Competing in Markets with Digital Convergence

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Abstract: This paper studies competition in the presence of digital convergence, a phenomenon that has been observed in markets as diverse as handheld computing, telecommunications, consumer electronics, networking, residential broadband and broadcast video. Convergence results in substantially more valuable and flexible products and services, but also leads to increased substitutability between products in previously distinct markets. This represents a critical trade-off for managers making technological and platform scope choices in these industries. We analyze this trade-off between product value and product substitutability by analyzing a new model of imperfect competition, which generalizes ideas from popular horizontal and vertical differentiation models, admitting endogenous product scope choices, variable substitutability across products and industry boundaries, and purchases of multiple converging products by individual customers.

We establish four different kinds of equilibria, each characterizing a different stage of product convergence. Our results show that early stages of convergence feature increasing prices and profitability, which eventually fall if the extent of product convergence progresses beyond a critical point. However, if firms can bilaterally and strategically control the degree of convergence in their industries, their equilibrium strategies result in sustained higher prices and profits even when industry boundaries blur. We also describe examples of equilibria in which consumers may buy multiple general-purpose products, using each for only a specialized subset of their requirements. The effect of changes in the fixed costs of expanding product scope, the variable costs of production, and the breadth of customer requirements are analyzed. Managerial guidelines based on each of our results are presented.

JEL Codes: D43, L13, L50

The extended appendix of this paper is available at

http://oz.stern.nyu.edu/papers/cmdc0604appx.pdf

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1. Introduction

In the last few years, rapid advances in information technologies have resulted in the digitization of product technologies across a wide variety of industries, a phenomenon that has been referred to as *digital convergence* (Yoffie 1997). Digital convergence is often accompanied by a shift in product design from rigid hardware architectures supporting narrow sets of functionalities towards *platform-based* architectures characterized by the use of powerful general-purpose digital hardware, and the reliance on a software platform to implement a broad set of functionalities. While this had led to more valuable and flexible products, the use of common underlying digital technologies across industries has also resulted in overlapping sets of functionalities being provided by products in previously distinct industries. Consequently, consumers now begin to view products across previously complementary industries as imperfect substitutes (Greenstein and Khanna 1997), thereby redefining the nature of choice and competition across these industries.

This tradeoff between increasing product value and increasing inter-industry substitutability is central to the economics of digital convergence, and the primary focus of our paper. It is illustrated well by a recent example. Cellular telephones in the early 1990's were limited analog voice communication devices; in contrast, the current generation of platform-based Symbian, PalmOS and Microsoft Smartphone based mobile handsets enable powerful IP-based data communications applications, as well as personal information management, multimedia and web browsing. A similar transition has occurred in the adjacent handheld computing industry, from hardwired Sharp Wizard organizers to the current generation of PocketPC and PalmOS-based personal digital assistants. These platform-based devices are far more effective at personal information management tasks, provide a robust interface with PC-based applications, and also have well-developed voice and data communications capabilities.

At their core, the former set of devices are still mobile telephones and the latter are still handheld computers. By virtue of their expanded features and versatility, devices in each industry are more valuable to consumers. However, due to the bilateral shift to a platform-based architecture and the accompanying expansion in their breadth of functionalities, converged products in the (neighboring) mobile handset and handheld computing industries are now viewed as substitutes. Moreover, the mobile telephony and computing industries are not isolated examples. Similar trends are currently prevalent in the consumer electronics (video game consoles, home entertainment gateways), residential Internet access, cable television, wireline telephony and networking equipment industries. As the IT component of products and services continues to increase across different industries, similar transformations are likely in other segments of the economy.

We study this trade-off between an increase in value (which we term the *value effect*) and an increase in substitutability (which we term the *substitution effect*) by developing a model of inter-industry competition between converging platform-based IT products. Our model specifically captures those effects of competition that are crucial to markets with digital convergence – consumers that demand different sets of product functionalities, products with variable product scope that have different levels of effectiveness at fulfilling these functionality needs, and firms that make strategic product scope and pricing choices.

We show that as the scope of platform-based products increases bilaterally across industries, the outcomes evolve through three different kinds of equilibria – *local-monopoly* equilibrium, *adjacent-markets* equilibrium, and *competitive* equilibrium – which describe pricing equilibria at low, intermediate and high levels of convergence. Under the former two equilibria, the value effect of digital convergence dominates the substitution effect, and both prices and profits rise as product scope increases. Under the competitive equilibrium, however, the substitution effect dominates the value effect, and prices and profits fall bilaterally across industries. We highlight the differential effects of changes in platform scope, the breadth of customer needs and the unit cost of products on pricing and profits under each equilibrium. Furthermore, if firms have complete strategic control over the scope of their products, we establish that any subgame-perfect outcome features relatively low levels of convergence, and that prices in the presence of inter-industry competition may in fact be higher than those in its absence. An appropriately chosen technology and product strategy can therefore sustain high levels of profitability even when industry boundaries threaten to blur.

Our extended appendix describes a fourth kind of outcome, the *non-exclusive* equilibrium, which occurs at very high levels of industry convergence. It predicts an interesting trajectory in purchasing patterns, wherein consumers initially buy multiple specialized (complementary) products at low levels of product scope, shift to buying one general-purpose device as the scope of each expands, and resume buying multiple substitutable products at very high levels of product scope.

The rest of the paper is structured as follows. Section 2 discusses existing research related to convergence and places our model of imperfect competition in the relevant literature. Section 3 outlines the model and briefly discusses some monopoly results. Section 4 characterizes the different kinds of equilibrium configurations, and analyzes symmetric convergence that is driven by exogenous factors. Section 5 describes optimal strategic choices of product convergence, establishing a family of subgame perfect equilibrium outcomes, and analyzes their sensitivity to changes in cost structure driven by technological progress. Section 6 concludes with a summary of the managerial implications of our results and a brief discussion of ongoing work. Section 8 presents an outline of the proofs of the paper's results; Section A of an extended appendix presents these proofs in detail. Section B of this extended appendix presents a base-case monopoly model in detail. Section C examines the effect of relaxing a number of the model's assumptions, and analyzes equilibria that involve non-exclusive product purchases.

2. Related research and modeling approach

A pioneering collection of essays in Yoffie (1997) forms the bulk of existing research on product convergence and its effects on industry structure. In particular, Greenstein and Khanna (1997) predict that convergence at the product level will lead to a higher intensity of competition. In contrast, summarizing a seminar on the telephone-cable TV convergence, Katz and Woroch (1997) observe that while convergence can be a source of increased competition if it creates new entry incentives and opportunities in each others' markets, increased concentration may reduce competition if there are significant economies of scale or scope across multiple markets. None of these papers present a formal model that capture these contrasting economic effects.

Product convergence is a relatively recent phenomenon. However, a similar transformation in manufacturing *processes* occurred in the early part of the twentieth century, during which identical production technologies such as machine tools were adopted across several different manufacturing industries. Ames and Rosenberg (1977) provide a good account of this transformation. A sizeable recent literature has followed them in studying the new generation of flexible manufacturing systems (Roller and Tombak 1993, Eaton and Schmitt 1994, among others). Our research differs from this stream in its level of analysis – *production-process* level versus *product* level – and in the consequent focus of analysis. The manufacturing convergence literature focuses on technology adoption patterns and on firms' product line and manufacturing strategies from the perspective of a firm *buying* and employing these converging technologies, while we focus on pricing and scope choices from the perspective of a firm *selling* converging products.

Analyzing these choices requires an underlying model of imperfect competition between converging industries. However, traditional economic models of imperfect competition (Dixit and Stiglitz 1976, Salop 1979, Shaked and Sutton 1982) were developed to analyze competition between *functionally similar* products in relatively *static* environments. Extensions developed to study technology markets include von Ungern-Sternberg (1988), who allows firms to choose their level of differentiation in a Salop-type model; Banker et al. (1998), who extend Dixit (1979) by incorporating product quality into the aggregate demand function, which simplifies the model considerably and lends itself to an elegant analysis, and Barua et al. (1991), who study information technology investments in a duopoly model with indirect sources of revenue. These models and their extensions cannot be readily applied to converging technology markets, since they do not permit a clear separation of a product's (variable) endowment of functionalities and a consumer's desire for these functionalities. This places undue restrictions on the kinds of products that can be represented. They also typically assume exclusionary choice – consumers are allowed to choose only one of the 'competing' products. To address these problems, we develop a new model that generalizes ideas from the standard horizontal and vertical differentiation models, and explicitly reintroduces features pioneered in the characteristics approach of Lancaster (1966, 1975). The next section presents this model.

3. Model

Our model is based on an underlying *functionality space*. Heterogeneous consumer preferences are specified as subsets of this functionality space, and products are represented as (effectiveness-adjusted) bundles of functionalities. A consumer's utility from a product is derived based on the extent to which their desired functionalities are supported by a product. This approach decouples a product's endowment of features from a consumer's desire for these features, and permits non-exclusionary choices.

3.1. The basic model

The basic model is presented in four parts: the functionality space, the definition of a product, the distribution and value functions of consumers, and the production technology available to firms.

Functionality space: The basis for product design and consumer preferences is a set of *functionalities*, where a functionality is any task or activity for which a consumer can potentially utilize a product. In the context of the mobile device industry discussed earlier, mobile voice communication, streaming video, video gaming, text messaging, scheduling, and mobile spreadsheet analysis are each distinct functionalities. A unit circle represents the functionality space, with every point on the circle representing a distinct functionality[2]. Two functionalities that are more similar in terms of the technology needed to realize them are closer to each other on the circle. The functionality space is assumed to be exogenous, and is not altered by the choices of the firms or consumers[3].

Products: Each product has a *core functionality*. This is the functionality it provides most effectively. In addition, each product has a level of product scope ($\frac{1}{t}$), which may be endogenously

[2] In reality, the set of functionalities that customers value would be finite. Our model therefore represents a continuous approximation of a discrete functionality space with a large number of ordered functionalities.

[3] Changes in functionality space are briefly discussed in section 6.

chosen, and which determines the effectiveness of a product on the other (*non-core*) functionalities. The effectiveness of a product with scope $\frac{1}{t}$, on a functionality at a distance $x \in [0, \frac{1}{2}]$ from its core functionality[4], is $u(x) = \max\{1 - tx, 0\}$. A product's effectiveness therefore decreases as one moves away from its core functionality. An increase in a product's scope (a decrease in t) increases the product's effectiveness on all non-core functionalities, but we assume that it does not alter its effectiveness on the core functionality. The terms 'product' and 'platform' are sometimes used interchangeably, to indicate that our model could directly represent the actual functionality provided by a specific device, or indirectly represent the potential value from the platform that the device is based on.

Returning to our example, the above definition of a product implies that a mobile phone handset is most effective at providing its core functionality (voice communication), and least effective on a diametrically opposite functionality (say, mobile spreadsheet analysis), while the converse is true for a PocketPC-based computer. The effectiveness of both products on intermediate functionalities such as interactive gaming and web browsing (which require both mobile communications and an OS-based application programming interface) is somewhere in between.

Two products with diametrically opposite core functionalities are considered to be in different industries. As product scope increases, the overlap in the sets of functionalities that are supported effectively by both products grows, the products become closer substitutes, and the industries converge. For brevity, the product whose core functionality is located at z on the unit circle is referred to as 'the product located at z'. To ensure that every product has non-zero effectiveness on every functionality[5], we assume that $1 - \frac{t}{2} \geq 0$ (or $t \leq 2$).

Consumers: Each consumer values a subset of the functionalities. We assume that these subsets are continuous and of equal size, and that the consumer values all functionalities within this set equally. However, different consumers require different sets of functionalities. Consequently, each consumer is represented as an arc on the functionality space, these arcs are of constant *breadth* $r \in (0, 1]$, and are

[4] Many of the results hold for more general positive, non-decreasing functions of distance, rather than just the linear one used here.

[5] The case of narrow-scope products ($t > 2$) is analyzed in the extended appendix.

uniformly distributed around the unit circle. The consumer whose arc of functionality requirements is centered at $y \in [0,1]$ on the unit circle is referred to as 'the consumer located at y'. The density of consumers on the unit circle is $\frac{n}{2}$.

Following Lancaster (1966, 1975), consumer utility is assumed to be additive over the set of functionalities. For each functionality, consumer utility is the product of the level of effectiveness that a product provides on that functionality and the value (0 or 1) that the consumer places on that functionality[6]. The value a consumer located at y derives from a product with slope $\frac{1}{t}$ is therefore computed by summing the effectiveness of the product over the set of functionalities that the consumer values:

$$U(y,t) = \int_{y-\frac{r}{2}}^{y+\frac{r}{2}} u(|x-z|)dx, \tag{3.1}$$

measured in monetary units. This is illustrated graphically in Figure 3.1. Note that in this figure, we have unfurled the circle and depicted it along the base of a rectangle. Moreover, the first half of the circle $[0, \frac{1}{2}]$ is replicated as $[1, \frac{3}{2}]$. This is both for clarity of illustration and for analytical convenience. The vertical dimension of the rectangle depicts the effectiveness of a product on the concerned functionality.

Technology: All firms have identical costs of production $C(q,t) = cq + F(t)$, where q is the quantity produced, and t is the reciprocal of product scope[7]. This separable form implies the following assumptions about $C(q,t)$.

1. Marginal cost of production is non-negative and constant: $C_1(q,t) = c \geq 0$, $C_{11}(q,t) = 0$.

2. Variable cost is independent of scope $C_{12}(q,t) = 0$.

In addition, the cost function is assumed to have the following properties:

3. Fixed cost is non-decreasing and convex in scope: $F_1(t) \leq 0, F_{11}(t) > 0$.

[6] This utility specification gives our representation of consumer types a combination of taste based heterogeneity (like the types in spatial models) and heterogeneity in their valuation of quality (effectiveness). Analogously, the specification of a product also combines notions of both horizontal and vertical differentiation.

[7] Different cost functions for different firms will result in asymmetric equilibria (rather than the symmetric equilibria we largely focus on in sections 4 and 5), but will not otherwise affect the qualitative nature of the results.

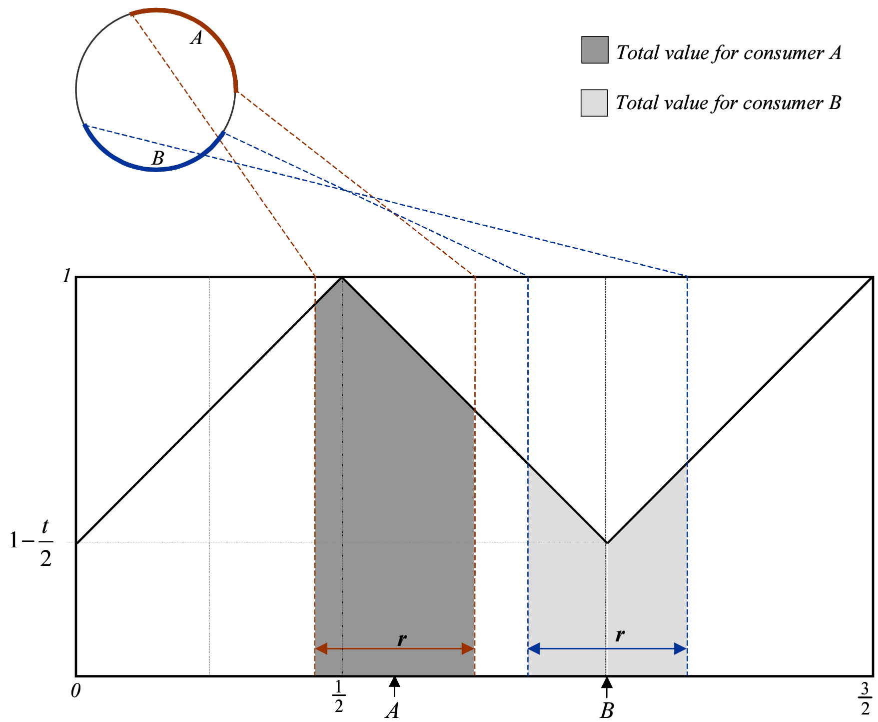

Figure 3.1: Graphically illustrates the value obtained by two consumers located at A and B, for a product whose core functionality is located at $\frac{1}{2}$.

4. Marginal cost is not too high: for $t \leq 2$, $c < U(y,t)$ for all y.

Numbered subscripts to functions represent partial derivatives with respect to the corresponding argument. Constant marginal costs and increasing convex fixed costs reflect the fact that an increase in product scope is typically achieved through a superior product design, an increase in the functionality of the software platform, or a combination of the two, both of which increased fixed costs, but do not alter variable production cost substantially. A partial summary of key notation (some of which is defined later in the paper) is provided in Table 3.1.

3.2. The value function, demand and some monopoly results

This subsection describes the consumer value function and inverse demand curve implied by the model, then presents some monopoly results. Without loss of generality, fix the location of a product at $\frac{1}{2}$, dividing consumers into two identical and symmetric segments $[0, \frac{1}{2}]$ and $[\frac{1}{2}, 1]$. At any price, demand from each segment will be identical, and we can focus our analysis on one of these segments $[\frac{1}{2}, 1]$.

Product-related	
t	Reciprocal of product scope. As t increases, scope $(1/t)$ decreases.
$u(x)$	Effectiveness of product on a functionality that is at a distance x from the core functionality
$F(t)$	Fixed cost at a level of scope $\frac{1}{t}$
c	Constant marginal cost
i, j	Indices for competing products. $i, j \in \{A, B\}$
Market-related	
$\frac{n}{2}$	Market size (density)
r	Consumers' breadth of functionality requirements
$U(.)$	Consumers' value function (willingness-to-pay)
Choice and outcome	
q	Variable representing normalized quantity. At a value q, actual demand is nq
$P^M(q,t)$	Monopoly inverse demand curve at a level of scope $(1/t)$
$P^C(q_i,t_i,t_j,p_j)$	Competitive inverse demand curve for firm i
$\pi^i(q_i,t_i,t_j,p_j)$	Gross profit (before accounting for fixed costs) for firm i
$\Pi^i(t_i,t_j)$	Net profits (after accounting for fixed costs)

Table 3.1: Summary of some of the notation used in the paper

In this paper, we consider the case where the set of functionalities demanded by consumers is not very large – more precisely, where $r \leq \frac{1}{2}$. The case of $r > \frac{1}{2}$ yields qualitatively similar results with the exception of an additional equilibrium, and is discussed in the extended appendix.

Computing the value function $U(y,t)$ in the interval $y \in [\frac{1}{2}, 1]$ yields the following piece-wise continuous form:

$$U(y,t) = \begin{cases} r - t\left[\frac{1}{2}[\frac{1+r}{2} - y]^2 + \frac{1}{2}[y - \frac{1-r}{2}]^2\right], & \frac{1}{2} \leq y \leq \frac{1+r}{2}; \\ r - t\left[\frac{1}{2}[y - \frac{1-r}{2}]^2 - \frac{1}{2}[y - \frac{1+r}{2}]^2\right], & \frac{1+r}{2} \leq y \leq \frac{2-r}{2}; \\ r - t\left[\frac{1}{4} - \frac{1}{2}[y - \frac{1+r}{2}]^2 - \frac{1}{2}[1 - [y - \frac{1-r}{2}]]^2\right], & \frac{2-r}{2} \leq y \leq 1; \end{cases} \tag{3.2}$$

The following lemma describes some useful properties of $U(y,t)$.

Lemma 1. *For all $y \in [\frac{1}{2}, 1], t \in [0, 2]$*

(a) $U(y,t)$ is continuous and decreasing in both y and t

(b) $U_1(y,t)$ is continuous and is piece-wise differentiable in both y and t.

(c) $U_2(y,t)$ is continuous and decreasing in y.

Part (a) of the lemma establishes that consumers located closer to the product derive more value from it, and an increase in scope increases value for all consumers. Part (c) establishes that the gross value from a product increases faster with product scope for those consumers who are farther away from it. An increase in product scope therefore increases value, but simultaneously reduces the heterogeneity in product value; this is consistent with the trade-off between the value effect and the substitution effect of convergence.

The value function $U(y,t)$ determines the *inverse demand function* $P^M(q,t) = U(q+\frac{1}{2},t)$ faced by a monopolist[8]. At a price $p = U(\hat{y},t)$, where $\hat{y} \in [\frac{1}{2},1]$, all consumers located between $\frac{1}{2}$ and $\hat{y}$ buy the product, resulting in a demand of $[\hat{y}-\frac{1}{2}]\frac{n}{2}$ from consumers located in $[\frac{1}{2},1]$, an identical demand from consumers located in segment $[0,\frac{1}{2}]$, and a total demand of $n[\hat{y}-\frac{1}{2}]$. As a benchmark, the optimal monopoly choices of price and scope are derived in Section B of the extended appendix. At relatively low levels of product scope ($\frac{1}{t} < \frac{r[2-\sqrt{2r}]}{2[r-c]}$), the monopolist prices such that a part of the market is excluded; however, at higher levels of product scope ($\frac{1}{t} \geq \frac{r[2-\sqrt{2r}]}{2[r-c]}$), the monopolist prices to sell to the entire market even in the absence of any regulatory mandate to do so. This may have implications in industries where universal access is a social priority, as discussed briefly in that section.

4. Competition between converging products

The next two sections model competition between two firms A and B, whose products' core functionalities are exogenously diametrically opposite each other, at $\frac{1}{2}$ and 1 respectively. This represents two firms in related but initially distinct industries, which begin to compete as their industries converge as a result of increases in product scope. For algebraic convenience, the demand of firm i is represented as nq_i, and characterized in terms of the normalized parameter $q_i \in [0,\frac{1}{2}]$, which is simply demand

[8] Note that the inverse demand function is $P^M(q,t) = U(y,t) = U(q+\frac{1}{2},t)$ and not $U(q,t)$ since $q \in [0,\frac{1}{2}]$ while $y \in [\frac{1}{2},1]$.

divided by n.

4.1. Duopoly demand and profit functions

We denote a firm's own choices using the subscript i, and those of its opponent with the subscript j. Given choices of scope t_A, t_B, and the opponent price p_j, the inverse demand curve of duopolist i, denoted $P_i(q_i, t_i, t_j, p_j)$, is composed of two functional forms – the *monopoly* inverse demand function $P^M(q, t_i) = U(q + \frac{1}{2}, t_i)$, and the *competitive* inverse demand function $P^C(q, t_i, t_j, p_j)$:

$$P^C(q, t_i, t_j, p_j) = U(q + \frac{1}{2}, t_i) - U(1 - q, t_j) + p_j. \tag{4.1}$$

Prior to specifying these demand functions formally, we provide the intuition that drives their specification. For a fixed product scope, duopoly demand is determined by a firm's price and the surplus consumers get from their opponent's product. For a sufficiently high value for the opponent's price p_j, customers close to firm i get no surplus from firm j's product, and their demand is therefore represented by the monopoly inverse demand curve for firm i. On the other hand, consumers close to product j who receive a positive net surplus from product j at a lower price p_j will purchase product i only if it provides them with at least an equivalent level of surplus. The competitive inverse demand function in (4.1) captures this additional aspect of demand from these consumers. The above intuition formalizes to the following specification for a duopolist's inverse demand curve.

$$P_i(q_i, t_i, t_j, p_j) = \begin{cases} P^M(q_i, t_i), & 0 \leq q_i \leq [1 - U^{-1}(p_j, t_j)]; \\ P^C(q_i, t_i, t_j, p_j), & [1 - U^{-1}(p_j, t_j)] \leq q_i \leq \frac{1}{2} \end{cases}$$

The inverse of U is with respect to its first argument (i.e. $U(U^{-1}(y, t), t) = y$). Figure 4.1 illustrates this demand curve. The inverse demand function has a kink at $\hat{q}_i = [1 - U^{-1}(p_j, t_j)]$, where it's slope changes discontinuously, and which represents a critical transition point beyond which the intensity of competition between the industries is substantially higher. The size of the monopoly and competitive regions for firm i therefore depends on its competitor's price p_j.

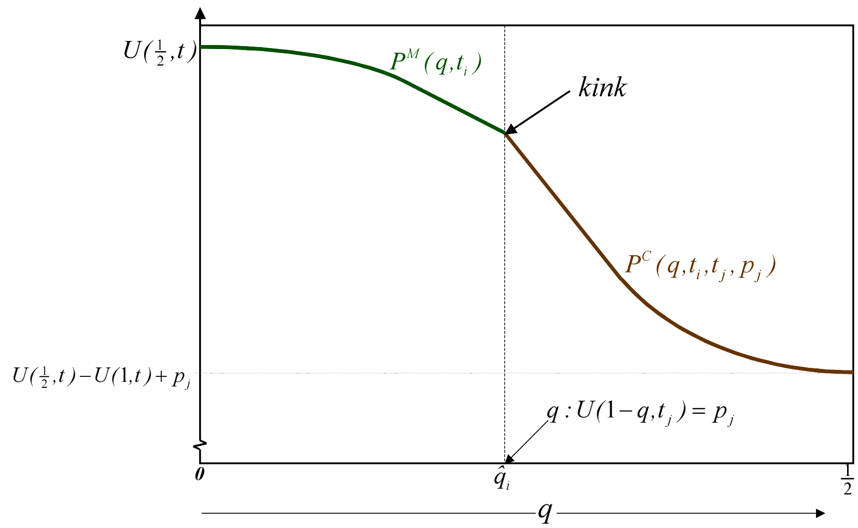

Figure 4.1: Illustrates the duopoly inverse demand curve. $\hat{q}_i = 1 - U^{-1}(p_j, t_j)$ corresponds to the indifferent consumer who gets exactly zero net surplus from product j at price p_j. All consumers to the left of this indifferent consumer have a reservation value of zero for product j and hence form the 'monopoly region' of the duopolist i's inverse demand curve, while consumers to the right of the indifferent consumer get positive net surplus, forming the 'competitive region'. The inverse demand function changes slope discontinuously at $\hat{q}_i$ and therefore has a kink, i.e. it is continuous but not differentiable, at this point.

The gross profit function for firm i, which represents payoffs after accounting for the variable cost of producing the product, but before accounting for the fixed cost of product scope, is

$$\pi^i(q_i, t_i, t_j, p_j) = nq_i[P_i(q_i, t_i, t_j, p_j) - c], \quad i = A, B, \tag{4.2}$$

and has the following property:

Lemma 2. *The function $\pi^i(q_i, t_i, t_j, p_j)$ has either no interior maximum, or a unique interior maximum in q_i.*

We refer to the portion of the gross profit function which corresponds to the monopoly region of $P_i(q_i, t_i, t_j, p_j)$ as the *monopoly portion* of the duopolist's gross profit function, and that which corresponds to the competitive region of $P_i(q_i, t_i, t_j, p_j)$ as the *competitive portion* of the duopolist's gross profit function.

4.2. Price equilibrium configurations

Depending on the values of product scope $1/t_A$, $1/t_B$, four different equilibrium configurations are possible for the second-stage pricing game. These are illustrated in Figure 4.2.

1. **Local-monopoly equilibrium:** Under this equilibrium, each firm prices in the monopoly region of their demand curves. A subset of consumers do not purchase either product, and at equilibrium $q_A + q_B < \frac{1}{2}$.

2. **Adjacent-markets equilibrium**: Under this equilibrium, both firms price *at the kink* between the monopoly and competitive regions of their respective demand curves, and the market is fully covered, that is $q_A + q_B = \frac{1}{2}$. It might seem that this equilibrium configuration is a knife's-edge case, but it is actually feasible across a significant range of product scope values and demand parameters.

3. **Competitive equilibrium**: This equilibrium configuration is analogous to the 'standard' equilibrium that one comes across in spatial models. The firms price in the competitive region of their demand curves and potentially compete for each others' marginal consumers. In equilibrium, each consumer buys a product, and therefore, $q_A + q_B = \frac{1}{2}$

4. **Non-exclusive equilibrium**: This equilibrium configuration involves multiple purchases by a subset of consumers. Since this occurs only when $r > \frac{1}{2}$, it is discussed in the extended appendix.

When $r < \frac{1}{2}$, no more than one of these configurations results for any given values of product scope $(1/t_A, 1/t_B)$. The exact product scope ranges, and more precise statement of the following proposition are specified in Section A.4 of the extended appendix.

Proposition 1. *For each pair of feasible values of product scope* $(1/t_A, 1/t_B)$ *for the competing products, there is a unique feasible equilibrium, as illustrated in Figure 4.3.*

(a) At low levels of product scope, the outcome is a local-monopoly equilibrium.

(b) At intermediate levels of product scope, the outcome is an adjacent-markets equilibrium.

(c) At higher levels of product scope, the outcome is a competitive equilibrium.

Part (a) of the proposition indicates that for a fairly significant range of overlap in product func-

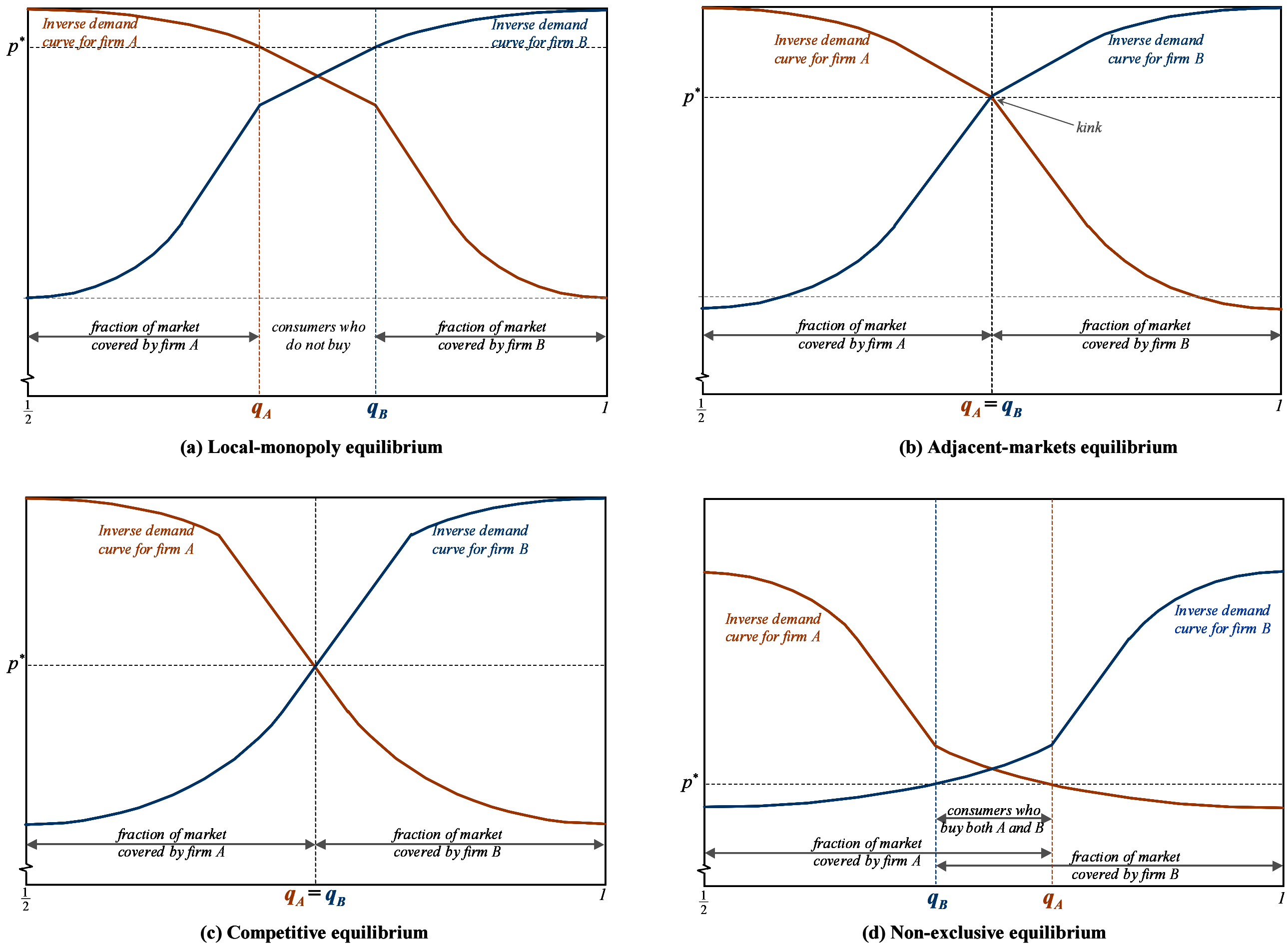

Figure 4.2: Illustrates the four feasible second-stage (price) equilibria. In the local-monopoly equilibrium, a positive fraction of customers do not purchase. The market is split evenly in the adjacent-markets and competitive equilibria, and in the non-exclusive equilibrium, which occurs only for $r > \frac{1}{2}$, a fraction of customers buy both products.

tionality, there ends up being no real strategic interaction between the price and demand choices of the two firms; each firm behaves like a monopolist, and a portion of the market is left unserved by either firm. This local-monopoly equilibrium persists for all pairs of scope values below the curve AM in Figure 4.3.

At any pair of scope values on the curve AM in Figure 4.3, all consumers purchase one or the other product. When product scope increases beyond this point, the resulting equilibrium configuration is an *adjacent-markets* equilibrium. The market is fully covered, yet neither firm finds it profitable to try and gain market share. Intuitively, this is because the benefit of capturing an opponent's consumer by

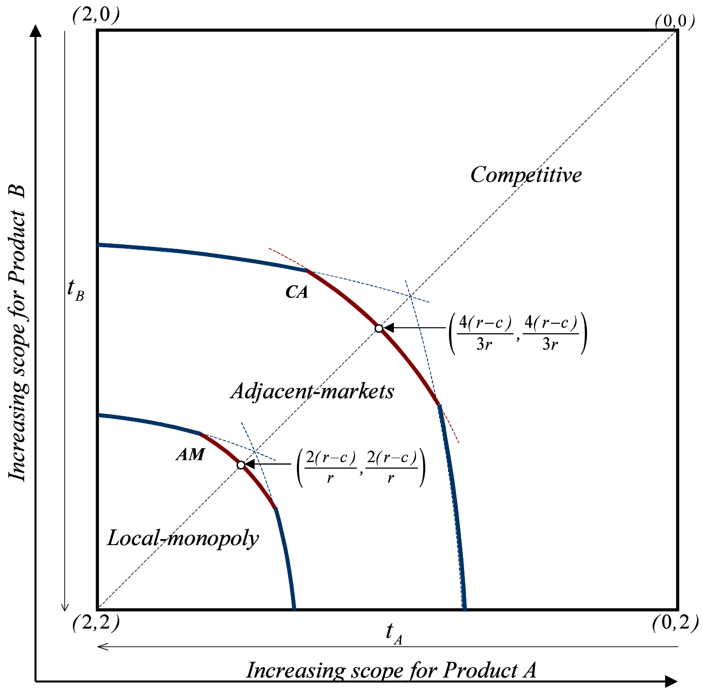

Figure 4.3: Illustrates the regions of the (t_A, t_B) space in which different equilibrium configurations occur, showing how as scope increases, the feasible configuration shifts from local-monopoly, to adjacent-markets, to competitive. The axes in the figure denote product scope. The origin corresponds to the lowest level of product scope required ($t = 2$) for each product to provide a positive level of effectiveness on the entire set of functionalities. The diagonally opposite vertex represents levels of product scope, where each product provides the entire set of functionalities at the highest level of effectiveness ($t = 0$). At this maximal level of product scope, the two products are effectively identical since each is equally effective on the entire set of functionalities.

dropping prices is outweighed by the corresponding loss in profit from one's existing customer base[9]. This behavior persists across a non-trivial range of values for product scope, between AM and CA.

Beyond CA, the outcome transitions to a competitive equilibrium configuration. Product scope is bilaterally high enough to make it profitable for each firm to unilaterally increase demand at the margin, even after accounting for the losses in profits from one's own customer base. Consequently, there is downward pressure on prices, which intensifies as product scope increases further.

[9] Some readers may have noted the similarity between our local monopoly and adjacent markets equilibria, and the corresponding local monopoly and kinked outcomes in Salop (1979). While the outcomes are indeed similar, their drivers are quite different. Salop's local monopoly and kinked equilibrium configurations arise due to differences in average costs. In our model, on the other hand, they arise primarily from the higher heterogeneity in consumer valuations at low product scopes that makes it unattractive for firms to increase demand at the 'kink'. Matutes and Regibeau (1988) have a similar evolution of equilibria between competing pairs of complementary products, where over an intermediate range of reservation values, firms producing incompatible substitutable systems engage in limit pricing in what they also term an 'adjacent markets' equilibrium.

Range of scope values	Equilibrium	Demand	Price	Gross Profit
$\frac{1}{2} \leq \frac{1}{t} \leq \frac{r^2}{r-c}$	Local-monopoly	$n\left[\sqrt{\frac{4[r-c]-r^2 t}{12t}}\right]$	$\left(\frac{2r+c}{3} - \frac{r^2 t}{6}\right)$	$\frac{n}{6}\sqrt{\frac{[4[r-c]-r^2 t]^3}{12t}}$
$\frac{r^2}{r-c} \leq \frac{1}{t} \leq \frac{r}{2[r-c]}$	Local-monopoly	$\frac{n[r-c]}{2rt}$	$\frac{r+c}{2}$	$\frac{n[r-c]^2}{4rt}$
$\frac{r}{2[r-c]} \leq \frac{1}{t} \leq \frac{3}{4}[\frac{r}{r-c}]$	Adjacent-markets	$\frac{n}{4}$	$r[1-\frac{t}{4}]$	$n[\frac{r-c}{4} - \frac{rt}{16}]$
$\frac{3}{4}[\frac{r}{r-c}] \leq \frac{1}{t}$	Competitive	$\frac{n}{4}$	$c + \frac{rt}{2}$	$\frac{nrt}{8}$

Table 4.1: Equilibria with symmetric values of product scope $1/t$ (scope increases from rows 1 to 4)

4.3. Symmetric equilibria with exogenous drivers of convergence

This section derives the Nash equilibrium prices for symmetric exogenously specified product scope $t_A = t_B = t$. This corresponds to a scenario where the level of convergence is not under the firm's strategic control, but is determined by industry-specific factors – for instance, due to progress in technology in an upstream industry (microprocessors and semiconductors in the case of computing and electronics devices), progress in a downstream industry (personal computers in the case of operating systems or application software), or exogenously specified *dejure* industry standards. Product scope could also be driven by competitive pressure from within one's industry, and our results might have some implications for this scenario, though the generality of this interpretation is limited by the fact that we do not explicitly model intra-industry competition.

The symmetric equilibria are those along the diagonal in Figure 4.3, their corresponding prices, demand and profits are presented in Table 4.1, and their essence is summarized in the following proposition.

Proposition 2. *At any symmetric level of product scope, there is a unique and symmetric Nash equilibrium pricing strategy. The equilibrium prices chosen by the firms are summarized in Table 4.1. As the two products converge due to an increase in scope* ($\frac{1}{t}$)*:*

(a) The price equilibrium transitions from local-monopoly to adjacent-markets to competitive equilibrium.

(b) Prices and gross profits are non-decreasing (and generally increasing) at low and intermediate levels of scope, but decrease progressively at high levels of scope.

Proposition 2 is illustrated in Figures 4.4 and 4.5, which also depicts the monopoly price and profit results (derived in the extended appendix) for comparison. At lower levels of product scope, there is a relatively low degree of convergence in the market (relatively low overlap in the set of functionalities supported by the two products), while at the extreme right, there is complete convergence (the two products are effectively identical).

As illustrated, exogenous convergence initially increases both prices and gross profits. This increase is driven first by the absence of a net substitution effect for $\frac{1}{2} \leq \frac{1}{t} \leq \frac{r}{2[r-c]}$ under the local-monopoly equilibrium, and the price increases are accompanied by a steady increase in demand for both products. Further product convergence transitions the outcome to an adjacent-markets equilibrium for $\frac{r}{2[r-c]} \leq \frac{1}{t} \leq \frac{3r}{4[r-c]}$. While there is real strategic interaction between the firm's choices of prices, the value effect continues to dominate the substitution effect, and convergence continues to bilaterally increase firm profits. Despite the converging products becoming increasingly *less* differentiated, prices *rise* rather than fall, even though the market is fully covered and there are no gaps in consumer demand that insulate the firms from competition. Strikingly, the duopoly prices in this region are even *higher* than the corresponding monopoly prices (the dotted line in Figure 4.4).

The economic intuition for this unusual result is as follows: the adjacent-markets equilibrium is at the transition point between the monopoly and the competitive portions of both firms' respective gross profit functions. The slope on the monopoly portion is positive, which would induce a demand increase by an unconstrained firm. However, the presence of the rival duopoly firm makes this demand increase impossible without reducing profits; due to the discontinuous change in the slope of inverse demand at the kink, the slope of the competitive portion of the gross profit function is still negative. The equilibrium response to this tension ends up being an increase in price (which keeps firms at the margin of the local-monopoly region), rather than in realized demand (which would move them to the competitive region). This also explains why even though duopoly prices are higher, duopoly profits are lower than monopoly profits.

As the products converge further, the outcome transitions to a competitive equilibrium, which

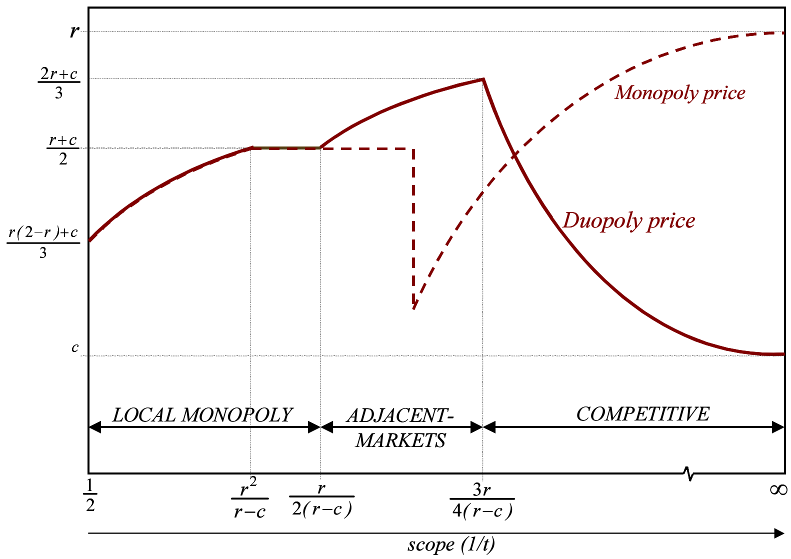

Figure 4.4: Equilibrium duopoly price as a function of scope $\frac{1}{t}$ for symmetric scope. Product scope increases along the x-axis from a level at which each product barely covers all the functionalities, to a level at which each product provides the entire set of functionalities at the maximum level of effectiveness.

is similar (though not identical) to that of the standard 'circle model' of horizontally differentiated duopoly. The firms split the market equally. As product scope increases, prices and profits fall, and the decline is inversely proportionate to product scope. When scope is infinite ($t = 0$), the two products are identical, prices converge to marginal cost and gross profits converge to zero as in the standard Bertrand duopoly.

An important strategic consequence of Proposition 2 is that whether digital convergence will be beneficial for a firm depends critically on how far it has progressed. The appropriate response to early-stage convergence is an increase in prices, despite an increase in product similarity, since the value effect is dominant, and convergence benefits firms in both industries. On the other hand, at a later stage of convergence, the substitution effect is dominant, and convergence should be resisted to the extent possible, since it is bilaterally profit-reducing. This trend can be illustrated by the mobile telephony and handheld computing industries. Following the launch of mainstream converged devices from both industries (the Palm-based Handspring Treo, and the Symbian-based Nokia Communicator), average prices for these devices were actually higher than those that prevailed when the flagship products from

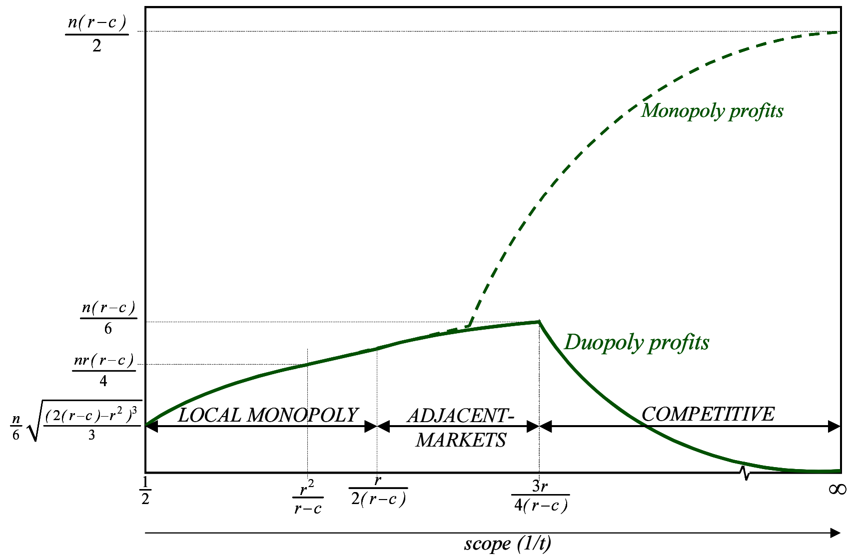

Figure 4.5: Equilibrium duopoly profits as a function of scope $\frac{1}{t}$ for symmetric scope. Product scope increases along the x-axis from a level at which each product barely covers all the functionalities, to a level at which each product provides the entire set of functionalities at the maximum level of effectiveness.

each of these device makers was more specialized. This is despite an evident increase in the overlap in their functionality; however, it unlikely to sustain if technological progress coupled with intense intra-industry competition drives outcomes to the point where products in the two industries are highly converged. Often, it is strategically sensible to exit industries that have converged too extensively with neighboring ones, since their profit potential is likely to have eroded significantly; with the increasing adoption of IP-based telephony, the wireline long-distance industry may be one such example.

The price expressions in Table 4.1 indicate that prices always *rise* as the breadth of functionality requirements of the consumers (r) *increases*. This is interesting, because an increase in r has two effects, both analogous to an increase in scope $\frac{1}{t}$ – it increases the gross value of each product for all consumers, but simultaneously reduces the effective level of product differentiation. Proposition 2 indicates that the former effect dominates the latter under all equilibrium outcomes[10]. From a managerial perspective, this highlights the benefits of inter-industry actions that increase the span of the average customer's needs, even if these reduce the effective level of product differentiation, since

[10] This is in contrast with the effect of an increase in product scope under competitive equilibrium. It highlights the importance of separating changes in the level of product differentiation that result from product scope choices, from those that result from changes in consumer needs.

Proposition 2 shows that the value effect of this kind of expansion dominates the substitution effect, independent of the extent of convergence between the industries.

Under the competitive equilibrium, any reduction in unit cost c translates into an equivalent price reduction, reflective of a high degree of competition. Under the local-monopoly equilibrium, cost decreases are shared by the firm and its consumers, and is accompanied by an increase in demand, which is consistent with the monopolistic nature of this outcome. However, under the adjacent-markets equilibrium configuration, wherein demand increases are not feasible, the entire benefit of a decrease in costs is captured by the duopolists, and prices remain unchanged[11]. This result has important implications for a firm's incentives to invest in technological progress that reduces their variable product costs. The benefit that a firm realizes from these investments is likely to be highest at intermediate stages of convergence (when prices are also at their maximum). In contrast, when industry convergence has progressed to the point where firms perceive active price competition from their neighboring industries, any variable cost reduction benefits will flow entirely to consumers.

5. Strategic product convergence

This section establishes that when firms can strategically control convergence by bilaterally making endogenous choices of product scope, their subgame perfect equilibrium choices result in relatively low levels of convergence. Specifically, the second-stage pricing subgame always results in either a local-monopoly or an adjacent-markets equilibrium. We also examine how equilibrium levels of product convergence vary as technological progress alters the fixed costs of increasing scope and the variable costs of production.

The timeline is as follows. In stage 1, firms simultaneously choose product scope $(1/t_A, 1/t_B)$. In stage 2, with perfect information about the first period choices, firms simultaneously choose prices.

[11] Beyond these direct effects within an equilibrium configuration, changes in r and c also affect the relative sizes of the regions of t under which each equilibrium is feasible. Since $(\frac{r-c}{r})$ is increasing in r and decreasing in c, an increase in the breadth of functionality requirements, or a reduction in unit costs both increase competitive intensity by decreasing the range of scope values at which the local monopoly and kinked equilibria are feasible, and increasing the ranges at which the outcome is a competitive equilibrium.

Denote the equilibrium second-stage choice of price by firm i as $P_i^*(t_i, t_j)$. These choices have been characterized in Section 4 and in the extended appendix. Firm i's first stage payoff function, or its net profit function after accounting for the cost of scope and the corresponding second-stage equilibrium choices of price and quantity, is:

$$\Pi^i(t_i, t_j) = \pi^i(q_i^*(t_i, t_j), t_i, t_j, P_j^*(t_j, t_i)) - F(t_i),$$

where $\pi^i(q_i, t_i, t_j, p_j)$ is the firm's gross profit as defined in (4.2). As established in Section 4, there are three potential equilibrium configurations in the second-stage subgame. Also, the firms' first stage best-response functions are neither continuous nor monotonic[12]. Therefore, to characterize the equilibria of the game, we adopt an indirect approach, examining ranges of product scope that correspond to each of the second stage subgames, and using the monotonicity of gross profits to derive the subgame perfect equilibrium. The following result establishes that within any second-stage subgame, the marginal effect of changing t_i on firm payoffs is always strictly monotonic.

Lemma 3. *Given any pair (t_A, t_B) for which $P_i^*(t_i, t_j)$, $i = A, B$ exists, if (t_A, t_B) is such that small unilateral changes in product scope do not change the equilibrium configuration of the second-stage subgame, then:*

(a) If this subgame corresponds to a local-monopoly or an adjacent-markets equilibrium, each firm's gross profits are increasing in its own product scope. That is, $\frac{d}{dt_i}\pi^i(q_i^*(t_i, t_j), t_i, t_j, P_j^*(t_j, t_i)) < 0$.

(b) If the subgame corresponds to a competitive equilibrium, each firm's gross profits are strictly decreasing in its own product scope. That is, $\frac{d}{dt_i}\pi^i(q_i^*(t_i, t_j), t_i, t_j, P_j^*(t_j, t_i)) > 0$.

The derivatives in Lemma 3 are total derivatives[13], including adjustments for the equilibrium

[12] A further complication is that there are portions of the (t_A, t_B) space (where the values of t_A and t_B are substantially different) in which no pure strategy equilibria may exist. It is possible that mixed-strategy equilibria do exist. However, since our payoff functions are not quasi-concave, the existence of mixed strategy equilibria is not guaranteed (Dasgupta and Maskin 1986) and their derivation is beyond the scope of this paper. A simple way around this problem (used by Economides 1984, for instance) is to restrict the first-stage action spaces of the firms to those values of scope for which second-stage pure strategy pricing equilibria exist. This is done by setting the payoffs of both firms to zero for values of (t_A, t_B) for which a pure strategy price equilibrium does not exist. Note that pure-strategy equilibria always exist for all symmetric pairs $t_A = t_B$ (and for pairs in their immediate neighborhood), and these are the equilibria we focus on.

[13] Although directionally similar, the result of Lemma 3 is distinct from that of Proposition 2, which describes the

second-stage changes in $q_i^*(t_i, t_j)$ and $P_j^*(t_j, t_i)$ that occur when one changes t_i. As an immediate consequence of Lemma 3(b), since $F_1(t) \leq 0$, a pair (t_A, t_B) that results in a competitive equilibrium configuration in the second stage can never be a first-stage equilibrium choice (since either firm can reduce scope by increasing t_i, thereby increasing gross profits and reducing fixed costs). This is formalized in the next proposition,

Proposition 3. *(a) Any value of product scope $(1/t_d^*)$ that satisfies the following conditions is a symmetric first-stage equilibrium choice of scope:*

$$\Pi_1^i(t_i, t)|_{t_i = t^+} \leq 0, \tag{5.1}$$

$$\Pi_1^i(t_i, t)|_{t_i = t^-} \geq 0, \text{ and} \tag{5.2}$$

$$\Pi_{11}^i(t_i, t)|_{t_i = t_d^*} < 0. \tag{5.3}$$

(b) For any non-negative fixed cost of scope $F(t) \geq 0$, the equilibrium strategic choices of product scope result in a relatively low level of product convergence. Specifically, equilibrium choices of (t_A, t_B) always result in second-stage outcomes that are either local-monopoly or adjacent-markets equilibria.

Note that only (5.1) and (5.2) are necessary conditions, and when $\Pi_1^i(t_i, t)$ is continuous at $t_i = t$, reduce to the more familiar first order condition

$$\Pi_1^i(t, t) = 0. \tag{5.4}$$

The special case where increases in scope are costless i.e. where fixed cost is independent of scope ($F_1(t) = 0$ for all t) is characterized in Proposition 4.

Proposition 4. *When changing product scope is costless, i.e. $F_1(t) = 0$:*

(a) There is potentially a continuum of asymmetric subgame-perfect Nash equilibria. The first-

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comparative statics of changing product scope symmetrically for both firms. Lemma 3 establishes the effect of changing a firm's own product scope while holding the (possibly different) scope level of its opponent constant.

stage choices under these equilibria correspond to those points on the line CA in Figure 4.3 (which defines the boundary between the adjacent-markets and competitive equilibrium regions) for which a second-stage pure-strategy price equilibrium exists.

(b) There is always a unique symmetric subgame-perfect Nash equilibrium. The two firms choose symmetric levels of product scope $\frac{1}{t^*} = \frac{3}{4}[\frac{r}{r-c}]$ *and price* $P^* = \frac{c+2r}{3}$*, split the market equally, and earn symmetric levels of profits* $\pi^* = \frac{1}{6}[r-c]$*. This corresponds to an adjacent-markets equilibrium outcome in the second stage pricing game.*

Proposition 4 can be viewed as the limiting case of a scenario where technological progress reduces the cost of increasing platform scope. For brevity, we focus our discussion on the symmetric equilibrium. It is somewhat striking that the equilibrium choices of scope (and the value provided by products) is limited even when scope is costless. In other words, not only is it strategically sensible to limit product convergence (as discussed following Proposition 2), but this choice is also an equilibrium strategy. The restricted level of convergence that emerges as subgame-perfect in Proposition 4 actually reflects the *optimal balance* between the value effect and the substitution effect that we have highlighted as characterizing digital convergence. Figure 4.4(a) shows that equilibrium duopoly prices are higher than they would be under a monopoly market structure at the same level of product scope; moreover, equilibrium profits are the highest possible under any symmetric duopoly equilibrium. The symmetric levels of convergence are therefore Pareto-efficient as well.

For industries in which firms' product scope choices are not entirely driven by industry standards or exogenous technology factors, an important strategic implication is therefore that after a point, these choices should be determined *purely* by strategic considerations. Managers should resist being influenced by the technological possibilities of designing highly versatile products that consumers value highly, and should limit the extent to which their products can fulfill customer needs even if more extensive fulfillment is technologically viable as well as cheap. We believe that strategy of this kind might have contributed to limit the extent of convergence between the cable and telecommunication industries, thereby enabling firms in both these industries to charge relatively high prices for their

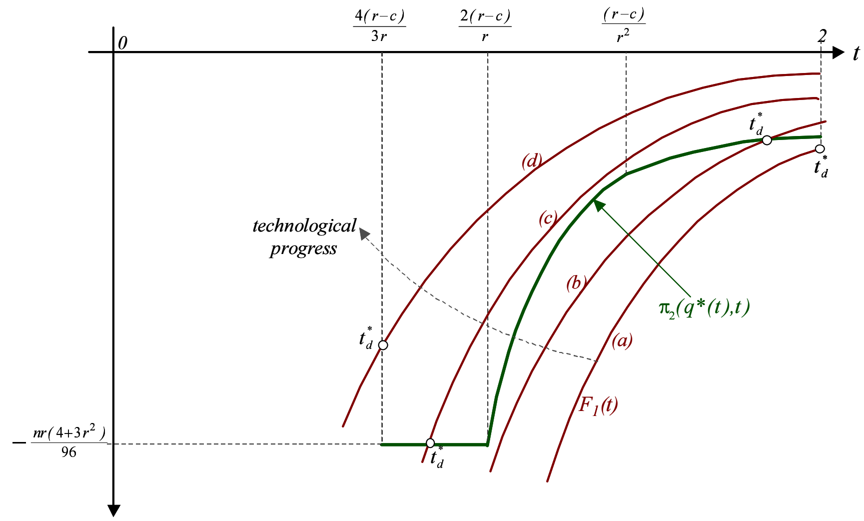

Figure 5.1: Equilibrium choice of product scope for different marginal costs of scope. $\pi_2(q^*(t), t)$ is marginal gross profit and the four $F_1(t)$ curves represent four different levels for marginal cost of scope.

broadband services. However, the recent entry of cable companies into offering local telephone service, and a corresponding move by Verizon into cable television delivered over fiber-optic lines suggests that they are moving away from their strategically sensible choices and towards ones driven by technological feasibility.

An immediate corollary of Proposition 4 is that while a reduction in variable costs c does lead to lower prices, firms capture a substantial fraction of this reduction. Further, this effect on price is entirely indirect, through the equilibrium change in their choice of product scope $\frac{1}{t^*}$. Recall from Section 4 that under an adjacent-markets equilibrium with fixed product scope, the duopolists did not reduce prices when c was reduced. Moreover, the equilibrium level of scope chosen by the duopolists is lower than the socially efficient level[14].

When fixed costs are strictly positive and increasing scope is costly, firms may find it profitable to choose a level of scope lower than the equilibrium value derived in Proposition 4. If there is a unique point t_d^* that satisfies conditions (5.1-5.3) from Proposition 3, this is the unique symmetric equilibrium.

[14] At the socially efficient level of scope, each product satisfies every functionality requirement of the consumer perfectly ($t = 0$), thus resulting in the highest amount of surplus possible. However, the products are also perfect substitutes, leading to undifferentiated Bertrand competition and marginal cost pricing. Interestingly, a monopolist would find it optimal to provide an infinite level of product scope, thereby achieving the first best outcome. However, the monopolist would also end up appropriating the entire value created, leaving no surplus to the consumers.

The curves *(b)* and *(c)* in Figure 5.1 illustrate two candidate marginal fixed costs curves, and their first-stage equilibrium levels of scope, which lead to second-stage local-monopoly and adjacent-markets equilibria respectively.

If feasible values of t that satisfy (5.1-5.3) do not exist, we need to examine the value of $\Pi_1^i(t,t)$ at the boundary points $t = \frac{4[r-c]}{3r}$ and $t = 2$. Curve *(d)* in Figure 5.1 illustrates one such an outcome where $F_1(t)$ lies entirely above the second-stage equilibrium marginal profit curve. Product scope at the unique symmetric subgame perfect equilibrium in this case is as obtained with costless scope, and strengthens Proposition 4 to the extent that it's result continues to hold if the marginal cost of scope is non-zero but small. Curve (a) in Figure 5.1 illustrates the case where $F_1(t)$ lies entirely below the second-stage equilibrium marginal profit curve. The symmetric subgame perfect equilibrium in this case corresponds to $t_d^* > 2$, and therefore, we do not interpret this further[15].

6. Discussion

This paper has presented a model of competition between platform-based products in converging information technology markets. The model provides a careful and detailed representation of how products fulfil diverse consumer functionality requirements, how the effectiveness of these products varies with scope, and how this affects value and product differentiation when industries converge. It represents a new contribution to the economics of information technology, generalizing standard horizontal differentiation and product characteristics models of imperfect competition, explicitly separating product design choices from consumers' preferences, characterizing four different kinds of pricing equilibrium outcomes, three of which are described in detail in this paper, and enabling the derivation of equilibria involving multiple purchases. A number of our results attest to the usefulness of these modeling enhancements.

Our key managerial insights are summarized below:

[15] If a unique value for t_d^* exists, that satisfies condition (5.4) of Proposition 4 but not (5.3), then it is a local minimum. In this case, either $t_d^* = \frac{4}{3}(\frac{r-c}{r})$ or $t_d^* > 2$ could be the equilibrium. Very little can be said for the case where there are multiple points that satisfy (5.4) and (5.3) and the equilibrium in this case depends on the specific form of the fixed cost function.

– At early and intermediate stages of digital convergence, the value effect dominates the substitution effect. Managers should expect a bilateral increase in price and profits as technology markets begin to converge; this increase will be despite a growing overlap in functionality between the converging products. However, at a later stage of convergence, the substitution effect dominates the value effect, bilaterally reducing prices and profits. When converge is driven by exogenous factors (technological progress in an upstream/downstream industry, technological standards, intra-industry competition), managers must therefore be careful not to overestimate the benefits or sustainability of the early gains their companies may realize from digital convergence.

– When the extent of digital convergence can be controlled endogenously by firms, it is crucial that product managers place strategic considerations ahead of technological possibilities. It is only through a careful and appropriate strategic control of the scope of one's product that a firm can sustain the higher revenues that convergence makes feasible. A restricted choice of scope under which both prices and profits are bilaterally maximum across both converging industries is supported as an equilibrium, even when increasing the scope of one's products is costless.

– Managerial incentives to invest in technology that reduces their variable costs of production (for instance, by shifting device logic from hardware to software) are maximum at an intermediate stage of digital convergence; a firm can capture a significant fraction of the reduction in variable cost their investment realizes. In contrast, at an advanced stage of convergence, any cost reduction from these investments will be competed away, captured by customers via lowered prices.

– Industry-wide efforts that result in an increase in the span of functionality that each customer values are likely to be more profitable over time than those which increase the scope of products. While both of these changes increase both the value of products as well as their substitutability, the former changes always enable firms to increase prices and profits, independent of the extent of digital convergence.

While prices in converging industries are often higher than their monopoly counterparts, this is partially a consequence of higher product variety and hence, a higher average product value. The contrast is with respect to a single-product monopolist; a monopolist supplying two products instead of one would actually charge prices exactly equal to the equilibrium duopoly prices. Depending on the shape of the fixed cost function, it may not be optimal for a monopolist to offer multiple products. Correspondingly, we have shown that firms who are duopolists on account of their industries converging and who can restrict their product scope will do so in equilibrium. In this scenario, convergence can reduce total surplus, since a monopolist would provide a much higher level of product scope when faced with the same demand and cost function; however, the monopolist would also appropriate most or all of this higher level of surplus.

Our analysis of the fourth kind of equilibrium – the non-exclusive equilibrium presented in our extended appendix – suggests an interesting trajectory in purchasing patterns that may accompany continued digital convergence when the span of customer requirements is relatively high. When products are relatively specialized, consumers are forced to purchase multiple products to satisfy their requirements, as no single product is sufficiently effective over the entire set of functionalities they desire. As product scope increases, each product becomes significantly more effective on a broader range of functionalities and consequently consumers shift to buying fewer products. Interestingly, at very high levels of convergence, consumers may once again switch to buying more products. This latter shift is caused by the fact that at very high levels of convergence, prices drop to the extent that purchases can be justified based solely on the incremental value each product provides. While each product is very effective at satisfying a consumer's entire span of desired functionalities, consumers still purchase multiple products and use these general-purpose products as if they were specialized; multi-computer households that buy both PCs and Macs are a good example of this phenomenon.

This trajectory of purchasing patterns may in fact be cyclical. The emergence of new functionalities or changes in quality expectations by consumers may alter the space of functionalities or the partial-equilibrium utility functions of consumers. This could result in firms' broad product scope choices as eventually being perceived as specialized again, and a repetition of this cycle when the next generation of technology emerges. Incorporating this kind of effect into a formal model remains work in progress.

A limitation of this paper is that it does not explicitly model intra-industry competition. Our analysis of exogenously varying product scope in section 4 alludes to this effect by explaining how firms will behave when the only strategic variable they can control is price, and suggests that if there are intra-industry competitive factors that dictate product scope choices, converging markets will intensify competition and prices will fall as product scope increases. In fact, converging into a different industry may be a strategic response to increased competition within one's own industry (this may explain, for instance, why Handspring initially chose to incorporate voice communication into their product via the Springboard). A more precise analysis of differentiated competition within each industry with the simultaneous threat of convergence from a neighboring industry remains an active direction of research in progress. We are also working on extending our model to incorporate a richer specification of product quality and admit a wider range of technological constraints on offering sets of functionalities within single products. We hope to address these open issues in the near future.

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8. Appendix: Outline of proofs

This appendix presents an outline of the proofs of our results. The complete proofs are available in our extended appendix.

Lemma 1: (a) The continuity of $U(y,t)$ is established by verifying its values from below and from above at $y = \frac{1+r}{2}$ and $y = \frac{2-r}{2}$, the two points at which its functional form changes. Computing the expressions for $U_1(y,t)$ and $U_2(y,t)$ verify that both are negative.

(b) The continuity of $U_1(y,t)$ is established by verifying its values from below and from above at $y = \frac{1+r}{2}$ and $y = \frac{2-r}{2}$, and inspection of its expression establishes that it it piece-wise differentiable.

(c) Computing the expression for $U_{12}(y,t)$ verifies that it is negative.

Lemma 2: The proof defines the following functions:

$$R^M(q,t) = nq[P^M(q,t) - c]. \tag{8.1}$$

$$R^C(q_i,t_i,t_j,p_j) = nq_i[P^C(q_i,t_i,t_j,p_j) - c]. \tag{8.2}$$

The function $\pi^i(q_i,t_i,t_j,p_j)$, $i = A,B$ takes either the form $R^M(q_i,t_i)$ or the form $R^C(q_i,t_i,t_j,p_j)$. The former is the monopoly profit function, which is shown in Lemma 4 to have at most one interior maximum in $[0,\frac{1}{2}]$. The function $R^C(q_i,t_i,t_j,p_j)$ is shown to be strictly concave for $q_i \leq \frac{1-r}{2}$ and to have no more than one interior maximum in $[0,\frac{1}{2}]$. Finally, the condition $R_1^M(q^*,t) = 0$ for any q^* is shown to imply that $R^C(q,t_i,t_j,p_j) < 0$ for $q > q^*$, which implies that both $R^M(q,t_i)$ and $R^C(q,t_i,t_j,p_j)$ cannot simultaneously have interior maxima.

Proposition 1: This is a fairly elaborate proof. Broadly, for any (t_A,t_B), the following two functions are defined:

$$\begin{aligned} Q^M(t_i) &= x : \frac{U(x+\frac{1}{2},t_i) - c}{-U_1(x+\frac{1}{2},t_i)} = x, \text{ if such an } x \text{ exists in } [0,\frac{1}{2}] \\ &= \frac{1}{2} \text{ otherwise.} \end{aligned} \tag{8.3}$$

$$\begin{aligned} Q^C(t_i,t_j) &= x : \frac{U(x+\frac{1}{2},t_i) - c}{-[U_1(x+\frac{1}{2},t_i) + U_1(1-x,t_j)]} = x, \text{ if such an } x \text{ exists in } [0,\frac{1}{2}] \\ &= \frac{1}{2} \text{ otherwise.} \end{aligned} \tag{8.4}$$

$Q^M(t_i)$ is the interior maximum in q (if it exists) of the *monopoly* portion of the profit function $\pi^i(q, t_i, t_j, p_j)$, and is $\frac{1}{2}$ otherwise. Analogously, $Q^C(t_i, t_j)$ is the interior local maximum of the *competitive* portion of the function $\pi^i(q, t_i, t_j, p_j)$ with $p_j = U(1 - Q^C(t_i, t_j), t_j)$. The proof of the proposition then shows the following relationship between these functions and the existence of the different kinds of equilibria:

- A local-monopoly equilibrium is feasible only if $Q^M(t_A) + Q^M(t_B) < \frac{1}{2}$
- An adjacent-markets equilibrium is feasible only if $Q^M(t_A) + Q^M(t_B) \geq \frac{1}{2}$ and $Q^C(t_A, t_B) + Q^C(t_B, t_A) \leq \frac{1}{2}$
- A competitive equilibrium is feasible only if $Q^C(t_A, t_B) + Q^C(t_B, t_A) \geq \frac{1}{2}$.

By definition, $Q^M(t_A) + Q^M(t_B) < Q^C(t_A, t_B) + Q^C(t_B, t_A)$, and this relationship therefore establishes that there is a maximum of one kind of equilibrium configuration for each (t_A, t_B).

The regions in the (t_A, t_B) space over which each of the configurations exist are established by (i) computing the explicit algebraic form of $Q^M(t_i)$ and $Q^C(t_i, t_j)$ for different ranges of (t_i, t_j) and (ii) computing the ranges of (t_i, t_j) over which each of the three conditions above are satisfied. The Nash equilibrium price pairs for each configuration are derived by computing the equilibrium q_A, q_B pairs, and are summarized in Table A.4 in the extended appendix.

Proposition 2: This follows directly from the proof of Proposition 1. When $t_A = t_B = t$, the expression for $Q^C(t, t)$ is considerably simplified, and applying the three conditions above yield the ranges of scope values in column 1 of Table 4.1. The equilibrium price pairs for each equilibrium have been computed in the proof of Proposition 1, and substituting $t_A = t_B = t$ yields the expressions in column 3 of Table 4.1. Once these prices and demand values are known, computing the payoffs is straightforward.

Lemma 3: (a) For a local-monopoly equilibrium, the result obtains from a direct application of the envelope theorem. For an adjacent markets equilibrium, the proof examines the changes in payoffs for a small decrease in scope, represented by a change in t_i to $t_i + \varepsilon$. Since there are multiple equilibria for most (t_A, t_B) pairs, it is assumed that if the choice of quantities prior to the increase of ε continues to be an equilibrium, the firms remain at this equilibrium, and the result follows from the fact that $R_2^C(q_i, t_i, t_j, p_j) < 0$. If not, the firms move to a new quantity pair that was not an equilibrium before the increase of ε; the proof shows that this must always lead to a decrease in the equilibrium demand for firm i, and consequently, payoffs decrease.

(b) An algebraic expression for the equilibrium profit function is computed in the proof, and its total derivative with respect to t_i is shown to be strictly positive.

Proposition 3: (a) These are simply first-order necessary and second-order sufficient conditions on the payoff functions for $(1/t_d^*)$ to be a first-stage Nash equilibrium.

(b) Lemma 3(b) shows that under any competitive equilibrium, a unilateral decrease in scope by firm i (that is, and increase in t_i) increases revenues; at best, this decrease in scope leaves costs unchanged (or reduces them). Therefore, firm i can increase profits by increasing t_i. As a consequence, the second-stage equilibrium has to be either local-monopoly or adjacent markets.

Proposition 4: (a) Lemma 3(a) shows that for any first-stage candidate pair (t_A, t_B), which corresponds to a local-monopoly or adjacent-markets equilibrium subgame, firms have a unilateral incentive to increase their scope if it leaves them in the same equilibrium configuration. Further, along the AM locus the payoff functions are continuous and decreasing in t_i. Therefore, these cannot be part of any subgame perfect equilibrium. Along the CA locus, an increase in t_i takes the firm into the adjacent-markets region while a decrease takes it into the competitive region. Both those changes strictly reduce payoffs. As a consequence, any pair (t_A, t_B) along the CA locus for which a pure-strategy second stage equilibrium exists is part of a subgame perfect equilibrium.

(b) Given (a), the only feasible symmetric subgame perfect Nash equilibrium is the one under consideration. For symmetric values of scope, a pure strategy second-stage price equilibrium always exists. The proof simply computes the appropriate algebraic expressions for equilibrium price and payoffs.