.

In other words, the real state of nature is unknown to the decision maker who has to learn about it through signals. However, instead of reacting in response to a signal provided by a single system, the DM requests signals from a number of sources and reacts only after examining the combined information. Here are some examples.

A manufacturing business of hi-tech electronic products wishes to place an offer in a bid for a new product. The CEO requests to prepare a cost estimate of the new product. This is performed by two independent teams. One team takes a 'micro' approach; the product is decomposed into major components which are further divided into items until the entire Bill-of Materials (see Monks[16], ch. 11) is exploded. Then the cost of each elementary component is assessed, and the total is summed up in order to obtain an aggregate estimate.

The second team takes a 'macro' approach (also known as Parametric Control). The team attempts to assess some overall traits of the new product; e.g., weight, volume, number of electronic cards (slots). Based on these few parameters and, of course, on past experience and historical data, the team calculates a rough estimate of the cost.

The CEO has to consider two distinct signals provided by two independent sources before deciding what he or she believes to be the real cost.

A similar example is very common in the construction industry. The cost of building a new house is usually estimated in two ways. One way is to try to list out all the necessary 'ingredients' of the house and sum up their cost; the other way is to calculate the 'magic number', normally the floor space, and multiply this number by the cost per square foot. The result will yield a quite good approximation of the cost of the building.

One last example is taken from a totally different area. When a person faces a crucial decision regarding his or her own health (e.g., undergoing a major surgery), most of the people will ask for a second opinion. A second opinion is indeed an additional signal, based on the same state of nature, but provided by an independent source.

The problem of multi-source information has been discussed in a number of articles. Winkler[18], for instance, examined the problem of combining several forecasts of a single variable. Morris [14,15] treated the same problem by a two-stage Bayesian process. However, the incorporation of the IS model (which is in fact based on Decision Theory) into the problem of multi-source information has not yet been explicitly presented.

This paper applies the IS model to the case where a decision maker has to consider a number of signals provided by "independent" (this term will be defined later more rigorously) information structures.

The paper addresses a number of questions. The first one is how independence of ISs can be treated? The paper coins a new term, '<u>orthogonal information structures</u>'; a new mathematical operator labeled '<u>orthogonal product</u>' is defined and its mathematical properties are analyzed.

The second question deals with the value of the information provided by an orthogonal IS. It is proven that the combined information collected from orthogonal systems is generally more informative than the information produced by each individual system; i.e., it is worthwhile to ask for a second opinion (subject, of course, to cost considerations).

The last part of the paper discusses the applicability of the orthogonal model. This discussion is assisted by a numerical example.

2. ORTHOGONAL INFORMATION STRUCTURES

We will first review briefly the traditional Information Structure (IS) model (McGuire[10]), and then incorporate orthogonality into the model .

Let E be a finite set of events (states of nature), $E=\{e_1,...,e_{n_E}\}$. Let p be a vector of a priori probabilities associated with the events in E, $p^t=(p_1,...,p_{n_E})$, where $\Sigma p_i=1$, $p_i\geq 0$, $i=1,...,n_E$, (the superscript t stands for a transpose operator).

Let Z be a finite set of signals, $Z=\{z_1,...,z_{n_Z}\}$.
An **information structure** Q is defined as an $n_E x n_Z$ Markov (stochastic) matrix of conditional probabilities in which signals of the set Z will be displayed at the occurrence of an event of E. Thus q_{ij} of Q is the probability that for a given event e_i, signal z_j will be displayed.

Let A be a finite set of actions that can be taken by the decision maker (DM), $A=\{a_1,...,a_{n_A}\}$. A cardinal payoff function U is defined from AxE to the real numbers, R^1, associating payoffs to pairs of actions and events, $U:AxE\text{-->}R^1$. The function U can be depicted by an $n_A x n_E$ matrix, denote U, whose each element a_{ij} reflects the payoff gained when an action a_i is taken and the event turns to be e_j.

The DM cannot observe the events but rather the signals, and chooses actions accordingly. The DM's strategy is delineated by an $n_Z x n_A$ Markov matrix D, whose each element d_{ij} determines the probability that the DM takes action a_j upon observing signal z_i. Obviously, the DM wishes to optimize D to obtain the maximum expected payoff. This is performed by the following algorithm:

Let p' be a square matrix containing the elements of p in its main diagonal and zeros elsewhere:

$$p' = \begin{vmatrix} p_1 & 0 \dots 0 \\ 0 & \cdot \quad \cdot \\ \cdot & \cdot \quad \cdot \\ \cdot & \cdot \quad 0 \\ 0 & \dots 0 \; p_{nE} \end{vmatrix}$$

The expected payoff gained from Q, U, and a decision rule D is given by tr(QDUp'), where 'tr' represents the trace operator. Maximization of the above is obtained by solving a Linear Programming problem for the elements of D constrained by the properties of a Markovian matrix (see Ahituv and Wand [3] for an elaborate discussion).

Given two ISs Q and R operating on the same set of events E, Q is defined to be **generally more informative** than R if the maximal expected payoff yielded by R is not larger than that yielded by Q for all payoff matrices U and all probability vectors p. A partial rank ordering of ISs is provided by

Blackwell Theorem [10] stating that Q is generally more informative than R if and only if there exists a Markov matrix M with appropriate dimensions such that QxM=R; M is called the garbling matrix. (Hereafter we will use the terms 'informativeness' and 'more informative' for the relationship 'generally more informative').

We will introduce now the notion of orthogonality. Intuitively, two (or more) ISs are considered to be orthogonal when they observe the same set of states of nature but generate signals independently; in other words, the likelihood of a signal to be generated by a certain IS does not depend on the signal produced by the other, but only on the conditional probabilities of the IS itself. This will be formulated now more rigorously.

Let Q and R be two information structures operating on the set of events E and producing the sets of signals $Z=\{z_1,...,z_{nz}\}$ and $W=\{w_1,...,w_{nw}\}$ respectively.

<u>Definition 1</u>: Signals z_j and w_k are orthogonal if and only if

$Pr((z_j/e_i)/(w_k/e_i)) = Pr(z_j/e_i) = q_{ij}$ and

$Pr((w_k/e_i)/(z_j/e_i)) = Pr(w_k/e_i) = r_{ik}$ for all i.

(i.e., the probability that z_j is triggered due to occurrence of e_i does not depend on whether w_k has been displayed or not, and vice versa).

Definition 2: Information Structure Q and R are orthogonal when all their signals are orthogonal one to each other.

It is obvious that the relationship of orthogonality is symmetric (by definition) and transitive. The next section shows how to compose an integrated IS out of orthogonal ones.

3. ORTHOGONAL PRODUCT

The purpose of this section is to show how one can combine the information provided by distinct orthogonal ISs. This will be done by defining a new mathematical operator, orthogonal product (or orthogonal multiplication) and inquiring its traits.

Let Q and R be two ISs defined in the same way as in the previous section.

<u>Definition 3</u>: S is called the orthogonal product of Q and R (denote: S = Q@R) if S is a matrix of n_E rows and $n_w x n_z$ columns whose elements are as follows: for every i, $i=1,\dots,n_E$

$$
\begin{aligned}
s_{11} &= q_{11}*r_{11} \\
s_{12} &= q_{11}*r_{12} \\
&\vdots \\
s_{1nw} &= q_{11}*r_{1nw} \\
s_{1nw+1} &= q_{12}*r_{11} \\
&\vdots
\end{aligned}
$$

The orthogonal product S maps a set of n_E events into a set of $n_w x n_z$ signals. The following numerical example clarifies the notion of orthogonal product.

<u>Example</u>: Let Q and R be the following 2x2 orthogonal ISs:

Q	z_1	z_2
e_1	.9	.1
e_2	.2	.8

R	w_1	w_2
	.8	.2
	.4	.6

Let S=Q@R. Then S is computed as below:

S	z_1&w_1	z_1&w_2	z_2&w_1	z_2&w_2
e_1	.72	.18	.08	.02
e_2	.08	.12	.32	.48

S observes the original set E and produces four signals which indicate what can be displayed to the DM: z_1 and w_1, z_1 and w_2, etc. It is now the task of the DM to devise the optimal decision rule for each individual pair of signals. But first let us discuss some mathematical properties of the orthogonal product.

<u>Property 1</u>: S is a Markovian matrix (this is the property of **closure**, i.e., the set of IS is closed under the operand of orthogonal multiplication; see Wylie and Barrett[19]).

Proof: Obviously, each element of S is non-negative; it is sufficient to show that the sum of all the elements in a row of S equals 1.

$$\sum_m s_{im} = \sum_j \sum_k q_{ij} * r_{ik} = \sum_j q_{ij} * \sum_k r_{ik} = 1*1=1$$

<u>Property 2</u>: The orthogonal product is an associative operation, i.e., Q@(R@L)=(Q@R)@L.

This property is a directly consequential to the definition of the operator.

<u>Property 3</u>: The unity element of the orthogonal product is the vector

$$\bar{1} = \begin{vmatrix} 1 \\ . \\ : \\ 1 \end{vmatrix}$$

since for any Q it yields $\bar{1}$@Q=Q.

Note that the unity element may have an informational interpretation as well. An IS of that kind does not provide any information; it always produces the same signal. Thus, combining it with another IS should not supply any additional knowledge.

Optimizing the decision rule for an orthogonal product matrix is similar to the process carried out for a 'simple' IS, namely through solving an LP problem. This is obvious since the product is indeed an IS. Let us continue with the previous example to demonstrate the optimization process.

Suppose Q, R and S are as follows:

$$Q=\begin{vmatrix} .8 & .2 \\ .5 & .5 \end{vmatrix} \qquad R=\begin{vmatrix} .5 & .5 \\ .8 & .2 \end{vmatrix} \qquad S=Q@R=\begin{vmatrix} .4 & .4 & .1 & .1 \\ .4 & .1 & .4 & .1 \end{vmatrix}$$

Suppose the a priori probabilities are $p^t=(.6\ .4)$, the set of actions is $A=\{a_1,a_2\}$, and the payoff matrix U is the following:

$$\begin{array}{c} \\ a_1 \\ a_2 \end{array} \begin{vmatrix} s_1 & s_2 \\ 20 & -15 \\ -30 & 40 \end{vmatrix} = U$$

Given Q alone, the optimal decision rule is $D_Q^*=\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}$

and the expected payoff would be $T_Q=11$.

For R, the optimal decision rule is $D_R^*=\begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix}$

and the expected payoff would be $T_R=8.6$.

Given both ISs, the optimal decision rule for S is

$$\begin{array}{c} \\ z_1\&w_1 \\ z_1\&w_2 \\ z_2\&w_1 \\ z_2\&w_2 \end{array} \begin{vmatrix} a_1 & a_2 \\ 1 & 0 \\ 0 & 1 \\ 1 & 0 \\ 1 & 0 \end{vmatrix} = D_S^*$$

and the expected payoff is $T_S=11.8$

Note that the marginal value of the information provided by the 'second opinion' (i.e., S) is 0.8 relative to Q, and 3.2 relative to R. However, before we elaborate on the value of orthogonal information, let us discuss two more properties of the orthogonal product.

Property 4: The 'maximum-entropy' matrix is the 'null-element' of the orthogonal product in terms of informativeness (i.e., contribution to the expected payoff). A maximum entropy matrix is a stochastic matrix whose elements are all equal, namely, suppose the matrix' dimensions are $n_E x n_Z$ then every element t_{ij} equals to $1/n_Z$ for all i and j.

For example, assume $T=\begin{vmatrix} .5 & .5 \\ .5 & .5 \end{vmatrix}$

and S=Q@T, where Q is taken from the previous example. S is in fact a flattening of Q:

$$S=\begin{vmatrix} .45 & .45 & .05 & .05 \\ .1 & .1 & .4 & .4 \end{vmatrix}$$

The optimal decision rule $D=\begin{vmatrix} 1 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 1 \end{vmatrix}$

will yield the same expected payoff as attained by Q.

Property 5: The matrices S=Q@R and S_1=R@Q are informatively equivalent.

Proof: It is obvious that the S and S_1 consist of the same column arranged, though, in a different order. It is proven in Marschak[9] that permutations of IS columns do not affect the informativeness of the IS.

Intuitively, the interpretation of Property 5 is clear. Knowing that the columns of the orthogonal matrix have been permuted, the DM has only to permute the rows of the decision matrix accordingly. This property implies that it really does not matter which opinion is considered first and which one is the second as long as two opinions are indeed asked for.

The next section will now inquire into the informativeness of the orthogonal product vis-a-vis its multipliers.

4. INFORMATIVENESS OF THE ORTHOGONAL PRODUCT

Is it always better to acquire a second opinion? A positive answer to that question is not quite intuitive. According to Blackwell Theorem, a garbled IS cannot perform better than the original matrix. An orthogonal product is after all an IS which has been produced from two original stochastic matrices, so maybe it might not perform better than its 'parent' matrices. However, unlike a garbled IS the orthogonal product is not generated through simple algebraic multiplication but rather by a novel operation. We will show now that an orthogonal product is generally more informative than each of the individual multipliers.

Theorem 1: Let Q and R be two ISs operating on a common set of events. Let S be the orthogonal product of Q and R. Then S is generally more informative than Q, and S is generally more informative than R.

Proof: The proof will be handled in a constructive fashion, namely we will build garbling matrices that transform S to Q or to R. This will constitute a sufficient condition to apply Blackwell Theorem to prove the above assertion.

Let $Q=\begin{vmatrix} q_{11}\ldots q_{1n_z} \\ : \\ q_{n_e1}\ldots q_{n_en_z} \end{vmatrix}$ $R=\begin{vmatrix} r_{11}\ldots r_{1n_w} \\ : \\ r_{n_e1}\ldots r_{n_en_w} \end{vmatrix}$

$$S=Q\partial R=\begin{vmatrix} q_{11}r_{11}\ldots q_{11}r_{1n_w}\ldots\ldots q_{1n_z}r_{1n_w} \\ : \\ q_{n_e1}r_{n_e1}\ldots q_{n_e1}r_{n_en_w}\ldots\ldots q_{n_en_z}r_{n_en_w} \end{vmatrix}$$

Let M_1 and M_2 be two Markov matrices each having $n_z x n_w$ rows and n_z or n_w columns respectively, constructed as follows:

$$M_1=\begin{vmatrix} 1 & 0 & . & . & . & 0 \\ 1 & 0 & . & . & . & 0 \\ : \\ 1 & 0 & . & . & . & 0 \\ \\ 0 & 1 & 0 & . & . & 0 \\ 0 & 1 & 0 & . & . & 0 \\ : \\ 0 & 1 & 0 & . & . & 0 \\ : \\ : \\ 0 & . & . & . & 0 & 1 \\ 0 & . & . & . & 0 & 1 \\ : \\ 0 & . & . & . & 0 & 1 \end{vmatrix}\begin{array}{l} \} \\ \}\text{first } n_w \text{ rows} \\ \} \\ \} \\ \\ \} \\ \}\text{second } n_w \text{ rows} \\ \} \\ \} \\ \\ \\ \} \\ \}\text{last } n_w \text{ rows} \\ \} \\ \} \end{array} \qquad M_2=\begin{vmatrix} 1 & 0 & . & . & . & 0 \\ 0 & 1 & 0 & . & . & 0 \\ : \\ 0 & . & . & . & 0 & 1 \\ \\ 1 & 0 & . & . & . & 0 \\ 0 & 1 & 0 & . & . & 0 \\ : \\ 0 & . & . & . & 0 & 1 \\ : \\ : \\ 1 & 0 & . & . & . & 0 \\ 0 & 1 & 0 & . & . & 0 \\ : \\ 0 & . & . & . & 0 & 1 \end{vmatrix}\begin{array}{l} \} \\ \}1^{st}\ n_z \\ \}\quad\text{rows} \\ \} \\ \\ \} \\ \}2_{nd}\ n_z \\ \}\quad\text{rows} \\ \} \\ \\ \\ \} \\ \}\text{last } n_z \\ \}\quad\text{rows} \\ \} \end{array}$$

It is easy to see that $S*M_1=Q$ and $S*M_2=R$. Hence, S is generally more informative than both Q and R.

Since sometimes two ISs can be equivalent in terms of their informativeness (e.g., $\begin{vmatrix} .9 & .1 \\ .2 & .8 \end{vmatrix}$ is equivalent to $\begin{vmatrix} .1 & .9 \\ .8 & .2 \end{vmatrix}$), it is important to note that this is not the case here, namely the relationship 'generally more informative' is a one-way relationship between the orthogonal product and its multipliers (except for some 'irregular' cases presented in the next section). In order to substantiate this proposition it is sufficient to provide a numerical example.

Examine the sample matrices Q and S of the previous section. It is easy to see that the matrix M_3 that solves the set of linear equations $Q*M_3=S$ is not Markovian, hence Q is not generally more informative than S.

We will examine now some immediate results of the above theorem.

5. THE VALUE OF A SECOND OPINION

Some immediate conclusions can derive from Theorem 1. First, it is clear that the orthogonal product of n+1 orthogonal ISs is generally more informative than the product of any n matrices out of them. This may imply that the acquisition of an additional opinion is always worthwhile. However, one has to consider the cost (moneywise as well as timewise) of obtaining the additional information vis-a-vis the marginal expected payoff.

Suppose the cost is not a constraining factor, then how far one should seek for additional opinions? A clear stopping rule is when one manages to obtain a 'complete and perfect' IS, i.e., the unity matrix. Such matrix provides the maximal expected payoff so there is no need for further inquiry.

This can also be displayed in terms of an orthogonal product:

Let $Q=\begin{vmatrix} .9 & .1 \\ .2 & .8 \end{vmatrix}$ $\quad I=\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}$

Their orthogonal product is $S_1=\begin{vmatrix} .9 & 0 & .1 & 0 \\ 0 & .2 & 0 & .8 \end{vmatrix}$

Suppose the optimal decision rule for I was $\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}$.

Obviously, a decision rule in the form of $\begin{vmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \\ 0 & 1 \end{vmatrix}$ will yield the same expected payoff.

The concept of orthogonality can also be associated with distributed systems. When two information systems are completely distributed, then from the point of view of top management seeking integrated data they can be perceived as orthogonal. This might imply that distributed systems are more informative than a centralized one. Again, one should be cautious in making hasty conclusions because distributed systems might degrade data reliability, or the integration of data might be costly. However, relating the concepts of orthogonality and distribution is an intriguing avenue for further research.

Finally, it was made clear in a previous section that the 'maximum entropy' matrix does not improve the expected payoff. Nevertheless, due to the above theorem it is now obvious that this matrix does not worsen the level of informativeness when it orthogonally multiplies a 'regular' IS. It maintains, in fact, the same level of informativeness; this of course is mathematically possible since the rank ordering set by Blackwell Theorem is a non-strict relationship.

The maximum entropy matrix can also be used to demonstrate the notion of 'little improvement'. For instance, the matrix $\begin{vmatrix} .5 & .5 \\ .5 & .5 \end{vmatrix}$ does not add to the informativeness of any existing IS, however if this matrix is slightly modified to look like

$$\begin{vmatrix} .5+\epsilon_1 & .5-\epsilon_1 \\ .5-\epsilon_2 & .5+\epsilon_2 \end{vmatrix}$$

where ϵ_1 and ϵ_2 are small numbers, its orthogonal multiplication with any other IS represents a 'little bit' of added knowledge. In ordinary words, the maximum entropy matrix represents a situation of maximum uncertainty, and any deviation from it will likely constitute an improvement.

The next section discusses possible application of orthogonal systems for cost estimation.

6. APPLYING THE MODEL

The purpose of this concluding section is to raise some ideas on possible application of the orthogonality model. This will be done in the contents of cost estimation problems.

Cost estimate of large (and usually unique, non-repetitive) projects is a severe problem in many various areas such as construction, ship building, aircraft industry, public utility companies, software development, electronics, high-tech industry, etc. Deviations that are three or four times larger than the initial estimate are not very seldom in those areas (see: Kharbanda and Stallworthy[7] for examples of hardware, software, power plants, and aircraft developments; McNichols[12] for examples of subway construction and military systems developments).

There are numerous reasons for the deviations. The prominent ones are: unforeseen exogenous factors (e.g., environmental, political, legal), mismanagement, deliberate deviation in order to get a contract, wrong estimation techniques.

We will show now how the concept of orthogonal IS can be applied to control the estimation process. This will be done by presenting the general notation in parallel to a numerical example.

Let E be a set of events representing the real cost of a project (ex-post !), $E=\{e_1,...e_{nE}\}$. For example, E={$100,000; $200,000; $300,000}, where the figures indicate possible costs of a project. Note that the fineness of event classification is subject to the DM judgment and may be revised. Without loss of generality, assume that the events are arranged such that the associate cost figures are sorted in an ascending order.

Let $p^t=(p_1,...,p_{n2})$ be the a priori probabilities assigned to the events of E. The probabilities may be subjective or based on past experience. For example, $p^t=(1/3, 1/3, 1/3)$.

The decision rule is simply to estimate the cost. It is assumed that the DM would like to tell what will be the real cost. Hence the decision rule is a matrix whose rows correspond to signals provided by an information system (which will be discussed later); the columns correspond to estimates of the costs (events), and the elements indicate the probability that the DM estimates a certain cost value under a given signal. For example:

	ESTIMATE			
	$100K	$200K	$300K	
signal 100,000	d_{11}	d_{12}	d_{13}	
signal 200,000	d_{21}	d_{22}	d_{23}	=D
signal 300,000	d_{31}	d_{32}	d_{33}	

The payoff matrix U displays a cardinal profit function which relates estimates to occurrences of real events (ex-post). It can be assumed that as the deviation increases so is the penalty the company pays. Therefore, the elements of the main diagonal of the matrix will be more in favor of the DM while 'remote' elements will reduce the profit (or increase the loss) monotonically. The following example delineates a matrix reflecting losses due to wrong estimates (note that underestimate and overestimate do not necessarily incur similar losses):

$$U=\begin{vmatrix} 0 & -150,000 & -180,000 \\ -20,000 & 0 & -120,000 \\ -70,000 & -50,000 & 0 \end{vmatrix}$$

The major objective of the DM is to determine the most appropriate decision rule, namely to estimate the cost as accurately as possible when a certain signal emerges from the IS. Note that the DM does not necessarily has to follow the signal; in other words, had the DM not trusted the IS, he or she could place an estimate not concurring with the signal emerging from the system (see Ahituv and Wand[3] for a case like that).

In order to perform a reasonable estimate, the DM employs some teams that ought to provide him or her with sufficient data. In the case of cost estimation, a very common approach is to decompose the project into components to obtain the 'Bill of Material' for the project. This method is labelled 'Bottom-Up' or 'Work Breakdown Structure' (W.B.S.); it was formulated by the U.S. Army in MIL STD 881A[13] (see also Buffa[4]). Once the elementary components have been identified and their cost has been determined, the figures are aggregated 'upward' to obtain the total cost, to which one has to add labor and other direct costs as well.

This method is considered to be relatively accurate, however it consumes much time and labor. Its accuracy deteriorates in R&D projects or in projects where human-power is a major factor (e.g., software development).

An alternative approach is called Parametric Costing (Dumas [6]). This method is based on identifying some crude parameters that constitute a significant statistical correlation with the cost of a project in a certain industry. For instance, the 'magic number' for the cost of a building is $500 per square meter; the cost of manufacturing an airborne radar can be estimated by parameters such as the volume and the weight of the instrument.

A well known method was developed by Large et al.[8] for estimating the cost of manufacturing a new aircraft. Their assessment for the cost of building 100 combat airplanes of the same model is given by a simple formula:

$$C=4.2*W^{.73}*S^{.74}$$

where C is the cost per 100 units, W is the aircraft weight, and S is its maximum speed.

A final example is a system named PRICE [17] which is an adaptive system. The user of the system can calibrate it to fit the organization's particular circumstances. PRICE can handle estimates in a number of areas, such as hardware, electronics, and software.

The problem with information systems for parametric costing is that they are not costless. They provide 'quick and dirty' information, namely faster but probably less accurate, at a certain additional cost. The DM can use them as Decision Support Systems (DSS) either to obtain fast responses or to crosscheck the signal provided by the regular IS. Still remains the question how much should one pay for a 'second opinion system'?

The orthogonal IS model cannot advise us how much to pay, but it can tell what is the worth of a second opinion by figuring out the marginal expected payoff emanating from the orthogonal product of the two system. The DM will then decide whether the price is worthwhile.

Let us turn now to the numerical example to demonstrate this.

Suppose the ISs for the bottom-up and the parametric approaches are the following matrices Q and R respectively:

$$Q=\begin{vmatrix} .9 & .1 & 0 \\ .05 & .9 & .05 \\ 0 & .1 & .9 \end{vmatrix} \qquad R=\begin{vmatrix} .7 & .2 & .1 \\ .1 & .8 & .1 \\ .1 & .2 & .7 \end{vmatrix}$$

The optimal decision rule for both Q and R will be the following matrix:

$$D^{*}=\begin{vmatrix} 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

The expected payoffs for Q and R are -8,000 and -22,000, respectively. In terms of gross expected payoff Q is preferred, however one has to take into account the cost of carrying out a bottom-up analysis and the time it might consume. Nevertheless, let us check now if taking Q or R as a second opinion yields some significant marginal payoff.

Let S be the orthogonal product, S=Q∂R.

$$S=\begin{vmatrix} .63 & .18 & .09 & .07 & .02 & .01 & 0 & 0 & 0 \\ .005 & .04 & .005 & .09 & .72 & .09 & .005 & .04 & .005 \\ 0 & 0 & 0 & .01 & .02 & .07 & .09 & .18 & .63 \end{vmatrix}$$

The optimal decision rule for this IS is:

$$D_{s}^{*}=\begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{vmatrix}$$

The expected payoff is -6,066.66. The marginal gross payoff is 1,933.34 relative to Q, and 15,933.34 relative to R. These values should be compared of course to the cost and time factors associated with obtaining the additional information. Note that intuitively it makes much sense that when R is available, the additional value of a second opinion provided by Q is greater than the other way around, since initially R appears to be less 'exact' than Q.

The last but not least important question is how to calibrate the model for practical use. This should be based on past experience and subjective judgment. However, once the model is programmed and installed on a computer (including the LP routine to solve the optimal decision rule), it can be used not only to assess the value of a second opinion but also to analyze the sensitivity of the solution to various assumptions regarding the model's components, i.e., the ISs, the payoffs, and the a priori probabilities. In fact the programmed model can serve as a Decision Support System for the solution as well as as for the initial assumptions.

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