Optimal Exercise of Executive Stock Options and Implications for Valuation

Jennifer N. Carpenter — New York University
Richard Stanton — U.C. Berkeley
Nancy Wallace — U.C. Berkeley

December 7, 2005[1]

[1]Preliminary and incomplete. Please do not cite without permission. We would like to thank Peter Carr and Peter Lakner for helpful comments and suggestions. We also thank Rik Sen for research assistance.

Optimal Exercise of Executive Stock Options and Implications for Valuation

Abstract

The cost of executive stock options to shareholders has become a focus of attention in finance and accounting. The difficulty is that the value of these options depends on the exercise policies of the executives. Because these options are nontransferable, the usual theory does not apply. We analyze the optimal exercise policy for a utility-maximizing executive and indicate when the policy is characterized by a critical stock price boundary. We provide a counterexample in which the executive exercises at low and high stock prices but not in between. We show how the policy varies with risk aversion, wealth, and volatility and explore implications for option value. For example, option value can decline as volatility rises.

With the explosive growth of executive stock options in corporate compensation, the cost of these options to shareholders has become a focus of attention in finance and accounting. Recent regulation requiring firms to recognize option expense after 2005 has intensified the demand for suitable valuation methods. The difficulty is that the value of these options depends crucially on the exercise policies of the option holders, but because these options are nontransferable, the usual theory does not apply.

In the case of an ordinary call, the holder can sell the option at any time, so his goal is presumably to maximize the option's present value. The value-maximizing exercise policy in a Black-Scholes world has been researched extensively (see Merton (1973), Van Moerbeke (1976), Roll (1977), Geske (1979), Whaley (1981), Kim (1990)). It calls for exercising the option once the stock price rises above a critical level. This critical level is increasing in the riskless rate, the stock return volatility, and the time remaining to maturity, and it is decreasing in the dividend rate, with no early exercise if the dividend rate is zero.

By contrast, the holder of an executive stock option must bear the risk of the option payoff, so simply maximizing the option's present value is generally not optimal. Indeed, evidence indicates that executives systematically exercise options on non-dividend paying stocks well before expiration. The executive presumably chooses an option exercise policy as part of a greater utility maximization problem that includes other decisions, such as portfolio and consumption choice and managerial strategy.

This paper studies the optimal exercise policy for an executive stock option under simple but appealing assumptions about the executive's choice set. We address the questions of when the policy can be described by a critical stock price boundary, as in the case of an ordinary option, and how this boundary varies with executive risk aversion, wealth and stock price volatility. We then explore the implications for option value.

The intuition that the need for diversification can lead an executive to sacrifice some option value by exercising it early is well understood in the literature, but explicit theory of the optimal exercise of ESOs is still developing. Huddart (1994), Marcus and Kulatilaka (1994), and Carpenter (1998) build binomial models of the utility-maximizing exercise decision with exogenous assumptions about how non-option wealth is invested. Detemple and Sundaresan (1999) extend these to allow for simultaneous option exercise and portfolio choice decisions. These papers establish the economic approach to ESO valuation, focusing on the optimality of early exercise and the fact that this makes ESOs worth less than their Black-Scholes value rather than an in-depth analysis of the exercise policy itself. In a continuous-time framework, Ingersoll (2006) develops a subjective option valuation methodology assuming the option is a marginal component of the executive's portfolio. Kadam, Lakner, and Srinivasan (2003) and Henderson (2004) model the optimal exercise policy for an infinite horizon option, but their models link the manager's consumption date to the option exercise date, which can distort the exercise decision, even in the absence of trading restrictions.

A number of papers model option value using exogenous specifications of the exercise policy. Jennergren and Naslund (1993), Carr and Linetsky (2000), and Cvitanic, Wiener, and Zapatero (2004) derive analytic formulas for option value assuming exogenously specified exercise boundaries and forfeiture rates. Hull and White (2004) propose a binomial model in which exercise occurs when the stock price reaches an exogenously specified multiple of the stock price and forfeiture occurs at an exogenous rate. Rubinstein (1994) and Cuny and Jorion (1995) also compute option value under exogenous assumptions about the timing of exercise.

Other authors have focused on the executive's private valuation of the option using certainty equivalents rather than on the market value of the option from the viewpoint of shareholders. These include Lambert, Larcker, and Verrechia (1991), Hall and Murphy (2002), Cai and Vijh (2004), and Miao and Wang (2005).

1 General formulation of the executive's problem

In the general version of the problem we consider, the executive has n finite-lived options with strike price K and expiration date T and additional wealth that can be invested subject to a prohibition on short sales of the stock. The investment set includes riskless bonds with constant riskless rate r, the underlying stock with price S_t, and a market portfolio with price M_t. These prices satisfy

$$\frac{dS_t}{S_t} = (\lambda - \delta)\, dt + \sigma\, dB_t \ , \tag{1}$$

$$\frac{dM_t}{M_t} = \mu\, dt + \sigma_m\, dB_t \ , \tag{2}$$

where B_t is a standard two-dimensional Brownian motion on a probability space equipped with the natural filtration and σ and σ_m are two-dimensional row vectors. The stock return volatility, σ, the stock dividend rate δ, and the mean and volatility of the market return, μ and σ_m are constant, and the mean stock return λ is equal to the normal return for the stock given its correlation with the market,

$$\lambda = r + \frac{\sigma\sigma_m'}{||\sigma_m||^2}(\mu - r) \ . \tag{3}$$

In particular, in the absence of the option, an optimal portfolio would contain no stock position beyond what is implicitly included in the market portfolio.

The executive simultaneously chooses an option exercise time τ, which is a stopping time of the filtration generated by the Brownian motion, and an investment strategy in the market and the stock, $\pi_t \equiv (\pi_t^m, \pi_t^s)$ satisfying $\mathrm{E} \int_{t=0}^{T} ||\pi_t||^2\, dt < \infty$. His goal is to maximize the expected utility of time T wealth:

$$\max_{\{\tau \le T, \pi^m, \pi^s \ge 0\}} \mathrm{E}\{V(W_\tau + n(S_\tau - K)^+, \tau)\} \tag{4}$$

subject to

$$dW_t = rW_t\,dt + \pi_t^m(\mu\,dt + \sigma_m\,dB_t) + \pi_t^s(\lambda\,dt + \sigma\,dB_t)\ , \tag{5}$$

where

$$V(W_t, t) \equiv \max_{\pi^m} \mathrm{E}_t\{U(W_T)\} \text{ s.t. } dW_u = rW_u\,du + \pi_u^m(\mu\,du + \sigma_m\,dB_u)\ , \tag{6}$$

and the utility function U is strictly increasing, strictly concave, and twice continuously differentiable.

This formulation entails a number of simplifications. The executive's portfolio does not include a position in restricted shares of stock (see Kaul, Liu, and Longstaff (2003) for a model of portfolio choice with restricted stock). It allows only for a single block exercise of the option, although the executive would probably prefer to exercise the options at a stochastic rate over time. The model also considers only a single grant of options when in practice, executives are granted new ten-year options every year and typically build up large inventories of options with different strikes and expiration dates. It would be useful to understand which options are most attractive to exercise first and how the anticipation of future grants of options and other forms of compensation affects current exercise decisions. In addition, the model does not account for any control the executive has over the underlying stock price process through the exertion of effort and through project and leverage choices; these choices may interact with the exercise decision. Despite these simplifications, we believe this formulation captures the essence of the executive stock option problem.

Intuition suggests that the optimal outside position in the stock in problem (4) is $\pi^s \equiv 0$, however this remains to be proved. The example in Evans, Henderson, and Hobson (2005) shows that results from traditional portfolio theory may fail to hold in the presence of an optimal stopping problem.

If the optimal investment policy π_t and the indirect utility function V satisfy, respectively, linear and polynomial growth conditions in W and S, then Theorem 3.1.8 of Krylov (1980) implies that the value function for the executive's problem,

$$f(W_t, S_t, t) \equiv \max_{\{t \le \tau \le T, \pi^m, \pi^s \ge 0\}} \mathrm{E}_t\{V(W_\tau + n(S_\tau - K)^+, \tau)\} \tag{7}$$

subject to

$$dW_u = rW_u\,dt + \pi_u^m(\mu\,du + \sigma_m\,dB_u) + \pi_u^s(\lambda\,du + \sigma\,dB_u)\ , \tag{8}$$

is continuous and satisfies $f(W_t, S_t, t) \ge V(W_\tau + n(S_\tau - K)^+, \tau)$ and $f(W_T, S_T, T) = U(W_T + n(S_T - K)^+)$.

2 Special case with outside wealth in riskless bonds

We start by analyzing the case in which the outside wealth is invested in riskless bonds and the stock appreciates at the riskless rate, as if the market return were riskless:

$$\frac{dS_t}{S_t} = (r - \delta)\,dt + \sigma\,dB_t\ . \tag{9}$$

We assume for now that the optimal investment policy entails no long position in the stock. The executive's problem at each time $t < T$ becomes

$$f(S_t, t) \equiv \max_{\{t \le \tau \le T\}} \mathrm{E}_t\{U(n(S_\tau - K)^+ e^{r(T-\tau)} + W)\}\ , \tag{10}$$

where the constant W is outside wealth at time T with $W > nKe^{rT}$ and $f : (0,\infty) \times [0,T] \to \mathcal{R}$ is a continuous function satisfying $f(S_t, t) \ge U(n(S_t - K)^+ + W)$ and $f(S_T, T) = U(n(S_T - K)^+ + W)$.

Note that

$$\begin{aligned}
\mathrm{E}[\sup_{0 \le t \le T} U(n(S_t - K)^+ e^{r(T-t)} + W)] &= \mathrm{E}[U(\max_{0 \le t \le T}(n(S_t - K)^+ e^{r(T-t)} + W))] && (11)\\
&\le U(\mathrm{E}[\max_{0 \le t \le T}(n(S_t - K)^+ e^{r(T-t)} + W)]) && (12)\\
&< \infty\ , && (13)
\end{aligned}$$

so Theorem D.12 of Karatzas and Shreve (1998) implies that an optimal exercise time is

$$\tau^* \equiv \inf\{t \in [0,T] : f(S_t, t) = U(n(S_t - K)^+ e^{r(T-t)} + W)\}\ . \tag{14}$$

The continuation region for the problem is

$$D \equiv \{(s,t) \in (0,\infty) \times [0,T] : f(s,t) > U(n(s - K)^+ e^{r(T-t)} + W)\}\ . \tag{15}$$

2.1 Existence of a critical stock price boundary

The first step in characterizing the exercise policy is to determine whether a single critical stock price boundary $\bar{s}(t)$ separates the continuation region below from the exercise region above, as is the case for ordinary American calls. This is often assumed to be true in executive stock option models with exogenously specified exercise policies, however, it remains to be proved that the utility-maximizing policy has this structure.

To formalize intuition about the various effects of waiting to exercise, let $g(s,t) \equiv U(n(s-K)^+ e^{r(T-t)} + W)$ denote the payoff function for the optimal stopping problem and note that on $(K,\infty) \times [0,T]$, g is $C^{2,1}$ and Itô's lemma implies that g has drift equal to $H(S_t, t)$ where

$$H(s,t) \equiv U'(h(s,t))(rK - \delta s)ne^{r(T-t)} + \frac{1}{2}U''(h(s,t))n^2 e^{2r(T-t)}\sigma^2 s^2 \tag{16}$$

and $h(s,t) \equiv n(s-K)e^{r(T-t)} + W$ is total time T wealth given exercise at time t and stock price s. This expression shows that when the option is in the money, the effects of waiting to exercise include the benefits of delaying payment of the strike price, the cost of losing dividends, and the cost of bearing stock price risk.

Proposition 2.1 *Suppose that H is nonincreasing in the stock price s. Then for each time $t \in [0,T)$, if there is any stock price at which exercise is optimal, then there exists a critical stock price $\bar{s}(t)$ such that it is optimal to exercise the option if and only if $S_t \geq \bar{s}(t)$.*

Proof Fix $t \in [0,T)$. Suppose (s_1,t) is a continuation point. We show that if $s_2 < s_1$ then (s_2,t) is also a continuation point. First note that it must be optimal to continue holding the option if $S_t \leq K$. Stopping then would guarantee a reward of $U(W)$, which is less than the expected utility of continuing, for example, until the first time the stock price rises to $K+c$, for some $c > 0$, or until expiration T.

So assume $s_1 > s_2 > K$. For $u \geq t$, let $S_u^{(i)}$ denote the stock price process starting from s_i at time t and note that $S_u^{(1)} > S_u^{(2)}$. Finally, let τ be the optimal stopping time given $S_t = s_1$. Since τ is a feasible strategy if $S_t = s_2$,

$$\begin{aligned} f(s_2,t) - f(s_1,t) &\geq \mathrm{E}_t\{U(n(S_\tau^{(2)}-K)^+e^{r(T-\tau)}+W) - U(n(S_\tau^{(1)}-K)^+e^{r(T-\tau)}+W)\} \\ &\geq \mathrm{E}_t\{U(n(S_\tau^{(2)}-K)e^{r(T-\tau)}+W) - U(n(S_\tau^{(1)}-K)e^{r(T-\tau)}+W)\} \\ &= g(s_2,t) - g(s_1,t) + \mathrm{E}_t\int_t^\tau (H(S_u^{(2)},u) - H(S_u^{(1)},u))du \\ &\geq g(s_2,t) - g(s_1,t) \ . \end{aligned} \tag{17}$$

Therefore, $f(s_2,t) - g(s_2,t) \geq f(s_1,t) - g(s_1,t) > 0$. $\square$

Remark The hypothesis is satisfied for constant relative risk averse utility functions with relative risk aversion less than or equal to one. Similarly, in the value maximization problem for an ordinary option, the second order term in H does not appear, the drift is nonincreasing in the stock price, and it follows that it is optimal to exercise if and only if the stock price has risen above a critical level. For executive stock options however, the risk aversion of the option holder gives rise to the second order term, and the drift need no longer be monotonic in the stock price.

Example with a split continuation region Figure 1 shows the optimal exercise policy for utility function

$$U(W) = \frac{W^{1-A}}{1-A} + cW \tag{18}$$

with $A = 10, c = 0.0001, K = 1, T = 10, r = 0.05, \sigma = 30\%$, and $\delta = 0$. The utility function is strictly increasing and strictly concave. As the figure shows, the executive continues for low and high stock prices, but exercises the option for intermediate stock prices. It is not clear, however, how much valuation error would be created by erroneously assuming the existence of a single critical exercise boundary. That would depend on how that single boundary was determined. In this example, if we ignore the presence of the upper boundary, the option value is 0.408 instead of the correct value of 0.432.

2.2 Dependence of the continuation region on the parameters

Understanding how executive stock option value varies with stock return volatility, executive wealth, and other parameters requires an understanding of how these parameters affect the exercise policy. With an ordinary American call option, the exercise boundary, or on other words, the set of stock prices at which the option holder would continue at a given point in time, is increasing with the stock volatility and the time to expiration and decreasing with the dividend rate. With executive stock options, the dependence of the continuation region on the parameters is less clear cut. This section describes how the continuation region changes with volatility, executive wealth and risk aversion, the stock dividend rate, and the time to expiration. Most of the results are drawn from numerical examples. All of the numerical examples use constant relative risk averse utility and a zero dividend rate. In all cases, even those in which the coefficient of relative risk aversion, A, is greater than one, the continuation region is characterized by a single critical stock price boundary.

2.2.1 Non-monotonicity with respect to the stock return volatility

A basic result in standard option pricing theory is that option value is increasing in volatility. This is also typically the case in executive stock option models with an exogenously specified exercise boundary that does not change with volatility (see for example Cvitanic, Wiener, and Zapatero, 2004). However, the utility-maximizing continuation region can shrink considerably with volatility and this can lead to option value declining in volatility.

Figure 2 plots exercise boundaries and option for various levels of stock return volatility. As volatility rises from 10% to 200%, the exercise boundary tends to fall first and then rise slightly. This is shown most clearly in Figure 2a, with risk aversion coefficient $A = 0.5$. The risk averse utility of the option payoff as a function of the stock price has both a convex region and a concave region, so in principle, an increase in volatility could either lead the executive to continue longer or exercise sooner. Apparently the concave portion dominates at low levels of volatility, making the executive exercise sooner as volatility rises. At higher levels of volatility, the convex portion seems to dominate and the boundary rises slightly. Empirically, Bettis, Bizjak and Lemmon (2005) find that options are exercised earlier at higher volatility firms.

At the lower levels of risk aversion shown in Figures 2a and 2b, executive stock option value, labeled "ESO" in the figures, is generally increasing in volatility, though not as fast as the Black-Scholes value, labeled "Max." However, at the higher levels of risk aversion shown in Figures 2c and 2d, executive stock option value is decreasing in volatility at low levels of volatility. Here the negative effect on value of the drop in the boundary of offsets the positive effect of extreme stock prices becoming more likely. In a model of real option values with the underlying following an arithmetic Brownian motion, Miao and Wang (2005) also find that the exercise boundary can fall with volatility, as can the certainty equivalent value of the option.

2.2.2 Risk aversion

Figure 3 shows how the exercise boundary and option value vary with executive risk aversion. In all of the examples, the dividend rate is set to zero so that the only motive for early exercise is the ability to transfer the option value to a more efficient portfolio, in this case, the riskless asset. Since this diversification motive would seem to be stronger with greater risk aversion, intuition would suggest that the exercise boundary, and thus option value, should decline with risk aversion. This is intuition is borne out in Figure 3, which examines the effect for four different parameterizations of wealth and volatility.

2.2.3 Wealth

With constant relative risk aversion, the executive's perception of riskiness of the option should be greater the greater portion of his total wealth the option represents. Thus, with more non-option wealth, the executive should have less incentive to reduce risk by exercising the option early. Figure 4 illustrates the effects of increasing wealth on the exercise boundary and option value. In all combinations of risk aversion and volatility, the exercise boundary and option value are increasing in non-option wealth.

2.2.4 Monotonicity with respect to the dividend rate

This section shows analytically that the executive's continuation region is larger the smaller the dividend rate on the stock, as is the case for an ordinary American option. This result holds regardless of the actual shape of the continuation region.

Proposition 2.2 *If a given state (s, t) is in the continuation when the dividend rate is δ_1, then it is also in the continuation region when the dividend rate is δ_2 for any $\delta_2 < \delta_1$.*

Proof Let $f(s, t; \delta)$ denote the value function and $S_t^{(\delta)}$ denote the stock price process when the dividend rate is δ. Let τ be the optimal stopping time for the problem with δ_1.

Then, since τ is a feasible choice for the problem with δ_2 and $S_\tau^{(\delta_2)}/S_t^{(\delta_2)} > S_\tau^{(\delta_1)}/S_2^{(\delta_1)}$,

$$\begin{aligned} f(s,t;\delta_2) - f(s,t;\delta_1) &\geq \mathrm{E}\{U(n(S_\tau^{(\delta_2)} - K)^+ e^{r(T-\tau)} + W) - \\ &\quad U(n(S_\tau^{(\delta_1)} - K)^+ e^{r(T_1-\tau)} + W) | S_t^{(\delta_1)} = S_t^{(\delta_2)} = s\} \\ &\geq 0 \end{aligned} \tag{19}$$

Therefore, $f(s,t;\delta_2) \geq f(s,t;\delta_1) > g(s,t)$ so (s,t) is in the continuation region for δ_2.

2.3 Boundedness of the continuation region

This section gives a conjecture about the boundedness of the continuation region. As a step toward this, the lemma below describes the evolution of g over the whole state space.

Lemma 2.1 *For $t \in [0,T)$ and τ a stopping time with $t \leq \tau \leq T$,*

$$\begin{aligned} g(S_\tau,\tau) - g(S_t,t) &= \int_t^\tau 1_{\{S_u>K\}} H(S_u,u)du + \\ &\quad \int_t^\tau 1_{\{S_u>K\}} U'(h(S_t,t)) n e^{r(T-u)} \sigma S_u \, dB_u + \\ &\quad \int_t^\tau U'(W) n e^{r(T-u)} \, d\Lambda_u(K) \end{aligned} \tag{20}$$

where $\Lambda_t(K)$ is the local time of the process S at the level K up to time t.

Proof Let

$$g_1(S_t,t) \equiv U(W) + U'(W) n e^{r(T-t)} (S_t - K)^+ \tag{21}$$

and let

$$g_2(S_t,t) \equiv g_1(S_t,t) - g(S_t,t) \ . \tag{22}$$

The second derivative of g_2 with respect to S exists almost everywhere so Itô's lemma can be extended to give

$$g_2(S_\tau,\tau) - g_2(S_t,t) = \int_t^\tau \frac{\partial g_2(S_u,u)}{\partial S} \, dS_u + \int_t^\tau \left(\frac{1}{2}\frac{\partial^2 g_2(S_u,u)}{\partial u^2}\sigma^2 S_u^2 + \frac{\partial g_2(S_u,u)}{\partial u}\right) du \tag{23}$$

as shown by Carr, Jarrow, and Myneni (1992, footnote 15). The nonconstant term in g_1 is the product of a differentiable function of time and a convex function of the stock price. Applying Karatzas and Shreve (1991, Theorem 7.1) for convex functions of semimartingales and then the product rule gives

$$\begin{aligned} g_1(S_\tau,\tau) - g_1(S_t,t) &= \int_t^\tau U'(W) n e^{r(T-u)} 1_{\{S_u>K\}} ((rK - \delta S_u)\, du + \sigma S_u \, dB_u) + \\ &\quad \int_t^\tau U'(W) n e^{r(T-u)} d\Lambda_u(K) \ . \end{aligned} \tag{24}$$

Subtracting (23) from (24) completes the proof of the lemma. □

Now consider a case in which the continuation region is characterized by a single critical stock price boundary $\bar{s}(t)$ as in Proposition 2.1 and set $\bar{s}(t) = \infty$ for $t \in [0, T)$ at which no exercise occurs.

Conjecture 2.1 *Suppose that the continuation region is characterized by a single critical stock price boundary $\bar{s}(t)$. Suppose further that there exists $\hat{s} > K$ and $L_1 < 0$ such that $H(S_t, t) < L_1$ for all $S_t > \hat{s}$ and all $t \in [0, T)$. Then the set of $t \in [0, T]$ for which $\bar{s}(t) = \infty$ contains no intervals.*

Sketch of proof Fix $t \in [0, T)$ and $S_t > \hat{s}$ and let

$$\hat{\tau} \equiv T \wedge \inf\{u > t : S_u = \hat{s}\} \ . \tag{25}$$

Then

$$\begin{aligned} f(S_t, t) - g(S_t, t) &= \mathrm{E}_t g(S_{\tau^*}, \tau^*) - g(S_t, t) \\ &= \mathrm{E}_t \int_t^{\tau^*} 1_{\{S_u > K\}} H(S_u, u) du + \\ &\quad \mathrm{E}_t \int_t^{\tau^*} U'(W) n e^{r(T-u)} \, d\Lambda_u(K) \qquad (26) \\ &= \mathcal{P}^{S_t}\{\hat{\tau} = T\} \mathrm{E}_t\{\int_t^{\tau^*} H(S_u, u) du | \hat{\tau} = T\} + \\ &\quad \mathcal{P}^{S_t}\{\hat{\tau} < T\} \mathrm{E}_t\{\int_t^{\tau^*} 1_{\{S_u > K\}} H(S_u, u) du | \hat{\tau} < T\} + \\ &\quad E_t \int_t^{\tau^*} U'(W) n e^{r(T-u)} \, d\Lambda_u(K) \qquad (27) \end{aligned}$$

where the probabilities above are conditional on the stock price starting at level S_t at time t. By assumption, $H(S_t, t)$ is bounded above on $\{(S_t, t) : S_t > K\}$ by some constant L_2. So

$$\begin{aligned} f(S_t, t) - g(S_t, t) &< \mathcal{P}^{S_t}\{\hat{\tau} = T\} L_1 \mathrm{E}_t\{\tau^* - t | \hat{\tau} = T\} + \mathcal{P}^{S_t}\{\hat{\tau} < T\} L_2 \mathrm{E}_t\{\tau^* - t | \hat{\tau} < T\} + \\ &\quad U'(W) n e^{r(T-t)} E_t(\Lambda_T(K) - \Lambda_t(K)) \ . \end{aligned} \tag{28}$$

From Carr and Jarrow (1990, Lemma A3),

$$E_t(\Lambda_T(K) - \Lambda_t(K)) = \frac{\sigma K}{2} \int_0^{T-t} \frac{1}{\sqrt{u}} N'(\frac{\log(K/S_t) - (r - \delta - \sigma^2/2)u}{\sigma\sqrt{u}}) \, du \ , \tag{29}$$

which converges to zero as $S_t \to \infty$. In addition, as $S_t \to \infty$, $\mathcal{P}^{S_t}\{\hat{\tau} < T\} \to 0$, so it seems the right-hand side of (28) converges to $L_1 \lim_{S_t \to \infty} \mathrm{E}_t(\tau^* - t) \leq 0$. On the other hand, the left-hand side must be nonnegative for all values of S_t. Therefore, $\mathrm{E}_t(\tau^* - t)$ should converge to zero as $S_t \to \infty$.

Remark Constant relative risk averse utility functions with relative risk aversion less than or equal to one satisfy the hypothesis of Proposition 2.1, as does risk-neutral utility with $\delta > 0$.

3 Summary and Conclusions

This paper seeks to advance the theory of executive stock option valuation with an in-depth study of the optimal exercise policy of a risk averse executive. Recent valuation models for executive stock options set the exercise policy exogenously, assuming a single critical stock price boundary. This paper shows that the optimal exercise policy need not be in that form. However, we prove the existence of a single critical boundary for constant relative risk averse utility functions with risk aversion coefficient less than or equal to one and find no counterexamples among our numerical results for constant relative risk averse utility functions with risk aversion coefficient greater than one.

Numerical examples show how the exercise boundary and option value vary with volatility, risk aversion, and wealth. The examples bear out the intuition that the exercise boundary and option value should be decreasing in executive risk aversion and increasing in the level of the executive's non-option wealth. However, in contrast to results from standard option theory, or from executive stock option valuation models with a fixed exercise boundary, executive stock option value can decline in stock return volatility when increases in volatility cause the optimal exercise boundary to drop sufficiently. These results underscore the importance of accurately characterizing the exercise policy for option valuation.

References

Bettis, J. Carr, John M. Bizjak, and Michael L. Lemmon, 2005, Exercise behavior, valuation, and the incentive effects of employee stock options, *Journal of Financial Economics* 76, 445-470.

Black, Fisher, and Myron Scholes, 1973, The pricing of options and corporate liabilities, *Journal of Political Economy* 31, 637-654.

Cai, Jie, and Anand M. Vijh, 2004, Executive stock and option valuation in a two-state variable framework: Allowing optimal investment of outside wealth in the riskfree asset and the market portfolio, University of Iowa working paper.

Carpenter, Jennifer N., 1998, The exercise and valuation of executive stock options, *Journal of Financial Economics* 48, 127-158.

Carr, Peter, and Robert Jarrow, 1990, The stop-loss start-gain paradox and option valuation: A new decomposition into intrinsic and time value, *Review of Financial Studies* 3, 469-492.

Carr, Peter, Robert Jarrow, and Ravi Myneni, 1992, Alternative characterizations of American put options, *Mathematical Finance* 2, 87-106.

Cuny, Charles J., and Philippe Jorion, 1993, Valuing executive stock options with an endogenous departure decision, *Journal of Accounting and Economics* 20, 193-205.

Detemple, Jerome, and Suresh Sundaresan, 1999, Nontraded asset valuation with portfolio constraints: A binomial approach, *Review of Economic Studies* 12, 835-872.

Evans, Jonathan, Vicky Henderson, and David Hobson, 2005, The curious incident of the investment in the market: Real options and a fair gamble, Princeton University working paper.

Geske, Robert, 1979, A note on an analytic valuation formula for unprotected American call options on stocks with known dividends, *Journal of Financial Economics* 7, 375-380.

Hall, Brian, and Kevin Murphy, 2002, Stock options for undiversified executives, *Journal of Accounting and Economics* 33, 3-42.

Henderson, Vicky, 2004, Valuing the option to invest in an incomplete market, Princeton University working paper.

Huddart, Steven, 1994, Employee stock options, *Journal of Accounting and Economics* 18, 207-231.

Hull, John, and Alan White, 2004, How to value employee stock options, *Financial Analysts Journal* 60, 114-119.

Ingersoll, Jonathan, 2006, The subjective and objective evaluation of incentive stock options, *Journal of Business* forthcoming.

Jennergren, L. Peter, and Bertil Naslund, 1993, A comment on "Valuation of executive stock options and the FASB Proposal," *The Accounting Review* 68, 179-183.

Kadam, Ashay, Peter Lakner, and Anand Srinivasan, 2003, Executive stock options:Value to the executive and cost to the firm, New York University working paper.

Karatzas, Ioannis, and Steven E. Shreve, 1998, Methods of Mathematical Finance (Springer-Verlag, New York).

Kaul, Matthias, Jun Liu, and Francis A. Longstaff, 2003, Paper millionaires: How valuable is stock to a stockholder who is restricted from selling it, *Journal of Financial Economics* 67, 385-410.

Kim, In Joon, 1990, The analytic valuation of American options, *Review of Financial Studies* 3, 547-572.

Krylov, Nikolai, 1980, Controlled Diffusion Processes (Springer-Verlag, New York).

Lambert, Richard A., David F. Larcker, and Robert E. Verrechia, 1991, Portfolio considerations in valuing executive compensation, *Journal of Accounting Research* 29, 129-149.

Marcus, Alan and Nalin Kulatilaka, 1994, Valuing employee stock options, *Financial Analysts Journal* 50, 46-56.

Merton, Robert C., 1973, Theory of rational option pricing, *Bell Journal of Economics and Management Science* 4, 141-183.

Miao, Jianjun, and Neng Wang, 2005, Investment, consumption and hedging under incomplete markets, Columbia University working paper.

Roll, Richard, 1977, An analytic valuation formula for unprotected American call options on stocks with known dividends, *Journal of Financial Economics* 5, 251-258.

Rubinstein, Mark, 1994, On the valuation of employee stock options, *The Journal of Derivatives* 3, 8-24.

Van Moerbeke, Pierre, 1976, On optimal stopping and free boundary problems, *Archives of Rational Mechanical Analysis* 60, 101-148.

Whaley, Robert E., 1981, On the valuation of American call options on stocks with known dividends, *Journal of Financial Economics* 9, 207-211.

Figure 1: Exercise Policy with Split Continuation Region

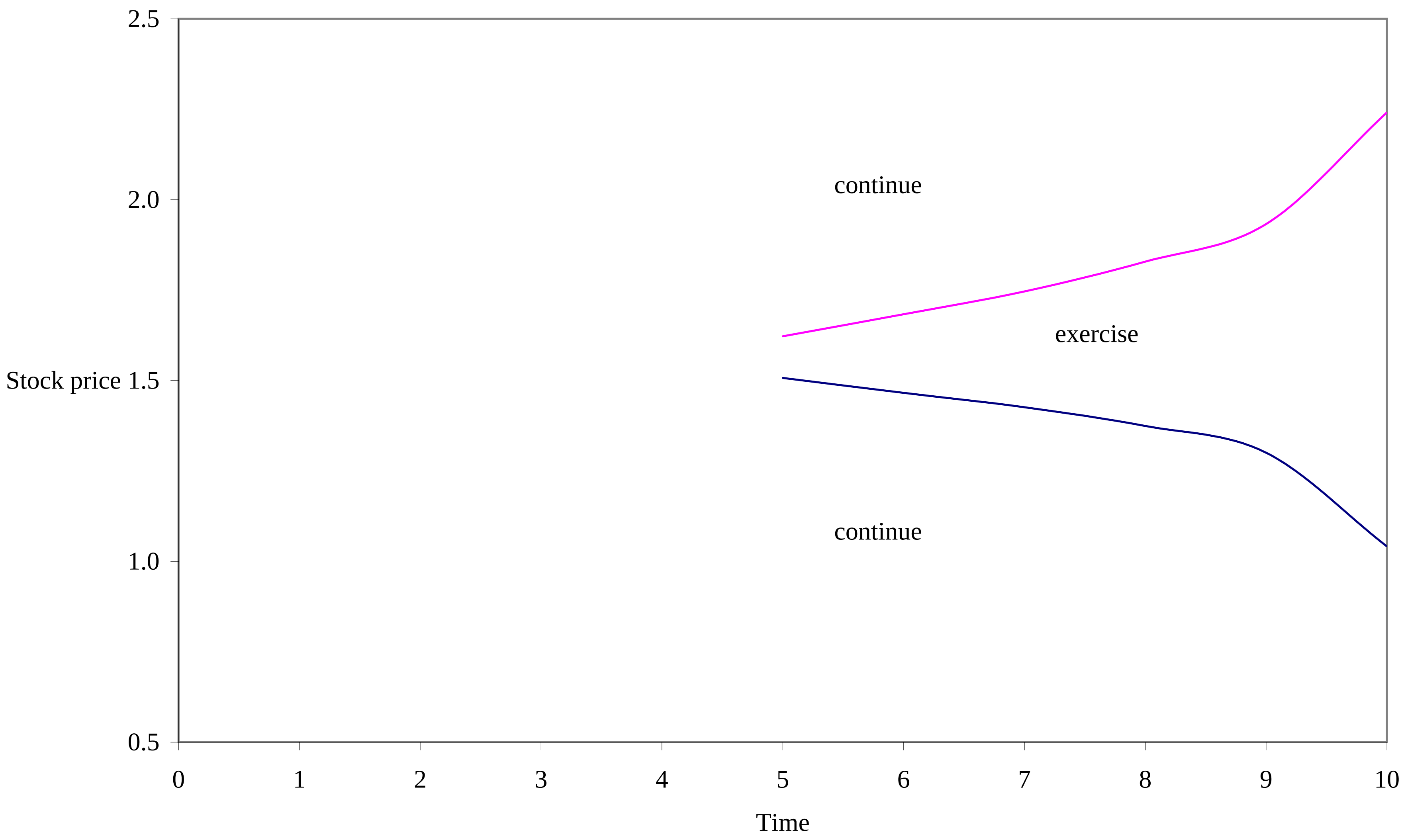
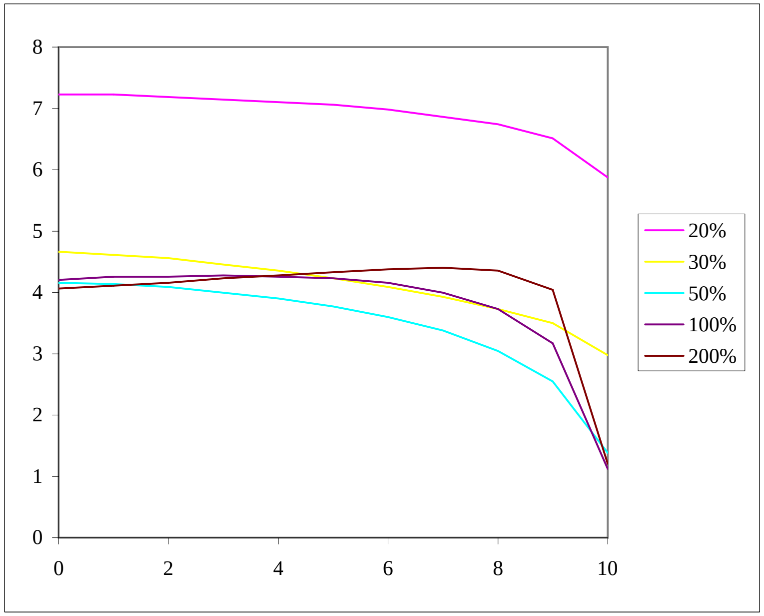
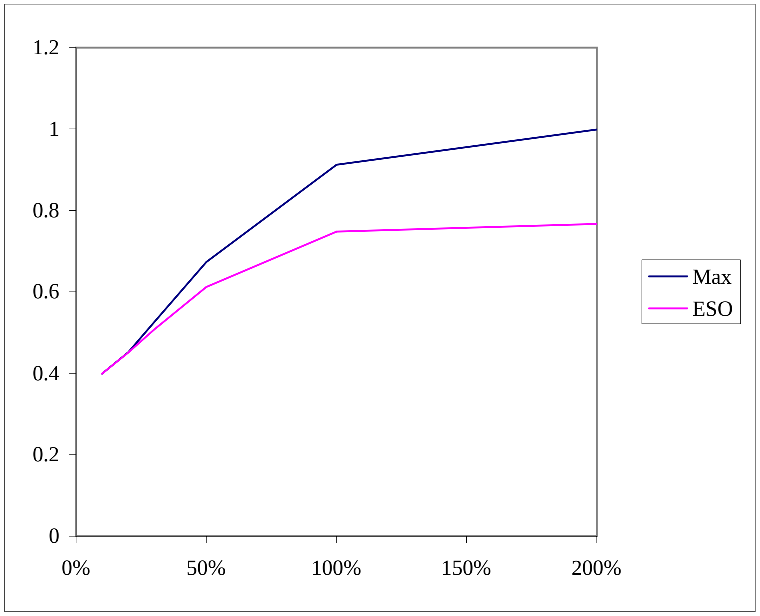


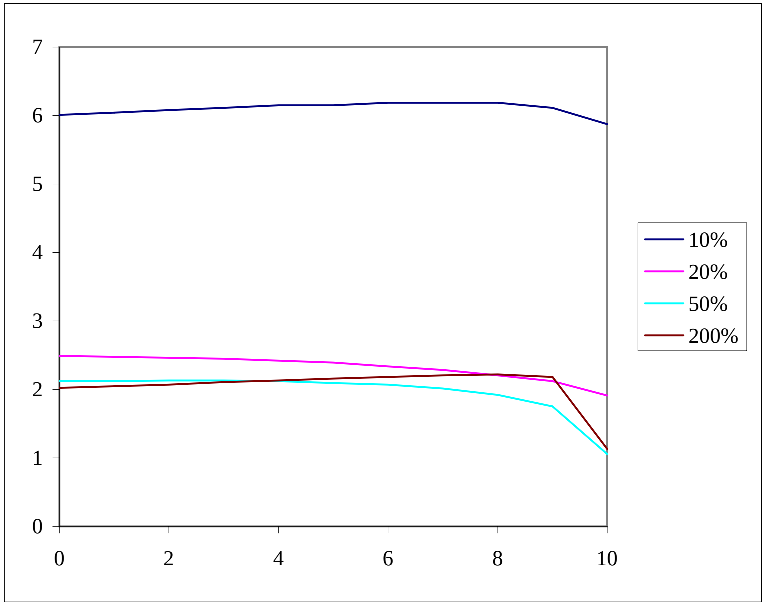
Figure 2: Exercise Boundaries and Option Values for Various Levels of Stock Volatility

a. Risk aversion coefficient = 0.5





b. Risk aversion coefficient = 2



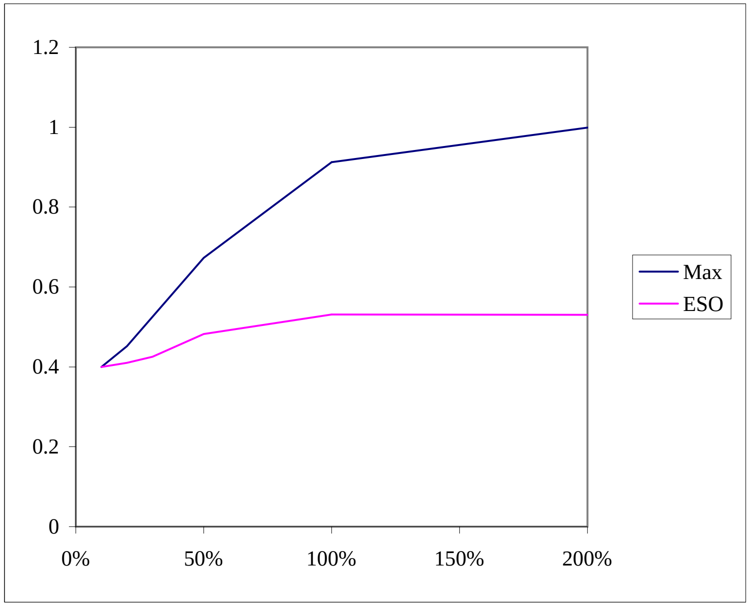
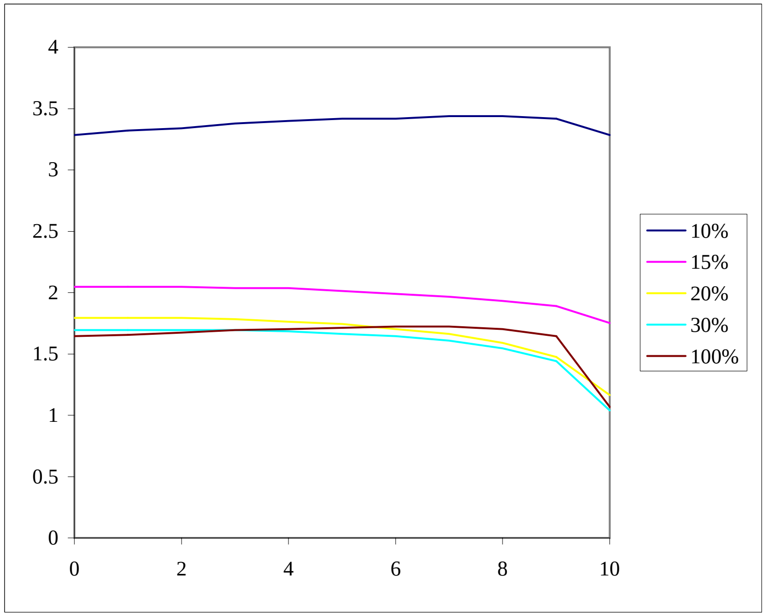
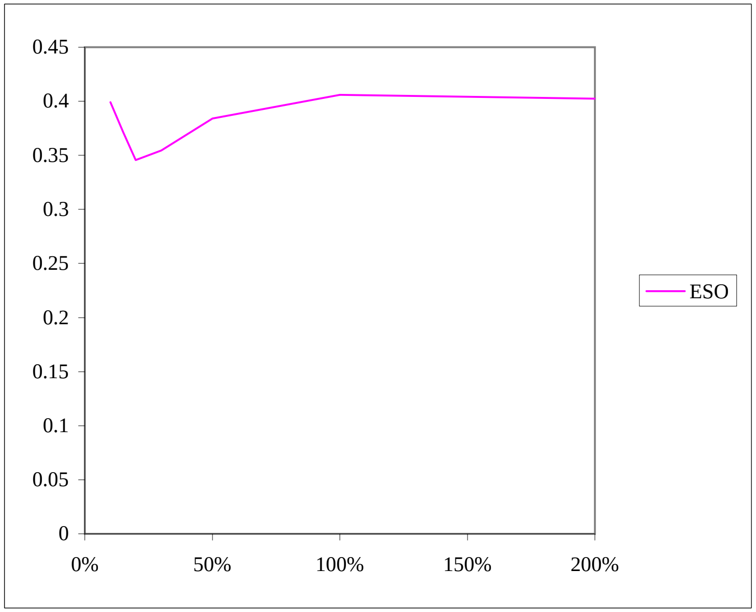


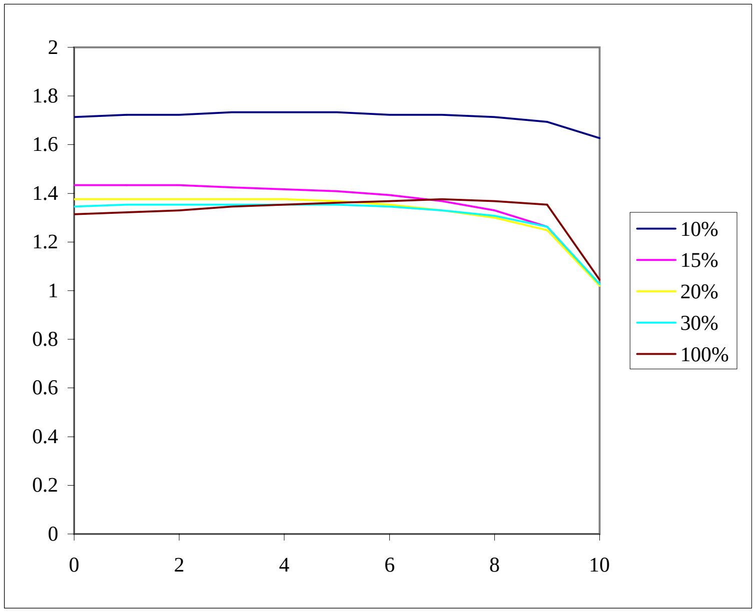
Figure 2 cont'd: Exercise Boundaries and Option Values for Various Levels of Stock Volatility

c. Risk aversion coefficient = 4





d. Risk averion coefficient = 10



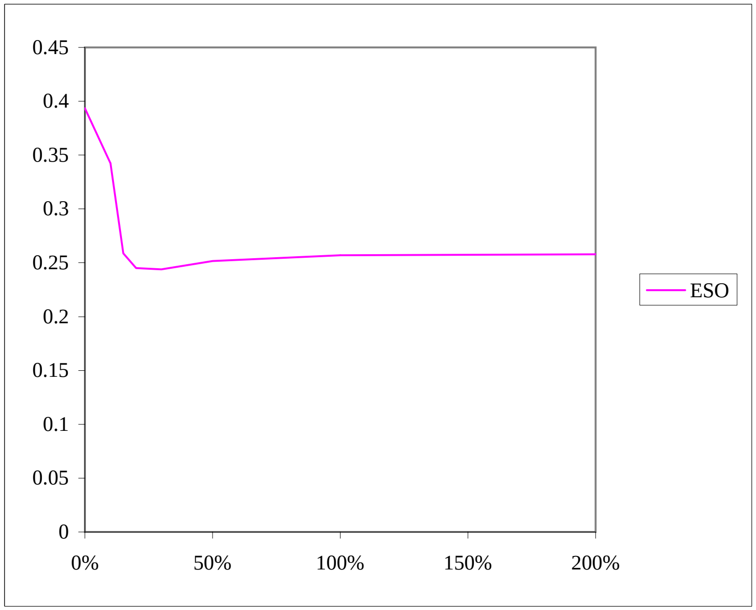
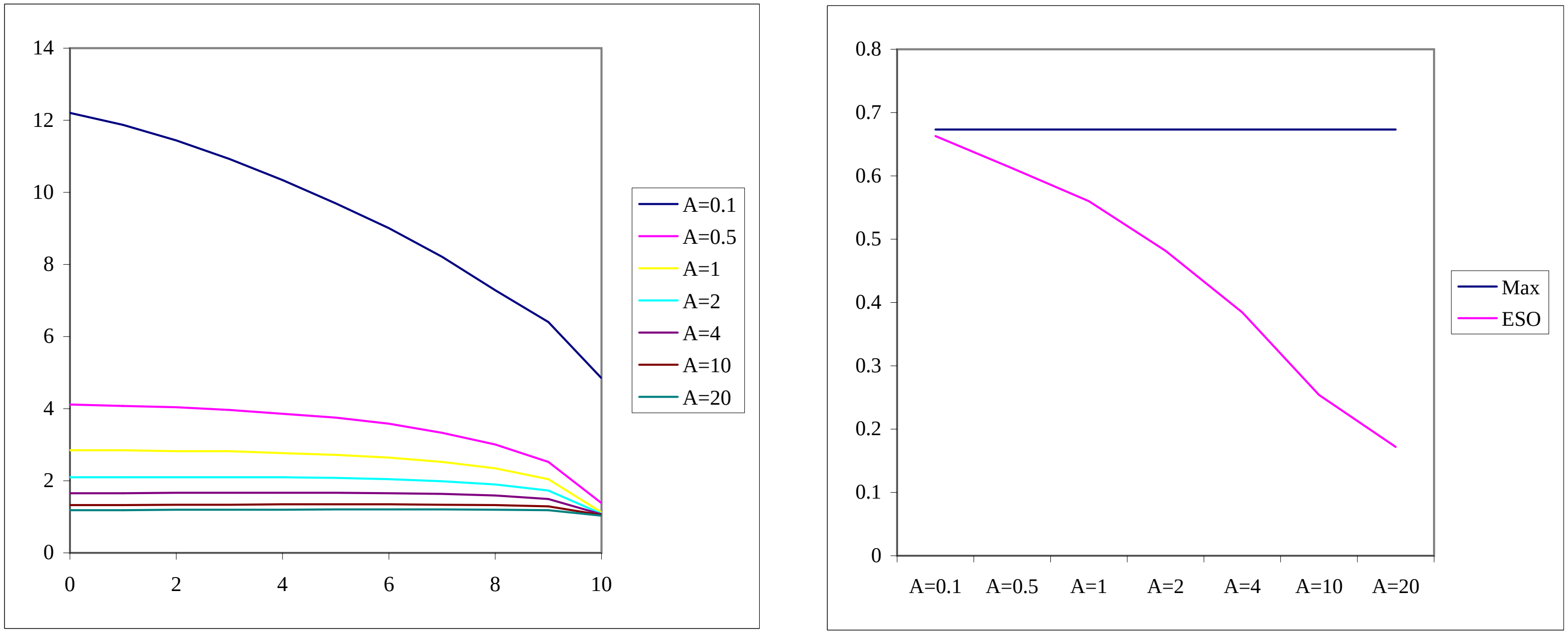


Figure 3: Exercise Boundaries and Option Values for Various Levels of Risk Aversion

a. Wealth = 2, Volatility = 50%



b. Wealth = 0.5, Volatility = 50%

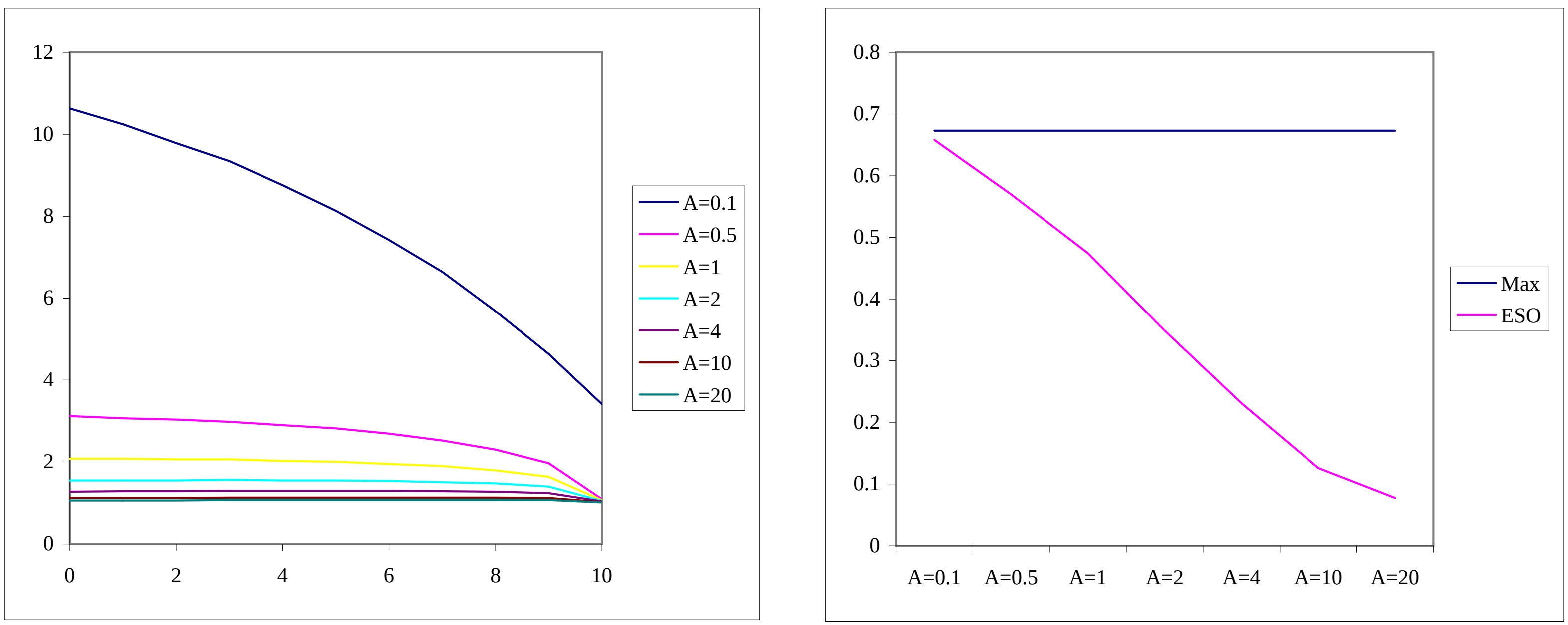
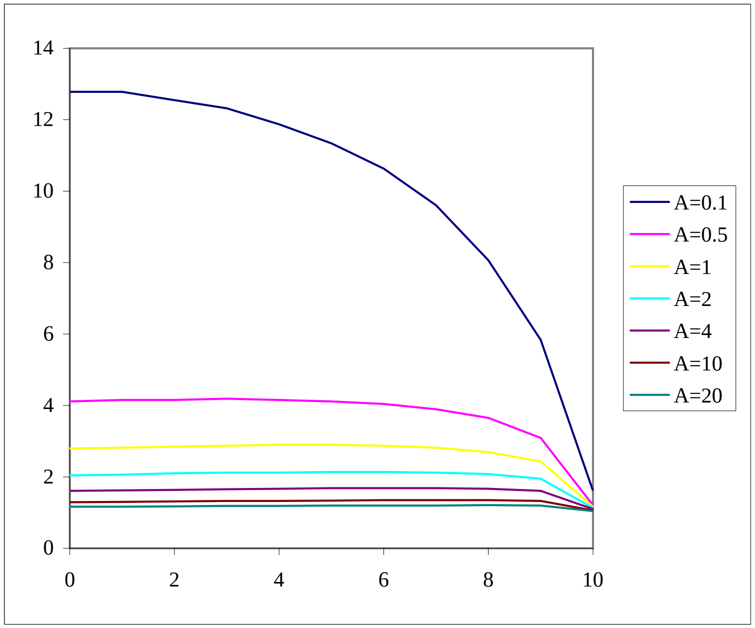
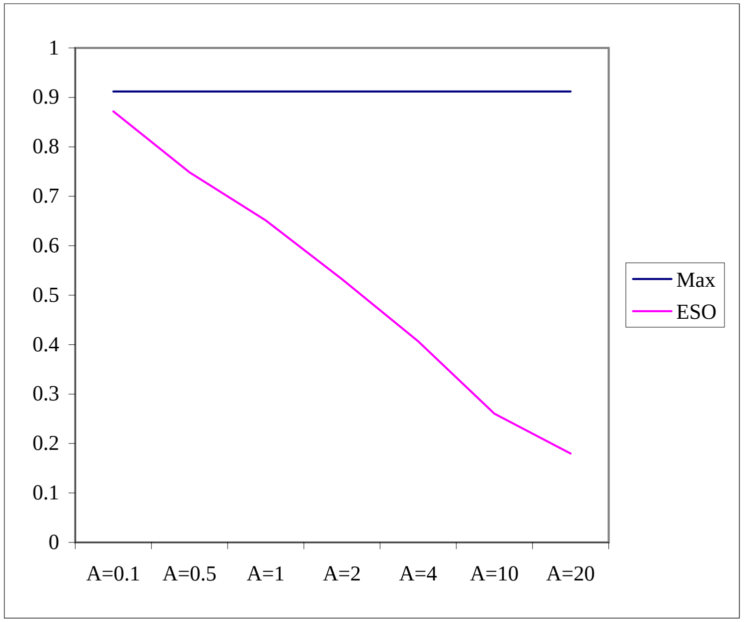


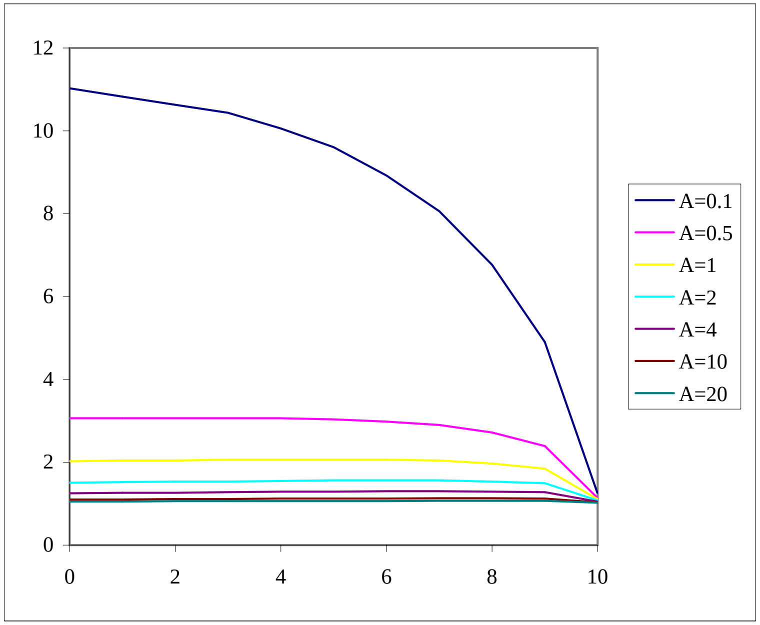
Figure 3 cont'd: Exercise Boundaries and Option Values for Various Levels of Risk Aversion

c. Wealth = 2, Volatility = 100%





d. Wealth = 0.5, Volatility = 100%



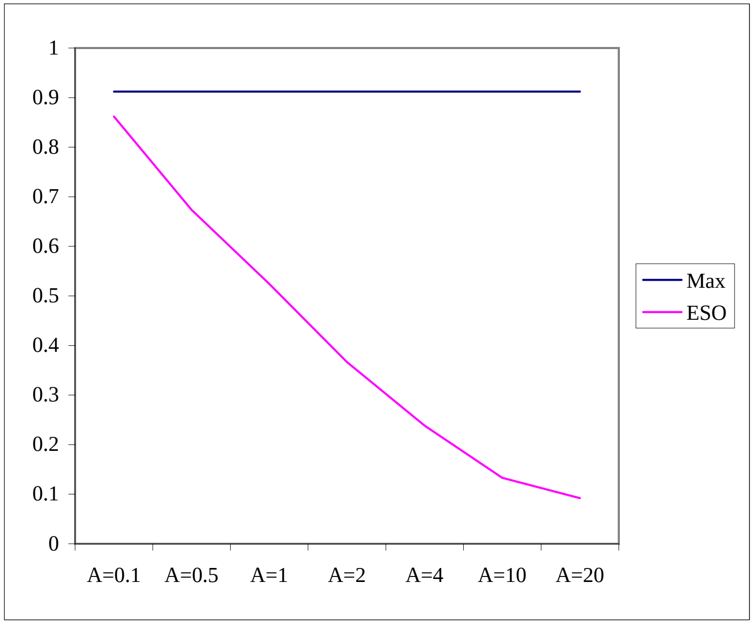
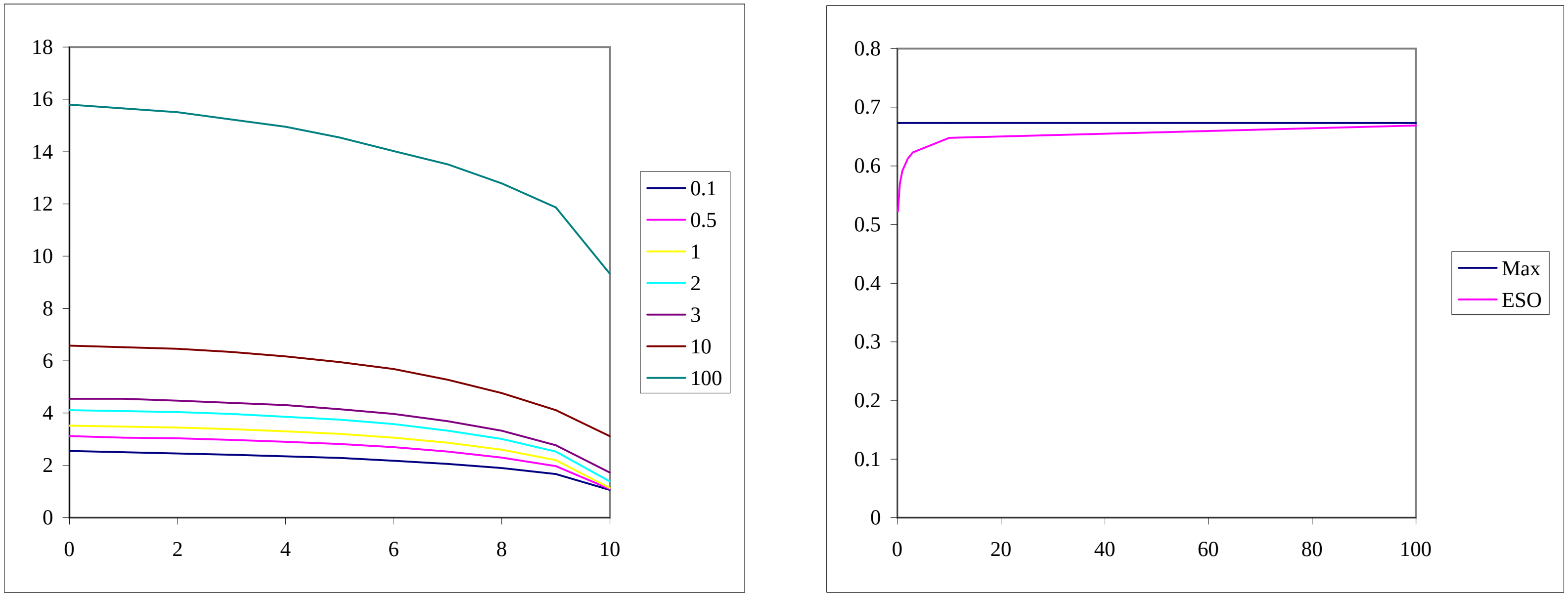


Figure 4: Exercise Boundaries and Option Values for Various Levels of Wealth

a. Risk aversion coefficient = 0.5, Volatility = 50%



b. Risk aversion coefficient = 2, Volatility = 50%

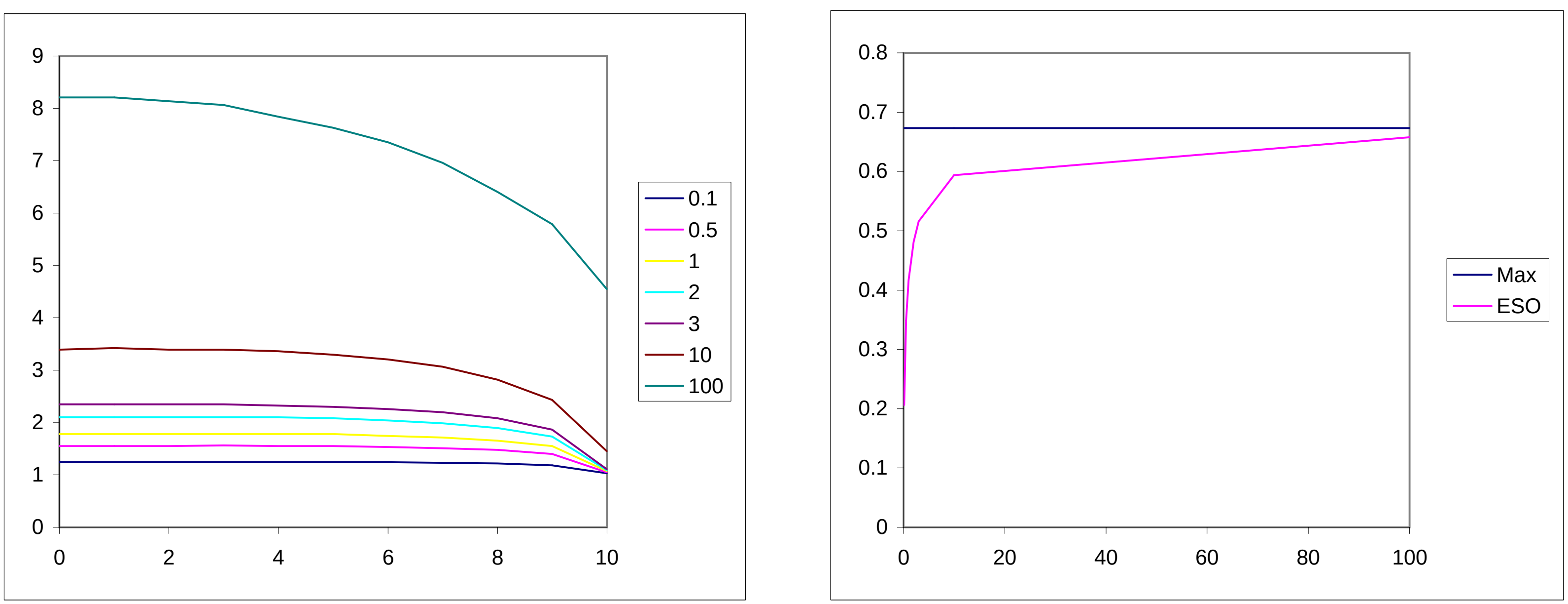
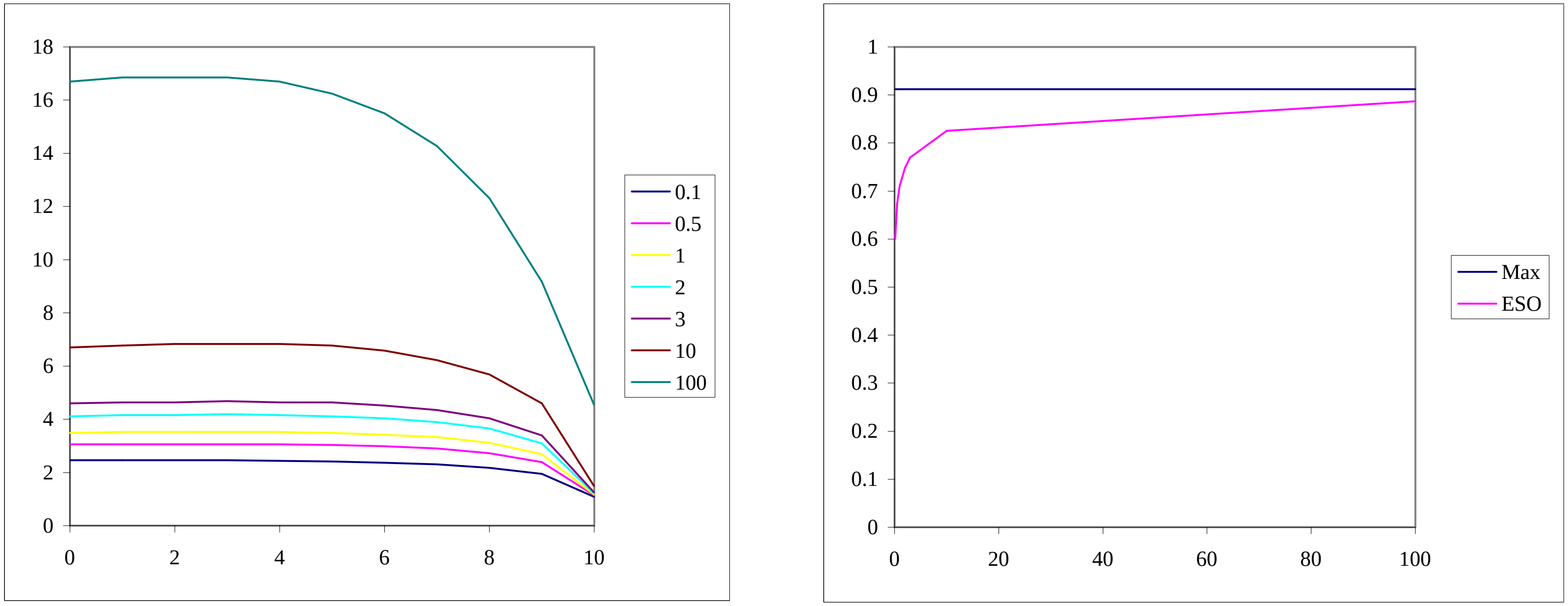


Figure 4 cont'd: Exercise Boundaries and Option Values for Various Levels of Wealth

c. Risk aversion coefficient = 0.5, Volatility = 100%



d. Risk aversion coefficient = 2, Volatility = 100%