Recruitment, training and career concerns*

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Abstract

We examine training and recruitment policies in a two-period model that nests two forms of production, "routine" work where ability and effort are substitutes and "creative" work where they are complements. Alternative ways of improving average ability have opposite implications for agents' career concerns. While teaching to the top (training complementary to ability) or identifying star performers increases agents' career concerns, teaching to the bottom has the opposite effect. The paper also makes more general comments relating to models of reputation.

Keywords: recruitment, training, career concerns, reputation

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> In the next decade and beyond, the ability to attract, develop, retain and deploy staff will be the single biggest determinant of a professional service firm's success.
>
> Maister [1997] p.189

1 Introduction

Popular press and academic literature have come to stress the importance of recruitment and development of staff in industries where human capital plays a critical role.[1] This popular literature tends to recommend recruiting the "best" and training them. There are, of course, costs as well as benefits to recruitment and training. In this paper, we highlight that there may be indirect costs and benefits through the effect on employees' incentives. Thus, the central contribution of this paper is to highlight that in addition to affecting the quality of staff, training and recruitment policies also play a role in affecting the behaviour of employees through their career concern incentives.

In human capital intensive industries including professional services such as the law, audit, consulting, and architecture, career concern incentives are of paramount importance. As discussed in Fama [1980] and Holmström [1982/99], agents may exert effort in trying to persuade future employers that they are of high ability, that is, they may be motivated by career concerns.[2] It is clear that their motivation will depend on the beliefs of potential future employers; and a principal contribution of this paper is to note that recruitment

[1] This literature includes Michaels et al. [2001], Maister [1997], Smart [1999], Hacker [2001] and no doubt many others.

[2] A wide literature has extended and considered applications of the career concerns framework. Most relevant to this paper, Dewatripont, Jewitt and Tirole [1999] provide a thorough analysis characterizing the impact of different information structures (mappings from ability and effort into observable outcomes). Others have focused on specific applications, whose primary effect is to alter such information structures, in particular through teamwork (Meyer [1994] and Jeon [1996]) and delegation of power (Ortega [2003], Blanes-i-Vidal [2007]). More recently Harstad [2007], Casas-Arce [2005] and Martinez [2005] consider different micro-foundations for career concerns models with non-linear returns functions and their implications.

and training policies will change these.

In particular, we highlight that while many different training and recruitment policies might have the same effect on the average level of ability of employees, they can have very different (and indeed exactly opposite) implications for incentives.

Our model allows us to present and discuss two different kinds of training or productivity enhancement. One that affects an employee's "core knowledge" that is valuable for tasks which do not require effort, and another that raises productivity for work that does require effort. We show that whichever policy is most effective in raising the overall productivity of those workers who are already most productive will lead to higher incentives for employees. Similarly, recruitment policies that are more focused on finding the very best workers lead to higher incentives for employees and recruitment policies that ensure that the least able are seldom recruited reduce employees' incentives.

There are two channels through which training can have an effect. First, training that is geared towards the most able increases the dispersion of the possible types of an employee so that observations are more informative. Second, training the top implies there is a greater pecuniary payoff to revealing yourself to be there. Recruitment policies that focus on identifying superstars rather than identifying inept performers have similar effects through both channels.

We can distinguish between different training policies and highlight that the key is the effect on the most productive since our model nests two models of production. One in which ability substitutes for effort (one might think in this case of ability as signifying knowledge of a routine task) and another in which ability and effort are complements (one might think of more able agents in this case as more likely to have the inspiration which allows hard work to reap rewards).

The career concerns literature and most of the reputation literature has viewed effort and ability as substitutes. However, more recent literature on reputational concerns in effect takes the opposite view and suggests that this view of reputation might lead to somewhat different effects.[3] In particular Mailath and Samuelson [2001] show that when reputation is a concern to avoid appearing inept then in a finite-horizon model reputation effects cannot arise. Further, Moav and Neeman [2005] suggest that more precise information can reduce incentives. We discuss at some length how our different views of the production process relate to this recent literature on reputation. Further, we make the methodological point that when effort and ability are not perfect complements, reputation effects do arise and are similar to the substitutes case.

2 Model

We introduce a two period model with a continuum of types of agents parameterized by $t \in [0, 1]$. Specifically in Period 1 a type t agent will have no strategic decision to make with probability t and in this case will succeed (for example by producing a high quality product) with probability μ and fail with probability $1 - \mu$. Otherwise (with probability $1 - t$) the agent must make an effort decision. Note that she only gets to make a strategic decision when her effort has an effect; or, equivalently, she knows which kind of task she is performing before she takes her effort decision.[4] In this latter case, when she chooses effort e, she succeeds with probability $\alpha + e$. Thus overall a t-type agent exerting effort e when given the opportunity to exert effort would succeed with probability $t\mu + (1-t)(\alpha+e)$ and fail otherwise. Effort is costly and, specifically, exerting effort e costs the agent $\frac{e^2}{2\gamma}$, where $\gamma < 1 - \alpha$.

[3] See, in particular, Tadelis [2002], Mailath and Samuelson [2001] and Bar-Isaac [2007].

[4] Similar results can be obtained when the agent does not know her own type and does not know which kind of task she faces. If she knows her own type but does not know the kind of task that she faces then different types would make different effort decisions.

Let $g(t)$ denote the distribution function for the types of agent and let T denote the average type (according to the *ex-ante* beliefs) $T = \int_0^1 tg(t)dt$ and let $V = \int_0^1 (t-T)^2 g(t)dt$ denote the variance of this prior distribution. This distribution function of types is common knowledge among the agent and employers.

Employers are risk neutral, value a success at 1 and a failure at 0 and they Bertrand compete for the agent's service in each period.[5] Moreover, outcomes are observable but effort is not observable and contracts are incomplete, so that in effect an agent is paid in advance at a wage which is simply the employers' common belief that the agent will produce a success.

There are two periods of trade, and outcomes are observed (and beliefs revised) in between the two periods.[6] Specifically, timing is as follows:

1. Period 1
    (a) employers Bertrand compete for the agent's service
    (b) the agent decides the level of effort if appropriate (that is if it is a task where effort will make a difference)
    (c) success/failure commonly observed
    (d) employers update beliefs according to Bayes rule
2. Period 2: employers Bertrand compete for the agent's service

Notice that in period 2, we could allow the agents the opportunity to exert effort but no agent would do so. Note that whether the agent knows her type or not would have

[5]It is sufficient to consider two employers bidding for the services of a single employee. More generally, the assumption that employers Bertrand compete for the product is not crucial, similar results would hold so long as the price paid was increasing in the customers' expected likelihood that the agent will be successful.

[6]One need not take the two periods of the model literally, rather the second period can be thought of as a reduced form payoff for a given reputation level, albeit one that is linear in the reputation.

no effect on this model since she has no ability to signal her type (we rule out long-term and outcome contingent contracts) and at the point where an agent has to make an effort decision then the problem is identical for all types.

We suppose that the agent weighs the two periods equally and maximizes the sum of profits for the two periods. We solve for the effort exerted in the Perfect Bayesian equilibrium.

2.1 Interpreting the model

It is worth highlighting that type t captures the agent's likelihood of getting an opportunity to engage in strategic behaviour. One cannot interpret a high t as low or high ability until all the parameters of the model are specified

The model is intended to reflect that agents might be confronted with a variety of different tasks, and the nature of the particular task undertaken is unobserved by employers. For example, employers hiring consultants find it difficult to determine the extent to which the project that they are assigning is a complex one or a simple one. Similarly, the difficulty of the project depends on the consultant's ability and experience. Depending on the value of μ, the model allows for somewhat different interpretations of the productive process and of the interpretation of high values of t as reflecting high or low ability.

Specifically, when μ is sufficiently high, high t reflects high ability; an agent with a high value of t finds it costless to succeed in a wide range of tasks, in this case ability and effort are substitutes and an agent would like employers to believe that she has a high value of t. One could think of this case as representing "routine" work, where more able workers know how to do more things and if they know how to perform the task that they are assigned they will succeed with high probability.[7] If they do not have the requisite knowledge they

[7] Another interpretation for the high μ case is that agents with a high t get it "right first time", but for all agents when they do not, there is the opportunity of exerting effort to try to "fix mistakes".

could exert effort to acquire it, by reading the appropriate manuals for example. Examples include translation work, mechanics, and routine work in low levels of management in an organizational hierarchy.

In contrast, when μ is sufficiently low, then one can think of agents for whom $(1-t)$ is higher as being more able, and effort and ability as complements. In this case even if the agent has some understanding of the task, exerting effort will still improve the outcome, however if she has no understanding of the task then she will surely fail. Here, one needs some ability in order to have any chance of success (one needs ability to have the flash of inspiration) but this in itself will not guarantee success, hard work is also required. This may be more appropriate for creative work, such as writing a Ph.D., writing advertisements or high level management which is not routine.

Note, that in the case that $\mu = 0$, the agent would prefer employers to believe that she is a type with a low value of t and so when $\mu = 0$, one should think of agents with low values of t as more able.

In application, of course many jobs will include elements of both creative and routine work, however for different jobs or at different times in an individual's career (or within an organizational hierarchy) it will be more appropriate to think of work as primarily of one or other of these two production types.

In both these cases we can think of more able agents as having facility in some tasks but not in others. The difference in productivity for a task in which one has facility and in which one does not when exerting no effort (which is the case in period 2) is simply given by $|\mu - \alpha|$.

3 Equilibrium analysis

Trivially, when faced with an effort decision, all agents make the same choice of effort. This follows since the benefits, as determined by equilibrium beliefs following success and failure, are identical for all agents and the costs are identical for all agents, even though the frequency with which they have to make such decisions alters.

Suppose that employers anticipate that agents exert effort x when they have an opportunity to exert effort in the first period. Then using Bayes' rule and rearranging terms, and as proven in Appendix A, the wages that employers would pay following a success and failure respectively are given by:

$$S(x) \quad : \quad = \alpha + (\mu - \alpha)\left[T + \frac{\mu - \alpha - x}{\alpha + x + T(\mu - \alpha - x)}V\right], \text{ and} \tag{1}$$

$$F(x) \quad : \quad = \alpha + (\mu - \alpha)\left[T - \frac{\mu - \alpha - x}{1 - \alpha - x - T(\mu - \alpha - x)}V\right] \tag{2}$$

An agent's problem, where relevant, is to choose e to maximize

$$(\alpha + e)S(x) + (1 - \alpha - e)F(x) - \frac{e^2}{2\gamma}. \tag{3}$$

The first order condition yields that $\frac{e}{\gamma} = S(x) - F(x)$ and a rational expectations equilibrium is defined by the effort level x that satisfies:

$$\frac{x}{\gamma} = S(x) - F(x). \tag{4}$$

Given this characterization, the following results ensue (and are proven in the Appendix).

Proposition 1 *Equilibrium effort is well-defined and unique. Further (i) The equilibrium effort* e^* *is lower than the efficient solution* $e^{fb} = \gamma$. *(ii) If* $\mu > \alpha$ *then* $\mu > \alpha + e^*$.

Note in particular that since equilibrium effort is unique, comparative statics exercises are well defined and can be explored. Note, further, as discussed at some length in Section 2.1, that in the case where μ is high ($\mu > \alpha$) or μ is low ($\mu < \alpha$) the model has natural interpretations as capturing "routine" and "creative" production respectively. Finally, note that when conducting these comparative statics results, we assume that parameters remain common knowledge among all agents, employers and potential employers; in particular, in the context of Sections 5.1 and 5.2, when we consider a firm choosing a recruitment or training strategy, it is assumed that these choices are known and understood by all market participants.

4 Comparative Static Results

The first result is a very intuitive one, if effort is less costly then the agent will exert more effort (when relevant) in equilibrium.

Proposition 2 *The equilibrium effort* e^* *is increasing in* γ

Proof. It is convenient to define $h(x) := S(x) - F(x) - \frac{x}{\gamma}$.

Note that by the arguments of the proof of Proposition 1, $h(x)$ is decreasing in the range $(0, \gamma)$. Using the Implicit Function Theorem $\frac{de^*}{da} = -\frac{\frac{\partial h(a)}{\partial a}}{\frac{\partial h(e)}{\partial e}}$ and since $\frac{\partial h(e)}{\partial e} < 0$, the sign of $\frac{de^*}{da}$ is simply the sign of $\frac{\partial h(x,a)}{\partial a}$ and so it is sufficient to consider that expression for $a = \gamma$, that is to consider $\frac{\partial h(x,\gamma)}{\partial \gamma}$. Recall

$$h(x) = -\frac{x}{\gamma} + S(x) - F(x). \tag{5}$$

and so taking the derivative with respect to γ, we obtain

$$\frac{\partial h(x,\gamma)}{\partial \gamma} = \frac{x}{\gamma^2} > 0 \tag{6}$$

Therefore, we conclude that the optimal effort e^* is increasing in γ. ■

Next we turn to comparative statics with respect to V. The intuition here is clear, the greater the variance in the distribution of types, the more scope that the observation of a success or failure has to shift beliefs and the associated rewards. This is a familiar intuition (from Holmström [1982/99] for example). Here we highlight that the result depends on V and no other characteristics of the distribution (in particular, we do not restrict that $g(.)$ be Normal).

Proposition 3 *The optimal effort e^* is increasing in V.*

Proof. As in the proof of Proposition 2, the sign of $\frac{de^*}{dV}$ is simply the sign of $\frac{\partial h(x,V)}{\partial V}$. Taking the derivative with respect to V yields:

$$\frac{\partial h}{\partial V} = \frac{\partial (S-F)}{\partial V} = \frac{(\mu-\alpha)(\mu-\alpha-x)}{[(\mu-\alpha-x)T+(\alpha+x)]\,[(1-\alpha-x)-(\mu-\alpha-x)T]} > 0. \tag{7}$$

■

Notice that while increasing V or γ has a clear monotonic effect on effort, the comparative statics with respect to other parameters depend on which of the two interpretations alluded to in the description of the model in Section 2 applies, that is, whether an agent's reputational concern is to try to convince employers that she is a "high t" type or a "low t" type.

First, we consider comparative statics with respect to α. Underlying the following result are two effects, first that it is more important to show oneself to be at the top of the ability distribution (or having facility in a greater range of tasks that is high t in the case when μ is high, low t in the case when μ is low) the greater the difference between the productivity of an agent in a task in which she has facility and her productivity in one which she does not, regardless of her level of effort (that is the greater is $|\mu - \alpha - x|$ for all effort levels x). Secondly as $|\mu - \alpha - x|$ increases then an observation of success or failure becomes more informative. We distinguish explicitly between these two effects in the discussion in Section 5.3. In particular therefore when μ is high, one would expect that an increase in α should reduce effort, but when μ is low, it would increase equilibrium effort. Note however that in all cases, increasing α raises the average productivity of the agent. Similar considerations apply with regard to comparative statics in μ. The proposition below demonstrates that these intuitions are borne out.

Proposition 4 *Equilibrium effort is increasing in α but decreasing in μ when $\mu < \alpha$ but decreasing in α and increasing in μ when $\mu > \alpha$.*

Proof. As in the proof of Proposition 2, it is sufficient to consider $\frac{\partial h}{\partial \alpha}$, and $\frac{\partial h}{\partial \mu}$.

$$\begin{aligned}\frac{\partial h(x,\alpha)}{\partial \mu} = \frac{\partial (S-F)}{\partial \mu} = \quad & \left[\frac{(\mu-\alpha-x)V}{(\mu-\alpha-x)T+\alpha+x} + \frac{(\mu-\alpha-x)V}{1-\alpha-x-(\mu-\alpha-x)T}\right] \\ & +(\mu-\alpha)\left[\frac{(\alpha+x)V}{((\mu-\alpha-x)T+\alpha+x)^2} + \frac{(1-\alpha-x)V}{(1-\alpha-x-(\mu-\alpha-x)T)^2}\right]\end{aligned} \tag{8}$$

Notice that the denominators are always positive. Since, by Proposition 1(ii) when $\mu > \alpha$ then also $\mu > \alpha + x$, it follows that in this case $\frac{\partial(S-F)}{\partial \mu} > 0$. If instead, $\mu < \alpha$, since by Proposition 1 $x > 0$ then $\frac{\partial(S-F)}{\partial \mu} < 0$.

Similarly

$$\begin{aligned}\frac{\partial h(x,\alpha)}{\partial \alpha} = \frac{\partial (S-F)}{\partial \alpha} = \quad & -\left[\frac{(\mu-\alpha-x)V}{(\mu-\alpha-x)T+\alpha+x} + \frac{(\mu-\alpha-x)V}{1-\alpha-x-(\mu-\alpha-x)T}\right] \\ & +(\mu-\alpha)\left[\frac{-V\mu}{((\mu-\alpha-x)T+\alpha+x)^2} + \frac{-V(1-\mu)}{(1-\alpha-x-(\mu-\alpha-x)T)^2}\right]\end{aligned} \tag{9}$$

Again the denominators are always positive and if $\mu > \alpha$, then $\frac{\partial(S-F)}{\partial\alpha} < 0$ since $(\mu-\alpha-x)$ and $(\mu-\alpha)$ are positive, similarly if $\mu < \alpha$, then $\frac{\partial(S-F)}{\partial\alpha} > 0$. ■

5 Discussion

5.1 Targeted training or productivity enhancement

Our model allows for different kinds of training, or productivity enhancing technology. In the model, increasing α and increasing μ can both increase the ability of agents, and can readily be interpreted as the result of training directed towards different types of agents or different kinds of productivity-enhancing technologies. A firm may wonder whether there is much difference, for example, in giving employees access to a database that expands their core knowledge (increasing μ) or giving them access to software that allows for more effective work (increasing α).

As demonstrated in Proposition 4, these two means of increasing average ability have exactly opposite effects for equilibrium effort. In particular for low values of μ when an agent would like employers to believe that she is a "low t" type then raising the ability of a "low t" type relatively more than raising the ability of a "high t" type (either by decreasing μ or by increasing α) heightens this reputational concern. As discussed below, it does so through two channels, by raising the pecuniary value of showing oneself to be a higher type and by making the outcome more informative about the agent's type.

Similarly in the case where μ is high, then agents with high t are the most productive and a greater distinction between the most able and the least able (here by increasing μ or decreasing α) will heighten the reputational concern for agents seeking to convince consumers that they are the most able.

Thus in all cases raising the productivity of the most able agents (or reducing the productivity of the least able) increases the equilibrium effort.

5.2 Recruitment policies and searching for superstars

While training as described above affects the productivity of a given type and thereby affects the prior beliefs about an agent's productivity, interviewing and recruitment policies directly affect the initial belief about the distribution of types $g(t)$. When employers seek recruitment policies which select better agents, there are various ways in which this can be achieved. Consider the case when μ is high (so that types with high values of t are the better agents); a recruitment policy that selects better agents will lead to a shift in the prior distribution from $g(t)$ with associated T and V, to a different prior distribution $g'(t)$ with associated $T' > T$ and V'. Following Proposition 2, among all policies with the same effect on average ability (that generate the same T'), an employer would prefer to choose a policy that raised rather than reduced the variance of the distribution. When superstars and disastrous potential recruits are rare, then it follows that employers concerned with employees' efforts would be better using recruitment policies that concentrated more on ensuring that any potential superstars were recruited than ruling out the worst of the applicants.[8]

[8] For example, suppose that with no recruitment policy, types are distributed according to the degenerate distribution $g(0) = \frac{1}{10}$, $g(\frac{1}{2}) = \frac{4}{5}$ and $g(1) = \frac{1}{10}$ so that $T = \frac{1}{2}$ and $V = \frac{1}{20}$. Now consider, two recruitment policies which raise the average ability, one does so by reducing the probability of recruiting disasters. Specifically Policy A leads to the distribution $g_A(0) = \frac{1}{20}$, $g_A(\frac{1}{2}) = \frac{17}{20}$, and $g_A(1) = \frac{1}{10}$ so that $T_A = \frac{21}{40}$ and $V_A = \frac{1}{20}(\frac{21}{40})^2 + \frac{17}{20}(\frac{1}{40})^2 + \frac{1}{10}(\frac{19}{40})^2 = \frac{59}{1600}$. Policy B increases the probability of identifying superstars and leads to the distribution $g_B(0) = \frac{1}{10}$, $g_B(\frac{1}{2}) = \frac{3}{4}$, and $g_B(1) = \frac{3}{20}$, then $T_B = \frac{21}{40}$ and $V_B = \frac{1}{10}(\frac{21}{40})^2 + \frac{3}{4}(\frac{1}{40})^2 + \frac{3}{20}(\frac{19}{40})^2 = \frac{99}{1600}$. Since $V_B > V_A$ it follows by Proposition 2 that, while both policies raise average ability equally, the latter policy would lead to greater equilibrium effort compared to the first and so would be preferred.

5.3 The information and value effects

By adapting the model slightly to suppose that in period 2 a type t agent succeeds with probability $t\mu' + (1-t)\alpha'$, we can distinguish between two channels through which changes in ability as discussed in Proposition 4 and Section 5.1 affect incentives. Specifically, in this modified model $S - F = (\mu' - \alpha')\left[\frac{(\mu-\alpha-x)V}{(\mu-\alpha-x)T+\alpha+x} + \frac{(\mu-\alpha-x)V}{1-\alpha-x-(\mu-\alpha-x)T}\right]$ and similar qualitative results apply.

Proposition 5 *Equilibrium effort is increasing in* α *and* α' *but decreasing in* μ *and* μ' *when* $\mu, \mu' < \alpha$ *but decreasing in* α *and* α' *and increasing in* μ *and* μ' *when* $\mu, \mu' > \alpha$.

Proof. Similar to the proof of Proposition 4, it is sufficient to consider $\frac{\partial(S-F)}{\partial\alpha}$, $\frac{\partial(S-F)}{\partial\alpha'}$, $\frac{\partial(S-F)}{\partial\mu}$,and $\frac{\partial(S-F)}{\partial\mu'}$.

First

$$\frac{\partial(S-F)}{\partial\mu'} = \frac{(\mu-\alpha-x)V}{(\mu-\alpha-x)T+\alpha+x} + \frac{(\mu-\alpha-x)V}{1-\alpha-x-(\mu-\alpha-x)T} \tag{10}$$

which is negative when $\mu < \alpha$ but positive when $\mu > \alpha$. Similarly,

$$\frac{\partial(S-F)}{\partial\alpha'} = -\frac{(\mu-\alpha-x)V}{(\mu-\alpha-x)T+\alpha+x} - \frac{(\mu-\alpha-x)V}{1-\alpha-x-(\mu-\alpha-x)T} \tag{11}$$

which is positive when $\mu < \alpha$ but negative when $\mu > \alpha$.

Next

$$\frac{\partial(S-F)}{\partial\mu} = (\mu'-\alpha')\left[\frac{(\alpha+x)V}{((\mu-\alpha-x)T+\alpha+x)^2} + \frac{(1-\alpha-x)V}{(1-\alpha-x-(\mu-\alpha-x)T)^2}\right], \text{ and} \tag{12}$$

$$\frac{\partial(S-F)}{\partial\alpha} = (\mu'-\alpha')\left[\frac{-VT}{((\mu-\alpha-x)T+\alpha+x)^2} + \frac{-V(1-\mu)}{(1-\alpha-x-(\mu-\alpha-x)T)^2}\right]. \tag{13}$$

Similarly to the proof of Proposition 4, the derivatives have the signs claimed in the relevant parameter ranges. ■

Thus Proposition 5 demonstrates that the overall effect of changing the abilities of types through changes to μ and α described in Proposition 4 can be decomposed into two distinct mechanisms.

First consider the comparative statics with respect to the second period productivities μ', and α'. Fixing some effort level, then the beliefs about the type of an agent following either a success or a failure do not change as μ' or α' changes. However, since the belief that the agent is excellent following a success is higher than it is following a failure, raising (lowering) μ' in the case where $\mu' > \alpha$ (where $\mu' < \alpha$) increases S—the agent's wage following a success—by more than it increases F, the wage following failure. Since incentives are stronger the greater the difference between S and F, an increase in μ' therefore raises incentives. A similar argument applies for α'. Notice that changing μ', and α' does not affect the inferences that employers draw from the outcomes (in equilibrium when they correctly anticipate x) but they affect the value to the agent of being thought of as a particular type. We therefore term this channel for influencing an agent's incentives a "value effect".

We now turn to the comparative statics with respect to the first period productivities through changes in μ, and α. If the beliefs about the type of the agent are fixed, then increasing μ, and α has no effect whatsoever on the value of the agent in Period 2. Changing μ and α, however, can affect the inferences that employers draw from an observation of success or failure, we therefore term such changes as having an "information effect". In particular, intuition can be drawn from the observation that for a fixed level of effort, increasing (reducing) μ in the case where $\mu > \alpha$ (where $\mu < \alpha$) increases the probability that "better types" generate success and decreases the probability that they generate failure. Therefore, conditional on observing on a success, employers believe that the agent is more likely to be at the top of the ability distribution and so S is higher, while conditional on observing a failure, employers believe that the agent is less likely to be at the top of the distribution and so F is lower. In particular, therefore, $(S - F)$ increases. Similar arguments apply with regard to changes in α.

5.4 Reputation for excellence or ineptitude

This paper relates to a wider literature on reputation. Much of the economic literature on reputation has focused on a reputation for excellence (trying to show that you are a type who always does well, or where reputation is about "who you'd like to be").[9] In these models, the "most able" are non strategic and implicitly, they are assumed to be somewhat unusual or scarce, so for example with that view of the world, one might expect general "untargeted" training to have little effect on them. Such training would reduce career concern incentives.

More recent literature (Mailath and Samuelson [2001], Tadelis [2002], Bar-Isaac [2007]) and common intuition suggests that often reputational concerns might also relate to avoiding a reputation for ineptitude (trying to show that you are not an inept type who always does badly or where reputation is about "who you're not") where the top of the distribution is the strategic type, and the bottom of the distribution is an inept types whom one might expect to be little affected by training.

The distinction between these two approaches to reputation has been forcibly made by Mailath and Samuelson [1998] who highlight, in particular, that the latter view of

[9] Following Kreps and Wilson [1982] and Milgrom and Roberts [1982] and later Fudenberg and Levine [1989], the formal economic literature on reputation has been used primarily to discuss beliefs about the type of the agent. Previous literature (Klein and Leffler [1981] for example) and a great deal of intuition has also used the term in a somewhat looser fashion to consider sustaining certain actions in infinitely repeated games. As highlighted in Fudenberg and Levine [1989] this corresponds closely to the notion of reputation where reputation is a concern to show that you're a "Stackelberg" type—that is a type whose behavior a strategic agent would like to promise to commit to—similar to what we term later in this note a reputation for excellence. See Bar-Isaac and Tadelis [2007] for a review of this literature and alternative approaches to reputation.

reputation leads to increasing certainty about the agent's type over time and so reputational incentives disappear over time unless type uncertainty is continually introduced.[10]

In practice, it is far from obvious whether it is more appropriate to think of agents as particularly concerned that others should think them to be excellent or that they should not think them to be inept. However, as we illustrate, modelling reputational concerns in these two ways can lead to opposite conclusions.

This paper highlights an important distinction between the two approaches in a simple two-period model. Specifically, following the intuition of the paragraphs above, making the strategic agent more efficient diminishes reputational concerns (reducing effort) when reputation is about excellence but increases reputational concerns when reputation is about ineptitude.

To see this more clearly consider setting $\mu = 1$ and the degenerate distribution $g(0) = 1-p$ and $g(1) = p$. This corresponds to a fairly typical model where type $t = 0$ corresponds to the strategic type whose reputational concern is to try to convince customers that she is the "excellent" or Stackelberg type. Following Proposition 2, in this case improving the strategic agent by raising α would reduce effort.

In contrast suppose that a strategic agent's reputational concern is to avoid a reputation for ineptitude. This corresponds to the model where $\mu = 0$ with $g(0) = 1 - p$ and $g(1) = p$ and in this case improving the strategic agent by raising α would increase equilibrium effort.

[10] Further in Mailath and Samuelson [1998], the model is constructed in such a way that there is unravelling so that if there are no reputational incentives at some point, there are no such incentives throughout.

The more general point on reputational incentives disappearing over time without some kind of replenishment of type uncertainty applies more widely. Indeed, Cripps, Mailath and Samuelson [2004] show this to be the case unless actions are perfectly observable, even in the case when reputation is about excellence and a competent agent can perfectly mimic an excellent agent (though incentives may disappear only in the very long run).

Bar-Isaac [2007] suggests an endogenous mechanism to maintain type uncertainty by allowing agents to choose to work in teams. Liu [2007] supposes that receivers must pay to observe history. This also provides an endogenous mechanism which limits an audience's certainty about type.

It is worth noting that in those papers that have considered something akin to the inept type we consider in this paper (in particular Diamond [1989], Mailath and Samuelson [1998], Tadelis [2002] and Bar-Isaac [2007]), there are no reputational incentives from trying to avoid a reputation for ineptitude or gain a reputation for competence *per se*.[11] Essentially this is because in the notation of this paper they have taken $\alpha = 0$.

5.5 Many periods

Extending the model beyond two periods is not straightforward. First, in a multi-period model, the agent's belief about herself (and in particular whether she knows her type) will make a difference. This belief will affect her expectations of continuation values in all periods up until the last one, and so a modeler must take a stance on this belief.[12] Second, with many time periods and different types, there are many equilibrium conditions that need to be simultaneously satisfied. For example, a decision on how much effort to exert with two periods remaining must take into account not only the agent's type but also expectations of what efforts she will take (and what efforts the public would anticipate and so compensate) in subsequent periods. Similarly, efforts in later periods depend on reputations that arise as a result of efforts in earlier periods. In contrast to the single equilibrium condition (4) that appears in this model, in a multi-period extension one needs to solve a system of non-linear equations. This makes an analytical characterization challenging, though for a finite-period (if not an infinite horizon) model one can proceed by backwards induction, where the analysis of the penultimate period would be identical

[11] In those papers, it is only useful to be thought of as a competent type if you can somehow also commit to exerting effort. Diamond also allows for an inept type that always fails, but has a strategic type for whom $\alpha = 0$, Cabral and Hortacsu [2004] is an exception in supposing that $\alpha \neq 0$.

[12] Note also that if the agent does not know her type initially, she will learn it at a different rate to customers, as she observes whether or not she has made a strategic decision and the level of effort (which would be relevant for considering off-equilibrium deviations) and not just the outcome, which is all that the public observes.

to our two-period model.

We numerically analyze a three period version of our model in a Appendix B, in which all the qualitative results of the two period model appear to be robust.[13] Nevertheless, although a number of forces operate in the same way and dominate in the many numerical parametrizations we have explored, there are some subtle and potentially counter-acting mechanisms. These can be understood by distinguishing between information and value effects,as we did in Section 5.3.

The information effect works as described above: increasing α in the case that effort and ability are complements or μ when they are substitutes suggests that, all else equal, outcomes are more informative signals on ability. As long as the value of entering the second period with a given reputation is non-decreasing, then since outcomes are more informative, the agent will be induced to exert more effort. In the case where effort and ability are substitutes, then following Proposition 1(ii) the agent always prefers that potential employers think she is better (and so more likely to be succeeding with probability μ) than exerting effort (in particular the very best agent $t = 1$ has no opportunity to exert effort). In this case therefore, the value with two remaining periods is indeed non-decreasing in reputation. It is theoretically possible however, that it can be non-monotonic when effort and ability are complements. In this case, even the best possible agent (that is one of type $t = 0$) would prefer to commit to exerting effort. With no means to do so, the agent may

[13] Note that there is an additional complication: it is not clear what would constitute an analogous result in the case of a mutli-period extension. There are a number of different efforts that characterize equilibrium (specifically equilibrium effort would be a function of the agent's belief about her type, the public's belief about the agent's type and the number of periods remaining). In considering comparative statics, it is not clear what the appropriate multi-period analogue of our results should be: effort in the first period given an initial reputation (prior distrinution), efforts in any period, given an initial reputation, or perhaps of greater interest might be the overall lifetime expected effort. Although we have not included it in the model, this would be particularly relevant in an environment where the initial employers competed both with respect to wages and recruitment/training policies. Employees would compete by offering workers recruitment/training policies that led to more efficient decisions for lifetime earnings.

Our results appear to be robust with respect to all of these measures.

benefit more from uncertainty about her ability which induces her to exert efficient effort than from having potential employers certain that she is excellent (and thereby dampening her incentives for effort). Although, this is a theoretical possibility, we have been unable to find numerical examples.[14]

In both cases, where effort and ability are substitutes or complements, the value effect can be decomposed into two effects. Consider, increasing α in the complements case for the second and third periods but not for the first period. First, one can think of a "direct" value effect, "better" agents are more productive as in the main model and so agents should seek to prove themselves to better, leading to similar comparative statics. However, there is an additional "indirect" value effect, which is that second period efforts are altered, as described in the main model. Specifically, these increase at all reputation levels. It is theoretically conceivable that second period efforts might increase so much more for a second period reputation that arises following a first period failure than a first period success that it dampens first period incentives.

Overall, therefore the "direct" value effect boost efforts, as will the information effect in the case where effort and ability are substitutes. Although, the "indirect" value effects (in both cases) and the information effect when effort and ability are complements might possibly work in the opposite direction, we have been unable to find examples in which our qualitative results on comparative statics are overturned in a three period extension of the model.[15]

[14] Further, there is intuition to suggest that this cannot arise. Such a result requires that the effort generated in equilibrium is valuable as compared to "raw ablity" (α). However, raw ability must be significant enough to generate equilibrium effort in the first place.

[15] Given the intuitions in this section, we have searched in particular in the effort and abilities as complements case (where the information effect is ambiguous) and in the case where the initial reputation is high (that is where the agent is very likely to be strategic) since here following first period success there is likely to be much less uncertainty (and so relatively low effort) as compared to failure.

6 Summary

At heart this paper highlights the simple observation that the distribution of prior beliefs is a crucial determinant of reputational incentives. There are numerous considerations which affect such prior beliefs, including for example contemporaries and social peers. Further, there are many policies that firms undertake (in particular, we have focused on training and recruitment policies) which affect the shape of the distribution of these priors. Different policies which affect the mean ability in the same way can have exactly opposite implications for reputational concerns.

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A Derivation of $S(x)$ and $F(x)$

Suppose that the equilibrium effort level is given by x. Then a type t agent generates success in the first period with probability

$$\mu t + (1-t)(\alpha + x) = (\mu - \alpha - x)t + (\alpha + x), \tag{14}$$

and generates a failure with probability

$$1 - \mu t - (1-t)(\alpha + x) = 1 - \alpha - x - t(\mu - \alpha - x). \tag{15}$$

By Bayes rule, the probability density function given a success and given the belief that agents exert effort x in the first period can be written down as

$$s(t,x) = \frac{(\mu - \alpha - x)t + \alpha + x}{\int_0^1 [(\mu - \alpha - x)t + \alpha + x]\, g(t)dt} g(t) = \frac{(\mu - \alpha - x)t + \alpha + x}{(\mu - \alpha - x)T + (\alpha + x)} g(t), \tag{16}$$

and the probability density function given a failure in the first period is

$$f(t,x) = \frac{1 - (\mu - \alpha - x)t - (\alpha + x)}{\int_0^1 [1 - (\mu - \alpha - x)t - (\alpha + x)]\, g(t)dt} g(t) = \frac{1 - (\mu - \alpha - x)t - (\alpha + x)}{1 - (\mu - \alpha - x)T - (\alpha + x)} g(t). \tag{17}$$

In the second period, an agent of type t will exert no effort and so succeed with probability $\mu t + (1-t)\alpha = t(\mu - \alpha) + \alpha$. In particular it follows, that if employers believed that the types were distributed according to $h(.)$ going into period 2 then they would be willing to pay the agent $\int_0^1 (t(\mu - \alpha) + \alpha)h(t)dt$.

It follows that the wage that employers would pay following success and failure respectively are given by:

$$S(x) = \int_0^1 (t(\mu - \alpha) + \alpha)s(t,x)dt = (\mu - \alpha)E[t|S,x] + \alpha, \text{ and} \tag{18}$$

$$F(x) = \int_0^1 (t(\mu - \alpha) + \alpha)f(t,x)dt = (\mu - \alpha)E[t|F,x] + \alpha, \tag{19}$$

where

$$E\left[t|S,x\right] = \int_0^1 ts(t,x)dt = T + \frac{(\mu-\alpha-x)V}{(\mu-\alpha-x)T+(\alpha+x)}, \text{ and} \tag{20}$$

$$E\left[t|F,x\right] = \int_0^1 tf(t,x)dt = T - \frac{(\mu-\alpha-x)V}{(1-\alpha-x)-(\mu-\alpha-x)T}. \tag{21}$$

Proof. of Proposition 1

We begin by proving (i) and (ii) for any equilibirium effort e^*.

(i) Notice that $S(x) < 1$ for all x and $F(x) > 0$ for all x, so in particular $S(\gamma) - F(\gamma) < 1$, then the equilibrium effort level

$$\frac{e^*}{\gamma} = S(e^*) - F(e^*) < 1 \Longrightarrow e^* < \gamma \tag{22}$$

(ii) For contradiction suppose that $\mu - \alpha - e^* < 0$, then consider

$$S(e^*) - F(e^*) = \frac{(\mu-\alpha)(\mu-\alpha-e^*)V}{\left[\mu T + (\alpha+e^*)(1-T)\right]\left[1-\alpha-e^*-(\mu-\alpha-e^*)T\right]} \tag{23}$$

Note that $1 - \alpha - e^* > T(\mu - \alpha - e^*)$ since $T < 1$ and $1 > \mu$ so the denominator is positive and since $\mu > \alpha$ and $\mu - \alpha - e^* < 0$ it follows that $S(e^*) - F(e^*) < 0$ but then it is impossible that $S(e^*) - F(e^*) = \frac{e^*}{\gamma} > 0$, which is our desired contradiction.

It suffices to show that there exists a unique solution for the equation (4), which is in the range $(0, \gamma)$.

Let $h(x) = -\frac{x}{\gamma} + S(x) - F(x)$. The equilibrium effort is then given by the solution of $h(e^*) = 0$ for $e^* \in (0, \gamma)$.

Note that $h(0) = S(0) - F(0) > 0$ and given (i) and (ii), $h(0) > 0$ and $h(\gamma) < 0$. Moreover, $h(x)$ is continuous, and thus there exists at least one solution in the range $(0, \gamma)$.

In order to demonstrate uniqueness, first take the derivative of $(S - F)$:

$$\frac{d(S-F)}{dx} = -(\mu-\alpha)V\left[\frac{\mu}{\left[(\mu-\alpha-x)T+(\alpha+x)\right]^2} + \frac{(1-\mu)}{\left[(1-\alpha-x)-(\mu-\alpha-x)T\right]^2}\right] \tag{24}$$

Then if $\mu > \alpha$, $\frac{d(S-F)}{dx} < 0$ and so $h(x)$ is monotonically decreasing in the range $(0, \gamma)$ and so the solution must be unique.

If $\mu < \alpha$ then $\frac{d(S-F)}{dx} > 0$ and so potentially, $h(x)$ could be increasing in some subset of $(0, \gamma)$ (note that since $h(0) > 0$ and $h(\gamma) < 0$ at must be decreasing in some of the range). However, we know that, when $\mu < \alpha$, $\frac{d^3h(x)}{dx^3} > 0$ since

$$\frac{d^3h(x)}{d^3x} = (\alpha-\mu)V\left[\frac{\mu 6(1-T)^2}{\left[(\mu-\alpha-x)T+\alpha+x\right]^4} + \frac{(1-\mu)6(1-T)^2}{\left[1-\alpha-x-(\mu-\alpha-x)T\right]^2}\right] > 0 \tag{25}$$

Suppose for a contradiction that $h(e^*) = 0$ has a number of solutions $0 < e_1 < \ldots < e_N < \gamma$. Then first note that since $h(0) > 0$ and $h(\gamma) < 0$ then N must be an odd number. In particular therefore if there are multiple solutions to $h(e^*)$ in the range then there must be at least three. However $h(0) > 0$ and $0 < e_1 < e_2 < e_3$ with $h(e_1) = h(e_2) = h(e_3) = 0$ requires $\frac{dh(e_1)}{dx} < 0$,

$\frac{dh(e_2)}{dx} > 0$ and $\frac{dh(e_3)}{dx} < 0$ which contradicts $\frac{d^3h(x)}{d^3x} > 0$. ■

B Three period, two type model

There are two types of agents parameterized by $0 \leq t \leq T \leq 1$.

Production is as in the main model. Specifically in each period, a type i agent will have no strategic decision to make with probability i and in this case will succeed (for example by producing a high quality product) with probability μ and fails with probability $1-\mu$. Otherwise (with probability $1-i$) the agent must make an effort decision. In this latter case, when she chooses effort e, she succeeds with probability $\alpha + e$. Thus overall an i-type agent exerting effort e when given the opportunity to exert effort would succeed with probability $i\mu + (1-i)(\alpha + e)$ and fail otherwise. Effort is costly and, specifically, exerting effort e costs the agent $\frac{e^2}{2\gamma}$.

Let r_j denote the public's belief that the agent is of type t at time j. Note that in contrast to the two period case, for multiple periods, it will make a difference whether we assume that agents know their own types or not. Here we assume that agents do not know their own types initially. Therefore if they observe that they have an effort decision to make they gain some information about their type. We can write the private, intermediate belief after observing an opportunity to exert effort as $p_1 = \frac{r_1(1-T)}{r_1(1-T)+(1-r_1)(1-t)}$.

As before, employers are risk neutral, value a success at 1 and a failure at 0 and they Bertrand compete for the agent's service in each period. Moreover, outcomes are observable but effort is not observable and contracts are incomplete, so that in effect an agent is paid in advance at a wage which is simply the employers' common belief that the agent will produce a success.

There are three periods of trade, and outcomes are observed (and beliefs revised) at the end of each period. Specifically timing is as follows:

1. Period 1
   (a) employers Bertrand compete for the agent's service
   (b) the agent observes whether or not a task where she has an opportunity to exert effort arises, and updates her belief about her ability and then decides the level of effort if appropriate (that is if it is a task where effort will make a difference)
   (c) success/failure commonly observed
   (d) employers update beliefs according to Bayes rule
2. Period 2
   (a) employers Bertrand compete for the agent's service
   (b) the agent decides the level of effort if appropriate (that is if it is a task where effort will make a difference)
   (c) success/failure commonly observed
   (d) employers update beliefs according to Bayes rule
3. Period 3: employers Bertrand compete for the agent's service

We suppose that agents weigh the periods equally and that agents maximize the sum of profits for the three periods and we solve for the effort exerted in the Perfect Bayesian equilibrium.

B.1 Solving the model

Work by backwards induction.

B.1.1 Period 3

In the final period neither type exerts effort.

Suppose that an agent enters this last period L with reputation r_3 (probability of being T type) then regardless of his type, he earns

$r_3(T\mu + (1-T)\alpha) + (1-r_3)(t\mu + (1-t)\alpha) = \alpha + (\mu-\alpha)t + (T-t)(\mu-\alpha)r_3$

So we can write the values for type t and T to entering the third period with reputation r_3 respectively as

$$V_3^t(r_3) = V_3^T(r_3) = V_3(r_3) = \alpha + (\mu-\alpha)t + (T-t)(\mu-\alpha)r_3. \tag{26}$$

B.1.2 Period 2

Following the analysis in the two-period model, then both types of agent will exert the same effort (that is $x_2^t = x_2^T = x_2$) which will be a function of r_2 the reputation at the start of period 2, which will in turn determine the equilibrium expectation of effort x_2^e (so that this is a function of r_2, though we often suppress this argument to avoid burdensome notation) and in particular this effort is

$$\frac{x_2(r_2)}{\gamma} = (T-t)(\mu-\alpha)(r_3^S(x_2^e, r_2) - r_3^F(x_2^e, r_2)),$$

where

$$\begin{aligned} r_3^S(r_2) &= \frac{r_2(T\mu + (1-T)(\alpha + x_2^e(r_2)))}{r_2(T\mu + (1-T)(\alpha + x_2^e(r_2))) + (1-r_2)(t\mu + (1-t)(\alpha + x_2^e(r_2)))}, \text{ and} \\ r_3^F(r_2) &= \frac{r_2(1 - T\mu - (1-T)(\alpha + x_2^e(r_2)))}{r_2(1 - T\mu - (1-T)(\alpha + x_2^e(r_2))) + (1-r_2)(1 - t\mu - (1-t)(\alpha + x_2^e(r_2)))}. \end{aligned}$$

Of course, as in the paper, one must worry about interiority and the existence and uniqueness of the solution, though for now we take these for granted and return later to verify them numerically.

We can then write down the second period value as a function of both the public belief r_2 and the agent's private belief, p_1

$$\begin{aligned} & r_2(T\mu + (1-T)(\alpha + x_2^e(r_2)) + (1-r_2)(t\mu + (1-t)(\alpha + x_2^e(r_2)))+ \\ V_2^t(r_2) = \; & p_1((T\mu + (1-T)(\alpha + x_2^e(r_2)))V_3(r_3^s) + (1 - T\mu - (1-T)(\alpha + x_2^e(r_2)))V_3(r_f^3) - (1-T)\tfrac{(x_2^e(r_2))^2}{2\gamma})+ \\ & (1-p_1)((t\mu + (1-t)(\alpha + x_2^e(r_2)))V_3(r_3^s) + (1 - t\mu - (1-t)(\alpha + x_2^e(r_2)))V_3(r_f^3) - (1-t)\tfrac{(x_2^e(r_2))^2}{2\gamma}) \end{aligned}$$

B.1.3 Period 1

Finally, if the initial reputation is r_1 and the public anticipates efforts x_1^e then we can write

$$r_2^s = \frac{r_1(T\mu + (1-T)(\alpha + x_1^e))}{r_1(T\mu + (1-T)(\alpha + x_1^e)) + (1-r_1)(t\mu + (1-t)(\alpha + x_1^e))}; \text{ and}$$
$$r_2^f = \frac{r_1(1 - T\mu - (1-T)(\alpha + x_1^e))}{r_1(1 - T\mu - (1-T)(\alpha + x_1^e)) + (1-r_1)(1 - t\mu - (1-t)(\alpha + x_1^e))}$$

Then first period effort would solve[16]:

$$\frac{x_1}{\gamma} = V_2(r_2^s) - V_2^T(r_2^f) \tag{27}$$

B.1.4 Equilibrium conditions

The conditions for equilibrium are

$$\begin{aligned} x_1 &= x_1^e \\ x_2(r_2^s) &= x_2^e(r_2^s) \\ x_2(r_2^f) &= x_2^e(r_2^f) \end{aligned}$$

Note in particular, that while the two-period model had a single (non-linear) equation to be satisfied. Here with two types and three periods, there are three non-linear equations which must be simultaneously satisfied. We were unable to solve that model analytically but it can be solved numerically..

B.2 Numerical Calculations

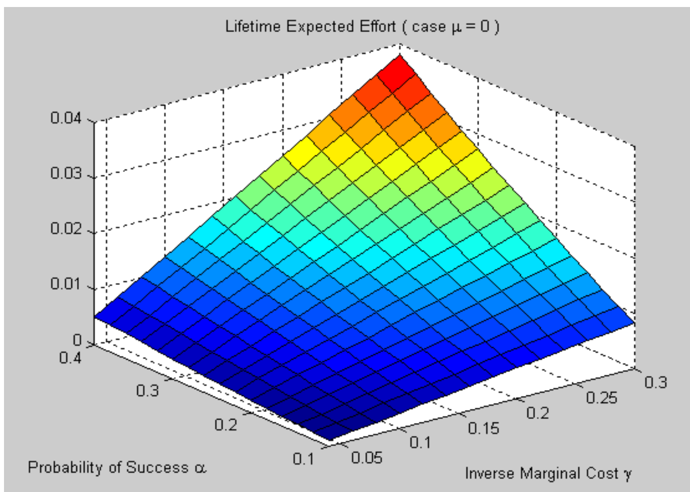
We have conducted many simulations by setting all parameters values of the model but one or two that we allow to vary over a specific range. For those specific values of the parameters we solved the previous system of non-linear equations and we have computed the exerted efforts levels that are the endogenous variables of the model. Finally, we have successfully checked the comparative statics results of the paper.[17] In doing so, it is important to notice that in the two period model of the paper there was only one level of exerted effort. However, in this extended version of the model we have to compute three effort levels: effort exerted in period one, effort exerted in period two after a success and effort exerted in period two after a failure. In order to be able to compare both models, we take an ex-ante point of view and we aggregate these three effort levels into a new variable "lifetime expected effort". This variable is the expected exerted effort in the three period, which is a weighted average of the three effort levels (weighted by the prior belief and probabilities of first period success and failure).

[16] Note that the agent's choice of action does not affect her private belief about herself p_1, rather her action in the second period is a function only of her public reputation. Therefore there is no gain from deviating in first period to try to generate private information or distort in that way, so we not need to consider such effects.

[17] Our Matlab code and instructions for the numerical calculations are available online at http://pages.stern.nyu.edu/~hbar-isa

In the following, we will provide two scenarios that illustrate two of the main results of the paper. Let $t = 0.2$ and $T = 0.8$ be the two types, meaning that type t (T)will have no strategic decision to make with probability 0.2 (0.8).

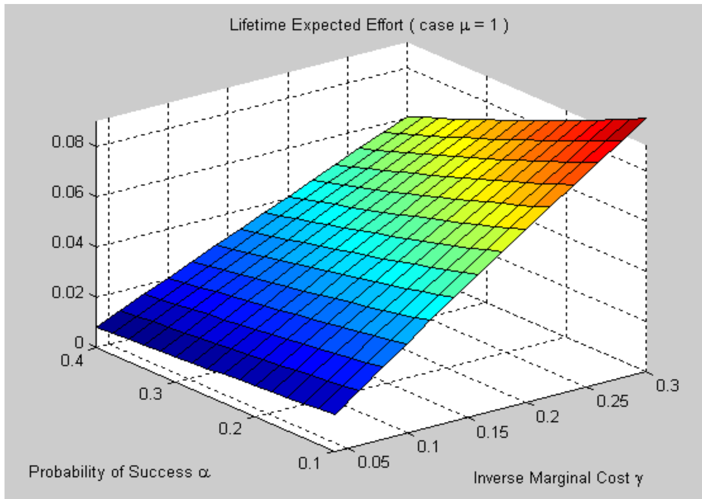
We consider first the case of "creative" work where effort and ability are complements. Suppose that when having no opportunity to exert effort then both types will succeed with probability $\mu = 0$. In case that the agent will have to exert effort, then an agent succeeds with probability $\alpha + e$, where α varies in this simulation between 0.1 and 0.4, and the inverse of the marginal cost of effort, γ, varies between 0.04 and 0.3. Finally, the public's belief that the agent is of type t at time 1 is $r_1 = 0.5$. Given this set of parameters, we solve the non-linear system numerically and we compute efforts for different values of γ and α. The figure below shows a two dimensional surface to illustrate how lifetime expected effort varies with the probability of success α and the inverse of the marginal cost γ



The mu=0 case

The figure above shows the lifetime effort jointly increases in both α and γ, as predicted in Propositions 2 and 4. Notice, that the relationship between lifetime effort and α, depends on whether or not, α is larger than μ. The previous figure illustrate the case in which $\alpha > \mu = 0$.

We analyze the opposite case (routine work or where effort and ability are substitutes) by keeping the same parameter values but for changing μ to $\mu = 1$. The theoretical prediction is that, the effort must be decreasing in α.



The mu=1 case

The figure above shows that in this case (that is when $\alpha < \mu = 1$) the lifetime effort decreases on α as is predicted in Proposition 4.

Our purpose with these examples is to show that the driving forces in the two period model play an important role in the extended model, and hence it is easy to find regular examples that are consistent with our predicted results. However, we have to acknowledge that there several caveats: i) We rely on the numerical Matlab procedure to solve our non-linear system, this procedure works well with interior solutions but not as well with extreme values of the parameters. ii) The numerical solutions are regular, and they behave smoothly with small changes in the parameters. Hence, we think that the solution is unique and hence our comparative static argument holds, however we do not have a formal proof of this uniqueness.