The Benefits of Franchising and Vertical Disintegration

in Monopolistic Competition for Locationally Differentiated Products

by

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Abstract

A model of franchising competition in locationally differentiated products is constructed. A franchisor (upstream firm) collects a marginal transfer fee per unit of output sold by a franchisee (downstream firm). For example, the marginal transfer fee can be realized as a markup on variable inputs supplied by the franchisor. A franchisor also collects a lump-sum rent (commonly called "franchising fee") from each franchisee. Acting in the first stage, a franchisor can manipulate the degree of competition in the downstream market through his choice of the marginal fee while keeping the franchisee's profits at zero through the lump sum rent. Franchisees choose prices for the final goods in the second stage. It is shown that, at the unique subgame-perfect equilibrium, the marginal fee is above marginal cost. Compared to a regime of vertically integrated firms, prices are higher, there are more numerous outlets when contractual costs are small, and social surplus is lower in the franchising regime.

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1. Introduction

The franchising arrangement falls between a price-determining market exchange and a hierarchical relationship within a firm. In a typical franchising arrangement, the franchisee has a free hand over levels of production and pricing, but is required to purchase a number of inputs from the franchisor and to pay him pre-specified franchising fees. The franchisor cannot force the franchisee to buy all inputs from him (and such contracts have been ruled invalid), but he can require the franchisee to buy from him inputs essential to the nature of the final product.[1] The franchisor can sell these inputs above cost at a range of markups specified in the franchising contract. For example, such inputs are frozen french fries or hamburger patties for McDonald's or Burger King and batter for Kentucky Fried Chicken or Chicken Delight. Sometimes they include the use of the building site of the retailer.[2] The use of the tradename and trademarks of the franchisor are important inputs to the franchisee; they are typically "sold" to the franchisee in return for a percentage of the franchisee's revenue and/or a lump-sum fee.

[1] Franchising "tying" contracts, where the franchisee is required to buy certain inputs from the franchisor, have been the issue of much litigation. The courts have recently ruled that a franchisor has the right to require that a franchisee buys essential inputs to the final good from him or designated licensees. See Klein and Saft (1985). The argument that this is essential to control the quality of output has been well accepted. However, requirements to buy inputs not essential to quality control requirements have been rejected. See Siegel v. Chicken Delight, Inc. 311 F. Supp. 847 (N.D. Cal. 1970), aff'd in part, 448 F.2d 43 (9th Cir. 1971), cert. denied, 405 U.S. 955 (1972).

[2] McDonald's typically ties the site lease to the franchising license. In a recent case, Principe v. McDonald's, 631 F.2d 303, 309 (1980), the court ruled that this practice is legal since "[the] lease is not separable from the McDonald's franchise to which it pertains. For a detailed analysis see Klein and Saft (1985).

The traditional analysis of franchising identifies two crucial benefits of the franchising arrangements.[3] First, a franchise can assure the buyers of minimum quality standards or of particular features of variety[4] in the product to be purchased. Thus, franchising utilizes efficiently information dissemination through the use of the single tradename of the franchisor.[5] Second, franchisees are usually better informed on the demand in their local market. Thus, they can adjust prices more efficiently to changing demand conditions. In this paper we abstract from both of these informational aspects of franchising contracts. Instead, we stress another aspect of the franchising arrangement: the influence of the vertical relationship between the franchisor and franchisee on market structure. Thus, we will compare a market structure of independent firms producing locationally differentiated products with a market structure of franchised firms.

What is the importance of the incentives resulting from the vertical relationship between franchisor and franchisees? First, note that in every franchising arrangement, a distinction is made between *fixed inputs* of the franchisee's production function, such as the name of the franchisor and the use of a building provided by him, and *variable inputs,* the quantity of which varies with the level of the franchisee's sales. The markup per unit of output, c, collected by the franchisor on variable inputs that he provides to the franchisee, is called *marginal transfer fee*. It appears as a marginal cost to the franchisee. The markup by the franchisor of fixed inputs,

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3 See Barbara Katz and Joel Owen (1992) for a recent review of the franchising literature.

4 For example, one reason for some customers to purchase from McDonald's is the assurance of the standardized *particular variety* of food offered, which these customers desire, independently of the quality features (ambiance, service, etc.) of the restaurant.

5 See Economides (1988) for a discussion of the informational benefits of use of tradenames and trademarks.

F, is called the *franchising fee*. It represents a lump-sum transfer from the franchisee to the franchisor. Thus, a franchising contract is typically a *two-part tariff* contract.[6]

Utilizing the fixed (franchising) fee of the two-part tariff, *a franchisor is able to extract the full profits of a franchisee*. It is assumed that there is a prevailing zero profit rate in the economy and therefore a prospective franchisee accepts a franchising contract that yields zero profits. To absorb downstream profits, a franchisor can use his two strategic variables, the marginal transfer fee c, and the lump sum fee F. Clearly, the franchising fee F plays no role in the *marginal* pricing decisions of the franchisee. However, by appropriate choice of the markup for variable inputs, the franchisor can manipulate the competitive environment in the output market to his advantage. We show that an increase of the marginal transfer fee by an upstream firm results in price increases by *all* downstream firms, including the competitors of the franchisees of the first mover. We show further, that the franchisor can achieve higher profits by choosing positive markups (above his marginal cost) on the inputs he supplies to his franchisees.[7]

The ability of a franchisor to manipulate competition in the downstream market has significant effects on market structure. We show that *prices are higher in the franchising regime* than in the regime of independent, vertically integrated firms. We proceed by modelling

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[6] In an alternative setup, a franchisor might link his marginal fees to sales or revenue of the franchisee. As far as affecting prices downstream, marginal fees on inputs or on sales will serve equally well. However, marginal fees on inputs are preferable to the franchisor because input use is easier to monitor.

[7] Recent literature on franchising has focused on informational asymmetry: the franchisee is better informed on local demand, as well as the potential for free riding by a franchisee on the national reputation of the franchise name and the quality level it implies. See Mathewson and Winter (1985) and Blair and Kaserman (1982). The model of this paper abstracts from both of these considerations and focuses on pricing by franchisees and the franchisors' influence on it. Pricing has been considered by Blair and Kaserman (1982) when the franchisee is a monopolist. The analysis of this paper requires more than one downstream competitor, and their strategic interaction is crucial for the results of this article.

competition among *franchisors* as *monopolistic competition*. In this environment, we show that, when contracting costs are small, *the number of retail outlets is larger in the franchising regime*. Further, we show that *social surplus is lower in the franchising regime*. Finally, in the franchising regime *the free entry equilibrium number of outlets exceeds the optimal one*, and the divergence between the equilibrium and the optimal number of outlets is larger under franchising than in the regime of vertically integrated firms.[8] In summary, the franchising arrangement results in significantly different prices as well as number of varieties in comparison with competition among vertically integrated firms.

We set up our model in the context of differentiated products. Consumers are located on a circle, which is an ideal representation of small diverse markets. This underscores the fact that demand for the products of our sector is generated by diverse consumers. The wide diversity of consumers has important implications on the equilibrium number of varieties and its relation to the optimal one. Our results can be interpreted as stating that a franchising regime is in a sense "better" in providing variety, but fails in surplus terms precisely because it over-provides variety.

The rest of the paper is organized as follows. Section 2 presents the model. Section 3.1 describes price competition among the franchisees. Location decisions of the franchisees are described in section 3.2. Equilibrium prices at the locational equilibrium are calculated in section 3.3. Section 4 analyzes the contracts that franchisors offer to franchisees and establishes the

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[8] The model of this paper can also be interpreted as describing a structure in which subsidiaries are paying marginal and lump-sum fees to a parent corporation. Then my results show that a firm has incentives to vertically disintegrate. It can create a downstream subsidiary, charge it marginal and lump-sum fees and generate higher profits for itself, as the downstream subsidiary charges higher prices than before. The present model could also be interpreted as a model of managerial incentives in which the proprietor inflates marginal costs to the manager. In this interpretation, our results are similar to Chaim Fershtman and Ken Judd (1987) and Steven Sklivas (1987), who have shown that when managers compete in prices, it pays for the proprietors to present them with higher marginal costs than the actual ones. Giacomo Bonanno and John Vickers (1988) have established independently that franchisors will choose a positive marginal fee. See also Patrick Rey and Joseph Stiglitz (1988). A positive marginal transfer fee also results in Katz and Owen (1992).

subgame-perfect equilibrium contract as well as the full market equilibrium under free entry. Section 5 compares the equilibrium market structure under franchising with the market structure of independent vertically integrated firms. Section 6 discusses optimality. Section 7 presents extensions and generalizations of the results. In Section 8 we present concluding remarks.

2. The Model

Competition among franchisors, franchisees and consumers is modelled as a multistage game. In stage 1, franchisors enter until there are no more profits to be made. Let m be the number of franchisors that have entered at the end of the first stage. In stage 2, each of the m franchisors, $i = 1, ..., m$, offers to n franchisees a *franchising contract* (c_i, F_i), where c_i is a transfer to the franchisor (upstream firm) per unit sold by the franchisee (downstream firm), and F_i is a lump sum transfer (rent).[9] In stage 3, franchisees accept contracts if they can make non-negative profits. In stage 4 franchisees choose locations. In stage 5, franchisees choose prices.

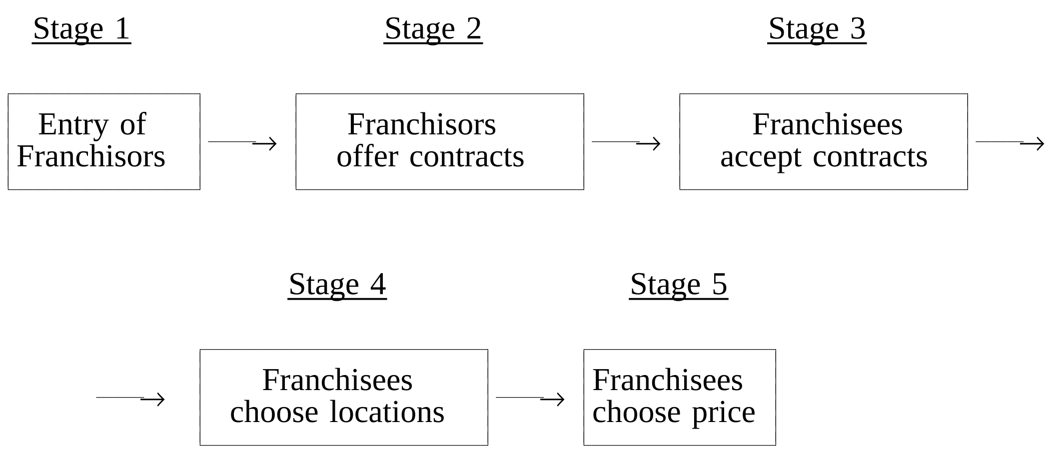

Figure 1

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[9] To be able to develop the model in a locational context, we restrict the number n of franchisees per franchisor to be the same for all franchises. The qualitative comparisons on prices, profits, and numbers of firms do not depend crucially on this assumption.

Franchisors know and anticipate the effects of contract changes on equilibrium prices and downstream profits. Thus, franchisors are able to manipulate the equilibrium outcome in the downstream market. This will prove to be crucial in the downstream market equilibrium.

The franchising contract stipulates that the franchisee will pay to the franchisor a lump sum rent F_i plus a marginal transfer rate of c_i dollars per unit of the good sold by the downstream firm.[10] In a typical franchising contract, the lump sum rent F_i is referred to as the "franchising fee". The marginal transfer fee c_i is realized as a markup by the franchisor of variable input materials the franchisee is required to buy from him. Let franchisor i have $j = 1, ..., n$ franchisees. If sales of a typical franchisee j of upstream firm (franchisor) 1 are $D_{1,j}$, the total revenue (transfer) collected by upstream firm 1 is

$$\Pi_1^u = \Sigma_{j=1}^n \ (c_1 D_{1,j} + F_1).$$

Let $p_{i,j}$ be the price quoted by franchisee j of franchisor i.[11] Let $\mathbf{p} = (\mathbf{p_1}, ..., \mathbf{p_n})$ be the vector of all prices, where $\mathbf{p_j} = (p_{1,j}, ..., p_{m,j})$. Thus, prices are ordered in the $\mathbf{p}$ vector with the first franchisees of franchisors 1 through m coming first, to be followed by the second franchisees of franchisors 1 through m, and so on. Similarly, let $\mathbf{c^0} = (c_1, ..., c_m)$ be the vector of all marginal transfer rates charged by franchisors 1 through m. We define $\mathbf{c} = (\mathbf{c^0}, ..., \mathbf{c^0})$ to be the vector of n repetitions of $\mathbf{c^0}$. Thus, $\mathbf{c}$ is the mn-long vector of marginal costs of all firms i,j, ordered as in the sequence of prices in the $\mathbf{p}$ vector.

Let the production technology for downstream firms (franchisees) have an entry cost E. It is assumed that the cost of writing a contract between an upstream and a downstream firm is K. Then the profit function of a franchisee (downstream firm) j of franchisor (upstream firm) i is

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[10] An extension of the results to non-linear contracts is presented in Section 6.

[11] If there are constant marginal costs in the production of the franchisee i,j other than c_i, $p_{i,j}$ can be interpreted as the difference between $p_{i,j}$ and the constant marginal costs other than c_i.

$$\Pi^d_{i,j}(p, c, F_i, K, E) = (p_{i,j} - c_i)D_{i,j}(p) - F_i - K - E. \qquad (1)$$

It is assumed that franchisees are located on a circumference in an interleaved pattern, so that a franchisee of type (franchise) 1 is followed by a franchisee of type 2, then of type 3 all the way to type m, and then the pattern is repeated n-1 times, starting with a franchisee of type 1, then of type 2, etc. See Figure 2. Formally, let $\mathbf{x} = (\mathbf{x_1}, ..., \mathbf{x_n})$, where $\mathbf{x_j} = (x_{1,j}, ..., x_{m,j})$, be the vector of all locations of consecutive franchisees. The definition of the vector of locations **x** corresponds appropriately to the earlier definitions of the vectors of prices and of marginal fees, **p** and **c**, so that franchisee i,j has the same position (that is, position $i + m(j-1)$) in all three vectors **x**, **p**, and **c**.

The locational arrangement implies that downstream firms belonging to the same franchise do not compete directly against each other. Thus, a retail store always has as its neighbors stores of other franchises. By spacing outlets of franchise Y between the outlets franchise X, we capture the fact that a franchisor can select the locations of his franchisees, and he benefits if his franchisees compete less against each other and more against the outlets of competing franchises.[12] For example, let there be three franchisors (m = 3), Burger King, McDonald's, and Wendy's, and that they have four franchisees each (n = 4). In the locational pattern, franchisees are located as follows: Burger King #1, McDonald's #1, Wendy's #1, Burger King #2, McDonald's #2, Wendy's #2, Burger King #3, McDonald's #3, Wendy's #3, Burger King #4, McDonald's #4, and Wendy's #4. Thus, there is always a McDonald's between a Burger King and a Wendy's. McDonald's outlets don't compete directly against each other.

Consumers are uniformly distributed on the circumference, and each one buys a unit of a differentiated product. The utility to the consumer located at z of consumption of one unit of product x, sold at price p is

[12] Franchisors also typically take steps to insure that their franchisees don't compete against each other by using territorial restraints to assign them to customers coming from specific territories.

$$U_z(x, p) = R - p - (z - x)^2.$$

The reservation price R is taken sufficiently large so that all consumers buy a differentiated good.[13]

3. Competition Among Franchisees

3.1 Price Equilibria

In the last stage of the game, the franchisees choose prices taking as given the contracts they have signed with the franchisors in the first stage of the game. Franchisee i,j is located at $x_{i,j}$ between franchisee $i-1,j$ located at $x_{i,j} - d$, and franchisee $i+1,j$ located at $x_{i,j} + d$. Its demand $D_{i,j}$ is generated by consumers located in the interval $[z^{\ell}_{i,j}, z^{r}_{i,j}]$, where (disregarding the j)

$$z^{\ell}_{i,j} = [x_i - x_{i-1} + (p_i - p_{i-1})/(x_i - x_{i-1})]/2,$$
$$z^{r}_{i,j} = [x_{i+1} - x_i + (p_{i+1} - p_i)/(x_{i+1} - x_i)]/2.$$

The demand for franchisee i,j is

$$D_{i,j}(p_{i,j}, p_{i-1,j}, p_{i+1,j}) = \int_{z^{\ell}_{i,j}}^{z^{r}_{i,j}} dz = [(p_{i+1} - p_i)/(x_{i+1} - x_i) - (p_i - p_{i-1})/(x_i - x_{i-1}) + x_{i+1} - x_{i-1}]/2.$$

The typical jth franchisee of the ith franchise has profit function

$$\Pi^{d}_{i,j}(p, c, F_i, K, E) = (p_{i,j} - c_i)D_{i,j}(p_{i,j}, p_{i-1,j}, p_{i+1,j}) - F_i - K - E. \quad (1)$$

Profit maximization implies,[14]

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[13] Thus, all downstream firms are in direct competition with their neighbors. In the terminology of Salop (1979) and Economides (1989), this paper describes "competitive" equilibria.

[14] Second order conditions are also satisfied.

$$\partial\Pi^d_{i,j}/\partial p_{i,j} = D_{i,j} + (p_{i,j} - c_i)(\partial D_{i,j}/\partial p_{i,j}) = 0, \quad (2)$$

where

$$\partial D_{i,j}/\partial p_{i,j} = -1/(x_{i+1} - x_i) - 1/(x_i - x_{i-1}) = -(x_{i+1} - x_{i-1})/[(x_{i+1} - x_i)(x_i - x_{i-1}).$$

Thus,

$$\partial\Pi^d_{i,j}/\partial p_{i,j} = (x_{i+1} - x_{i-1})/2 - p_i[1/(x_{i+1} - x_i) + 1/(x_i - x_{i-1})] + p_{i+1}/[2(x_{i+1} - x_i)] + p_{i-1})/[2(x_i - x_{i-1})] + c_i\{1/[2(x_{i+1} - x_i)] + 1/[2(x_i - x_{i-1})]\} = 0$$

Solving this for p_i gives

$$p^*_i = \{(x_{i+1} - x_i)(x_i - x_{i-1})/[2(x_{i+1} - x_{i-1})]\}\{p^*_{i+1}/(x_{i+1} - x_i) + p^*_{i-1})/(x_i - x_{i-1}) + x_{i+1} - x_{i-1} + c_i(x_{i+1} - x_{i-1})/[(x_{i+1} - x_i)(x_i - x_{i-1})]\}. \quad (3)$$

The system of these conditions for $i = 1, ..., m$, and $j = 1, ..., n$, can be written as

$$\mathbf{A}\cdot(\mathbf{p}^*(\mathbf{x}) - \mathbf{c}) = \mathbf{g}(\mathbf{x}) \quad (4)$$

where $\mathbf{A}$ is an $mn \times mn$ matrix with $a_{k,k} = 1$, $a_{k,k-1} = (x_{i+1} - x_i)/[2(x_{i+1} - x_{i-1})]$, $a_{k,k+1} = (x_i - x_{i-1})/[2(x_{i+1} - x_{i-1})]$, with $i = \underset{m}{\mathrm{mod}}\, k$, and zeros elsewhere, and $g_k = [x_{i+1} - x_{i-1} - c_i]/2$. The locational nature of competition implies that each franchisee competes only with its two closest neighbors. Thus, except for the two extreme corners, matrix $\mathbf{A}$ has non-zero entries only on the main diagonal and on the diagonals immediately above and below it.

To find the equilibrium prices $\mathbf{p}^*(\mathbf{x})$, we multiply (4) by $\mathbf{A}^{-1}$, so that

$$\mathbf{p}^*(\mathbf{x}) = \mathbf{c} + \mathbf{A}^{-1}\cdot\mathbf{g}(\mathbf{x}). \quad (5)$$

3.2 Location Equilibrium

Profits at the price equilibrium can be written (by substitution of $D_{i,j}$ from (2)) as

$$\Pi^*_i = (p^*_i - c_i)D^*_i = (p^*_i - c_i)^2(\partial D_{i,j}/\partial p_{i,j}) - F_i - K - E \equiv \Pi_i(\mathbf{x}, \mathbf{p}^*(\mathbf{x})) \quad (6)$$

where $\mathbf{p}^*(\mathbf{x})$ is defined in (5). In stage 4, franchisees choose locations, expecting the equilibrium prices of the subgame of stage 5. Thus, franchisee i uses the equilibrium profits of the subgame, $\Pi_i(\mathbf{x}, \mathbf{p}^*(\mathbf{x}))$, as his objective function. We can establish the following Lemma.

<u>*Lemma 1*</u>*: Given* $x_{k+1} - x_k = d$ *for all* $k \neq i, k \neq i+1,$ *, and* $x_{k+1} - x_{k-1} = 2d$*, the function* $\Pi_i(\mathbf{x}, \mathbf{p}^*(\mathbf{x}))$ *is quasi-concave in* x_i *and it is maximized at* $x_i = (x_{i+1} + x_{i-1})/2$*.*

This Lemma is the same as Lemma 2 of Economides (1989). Without repeating the proof, we note the main points. We express $\Pi_i(\mathbf{x}, \mathbf{p}^*(\mathbf{x}))$ as a function of z, the relative deviation of franchisee j from the symmetric pattern. Defining $z = (x_i - x_{i-1})/d - 1$, we have

$$\Pi_i(\mathbf{x}, \mathbf{p}^*(\mathbf{x})) = \Phi(d, z) = (p_i^*(z) - c_k)^2/[d(1 - z^2)] - F_i - K - E,$$

where $p_i^*(z)$ is the solution of (4), which can be rewritten in terms of z as

$$\mathbf{A}(z) \cdot (\mathbf{p}^*(z) - \mathbf{c}) = \mathbf{g}(z).$$

The locational nature of competition implies that z enters in very few elements of the matrix $\mathbf{A}(z)$. The affected elements are positioned near element a_{jj}. We can utilize this by writing

$$\mathbf{A}(z) = \mathbf{U} + \mathbf{V}(z),$$

where $\mathbf{U}$ is independent of z and has a well known factorization $\mathbf{U} = \mathbf{M}\mathbf{M}'/s$, where s is a scalar. Then we can approximate $\mathbf{A}^{-1}(z)$ as

$$\mathbf{A}^{-1}(z) \simeq \mathbf{U}^{-1} - \mathbf{U}^{-1}\mathbf{V}(z)\mathbf{U}^{-1} + \mathbf{U}^{-1}\mathbf{V}(z)\mathbf{U}^{-1}\mathbf{V}(z)\mathbf{U}^{-1}.$$

We then write $\mathbf{p}^*(z) = \mathbf{c} + \mathbf{A}^{-1}(z) \cdot \mathbf{g}(z)$, and derive the quasi-concavity of $\Pi_i(\mathbf{x}, \mathbf{p}^*(\mathbf{x}))$ in z (and therefore in x_i) by direct computation.

From Lemma 1, we know that if all other franchisees have located symmetrically, then franchisee k also decides to locate at the symmetric location. Therefore the symmetric configuration, $x_{i+1} - x_i = d$, all i, is an equilibrium. This is summarized in the following Theorem.

Theorem 1: A locationally symmetric equilibrium exists in the fourth stage of the game.

3.3 Prices at the Location Equilibrium

We now calculate the equilibrium prices for the symmetric locations equilibrium that has been established in stage 4. For symmetric locations, the $mn \times mn$ matrix **A** of the system of the first order conditions (4) takes a simple form:

$$A = \left\| \begin{array}{cccccccc} 1 & -1/4 & 0 & & & & & -1/4 \\ -1/4 & 1 & -1/4 & & & & & \\ 0 & -1/4 & 1 & -1/4 & & & & \\ & & & & & & & \\ & & & & & & & \\ & & & & & -1/4 & 1 & -1/4 \\ -1/4 & & & & & 0 & -1/4 & 1 \end{array} \right\| .$$

We can now rewrite (4) as

$$\mathbf{A} \cdot \mathbf{p}^* = \mathbf{e} \qquad (7)$$

where $\mathbf{e} = \mathbf{A} \cdot \mathbf{c} + \mathbf{g}(\mathbf{x})$, so that $\mathbf{e} = (e_1, ..., e_n)$, $e_j = (e_{1,j}, ..., e_{m,j})$, $e_{i,j} = (d^2 + c_i)/2$.

The symmetric nature of competition at the symmetric locational equilibrium implies that each row of **A** is a translation to the right of the row above it. This means that matrix **A** is *circulant* and *symmetric*. Its inverse, **B**, is circulant and symmetric as depicted below.

$$\| b_1 \quad b_2 \quad b_3 \qquad\qquad\qquad b_{mn} \quad \|$$

$$\mathbf{B} = \left\| \begin{array}{cccccccc} b_2 & b_1 & b_2 & b_3 & & & & \\ b_3 & b_2 & b_1 & b_2 & b_3 & & & \\ & & & & & & & \\ & & & & & & & \\ & & & & & b_1 & b_2 & b_3 \\ & & & & & b_2 & b_1 & b_2 \\ b_{mn} & b_{mn-1} & & & & b_3 & b_2 & b_1 \end{array} \right\| .$$

Lemma 2: *For symmetric locations, the inverse of A is a circulant, symmetric matrix $B \equiv A^{-1}$. An element of an upper or lower diagonal at distance $k-1$ from the main diagonal is, $b_k = C_1(\rho_1)^k + C_2(\rho_2)^k$, $k = 1, ..., mn$, where $\rho_1 = 2 + \sqrt{3}$, $\rho_2 = 2 - \sqrt{3}$, $C_1 = 4(\rho_2 - \rho_2^{mn+1})/D > 0$, $C_2 = 4(\rho_1^{mn+1} - \rho_1)/D > 0$, $D = \rho_1^{mn+1} - \rho_1^{mn-1} + 2\rho_1 - 2\rho_2 - \rho_2^{mn+1} + \rho_2^{mn-1} > 0$. The sum of all elements of B across a row or column is $\Sigma_{k=1}^{mn} b_k = 2$. All elements of B are positive, $b_k > 0$, and all elements in the main diagonal are larger than one, $b_1 > 1$.*

Proof: See the Appendix.

It follows that a non-cooperative price equilibrium exists and prices are given by

$$\mathbf{p}^* = \mathbf{B} \cdot \mathbf{e}. \tag{8}$$

This formula makes transparent that the effects of location or marginal cost changes are going to be localized in nature, being significantly felt near the point of disturbance, and much less felt further away. **e** is linear in locations and marginal costs; to derive the equilibrium price, we weigh **e** by the coefficients of **B**; and these coefficients, although all positive, are larger than one only on the diagonal. Thus, the effects of changes in the location or marginal cost of franchisee i will be felt at most at product i and its neighborhood.

Expanding equation (8) for row $i + m(j-1)$ gives the equilibrium price $p_{i,j}$ of the jth franchisee in the ith franchise as,[15]

$$p^*_{i,j}(\mathbf{c}) = d^2 + [\Sigma^{m-1}_{k=0}\ c_{i+k}\ \Sigma^{n-1}_{\ell=0}\ b_{1+k+\ell m}]/2. \qquad (9)$$

Note that the equilibrium price for franchisee i,j is independent of the index j, so that all franchisees in the same franchise charge the same price, $p^*_{i,j}(c) = p^*_i(c)$.

Theorem 2: *In the last two stages of the game, there exists a subgame-perfect equilibrium where franchisees locate symmetrically and their equilibrium prices are given by (9). All downstream firms belonging to the same franchise charge the same price at equilibrium.*

Equilibrium prices are an increasing function of the distance between consecutive franchisees plus a positively weighted sum of the marginal transfer fees of all franchisors. Thus, the price of every downstream firm increases in the marginal fees of all franchisors. It is interesting to note that, although a franchisee competes with only the two closest competitors, changes in the strategies of other franchisees or franchisors will also affect this franchisee's equilibrium price.

Corollary 1: *The equilibrium price of any franchisee increases in the marginal transfer fee of _every_ franchisor, i.e.,* $dp^*_{i,j}/dc_k > 0$, *for all* k. *However, the price-to-cost-margin of a franchisee in the ith franchise decreases in the transfer fee* c_i, *i.e.,* $1 > dp^*_{i,j}/dc_i > 0$.

These results come directly from equation (9). The first result follows from the fact that all coefficients b_k are positive. For the second, note that $dp^*_{i,j}/dc_i = (\Sigma^{n-1}_{\ell=0}\ b_{1+\ell m})/2 < 1$, since

15 $p^*_{i,j}(\mathbf{c}) = \Sigma^{mn}_{k=1}\ b_k e_{\underset{m}{\mathrm{mod}}\ (i+j+k-1)} = d^2 + [\Sigma^{mn}_{k=1}\ b_k c_{\underset{m}{\mathrm{mod}}\ (i+j+k-1)}]/2 = d^2 + [\Sigma^{m-1}_{k=0}\ c_{i+k}\ \Sigma^{n-1}_{\ell=0}\ b_{1+k+\ell m}]/2.$

the sum of the elements of a column of **B** is less than 2. By increasing his marginal fee, a franchisor is able to reduce competition in the downstream market as signified by the resulting higher prices. This is an important influence of the franchisor in the final goods market. However, the effects of higher marginal fees *on profits* are not immediately clear. The determination of the effects on profits will come as the solution of the problem of the simultaneous non-cooperative choices of marginal fees by the franchisors to which we turn next.

4. The Franchisor's Choice of Franchising Contracts to Offer

It is to the interest of a franchisor to absorb all the profits of a franchisee by offering the appropriate franchising contract. Further, the upstream firm can influence and determine, to an extent, the degree of competition in the downstream market, as signified by $p^*(c)$, through its choice of c_i, the transfer per unit of sales that it collects from its franchisees. It is assumed that all franchisors choose simultaneously and non-cooperatively the contracts (c_i, F_i), $i = 1, ..., m$, that they offer. Of course, the ability of an upstream firm to determine the nature of downstream competition is limited by the choices of other upstream firms.

The ith upstream firm franchising to n downstream firms collects total fees $n(F_i + c_i D_i)$. Since it absorbs all the profits of each of its franchisees, from equation (1) it follows that the franchising fee collected by the ith franchisor from each of its franchisees is[16]

$$F_i = (p_i^* - c_i)D_i(p^*) - K - E. \quad (10)$$

Thus, the profits of franchisor i are

$$\Pi_i^u(c, F_i) = n(D_i(p^*)c_i + F_i) = n[D_i(p^*(c))p_i^*(c) - K - E], \quad (11)$$

[16] Realized profits for a downstream firm in the ith franchise are $\Pi_{ij}^{d*} = \Pi_i^{d*} = (p_i^* - c_i)D_i^*(p\ ) - F_i - K - A$.

by substitution from (10). Substituting the equilibrium prices from (4), profits at the equilibrium of the subgame are

$$\Pi^u_i(c) = n[(d^2 - c_i + (\Sigma^{m-1}_{k=0} c_{i+k} \Sigma^{n-1}_{\ell=0} b_{1+k+\ell m})/2)(d^2 + (\Sigma^{m-1}_{k=0} c_{i+k} \Sigma^{n-1}_{\ell=0} b_{1+k+\ell m})/2)/d - K - E].$$

The ith franchisor maximizes his profits by choosing c_i non-cooperatively while recognizing the effect of his choice on the equilibrium prices of all downstream firms. This results in a unique subgame-perfect equilibrium where all franchisors charge marginal transfer fee

$$c^* = d^2[\Sigma^{n-1}_{\ell=0} b_{1+\ell m} - 1]/[1 - (\Sigma^{n-1}_{\ell=0} b_{1+\ell m})/2] \equiv d^2\phi > 0, \tag{12}$$

where ϕ is defined as

$$\phi = [\Sigma^{n-1}_{\ell=0} b_{1+\ell m} - 1]/[1 - (\Sigma^{n-1}_{\ell=0} b_{1+\ell m})/2]. \tag{13}$$

Because all b_i are positive, they add to 2, and $b_1 > 1$, both the numerator and the denominator of (8) are positive, and therefore both ϕ and the marginal fee c^* are positive.

<u>Theorem 3</u>: *The game of (upstream) franchisors offering franchising contracts to downstream firms has a unique and symmetric subgame-perfect equilibrium where the marginal transfer fee c^* charged by an upstream firm is above its marginal cost. c^* is given by equation (12).*

<u>Proof</u>: See the Appendix.

The implied equilibrium prices in the subgame are (by substitution of (12) in (9)),

$$p^*_{i,j}(c^*) = p^*(c^*) = d^2 + c^* = d^2(1 + \phi).$$

The realized profits for an upstream firm are

$$\Pi^u_i(c^*) = n[p^*(c^*)D_{i,j}(p^*(c^*)) - K - E] = n[(1 + \phi)/(nm)^3 - K - E],$$

where the equilibrium prices have been substituted and the fact that the distance between outlets is the inverse of their number, $d = 1/(nm)$ (since they are all equispaced in a circumference of length 1), has been used.

Free entry in the franchisors market drives their profits to zero, and determines the total number of downstream outlets at the monopolistic competition equilibrium in the franchisors market as

$$N^* = (1 + \phi)^{1/3}/(K + E)^{1/3}, \tag{14}$$

where $N = nm$ is the total number of downstream outlets (franchisees).[17] The implied marginal transfer fee charged by franchisors to franchisees per unit of output is

$$c^*(N^*) = \phi(K + E)^{2/3}(1 + \phi)^{2/3}, \tag{15}$$

and the implied equilibrium price in the final goods market is

$$p^*(c^*(N^*)) = (1 + \phi)^{1/3}(K + E)^{2/3}. \tag{16}$$

Theorem 4: The free-entry equilibrium number of downstream outlets, N^*, their prices, p^*, and the marginal transfer rates, c^*, are given by equations (14)-(16).

5. Comparisons with Vertically Integrated Firms

It is instructive to compare the franchising equilibrium with the one of vertically integrated firms. Each vertically integrated firm has the same technology as the franchisees, facing an entry cost E. Its maximization problem can be thought of as a special case of the

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[17] The number N^* should be thought of only as an approximation to the true equilibrium number of firms which is the integer part of N^*, i.e., $I[N^*]$. Keeping in mind the truncations of decimals that may arise, the subsequent inequalities in the comparisons may only hold as weak inequalities.

problem of downstream firms in franchising with all $c_i = 0$, $F_i = 0$ and $K_i = 0$. It is immediate from equation (9) that the equilibrium price for every vertically integrated firm is

$$p^{**} = d^2,$$

and equilibrium profits for all firms are

$$\Pi_i = d^3 - E.$$

Given that $d = 1/n$ under free entry, there will be

$$n^{**} = 1/E^{1/3} \tag{17}$$

active firms, and their equilibrium price will be[18]

$$p^{**} = E^{2/3}. \tag{18}$$

We now compare the equilibrium of vertically integrated firms with the franchising one. Comparing (16) and (18), note that prices are always higher in the franchising regime,

$$p^*/p^{**} = (1 + \phi)^{1/3}(K + E)^{2/3}/E^{2/3} \geq (1 + \phi)^{1/3} > 1.$$

This is the result of two factors. First, prices are higher in the franchising regime because of the strategic setting of marginal transfer fees. Second, prices may be even higher because the transaction cost K of writing a contract between an upstream and a downstream firm restricts to some extent the number of outlets in the franchising regime.

<u>*Corollary 2*</u>*: Prices of final goods are higher in the franchising regime than in the regime of vertically integrated firms.*

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[18] These results on independent firms have been established in Economides (1989).

Comparing the number of *outlets* in equations (14) and (17), note that the franchising regime could have more or fewer outlets than the one of vertically integrated firms, depending on the cost of writing the franchising contract, K, since

$$N^* > n^{**} \Leftrightarrow \phi > K/E.$$

When the cost of contracting, K, is zero, there is a larger number of outlets in the franchising regime. With K = 0, for the same number of outlets, prices and profits are higher in the franchising regime. It follows that under free entry the number of outlets will be larger in the franchising regime. However, when the contracting costs are large compared to the fixed production cost, the franchising regime has fewer outlets than the regime of vertically integrated firms. Typically we expect that the contracting costs are small and hence there are more outlets in the franchising regime.

Corollary 3: The number of outlets is larger (smaller) in the franchising regime than in the regime of vertically integrated firms when the cost of contracting between franchisor and franchisee is small (large).

6. Optimality

One criterion for the selection of regimes (market structures) is their contribution to social surplus. Next, we compute the surplus under franchising, and we compare it at equilibrium with the surplus at the free-entry equilibrium in the regime of vertically integrated firms. The optimal and equilibrium numbers of outlets in each regime are also compared.

The surplus for one firm in the regime of vertically integrated firms is

$$s(d) = 2[Rd/2 - \int_0^{d/2} z^2 \, dz] - E = Rd - d^3/12 - E.$$

Social surplus in a market of $n = 1/d$ firms is

$$S(n) = n\, s(1/n) = R - 1/(12n^2) - nE. \qquad (19)$$

Social surplus is maximized at

$$n^o = 1/(6E)^{1/3}.$$

Social surplus in the franchising regime is[19]

$$S_f(N) = S(N) - KN = R - 1/(12N^2) - N(E + K), \qquad (20)$$

Social surplus under franchising is maximized at

$$N^o = 1/[6(E + K)]^{1/3}.$$

In the regime of independent firms, it is known that the number of varieties at the free-entry equilibrium far exceeds the surplus maximizing number, i.e., $n^o < n^{**}$.[20] In the franchising regime, this is true too, and the divergence between the optimal and the equilibrium number of varieties is accentuated,

$$N^*/N^o = 6^{1/3}(1 + \phi)^{1/3} > 6^{1/3} = n^{**}/n^o > 1.$$

Next the realized social surpluses at the two equilibria are compared. Substitution of (14) in (20) and of (17) in (19) yields

$$S_f(N^*) = R - (K + E)^{2/3}[1 + 12(1 + \phi)]/[12(1 + \phi)^{2/3}],$$

$$S(n^{**}) = R - 13E^{2/3}/12.$$

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[19] Note that for each downstream firm we have subtracted from total surplus the extra cost K of writing the franchising contract.

[20] See Salop (1979) and Economides (1989).

Direct comparison of these two expressions reveals that for all $K \geq 0$,[21]

$$S(n^{**}) > S_f(N^*).$$

Theorem 5: Irrespective of the size of franchising contracting costs, social surplus is lower in the franchising regime than in the regime of vertically integrated firms.

To understand these results, first consider the case of zero contracting costs. Then higher profits in the franchising regime for any fixed number of firms implies a larger number of active firms at the free entry equilibrium under franchising than is optimal. This in turn implies a larger distortion compared to optimality in the number of available varieties at the free entry of the franchising regime than in the regime of vertically integrated firms. It follows that social surplus is lower under franchising. For the case of positive contracting costs, observe that social surplus decreases with the cost of contracting. Therefore, the comparison of surpluses is even less favorable to the franchising regime when contracting costs are positive.

7. Extensions

7.1 General Demand Functions

The result that a positive marginal transfer fee will be charged by the upstream firm (Theorem 3) is not limited to the specification described above. Consider a general formulation where m franchisors have n franchisees each. Each franchisee of type i has profit function

$$\Pi_i^d = (p_i - c_i)D_i(p) - V_i(D_i(p)) - F_i - K - E,$$

[21] $S_f(N^*) < S(n^{**}) \Leftrightarrow (1 + K/A)^{2/3}[1 + 12(1 + \phi)]/[13(1 + \phi)^{2/3}] > 1$. Let $f(\phi) = [1 + 12(1 + \phi)]/[13(1 + \phi)^{2/3}]$. Note that $f(0) = 1$, and $f'(\phi) = (10 + 12\phi)/[39(1 + \phi)^{5/3}] > 0$. Thus $f(\phi) > 1$ for all $\phi > 0$, including the positive ϕ defined in equation (13). The inequality is further enhanced when $f(\phi)$ is multiplied by $(1 + K/A)^{2/3}$ since $K \geq 0$.

where c_iD_i and F_i are the marginal and lump-sum transfers to the upstream firm and $V_i(D_i)$ represents other variable costs of the franchisee. The profit function of upstream firm i is

$$\Pi_i^u = n[(p_i - c_i)D_i + F_i] = n[p_iD_i(p) - V_i(D_i(p)) - K - E].$$

Let the non-cooperative equilibrium prices of a downstream firm of type i be $p_i^*(\mathbf{c})$, where $\mathbf{c} = (c_1, ..., c_m)$. Then the effect on profits of upstream firm i of an increase in the marginal transfer fee charged by this firm to its franchisors is

$$(\partial\Pi_i^u/\partial c_i)/n = D_i dp_i^*/dc_i + (p_i - dV_i/dD_i)[(\partial D_i/\partial p_i)(dp_i^*/dc_i) + \Sigma_{i\neq j} (\partial D_i/\partial p_j)(dp_j^*/dc_i)]. \qquad (21)$$

Utilizing the first order condition for maximization with respect to price by downstream firm i,

$$(p_i - dV_i/dD_i)(\partial D_i/\partial p_i) + D_i = c_i(\partial D_i/\partial p_i),$$

(21) can be re-written as

$$(\partial\Pi_i^u/\partial c_i)/n = c_i(dp_i^*/dc_i)(\partial D_i/\partial p_i) + (p_i - dV_i/dD_i)\Sigma_{i\neq j} (\partial D_i/\partial p_j)(dp_j^*/dc_i).$$

Evaluating $\partial\Pi_i^u/\partial c_i$ at $c_i = 0$, we have

$$\partial\Pi_i^u/\partial c_i = n(p_i - dV_i/dD_i)\Sigma_{i\neq j} (\partial D_i/\partial p_j)(dp_j^*/dc_i). \qquad (22)$$

In this expression, coefficient $(p_i - dV_i/dD_i)$ is positive because price exceeds marginal cost. Inside the summation, the first term $(\partial D_i/\partial p_j)$ is positive, since an increase in the price of an opponent increases own demand. Therefore $\partial\Pi_i^u/\partial c_i > 0$ at $c_i = 0$ if $dp_j^*/dc_i > 0$ for $i \neq j$, i.e., if an increase in the marginal transfer fee of firm i has a positive effect on prices of other firms. This is the necessary and sufficient condition for the appearance of positive marginal transfer fees in the franchising regime.[22]

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[22] In the differentiated products model of the earlier sections, it is clear from equation (9) that every price increases in all c_i.

Theorem 6: Upstream firms (franchisors) will choose positive marginal transfer fees provided that these have a positive effect on the prices of franchisees of rival franchisors.

The incentive to vertically disintegrate can be seen as a special case of the above analysis. Fix n to equal one. In the regime of vertical disintegration, there are m upstream firms with one subsidiary each. In the regime of integration, there are m independent firms. As noted before, competition in this regime can be thought of as a special case of competition in the regime of vertical disintegration with all transfer costs zero, and therefore $c_i = 0$ for all i. Inspecting equation (21), we see that, provided that $dp_j^*/dc_i > 0$, an upstream firm has an incentive to charge positive c_i irrespective of the choices of competitors. Therefore, provided that $dp_j^*/dc_i > 0$, each firm finds it desirable to charge a positive c_i and thus vertically disintegrate.

Theorem 7: Firms have incentives to vertically disintegrate and use a positive marginal transfer fee, provided that marginal transfer fees have a positive effect on prices of rivals.

Theorem 7 provides new motive for franchising over and above the traditional reasons. Note, however, that the general results, as well as the particular ones of previous sections, depend crucially on *the existence of downstream competitors* and do not hold for a downstream monopolist. If a franchisee is a monopolist, it is best for his franchisor to have no markup on variable inputs, so that the franchisee achieves the monopoly price. When there are no downstream competitors, the right hand side of (21) is zero, and therefore the choice of $c_i = 0$ maximizes upstream profits. Any positive marginal fee would result in the franchisee choosing a price above the zero-marginal-fee monopoly price, which is not optimal from the point of view of the franchisor.[23]

7.2 Non-Linear Marginal Fee Contracts

The analysis so far has been restricted to franchising contracts (c_i, F_i) where the total transfer to the upstream firm was linear in the quantity sold, $c_iD_i + F_i$. This linear contract can be thought of as a special case of a contract that absorbs $C_i(D_i) + F_i$ from the franchisee, where $C_i(D_i)$ is a general function of sales. Consider the maximization problem of the downstream firm given the contract $(C_i(D_i), F_i)$. It chooses p_i to maximize

$$\Pi_i^d = p_iD_i - C_i(D_i) - V_i(D_i) - F_i - K - E.$$

Its first order condition is

$$D_i + (p_i - dC_i'(D_i) - V_i'(D_i))(\partial D_i/\partial p_i) = 0,$$

where only the derivative of $C_i(D_i)$ appears. Thus, specification of the appropriate derivative of $C_i(D_i)$, $c_i \equiv C_i'(D_i)$, as the marginal transfer fee by the upstream firm in the linear contract will have the same effect on prices as specifying the function $C_i(D_i)$ in the general case. Furthermore, the upstream firm can realize the whole profits of a downstream firm (through appropriate choice of F_i) irrespective of the functional form of $C_i(D_i)$. Therefore it is sufficient to use the linear form of the transfer contract (c_i, F_i) as was done above.

Theorem 7: Use of general (non-linear) marginal transfer contracts by upstream firms results in the same equilibrium as when linear marginal transfer contracts are used.

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[23] The fact that a monopolist will use a zero marginal fee has been derived as Proposition 1 by Blair and Kaserman (1982) for the case of a linear demand. They also show that under uncertainty concerning future levels of demand (or if the discount rate of the franchisee is higher than the discount rate of the franchisor), the franchisor will levy a positive marginal fee to a downstream monopolist.

8. Concluding Remarks

In the context of a model of locationally differentiated products, it has been shown that franchisors have incentives to overcharge franchisees for variable inputs, as long as the latter choose prices non-cooperatively in an oligopolistic setting. Franchisors choose to charge more for variable inputs so that they can benefit from the resulting higher equilibrium prices in the downstream market. The franchising arrangement results in higher prices and profits for the franchisors in the short run. In a long run free-entry zero-profits monopolistic competition equilibrium for franchisors, there are more numerous outlets in the franchising regime than in a regime of vertically integrated firms. This implies a larger divergence between optimal and equilibrium product diversity in the franchising regime than in a regime of vertically integrated firms. Further, the realized social surplus at equilibrium is lower in the franchising regime than in a regime of vertically integrated firms. Thus, we have another motive for franchising; but, unlike the information-related motive, this one pushes the market in a non-optimal direction.

Appendix

Proof of Lemma 2:

The circulant and symmetric nature of **B** is immediate. It is sufficient to establish a row of **B**. Consider

$$b_k = C_1(\rho_1)^k + C_2(\rho_2)^k,$$

where $\rho_1 = 2 + \sqrt{3} > 1$, $0 < \rho_2 = 2 - \sqrt{3} < 1$, $\rho_1\rho_2 = 1$, are the solutions of

$$-1/4 + x - x^2/4 = 0.$$

Then the product of the first row of **B** with any column of **A**, except for the first and the last, yields immediately zero. To establish that $\mathbf{B}\cdot\mathbf{A} = \mathbf{I}$ it is sufficient to have

$$b_1 - b_2/4 - b_{mn}/4 = 1$$

and

$$b_{mn} - b_{mn-1}/4 - b_1/4 = 0.$$

Clearly $b_1 - b_2/4 = b_0/4$ and $b_{mn} - b_{mn-1}/4 = b_{mn-1}/4$. Therefore it is sufficient to have $b_0 - b_{mn} = 4$ and $b_{mn+1} = b_1$, i.e.,

$$C_1(1 - \rho_1^{mn}) + C_2(1 - \rho_2^{mn}) = 4,$$
$$C_1(\rho_1^{mn+1} - \rho_1) + C_2(\rho_2^{mn+1} - \rho_2) = 0,$$

the solution of which yields

$$C_1 = 4(\rho_2 - \rho_2^{mn+1})/D > 0,\ C_2 = 4(\rho_1^{mn+1} - \rho_1)/D > 0,$$
$$D = \rho_1^{mn+1} - \rho_1^{mn-1} + 2\rho_1 - 2\rho_2 - \rho_2^{mn+1} + \rho_2^{mn-1} > 0.$$

Since $C_1, C_2 > 0$, it follows that $b_k > 0$, for all k. Then, $b_1 = 1 + (b_2 + b_{mn})/4 > 1$. Q.E.D.

Proof of Theorem 3:

$$(d/n)\partial\Pi_i^u/\partial c_i = [-1 + (\Sigma_{\ell=0}^{n-1} b_{1+\ell m})/2][d^2 + (\Sigma_{k=0}^{m-1} c_{i+k} \Sigma_{\ell=0}^{n-1} b_{1+k+\ell m})/2]$$
$$+ [d^2 - c_i + (\Sigma_{k=0}^{m-1} c_{i+k} \Sigma_{\ell=0}^{n-1} b_{1+k+\ell m})/2][\Sigma_{\ell=0}^{n-1} b_{1+\ell m}]/2 = 0.$$

Evaluating $\partial\Pi_i^u/\partial c_i$ at $c_i = c^*$ and using $\Sigma_{k=1}^{mn} b_k = 2$, it follows that

$$[-1 + (\Sigma_{\ell=0}^{n-1} b_{1+\ell m})/2][d^2 + c^*] + d^2(\Sigma_{\ell=0}^{n-1} b_{1+\ell m})/2 = 0,$$

or equivalently,

$$c^* = d^2[-1 + \Sigma_{\ell=0}^{n-1} b_{1+\ell m}]/[1 - (\Sigma_{\ell=0}^{n-1} b_{1+\ell m})/2].$$

Next it is shown that this equilibrium is unique. For uniqueness, it is sufficient to show that

$$\partial^2\Pi_i^u/\partial c_i^2 + \Sigma_{s\neq i} \partial^2\Pi_i^u/\partial c_s^2 < 0,$$

so that the best reply mapping is a contraction. Now,

$$(d/n)\partial^2\Pi_i^u/\partial c_i^2 = 2[-1 + (\Sigma_{\ell=0}^{n-1} b_{1+\ell m})/2][\Sigma_{\ell=0}^{n-1} b_{1+\ell m}]/2 < 0,$$

so that second order conditions for maximization are satisfied. Further,

$$(d/n)\partial^2\Pi_i^u/\partial c_{i+t}^2 = [-1 + \Sigma_{\ell=0}^{n-1} b_{1+\ell m}][\Sigma_{\ell=0}^{n-1} b_{1+t+\ell m}]/2,$$

and therefore,

$$(d/n)\Sigma_{s\neq i}\partial^2\Pi_i^u/\partial c_s{}^2 = [-1 + \Sigma_{\ell=0}^{n-1} b_{1+\ell m}][\Sigma_{\ell=0}^{n-1} b_{2+\ell m} + b_{3+\ell m} + ... + b_{m+\ell m}]/2 =$$
$$= [-1 + \Sigma_{\ell=0}^{n-1} b_{1+\ell m}][2 - \Sigma_{\ell=0}^{n-1} b_{1+\ell m}]/2.$$

Thus,

$$\partial^2\Pi_i^u/\partial c_i^2 + \Sigma_{s\neq i} \partial^2\Pi_i^u/\partial c_s^2 = n[-2 + \Sigma_{\ell=0}^{n-1} b_{1+\ell m}]/(2d) < 0$$

because all b_i are positive and they add to 2. Q.E.D.

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