Personalized Pricing and Quality Design[1]

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Abstract

We develop an analytical framework to investigate the competitive implications of personalized pricing and quality allocation (PPQ), whereby firms charge different prices and offer different qualities to different consumers, based on their willingness to pay. We embed PPQ in a model of spatial differentiation, and show how information about consumer preferences affects multi-product firms' choices over pricing schedules and product line offerings. We show that firms' optimal pricing strategies with PPQ will be non-monotonic in consumer valuations. Our model sheds light on the different product quality schedules offered by firms, given that one or both firms implement PPQ.

Contrary to prior literature on one-to-one marketing, we show that even symmetric firms can avoid the well-known Prisoner's Dilemma problem due to the quality enhancement effect at the individual consumer level. The rent extraction effect due to quality enhancement dominates the adverse effect of price competition. Moreover, this result is stronger when firms have a larger proportion of loyal consumers. When both firms have PPQ, consumer surplus is non-monotonic in valuations such that some low valuation consumers get higher surplus than high valuation consumers.

For a wide range of fixed costs, we also demonstrate some results on the profitability of adopting PPQ and show the emergence of asymmetric equilibria, where one firm adopts PPQ and the other firm does not when the number of loyal customers is less than a critical value. We extend our analysis to asymmetric firms and show that when one firm adopts PPQ, it always increases its quality level while the other firm keeps its quality schedule unchanged compared to when neither firm has PPQ. We demonstrate that a firm with an ex-ante, smaller loyal segment can be better off with PPQ.

Keywords: *Competitive strategy, Personalized marketing, Non-linear pricing, Price discrimination, Quality design.*

1 Introduction

Personalized pricing has often been defined as gauging a shopper's desire, measuring his means, and then charging accordingly. This requires knowledge of each consumer's preferences and an ability to charge different prices to different consumers. The price offered to a consumer whose valuation for a product or service is known may be higher or lower than the posted uniform price charged by firms who lack the sophistication to target individual consumers. Various technologies exist today that allow firms to identify and track individual customers. This leads to the creation of consumer profiles, matching of consumer identities with relevant demographic information, and comparison with the preferences of similar customers through various collaborative and content filtering techniques. Based on such information, firms deploy algorithms to determine prices that approach first degree price discrimination.

There are several examples of personalized pricing. These include major providers of long distance telephone service (such as AT&T, MCI and Sprint), mail order companies like Land's End and L.L. Bean, who have individual specific catalog prices, the online data provider Lexis-Nexis, which "sells to virtually every user at a different price" (Shapiro and Varian 1999), and firms in financial services and banking such as Wells Fargo and MBNA, who engage in individualized pricing through personalized discounts on card fees (Zhang 2003).

Quality has often been defined as a broad notion that encompasses any feature that may affect a consumer's willingness to pay for a good. This could include features intrinsic to the product itself (such as durability, functionality or configuration) or those related to the quality of the shopping experience, or the service level provided by the firm (such as warranties, return policies, delivery schedule and customer service). In the context of customer service, when a firm renders a personalized service to each customer based on his profile, that is an example of a personalized quality design. It is quite common in the financial services industry to provide a differentiated service to customers based on their net worth, which is a good proxy for willingness to pay. For example, when a call comes into a call-center, the customer's profile pops up on the service representative's screen and the call is addressed accordingly. Retailers like Lands End and L.L. Bean are also well known for using such relationship management technologies for delivering personalized customer service. This is increasingly becoming common in the hotel industry wherein hotels personalize the frills provided to customers based on their profiles (Bailor 2005). The market for computer servers, storage devices and workstations combines personalized pricing and targeted quality allocation. Major players such as IBM, Hewlett Packard, and Sun Microsystems use personalized discounting for different customers. PC vendors like Dell offer computers of varying configurations to customers, which differ in their speed and performance due to the presence of different processors and memory modules. Consumers can either select a brand with a particular configuration themselves, or a firm's sales representative can recommend a specific product based on their interactions with consumers. Similarly, it is quite common for consumers to choose extended warranties or delivery options from a menu of choices, either by themselves or based on a specific recommendations by sales representatives. In the enterprise software applications market, there is also a trend towards customizing the product to suit clients' needs as well as offer a personalized level of service quality through the use of one-to-one repair schedules and uptime guarantees.

Many firms believe that the concept of making the right offer (price and quality) to the right customer would be the way of the future. Hence, they are investing in technologies and processes which enable the use of consumer information to tailor prices and services. In this paper, we use the term personalized pricing and quality, or PPQ, to refer to the case in which a firm can implement a pricing policy and offer a quality schedule based on complete knowledge of the willingness to pay of each consumer. Since, the amount of information required for implementing PPQ is high, in practice firms may not know valuations precisely. Hence, our results should be interpreted as the solution to an important limiting case which provides a useful benchmark – the case of perfect information. Hence, we ignore the possibility of mistargeting, which results, for example, when a firm mistakenly perceives some price-sensitive customers as price-insensitive and charges them high prices.[2] We examine the following questions:
(i) How does the presence of technologies which facilitate PPQ, affect equilibrium price and quality schedules? (ii) when do firms competing on the quality of value-added services benefit from personalized pricing and quality design, and how does this depend on firm size? (iii) what are the incentives for competing firms to adopt such technologies, and (iv) how is consumer surplus and overall social welfare affected by the adoption of PPQ technologies?

1.1 Prior Literature

A number of recent papers (Shaffer and Zhang 1995, Bester and Petrakis 1996, Chen 1997, Fudenberg and Tirole 2000), have shown that when firms offer one-to-one promotions or other forms of customized pricing, it generally leads to a Prisoner's Dilemma which leaves all firms worse-off

[2] Chen, Narasimhan & Zhang (2001), have shown that mistargeting can have an important effect. It softens price competition in the market, and qualitatively changes the incentives for competing firms engaged in individual marketing. Liu and Serfes (2004) also consider imperfect information in a spatial price discrimination model and find that when the quality of information is low, firms unilaterally commit not to price discriminate.

compared to the scenario when they do not offer customized pricing. These papers are based on ex-ante symmetric firms. Corts (1998) and Shaffer and Zhang (2000) find that targeted promotions need not necessarily lead to a Prisoner's Dilemma. However, they allow for at most one promotional price by symmetric firms, and their result accrues due to an alleviation of price competition. A closely related paper is that by Shaffer and Zhang (2002) who consider perfect price discrimination by competing firms in a model that includes both horizontal and vertical differentiation, with a positive cost of targeting customers. They are the first to show that a Prisoner's Dilemma can be avoided with one-to-one promotions but only with asymmetric firms (when firms are dissimilar in market size, ex-ante).

We show that even *symmetric firms* are better off when they engage in one-to-one pricing and product allocations, and can thus avoid the Prisoner's Dilemma. In our model, this result arises because of the "quality enhancement" effect from offering a continuum of qualities in the market. With PPQ, firms can provide higher qualities to each consumer without the fear of intra-firm product cannibalization which occurs in situations with self-selection. This occurs because PPQ enables a firm to allocate a pair of price and quality to each individual consumer. This kind of targeting leads to a higher rent extraction ability for each firm. This effect offsets the price competition effect and makes it profitable for symmetric firms to engage in PPQ. Moreover, in contrast to Shaffer and Zhang (2002) we show that when firms are asymmetric in size, even the smaller firm can gain when both firms adopt PPQ.

Recent work on customer recognition and behavior-based price discrimination includes Villas-Boas (1999, 2004), Feinberg, Krishna and Zhang (2002) and Acquisti and Varian (2005). Much of the recent work on perfect price discrimination has been done either in the context of horizontal product differentiation (Thisse and Vives 1988, Shaffer and Zhang 1995, Chen and Iyer 2002, Bhaskar and To 2004, Liu and Serfes 2004). In the context of channel management, Liu and Zhang (2005) analyze the benefit of personalized pricing for a retailer. Our paper is also related to the work of Choudhary et al., (2005) who look at the impact of personalized pricing in a vertically differentiated duopoly setting with each firm offering a single quality. Our model is different since we incorporate a continuum of qualities and prices, and firms are able to customize both prices and qualities. Moreover, we also explicitly analyze the incentives that firms have for adopting PPQ, when adopting PPQ entails some fixed costs.

Our work is also related to the emerging stream of research on product customization. Dellaert and Syam (2002) bring into focus the issues surrounding mass-customization via an analysis of consumer-producer interaction. In Dewan et al.(2003), both firms make symmetric investments in product customization technology which leaves them worse-off in a simultaneous mode game. In a duopoly model of horizontal differentiation, Bernhardt et al. (2005) show that despite an increase in efficiency, firms do not make symmetric investments in product customization technology. Syam et al., (2005) show that firms find it profitable to customize only one of a product's two attributes and each firm chooses the same attribute. Syam and Kumar (2005) examine firms' incentives to offer customized products when the prices of all customized offerings are the same and the degree of customization is endogenously determined. An interesting result from their paper is that customization helps firms increase the prices of the standard products as well and thus firms can increase profits by offering both standard and customized products. They also find conditions under which ex-ante symmetric firms will adopt asymmetric strategies.

A common theme in the customization literature is that firms can customize their product to eliminate product differentiation, which leads to fierce price competition. Further these papers also differ based on whether firms customize prices or not. Our work is different from all of these papers because firms in our model do not make decisions between offering standardized vs. customized products. They always produce the same number of products, i.e. the length of the product line is fixed. What changes with PPQ technology is the level of quality offered to each consumer, and the corresponding price charged. Basically, firms can choose to decide whether they allow consumers to self-select from the (price, quality) menu or whether they proactively target each consumer with a (price, quality) schedule. This ensures that even though firms know individual customer types, there still exists sufficient product differentiation. More importantly, unlike prior work, our paper combines both personalized pricing and one-to-one quality allocation in the same theoretical framework.[3]

1.2 Overview of Results

We highlight a number of findings. First, in a duopoly setting, we characterize firms' optimal price and quality schedules, as well as consumer surplus and social welfare, when, neither firm, one firm or both firms have PPQ. Second, in contrast to prior work, we show that quality enhancement through the allocation of a targeted quality schedule to each consumer leads to less aggravated price competition by strengthening the opportunities for rent extraction. Thus, the adoption of

[3] A simple example of this set up is as follows: Consider a consumer who wants to buy a computer from her favorite brand (Dell or Apple). She can select the desired configuration of her computer (memory modules, processor speed, graphics cards, etc) and pay the corresponding price from a catalog or from the Internet. Alternatively, one can have a scenario where the consumer is not exactly aware of the precise configuration that she wants for her computer. She walks into a physical store, and talks to a sales representative. The sales representative based on the customer interaction process and other information sources (such as purchase history of the customer) recommends a specific configuration. This would be an example of targeted quality allocation.

PPQ technologies by competing firms can make even symmetric (or identical) firms better-off. Even after explicitly accounting for the costs of PPQ, we find regions where symmetric firms are better-off after adopting PPQ. This has important managerial implications for firms which practice one-to-one marketing and are considering making investments in CRM technologies such as those sold by Siebel, Terradata etc.

Third, we show that the adoption of PPQ by both firms has a differential impact on average consumer surplus as well as on the surplus accruing to a consumer at a given location. While the adoption of PPQ results in a lower average consumer surplus, interestingly, we find that some consumers are actually better off when both firms adopt PPQ. In particular, consumers located closer to the middle of the market–who are the least loyal to either firm or are the least likely to buy from either firm, are the ones who are the most better-off (in terms of their surplus) when both firms adopt PPQ technologies. Intutively, in the absence of PPQ, it's important for firms to leave some information rents for their most loyal (higher valuation) consumers so that it can prevent cannibalization wherein the higher type consumers buy lower quality products. This leads to positive surplus for the higher valuation consumers. However, with PPQ there is no potential for such cannibalization and as a result, firms do not need to leave any information rents for consumers. Consequently, these loyal consumers are left with no surplus.

Fourth, we consider asymmetric firms (in market size) and show that, compared to the No-PPQ scenario, when one firm adopts PPQ, it always increases its quality level while the other firm keeps its quality schedule unchanged. Conversely, compared to the Both-PPQ scenario, when a firm drops PPQ, it always decreases its quality schedule while the other firm keeps its quality schedule unchanged.

For a wide range of cost parameters, we further demonstrate some results on the profitability of adopting PPQ. An interesting result is the emergence of an asymmetric equilibrium: situation where one firm adopts PPQ and the other firm does not despite both firms being symmetric in the size of their loyal segments. This occurs because in some cases, once a firm adopts PPQ, its rival's benefit from adopting it does not outweigh its costs. We also find that starting from asymmetric firms (in the size of their loyal customer segments) when firms progressively become symmetric, the adoption of PPQ technologies increasingly becomes beneficial to both firms. The rent extraction effect due to quality enhancement dominates the adverse effect of price competition and this result is stronger when firms have a larger proportion of loyal consumers.

The rest of the paper is as follows. Section 2 describes the model in detail. Section 3 presents a preliminary result that acts as a benchmark for comparative statics. We then proceed to Section

4 and 5 to analyze the equilibrium when one or both firms have PPQ, respectively. Section 6 demonstrates the impact of PPQ on asymmetric firms. In Section 7, we provide some interesting observations with the help of numerical analysis. Managerial implications of our findings are presented in Section 8. All proofs are relegated to the Appendix.

2 Model

We consider personalized pricing and quality design in a duopoly model.[4] Two multi-product firms compete in both the quality and price of the products they offer. Each firm's product line consists of a continuum of qualities, as in prior literature (Mussa and Rosen 1978). In this framework, a firm's focus is on the choice of price as a function of quality rather than the choice of quality levels itself. This is because the implicit assumption in such models is that a firm's product line length is fixed: all possible quality levels are produced by firms.

When neither firm has access to PPQ, prices are chosen simultaneously by both firms. When only one firm has access to PPQ, the firm without PPQ chooses its price first. After observing this firm, the firm with access to PPQ sets a menu of prices. This setting is widely adopted in the literature (see for example, Thisse and Vives 1988, Choudhary et al. 2005, Liu and Zhang 2006) for two reasons. First, a simultaneous choice of pricing in this asymmetric game does not lead to a pure strategy Nash equilibrium. Second, in practice firms with PPQ can offer discounts to each consumer given opponents' uniform prices for standard (non-customized) products. Personalized pricing is executed for each consumer at the point of sale. Hence, a firm which engages in PPQ chooses its price after a rival that has a uniform pricing policy (which must be posted and committed to before sales occur). In other words, the flexibility implied by personalized pricing incorporates an implicit assumption on flexibility in timing as well. When both firms have PPQ, the order of moves at stage 2 does not affect the outcome; we again posit that prices are chosen simultaneously. Once prices are chosen, at the last stage of the game (stage 3), consumers decide which, if any, product to buy.[5]

Two firms located at the two ends of a straight line from 0 to 1, offer a continuum of products differentiated in quality. The firm located at the left is denoted as firm L while that located on the

[4] We present an analysis of the monopoly model with or without PPQ in the supplementary technical Appendix B. Note that the No-PPQ results in a monopoly are very similar to those with competition. This highlights the "local monopolist" nature of each firm.

[5] Note that with PPQ each consumer receives a single (price, quality) offering from the firm in accordance with their types. Hence, it's not critical for consumers to observe the menu before purchase in scenarios with PPQ. In contrast, when a firm does not have PPQ, a consumer can choose any pair from a menu of prices and qualities. In this case consumers do need to observe the menu of prices and qualities. This is feasible and common in practice too. For example, using sources on the internet, or catalogues consumers can observe the different prices firms charge for different possible configurations of a computer by changing the kind of processors, memory modules, and so on.

right is denoted as firm R. Consumer types are denoted by the parameter θ where $\theta \in [0,1]$ with a uniform distribution. A consumer has positive utility for one unit only. The type parameter θ indicates a consumer's marginal valuation for quality. If either of the two products offers a positive net utility, the consumer buys the one that maximizes their surplus. Otherwise, they choose not to buy any product. The utility to a consumer with type θ buying from the firm located at 0, firm L, is

$$u^L(q,\theta) = q \times (1-\theta),$$

while his utility in buying from the firm located at 1, firm R, is

$$u^R(q,\theta) = q \times \theta.$$

Thus, for a given consumer, θ is analogous to a "transportation cost" of buying from firm L and $1-\theta$ is analogous to a "transportation cost" of buying from firm R (Spulber 1989, Stole 1995).[6] Thus, the intensity of a consumer's preference for a firm is inversely proportional to the the distance between a consumer and the firm; the consumer located at 0 values firm L the most while the consumer located at 1 values R the most. Without PPQ, firms are unable to observe each consumer's most preferred product. However, they know the distribution of consumer preferences.

In the case of PPQ, we allow one or both firms to be equipped with a technology that perfectly reveals the consumer's type before a given price and a given quality is offered to the consumer. Both firms know which firm has PPQ before the game is played. In practice, implementing PPQ may well require some fixed costs. However, if such costs are independent of the quality of the product being offered by the firm, they do not affect the qualitative nature of the results. For simplicity, we treat these costs as zero.[7] We consider pure strategy Nash equilibria of this game.

Consistent with the prior literature, we assume that firms have a marginal cost of production which is invariant with the quantity, but depends on the quality of the product (Moorthy 1988). That is, both firms have the same cost function, but depending on the quality schedules they choose, their marginal costs may differ in equilibrium. Each firm has a constant marginal cost for producing

[6] This is a very common setup in the non-linear pricing literature and is quite intuitive. The term, θq or $(1-\theta)q$, can be regarded as the quality weighted transportation costs that is common in models with a horizontally differentiated market. This setup matches the scenario where two firms sell branded products and have groups of loyal customers. Typical examples are fashion industry (apparel, jewelry, computers, shoes, luxury cars...etc.). In these industries, brand preferences and product quality are often fused together in consumer's willingness-to-pay (WTP). Basically a straightforward interpretation of our model is that there exist customers who have very high marginal WTP for quality for the products of one firm but not for products of the other firm. Customers who like Microsoft's products may not like Apple's products because their inherent preferences for these brand are very different. Moreover, this setup also captures the fact that customers who do not have loyalty towards any particular brand, have a low marginal willingness-to-pay (WTP) for both brands.

[7] In Section 7, we provide guidelines as to when firms should or should not invest in PPQ if the fixed costs of investing in PPQ are non-zero.

the good, denoted by c. Further $c(\cdot)$ is twice differentiable, strictly increasing and strictly convex in q. That is, $c'(q) > 0$ and $c''(q) > 0$. For analytical tractability in solving the general model and to highlight the impact of the cost function on different decision variables, we use the following function: $c(q) = q^\alpha/\alpha$. This function satisfies all of the above properties for $\alpha > 1$.

3 Neither firm has PPQ

Consider the benchmark case when neither firm has access to PPQ (we call this the No-PPQ case). Basically each firm offers a menu of prices, $p(q)$, for all consumer types θ. The decision variable $p(q)$ of the firm can be equivalently written as $q(\theta)$ and $p(\theta)$ since each consumer will self-select the contract designed for his type in equilibrium.[8]

As shown by Spulber (1989), in equilibrium, each firm occupies half of the market. Basically, the equilibrium pricing menu is similar to that of Mussa and Rosen (1978) since both firms compete by lowering the price by a constant while keeping the quality schedule at the same level. We use superscripts to denote the variables of firm R or L. Let π_N^L and π_N^R denote the profit of firm L and firm R, respectively in the No-PPQ case.

The objective function of firm R is given by

$$\max_{p^R(\theta),\, q^R(\theta)} \pi_N^R, \text{ where } \pi_N^R = \int_B^1 \left[p^R(\theta) - \frac{(q^R)^\alpha(\theta)}{\alpha} \right] d\theta, \tag{1}$$

subject to the following constraints:

- **(IC):** Each consumer of type θ chooses the $q^R(\theta)$ and $p^R(\theta)$ that the seller designed for him. $\theta = \arg\max_t \theta \times q^R(t) - p^R(t), \ \ \forall \theta \in [0, 1]$.

- **(IR1):** Each consumer of type θ receives a utility level that is higher than 0. $s^R(\theta) \geq 0$.

- **(IR2):** The marginal consumer B gets the same surplus from each firm and hence, is indifferent between buying from firm R and firm L. That is, $s^R(B) = s^L(B)$.

Intuitively, the IC constraint ensures that a consumer prefers the contract that was designed for him, and the IR constraint guarantees that each consumer accepts his designated contract. Firms set a quality schedule $q(\theta)$ and compete for the marginal consumer by offering prices that

[8] Rather than considering all possible pricing functions, the revelation principle ensures that the firm can restrict its attention to direct mechanisms—that is, contracts in which one specific quality-price pair is designed for each consumer, and in which it is rational and optimal for the consumer to choose the price and quality pair that was designed for him or her. This type of transformation is standard in models of price screening (see, for instance, Armstrong 1996).

progressively get lower as one moves towards the middle of the market. The lowest price is offered to the customer at $\theta = 0.5$.

Given the above utility function, the net surplus of each consumer following a standard transformation from the non-linear pricing literature (Armstrong 1996) is given by $s(\theta) \equiv u(q(\theta), \theta) - p(\theta)$. Following the approach in the nonlinear pricing literature (Mussa and Rosen 1978, Maskin and Riley 1984, Sundararajan 2004), we substitute the pricing schedule, $p^R(\theta)$, by the consumer surplus function, $s^R(\theta)$ given that $p^R(\theta) = u^R(q^R(\theta), \theta) - s^R(\theta)$. Thus, we consider $q^R(\theta)$ and $s^R(\theta)$ as the decision variables. Recall that each firm offers a continuous menu of prices and qualities. Since consumers choose any contract $(p(\theta), q(\theta))$ from the menu, the incentive compatibility (IC) condition for consumers is given by

$$s^R(\theta) = \max_t \ \theta \times q^R(t) - p^R(t). \tag{2}$$

From the first order condition of (2) and using the envelope theorem, we have the following Lemma.

Lemma 1 $\frac{ds^R(\theta)}{d\theta} = q^R(\theta)$ *and* $\frac{ds^L(\theta)}{d\theta} = -q^L(\theta)$.

The proof of this Lemma and all other results is relegated to the Appendix. This Lemma implies that

$$s^R(\theta) = s^R(B) + \int_B^\theta q^R(t)dt, \tag{3}$$

$$s^L(\theta) = s^L(B) + \int_\theta^B q^L(t)dt. \tag{4}$$

It follows that due to the presence of the incentive compatibility constraint, the slope of the surplus function offered by firm R, $s^R(\theta)$ is determined by its quality schedule, $q^R(\theta)$. In this model, note that competition between these two firms only affects the surplus offered to the consumer at the boundary given by $s^R(B)$. Basically, this implies that these two firms compete by lowering the pricing schedule by a constant, $s^R(B)$. Given the continuous product lines (where there is quality level available for every possible consumer type θ), there is a fear of cannibalization because some high valuation consumers might end up buying the lower quality product. Consequently, firms need to leave some information rents for the high valuation consumers (consumers located closer to 0 or 1) in order to prevent them from buying lower quality products. Basically without PPQ, firms have to "reward" their loyal customers to prevent them from buying lower quality products. As a result, the firm's decision variables can be further simplified into $q(\theta)$ and $s(B)$, where $s(B)$ is the surplus of the marginal consumer who is indifferent between buying from either of the two firms.

Based on equation (1), the simplified objective function for firm R can be rewritten as

$$\max_{q^R(\theta),\, s^R(\theta)} \pi_N^R, \text{ where } \pi_N^R = \int_B^1 \left[\theta q^R(\theta) - s^R(\theta) - \frac{(q^R)^\alpha(\theta)}{\alpha}\right] d\theta, \tag{5}$$

Similarly, the optimization problem for firm L can be derived as follows:

$$\max_{q^L(\theta),\, s^L(\theta)} \pi_N^L, \text{ where } \pi_N^L = \int_0^B \left[(1-\theta) q^L(\theta) - s^L(\theta) - \frac{(q^L)^\alpha(\theta)}{\alpha}\right] d\theta. \tag{6}$$

Both (5) and (6) are subject to the same constraints as before. The detailed derivations are provided in the Appendix. This leads to our first result.

Proposition 1 *The optimal prices, quality schedules and surplus functions of the No-PPQ case are as follows:*

$$\begin{aligned}
q^L(\theta) &= (1-2\theta)^{1/(\alpha-1)}, \theta \in [0, 1/2], \\
q^R(\theta) &= (2\theta-1)^{1/(\alpha-1)}, \theta \in [1/2, 1], \\
s^L(\theta) &= \frac{\alpha-1}{2\alpha}(1-2\theta)^{\alpha/(\alpha-1)}, \theta \in [0, 1/2], \\
s^R(\theta) &= \frac{\alpha-1}{2\alpha}(2\theta-1)^{\alpha/(\alpha-1)}, \theta \in [1/2, 1], \\
p^L(\theta) &= (1-2\theta)^{1/(\alpha-1)}\left(\frac{-2\theta+\alpha+1}{2\alpha}\right), \ \theta \in [0, 1/2], \\
p^R(\theta) &= (2\theta-1)^{1/(\alpha-1)}\left(\frac{2\theta+\alpha-1}{2\alpha}\right), \ \theta \in [1/2, 1].
\end{aligned}$$

Since each firm covers half the market, the indifferent customer is located at $\theta = 0.5$. Note that the total surplus generated by firm R is $\theta q^R(\theta) - \frac{(q^R)^\alpha(\theta)}{\alpha}$. This implies that the socially optimal quality level (first-best solution) is given by $q^R(\theta) = \theta^{(\alpha-1)}$. By comparing this quality level with the optimal quality schedule actually offered by the firm, we find that the quality received by each consumer is lower than the socially optimal level (except for the highest type whose $\theta = 1$). This degradation of quality happens because of the potential for cannibalization. Basically, due to the nature of the self-selection problem, higher the offered quality by the firm to a consumer, more is the information rent needed to be given to higher valuation consumers in order to prevent them from deviating to buy its lower quality products. This causes the firm to distort the quality of the product offered to each consumer.

4 Only One Firm Has PPQ

Next, we analyze a situation in which only one firm has access to technologies which facilitate PPQ. Without loss of generality, we assume that among these two firms, only firm R has PPQ. To solve this game, we analyze a setting in which each firm makes its pricing decision sequentially rather than simultaneously. At stage 1, firm L (the firm without PPQ) announces its menu and allows consumers to self-select a particular quality (and price) from its product line. At stage 2, firm R (the firm with PPQ) targets every consumer with a specific quality (and price) in accordance with their type. In the final stage, consumers choose which firm to buy from and demand is realized. The solution concept of this section is subgame perfect Nash equilibrium.

Given any strategy of firm L, in equilibrium, firm R will offer the socially optimal level of quality to maximize its profit because it can perfectly target consumers to avoid cannibalization. Generally, whenever one firm acquires PPQ, it does not need to consider the cannibalization problem since consumers can now be allocated the price and quality pair exactly in accordance with their valuation. Let π_R^L and π_R^R denote the profit of firm L and firm R, respectively in the this case. Formally, the maximization problem of firm R can be written as

$$\max_{q^R(\theta),\ s^R(\theta)} \pi_R^R(\theta), \text{ where } \pi_R^R(\theta) = \ \theta q^R(\theta) - s^R(\theta) - \frac{\left(q^R\right)^\alpha(\theta)}{\alpha}, \forall\theta \in [0,1]. \tag{7}$$

Firm R sets the price, or equivalently, sets the surplus function $s^R(\theta)$, such that each consumer's surplus exactly matches his/her surplus from the outside opportunity, which is either equal to zero or equal to the surplus from buying from firm L. Given R's strategy described above, L's optimization problem is the same as that in the No-PPQ case given by equation (6) except that (IR2), is replaced by the socially optimal surplus curve of firm R given as follows:

$$s^L(B) = \max_{q^R(\theta)} \left[\theta q^R(\theta) - \frac{\left(q^R\right)^\alpha(\theta)}{\alpha}\right] |_{\theta=B}. \tag{8}$$

If firm L were to offer less than the socially optimal surplus of firm R, then firm R could potentially poach L's consumers by offering lower prices and by adjusting quality. The potential for poaching exists since firm R can perfectly identify each consumer, and in particular, it can lower its price to marginal cost for the consumer at the boundary. Thus, firm L can retain the marginal consumer at B (that is, maintain its market share) only if its surplus $s^L(B)$ equals the socially optimal surplus offered by firm R. This leads to the following proposition.

Proposition 2 *In the case when only one firm has PPQ, the optimal prices, quality schedules and* *surplus functions are as follows:*

$$
\begin{aligned}
q^L(\theta) &= (1-2\theta)^{1/(\alpha-1)},\ \theta \in [0,B],\\
q^R(\theta) &= \theta^{1/(\alpha-1)},\ \theta \in [B,1],\\
s^L(B) &= (1-\frac{1}{\alpha})(1-2B)^{\alpha/(\alpha-1)} - B^{\alpha/(\alpha-1)},\\
s^L(\theta) &= s^L(B) - \frac{\alpha-1}{2\alpha}(1-2B)^{\alpha/(\alpha-1)} + \frac{\alpha-1}{2\alpha}(1-2\theta)^{\alpha/(\alpha-1)},\ \theta \in [0,B],\\
s^R(\theta) &= \max\ [0,(1-\theta)q^L(B) - p^L(B)],\ \theta \in [B,1]\\
p^L(\theta) &= (1-\theta)(1-2\theta)^{1/(\alpha-1)} - s^L(\theta),\ \theta \in [0,B],\\
p^R(\theta) &= \theta^{\alpha/(\alpha-1)} - s^R(\theta),\ \theta \in [B,1].
\end{aligned}
$$

The marginal consumer's type is given by $B = \left[(\frac{2\alpha-1}{\alpha-1})^{\frac{\alpha-1}{\alpha}} + 2\right]^{-1}$. For the quadratic cost function case this turns out to be $B = 0.27$. Although a general expression of $s(B)$ and prices are analytically tractable, the math is not easily parsable and so we do not present it in the main body of the paper. However, we do derive several interesting results in the latter sections.

5 Both Firms Have PPQ

In this case, both firms have complete knowledge of each consumer's type and are able to implement PPQ. We term this the Both-PPQ case and derive the Nash equilibrium of this game. Since both firms have full information about consumer preferences for price and quality, they engage in a Bertrand-type price competition. Consequently, in equilibrium both firms offer a socially optimal level of quality. A firm located closer to a given consumer will set a price schedule such that it can exactly match the consumer surplus offered by its rival. The firms' profit functions are given by

$$\max_{q^L(\theta),\ s^L(\theta)} \pi^L_{Both}(\theta),\ \text{where } \pi^L_{Both}(\theta) = (1-\theta)q^L(\theta) - s^L(\theta) - \frac{(q^L)^\alpha(\theta)}{\alpha}, \tag{9}$$

$$\max_{q^R(\theta),\ s^R(\theta)} \pi^R_{Both}(\theta),\ \text{where } \pi^R_{Both}(\theta) = \ \theta q^R(\theta) - s^R(\theta) - \frac{(q^R)^\alpha(\theta)}{\alpha}. \tag{10}$$

where $s^L(\theta)$ and $s^R(\theta)$ are equal to the socially optimal surplus offered by the rival firm. Formally,

$$s^R(\theta) = \max_{q^R(\theta)} \left[(1-\theta)q^R(\theta) - \frac{\left(q^R\right)^\alpha(\theta)}{\alpha}\right] = (1-\frac{1}{\alpha})(1-\theta)^{\alpha/(\alpha-1)}, \quad \theta \in [1/2,1], \tag{11}$$

$$s^L(\theta) = \max_{q^L(\theta)} \left[\theta q - \frac{\left(q^L\right)^\alpha(\theta)}{\alpha}\right] = (1-\frac{1}{\alpha})\theta^{\alpha/(\alpha-1)}, \quad \theta \in [0,1/2]. \tag{12}$$

Note that firm R offers a surplus which is equal to the socially optimal surplus of firm L. If R's surplus is less than the socially optimal surplus offered by L, L would be able to poach on R's consumers by increasing quality or decreasing price. If R's surplus is more than that of L, it is not maximizing its profit. Hence, it is optimal for firm R to increase its price to the profit maximizing level.

Given the kind of price competition that will ensue between the two firms, we can determine the surplus functions $s^L(\theta)$, $s^R(\theta)$, and hence point out the optimal price schedules. All consumers whose $\theta \in [1/2, 1]$, buy from firm R in equilibrium. Similarly, all consumers whose $\theta \in [0, 1/2]$, buy from firm L in equilibrium. Basically, the equilibrium price from firm R (or from firm L) is set so that consumers feel indifferent between buying from firm R and from firm L. The equilibrium price offered by each firm to its rivals' consumers is set to marginal cost due to Bertrand price competition. This leads to the following result.

Proposition 3 *The optimal prices, quality schedules and surplus functions when both firms have PPQ are as follows:*

$$
\begin{aligned}
q^L(\theta) &= (1-\theta)^{1/(\alpha-1)},\ \theta \in [0, 1/2], \\
q^R(\theta) &= \theta^{1/(\alpha-1)},\ \theta \in [1/2, 1], \\
s^L(\theta) &= (1-\frac{1}{\alpha})\theta^{\alpha/(\alpha-1)},\ \theta \in [0, 1/2], \\
s^R(\theta) &= (1-\frac{1}{\alpha})(1-\theta)^{\alpha/(\alpha-1)},\ \theta \in [1/2, 1], \\
p^L(\theta) &= (1-\theta)^{\alpha/(\alpha-1)} - (1-\frac{1}{\alpha})\theta^{\alpha/(\alpha-1)},\ \theta \in [0, 1/2], \\
p^R(\theta) &= \theta^{\alpha/(\alpha-1)} - (1-\frac{1}{\alpha})(1-\theta)^{\alpha/(\alpha-1)},\ \theta \in [1/2, 1].
\end{aligned}
$$

Note again that since both firms are symmetrically equipped with PPQ, they share one-half of the whole market, similar to the No–PPQ case. In equilibrium, both firms offer a socially optimal level of quality. Further, note that compared to the No-PPQ case the adoption of PPQ actually decreases the quality difference *between* the products of a firm. However, since qualities and prices are now targeted (with PPQ), firms do not need to degrade qualities. Intuitively this occurs because from a firm's perspective, there is no fear of cannibalization in this case. Recall that since firms can allocate qualities by targeting consumers directly with PPQ, there are no consumer self-selection problems. As such there is no competition *between the* products of a given firm. Consequently, firms do not have any incentive to degrade qualities offered to their customers. Thus, they provide their loyal customers products with better quality which results in higher prices as well. This leads to higher profits than the No-PPQ case. On the other hand, despite offering their competitor's loyal customers with higher qualities and lower prices (both firms' prices fall to marginal cost in their respective rival's turfs) they are unable to poach on their competitor's territory.[9] A comparison of quality schedules offered reveals that when a firm adopts PPQ, it increases the quality offered to each consumer. However, the firm without PPQ keeps its quality schedule unchanged. When both firms adopt PPQ, their qualities are always higher than the No–PPQ qualities. This enables them to offer a higher quality than in the No-PPQ case and charge higher prices. We discuss these results in detail in the following sections.

5.1 Prices

We plot the price curves for quadratic and cubic cost functions in Figures 1 and 2 for each of the three cases: (i) neither firm has PPQ, (ii) one firm (firm R, without loss of generality) has PPQ and (iii) both firms have PPQ. Interestingly, note that when $\alpha = 2$, the price functions are convex, while when $\alpha = 3$, the price functions are concave.[10] The thick continuous U-shaped curves indicate the price function when both firms have PPQ or when neither firm has PPQ. It is immediate to see that firm prices are always higher in the Both–PPQ case. The dotted discontinuous curve represents the price function for the case when only one firm (firm R) has PPQ. Note that when only firm R has PPQ, firm L offers a higher price compared to the No-PPQ case but lower than the Both-PPQ case. On the other hand, firm R's price is higher than its price in the No-PPQ and the Both-PPQ cases. This leads to the following corollary.

Corollary 1 *Suppose the cost function is quadratic ($\alpha = 2$). (i) Then, the adoption of PPQ by both firms leads to higher prices for all consumers compared to the No-PPQ case. (ii) When only one firm adopts PPQ, the firm without PPQ increases its price to all its consumers, compared to the No-PPQ case. However, some potential consumers of the firm without PPQ, buy from the PPQ firm at lower prices than in the No-PPQ case.*

[9] The implicit notion here is that consumers buy from the firm offering a higher quality product even if the surplus offered by both firms is exactly the same.

[10] The intuition behind this comes from the fact that a price charged to a consumer is determined by two effects: (i) that of the offered quality (quality effect) and, (ii) that of the information rent left for the consumers. These two forces have countervailing effects and thus the net shape of the pricing function depends on which of the two forces dominate. Moreover, as α increases it becomes relatively more costly to offer higher quality products. Hence, quality scheudles become more concave, and the pricing function also becomes more concave.

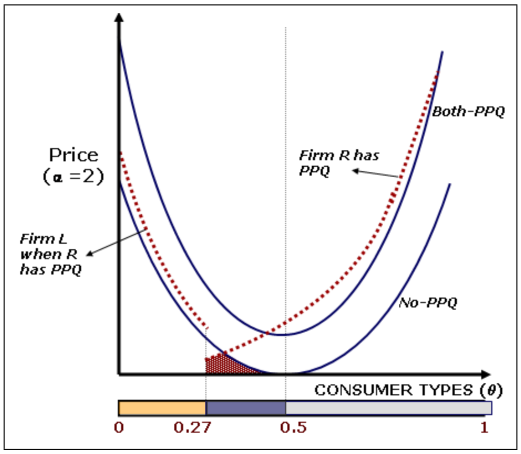

Figure 1: Price with or without PPQ

(α=2)

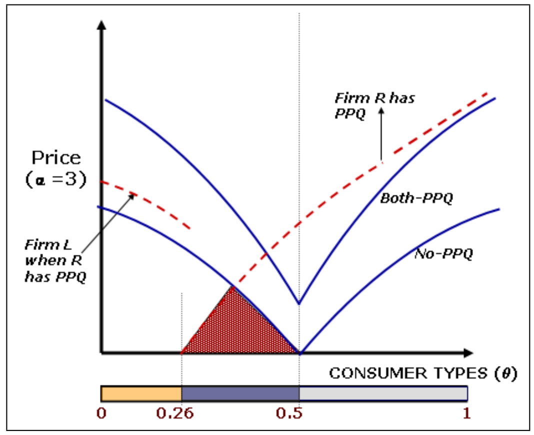

Figure 2: Price with or without PPQ

(α=3)

Consider the case when $\alpha = 2$. When R is the only firm that offers PPQ, its market coverage extends across the region where $\theta \in [0.27, 1]$ while firm L covers the market where $\theta \in [0, 0.27]$. Notice that when firm R has PPQ, firm L's price is always higher than its No-PPQ price. However, firm R's price is lower than firm L's No-PPQ price in the region of $\theta \in [0.27, 0.38]$. Thus, consumers in this region get a lower price. Essentially the intuition is as follows: Since firm R (the firm with PPQ) knows the preferences of each consumer, it has the flexibility to target some of its rival's consumers. Firm L (the firm without PPQ) knows that firm R can offer a lower quality and lower price at the margin, and thus lure away some of its own consumers, especially those with relatively weaker preferences for its products (customers whose type $\theta \in$ [0.27, 0.5], given by the triangular shaded regions in each figure). Although firm L can respond strategically by lowering its price to prevent this poaching, it is less profitable for firm L to do so, and hence it does not find it optimal to sell to all of its own potential consumers by lowering its price. On the contrary, by increasing its price it is able to extract a higher surplus from its loyal customers (customers whose type $\theta \in$ [0, 0.27]) who have a stronger preference for its products. This results in higher overall profits than those accruing from undercutting firm R and engaging in a head-head competition for some less profitable customers. Consequently, firm L offers a higher price compared to the No-PPQ case. Thus, the adoption of PPQ by one firm alleviates the price competition between firms and raises the average prices. Indeed as we see from the figure above, when one firm (firm R) has PPQ, for a wide space in the parameter region of θ its price is higher than the price it offers in the No-PPQ and the Both-PPQ cases.

5.2 Consumer Surplus

Proposition 4 *(i) When both firms have PPQ, consumer surplus is non-monotonic in valuations in that low valuation consumers get higher surplus compared to high valuation consumers. Specifically, for all $\theta \in [0, \widehat{\theta}]$, and for all $\theta \in [1-\widehat{\theta}, 1]$, consumers get lower surplus when both firms have PPQ in contrast to the No-PPQ scenario. Thus, when both firms have PPQ consumers located in the middle of the market have the highest surplus in contrast to the No-PPQ scenario wherein these consumers (in the middle) have the lowest surplus. (ii) Further, when one firm has PPQ, some of its consumers are left with positive surplus.*

In the No-PPQ scenario, the fact that consumers in the middle (or the ones which have the lowest inclination to buy from either firm) have the lowest surplus comes from Lemma 1 based on the incentive compatible constraints. This is similar to the non-linear pricing literature (Mussa & Rosen 1978) where the lowest consumer type gets a *zero* surplus since each firm acts as a local monopolist. Here as well, the local monopolist captures the entire surplus of the consumer at the boundary ($\theta = 0.5$) as seen from figure 3. On the other hand, in the Both-PPQ scenario, consumer surplus provided by one firm is determined by its rival's socially optimal welfare curve. We can indeed verify that the surplus provided by each firm to a given consumer increases as the consumer's location gets closer to the rival firm as stated in the beginning of this section. As a result, consumers located in the middle receive a higher consumer surplus, with the highest surplus accruing to the consumer located at $\theta = 0.5$.

Interestingly, this result suggests that consumers who are the least loyal to either firm, are the ones who are the most well-off when both firms adopt PPQ. Thus, we show in figure 3 that consumer surplus is monotonic (non-monotonic) in valuations depending on whether firms don't have (have) access to such PPQ technologies.[11] Moreover, we note that as the cost of quality decreases (α increases), the optimal quality offered to any consumer also increases. Hence, the surplus accruing to any consumer also increases with α. This is true when both firms have PPQ as well as when neither firm has PPQ (except for the consumer located at $\theta = 0.5$).

[11] Prior literature in Hotelling models (for example, Ulph and Vulkan 2000) have shown that if transportation costs do not increase fast with distance then all consumers get lower prices (and higher surplus) when firms practice personalized pricing. This is in contrast to our results where we show that the most loyal consumers get zero surplus while the least loyal consumers get positive surplus, and that the size of these "loyal" segments is driven by the convexity of the cost function (α) parameter.

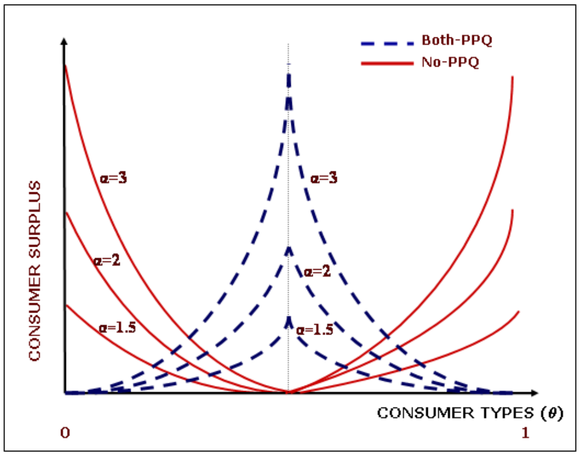

Figure 3: Consumer surplus for different α.

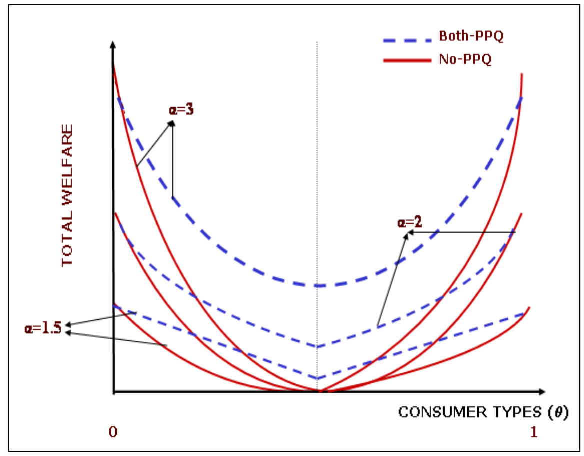

Figure 4: Social welfare for different α.

We find that the total consumer surplus is highest when neither firm has access to PPQ. Thus, the adoption of PPQ enables the firms to extract the maximum rent from consumers. Once again, the additional rents from quality enhancement outweigh the price competition effect from personalized pricing leading to a lower consumer surplus.[12]

We find that when one firm adopts PPQ, the consumer surplus from its rival is higher than that in the Both-PPQ case, but lower than in the No-PPQ case. The intuition is driven by the increase in average prices when one firm adopts PPQ. Further, note that when one firm (firm R, for example) has PPQ, not all of its consumers are left with zero surplus. Of course, its own immediate consumers (those whose type $\theta \in [0.5, 1]$) do not get any surplus at all. However, there are some consumers in firm L's territory, that R is able to capture by offering them lower qualities at lower prices. These consumers in the region whose type $\theta \in (0.27, 0.5)$, are served by the PPQ firm, and consequently a small proportion of them get a positive surplus. Specifically, when $\alpha = 2$ consumers in the region where $\theta \in (0.27, 0.345)$ get positive surplus whereas the remaining consumers are left with no surplus.

5.3 Welfare

We plot the welfare curves in Figure 4 for each of the two cases as before: neither firm has PPQ, and both firms have PPQ. We define welfare of a consumer as the sum of the firm's profit from

[12] Note that this is in contrast to prior work in personalized pricing (for example, Choudhary et al. 2005) who show that total consumer surplus is highest when both firms engage in personalized pricing. In their model this result occurs since firms could only personalize prices–the products offered to all consumers were the same. Hence, the competitive effect of aggravated price competition led to lower prices than in the scenario when firms did not practice personalized pricing.

that consumer and the surplus accruing to that consumer. Note from figure 4 that the total welfare is highest when both firms adopt PPQ. Next, we show that the adoption of PPQ by one firm (for example, firm R) has interesting welfare implications.

Corollary 2 *Suppose the cost function is quadratic ($\alpha = 2$). (i) When only one firm adopts PPQ, social welfare may be lower than the No–PPQ and the Both-PPQ cases because of the "misallocation" effect. (ii) When both firms adopt PPQ, social welfare is highest.*

The intuition for this result is similar to that for Corollary 1. The misallocation effect arises because in a socially optimal situation consumers whose $\theta \in [0.27, 0.5]$ should have ideally bought from firm L while those customers whose $\theta \in [0.5, 1]$ should have bought from firm R. However, when firm R has PPQ, it induces some of L's consumers (those with $\theta \in [0.27, 0.5]$) to buy from it by offering them lowering qualities at lower prices. This lowering of offered quality to each consumer (from the first-best solution wherein $q(\theta) = \theta$) results in a welfare loss compared to the socially optimal scenario.

In general, in the No-PPQ scenario only the highest consumer type (that located at $\theta = 1$ or $\theta = 0$) gets the socially optimal quality. In the Both-PPQ case all consumers get the socially optimal quality. Since both firms can identify each consumer, they do not need to degrade the offered qualities in order to prevent possible cannibalization, wherein the higher consumer types choose lower qualities. That is, firms can maintain the incentive compatibility constraints without having to lower the quality offered to a given consumer.

When one firm has PPQ (say firm R for example), while all the immediate consumers of the PPQ firm (those located between 0.5 and 1) get a socially optimal quality, only the highest type of the firm without PPQ (firm L in this case) gets the socially optimal quality. The remaining consumers of firm L (located between 0 and 0.27) as well those consumers of firm L (located between 0.27 and 0.5) who have been poached by firm R get less than socially optimal quality. In sum, although the quality $q^R(\theta)$ increases up to the socially optimal level, not all consumers served by firm R receive a higher quality product. Consequently, social welfare will be lower from transacting with some consumer types within the region where $\theta \in [0.27, 0.5]$.

5.4 Firm Profits

Proposition 5 *The adoption of PPQ does not lead to a Prisoner's Dilemma. Both firms are always better off adopting PPQ compared to the No-PPQ case.*

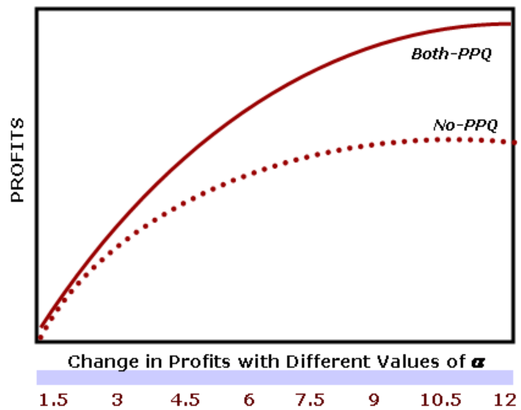

Figure 5: Profits with or without PPQ for different α

From figure 5, we can observe that the profits in the Both-PPQ case are always higher than that in the No-PPQ case for any value of α. This result arises because of the quality enhancement effect. Each firm offers a continuum of qualities, and then allocates a personalized quality at a personalized price for each consumer.[13]This leads to a higher rent extraction ability from the loyal consumers of each firm since it acts as a local monopolist. Even though the firm leaves some surplus to consumers in the middle, the positive quality enhancement effect offsets the negative price competition effect, and thereby makes it possible for symmetric firms to increase profits after adopting PPQ. Basically when both firms have PPQ, they do not have any incentive to leave any surplus for their loyal consumers that is higher than the surplus from the "outside opportunity"(in the Both-PPQ case the outside opportunity is the surplus offered by the rival firm). Since these loyal consumers have minimal valuation for the rival firm's products, neither firm has an incentive to offer them any positive surplus. Therefore, they end up charging higher prices and reaping greater profits.

6 Asymmetric firms (Market size)

In this section we consider the case in which firms are asymmetric in size such that one firm has a larger "loyal customer base" than the other firm. We model this in the following way. Firms are still located at 0 and 1 as before. However, in contrast with the prior section, customers are uniformly distributed from 0 to r, $0.5 \leq r \leq 1$.[14] We are interested in analyzing the impact of a

[13]Recall that each firm offers a continuous menu of price and quality pairs. We use the phrase "continuum of qualities" to refer to the same phenomenon.

[14]Note that when r <0.5, only firm L can exist in the market when both firms have PPQ, and thus a comparison becomes moot.

loyal customer segment, which is determined by the value of r. As before, we have three cases: (i) Neither firm has PPQ, (ii) Only one firm has PPQ, and (iii) Both firms have PPQ. The solution concept is exactly the same as that in the benchmark case and is omitted here for brevity. We list the optimal quality schedules as follows:

Proposition 6 *The optimal quality schedules are given as follows:*

(1) Neither firm has PPQ:

$$\boxed{q^L(\theta) = (1-2\theta)^{1/(\alpha-1)}, \theta \in [0, B_1],}$$
$$\boxed{q^R(\theta) = (2\theta - r)^{1/(\alpha-1)}, \theta \in [B_1, r].}$$

(2) Only L has PPQ:

$$\boxed{q^L(\theta) = (1-\theta)^{1/(\alpha-1)}, \theta \in [0, B_2],}$$
$$\boxed{q^R(\theta) = (2\theta - r)^{1/(\alpha-1)}, \theta \in [B_2, r].}$$

(3) Only R has PPQ:

$$\boxed{q^L(\theta) = (1-2\theta)^{1/(\alpha-1)}, \theta \in [0, B_3],}$$
$$\boxed{q^R(\theta) = \theta^{1/(\alpha-1)}, \theta \in [B_3, r].}$$

(4) Both firms have PPQ:

$$\boxed{q^L(\theta) = (1-\theta)^{1/(\alpha-1)}, \theta \in [0, B_4],}$$
$$\boxed{q^R(\theta) = \theta^{1/(\alpha-1)}, \theta \in [B_4, r].}$$

This leads to the following results about how firms change their quality schedules with the adoption of PPQ by either one or both firms. The values of the marginal customer (B_i) are derived in the Appendix.

Proposition 7 *Compared to the No-PPQ case, (i) when the larger firms gets PPQ, it always increases its quality level while the smaller firm keeps its quality schedule unchanged. (ii) when the smaller firm gets PPQ, it always increases its quality while the larger firm keeps its quality schedule unchanged.*

The intuition for this result is similar to that in Corollary 2. Basically, any firm which gets PPQ will not have an incentive to degrade qualilties because it no longer has to worry about consumer self-selection and product cannibalization. Hence, it increases its quality schedule.

Proposition 8 *Compared to the Both-PPQ case, (i) when the smaller firm gets PPQ, the larger firm decreases its quality while the smaller firm keeps its quality schedule unchanged. (ii) when the larger firm gets PPQ, the smaller firm decreases its quality while the larger firm keeps its quality schedule unchanged.*

An interesting observation is that when one firm acquires PPQ, it changes its quality compared to the No-PPQ case but keeps it unchanged compared to the Both-PPQ case. Intuitively this occurs because competition between firms only determines the surplus function, and consequently the optimal price functions. Notice that when both firms have PPQ, we find that optimal prices, quality schedules and surplus functions are independent of the range of θ between which consumers are distributed (i.e. the quality and price schedules are independent of r). This is because firms know the preferences of each consumer and are able to offer them the corresponding (price, quality) schedule in accordance with their type. When one firm acquires PPQ, it increases its quality schedule to all types in comparison to what it was offering in the absence of PPQ. As a consequence, the total welfare will be higher for customers buying from the firm with PPQ.

7 PPQ Technology Adoption Decision

Next, we investigate when and which firm will adopt PPQ, when adopting PPQ entails a cost. Suppose in the very first stage, each firm decides whether or not to adopt the PPQ technology at a fixed cost of F. In the second stage, similar to the previous analysis, firms play a simultaneous pricing game when both firms have PPQ (or when both firms do not have PPQ). They play a sequential pricing game when only one firm has PPQ. We are interested in determining the range of fixed costs over which the adoption of PPQ leads to a positive outcome for both firms or a negative outcome such as a Prisoner's Dilemma where both firms are worse-off in comparison to the scenario when neither of them have PPQ. In order to determine the impact of market size and customer loyalty on each firm's optimal strategies, we generalize the range over which customers are uniformly distributed. In particular, we consider two stylized examples; one in which customer type θ is distributed between $[1-r, r]$ (which we refer to as the symmetric case), and the other in which customer type θ is distributed from $[0, r]$ (which we refer to as the asymmetric case). From the symmetric case, we are able to analyze the situation when each firm's loyal segment changes equally. In the asymmetric case, the size of the loyal segment is different for each firm.[15]

7.1 Symmetric Case

From the expressions stated in the supplementary technical Appendix C, we can solve the total profit of each case. When the customer type is uniformly distributed from $[1-r, r]$, we have the following payoff matrix.

Pay Off	**R, No-PPQ**	**R, PPQ**
L, No-PPQ	(π_N^L, π_N^R)	$(\pi_R^L, \pi_R^R - F)$
L, PPQ	$(\pi_L^L - F, \pi_L^R)$	$(\pi_{Both}^L - F, \pi_{Both}^R - F)$

[15] We provide the detailed derivations of the expressions for firm profits in Appendix C.

The firms' payoffs with a change in the value of r are shown in figure 6a. A complete characterization of all Nash Equilibria (NE) is depicted in figure 6b.

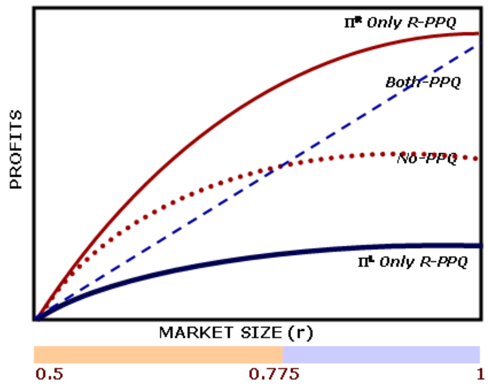

Figure 6a: Firm Payoffs with PPQ

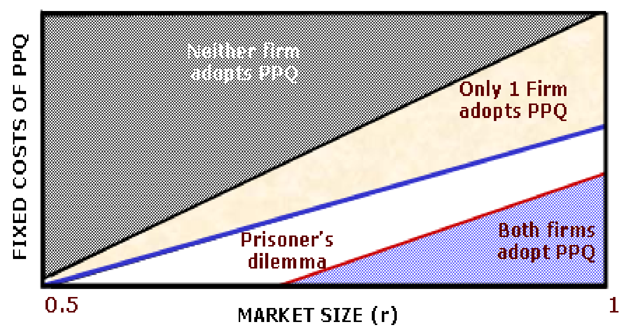

Figure 6b: PPQ Adoption When Firms are Symmetric.

Given figures 6a and 6b, we first have the following result.

Observation 1: *(i) When the customer types are uniformly distributed in $[1-r, r]$ and $r > 0.775$, the profit of each firm is higher after both firms adopt PPQ. When $r \leq 0.775$, the profit of each firm is smaller after both firms adopt PPQ. (ii) Moreover, it is not a dominant strategy for a firm to adopt PPQ even if its competitor were to have PPQ.*

We can observe that when r is larger than 0.775, it is possible to have situations in which both firms are better off after the adoption of PPQ. From this result, we conclude that when both firms have a larger loyal segment, it is less likely that the adoption of PPQ will lead to a Prisoner's Dilemma. On the other hand, if both firms have few loyal customers, the adoption of PPQ will lead to a Prisoner's Dilemma. This result is in contrast with that of Shaffer and Zhang (2002) who show that the firm with a smaller market size is always worse-off after the adoption of PPQ. This happens because in their model the price competition effect is stronger than the market share effect. In contrast, in our model the rent extraction effect due to quality enhancement dominates the adverse effect of price competition and this result is stronger when firms have a larger proportion of loyal consumers (when r increases) because the marginal benefit from the quality enhancement effect will be higher for such firms.

In figure 6b, the thick black line below the triangular region is the difference between π_R^R and π_N^R. When the fixed cost of PPQ technology, F, is higher than this level neither firm will adopt PPQ. The second line in the middle of this figure is the difference in the profit of firm L when both firms have PPQ and when only firm R has PPQ, i.e. the difference between π_{Both}^L and π_R^L. When F is higher than this level, if R adopts PPQ, L will not adopt it to facilitate a level-playing field and vice-versa. In other words, in that region even with symmetric firms we have two asymmetric

Nash equilibria in which only one firm adopts PPQ. Thus, we find that depending on the costs of adopting PPQ, it is not a dominant strategy for a firm to adopt PPQ even if its competitor adopts PPQ. The lowest line at the bottom is the difference of π^L_{Both} and π^L_N. When F is below it, both firms are better off after adopting PPQ. We provide a numerical example below to characterize the regions where firms adopt PPQ and the corresponding value of the cost of PPQ. Note that the numbers in this example are exactly equal to the firms' profits under each scenario (No-PPQ, Only one firm PPQ and Both-PPQ) when $\alpha = 2$.

Example 1: When $r = 1$, the payoff matrix is given as follows:

Profits	**R, No-PPQ**	**R, PPQ**
L, NO-PPQ	$(0.083, 0.083)$	$(0.0654, 0.162 - F)$
L, PPQ	$(0.162 - F, 0.0654)$	$(0.125 - F, 0.125 - F)$

Hence, the potential cases are as follows:

1. $F \geq 0.079$, neither firm adopts PPQ in equilibrium.
2. $F \in [0.06, 0.079]$, there are two NEs: either firm R or firm L adopts PPQ in equilibrium.
3. $F \in [0.042, 0.06]$, the equilibrium is (PPQ, PPQ). Prisoner's Dilemma oocurs in equilibrium.
4. $F \leq 0.042$, the equilibrium is (PPQ, PPQ). Both firms are better off after adoption of PPQ.

7.2 Asymmetric Case

In general, when a firm has a larger loyal segment, its incremental benefit from adopting PPQ is higher compared to the firm with a smaller loyal segment. Formally, when r is close to 1, the benefit of simultaneous adoption of PPQ by both firms is higher for firm R (smaller firm).

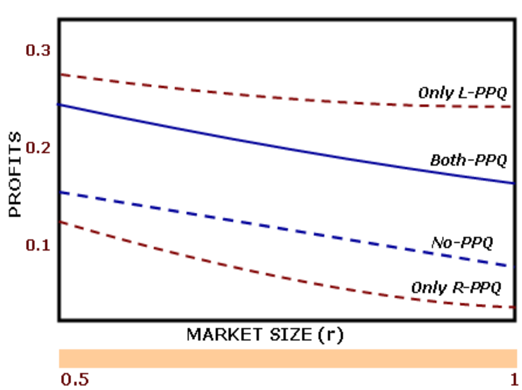

Figure 7a: Profits of firm L (larger firm).

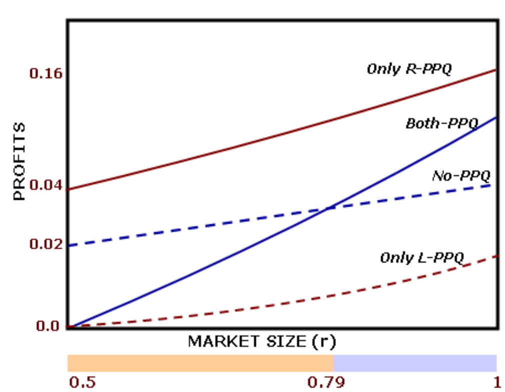

Figure 7b: Profits of firm R (smaller firm).

Notice from figures 7a and 7b that with an increase in the size of the loyal segment (r), the adoption of PPQ always leads to lower profits for the larger firm and higher profits for the smaller firm. From figure 7b, note that when the larger firm has PPQ, the adoption of PPQ may in some cases be detrimental for the smaller firm compared with the No-PPQ case, especially when $r \leq 0.79$. By comparing the Both-PPQ and Only L-PPQ curves, we can conclude that adopting PPQ is not a dominant strategy for the smaller firm especially when the larger firm has PPQ and the number of loyal customers it has is less than a certain threshold. The Nash equilibria are summarized in figure 8. We demonstrate the impact of the size of the loyal segment (r) on firms' PPQ adoption decisions with the following examples, which leads to Observation 2.

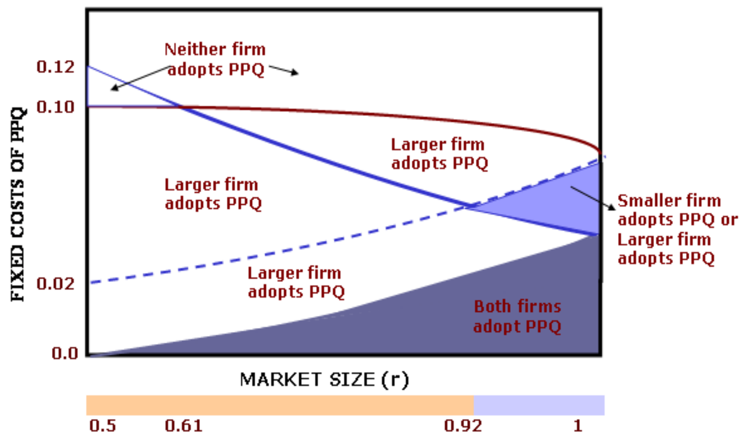

Figure 8: PPQ Adoption When Firms are Asymmetric.

Example 2: When $r = 0.95$, the payoff matrix is given as follows:

Profits	**R, No-PPQ**	**R, PPQ**
L, No-PPQ	$(0.087, 0.075)$	$(0.069, 0.145 - F)$
L, PPQ	$(0.169 - F, 0.055)$	$(0.132 - F, 0.106 - F)$

Hence, the potential cases are as follows:

1. $F \geq 0.081$, the equilibrium is (No-PPQ, No-PPQ).
2. $F \in [0.07, 0.081]$, the equilibrium is (PPQ, No-PPQ).
3. $F \in [0.063, 0.07]$, there are two equilibria: (PPQ, No-PPQ) or (No-PPQ, PPQ).
4. $F \in [0.051, 0.063]$, the equilibrium is (PPQ, No-PPQ). PPQ is a dominant strategy for L.
5. $F \in [0.045, 0.051]$, the equilibrium is (PPQ, PPQ). Prisoner's Dilemma occurs in equilibrium.
6. $F \in [0.031, 0.045]$,the equilibrium is (PPQ, PPQ). Firm L is better off and firm R is worse-off.
7. $F < 0.31$,the equilibrium is (PPQ, PPQ). Both firms are better off after adoption of PPQ.

By comparing example 1 (point 4) in the symmetric case and example 2 (point 7) in the asymmetric case, we can make the following conclusion.

Observation 2: *As the extent of asymmetry in market size increases (as r decreases), the range of values over which both firms are better off by adopting PPQ decreases.*

8 Managerial Implications and Conclusion

Firms' are increasingly realizing that the ability to establish attractive value propositions and turn them into personalized and compelling offers across the right channel for the right customer at the most opportune moment - drives customer relationships, and profits. This has led to a widespread adoption of CRM and personalization technologies by firms in different industries such as long distance telecommunications, industrial products, mobile telephone service, hotels, IT hardware, financial services, online retailing, credit cards, etc., in order to influence their customer acquisition and retention strategies. Moreover increasing availability of flexible manufacturing technologies is facilitating quality enhancement through customization.

Our novelty consists in combining both personalized pricing and targeted quality allocation in the same theoretical framework. Our model highlights how firms should allocate product or service qualities, and prices, and how in turn, such targeting decisions impact the surplus of consumers, and overall social welfare. In contrast to prior work, we show that quality enhancement through targeted quality allocation leads to less aggravated price competition by strengthening the opportunities for rent extraction for firms, when firms are able to personalize prices as well. Thus, the adoption of PPQ technologies such as customer relationship management systems (CRM) and flexible manufacturing systems (FMS) by competing firms can make even *symmetric* firms better-off. That is, when firms can better target the allocation of qualities and prices, and offer a broader product line, competition becomes less intense because a greater proportion of the potential consumers now has a higher willingness to pay for the firms' products. We account for the cost of PPQ technologies which can include, for instance, the cost of FMS in the case that the product quality is enhanced. Another example of such a cost could be those incurred in providing personalized services when it is the quality of service that is being personalized for each consumer. Even after explicitly accounting for such costs, we find regions where symmetric firms are better-off after engaging in PPQ. Prior work (Shaffer and Zhang 2002) has identified situations where asymmetric firms can avoid the Prisoner's Dilemma through the market share effect. We show that even *symmetric* firms can avoid a Prisoner's Dilemma because of the quality enhancement effect.

An interesting result is the emergence of asymmetric equilibria: situations where one firm adopts PPQ and its rival does not, despite both firms being ex-ante symmetric. This is driven by the presence of fixed costs of PPQ adoption. This has important implications since in many industries we do see such disparities in technology investments by firms of similar size and market share. Another result from our analysis is that starting from asymmetric firms (in the size of their loyal segments) when firms become progressively symmetric, then the adoption of PPQ technologies is increasingly beneficial to both firms. This implies that industries with a higher level of firm concentration will have greater incentives to adopt such technologies and invest in loyalty building measures.

An implication of our analysis is that the adoption of CRM technologies leads to an increase in the quality level of the entire product line of a firm. This is relevant for a firm's pricing and product line decision since the adoption of PPQ negates the threat from intra-firm competition that was prevalent in the absence of PPQ. Basically, firms which adopt PPQ only need to consider inter-firm competition, and hence it is optimal for them to offer a significant product quality/service improvement.

Our paper also offers insights on the different product quality schedules offered by firms, given that one or both firms can implement PPQ. Compared to the No-PPQ scenario, when one firm adopts PPQ, it always increases its quality level while the other firm keeps its quality schedule unchanged. Conversely, compared to the Both-PPQ scenario, when a firm drops PPQ, it always decreases its quality schedule while the other firm keeps its quality schedule unchanged. Such changes in offered qualities are often seen in practice. In the IT hardware industry, these kinds of changes in quality schedules are often done through stripping off some value-added customer service, such as next-day on-site repair versus same-day 8-hour repair, or a 99% uptime guarantee versus a 99.99%. Another example is that of hardware vendors like HP and IBM who differentiate themselves by providing superior warrantees, new generation web-based applications, as well as clustering and security management software embedded in the same hardware box. Similarly, in many industrial products markets, to add value to customers beyond the core product, suppliers offer additional services as educational programs, 24-hour repair, consulting services, quality control assurance and testing, just-in-time (JIT) delivery, either separately or via some combination of the above. Many professional services such as IT or management consultants offer a wide array of differentiated services to their clients and charge different prices.

The adoption of PPQ by both firms has a differential impact on average consumer surplus as well as on the surplus accruing to any one consumer beyond a certain location. While PPQ adoption leads to lower average consumer surplus, interestingly, we find that some consumers are actually better off when both firms adopt PPQ. That is, there is a transfer of surplus among consumers. In particular, consumers located closer to the middle of the market–who are the least loyal to either firm or have the lowest willingness to pay for either firm's products, are the ones who are the most better-off when both firms adopt PPQ technologies. This is in contrast to a scenario when neither firm has a PPQ technology, when the very same consumers who are least likely to buy either firms' products, are the most worse-off. Basically, firms engaging in PPQ are able to extract more surplus from consumers who have the strongest preferences for their products. In the absence of the ability to discriminate, firms were extracting less surplus from them.

From a public policy perspective, our analysis of social welfare highlights that social welfare is highest when both firms adopt PPQ. Indeed even if one firm adopts PPQ, social welfare is higher than the situation where neither firm has PPQ. However in such a case, the total welfare for some consumers can be lower because of the misallocation of the products. In particular, because some customers of the firm without PPQ end up buying from the firm with PPQ at lower prices and lower qualities, we see a decrease in social welfare for those regions.

Our paper has several limitations, some of which can be fruitful areas of research. For example, we have only considered symmetric cost functions for both firms. Some firms may have operational efficiencies which can give rise to less convex production costs when customizing quality. It would be interesting to see how firms' strategies change under such scenarios. Another interesting extension would be to study competition in markets with discrete segments such as loyals and switchers, when firms adopt non-linear pricing schedules. A third area of related research would be to allow competing firms to invest in loyalty building measures, such as switching costs, before they invest in PPQ. Finally, we do not consider consumers making strategic choices in revealing information about their preferences. One could consider a scenario where higher valuation consumers might want to mimic lower types and vice-versa, in anticipation that some consumers are left with positive surplus while others are not when firms engage in PPQ. Incorporating such a situation is beyond the scope of this paper but it might be an interesting extension to pursue in a related framework. We hope our research paves the way for more future work in this domain.

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9 Appendix A

9.1 Neither Firm has PPQ

Proof of Proposition 1

We proceed in a series of steps by first stating and proving several lemmas.

Lemma 1 $\frac{ds^R(\theta)}{d\theta} = q^R(\theta)$ and $\frac{ds^L(\theta)}{d\theta} = -q^L(\theta)$.

First, recall that each firm maintains a menu of prices and qualities. Since consumers choose any contract from the menu, the incentive compatibility condition for consumers is given by

$$s^R(\theta) = \max_t \; \theta \times q^R(t) - p^R(t). \tag{13}$$

The first order condition is

$$\theta \times \frac{\partial q^R(t)}{\partial t} - \frac{\partial p^R(t)}{\partial t} = 0. \tag{14}$$

This equation holds at $t = \theta$ because consumers self-select the price and quality pair designed for them. By differentiating equation (13), we have

$$\frac{ds^R(\theta)}{d\theta} = q^R(\theta) + \theta \times \frac{\partial q^R(\theta)}{\partial \theta} - \frac{\partial p^R(\theta)}{\partial \theta},$$
$$\Rightarrow \frac{ds^R(\theta)}{d\theta} = q^R(\theta). \tag{15}$$

In the second equation, the last two terms are zero because of the first-order condition as shown above in equation (14). Using the same procedure, it can be shown that

$$\frac{ds^L(\theta)}{d\theta} = -q^L(\theta).$$

This Lemma implies that

$$s^R(\theta) = s^R(B) + \int_B^\theta q^R(t)dt$$
$$s^L(\theta) = s^L(B) + \int_\theta^B q^L(t)dt.$$

Note that the IC constraint ensures that a consumer prefers the contract that was designed for him, and the IR constraint guarantee that each consumer type accepts his designated contract. Hence, in this case (IC) implies that the slope of $s^R(\theta)$ is equal to $q^R(\theta)$ as shown in Lemma 1. In this model, competition between two firms affects only the surplus to the consumer at the boundary (which for example is equal to $s^R(B)$ for firm R), which is a constant. This implies that two firms compete by lowering the pricing schedule by a constant, $s^R(B)$. Higher consumer types will receive higher surplus; this is termed as information rent in the non-linear pricing literature. This implies that whenever the firm increases the quality offered to any consumer, it has to leave higher information rents to higher consumer types in order to avoid cannibalization during self-selection.

As a result, our decision variables can be further simplified as $q(\theta)$ and $s(B)$, where $s(B)$ is the surplus of the marginal consumer who is indifferent between buying from two firms. Substituting for $s^R(\theta)$, the simplified objective function for firm R can be rewritten as

$$\max_{q^R(\theta),\ s^R(\theta)} \pi_N^R, \text{ where } \pi_N^R = \int_B^1 \left[\theta q^R(\theta) - s^R(\theta) - \frac{(q^R)^\alpha(\theta)}{\alpha}\right] d\theta,$$
$$s.t.\ s^R(\theta) \geq 0,\ s^L(B) = s^R(B).$$

After substituting for the value of $s^R(\theta)$, the optimization problem becomes equal to

$$\max_{q^R(\theta),\ s^R(B)} \pi_N^R, \text{ where } \pi_N^R = \int_B^1 \left[\theta q^R(\theta) - \frac{(q^R)^\alpha(\theta)}{\alpha} - s^R(B) - \int_B^\theta q^R(t)dt\right] d\theta. \tag{16}$$

Changing the order of integration of the last term in the bracket[16], we can simplify the objective function as

$$\pi_N^R = \int_B^1 \left[\theta q^R(\theta) - \frac{(q^R)^\alpha(\theta)}{\alpha} - s^R(B) - q^R(\theta)(1-\theta)\right] d\theta, \tag{17}$$
$$= \int_B^1 \left[(2\theta-1)q^R(\theta) - \frac{(q^R)^\alpha(\theta)}{\alpha} - s^R(B)\right] d\theta. \tag{18}$$

Similarly, the optimization problem for firm L is given as follows:

$$\max_{q^L(\theta), s^L(\theta)} \pi_N^L, \text{ where } \pi_N^L = \int_0^B \left[(1-\theta)q^L(\theta) - s^L(\theta) - \frac{(q^L)^\alpha(\theta)}{\alpha}\right] d\theta, \tag{19}$$
$$s.t.\ s^L(\theta) \geq 0,\ s^L(B) = s^R(B).$$

After substituting for the value of $s^l(\theta)$, the optimization problem becomes equal to

$$\max_{q^L(\theta),\ s^L(B)} \pi_N^L, \text{ where } \pi_N^L = \int_0^B \left[(1-2\theta)q^L(\theta) - \frac{(q^L)^\alpha(\theta)}{\alpha} - s^L(B)\right] d\theta. \tag{20}$$

The optimal quality schedule can be determined by maximizing the integrand point-wise (the terms in the bracket). This leads to the following Lemma.

Lemma 2 *The equilibrium quality schedules are* $q^R(\theta) = (2\theta-1)^{1/(\alpha-1)}$ *and* $q^L(\theta) = (1-2\theta)^{1/(\alpha-1)}$.

Differentiating terms in the bracket of (17) with respect to $q^R(\theta)$, we have

$$\theta - (q^R)^{\alpha-1}(\theta) - (1-\theta) = 0. \tag{21}$$
$$\Longrightarrow q^R(\theta) = (2\theta-1)^{1/(\alpha-1)}$$

The solution of firm L can be derived in a similar manner. To find the solution of $s^R(B)$, we differentiate the objective functions w.r.t. $s^R(B)$ and derive the following Lemma by Leibniz Theorem.

[16] $\int_B^1 \left[\int_B^\theta q^R(t)dt\right] d\theta = \int_B^1 \left[\int_t^1 q^R(t)d\theta\right] dt = \int_B^1 q^R(t)(1-t)dt = \int_B^1 q^R(\theta)(1-\theta)dt$

Lemma 3 *The equilibrium consumer surplus at the boundary is given by $s^R(B) = s^L(B) = 0$.*

Define the terms in the bracket of (18) as X. Using Leibniz Theorem, we have

$$\frac{d\pi_N^R}{ds^R(B)} = \int_B^1 \frac{\partial X}{\partial s^R(B)} d\theta - X|_{\theta=B} \times \frac{dB}{ds^R(B)}.$$

As a result, by differentiating (18) with respect to $s^R(B)$, we have

$$\underbrace{\int_B^1 -1 \cdot d\theta}_{1^{\text{st}} \text{ term}} - \underbrace{\left[(2B-1)q^R(B) - \frac{(q^R)^\alpha (B)}{\alpha} - s^R(B)\right]}_{2^{\text{nd}} \text{ term}} \underbrace{\frac{dB}{ds^R(B)}}_{3^{rd} \text{ term}} = 0. \tag{22}$$

These terms represents the costs and benefits that accrue to firm R if it changes its price by one unit. Intuitively, when price is lowered by 1 unit, the first term represents the aggregate loss in revenue from all existing consumers of firm R. The second and third terms together represent the gain in revenue from attracting some potential consumers in firm L's territory. Specifically, the second term represents the profit from the marginal consumer and the third term represents the gain in market share from infra-marginal consumers that occurs by lowering price by one unit. From Lemma 1, we know that

$$\begin{aligned}\frac{dB}{ds^R(B)} &= \frac{dB}{ds^L(B)}, \text{ (given that } s^R(B) = s^L(B)) \\ &= \frac{1}{-q^L(B)}\end{aligned}$$

Substituting this back to (22), we have

$$(B-1) - \left[(2B-1)q^R(B) - \frac{(q^R)^\alpha (B)}{\alpha} - s^R(B)\right] \frac{1}{-q^L(B)} = 0.$$

After rearranging terms the above equation can be written as

$$s^R(B) = q^L(B)(B-1) + (2B-1)q^R(B) - \frac{(q^R)^\alpha (B)}{\alpha}. \tag{23}$$

In the symmetric equilibrium, $B = 1/2$. Moreover, from Lemma 3 we know that $q^L(B) = q^R(B) = 0$. Substituting these in equation (23) we have

$$s^R(B) = 0.$$

The complete solutions are summarized in the statement of the proposition. Quality schedules are derived in Lemma 2. By definitions, $s^L(\theta) = 0 + \int_\theta^{1/2} q^L(t)dt$ and $s^R(\theta) = 0 + \int_{1/2}^\theta q^R(t)dt$. Hence, the optimal surplus functions are given by

$$s^L(\theta) = \frac{\alpha-1}{2\alpha}(1-2\theta)^{\alpha/(\alpha-1)}.$$

$$s^R(\theta) = \frac{\alpha-1}{2\alpha}(2\theta-1)^{\alpha/(\alpha-1)}.$$

The optimal price schedules are derived by substituting $p^L(\theta) = (1-\theta)q^L(\theta) - s^L(\theta)$ and $p^R(\theta) = \theta q^R(\theta) - s^R(\theta)$. Hence, the optimal prices are given by

$$\begin{aligned} p^L(\theta) &= (1-\theta)(1-2\theta)^{1/(\alpha-1)} - \frac{\alpha-1}{2\alpha}(1-2\theta)^{\alpha/(\alpha-1)}, \\ &= (1-2\theta)^{1/(\alpha-1)}\left(\frac{-2\theta+\alpha+1}{2\alpha}\right). \end{aligned} \tag{24}$$

$$\begin{aligned} p^R(\theta) &= \theta(2\theta-1)^{1/(\alpha-1)} - \frac{\alpha-1}{2\alpha}(2\theta-1)^{\alpha/(\alpha-1)}, \\ &= (2\theta-1)^{1/(\alpha-1)}(\frac{2\theta+\alpha-1}{2\alpha}). \end{aligned} \tag{25}$$

9.1.1 Total Welfare, Surplus and Profits

Since firms are symmetric, it is sufficient to present the results for any one firm. Without loss of generality, consider firm L. Then the total surplus is given by

$$s_N^L = s_N^R = \int_0^{1/2} \frac{\alpha-1}{2\alpha}(1-2\theta)^{\alpha/(\alpha-1)} d\theta = \frac{(\alpha-1)^2}{4\alpha(2\alpha-1)}.$$

The total welfare function is given by

$$\begin{aligned} w_N^L &= \int_0^{1/2} [u^L(q(\theta),\theta) - c(q(\theta))] d\theta, \\ &= \int_0^{1/2} [(1-\theta)(1-2\theta)^{1/(\alpha-1)} - \frac{1}{\alpha}(1-2\theta)^{\alpha/(\alpha-1)}] d\theta, \\ &= \frac{(2\frac{1}{\alpha-1}+3)}{4(\frac{1}{\alpha-1}+1)(\frac{1}{\alpha-1}+2)} - \frac{\alpha-1}{2\alpha(2\alpha-1)}, \\ &= \frac{3\,(\alpha-1)^2}{4\alpha(2\alpha-1)}. \end{aligned}$$

Finally, profits are given by

$$\pi_N^L = w_N^L - s_N^L = \frac{(\alpha-1)^2}{2\alpha(2\alpha-1)}.$$

9.2 Only One Firm has PPQ

Proof of Proposition 2

In this case, recall that we solve a sequential pricing game since the simultaneous pricing game does not have a pure strategy Nash Equilibrium. Without loss of generality, let R be the firm with PPQ. In stage 2, given firm L's quality and pricing schedules, firm R will set its quality schedule equal to the socially optimal quality schedule. Basically, firm R will set the price so that the consumers feel indifferent between buying from L or R.[17] Formally, the problem of firm R in this case is

$$\max_{q^R(\theta),s^R(\theta)} \pi_R^R(\theta), \text{ where } \pi_R^R(\theta) = \theta q^R(\theta) - s^R(\theta) - \frac{(q^R)^\alpha(\theta)}{\alpha}, \forall \theta \in [0,1]. \tag{26}$$

The equilibrium quality schedule, $q^R(\theta)$, can be determined by the first order condition given by

$$\frac{\partial \pi_R^R(\theta)}{\partial q^R(\theta)} = \theta - (q^R)^{\alpha-1}(\theta) = 0.$$
$$\implies q^R(\theta) = \theta^{1/(\alpha-1)}$$

L's optimization problem is the same as that in the No-PPQ case except that the individual rationality constraint (IR2) is now different (please see below). Since this does not affect the optimal quality schedule for firm L, it is the same as that in the No-PPQ case and is equal to the following:

$$q^L(\theta) = (1-2\theta)^{1/(\alpha-1)}.$$

Next we determine the surplus function of firm L. Note that the surplus offered by firm L will depend on firm R's socially optimal surplus curve. If firm L were to offer less than the socially optimal surplus of firm R, then firm R could potentially poach L's consumers by offering lower prices. The potential for poaching exists since R can perfectly identify each consumer. L's optimization problem is the same as that in the No-PPQ case except that the individual rationality constraint (IR2), instead of being given by $s^L(B) = s^R(B)$, is replaced by the socially optimal surplus curve of firm R. Specifically, it is given by

$$s^L(B) = \max_{q^R(\theta)} \left[\theta q^R(\theta) - \frac{(q^R)^\alpha(\theta)}{\alpha}\right]|_{\theta=B}.$$

Similar to the Proof of Lemma 3, differentiating (20) with respect to $s^L(B)$, we have

$$\frac{\partial \pi_R^L(\theta)}{\partial s^L(B)} = -B + \left[(1-2B)q^L(B) - s^L(B) - \frac{(q^L)^\alpha(B)}{\alpha}\right]\frac{dB}{ds^L(B)} = 0. \tag{27}$$

Given that

$$s^L(B) = \max_{q^R(B)} \; Bq^R(B) - \frac{(q^R)^\alpha(B)}{\alpha} = (1-\frac{1}{\alpha})B^{\alpha/(\alpha-1)},$$

we have

$$\frac{dB}{ds^L(B)} = B^{-1/(\alpha-1)}. \tag{28}$$

Substituting this back in (27), it follows that

$$s^L(B) = -B \cdot B^{1/(\alpha-1)} + \left[(1-2B)q^L(B) - \frac{(q^L)^\alpha(B)}{\alpha}\right],$$
$$= (1-\frac{1}{\alpha})(1-2B)^{\alpha/(\alpha-1)} - B^{\alpha/(\alpha-1)}.$$

[17] There could exist multiple SPNE in this game because firm L can offer several $p(q(\theta))$(outside opportunity) to firm R's customers as long as the incentive compatible constraints are satisfied. Here, we assume that L does not offer any additional discounts to R's consumers. In other words, the outside opportunity of R's customers is equal to $(1-\theta)q^L(B) - p^L(B)$.

Given that the marginal consumer feels indifferent between buying from firm L and firm R, we have

$$\begin{aligned}
s^L(B) &= w^R(B) \\
&\Leftrightarrow (1-\frac{1}{\alpha})(1-2B)^{\alpha/(\alpha-1)} - B^{\alpha/(\alpha-1)} = (1-\frac{1}{\alpha})B^{\alpha/(\alpha-1)}, \\
&\Leftrightarrow B = \left[(\frac{2\alpha-1}{\alpha-1})^{(\alpha-1)/\alpha} + 2\right]^{-1}.
\end{aligned}$$

As a consequence, the consumer surplus function of firm L is given by

$$\begin{aligned}
s^L(\theta) &= s^L(B) + \int_\theta^B (1-2t)^{1/(\alpha-1)} dt, \\
&= s^L(B) + (\frac{\alpha-1}{2\alpha})(1-2B)^{\alpha/(\alpha-1)} - (\frac{\alpha-1}{2\alpha})(1-2\theta)^{\alpha/(\alpha-1)}.
\end{aligned}$$

Next we derive the consumer surplus function for firm R. Firm R sets the price, equivalently $s^R(\theta)$, such that each consumer's surplus exactly matches his/her surplus from the outside opportunity. Recall that the outside opportunity of R's consumers is either 0 or equal to the surplus offered by firm L which is determined by the contract offered to the marginal consumer $(q^L(B), s^L(B))$. As a result, the consumer surplus function of firm R is given by

$$s^R(\theta) = \max\ (0, (1-\theta)q^L(B) - p^L(B)).$$

Note that we already have derived the expressions for $(q^L(\theta), s^L(\theta))$ and $(q^R(\theta), s^R(\theta))$. Hence, by substituting the relevant expressions in $p(\theta) = u(q(\theta), \theta) - s(\theta)$,we have

$$\begin{aligned}
p^L(\theta) &= (1-\theta)(1-2\theta)^{1/(\alpha-1)} - s^L(\theta), \quad \theta \in [0, B], \\
p^R(\theta) &= \theta^{\alpha/(\alpha-1)} - s^R(\theta), \quad \theta \in [B, 1].
\end{aligned}$$

9.2.1 Total Welfare, Surplus and Profits

Due to the fact that B, $s^L(B)$, and $s^L(\theta)$ don't have simple closed form solutions, we cannot present the prices and profits in closed-form solutions. However, we can derive the relevant expressions for a given value of α. For example, when $\alpha = 2$, we find that

$$B = 2 - \sqrt{3} = 0.27$$

Since firm L moves first, we derive the relevant expressions for surplus, price, and welfare functions respectively as follows:

$$s^L(B) = \frac{7}{2} - 2\sqrt{3}.$$

$$s^L(\theta) = \frac{1}{2}\left(-2\theta + 2\theta^2 + 2\sqrt{3} - 3\right).$$

$$p^L(\theta) = -2\theta + \theta^2 - \sqrt{3} + \frac{5}{2}.$$

The total surplus, welfare and profit functions for firm L are given as follows:

$$s_R^L = \int_0^B s^L(\theta)d\theta = \frac{1}{2}\sqrt{3} - \frac{5}{6}.$$
$$w_R^L = \int_0^B w^L(\theta)d\theta = \frac{3}{2}\sqrt{3} - \frac{5}{2}.$$
$$\pi_R^L = \int_0^B \pi^L(\theta)d\theta = 5 - (3)^{\frac{3}{2}}.$$

Given all these solutions in the first stage, we can derive the optimal consumer surplus schedule of firm R. Firm R offers zero surplus to some of its consumers and then offers positive surplus to those consumers who are located closer to firm L. Hence, we need to derive the location of the marginal consumer of firm R who obtains a positive surplus. This is given by the equating the surplus from outside opportunity (in this case the surplus offered by firm L) to zero.

$$\begin{aligned}
0 &= (1-\theta^M)q^L(B) - p^L(B) \\
&= (1-\theta^M)(1-2B) - p^L(B) \\
&= (1-\theta^M)\left[1-2(2-\sqrt{3})\right] - \left[-2(2-\sqrt{3}) + (2-\sqrt{3})^2 - \sqrt{3} + \frac{5}{2}\right], \\
\Rightarrow \theta^M &= \left\{\frac{1}{2\sqrt{3}-3}\left[\sqrt{3} - \left(2-\sqrt{3}\right)^2 - \frac{3}{2}\right]\right\} = 0.345.
\end{aligned}$$

Consequently, the total consumer surplus, welfare and profit of firm R are

$$s_R^R = \int_{(2-\sqrt{3})}^{0.34530}\left[(1-\theta)\cdot(1-2B) - (-2B + B^2 - \sqrt{3} + \frac{5}{2})\right]d\theta = 0.01\,38.$$
$$w_R^R = \int_B^1 w^R(\theta)d\theta = \frac{5}{6}\left(\sqrt{3}-1\right)\left(2-\sqrt{3}\right).$$
$$\pi_R^R = \frac{1}{72}\left(738\sqrt{3} - 1263\right).$$

9.3 Both Firms have PPQ

Proof of Proposition 3

In this case, both firms know exactly each consumer's type. These two firms engage in a competition similar to Bertrand competition. In equilibrium, both firms offer a socially optimal level of quality.

The firm located closer to a consumer will set the price such that the consumer surplus exactly matches the highest possible consumer surplus offered by the other firm. The rival firm sets price at marginal cost. Neither firm will deviate by offering a lower price to its rivals' customers since no such action can bring in additional profit. Hence, the profit functions of the firms are given as follows:

$$\max_{q^L(\theta),\, s^L(\theta)} \pi_{Both}^L(\theta), \text{ where } \pi_{Both}^L(\theta) = (1-\theta)q^L(\theta) - s^L(\theta) - \frac{(q^L)^\alpha(\theta)}{\alpha}, \tag{29}$$

$$\max_{q^R(\theta),\, s^R(\theta)} \pi_{Both}^R(\theta), \text{ where } \pi_{Both}^R(\theta) = \theta q^R(\theta) - s^R(\theta) - \frac{(q^R)^\alpha(\theta)}{\alpha}. \tag{30}$$

Note that as before, firms still optimize with respect to both quality and surplus. Moreover, due to the perfect targeting of consumers there are no self-selection problems, and thus there is no potential for cannibalization. Hence, firms do not have to consider any IC constraints from the consumers' point of view. Therefore, the optimal quality schedules are determined by

$$\begin{aligned}\frac{\partial \pi^L_{Both}(\theta)}{\partial q^L(\theta)} &= (1-\theta) - \left(q^L\right)^{\alpha-1}(\theta) = 0,\\ &\Leftrightarrow q^L(\theta) = (1-\theta)^{1/(\alpha-1)}.\\ \frac{\partial \pi^R_{Both}(\theta)}{\partial q^R(\theta)} &= \theta - \left(q^R\right)^{\alpha-1}(\theta) = 0,\\ &\Leftrightarrow q^R(\theta) = \theta^{1/(\alpha-1)}.\end{aligned}$$

Both of these are the socially optimal quality schedules (first-best solutions).

Given the nature of the price competition between the two firms, we can determine $s^L(\theta)$, $s^R(\theta)$ and hence demonstrate the optimal price schedules. When $\theta \in [1/2, 1]$, consumers buy from firm R in equilibrium. At the same time, the equilibrium price from firm L is equal to its marginal cost, $\frac{(q^L)^\alpha(\theta)}{\alpha}$, because of Bertrand price competition. The equilibrium price from firm R is set at a level so that consumers feel indifferent between buying from firm R and firm L.

$$\begin{aligned}s^R(\theta) &= (1-\theta)q^L(\theta) - p^L(\theta),\\ &= (1-\theta)(1-\theta)^{1/(\alpha-1)} - \frac{(q^L)^\alpha(\theta)}{\alpha}\\ &= (1-\frac{1}{\alpha})(1-\theta)^{\alpha/(\alpha-1)}.\end{aligned}$$

Similarly, we can derive the consumer surplus function of firm L.This is given by

$$\begin{aligned}s^L(\theta) &= \theta q^R(\theta) - p^R(\theta),\\ &= (1-\frac{1}{\alpha})\theta^{\alpha/(\alpha-1)}.\end{aligned}$$

The social welfare functions are given by

$$\begin{aligned}w^L(\theta) &= u^L(q(\theta),\theta) - c(q(\theta))\\ &= (1-\theta)q^L(\theta) - \frac{(q^L)^\alpha(\theta)}{\alpha}\\ &= (1-\frac{1}{\alpha})(1-\theta)^{\alpha/(\alpha-1)},\ \theta \in [0,1/2].\end{aligned}$$

$$\begin{aligned}w^R(\theta) &= u^R(q(\theta),\theta) - c(q(\theta))\\ &= \theta q^R(\theta) - \frac{(q^R)^\alpha(\theta)}{\alpha}\\ &= (1-\frac{1}{\alpha})\theta^{\alpha/(\alpha-1)},\ \theta \in [1/2,1].\end{aligned}$$

Since $p(\theta) = u(q(\theta),\theta) - s(\theta)$, the price charged by each firm is given by

$$p^L(\theta) = (1-\theta)^{\alpha/(\alpha-1)} - (1-\frac{1}{\alpha})\theta^{\alpha/(\alpha-1)}, \theta \in [0,1/2], \tag{31}$$

$$p^R(\theta) = \theta^{\alpha/(\alpha-1)} - (1-\frac{1}{\alpha})(1-\theta)^{\alpha/(\alpha-1)},\ \theta \in [1/2,1]. \tag{32}$$

9.3.1 Total welfare, surplus and profits

Next, we present the closed-form solutions for the total welfare, surplus and profits. Since firms are symmetric, it is sufficient to present the results from firm L. The total welfare in this case is given by

$$\begin{aligned} w^L_{Both} &= \int_0^{1/2} [(1-\frac{1}{\alpha})(1-\theta)^{\alpha/(\alpha-1)}]d\theta \\ &= \frac{(\alpha-1)^2}{\alpha(2\alpha-1)}(1-2^{-(2\alpha-1)/(\alpha-1)}). \end{aligned}$$

The total consumer surplus is given by

$$\begin{aligned} s^L_{Both} &= \int_0^{1/2} \left[(1-\frac{1}{\alpha})\theta^{\alpha/(\alpha-1)}\right] d\theta \\ &= \frac{(\alpha-1)^2}{\alpha(2\alpha-1)} \cdot 2^{-(2\alpha-1)/(\alpha-1)}. \end{aligned}$$

The total profit is given by

$$\begin{aligned} \pi^L_{Both} &= w^L_{Both} - s^L_{Both} \\ &= \frac{(\alpha-1)^2}{\alpha(2\alpha-1)}(1-2^{-\alpha/(\alpha-1)}). \end{aligned}$$

Proof of Corollary 1

First, by comparing the prices of firm L in the No-PPQ and Both-PPQ cases from equations (24) and (31), we can show the difference when $\alpha = 2$ is given by the following equation:

$$(1-2\theta)\left(\frac{-2\theta+3}{4}\right) - \left[(1-\theta)^2 - \frac{1}{2}\theta^2\right] = \frac{1}{4}\left(2\theta^2-1\right) < 0.\ \forall\theta \in [0, \frac{1}{2}]$$

Similarly, we can show that the price of firm R in the Both-PPQ case is higher than that in the No-PPQ case.

For the case when $\alpha = 2$, and firm R has PPQ, the price function of firm L is given by

$$p^L_R(\theta) = \theta^2 - 2\theta - \sqrt{3} + \frac{5}{2}, \quad \theta \in [0, 2-\sqrt{3}]. \tag{33}$$

The price of firm L in the No-PPQ case is given by

$$(1-2\theta)\left(\frac{-2\theta+3}{4}\right) \tag{34}$$

Comparing these two equations, we have

$$\left(\theta^2 - 2\theta - \sqrt{3} + \frac{5}{2}\right) - (1-2\theta)\left(\frac{-2\theta+3}{4}\right) = \frac{7}{4} - \sqrt{3} = 0.0179.$$

The last part of this corollary states that in the case when only firm R has PPQ, some consumers in L's market segment may receive lower prices from R. We can verify this by looking at the price of the marginal consumer located very close to $\theta = 2-\sqrt{3}$. This is given by

$$\begin{aligned} p^R_R(\theta) &= \theta^2 - \max\ (0, 3\theta + 5\sqrt{3} - 2\theta\sqrt{3} - \frac{17}{2}), \\ &\Longrightarrow p^R_R(2-\sqrt{3}) = 0.0359 < p^L_N(2-\sqrt{3}) = 0.286. \end{aligned}$$

Proof of Proposition 4

We first show that the surplus is lowest at $\theta = 1/2$ in the No-PPQ case.

$$\frac{ds^L(\theta)}{d\theta} = \frac{d}{d\theta}\left[\frac{\alpha-1}{2\alpha}(1-2\theta)^{\alpha/(\alpha-1)}\right] = -(1-2\theta)^{1/(\alpha-1)} < 0, \forall\theta \in [0, \frac{1}{2}).$$

$$\frac{ds^R(\theta)}{d\theta} = \frac{d}{d\theta}\left[\frac{\alpha-1}{2\alpha}(2\theta-1)^{\alpha/(\alpha-1)}\right] = (2\theta-1)^{1/(\alpha-1)} > 0, \forall\theta \in (\frac{1}{2}, 1].$$

Next, we show that the surplus is highest at $\theta = 1/2$ in the Both-PPQ case.

$$\frac{ds^L(\theta)}{d\theta} = \frac{d}{d\theta}\left[(1-\frac{1}{\alpha})\theta^{\alpha/(\alpha-1)}\right] = \theta^{1/(\alpha-1)} > 0, \forall\theta \in [0, \frac{1}{2}). \tag{35}$$

$$\frac{ds^R(\theta)}{d\theta} = \frac{d}{d\theta}\left[(1-\frac{1}{\alpha})(1-\theta)^{\alpha/(\alpha-1)}\right] = -(1-\theta)^{1/(\alpha-1)} < 0, \forall\theta \in (\frac{1}{2}, 1]. \tag{36}$$

Lastly, we derive the regions in which the consumer surplus from buying from firm L is higher in the No-PPQ case than that in the Both-PPQ case.

$$\frac{\alpha-1}{2\alpha}(1-2\theta)^{\alpha/(\alpha-1)} \geq (1-\frac{1}{\alpha})\theta^{\alpha/(\alpha-1)}$$
$$\Leftrightarrow \theta \leq \frac{1}{2+2^{(\alpha-1)/\alpha}}, \forall\ \theta \in [0, \frac{1}{2}).$$

As a result, the value of $\widehat{\theta}$ in the main text is $\frac{1}{2+2^{(\alpha-1)/\alpha}}$. Similarly, by symmetry, we can show that when $\theta \geq \widehat{\theta}$, the consumer surplus from buying from firm R is higher in the No-PPQ case than that in the Both-PPQ case.

Proof of Corollary 2

We define welfare of a consumer as the sum of the firm's profit from that consumer and the surplus accruing to that consumer. First note that when only one firm has PPQ, there are three regions in the market which we need to consider in order to derive the stated result. In the first region where $\theta \in [0, B]$, the welfare generated from firm L is the same as that in the No–PPQ case. This is because the quality schedule of the firm L (the No–PPQ firm) remains the same in each case. The second region under consideration extends from $\theta \in [B, \frac{1}{2}]$. We analyze the welfare in this region at the end. In the third region where $\theta \in [\frac{1}{2}, 1]$, the welfare generated by firm R is higher in this case compared to the No–PPQ case. This is because these consumers are located closer to R and get the socially optimal quality from firm R. Given these results, it is sufficient for us to compare the welfare in the second region. When $\alpha = 2$, the corresponding expressions for firm L and for firm R, respectively are given by:

$$w_N^L(\theta) = (1-\theta)(1-2\theta) - \frac{1}{2}(1-2\theta)^2,\ \theta \in [B, \frac{1}{2}],$$
$$w_R^R(\theta) = \theta \cdot \theta - \frac{1}{2}\theta^2,\ \theta \in [B, \frac{1}{2}].$$

Recall that $B = 2 - \sqrt{3}$. If we compare the welfare of the marginal consumer in the case when only R has PPQ, we can find that the welfare of this consumer is lower than what (s)he gets in the No-PPQ case as given by the following equation:

$$\left[w_N^L(\theta) - w_R^R(\theta)\right]|_{\theta=B} = \left(-\frac{1}{2}\right)\left(2\theta + \theta^2 - 1\right)|_{\theta=2-\sqrt{3}} = 0.196.$$

The proof of the Part (ii) of the result that each consumer in the Both-PPQ case has the highest welfare is immediate because the quality is the first-best solution and each consumer buys from the firm situated closer to him. This proves the corollary.

Proof of Proposition 5

It is sufficient to compare the profits in the case in which both firms adopt PPQ with that when neither firm adopts PPQ. The profit in the No-PPQ case is given by

$$\pi_N^L = \int_0^B \left[(1-2\theta)q^L(\theta) - \frac{(q^L)^\alpha(\theta)}{\alpha} - s^L(B)\right] d\theta$$

$$= \int_0^{1/2} [(1-2\theta)^{1/(\alpha-1)}\left(\frac{2\theta+\alpha-1}{2\alpha}\right)]d\theta \tag{37}$$

$$= \frac{(\alpha-1)^2}{2\alpha(2\alpha-1)}. \tag{38}$$

The profit in the Both-PPQ case is given by

$$\pi_{Both}^L = \int_0^{1/2} \left[(1-\theta)q^L(\theta) - s^L(\theta) - \frac{(q^L)^\alpha(\theta)}{\alpha}\right] d\theta$$

$$= \int_0^{1/2} [(1-\frac{1}{\alpha})((1-\theta)^{\alpha/(\alpha-1)} - \theta^{\alpha/(\alpha-1)})]d\theta \tag{39}$$

$$= (2-2^{\frac{1-\alpha}{\alpha-1}})\frac{(\alpha-1)^2}{2\alpha(2\alpha-1)}. \tag{40}$$

Since the first term in (40) is greater than one for all $\alpha > 1$, we find that (40) is always greater than (38). Thus, the profit of the Both-PPQ case is always higher.

Proof of Proposition 6

In the following paragraphs, we solve for the optimal quality schedules and the location of the marginal consumer in each case. For the quality schedule of the PPQ firm, all of the results in the Both-PPQ case still apply because the two firms compete for each individual consumer (each θ). Hence, the results do not depend on the distribution and range of θ.

- **Determining B_1(No-PPQ Case)**

First, note that the proof of $\frac{ds^R(\theta)}{d\theta} = q^R(\theta)$ and $\frac{ds^L(\theta)}{d\theta} = -q^L(\theta)$ in Lemma 1 still applies because the proof does not depend on the value of the upper bound, r. Consider first the objective function of firm R. This is given by

$$\max_{q^R(\theta),\, s^R(B_1)} \pi_N^R \text{ where } \pi_N^R = \int_{B_1}^r \left[\theta q^R(\theta) - \frac{(q^R)^\alpha(\theta)}{\alpha} - s^R(B_1) - \int_{B_1}^\theta q^R(t)dt\right] d\theta.$$

The last term in the integrand can be simplified as follows:

$$\int_{B_1}^r \left[\int_{B_1}^\theta q^R(t)dt\right] d\theta = \int_{B_1}^r \left[\int_t^r q^R(t)d\theta\right] dt = \int_{B_1}^r q^R(t)(r-t)dt = \int_{B_1}^r q^R(\theta)(r-\theta)dt.$$

Note that the only difference between this case and our benchmark symmetric No-PPQ case is that the upper bound of integral here is r rather than 1. Substituting this term back in the objective function, we have

$$\begin{aligned}\pi_N^R &= \int_{B_1}^{r}\left[\theta q^R(\theta) - \frac{(q^R)^\alpha(\theta)}{\alpha} - s^R(B_1) - q^R(\theta)(r-\theta)\right]d\theta,\\ &= \int_{B_1}^{r}\left[(2\theta - r)q^R(\theta) - s^R(B_1) - \frac{(q^R)^\alpha(\theta)}{\alpha}\right]d\theta.\end{aligned}$$

The objective function of firm L is the same as that in the benchmark case and can be rewritten as

$$\max_{q^L(\theta),\, s^L(B_1)} \pi_N^L \quad \text{where } \pi_N^L = \int_0^{B_1}\left[(1-2\theta)q^L(\theta) - s^L(B_1) - \frac{(q^L)^\alpha(\theta)}{\alpha}\right]d\theta.$$

The optimal quality schedule can be determined by maximizing the integrand pointwise (the terms in the bracket). The results are listed in Proposition 6.

Differentiating the objective function of firm R with respect to $s^R(B_1)$, we have

$$\int_{B_1}^{r} -1\cdot d\theta - \left[(2B_1 - r)q^R(B_1) - s^R(B_1) - \frac{(q^R)^\alpha(B_1)}{\alpha}\right]\frac{dB_1}{ds^R(B_1)} = 0. \tag{41}$$

As before, $\frac{dB_1}{ds^R(B_1)}$ can be derived by equating the consumer surplus from two firms offered at the boundary to the marginal consumer.

$$\frac{dB_1}{ds^R(B_1)} = \frac{1}{-q^L(\theta)}.$$

Substituting this back into (41), we have

$$(B_1 - r) - \left[(2B_1 - r)q^R(B_1) - \frac{(q^R)^\alpha(B_1)}{\alpha} - s^R(B_1)\right]\frac{1}{-q^L(B_1)} = 0. \tag{42}$$

This implies that

$$s^R(B_1) = q^L(B_1)(B_1 - r) + \left[(2B_1 - r)q^R(B_1) - \frac{(q^R)^\alpha(B_1)}{\alpha}\right]. \tag{43}$$

Similarly, we can derive a necessary condition of firm L which is given by

$$-B_1 + \left[(1-2B_1)q^L(B_1) - \frac{(q^L)^\alpha(B_1)}{\alpha} - s^L(B_1)\right]\frac{1}{q^R(B_1)} = 0. \tag{44}$$

This implies that

$$s^L(B_1) = -q^R(B_1)B_1 + \left[(1-2B_1)q^L(B_1) - \frac{(q^L)^\alpha(B_1)}{\alpha}\right]. \tag{45}$$

Since $s^R(B_1) = s^L(B_1)$, we can equate (43) and (45) to derive the following equation.

$$q^L(B_1)(B_1 - r) + \left[(2B_1 - r)q^R(B_1) - \frac{(q^R)^\alpha(B_1)}{\alpha}\right] = -B_1 q^R(B_1) + \left[(1-2B_1)q^L(B_1) - \frac{(q^L)^\alpha(B_1)}{\alpha}\right].$$

Rearranging the terms and substituting $q^L(B_1) = 1 - 2B_1$ and $q^R(B_1) = 2B_1 - r$, we have

$$\begin{aligned}&\left[(3B_1 - r)q^R(B_1) - \frac{(q^R)^\alpha(B_1)}{\alpha}\right] = \left[(1 - 3B_1 + r)q^L(B_1) - \frac{(q^L)^\alpha(B_1)}{\alpha}\right],\\ \Leftrightarrow &\left[(3B_1 - r)(2B_1 - r) - \frac{(2B_1 - r)^\alpha}{\alpha}\right] = \left[(1 - 3B_1 + r)(1 - 2B_1) - \frac{(1-2B_1)^\alpha}{\alpha}\right],\\ \Leftrightarrow &\left[\alpha(3B_1 - r)(2B_1 - r) - (2B_1 - r)^\alpha\right] = \left[\alpha(1 - 3B_1 + r)(1 - 2B_1) - (1 - 2B_1)^\alpha\right].\end{aligned} \tag{46}$$

B_1 is the solution that satisfies this equation. A closed-form solution of the general case is not tractable. However, it is can be derived numerically for specific values of α.

- **Determining B_2(Only Firm R PPQ Case)**

In this case, the procedure to derive B_2 is similar to that for deriving B_1 except that firm R's quality schedule is now different. From (45), from firm L's perspective, $s^L(B_2)$ is given by

$$s^L(B_2) = -q^R(B_2)(B_2) + \left[(1-2B_2)q^L(B_2) - \frac{(q^L)^\alpha(B_2)}{\alpha}\right]. \tag{47}$$

$s^R(B_2)$ is determined by the socially optimal surplus function of buying from firm R. Since

$$\begin{aligned} s^L(B_2) &= s^R(B_2), \\ \iff s^L(B_2) &= \max_{q^R(B_2)} \quad B_2\, q^R(B_2) - \frac{(q^R)^\alpha(B_2)}{\alpha}, \\ \iff s^L(B_2) &= (1-\frac{1}{\alpha})B_2^{\alpha/(\alpha-1)}. \end{aligned} \tag{48}$$

Intuitively if $s^L(B_2) < (1-\frac{1}{\alpha})B_2^{\alpha/(\alpha-1)}$, because of firm R's ability to target consumers, the marginal consumer at B_2 will end up buying from firm R. Hence, for all consumers of firm L, $s^L(B_2)$ must be greater than $(1-\frac{1}{\alpha})B_2^{\alpha/(\alpha-1)}$. The marginal consumer is determined by equating the two surplus functions. From equations (47) and (48), we have

$$-B_2 \cdot q^R(B_2) + \left[(1-2B_2)q^L(B_2) - \frac{(q^L)^\alpha(B_2)}{\alpha}\right] = (1-\frac{1}{\alpha})B_2^{\alpha/(\alpha-1)}.$$

Substituting $q^L(B_2)$ by $(1-2B_2)^{1/(\alpha-1)}$ and $q^R(B_2)$ by $B_2^{1/(\alpha-1)}$, we have

$$-B_2 \cdot B_2^{1/(\alpha-1)} + \left[(1-2B_2)^{\alpha/(\alpha-1)} - \frac{(1-2B_2)^{\alpha/(\alpha-1)}}{\alpha}\right] = (1-\frac{1}{\alpha})B_2^{\alpha/(\alpha-1)},$$

which on further simplification can be written as

$$\begin{aligned} (1-\frac{1}{\alpha})(1-2B_2)^{\alpha/(\alpha-1)} &= (2-\frac{1}{\alpha})B_2^{\alpha/(\alpha-1)}, \\ \Leftrightarrow \frac{1-2B_2}{B_2} &= \left(\frac{2\alpha-1}{\alpha-1}\right)^{(\alpha-1)/\alpha}, \\ \Leftrightarrow B_2 &= \left[2+\left(\frac{2\alpha-1}{\alpha-1}\right)^{(\alpha-1)/\alpha}\right]^{-1}. \end{aligned} \tag{49}$$

- **Determining B_3 (Only Firm L PPQ Case)**

Similar to the procedure of determining B_2, firm R's consumer surplus at B_3 is derived by (43) and given as

$$s^R(B_3) = q^L(B_3)(B_3 - r) + \left[(2B_3 - r)q^R(B_3) - \frac{(q^R)^\alpha(B_3)}{\alpha}\right].$$

Since firm L has PPQ, the optimal quality schedule is given by

$$q^L(B_3) = (1 - B_3)^{1/(\alpha-1)}.$$

Hence, the socially optimal consumer surplus curve of firm L at B_3 is given by

$$s^L(B_3) = (1 - \frac{1}{\alpha})(1 - B_3)^{\alpha/(\alpha-1)}.$$

Since $s^R(B_3) = s^L(B_3)$, we have

$$(B_3 - r)q^L(B_3) + \left[(2B_3 - r)q^R(B_3) - \frac{(q^R)^\alpha(B_3)}{\alpha}\right] = (1 - \frac{1}{\alpha})(1 - B_3)^{\alpha/(\alpha-1)}.$$

After substituting $q^L(B_3)$ by $(1-B_3)^{1/(\alpha-1)}$ and $q^R(B_3)$ by $(2B_3-r)^{1/(\alpha-1)}$, this equation can be written as

$$(B_3 - r)(1 - B_3)^{1/(\alpha-1)} + (1 - \frac{1}{\alpha})(2B_3 - r)^{\alpha/(\alpha-1)} = (1 - \frac{1}{\alpha})(1 - B_3)^{\alpha/(\alpha-1)}. \tag{50}$$

B_3 is the solution that satisfies this equation but the closed-form solution of B_3 is intractable. However, for any given value of α and r, B_3 can be solved for numerically.

- **Determining B_4 (Both PPQ Case)**

Both firms compete at the individual consumers level. In mathematical terms, they maximize their objective function as if θ is given. Hence, the solutions do not depend on the distribution of θ at all. As a result, it is immediate that $B_4 = 1/2$ just as we have in the baseline case.

9.4 Specific example ($\alpha = 2$)

For illustrative purposes we solve the cases when the cost function is quadratic.

Case 1: B_1.

When $\alpha = 2$, equation (46) can be further simplified as

$$\begin{aligned}
&\left[2(3B_1 - r)(2B_1 - r) - (2B_1 - r)^2\right] = \left[2(1 - 3B_1 + r)(1 - 2B_1) - (1 - 2B_1)^2\right], \\
&\Leftrightarrow \left(2(3B_1 - r)(2B_1 - r) - (2B_1 - r)^2\right) - \left(2(1 - 3B_1 + r)(1 - 2B_1) - (1 - 2B_1)^2\right) = 0, \\
&\Leftrightarrow \left(2r - 6B_1 + 2rB_1 - r^2 + 1\right) = 0, \\
&\Leftrightarrow B_1 = \frac{2r - r^2 + 1}{6 - 2r}.
\end{aligned} \tag{51}$$

Case 2: B_2.

Equation (49) can be further simplified as

$$B_2 = 1/\left[2 + (3)^{1/2}\right] = 2 - \sqrt{3}.$$

Case 3: B_3.

When $\alpha = 2$, equation (50) is equivalent to

$$(B_3 - r)(1 - B_3) + \frac{1}{2}(2B_3 - r)^2 = \frac{1}{2}(1 - B_3)^2.$$

The solution of this equation is given by $B_3 = r + \sqrt{5 - 2r} - 2$.

When $r = 1$, $B_3 = \sqrt{3} - 1$. Comparing this to the solution of B_2, by the symmetry of the game we must have

$$\begin{aligned} & B_3 = 1 - B_2, \\ & \Leftrightarrow \sqrt{3} - 1 = 1 - \left(2 - \sqrt{3}\right), \end{aligned}$$

which verifies our derivations.

Proof of Propositions 7 and 8

The proofs of Propositions 7 and 8 follow directly from the results of Proposition 6, by comparing the different quality schedules. We only need to show that

$$\begin{aligned} & (1 - \theta)^{1/(\alpha - 1)} \geq (1 - 2\theta)^{1/(\alpha - 1)} \\ & \qquad \Leftrightarrow (1 - \theta) \geq (1 - 2\theta) \Leftrightarrow \theta \geq 0. \end{aligned}$$

Further, we need to show that

$$\begin{aligned} & \theta^{1/(\alpha - 1)} \geq (2\theta - r)^{1/(\alpha - 1)} \\ & \qquad \Leftrightarrow \theta \geq (2\theta - r) \Leftrightarrow r \geq \theta. \end{aligned}$$

The last equality is true because θ is uniformly distributed between $[0, r]$.