The Emergence of Concentrated Ownership and the Rebalancing of Portfolios due to Shareholder Activism in a Financial Market Equilibrium*

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Abstract

Consider a financial market equilibrium with correlated firms and risk averse investors holding diversified portfolios. When an activist investor has the ability to perform value-enhancing activities in a single firm, and these activities increase with ownership, we show that optimizing behavior by all investors leads to a concentration of shares in the hands of this activist. This concentration arises in the presence of complete information and is a consequence of Walrasian equilibrium mechanisms that include all investors and give no special powers to any of them in the equilibrium process. By yielding more ownership to the activist, all investors alter the risk profiles of their holdings, ending with less balanced portfolios. This rebalancing effect is accompanied by an increase in the price of the security that the activist can affect, as well as in the total value of the market. When the activist can affect more than one firm, rebalancing of all portfolios again occurs. Although the activist may not acquire increased concentration in all the firms she might affect, prices change for all those firms, and we give conditions under which at least one price must increase. We find that equilibrium results in a sharing of the costs and benefits of activism among all market participants, mitigating the free-rider problem. When we study multiple activists in many firms, we show that concentration can occur for several activists, and rebalancing occurs for all investors. Predictions on investor-specific concentration are difficult and excessive portfolio churning is present. The introduction of asymmetric information concerning activism again results in rebalancing and in concentration of ownership, but not necessarily in the hands of the activist.

The Emergence of Concentrated Ownership and the Rebalancing of Portfolios due to Shareholder Activism in a Financial Market Equilibrium*

Consider a financial market equilibrium with many firms and with risk averse investors holding diversified portfolios. Are there conditions under which these investors would forego some of the advantages of diversification, preferring instead a more concentrated ownership structure among themselves?

In this paper we show that when some of these investors have the ability to perform value-enhancing activities, and when this ability increases with ownership, optimizing behavior by all investors leads to a rebalancing of all market portfolios held by all market participants. This occurs in the presence of complete information and is a consequence of Walrasian equilibrium mechanisms that include all investors. To obtain our results, we do not give prior power to an originating entrepreneur to assign large block holdings, we do not permit any shareholders a strategic role in price determination and we do not require liquidity or noise traders.

When the potential activist is known to be able to affect only a single firm, we show that the rebalancing effect implies an increase in concentration of ownership in that firm in the hands of that activist. We also show that the rebalancing effect is accompanied by an increase in the price of the security that the activist can affect, as well as in the total value of the market. When we study the problem of an potential activist who is able to affect more than one firm, and we again find that the rebalancing of all portfolios for all investors occurs. However, unlike the case when the potential activist can affect only one firm, the activist may not acquire a concentration of ownership in all, or indeed any, of the firms that she might affect. Nonetheless, prices change for all the firms that the activist can affect, and we establish conditions under which some of these prices must increase. Since the activist alone is assumed to bear the costs of activism, a free-rider problem arises. We show that due to the prices established in the Walrasian equilibrium, this problem is partially mitigated.

We also model the problem of multiple activists in many firms. In this more complicated case, we show that rebalancing continues to occur in the portfolios of all investors, and concentration can occur for several activists. Gaming and other effects make the results unpredictable and lead to excessive portfolio churning.

When we allow asymmetric information regarding the ability of the activist to affect the performance of a single firm, concentration of ownership occurs, but it does not necessarily occur in the hands of the activist. We investigate the misallocations that may result from the rebalancing of the portfolios.

The remainder of the paper is organized in the following way. Section I reviews the relevant literature and discusses the relationship of our model to this literature. We present our model of a single activist with the potential to affect an arbitrary number of firms, and give its general solution, in Section II. In Section III we explore the impact of activism on ownership concentration. We show that it leads to the rebalancing of all portfolios of all market participants and that, consequently, no investors hold the market portfolio. In Section IV we study the impact of activism on equilibrium prices. We show that activism causes equilibrium prices to change, and derive the conditions for which these prices increase. Then, we discuss the increase in the value of the market resulting from activism. The issue of free-ridership is analyzed in Section V, where we find that equilibrium results in a sharing of the costs and benefits of activism among all the market participants. We then extend our model to incorporate the possibility that an arbitrary number of activists have the potential to improve an arbitrary number of firms in Section VI, and derive the ownership structure and equilibrium prices in this setting. Asymmetric information concerning the abilities of the activist is modeled in Section VI, and its impact on ownership structure and equilibrium prices is studied. In Section VIII we present our summary and conclusions.

1 Discussion of the Literature

Our paper builds on ideas developed and discussed in the literature relating to takeovers and to shareholder activism. In their seminal paper, Grossman and Hart (1980) study the conditions under which a tender offer might succeed, thereby leading to a concentration of ownership. They argue that corporate takeovers will not occur unless some means, e.g., dilution, can be devised to make the value of a share worth more to the raider than to an existing shareholder. Shleifer and Vishny (1986) show that an ownership structure containing a large shareholder is enough to induce increased takeover activity. However, starting from a diffuse ownership situation, they find that only a type of asymmetric information would lead to a large shareholder. Both the Grossman and Hart and Shleifer and Vishny papers assume a structure in which risk neutral investors in a single firm face the prospect of selling their shares in a tender offer. Kyle and Vila (1991) continue to study takeovers in the context of a single firm and risk neutral investors and, in introducing noise traders, move the study of takeovers into a market equilibrium context. The equilibrium is one in which the informed trader has monopsonistic power, and acts strategically to maximize expected profits. Using the equilibrium concept of Kyle (1985), Kyle and Vila show that, in an equilibrium with noise traders, neither dilution nor a large shareholder is needed to induce the informed trader to find it profitable to declare a takeover.

The focus on market equilibrium is maintained in the models of shareholder activism of Admati, Pfleiderer and Zechner (1994), Maug (1998), Bolton and von Thadden (1998) and Kahn and Winton (1998), although the definition of equilibrium employed differs. Admati, Pfleiderer and Zechner place the question of the concentration of ownership in a market context with risk averse investors. They assume one risk averse large investor, a continuum of risk averse small investors and multiple securities. In their model, the large investor, who can alter the outcome of the firm, has strategic power enabling him to supply information concerning the future value of the firm to the continuum of small investors. These small investors, among themselves, use this information to determine the division of ownership between themselves and the large investor. Knowing this mechanism, the large investor chooses the optimum information to supply. As a result, Admati, Pfleiderer and Zechner find that the large investor does not hold the market portfolio. Maug (1998) explicitly raises the question of the emergence of a large shareholder with the incentive to be active. He does so in the context of a model with a single firm and risk neutral investors. As in Admati, Pfleiderer, and Zechner, Maug allows the large shareholder to alter the flow of information to the marketplace for his own advantage. He posits liquidity traders and a market maker in a single firm, similar to Kyle and Vila, but then allows the large shareholder to optimize the size of his holdings. Maug's analysis focuses on the role of market liquidity in determining the concentration of ownership. Bolton and von Thadden (1998) also consider the problem of the emergence of a large active shareholder in the context of a single firm and risk neutrality. The focus of their paper, as Maug's, is the role of liquidity and its relationship to ownership concentration. In the Bolton and von Thadden model, the owner of a firm distributes shares of that firm to a finite number of investors. In the next period, after observing the performance of the firm, these owners, together with a new group of investors, renegotiate their holdings in a bidding game. The value of the firm at this time depends on the ownership structure determined by the original owner. Bolton and von Thadden establish the conditions under which the original owner would choose to concentrate ownership in the hands of one investor, and relate this decision to the liquidity of the market. Emergence of ownership concentration is also a focus of Kahn and Winton (1998), who study large shareholder intervention in the context of a single firm with risk neutral investors. Liquidity traders, large informed investors, and market makers participate in the equilibrium considered by Kahn and Winton. They explore the decision of a large owner who must choose whether to intervene and improve the performance of a firm or to sell its shares in the firm.

Related papers also include Huddard (1993) and Cuoco and Cvitanic (1998). In Huddard activism is limited to the enforcement of an incentive contract on the manager. He examines the role of a risk averse active shareholder and a finite number of risk averse small investors in one security. In considering the balance between added return and added idiosyncratic risk, Huddard shows that a large monopsonistic shareholder will increase his holdings. Cuoco and Cvitanic examine an optimum consumption and investment problem in continuous time. They assume the existence of a large investor who acts monoposonistically and whose portfolio choices have an impact on the expected returns of assets. By modeling the relationship between prices and the investments of the large shareholder, Cuoco and Cvitanic establish the existence of optimal consumption and investment policies.

Our model differs from others in the literature in five ways. First, by assuming that share demands are resolved by all participants in a Walrasian equilibrium, our analysis proceeds without either liquidity or noise traders or monopsonistic power on the part of the activist. Second, we link the effectiveness of an activist directly to the ownership she acquires in the equilibrium setting. Third, by examining a market for correlated securities when investors are risk averse, we are able to analyze the general portfolio impacts resulting from activism. Fourth, we investigate the implications of allowing the investor or investors to be potential activists in more than one firm. Fifth, we allow for asymmetric information between the activist and the other investors.

In our model, we begin with an equilibrium in which all investors hold diversified portfolios, and focus on the subsequent emergence of a large shareholder and the consequences of this emergence on the other market participants, prices and holdings. We assume a finite number of risk averse investors, one (or several) of whom can influence the random returns of firms by her (their) activism. All investors are seeking to invest in a finite number of firms which are assumed to be interdependent. We assume that the degree of success of an activist within a firm is monotonic in the amount of ownership acquired by that individual, and analyze the Walrasian equilibrium of such a system when all parties, including the activist, must compete for shares in the marketplace. We do not give the activist any strategic role in the determination of price. Through our analysis, we wish to derive conditions under which the marketplace would "cede" additional ownership to the activist investor.

Whereas our model attempts to answer some of the same questions concerning the emergence of large shareholders that Bolton and von Thadden, Maug, and Kahn and Winton address, our model is most closely related to that of Admati, Pfleiderer and Zechner (1994). Their model, as does ours, investigates the market implications of a risk averse activist in the context of risk averse investors maximizing end-of-period wealth. Our model differs from theirs in that we do not give monopsonistic power to the activist as they do. Although they introduce the possibility that the return of securities might depend on ownership, they investigate what they call the case of an allocation-neutral monitoring technology in which an activist's effectiveness does not depend on ownership. We, on the other hand, assume in our model that the effectiveness of the activist is related to her level of ownership. Also, we go beyond their results in analyzing the spillover effect on portfolio holdings, as well as the equilibrium prices, resulting from activism.

2 The Model and its General Solution

We assume that there are M investors, F_j, $j = 1, ..., M$, each of whom has an exponential utility function u_j, with Pratt-Arrow coefficient of absolute risk aversion a_j. Each investor F_j is assumed to be a von-Neumann Morgenstern expected utility of end-of-period wealth maximizer. Our model considers three moments of time, t = 0, 1, and 2. At t = 0, the group of M risk averse investors, sharing common information about the future prospects of N risky interdependent firms at t = 2, vie for shares in those risky firms in a competitive market. At t = 1, three pieces of new information become known to all market participants: (1) that some of the firms could benefit from reorganization, (2) that one (or several) of the investors has a unique ability to affect this reorganization and improve the performances of these firms, and (3) that the success in implementing these improvements is monotonic in the amount of ownership possessed by these skilled investors. With this new common information, all investors again vie for shares in the N firms in a competitive market. At t = 2, all random variables are realized and all firms are liquidated. By the phrase "improve the performances of these firms," we have in mind that the particularly skilled investors can engage in value-enhancing activities relating to firm decision-making. When an investor is able to innovate and restructure a firm, we refer to that investor as being skilled, or more generally, as being an activist shareholder.

We now proceed to examine each of these moments of time in more detail. At t = 0, each F_j invests in a portfolio of N risky securities as well as in the risk-free asset. Having allowed the possibility that firms might benefit from reorganization at t = 1, we assume at t = 0 that each F_j takes into account the possibility of intervening events at t = 1. These intervening events might lead to unanticipated improvements in the performances of the firms or to unanticipated deteriorations in those performances. As a result, we assume that at t = 0 each F_j cannot specify the precise expected value of each security, but must instead consider each expected value as a random variable. Thus, the random price per share of the risky securities at t = 2, as perceived by all investors at t = 0, is assumed to be given by the $n \times 1$ random vector $\widetilde{\boldsymbol{\mu}_0} = \widetilde{\boldsymbol{\tau}} + \widetilde{\boldsymbol{\varepsilon}_0}$ where $\widetilde{\boldsymbol{\varepsilon}_0}$, the vector of errors, is normally distributed with mean $\mathbf{0}$ and non-singular covariance matrix $\boldsymbol{\Omega}$, and $\widetilde{\boldsymbol{\tau}}$ is the vector of the random expected values of the securities. Furthermore, the random vector $\widetilde{\boldsymbol{\tau}} = \boldsymbol{\mu}_0 + \widetilde{\boldsymbol{\varepsilon}_\tau}$ where $\boldsymbol{\mu}_0$ is the expected value of $\widetilde{\boldsymbol{\tau}}$ and $\widetilde{\boldsymbol{\varepsilon}_\tau}$ is normally distributed with mean $\mathbf{0}$, non-singular covariance matrix $\boldsymbol{\Omega}_\tau$, and is independent of $\widetilde{\boldsymbol{\varepsilon}_0}$. In summary, all investors assume that the random price per share of the risky securities at t = 2, as perceived at t = 0, is normally distributed with mean vector $\boldsymbol{\mu}_0$ and covariance matrix $\boldsymbol{\Omega}_0 = \boldsymbol{\Omega} + \boldsymbol{\Omega}_\tau$ where $\boldsymbol{\Omega}_\tau$ describes the additional uncertainty due to the inability of investors to forecast precisely the firms' expected values. We assume that the rate of return of the risk-free asset is 0 and we permit investors to borrow and lend at this rate. Having started with initial wealth h_{j0}, and taking the $n \times 1$ price vector $\mathbf{P}$ as given, F_j seeks to determine $\mathbf{x}^*_{j0}$, the $n \times 1$ vector of shares in each of the risky assets, and z_{j0}, the amount borrowed or lent for these purchases. The $\mathbf{x}_{j0}^*$ satisfy $\arg\max Eu_j[z_{j0} + \mathbf{x}_{j0}^{'}\widetilde{\boldsymbol{\mu}_0}]$ subject to $z_{j0} + \mathbf{x}_{j0}^{'}\mathbf{P} = h_{j0}$ where Eu_j is the expected utility of F_j. An equilibrium $n \times 1$ price vector $\mathbf{P}_0$ yields the demands $\mathbf{x}_{j0}^*$ so that all shares are sold, i.e., $\sum_{j=1}^{M} \mathbf{x}_{j0}^* = \mathbf{Q}$ where $\mathbf{Q}$, with elements q_i, is the $n \times 1$ vector of the total number of shares in each of the N risky firms.

The situation at t = 0 just described is the classical formulation whose solution is well-known. We present this result without proof in Proposition 1. Using the notation $s = \sum_{j=1}^{M} \frac{1}{a_j}$ and $\gamma_j = \frac{1}{a_j s}$, we have the following result.

Proposition 1 *In equilibrium at $t = 0$, investor F_j, $j = 1, ..., M$, acquires a fixed proportion of ownership in each of the N risky firms. The price paid for this acquisition is the expected price per share at $t = 2$, corrected by a risk premium. More precisely, the unique equilibrium solution is given by $\mathbf{x}_{j0}^* = \gamma_j \mathbf{Q}$ and $\mathbf{P}_0 = \boldsymbol{\mu}_0 - \frac{2}{s}\boldsymbol{\Omega}_0\mathbf{Q}$, where $\boldsymbol{\Omega}_0 = \boldsymbol{\Omega} + \boldsymbol{\Omega}_\tau$.*

As a result of Proposition 1, at t = 0, all investors are participants in the market and each holds the market portfolio. Assuming that the mean vector is normally distributed is a convenience permitting us to establish a familiar benchmark against which to compare holdings at t = 1. Our development could have proceeded without this assumption by comparing the holdings at t = 1 to arbitrary initial holdings. However, we chose this familiar benchmark to emphasize the process by which a concentration of ownership might arise in equilibrium from a group of well-diversified investors.

At t = 1, new information becomes available to all F_j. This new information is that one or more of the firms could benefit from reorganization, that one investor, F_1, has the skill to restructure the firm or firms so as to improve their performances, and the degree to which F_1

will be successful in this endeavor depends on the number of shares that she will own at t = 1 in these firms. Consequently, at t = 1, all investors update their appraisals of the price per share at t = 2 as $\widetilde{\boldsymbol{\mu}_1} = \boldsymbol{\mu}_1 + \boldsymbol{\mu}(\mathbf{x}_{11}) + \widetilde{\boldsymbol{\varepsilon}_1}$ where $\boldsymbol{\mu}_1$ is the $n \times 1$ mean vector absent any further intervention by F_1 and $\boldsymbol{\mu}(\mathbf{x}_{11})$, an $n \times 1$ vector, is the incremental improvement based on F_1 owning $\mathbf{x}_{11}$ shares of the N securities at t = 1. The random vector $\widetilde{\boldsymbol{\varepsilon}_1}$ is normally distributed with mean $\mathbf{0}$ and non-singular covariance matrix $\boldsymbol{\Omega}_1$ and is assumed to be independent of $\widetilde{\boldsymbol{\varepsilon}_0}$ and $\widetilde{\boldsymbol{\varepsilon}_\tau}$. With this new information, each F_j adjusts her portfolio holdings as follows. Starting this period with wealth h_{j1}, and taking prices $\mathbf{P}$ as given, F_j seeks to determine $\mathbf{x}^*_{j1}$, the $n \times 1$ vector of shares in each of the risky assets and z_{j1}, the amount borrowed or lent for these purchases. The $\mathbf{x}^*_{j1}$ satisfy $\arg\max Eu_j[z_{j1} + \mathbf{x}'_{j1}\widetilde{\boldsymbol{\mu}_1}]$ subject to $z_{j1} + (\mathbf{x}_{j1} - \gamma_j\mathbf{Q})'\mathbf{P} = h_{j1}$. An equilibrium price vector at t = 1, $\mathbf{P}_1$, yields the demands $\mathbf{x}^*_{j1}$ so that all shares are sold, i.e., $\sum_{j=1}^{M} \mathbf{x}^*_{j1} = \mathbf{Q}$. The relationships of the parameters $\boldsymbol{\mu}_1$ and $\boldsymbol{\Omega}_1$ to the parameters $\boldsymbol{\mu}_0$ and $\boldsymbol{\Omega}_0$ will be addressed below. The case of more than one activist will also be explored later.

We now investigate the response of the investors to the presence of an activist, F_1, who has the skill to affect f firms, which are assumed to be firms 1 through f. We assume in what follows that the $n \times 1$ vector of improvements or innovations $\boldsymbol{\mu}(\mathbf{x}_{11}) = \mathbf{D}\mathbf{x}_{11}$ where $\mathbf{D}$ is a diagonal $n \times n$ matrix whose positive diagonal elements d_i, $i = 1, ..., f$, represent the incremental improvement per share that F_1 can generate in firm i. The remaining diagonal elements of $\mathbf{D}$ are zero. Also, let $\mathbf{C}$ be an $n \times n$ diagonal matrix whose positive entries c_i, $i = 1, ..., f$, represent the cost per share of the improvements in firm i. The remaining diagonal elements of $\mathbf{C}$ are zero. We assume that these costs, which are borne entirely by F_1, are known only to F_1, whereas $\mathbf{D}$ is known to all market participants. We next show that this information causes trading to take place at t = 1 and, as a result, no investor holds the market portfolio. In what follows, we let $s_{-1} = s - \frac{1}{a_1}$.

Proposition 2 ***The General Solution.*** *In equilibrium at $t = 1$, investor F_j, $j = 1, ..., M$, in general, acquires a different proportion of ownership in each of the N risky firms. The price paid for each firm's shares is the expected price per share at $t = 2$ corrected by a risk premium which depends on the impact of the activist. More precisely, if $a_1\mathbf{\Omega}_1 - (\mathbf{D} - \mathbf{C})$ is a positive definite matrix, then the unique equilibrium solution is given by*

$$\begin{aligned} [\mathbf{\Omega}_1 - \frac{\gamma_1 s_{-1}}{2}(\mathbf{D} - 2\mathbf{C})]\mathbf{x}_{11}^* &= \gamma_1\mathbf{\Omega}_1\mathbf{Q} \\ \mathbf{P}_1 &= \boldsymbol{\mu}_1 - 2[a_1\mathbf{\Omega}_1 - (\mathbf{D} - \mathbf{C})]\mathbf{x}_{11}^* \\ \mathbf{x}_{j1}^* &= \frac{1}{a_j s_{-1}}(\mathbf{Q} - \mathbf{x}_{11}^*) \quad j > 1. \end{aligned}$$

Proof. See Appendix.

Comparing the results of Proposition 2 to those of Proposition 1, we see that, in general, all of the holdings of all the F_j change simultaneously. Importantly, at t = 1, no investor holds the market portfolio. The equations $\mathbf{x}_{j1}^*$ defining the holdings of F_j, $j > 1$, have the same form as those of Admati, Pfleiderer and Zechner; however, the argument by which their results were derived is entirely different from ours. Whereas Admati, Pfleiderer and Zechner allow the activist to choose his holdings as a monopsonist, in our model all investors are price takers and $\mathbf{x}_{11}^*$, the vector of the holdings of the activist at t = 1, is determined simultaneously with all other holdings.

In our model, we place the impact of activism in the mean of the distribution of future prices, i.e., $\boldsymbol{\mu}(\mathbf{x}_{11}) = \mathbf{D}\mathbf{x}_{11}$. However, as can be seen in the proof of Proposition 2, our approach to modeling activism is equivalent to assuming that the covariance matrix is reduced by the amount $\mathbf{D}$ as a consequence of F_1's involvement. Therefore, without much additional effort, we could have modeled the impact of F_1 on both the mean and the covariance matrix without substantially changing the results.

We now present the explicit solutions to the equations in Proposition 2 and examine their consequences. We use the following notation. Let $(\mathbf{D}-\mathbf{C})_f$ be the $f \times f$ non-singular diagonal matrix with elements $d_i - c_i$, $i = 1, ..., f$. Similarly, let $\mathbf{\Delta}_f$ be the $f \times f$ non-singular diagonal matrix with elements $d_i - 2c_i$, $\mathbf{W}_f$ be the upper $f \times f$ submatrix of $\mathbf{\Omega}_1^{-1}$, $\mathbf{Q}_f$ be the $f \times 1$ vector of the first f elements of $\mathbf{Q}$ and $\mathbf{I}_f$ be the identity matrix of dimension f. Finally, the i^{th} column of $\mathbf{\Omega}_1^{-1}$ is denoted by $\boldsymbol{\omega}^i$.

3 The Impact of Activism on Concentration of Ownership

Proposition 3 ***Rebalancing Result.*** *The result of F_1's potential activism in firms 1 through f produces a rebalancing of all the holdings of all the investors. Consequently, no investor holds the market portfolio. More precisely, under the conditions of Proposition 2,*

$$\mathbf{x}_{11}^{*} - \gamma_1 \mathbf{Q} = (\boldsymbol{\omega}^1, ..., \boldsymbol{\omega}^f)\mathbf{k}$$

where $\mathbf{k} = \frac{\gamma_1 s_{-1}}{2}[\mathbf{\Delta}_f^{-1} - \frac{\gamma_1 s_{-1}}{2}\mathbf{W}_f]^{-1}\mathbf{Q}_f$ *and*

$$\mathbf{x}_{j1}^{*} - \gamma_j \mathbf{Q} = -\frac{1}{a_j s_{-1}}(\boldsymbol{\omega}^1, ..., \boldsymbol{\omega}^f)\mathbf{k}, \ j > 1.$$

Proof. See Appendix.

At optimality each investor's holdings in all securities change. We call the changes in the portfolios of F_j, $j = 1, ..., M$, resulting from activism the "rebalancing effect." If we think of the elements of $\mathbf{Q}$ (properly normalized) as the coefficients of a market portfolio and if we think of the elements $(\boldsymbol{\omega}^1, ..., \boldsymbol{\omega}^f)\mathbf{k}$ (properly normalized) as a rebalancing portfolio, then the solution given in Proposition 3 states that, at optimality, all participants hold different mixtures of the two risky portfolios.

In the special case when firms' performances are independent of one another, the solution in Proposition 3 simplifies: no changes in holdings occur in firms other than in those in which F_1 is a potential activist and, given our assumptions, F_1 acquires additional ownership in all the f firms when all $d_i - 2c_i > 0$. When independence does not hold, neither of these two simplifications can be made. In general, rebalancing involves the entire portfolio. The circumstances under which F_1 acquires greater ownership in all, some, or none of the the f firms is dealt with below.

We now show that when F_1 is capable of being an activist only in firm 1, F_1 always acquires more ownership in that firm.

Corollary 1 *If F_1 is a potential activist only in firm 1, then as a result of the equilibrium at t = 1, F_1 acquires a larger share of ownership in firm 1 and all investors in the market including F_1 choose to rebalance their entire portfolios as a result. More precisely, we have the following result. Under the conditions of Proposition 2, and if Δ_f is positive definite, then*

$$\mathbf{x}_{11}^{*} - \gamma_1 \mathbf{Q} = k_1 \boldsymbol{\omega}^1 \text{ with } k_1 > 0$$

and where the first element of $\boldsymbol{\omega}^1$, ω_1^1, is positive.

Proof. See Appendix.

Unlike other attempts to explain concentration of ownership, Corollary 1 shows that concentration arises in a Walrasian equilibrium context with complete information and with no special market power accorded to F_1.

As a consequence of the rebalancing effect, the potential activist holds a riskier portfolio at optimality. This follows since $cov[\widetilde{\boldsymbol{\mu}_1'}\boldsymbol{\omega}^1,\ \widetilde{\boldsymbol{\mu}_1'}\mathbf{Q}] = q_1$ and thus the covariance between these two risky portfolios is positive. This in turn implies that the variance of the portfolio determined by the vector $\mathbf{x}_{11}^*$ is greater than the variance of the portfolio $\gamma_1\mathbf{Q}$. Thus, F_1 takes on additional risk to acquire additional profit.

When F_1 is a potential activist in firms 1 and 2, the change in holdings for F_1 is $k_1\boldsymbol{\omega}^1 + k_2\boldsymbol{\omega}^2$ where k_i are the elements of the vector $\mathbf{k}$ in Proposition 3, the Rebalancing Result. When we examine the implied change in F_1's holdings in both of these firms, we no longer have the guarantee that an increase in holdings will occur in both firms. This follows from the fact that, since $\boldsymbol{\Omega}_1$ is positive definite, the i^{th} element of $\boldsymbol{\omega}^i$ is positive, but all other elements in this vector may be of either sign. Thus, the change in holdings for F_1 depends on two opposing forces, one that yields F_1 more shares because of skill, and the other that yields F_1 less shares due to diversification issues. This tension, which we see for the case $f = 2$, is intensified in the general case of arbitrary f. We call the result of these opposing forces the confounding effect. The conditions that yield F_1 greater ownership in at least one of the firms in which she is a potential activist are given next.

Proposition 4 ***Concentration of Ownership.*** *Under the conditions of Proposition* 3, *and if the set of firms in which* F_1 *is a potential activist is not too correlated with the remaining firms, then at optimality* F_1 *holds a greater concentration of ownership in at least one of the firms in which she is a potential activist. More precisely, under the conditions of Proposition* 2, *and if* $\mathbf{W}_f^{-1} - \frac{\gamma_1 s_{-1}}{2}\boldsymbol{\Delta}_f$ *is positive definite, then at least one of the first* f *components of* $\mathbf{x}_{11}^* - \gamma_1\mathbf{Q}$ *of Proposition* 3 *is positive.*

Proof. See Appendix.

The matrix $\mathbf{W}_f$ is the upper left $f \times f$ submatrix of $\mathbf{\Omega}_1^{-1}$. As such, it can be written as $[\mathbf{\Omega}_{f,f} - \mathbf{\Omega}_{f,N-f}\mathbf{\Omega}_{N-f,N-f}^{-1}\mathbf{\Omega}_{N-f,f}]^{-1}$ where $\mathbf{\Omega}_{i,j}$ are the matrix elements of the partioned $\mathbf{\Omega}_1$ matrix with, for example $\mathbf{\Omega}_{f,f}$ being the upper $f \times f$ submatrix of $\mathbf{\Omega}_1$. Thus, $\mathbf{W}_f^{-1} = \mathbf{\Omega}_{f,f} - \mathbf{\Omega}_{f,N-f}\mathbf{\Omega}_{N-f,N-f}^{-1}\mathbf{\Omega}_{N-f,f}$. The requirement in this proposition that $\mathbf{W}_f^{-1} - \frac{\gamma_1 s_{-1}}{2}\mathbf{\Delta}_f$ be positive definite can therefore be written as $(\mathbf{\Omega}_{f,f} - \frac{\gamma_1 s_{-1}}{2}\mathbf{\Delta}_f) - \mathbf{\Omega}_{f,n-f}\mathbf{\Omega}_{N-f,N-f}^{-1}\mathbf{\Omega}_{N-f,f}$ being positive definite. Since $\mathbf{\Omega}_1 - \frac{\gamma_1 s_{-1}}{2}\mathbf{\Delta}$ is positive definite (see proof of Corollary 1), $\mathbf{\Omega}_{f,f} - \frac{\gamma_1 s_{-1}}{2}\mathbf{\Delta}_f$ is positive definite and the requirement would be met if the elements of $\mathbf{\Omega}_{f,N-f}\mathbf{\Omega}_{N-f,N-f}^{-1}\mathbf{\Omega}_{N-f,f}$ are not too large. Since $\mathbf{\Omega}_{f,N-f}$ is the matrix of covariances between the sets of securities, it follows that when the first f securities are close to being uncorrelated with the remaining $N - f$ securities, increased concentration occurs. The requirement that the correlations are small can be seen as an enhancement of diversification, that is, the reduction of risk. We summarize these observations in the following corollary.

Corollary 2 *If the first f firms are uncorrelated with the remaining firms then, at optimality, F_1 acquires more ownership in at least one of the f firms. More precisely, if $\mathbf{\Omega}_{f,N-f} = \mathbf{0}$, then at least one of the first f components of $\mathbf{x}_{11}^* - \gamma_1\mathbf{Q}$ is positive.*

To illustrate Corollary 2, one can imagine an activist whose skill is applicable to all of the firms in a particular industry. If that industry happens to be uncorrelated, or only slightly correlated, with the other firms in the market, then at optimality a concentration of ownership will accrue to the activist in at least one of those firms in the industry in which the activist is skilled, but not necessarily in all of the firms in that industry. Since it is reasonable to assume that the firms within the industry are highly correlated, then risk considerations keep the activist from getting a concentration of ownership in all the firms of that industry. In fact, in this situation the activist will sell some of the shares in firms in which she has the skill to improve the performance.

Another illustration, albeit rather extreme, is one in which the activist's skill is applicable to all $f = N$ firms. Here, by default, $\boldsymbol{\Omega}_{f,N-f} = \mathbf{0}$, and we again find a concentration of ownership in at least one firm and the possibility of selling shares in others.

We now turn our attention to the impact of activism on security prices.

4 The Impact of Activism on Prices

Proposition 5 ***Equilibrium Prices.*** *When F_1 is a potential activist in f firms, then at t = 1, the equilibrium prices of securities* 1 *through f change and the prices of the other securities may also change. More precisely, we have*

$$\mathbf{P}_1 - \mathbf{P}_0 = (\boldsymbol{\mu}_1 - \boldsymbol{\mu}_0) - \frac{2}{s}(\boldsymbol{\Omega}_1 - \boldsymbol{\Omega}_0)\mathbf{Q} + \frac{4}{\gamma_1 s_{-1}}\begin{pmatrix}\mathbf{I}_f \\ \mathbf{0}\end{pmatrix}[(\mathbf{D}-\mathbf{C})_f \boldsymbol{\Delta}_f^{-1} - \frac{s_{-1}}{2s}\mathbf{I}_f]\mathbf{k}.$$

To interpret this general result on equilibrium prices, we must relate the parameters at t = 1 to those at t = 0. We assume in what follows that $\boldsymbol{\mu}_1 = \boldsymbol{\mu}_0$, which implies that activism provides an incremental improvement over the initial parameter values. We now consider two ways in which uncertainty at t = 1 relates to uncertainty at t = 0. First, for comparative purposes we consider that uncertainty has remained unchanged, i.e., $\boldsymbol{\Omega}_1 = \boldsymbol{\Omega}_0$. Second, we assume that part of the uncertainty that existed at t = 0 has been resolved at t = 1. Since we assumed that $\boldsymbol{\Omega}_0 = \boldsymbol{\Omega} + \boldsymbol{\Omega}_\tau$ where $\boldsymbol{\Omega}_\tau$ represented the uncertainty in specifying an appropriate mean vector, we assume $\boldsymbol{\Omega}_1 = \boldsymbol{\Omega}$. Using these specifications we have the following corollaries.

Corollary 3 *If $\boldsymbol{\mu}_1 = \boldsymbol{\mu}_0$ and $\boldsymbol{\Omega}_1 = \boldsymbol{\Omega}_0$ then only the equilibrium prices of the first f securities change and the prices of the remaining securities remain unchanged. Moreover, at least one of the first f prices increases. More precisely, if $\boldsymbol{\mu}_1 = \boldsymbol{\mu}_0$ and $\boldsymbol{\Omega}_1 = \boldsymbol{\Omega}_0$ and if Δ_f is positive definite, then*

$$\mathbf{P}_1 - \mathbf{P}_0 = \frac{4}{\gamma_1 s_{-1}} \begin{pmatrix} \mathbf{I}_f \\ \mathbf{0} \end{pmatrix} [(\mathbf{D}-\mathbf{C})_f \boldsymbol{\Delta}_f^{-1} - \frac{s_{-1}}{2s} \mathbf{I}_f]\mathbf{k}$$

and at least one element of $[(\mathbf{D}-\mathbf{C})_f \boldsymbol{\Delta}_f^{-1} - \frac{s_{-1}}{2s} \mathbf{I}_f]\mathbf{k}$ is positive.

Proof. See Appendix.

We assumed that $\boldsymbol{\mu}_1 = \boldsymbol{\mu}_0$ and that $\boldsymbol{\Omega}_1 = \boldsymbol{\Omega}_0$ in order to compare the equilibrium prices at t = 0 and t = 1. Corollary 3 states that the share prices of the f firms change at t=1, whereas the prices of all the other securities remains unchanged. Nonetheless, given the rebalancing effect described above, the other investors take profits on at least one of the first f firms and rearrange their holdings in all the other firms for diversification purposes. We note that the rearrangement of shares in securities other than the first f takes place at neither a profit nor a loss for all investors since prices do not change for these securities. On the other hand, F_1 incurs the cost $(\mathbf{P_1} - \mathbf{P_0})'(\mathbf{x}^*_{\mathbf{11}} - \gamma_1 \mathbf{Q})$ for additional shares, thereby taking on additional risk with a prospect of additional return. As a consequence of the rebalancing, none of the F_j holds the market portfolio at t = 1.

Corollary 4 *If $\boldsymbol{\mu}_1 = \boldsymbol{\mu}_0$ and the uncertainty at t = 1 has decreased, i.e., $\boldsymbol{\Omega}_1 = \boldsymbol{\Omega}$, then all security prices change and at least one of the first f security prices increases. More precisely, we have if $\boldsymbol{\mu}_1 = \boldsymbol{\mu}_0$ and that $\boldsymbol{\Omega}_1 = \boldsymbol{\Omega}$ then*

$$\mathbf{P}_1 - \mathbf{P}_0 = \frac{2}{s} \boldsymbol{\Omega}_\tau \mathbf{Q} + \frac{4}{\gamma_1 s_{-1}} \begin{pmatrix} \mathbf{I}_f \\ \mathbf{0} \end{pmatrix} [(\mathbf{D}-\mathbf{C})_f \boldsymbol{\Delta}_f^{-1} - \frac{s_{-1}}{2s} \mathbf{I}_f]\mathbf{k}$$

and at least one element of $[(\mathbf{D}-\mathbf{C})_f \boldsymbol{\Delta}_f^{-1} - \frac{s_{-1}}{2s} \mathbf{I}_f]\mathbf{k}$ is positive.

Proof. See Appendix.

The risk premium is defined to be $\boldsymbol{\mu}_i - \mathbf{P}_i$, $i = 0, 1$. At t = 0, $\boldsymbol{\mu}_0 - \mathbf{P}_0$ has the familiar form, which can be interpreted as the covariance of the firms' share prices with the market. At t = 1, in the context of an activist shareholder, this simple interpretation is no longer valid. The change in risk premia from t = 0 to t = 1 when $\boldsymbol{\mu}_1 = \boldsymbol{\mu}_0$ is $\mathbf{P}_1 - \mathbf{P}_0$. Thus, the previous two corollaries can be interpreted as the change in risk premia due to the potential intervention of an activist and the possible reduction of uncertainty. As the corollaries demonstrate, this change does not depend entirely on the covariance structure since the term describing the impact of activism does not have this form. Therefore, the risk premium at t = 1 also does not have this form.

The case when F_1 is a potential activist only in one firm, $f = 1$, yields the following unambiguous result. We let $\mathbf{e}_i$ be the $n \times 1$ vector whose i^{th} element is 1 and whose other elements are zero.

Corollary 5 *Under the conditions of Corollary 3, at t = 1, when F_1 is a potential activist only in firm 1, then at optimality the share price of firm 1 increases. More precisely, we have, if $f = 1$, $\mathbf{e}_1'(\mathbf{P}_1 - \mathbf{P}_0) > \mathbf{0}$.*

Proof. See Appendix.

As a consequence of the optimality in our model, Corollary 5 shows that at t = 1, when F_1 is an activist only in firm 1, F_1 acquires additional shares in that firm even though the price of those shares increases. Obviously this increase in shares both increases F_1's risk and also her potential for gain. This result differs from the point made in the literature, by, for example, Admati, Pfleiderer and Zechner, Maug, and Kahn and Winton, that F_1 can only make a profit on the shares purchased at t = 0 since the prices of shares at t = 1 will have risen to absorb all of the potential benefit from F_1's activism.

When some of the uncertainty is resolved at t = 1, all prices change as a result of this resolution and at least one of the first f prices increases due to activism. Importantly, we can use these results to evaluate the overall impact of activism on the market. The quantity $\mathbf{Q}'(\mathbf{P}_1 - \mathbf{P}_0)$ represents the change in the total value of the market. By Corollary 4, this change is equal to $\frac{2}{s}\mathbf{Q}'\mathbf{\Omega}_\tau\mathbf{Q} + \frac{4}{\gamma_1 s_{-1}}\mathbf{Q}'_f[(\mathbf{D}-\mathbf{C})_f\mathbf{\Delta}_f^{-1} - \frac{s_{-1}}{2s}\mathbf{I}_f]\mathbf{k}$. Since $\mathbf{\Omega}_\tau$ is positive definite, the first term is positive and, as was shown in the proof of Corollary 5, when F_1 is an activist in only firm 1, then the second term is positive. Thus, in this case we can interprete the overall increase in market value as being due to two sources: the reduction of uncertainty and the impact of activism. We summarize these remarks in the following corollary.

Corollary 6 *Under the conditions of Corollary 4, and if F_1 is an activist in only firm 1, the potential for activism always produces an increase in the value of the market.*

When F_1 is a potential activist in more than one firm, it is possible that the term $\frac{4}{\gamma_1 s_{-1}}\mathbf{Q}'_f[(\mathbf{D}-\mathbf{C})_f\mathbf{\Delta}_f^{-1} - \frac{s_{-1}}{2s}\mathbf{I}_f]\mathbf{k}$ may be negative. If this term were negative, it would imply that the total value of the market could decline in the presence of this activist.

5 A Note on Free-Ridership

We assumed in the model above that the potential activist bore the entire costs of her activism. As such, all other investors derived the benefits of activism without paying for them, creating a free-rider problem. We show in the next proposition that the free rides are not so free.

Proposition 6 *Not so Free-Riders. Under the conditions of the Equilibrium Prices Proposition, Proposition* 5, *and if* $f = 1$, *the price of a share of the first security is decreasing in* c_1, *that is* $\frac{d(\mathbf{e}_1'\mathbf{P}_1)}{dc_1} < 0$.

Proof. See Appendix.

When F_1 is a potential investor only in firm 1, it was established above that, at optimality, her holdings in the first firm increase. The additional shares were sold to F_1 by the other investors at the price of $\mathbf{e}_1'\mathbf{P}_1$ per share. Therefore, as c_1 increases, the profit per share of the other investors decreases and these other investors indirectly bear some of the private costs that F_1 incurs for her activism. On the other hand, although F_1 pays c_1, she recoups part of this cost by paying less for each additional share acquired in firm 1. This feature mitigates the free-rider problem to some extent.

6 Multiple Activists in Many Firms

So far we have considered the case of a single activist. This is the same assumption employed by the authors of the studies we discussed above. Above, we have generalized these other studies by considering activism in several firms. Now, now consider the further case in which multiple activists might affect the same firms at the same time.

The assumption of multiple activists raises two new issues. One of these issues is the game-theoretic one. This involves the determination of coalitions that might evolve within the firm, as well as the degree of power each activist might have. The second, which builds on the first, is the ways in which the resulting power structure might have an impact on the value of the firm. While we do not deal with the first of these in this paper, we abstract the gaming considerations by assuming that the results yield a known ranking of power among activists. On the second issue, we assume that the impact on holdings and prices is affected by each activist in proportion both to the shares acquired in equilibrium and to the index of coalitional strength. For some results on the formation of coalitions within a firm see Zwiebel (1995) and Fluck (1999).

We let $\mathbf{D}_j$, $j = 1, ..., M$ be an $n \times n$ diagonal matrix whose elements are non-negative. The i^{th} diagonal element represents the per share contribution (strength) that potential activist j may exert on firm i. With this notation that allows all elements of $\mathbf{D}_j$ to be zero, all investors are potential activists. Let $\mathbf{C}_j$ be an $n \times n$ diagonal matrix whose non-negative i^{th} diagonal element represents the cost per share that F_j bears when she is active in firm i. We assume that these costs are borne entirely by F_j. Whereas $\mathbf{D}_j$ is known to all investors, $\mathbf{C}_j$ is known to F_j alone. We let $\mathbf{\Delta}_j = \mathbf{D}_j - 2\mathbf{C}_j$. We now model the impact of multiple activist on prices. We assume that at t = 1, $\widetilde{\boldsymbol{\mu}_1} = \boldsymbol{\mu}_1 + \boldsymbol{\mu}(\mathbf{x}_{11}, \mathbf{x}_{21}, ..., \mathbf{x}_{M1}) + \widetilde{\boldsymbol{\varepsilon}_1}$ where $\boldsymbol{\mu}(\mathbf{x}_{11}, \mathbf{x}_{21}, ..., \mathbf{x}_{M1}) = \sum_{i=1}^{M} \mathbf{D}_i \mathbf{x}_{i1}$ and $\widetilde{\boldsymbol{\varepsilon}_1}$ is a random vector whose distribution is normal with zero mean, covariance matrix $\mathbf{\Omega}_1$ and is independent of $\widetilde{\boldsymbol{\varepsilon}_0}$. As before, the optimization at t = 1 determines $\mathbf{x}^*_{j1} = \arg\max Eu_j[z_{j1} + \mathbf{x}'_{j1}\widetilde{\boldsymbol{\mu}_1}]$ $s.t.$ $z_{j1} + (\mathbf{x}_{j1} - \gamma_j \mathbf{Q})'\mathbf{P}_1 + \mathbf{x}'_{j1}\mathbf{C}_j \mathbf{x}_{j1} = h_{j1}$ and the equilibrium price vector $\mathbf{P}_1$.

Proposition 7 ***The General Solution with Multiple Activists.*** *In equilibrium at t = 1 with M potential activists, each investor F_j, $j = 1, ..., M$, in general acquires a different proportion of ownership in each of the N risky firms. That is, at optimality, some potential activists sell shares to other potential activists. The price paid for each firm's shares is the expected price per share at $t = 2$ corrected by a risk premium that depends on the impact of all the activists. More precisely, if we let $[\mathbf{a}_j \mathbf{\Omega}_1 - \frac{1}{2}\mathbf{\Delta}_j]$ be a positive definite matrix for each j, $j = 1, ..., M$, then the unique equilibrium solution is given by*

$\mathbf{x}^*_{j1} = [\mathbf{a}_j\boldsymbol{\Omega}_1 - \frac{1}{2}\boldsymbol{\Delta}_j]^{-1}[\sum_i(\mathbf{a}_j\boldsymbol{\Omega}_1 - \frac{1}{2}\boldsymbol{\Delta}_j)^{-1}]^{-1}\mathbf{Q}$ *and*

$$\boldsymbol{\mu}_1 - \mathbf{P}_1 = [\mathbf{I} - \sum_i \mathbf{D}_i(\mathbf{a}_j\boldsymbol{\Omega}_1 - \tfrac{1}{2}\boldsymbol{\Delta}_j)^{-1}][\sum_i (\mathbf{a}_j\boldsymbol{\Omega}_1 - \tfrac{1}{2}\boldsymbol{\Delta}_j)^{-1}]^{-1}\mathbf{Q}.$$

Proof. See Appendix.

Examination of the optimal holdings equation $\mathbf{x}^*_{j1}$ shows that when the $\boldsymbol{\Delta}_j$ are different, the corresponding investors hold entirely different portfolios. Since each investor was initially holding the market portfolio, it follows that some potential activists sold at optimality, while others acquired larger holdings. Which potential activists sold, and which acquired more, depends on a complicated relationship of relative benefits, relative costs, and attitudes towards risk. Without information on these characteristics, it is hard to predict which activists will acquire a concentration of ownership. This lack of predictability is added to the lack of predictability due to the confounding effect described above, which is also present here.

Prices, too, suffer from the lack of predictability since the risk premium also depends on the relative benefits, relative costs, and attitudes towards risk. Since costs are assumed to be known only by the parties that expend them, it seems unlikely that the risk premium could be evaluated, suggesting that, in practice, the uncertainty of prices would increase.

7 Asymmetric information concerning the activism of F_1 in Firm 1.

So far we have assumed that all investors correctly perceive that F_1 has the skill to improve performance. We now introduce the idea of uncertainty regarding the activist's skills. Specifically, we investigate the situation in which F_j, $j > 1$, is unsure about the ability of F_1 and assigns probability λ to the event that F_1 has such skill, and probability $(1 - \lambda)$ otherwise. We will compare three equilibria under these conditions, corresponding to the cases

(a) F_1 has the skill, (b) F_1 does not have the skill and (c) the "rational expectations" case in which F_1 is drawn from a distribution such that the probability that F_1 has the skill is, in fact, λ. We refer to the holdings in these three cases as $\mathbf{x}_{11}^*(D)$, $\mathbf{x}_{11}^*(0)$, $\mathbf{x}_{11}^*(\lambda)$, respectively. Throughout this section, we employ the assumption that F_1's special skill pertains only to firm 1.

Proposition 8 ***Asymmetric Information.*** *If the investors F_j, $j > 1$, believe F_1 to be an effective activist with probability λ and not an effective activist with probability $1-\lambda$, then the equilibrium solution yields F_1 a larger share of ownership in firm 1 if either F_1 is an effective activist or if F_1 is drawn from a population of activists and non-activists with probability λ. When F_1 does not have this skill, F_1 acquires a smaller share of ownership in firm 1. In all cases, all investors, including F_1, choose to rebalance their portfolios. More precisely, we have the following result. Under the conditions of Corollary 1 and if $\boldsymbol{\mu}(\mathbf{x}_{11}) = \mathbf{D}\mathbf{x}_{11}$ with probability λ and $\boldsymbol{\mu}(\mathbf{x}_{11}) = 0$ otherwise, then $\mathbf{e}_1'\mathbf{x}_{11}^*(D) > \mathbf{e}_1'\mathbf{x}_{11}^*(\lambda) > \mathbf{e}_1'\mathbf{x}_{11}^*(0)$ and $\mathbf{e}_1'(\mathbf{x}_{11}^*(D) - \gamma_1\mathbf{Q}) > 0$, $\mathbf{e}_1'(\mathbf{x}_{11}^*(\lambda) - \gamma_1\mathbf{Q}) > 0$, and $\mathbf{e}_1'(\mathbf{x}_{11}^*(0) - \gamma_1\mathbf{Q}) < 0$.*

Proof. See Appendix.

When either F_1 has the skill to improve the performance of the firm, or if the other market participants correctly assess the probability that F_1 has this skill, then the optimum solution yields a concentration of ownership in the hands of F_1. Of course, this concentration is less than in the absence of asymmetric information. However, when F_1 does not have the skill, a concentration of ownership still occurs but in the wrong hands. All investors but F_1 acquire additional shares in firm 1. At the same time, all investors rebalance their portfolios so that when $\boldsymbol{\mu}(\mathbf{x}_{11}) = \mathbf{0}$ we have the extreme effect that all market participants take an additional risk with no promise of additional return.

We now address the equilibrium prices under the various assumptions. Let $\mathbf{P}_1(D)$, $\mathbf{P}_1(0)$, and $\mathbf{P}_1(\lambda)$ be the equilibrium vectors corresponding to the three assumptions about F_1.

Proposition 9 *Under the assumptions of Proposition 7, at $t = 1$, the equilibrium price of security 1 changes regardless of whether F_1 has the skill to be an activist or not. All other equilibrium prices remain unchanged. If $\boldsymbol{\mu}_0 = \boldsymbol{\mu}_1$ and $\boldsymbol{\Omega}_0 = \boldsymbol{\Omega}_1$, then at $t = 1$, the equilibrium price of security 1 increases regardless of whether F_1 has the skill to be an activist or not. More precisely, under the conditions of Proposition 7 and if $\boldsymbol{\mu}_0 = \boldsymbol{\mu}_1$ and if $\boldsymbol{\Omega}_0 = \boldsymbol{\Omega}_1$, there exist constants $K_D > 0$, $K_\lambda > 0$, $K_0 > 0$ such that*

$$
\begin{aligned}
a. \quad \mathbf{P}_1(D) - \mathbf{P}_0 &= K_D \mathbf{e}_1 \\
b. \quad \mathbf{P}_1(\lambda) - \mathbf{P}_0 &= K_\lambda \mathbf{e}_1 \\
c. \quad \mathbf{P}_1(0) - \mathbf{P}_0 &= K_0 \mathbf{e}_1.
\end{aligned}
$$

Proof. See Appendix.

Since these solutions are simultaneous solutions for all F_j, F_1 cannot, as in other models, take advantage of the knowledge of her own situation. Nonetheless, there are benefits to F_1. In case (a), F_1 acquires additional shares in firm 1, albeit not as many as in the complete information case and the demand for these additional shares increases the share price. In case (b), F_1 also acquires additional shares and, although her demand is lower for these shares, it is still sufficient to increase the price per share. In case (c), where F_1 would have been content to stay with her initial holdings $\gamma_1\mathbf{Q}$, the other investors mistakenly increase their demands for shares in firm 1, thereby again increasing the price per share.

8 Conclusions

In this paper we investigate the impact of activism on the market for correlated securities when investors are risk averse. When the activist is known to be able to have an impact on only one firm, $f = 1$, we show that in a Walrasian equilibrium context which includes the activist, the activist acquires a concentration of ownership in that firm. This concentration of ownership arises from a market whose participants hold diversified portfolios. By yielding more ownership to the activist, all investors alter the risk profiles of their holdings, ending with less balanced portfolios. This rebalancing effect is accompanied by an increase in both the price of the security that the activist can affect, as well as in the total value of the market. Unlike other explanations for the concentration of ownership, our explanation only requires that all investors are price takers who are aware of the existence of the activist and that the effectiveness of activism depends on the degree of ownership concentration.

In the case when the activist has the potential to have an impact on more than one firm, $f > 1$, again we find that the rebalancing of portfolios for all investors occurs. However, unlike the case when $f = 1$, the activist may not acquire a concentration of ownership in all the firms that she might affect or indeed any concentration in those firms. For the special case in which the f firms are not too correlated with the other firms in the market, we show that the activist increases her holdings in at least one of the f firms. We discuss the circumstances leading the activist with the skill to improve several firms to increase her holdings, at optimality, in only a subset of these f, while decreasing her holdings in the others.

In a market with correlated securities and with a potential activist shareholder present, we show that concentration of ownership arises as a natural consequence of the individual optimizations of all of the risk averse investors. Proving that concentration arises endogenously in a complete information Walrasian equilibrium in which all investors including the activist are price takers is a major contribution of our paper and differentiates our results from those in the literature. Another finding is that the existence of a potential activist causes all investors at optimality to rebalance their portfolios.

Prices also change as a consequence of the presence of a potential activist: if the activist can improve f firms, we find that the prices of the securities of all the f firms change while the prices of the remaining securities may also change. We establish the conditions under which at least one of the security prices of the f firms must increase, and in the special case in which $f = 1$, we show that the price of the corresponding security must increase.

Having established the conditions under which the presence of an activist leads to an increase in concentration in the hands of that activist and to an increase in the price of at least one of the securities in which the activist can improve performance, we have also shown that the total market value of all the securities has risen as a consequence of the activist. However, the source of this increase in market value is the rebalancing of all portfolios that occurs at optimality in the presence of an activist. Transaction and other costs associated with this "churning" are ignored in our model and thus it is impossible for us to draw the conclusion that an increase in the value of the market is beneficial in a more general context. Within our model, of course, the rebalancing is an optimal response to the presence of the acitivist.

Usual discussions assert that equilibrium price increases fully reflect the benefits associated with the improvements that the activist can make. These discussions conclude that the activist cannot make a profit on new shares purchased as the other investors will have anticipated the activist's improvments and will have bid up the price accordingly. We find a somewhat different result. Since at optimality we can establish the conditions under which the activist will acquire additional shares of the firm or firms she can improve, the new holdings must be associated with an increase in the expected utility of the activist. Thus, some gain from the purchase of additional shares must accrue to the activist under these conditions.

Since only the activist pays the cost of activism, a free-rider problem arises. But, we find that an increase in the activist's costs is associated with a decrease in the equilibrium price of the security. This means that for larger costs incurred by the activist, the price that the activist must pay to acquire additional shares is lower. On the other side of the trade, however, the other investors receive a lower price for the shares that at optimality they choose to sell. Thus, in equilibrium, the costs of activism are shared in some manner by all investors and not solely borne by the activist. We interprete this result as mitigating the free-rider problem to some extent.

When we consider many potential activists, our results show that, beside the usual rebalancing, concentration of ownership could occur for several activists. Unlike the situation where only a single activist is present, the confounding effect and additional gaming effects become more important with multiple activists. These, in turn, lead to unpredictable results and excessive churning.

When we broaden our model to include the impact of asymmetric information concerning activism, we again find the rebalancing effect. But here we show that concentration of ownership may fall into the wrong hands.

The balance sheet on activism has both positive and negative entries. The positive entry is that we can show that the presence of an activist increases the total value of the market. The negative entries concern the rebalancing or churning that is generated by activism, as well as the results of what we call the confounding effect. This latter effect follows from the relationship between the risk characteristics of the activist and the covariance structure between the firms she can improve and those she cannot affect. Because of correlations between the share prices of these two groups of firms, the activist may find it optimal to sell shares in firms that she could have improved.

Although we have studied a two period model, we might speculate as to the after effects of activism. Suppose we imagine that at t = 2 the activist improves the performances of the firms in which she has acquired additional ownership at t = 1. Having completed these improvements at t = 2, all investors know that the activist can make no additional improvements. Thus if all investors were now at t= 2, speculating about t = 3, there would be no thought of a potential activist and the problems facing the investors would be the same as those at t = 0. Thus the equilibirum prices at t = 2 would revert to those of t = 0 and all investors would rebalance their portfolios again and at optimality would once again hold the market portfolio. Thus, the impact of activism would have been to cause an initial rebalancing in portfolios as investors moved from holding the market portfolio to their optimal portfolio in the presence of activism, only to be followed by another round of rebalancing after the activist had in fact improved the firm and investors once again found it optimal to hold the market portfolio. Prices also would be affected, going up at t = 1 and then down again at t = 2. This zigzag pattern of prices, accompanied by a pattern of the resulting buying and selling, would be the anticipated course of events in the presence of activism.

9 Appendix

9.1 Proof of Proposition 2.

Each investor F_j $(j > 1)$ seeks $\mathbf{x}^*_{j1}$ such that $\mathbf{x}^*_{j1} = \arg\max Eu_j[z_{j1} + \mathbf{x}'_{j1}\widetilde{\boldsymbol{\mu}_1}]$ subject to $z_{j1} + \mathbf{x}'_{j1}\mathbf{P}_1 = h_{j1} + \gamma_j \mathbf{Q}'\mathbf{P}_1$. Substituting for z_{j1} we have $\mathbf{x}^*_{j1} = \arg\max Eu_j[h_{j1} + \gamma_j \mathbf{Q}'\mathbf{P}_1 + \mathbf{x}'_{j1}(\widetilde{\boldsymbol{\mu}_1} - \mathbf{P}_1)]$. We can replace the expected utility by its certainty equivalent and since, by assumption, $\widetilde{\boldsymbol{\mu}_1}$ is normally distributed and u_j is exponential with parameter a_j, it follows that $\mathbf{x}^*_{j1} = \arg\max[h_{j1} + \gamma_j \mathbf{Q}'\mathbf{P}_1 + \mathbf{x}'_{j1}(\boldsymbol{\mu}_1 + \mathbf{D}\mathbf{x}_{11} - \mathbf{P}_1) - a_j \mathbf{x}'_{j1}\boldsymbol{\Omega}_1\mathbf{x}_{j1}]$. The first-order conditions are $\boldsymbol{\mu}_1 + \mathbf{D}\mathbf{x}_{11} - \mathbf{P}_1 = 2a_j\boldsymbol{\Omega}_1\mathbf{x}_{j1}$. Dividing by a_j and summing over j, $j > 1$, with $\sum_{j>1}\frac{1}{a_j} = s_{-1}$, we have $s_{-1}(\boldsymbol{\mu}_1 + \mathbf{D}\mathbf{x}_{11} - \mathbf{P}_1) = 2\boldsymbol{\Omega}_1(\mathbf{Q} - \mathbf{x}_{11})$. Thus, $\mathbf{x}^*_{11}$ must satisfy $(2\boldsymbol{\Omega}_1 + s_{-1}\mathbf{D})\mathbf{x}^*_{11} = 2\boldsymbol{\Omega}_1\mathbf{Q} - s_{-1}(\boldsymbol{\mu}_1 - \mathbf{P}_1)$.

Investor F_1 seeks $\mathbf{x}^*_{11}$ such that $\mathbf{x}^*_{11} = \arg\max[z_{11} + \mathbf{x}'_{11}\widetilde{\boldsymbol{\mu}_1}]$ subject to $z_{11} + \mathbf{x}'_{11}\mathbf{P}_1 + \mathbf{x}'_{11}\mathbf{C}\mathbf{x}_{11} = h_{11} + \gamma_1\mathbf{Q}'\mathbf{P}_1$. Substituting, as above, and again using the certainty equivalent, we have that $\mathbf{x}^*_{11} = \arg\max\{h_{11} + \gamma_1\mathbf{Q}'\mathbf{P}_1 + \mathbf{x}'_{11}[\boldsymbol{\mu}_1 - \mathbf{P}_1 + (\mathbf{D} - \mathbf{C})\mathbf{x}_{11}] - a_1\mathbf{x}'_{11}\boldsymbol{\Omega}_1\mathbf{x}_{11}\}$. The first-order conditions are $\boldsymbol{\mu}_1 - \mathbf{P}_1 = 2[a_1\boldsymbol{\Omega}_1 - (\mathbf{D} - \mathbf{C})]\mathbf{x}^*_{11}$. Substituting $\boldsymbol{\mu}_1 - \mathbf{P}_1$ in the previous equation for $\mathbf{x}^*_{11}$, we have $[\boldsymbol{\Omega}_1(1 + a_1 s_{-1}) - \frac{s_{-1}}{2}(\mathbf{D} - 2\mathbf{C})]\mathbf{x}^*_{11} = \boldsymbol{\Omega}_1\mathbf{Q}$. Multiplying both sides by $\frac{1}{1+a_1 s_{-1}}$ while noting that $\frac{1}{1+a_1 s_{-1}} = \gamma_1$, we have $[\boldsymbol{\Omega}_1 - \frac{\gamma_1 s_{-1}}{2}(\mathbf{D} - 2\mathbf{C})]\mathbf{x}^*_{11} = \gamma_1\boldsymbol{\Omega}_1\mathbf{Q}$.

Finally, from the equation resulting from summing over the first-order conditions for F_j, $j > 1$, $2a_j\boldsymbol{\Omega}_1\mathbf{x}^*_{j1} = \boldsymbol{\mu}_1 + \mathbf{D}\mathbf{x}^*_{11} - \mathbf{P}_1 = \frac{2}{s_{-1}}\boldsymbol{\Omega}_1(\mathbf{Q} - \mathbf{x}^*_{11})$. Multiplying through by $\frac{2}{a_j}\boldsymbol{\Omega}_1^{-1}$, we have $\mathbf{x}^*_{j1} = \frac{1}{a_j s_{-1}}(\mathbf{Q} - \mathbf{x}^*_{11})$ for $j > 1$.

Since it is assumed that $a_1\boldsymbol{\Omega}_1 - (\mathbf{D} - \mathbf{C})$ is positive definite, it follows that all the objective functions of investors are concave, making the solution of the first-order conditions the unique equilibrium solution. ♣

9.2 Proof of Proposition 3

For $\mathbf{\Omega}_1$ non-singular, it is easily checked that $[\mathbf{\Omega}_1 - \frac{\gamma_1 s_{-1}}{2}(\mathbf{D} - \mathbf{2C})]^{-1}$

$= \mathbf{\Omega}_1^{-1} + \frac{\gamma_1 s_{-1}}{2}(\boldsymbol{\omega}^1, ..., \boldsymbol{\omega}^f)\mathbf{\Delta}_f[\mathbf{\Delta}_f - \frac{\gamma_1 s_{-1}}{2}\mathbf{\Delta}_f\mathbf{W}_f\mathbf{\Delta}_f]^{-1}\mathbf{\Delta}_f(\boldsymbol{\omega}^1, ..., \boldsymbol{\omega}^f)'$

$= \mathbf{\Omega}_1^{-1} + \frac{\gamma_1 s_{-1}}{2}(\boldsymbol{\omega}^1, ..., \boldsymbol{\omega}^f)[\mathbf{\Delta}_f^{-1} - \frac{\gamma_1 s_{-1}}{2}\mathbf{W}_f]^{-1}(\boldsymbol{\omega}^1, ..., \boldsymbol{\omega}^f)'$. Postmultiplying the right-hand-side of the last expression by $\gamma_1\mathbf{\Omega}_1\mathbf{Q}$ and noting that $\mathbf{\Omega}_1(\boldsymbol{\omega}^1, ..., \boldsymbol{\omega}^f) = \binom{\mathbf{I}_f}{\mathbf{0}}$, it follows that

$\mathbf{x}_{11}^* = \gamma_1\mathbf{Q} + \frac{\gamma_1^2 s_{-1}}{2}(\boldsymbol{\omega}^1, ..., \boldsymbol{\omega}^f)[\mathbf{\Delta}_f^{-1} - \frac{\gamma_1 s_{-1}}{2}\mathbf{W}_f]^{-1}[\mathbf{I}_f, \mathbf{0}]\mathbf{Q}$

$= \gamma_1\mathbf{Q} + \frac{\gamma_1^2 s_{-1}}{2}(\boldsymbol{\omega}^1, ..., \boldsymbol{\omega}^f)[\mathbf{\Delta}_f^{-1} - \frac{\gamma_1 s_{-1}}{2}\mathbf{W}_f]^{-1}\mathbf{Q}_f$

$= \gamma_1\mathbf{Q} + (\boldsymbol{\omega}^1, ..., \boldsymbol{\omega}^f)\mathbf{k}$ where $\mathbf{k} = \frac{\gamma_1^2 s_{-1}}{2}[\mathbf{\Delta}_f^{-1} - \frac{\gamma_1 s_{-1}}{2}\mathbf{W}_f]^{-1}\mathbf{Q}_f$. Furthermore, $\mathbf{x}_{j1}^* = \frac{1}{a_j s_{-1}}(\mathbf{Q} - \mathbf{x}_{\mathbf{11}}^*) = \gamma_j\mathbf{Q}$ $\frac{1}{a_j s_{-1}}(\boldsymbol{\omega}^1, ..., \boldsymbol{\omega}^f)\mathbf{k}$. ♣

9.3 Proof of Corollary 1

From Proposition 3, we have

$\mathbf{x}_{11}^* - \gamma_1\mathbf{Q} = k_1\boldsymbol{\omega}^1$ where

$k_1 = \frac{\gamma_1^2 s_{-1}}{2}(\frac{1}{d_1 - 2c_1} - \frac{\gamma_1 s_{-1}}{2}\frac{1}{\sigma_1^2})^{-1}q_1$

$= \frac{\gamma_1^2 s_{-1}}{2}\frac{\sigma_1^2(d_1 - 2c_1)}{\sigma_1^2 - \frac{\gamma_1 s_{-1}}{2}(d_1 - 2c_1)}$ where σ_1^2 is the upper diagonal element of $\mathbf{\Omega}_1$. The positive definiteness of $a_1\mathbf{\Omega}_1 - (\mathbf{D} - \mathbf{C})$ implies the positive definiteness of $[\mathbf{\Omega}_1 - \frac{\gamma_1 s_{-1}}{2}(\mathbf{D} - \mathbf{2C})]$ which in turn guarantees that $\sigma_1^2 - \frac{\gamma_1 s_{-1}}{2}(d_1 - 2c_1) > 0$. This, together with the assumption that $d_1 - 2c_1 > 0$, guarantees that $k_1 > 0$. Also, since $\mathbf{\Omega}_1$ is positive definite the diagonal elements of $\mathbf{\Omega}_1^{-1}$ are positive so the first element of $\boldsymbol{\omega}^1$ is positive. Therefore, the first element of $\mathbf{x}_{11}^* - \gamma_1\mathbf{Q}$ is positive. ♣

9.4 Proof of Proposition 4

From Proposition 3, the impact on the first f elements of $\mathbf{x}_{11}^*$ is $\mathbf{W}_f\mathbf{k} = \frac{\gamma_1^2 s_{-1}}{2}\mathbf{W}_f[\mathbf{\Delta}_f^{-1} - \frac{\gamma_1 s_{-1}}{2}\mathbf{W}_f]^{-1}\mathbf{Q}_f = \frac{\gamma_1^2 s_{-1}}{2}[\mathbf{W}_f^{-1} - \frac{\gamma_1 s_{-1}}{2}\mathbf{\Delta}_f]^{-1}\mathbf{\Delta}_f\mathbf{Q}_f$. If all the elements of $\mathbf{W}_f\mathbf{k}$ are negative, then $\mathbf{Q}_f'\mathbf{\Delta}_f\mathbf{W}_f\mathbf{k}$ must be negative since the elements of $\mathbf{\Delta}_f\mathbf{Q}_f$ are positive. But $\mathbf{Q}_f'\mathbf{\Delta}_f\mathbf{W}_f\mathbf{k}$ is a quadratic form in the matrix $[\mathbf{W}_f^{-1} - \frac{\gamma_1 s_{-1}}{2}\mathbf{\Delta}_f]^{-1}$. Since, by assumption, $[\mathbf{W}_f^{-1} - \frac{\gamma_1 s_{-1}}{2}\mathbf{\Delta}_f]$ is positive definite, $\mathbf{Q}_f'\mathbf{\Delta}_f\mathbf{W}_f\mathbf{k}$ must be positive. Therefore, not all elements of $\mathbf{W}_f\mathbf{k}$ can be negative. ♣

9.5 Proof of Proposition 5

In the proof of Proposition 2, we established that $\boldsymbol{\mu}_1 - \mathbf{P}_1 = 2[a_1\mathbf{\Omega}_1 - (\mathbf{D} - \mathbf{C})]\mathbf{x}_{11}^*$. We now evaluate the right-hand-side of this equation. Using the result of Proposition 3, we have that

$$
\begin{aligned}
&2[a_1\mathbf{\Omega}_1 - (\mathbf{D} - \mathbf{C})]\mathbf{x}_{11}^* \\
&\quad= 2[a_1\mathbf{\Omega}_1 - (\mathbf{D} - \mathbf{C})][\gamma_1\mathbf{Q} + (\boldsymbol{\omega}^1, ..., \boldsymbol{\omega}^f)\mathbf{k}] \\
&\quad= \tfrac{2}{s}\mathbf{\Omega}_1\mathbf{Q} - 2(\mathbf{D} - \mathbf{C})\gamma_1\mathbf{Q} + 2a_1\binom{\mathbf{I}_f}{\mathbf{0}}\mathbf{k} - 2(\mathbf{D} - \mathbf{C})(\boldsymbol{\omega}^1, ..., \boldsymbol{\omega}^f)\mathbf{k} \\
&\quad= \tfrac{2}{s}\mathbf{\Omega}_1\mathbf{Q} - 2\gamma_1\binom{\mathbf{I}_f}{\mathbf{0}}(\mathbf{D} - \mathbf{C})_f\mathbf{Q}_f + 2a_1\binom{\mathbf{I}_f}{\mathbf{0}}\mathbf{k} - 2\binom{\mathbf{I}_f}{\mathbf{0}}(\mathbf{D} - \mathbf{C})_f\mathbf{W}_f\mathbf{k} \\
&\quad= \tfrac{2}{s}\mathbf{\Omega}_1\mathbf{Q} - 2\binom{\mathbf{I}_f}{\mathbf{0}}[\tfrac{2}{\gamma_1 s_{-1}}(\mathbf{D} - \mathbf{C})_f(\mathbf{\Delta}_f^{-1} - \tfrac{\gamma_1 s_{-1}}{2}\mathbf{W}_f) - a_1\mathbf{I}_f + (\mathbf{D} - \mathbf{C})_f\mathbf{W}_f]\mathbf{k} \\
&\quad= \tfrac{2}{s}\mathbf{\Omega}_1\mathbf{Q} - \tfrac{4}{\gamma_1 s_{-1}}\binom{\mathbf{I}_f}{\mathbf{0}}[(\mathbf{D} - \mathbf{C})_f\mathbf{\Delta}_f^{-1} - \tfrac{s_{-1}}{2s}\mathbf{I}_f]\mathbf{k}.
\end{aligned}
$$

Thus, the result for $\boldsymbol{\mu}_1 - \mathbf{P}_1$ follows. The expression for $\mathbf{P}_1 - \mathbf{P}_0$ follows from the additional fact, established in Proposition 1, that $\boldsymbol{\mu}_0 - \mathbf{P}_0 = \frac{2}{s}\mathbf{\Omega}_0\mathbf{Q}$. ♣

9.6 Proof of Corollary 3

That only the first f components of $\mathbf{P}_1 - \mathbf{P}_0$ are affected follows by evaluating the corresponding expression in Proposition 5 at $\boldsymbol{\mu}_1 = \boldsymbol{\mu}_0$ and $\mathbf{\Omega}_1 = \mathbf{\Omega}_0$. We now show that at least one element of $[(\mathbf{D}-\mathbf{C})_f\mathbf{\Delta}_f^{-1} - \frac{s_{-1}}{2s}\mathbf{I}_f]\mathbf{k}$ is positive. First, the diagonal elements of $[(\mathbf{D}-\mathbf{C})_f\mathbf{\Delta}_f^{-1} - \frac{s_{-1}}{2s}\mathbf{I}_f]$ are positive, since $d_i - 2c_i$ are assumed positive and therefore $\frac{d_i-c_i}{d_i-2c_i} - \frac{s_{-1}}{2s} = \frac{1}{d_i-2c_i}[d_i(1-\frac{s_{-1}}{2s}) - 2c_i(\frac{1}{2}-\frac{s_{-1}}{2s})] > \frac{1}{2}(1-\frac{s_{-1}}{s}) > 0$. Thus, the vector $[(\mathbf{D}-\mathbf{C})_f\mathbf{\Delta}_f^{-1} - \frac{s_{-1}}{2s}\mathbf{I}_f]^{-1}\mathbf{Q}_f$ is a vector of positive elements. Second, since $\mathbf{\Omega}_1 - \frac{\gamma_1 s_{-1}}{2}(\mathbf{D}-\mathbf{C})$ was shown to be positive definite, so is its inverse. Using the explicit form of the inverse given in Proposition 3, it follows that $\mathbf{\Delta}_f[\mathbf{\Delta}_f - \frac{\gamma_1 s_{-1}}{2}\mathbf{\Delta}_f\mathbf{W}_f\mathbf{\Delta}_f]^{-1}\mathbf{\Delta}_f$ and thus $[\mathbf{\Delta}_f^{-1} - \frac{\gamma_1 s_{-1}}{2}\mathbf{W}_f]^{-1}$ is positive definite. If all the elements of $[(\mathbf{D}-\mathbf{C})_f\mathbf{\Delta}_f^{-1} - \frac{s_{-1}}{2s}\mathbf{I}_f]\mathbf{k}$ were negative, then from the preceeding remarks, $\mathbf{Q}_f'[(\mathbf{D}-\mathbf{C})_f\mathbf{\Delta}_f^{-1} - \frac{s_{-1}}{2s}\mathbf{I}_f]^{-1}[(\mathbf{D}-\mathbf{C})_f\mathbf{\Delta}_f^{-1} - \frac{s_{-1}}{2s}\mathbf{I}_f]\mathbf{k}$ must be negative or $\mathbf{Q}_f'\mathbf{k}$ must be negative. But $\mathbf{Q}_f'\mathbf{k} = \frac{\gamma_1^2 s_1}{2}\mathbf{Q}_f'[\mathbf{\Delta}_f^{-1} - \frac{\gamma_1 s_{-1}}{2s}\mathbf{W}_f]^{-1}\mathbf{Q}_f$ is a quadratic form with a positive definite matrix which must be positive. This contradiction implies that at least one element of $[(\mathbf{D}-\mathbf{C})_f\mathbf{\Delta}_f^{-1} - \frac{s_{-1}}{2s}\mathbf{I}_f]\mathbf{k}$ must be positive. ♣

9.7 Proof of Corollary 4

This proof is the same as the one for Corollary 3.

9.8 Proof of Corollary 5

Since we showed in Corollary 3 that at least one of the f prices increases, when $f = 1$, it follows that the first price increases and thus $\mathbf{e}_1'(\mathbf{P}_1 - \mathbf{P}_0) > \mathbf{0}$. ♣

9.9 Proof of Proposition 6

It follows from Proposition 5 that $\mathbf{P}_1 = \boldsymbol{\mu}_1 - \frac{2}{s}\mathbf{\Omega}_1\mathbf{Q} + \frac{4}{\gamma_1 s_{-1}}\binom{\mathbf{I}_f}{\mathbf{0}}[(\mathbf{D}-\mathbf{C})_f\mathbf{\Delta}_f^{-1} - \frac{s_{-1}}{2s}\mathbf{I}_f]\mathbf{k}$. Writing the first element of $\mathbf{P}_1$ in terms of c_1, we have for $f = 1$,

$$\mathbf{e}_1'\mathbf{P}_1 = \text{constant} + \frac{4}{\gamma_1 s_{-1}}[\frac{d_1-c_1}{d_1-2c_1} - \frac{s_{-1}}{2s}]\frac{\gamma_1^2 s_{-1}}{2}[\frac{1}{d_1-2c_1} - \frac{\gamma_1 s_{-1}}{2}\omega_1^1]^{-1}q_1$$

$= \text{constant } +2\gamma_1[\frac{d_1 - c_1 - \frac{s_{-1}}{2s}(d_1 - 2c_1)}{d_1 - 2c_1}][\frac{d_1 - 2c_1}{1-(d_1-2c_1)\frac{\gamma_1 s_{-1}}{2}\omega_1^1}]$

$= \text{constant } +2\gamma_1[\frac{d_1(1-\frac{s_{-1}}{2s}) - c_1(1-\frac{s_{-1}}{s})}{1-d_i\frac{\gamma_1 s_{-1}}{2}+\omega_1^1+c_1\gamma_1 s_{-1}\omega_1^1}]$.

Since $\frac{s_{-1}}{s} < 1$, the numerator of the last bracketed expression is decreasing in c_1. Since ω_1^1, the first element of $\boldsymbol{\omega}^1$, is positive, the coefficient of c_1 in the denominator is positive and thus the denominator is increasing. Thus, the bracketed expression is decreasing in c_1. ♣

9.10 Proof of Proposition 7

Each investor other than F_1 believes that F_1 is effective, i.e., $\boldsymbol{\mu}(\mathbf{x}_{11}) = \mathbf{D}\mathbf{x}_{11}$, with probability λ and not effective, $\boldsymbol{\mu}(\mathbf{x}_{11}) = 0$, with probability $(1-\lambda)$. Thus, each investor F_j, $j > 1$, seeks $\mathbf{x}_{11}^*$ such that $\mathbf{x}_{j1}^* = \arg\max_{\mathbf{x}_{j1}}\{\lambda[h_{j1} + \gamma_j \mathbf{Q}'\mathbf{P}_1 + \mathbf{x}_{j1}'(\boldsymbol{\mu} + \mathbf{D}\mathbf{x}_{11} - \mathbf{P}_1) - a_j \mathbf{x}_{j1}'\boldsymbol{\Omega}_1\mathbf{x}_{j1}]+$

$(1-\lambda)[h_{j1} + \gamma_j \mathbf{Q}'\mathbf{P}_1 + \mathbf{x}_{j1}'(\boldsymbol{\mu}_1 - \mathbf{P}_1) - a_j \mathbf{x}_{j1}'\boldsymbol{\Omega}_1\mathbf{x}_{j1}]\} =$

$\arg\max[h_{j1} + \gamma_j \mathbf{Q}'\mathbf{P}_1 + \mathbf{x}_{j1}'(\boldsymbol{\mu}_1 + \lambda\mathbf{D}\mathbf{x}_{11} - \mathbf{P}_1) - a_j \mathbf{x}_{j1}'\boldsymbol{\Omega}_1\mathbf{x}_{j1}]$.

Since this is the same as was worked out in the proof of Proposition 2 with $\mathbf{D}$ replaced by $\lambda\mathbf{D}$, summing over j, $j > 1$, implies that x_{11}^* must satisfy $(2\boldsymbol{\Omega}_1 + s_{-1}\lambda\mathbf{D})\mathbf{x}_{11}^* = 2\boldsymbol{\Omega}_1\mathbf{Q} - s_{-1}(\boldsymbol{\mu}_1 - \mathbf{P}_1)$.

If $\boldsymbol{\mu}(\mathbf{x}_{11}) = \mathbf{D}\mathbf{x}_{11}$, then the optimization problem for F_1 is the same as before. This implies that x_{11}^* must satisfy $\boldsymbol{\mu}_1 - \mathbf{P}_1 = 2[a_1\boldsymbol{\Omega}_1 - (\mathbf{D} - \mathbf{C})]\mathbf{x}_{11}^*$. Combining both equations for $\mathbf{x}_{11}^*$, we have $[2\boldsymbol{\Omega}_1(1 + a_1 s_{-1}) - s_{-1}(\mathbf{D}(2-\lambda) - 2\mathbf{C})]\mathbf{x}_{11}^* = 2\boldsymbol{\Omega}_1\mathbf{Q}$. Dividing, we have $[\boldsymbol{\Omega}_1 - \frac{\gamma_1 s_{-1}}{2}((2-\lambda)\mathbf{D} - \mathbf{2C})]\mathbf{x}_{11}^*(D) = \gamma_1\boldsymbol{\Omega}_1\mathbf{Q}$. This is the same equation that we solved in Proposition 3 with $\mathbf{D}$ replaced by $(2-\lambda)\mathbf{D}$. Thus, for $f = 1$, $\mathbf{x}_{11}^*(D) - \gamma_1\mathbf{Q} = \frac{\gamma_1^2 q_1 s_{-1}}{2}\frac{[d_1(2-\lambda) - 2c_1]}{1-\frac{\gamma_1 s_{-1}}{2}[d_1(2-\lambda)-2c_1]\omega_1^1}\boldsymbol{\omega}^1 = \beta_D\boldsymbol{\omega}^1$.

If $\boldsymbol{\mu}(\mathbf{x}_{11}) = 0$, then $\mathbf{D} = \mathbf{C} = 0$ and the first-order conditions for F_1 are $\boldsymbol{\mu}_1 - \mathbf{P}_1 = 2a\boldsymbol{\Omega}_1\mathbf{x}_{11}^*$. Combining this equation with those of the F_j, $j > 1$, x_{11}^* must satisfy $[2\boldsymbol{\Omega}_1(1 +$

$a_1 s_{-1}) + s_{-1}\lambda\mathbf{D}]\mathbf{x}_{11}^* = 2\mathbf{\Omega}_1\mathbf{Q}$ or $[\mathbf{\Omega}_1 + \frac{\gamma_1 s_{-1}}{2}\lambda D]\mathbf{x}_{11}^*(0) = \gamma_1\mathbf{\Omega}_1\mathbf{Q}$. This, again, is the same equation solved in Proposition 3 with D replaced by $-D\lambda$ and $C = 0$. Thus, for $f = 1$, $\mathbf{x}_{11}^*(0) - \gamma_1\mathbf{Q} = -\frac{\gamma_1^2 q_1 s_{-1}}{2}\frac{d_1\lambda}{1+\frac{\gamma_1 s_{-1}}{2}d_1\lambda\omega_1^1}\boldsymbol{\omega}^1 = -\beta_0\boldsymbol{\omega}^1$.

If F_1 is chosen from a population where with probability λ $\boldsymbol{\mu}(\mathbf{x}_{11}) = \mathbf{D}\mathbf{x}_{11}$ and with probability $(1-\lambda)$ $\boldsymbol{\mu}(\mathbf{x}_{11}) = \mathbf{0}$, then the solution to the problem is the same as in Proposition 3 with $\mathbf{D}$ and $\mathbf{C}$ replaced by $\lambda\mathbf{D}$ and $\lambda\mathbf{C}$, respectively. Letting $\mathbf{x}_{11}^*(\lambda)$ be this solution, we have $\mathbf{x}_{11}^*(\lambda) - \gamma_1\mathbf{Q} = \frac{\gamma_1^2 q_1 s_{-1}}{2}\frac{[d_1-2c_1]\lambda}{1-\frac{\gamma_1 s_{-1}}{2}\lambda(d_1-2c_1)\omega_1^1}\boldsymbol{\omega}^1 = \beta_\lambda\boldsymbol{\omega}^1$.

The first element of $\boldsymbol{\omega}^1$, i.e., ω_1^1, is positive and $1 - \frac{\gamma_1 s_{-1}}{2}\lambda(d_1 - 2c_1)\omega_1^1$ is positive as a result of the positive definite assumptions of Proposition 2. It now follows since $d_1 - 2c_1 > 0$ that $\beta_\lambda > 0$ and $\beta_D > 0$. Since β_0 involves only positive terms, $\beta_0 > 0$. Since $2 - \lambda > 1$, it follows that $\beta_D > \beta_\lambda$. Therefore, $\mathbf{e}_1'\mathbf{x}_{11}^*(D) > \mathbf{e}_1'\mathbf{x}_{11}^*(\lambda) > \mathbf{e}_1'\mathbf{x}_{11}^*(0)$. With $\omega_1^1 > 0$, it follows that the first elements of $\mathbf{x}_{11}^*(D) - \gamma_1\mathbf{Q}$ and $\mathbf{x}_{11}^*(\lambda) - \gamma_1\mathbf{Q}$ are positive while the first element of $\mathbf{x}_{11}^*(0) - \gamma_1\mathbf{Q}$ is negative. ♣

9.11 Proof of Proposition 8

From Proposition 5 and the assumption that $\boldsymbol{\mu}(\mathbf{x}_{11}) = \mathbf{D}\mathbf{x}_{11}$, we have, for $f = 1$, $\boldsymbol{\mu}_1 - \mathbf{P}_1(\mathbf{D}) = 2[a_1\mathbf{\Omega}_1 - (\mathbf{D} - \mathbf{C})]x_{11}^*(\mathbf{D}) = 2[a_1\mathbf{\Omega}_1 - (d_1 - c_1)\mathbf{e}_1\mathbf{e}_1'][\gamma_1\mathbf{Q} + \beta_D\boldsymbol{\omega}^1]$

$= \frac{2}{s}\mathbf{\Omega}_1\mathbf{Q} + 2[a_1\beta_D - \gamma_1 q_1(d_1 - c_1) - (d_1 - c_1)\omega_1^1\beta_D]\mathbf{e}_1$

$= \frac{2}{s}\mathbf{\Omega}_1\mathbf{Q} - K_D\mathbf{e}_1$ with $K_D = 2(d_1 - c_1)(\gamma_1 q_1 + \omega_1^1\beta_D] - 2a_1\beta_D$.

Similarly, $\boldsymbol{\mu}_1 - \mathbf{P}_1(\lambda) = 2[a_1\mathbf{\Omega}_1 - \lambda(\mathbf{D} - \mathbf{C})]x_{11}^*(\lambda)$

$= 2[a_1\mathbf{\Omega}_1 - \lambda(d_1 - c_1)\mathbf{e}_1\mathbf{e}_1'][\gamma_1\mathbf{Q} + \beta_\lambda\boldsymbol{\omega}^1]$

$= \frac{2}{s}\mathbf{\Omega}_1\mathbf{Q} - K_\lambda\mathbf{e}_1$ with $K_\lambda = 2\lambda(d_1 - c_1)(\gamma_1 q_1 + \omega_1^1\beta_\lambda] - 2a_1\beta_\lambda$.

Finally, when $\boldsymbol{\mu}(\mathbf{x}_{11}) = \mathbf{0}$, then, $\boldsymbol{\mu}_1 - \mathbf{P}_1(\mathbf{0}) = 2a_1\mathbf{\Omega}_1 x_{11}^*(\mathbf{0})$

$= 2a_1\boldsymbol{\Omega}_1[\gamma_1\mathbf{Q} + \beta_0\boldsymbol{\omega}^1]$

$= \frac{2}{s}\boldsymbol{\Omega}_1\mathbf{Q} - K_0\boldsymbol{\omega}^1$ with $K_0 = 2a_1\beta_0$.

The scaler $\frac{K_D}{2} = (d_1 - c_1)(\gamma_1 q_1 + \omega_1^1\beta_D] - a_1\beta_D = \beta_D[(d_1 - c_1)(\gamma_1 q_1\beta_D^{-1} + \omega_1^1] - a_1$

$= \beta_D\frac{2}{\gamma_1 s_{-1}[d_1(2-\lambda)-2c_1]}[d_1 - c_1 - \frac{s_{-1}}{2s}(d_1(2-\lambda) - 2c_1)]$

$= \beta_D\frac{2}{\gamma_1 s_{-1}[d_1(2-\lambda)-2c_1]}[d_1(1 - \frac{s_{-1}}{2s} + \frac{\lambda s_{-1}}{2s}) - c_1(1 - \frac{s_{-1}}{s})] > 0.$

Since the equation for $\boldsymbol{\mu}_1 - \mathbf{P}_1(\lambda)$ has replaced $\mathbf{D}-$with $\lambda\mathbf{D}$ and $\mathbf{C}$ with $\lambda\mathbf{C}$ the results of Corollary 5 hold and $K_\lambda > 0$. Finally, $K_0 > 0$ since a_1 and β_0 are positive. Since $\boldsymbol{\mu}_1 = \boldsymbol{\mu}_0$ and $\boldsymbol{\Omega}_1 = \boldsymbol{\Omega}_0$, $\mathbf{P}_0 = \boldsymbol{\mu}_1 - \frac{2}{s}\boldsymbol{\Omega}_1\mathbf{Q}$ so $\mathbf{P}_1(D) - \mathbf{P}_0 = K_D\mathbf{e}_1$, $\mathbf{P}_1(\lambda) - \mathbf{P}_0 = K_\lambda\mathbf{e}_1$, and $\mathbf{P}_1(0) - \mathbf{P}_0 = K_0\boldsymbol{\omega}^1$. ♣

9.12 Proof of Proposition 9

Investor F_j seeks $\mathbf{x}_{j1}^*$ such that $\mathbf{x}_{j1}^* = \arg\max[z_{j1} + \mathbf{x}_{j1}'\widetilde{\boldsymbol{\mu}_1}]$ subject to $z_{j1} + \mathbf{x}_{j1}'\mathbf{P}_1 + \mathbf{x}_{j1}'\mathbf{C}_j\mathbf{x}_{j1} = h_{j1} + \gamma_j\mathbf{Q}\mathbf{P}_1$. Substituting as in Proposition 2 and again using the certainty equivalent, we have $\mathbf{x}_{j1}^* = \arg\max\{h_{j1} + \gamma_j\mathbf{Q}'\mathbf{P}_1 + \mathbf{x}_{j1}'[\boldsymbol{\mu}_1 - \mathbf{P}_1 + \sum_{i\neq j}\mathbf{D}_i\mathbf{x}_{i1} + (\mathbf{D}_j - \mathbf{C}_j)\mathbf{x}_{j1}] - a_j\mathbf{x}_{j1}'\boldsymbol{\Omega}_1\mathbf{x}_{j1}\}$.

The first-order conditions for F_j are $\boldsymbol{\mu}_1 - \mathbf{P}_1 + \sum_{i\neq j}\mathbf{D}_i\mathbf{x}_{i1} + 2(\mathbf{D}_j - \mathbf{C}_j)\mathbf{x}_{j1} - 2a_j\boldsymbol{\Omega}_1\mathbf{x}_{j1} = \mathbf{0}$. We can rewrite the first-order conditions as $\boldsymbol{\mu}_1 - \mathbf{P}_1 + \sum_i\mathbf{D}_i\mathbf{x}_{i1} - 2[a_j\boldsymbol{\Omega}_1 - \frac{1}{2}\boldsymbol{\Delta}_j]\mathbf{x}_{j1} = \mathbf{0}$. It now follows that $[a_j\boldsymbol{\Omega}_1 - \frac{1}{2}\boldsymbol{\Delta}_j]\mathbf{x}_{j1} = [a_M\boldsymbol{\Omega}_1 - \frac{1}{2}\boldsymbol{\Delta}_M]\mathbf{x}_{M1}$ for $j = 1, ..., M$. Therefore, solving for $\mathbf{x}_{j1}$ and imposing the constraint that $\sum_j\mathbf{x}_{j1} = \mathbf{Q}$, we have that $\mathbf{x}_{j1}^* = (a_j\boldsymbol{\Omega}_1 - \frac{1}{2}\boldsymbol{\Delta}_j)^{-1}[\sum_i(a_i\boldsymbol{\Omega}_1 - \frac{1}{2}\boldsymbol{\Delta}_i)^{-1}]^{-1}\mathbf{Q}$.

Substituting these values into the first order conditions for F_j, and solving for $\boldsymbol{\mu}_1 - \mathbf{P}_1$ we have $\boldsymbol{\mu}_1 - \mathbf{P}_1 = 2(a_j\boldsymbol{\Omega}_1 - \frac{1}{2}\boldsymbol{\Delta}_j)\mathbf{x}_{j1} - \sum_i\mathbf{D}_i\mathbf{x}_{i1}$

$= 2[\sum_i(a_i\boldsymbol{\Omega}_1 - \frac{1}{2}\boldsymbol{\Delta}_i)^{-1}]^{-1}\mathbf{Q} - \sum_i\mathbf{D}_i(a_i\boldsymbol{\Omega}_1 - \frac{1}{2}\boldsymbol{\Delta}_i)^{-1}[\sum_i(a_i\boldsymbol{\Omega}_1 - \frac{1}{2}\boldsymbol{\Delta}_i)^{-1}]^{-1}\mathbf{Q}$.

Factoring, we have $\boldsymbol{\mu}_1 - \mathbf{P}_1 = [2\mathbf{I} - \sum_i\mathbf{D}_i(a_i\boldsymbol{\Omega}_1 - \frac{1}{2}\boldsymbol{\Delta}_i)^{-1}][\sum_i(a_i\boldsymbol{\Omega}_1 - \frac{1}{2}\boldsymbol{\Delta}_i)^{-1}]^{-1}\mathbf{Q}$.♣

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