The Effect of External Finance and Internal Capital Markets on the Equilibrium Allocation of Capital*

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Abstract

We model the equilibrium allocation of capital in the presence of imperfect institutional development. In our model, imperfect institutional development reduces external market activity by limiting capital flows among firms, thus compromising the efficiency of economy wide capital allocation. We show that the efficiency of capital allocation increases with firms' external financing requirements because higher external financing requirements lead to more liquidation of projects and a more active external capital market. Furthermore, a higher degree of conglomeration reduces external market activity because conglomerates allocate capital internally rather than supply it to the market. Thus, the efficiency of capital allocation may decrease in the presence of conglomerates, even when they allocate capital internally to their most productive units. Our results help explain why countries that rely heavily on external finance also have high productivity and growth, and they provide a new perspective on the recent debate about the desirability of dismantling business groups in developing countries. The fact that the main results of the paper run against the intuition obtained from partial equilibrium models shows the importance of modeling financial imperfections in an equilibrium framework in order to derive implications about overall economic efficiency.

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1 Introduction

Recent research reveals that the level of institutional development is an important determinant of financial variables. In a series of papers, La Porta, Lopez de Silanes, Shleifer and Vishny (1997, 1998, 1999, 2000a, 2002) show that the extent of legal protection of outside investors against expropriation by the manager or 'insiders' affects the size of the external capital markets, ownership and control concentration, dividend policy, the number and value of publicly traded firms, etc.[1] Moreover, institutional development seems to affect real variables such as the efficiency of capital allocation in an economy. Demirguc-Kunt and Maksimovic (1998) and Rajan and Zingales (1998) show that firms or industries that are in need of relatively more finance grow faster in countries with active stock markets and developed legal systems. Wurgler (2000) shows that better investor protection leads to a more efficient use of capital in the economy.[2]

Our understanding of the ways institutional development affects an individual firm's decisions has advanced considerably. La Porta et al (1999) and Bebchuck (1999) explain why poor investor protection leads to concentrated control. La Porta et al. (2002) show that firm valuation is higher in countries with better legal protection of shareholders. Shleifer and Wolfenzon (2002) present a model that shows how investor protection affects ownership concentration, dividend policy, the size of the external equity market, firm valuation, the size of private benefits of control, and the number of firms going public. However, the theoretical link between institutional development and the efficiency of economy-wide capital allocation has received less attention.[3] In this paper we propose a framework to study this link. Our starting point is that institutional development affects activity in external capital markets. In particular, lack of institutional development reduces reallocation of capital across firms and projects in the economy. Our main contribution is to show that in a situation where capital reallocation is distorted by a lack of institutional development, an increase in firms' external financing needs, or a decrease in the

[1]See also Claessens, Djankov, and Lang (2000a), Kumar, Rajan, and Zingales (1999), and Nenova (2002), among others.

[2]Perhaps as a result of this better allocation of capital, countries with 'better' institutions have also been shown to grow faster (King and Levine, 1993, Levine and Servos, 1998 and Beck, Levine and Loayza, 2000).

[3]One exception is Shleifer and Wolfenzon (2002). Although their main focus is not the efficiency of capital allocation, some of their results are relevant to it. We discuss the relation between our paper and Shleifer and Wolfenzon (2002) after we have explained our results.

level of conglomeration in the economy, may improve the allocation of capital by stimulating market activity.

In an environment with imperfect institutional development, entrepreneurs with high productivity projects cannot credibly commit to paying back the entire cash flows that their firms generate since, once they have taken the investors' money, there may be little investors can do to get it back. A direct result of this limited pledgeability is that the investments of high productivity firms are constrained.[4] However, this is not the only distortion introduced. Due to limited pledgeability, firms with mediocre projects are not properly compensated when they supply their capital to the external market and, as a result, these firms continue their projects rather than liquidate them. Therefore, even when the demand for capital of high productivity firms is not satisfied, capital is retained in mediocre projects. That is, lack of institutional development reduces market activity.

Our main results are as follows. First, if existing projects have raised enough external finance in the past, they will be liquidated too often (as in Diamond, 1991). This excessive liquidation is suboptimal for a firm in isolation, but it allows the released capital to flow from mediocre to high productivity projects, improving the equilibrium allocation. This result suggests an additional reason why countries that rely heavily on external finance are also countries with high productivity and growth. The traditional explanation for this correlation is that external finance is beneficial because, without it, firms would not be able to make the necessary investments. This argument implies that external finance is beneficial when there are no other sources of finance. While undoubtedly this is an important role of external finance, our model suggests that external finance can independently affect the equilibrium allocation of capital by stimulating market activity.

Our second result is that an increase in the degree of conglomeration (the fraction of projects in the economy that are in multi-project firms) can decrease the efficiency of capital allocation, even when these conglomerates allocate capital internally to their best units. A stand alone

[4]The assumption of limited pledgeability can be justified by the framework in Shleifer and Wolfenzon (2002), who show that higher levels of investor protection (i.e., higher costs of expropriation) lead to lower expropriation and consequently higher pledgeability. Even though we focus on the institutional environment, limited pledgeability can also be a consequence of the inalienability of human capital (Hart and Moore 1994), or moral hazard in project choice due to private benefits (Holmstrom and Tirole, 1997).

firm with a worthless project has no better option than to supply the project's capital to the external market. This capital can then find its way to a high productivity project. However, a conglomerate allocates the capital of a worthless project to its best unit, even if this unit is of mediocre productivity. A conglomerate prefers this internal reallocation even when there are higher productivity projects in the economy in need of capital. This because the best projects in the economy cannot properly compensate the conglomerate for its capital, due to limited pledgeability. In this case, the presence of conglomerates further distorts the equilibrium capital allocation by reducing the supply of capital to the external market.

According to this argument, the observation that conglomerates allocate capital to their most productive projects is not a sufficient condition to advocate the presence of conglomerates. For example, it has been documented that one of the roles of business groups in developing countries is to allocate capital among member firms (Leff, 1976 and Khanna and Palepu, 1997). Khanna and Palepu (1999) argue that because business groups provide an important substitute for the lack of active external capital markets in developing countries, calls for their dismantlement should be ignored. The result in this model suggests that business groups can simultaneously be detrimental to capital allocation due to their negative effect on external market activity and be efficient at allocating capital internally. The dismantling of efficient conglomerates may improve overall capital allocation by stimulating external market activity.

In general, our two results indicate that one cannot simply extrapolate results obtained in a partial equilibrium framework. We show that an increase in external finance introduces inefficiencies at the firm level, yet it can improve the efficiency of economy wide capital allocation. Similarly, conglomerates can allocate capital to their best units, yet dismantling them could stimulate external market activity and lead to better capital allocation. This is because changes that take place at the firm level (e.g., a change in the degree of pledgeability or a need for external finance) affect firms' actions directly and also indirectly through their effect on the external market.

There are several strands of literature related to our paper, the first of which is the literature on the benefits and costs of liquidation in an equilibrium context. According to the 'liquidationist' view (De Long, 1990), the liquidation of existing firms is healthy for the economy since it releases factors of production that can be used more productively for new projects. This position goes back to Schumpeter's (1942) idea of 'Creative Destruction'. Contrary to this view, Caballero and Hammour (1996, 1998, 1999) argue that in an economy with frictions, liquidation might be excessive since the released factors will end up being unemployed.[5] Our model is consistent with both views. As we show in the paper, for low levels of pledgeability the reallocation market for capital works very poorly, and the capital released by mediocre projects cannot reach the high productivity ones. In such a case, the liquidation induced by external finance and deconglomeration cannot improve the equilibrium allocation, as argued by Caballero and Hammour. However, at higher levels of pledgeability, the reallocation market works better and the released resources find their way to more productive activities (consistent with De Long and Schumpeter). In our set up, the degree of frictions (as measured by pledgeability) determines whether liquidation is beneficial.

Our results are also related to the recent debate about the costs and benefits of internal capital markets in conglomerates (see Stein, 2001, for a survey). This literature has suggested that a potential benefit of an internal capital market is its ability to allocate capital efficiently across divisions (Gertner, Scharfstein and Stein, 1994, Stein 1997, Matsusaka and Nanda, 2002). The opposite view is that internal capital markets do not allocate capital efficiently, but rather in a 'socialistic' way (Shin and Stulz, 1998, Rajan, Servaes and Zingales, 2000; Scharfstein and Stein 2000). These papers analyze conglomerates in isolation and thus have no equilibrium implications.[6] Our point is that even when conglomerates improve the internal allocation of capital, they can decrease the efficiency of the economy-wide allocation because of their negative impact on market activity. In fact, our argument does not depend on whether internal allocation of capital in a conglomerate is efficient. The crucial feature driving our results is that, due to limited pledgeability, conglomerates have a preference towards internal reallocation of capital, vis-à-vis external reallocation in the capital market.

The role of active markets has received attention in the literature. Large and more liquid markets increase incentives for agents to acquire information about firms because when markets

[5]This is similar to the arguments of Shleifer and Vishny (1992), who argue that specific assets which are sold because of liquidity constraints might not be redeployed to the buyers with the highest valuations because such buyers might not have enough funds to finance the acquisition.

[6]Maksimovic and Phillips (2001, 2002) are an exception: they analyze allocation decisions by conglomerates in an equilibrium context, but with no role for financial imperfections.

are large it is easier for agents to profit from private information (Kyle, 1984; Holmstrom and Tirole, 1993). Alternatively, large and active markets can absorb large trades with smaller adverse price changes thus increasing potential gains from trade (Pagano, 1989). Our paper points to an additional reason why an active external market for capital can be beneficial: active markets help the economy overcome frictions introduced by the limited pledgeability of cash flows, allowing capital to flow to the highest productivity users.

A recent theoretical paper that also analyzes the effect of institutional development (legal protection) on the allocation of capital is Shleifer and Wolfenzon (2002). Their focus is on the allocation decision of investors, while the mechanism explored in this paper deals with the allocation of capital among firms (our reallocation market for capital). In fact, our main results stem from the bias for internal investment of mediocre firms and conglomerates, which is not analyzed by Shleifer and Wolfenzon. It is this bias that generates the potential benefits of external finance as well as the costs of internal capital markets, which are the novel results of this paper. Perhaps most importantly for empirical purposes, these results suggest that, even though lack of institutional development is crucial, it is not the only factor that influences the efficiency of capital allocation. External finance and internal capital markets may also have independent effects.

In the next section we characterize the effect of limited pledgeability in the equilibrium allocation of capital, and analyze the effect of changes in the external financing requirements of investment projects. As we show in section **??**, this model can be easily extended to analyze the effects of conglomeration and internal capital markets on the equilibrium allocation. Section **??** presents our final remarks. All the proofs are relegated to the Appendix.

2 Limited pledgeability, external finance and the equilibrium allocation of capital

In this section, we develop our theoretical framework to analyze the effect of institutional development on the equilibrium allocation of capital. We then analyze how the equilibrium allocation changes with firms' external financing needs. In the model, entrepreneurs issue financial contracts to raise the necessary capital to pay the investment cost of their projects at date t_0.

Information about the profitability of projects arrives at a later date t_1. In light of the new information, capital can be reallocated among projects. The efficiency of this 'reallocation market' is the main object of our analysis.

In our set up, institutional development affects the fraction of cash flows that firms can pledge to outside investors. Better institutions (e.g., laws that protect outsiders and regulators that enforce these law) lead to higher pledgeability of cash flows. This limited pledgeability drives a wedge between the true productivity of the projects and the 'pledgeable return', the return that an entrepreneur can credibly commit to pay. We show that this wedge distorts the functioning of the reallocation market in two important ways. First, some high productivity projects cannot raise capital, and second, less productive projects are continued and their capital is not supplied to more productive projects.

We then analyze the relation between external financing needs at date t_0 and the efficiency of the reallocation market at date t_1. We vary the degree of date t_0 external financing needs by varying the initial wealth of entrepreneurs. To pay back investors, entrepreneurs enter contracts that require liquidation at date t_1. The higher the financing needs, the more frequently projects need to be liquidated. For a firm in isolation, this increased liquidation is an inefficiency brought about by external finance (as in Diamond, 1991). This result is consistent with most of the corporate finance literature in that external finance introduces inefficiencies (Jensen and Meckling, 1976, Myers, 1977, Myers and Majluf, 1984). However, once we embed these firms in a market equilibrium framework in which capital allocation is already distorted, this 'excessive' liquidation can be socially beneficial because the capital released by liquidation can find its way to higher productivity projects. Thus, even though it always introduces inefficiencies at the firm level, external finance can be socially beneficial when its equilibrium effects are taken into account.

2.1 The model

The timing of events is shown in Figure 1. There are three periods in the model, t_0, t_1, and t_2, and two types of agents. There is a set J with measure 1 of agents ('entrepreneurs'), each with one project opportunity (projects are described below) and a different set of agents with no project opportunities ('investors'). All agents are risk-neutral and do not discount the future.

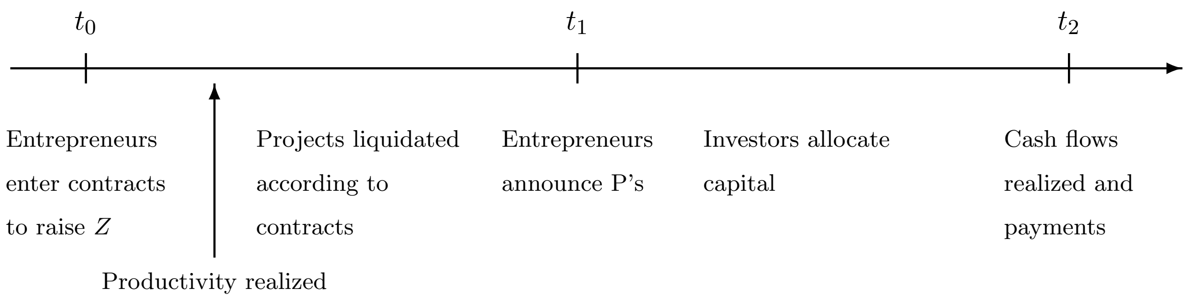

Figure 1: Timing of events

Thus, agents maximize their consumption in the final period (t_2).

There is a single good in this economy ('capital'). At date t_0 the aggregate amount of capital $1 + K$ (with $K > 1$) is held by the two types of agents. Each entrepreneur has an endowment of $1 - Z$ units (with $0 \leq Z < 1$). The remaining $K + Z$ is distributed among investors.

Technologies Capital can be stored with no depreciation from date t_0 to date t_1 and from date t_1 to date t_2. In addition, capital can be invested in two types of technologies that pay off in period t_2.

The first technology ('general technology') is available to all agents in the economy at date t_1. We define $x(\omega)$ as the *per-unit* pay off of the general technology with ω being the aggregate amount of capital invested in it.

Assumption 1 *The general technology satisfies:*

a. $\frac{\partial[\omega x(\omega)]}{\partial \omega} \geq 0$ *c.* $x(1+K) > 1$

b. $x'(\omega) \leq 0$

We assume that total output is increasing in the amount invested in the general technology (assumption **??**(a)) but that the per-unit payoff is decreasing (assumption **??**(b)). Assumption **??**(c) guarantees that no capital is invested in storage at date t_1 since the general technology offers a higher return regardless of the amount invested in it.

We refer to the second type of technologies as projects. Each project is of infinitesimal size. At date t_0 a project requires an investment of one unit of capital. At date t_1, a project can be liquidated, in which case the entire unit of capital is recovered.[7] If a project is not liquidated, it can receive additional capital or can simply be continued with no change until date t_2. At date t_2, projects generate cash flows.

Projects have different productivities. Each project's type, s, can take three values: H (high productivity or 'good' projects), M (medium productivity or 'medium' projects), and L (low productivity or 'bad' projects) with probabilities p_H, p_M, and p_L (with $p_H + p_M + p_L = 1$), respectively. The projects' type is not known at date t_0 (not even to the entrepreneur who owns it) but it is revealed to everyone in the economy at the beginning of date t_1. The probability distribution is independent across projects so that, at date t_1, exactly a fraction p_H, p_M and p_L of the projects are of type H, M and L, respectively.

For simplicity, we assume drastic decreasing returns to scale. Each project generates Y_H, Y_M or Y_L (with $Y_H > Y_M > Y_L \equiv 0$) *per unit* invested when the project is good, medium, or bad, respectively, but only for the *first two units* (i.e., the unit invested at t_0 and the unit that is potentially invested at t_1). Additional units invested generate no cash flows. To rank the productivity of the projects, we make the following assumption:

Assumption 2 $Y_M > x(0)$

The most productive technologies are good projects, followed by medium projects. By assumption **??**, medium projects are more productive than the general technology regardless of the amount invested in the latter (since $x(0) > x(\omega)$ by assumption **??**(b)). Finally, the bad projects are the least productive.

Capital market at date t_0 In the capital market at date t_0, entrepreneurs raise Z from outside investors in exchange for a set of promised payments. To focus on imperfections brought about by limited pledgeability of payoffs (described below), we assume that initial contracts can be made fully contingent on dates and type of project.

A contract offered by entrepreneur j is defined by a vector $\{q_s^j, D_{1s}^j, D_{2s}^j\}_{s=L,H,M}$ where q_s^j

[7] The only technology to transfer capital from date t_0 to date t_1 is storage. Since an investment in a project can always be liquidated at date t_1 and the entire unit recovered, there is no cost in starting a project at date t_0. Therefore, we assume that all projects are set up.

is the probability that the project is liquidated, D_{1s}^j is the payment to investor at date t_1 when the project is liquidated (since projects generate no cash flow at date t_1, the payment D_{1s}^j can be positive only when there is liquidation), and D_{2s}^j is the payment to the investor at date t_2.

Entrepreneurs approach investors and make them a take-it-or-leave-it offer.[8] Since initial investors can always store their capital from date t_0 to date t_1 and then invest it in the reallocation market from date t_1 to date t_2, they accept the contract only if it offers them a payoff of at least R^*Z, where R^* is the expected return in the date t_1 reallocation market. Entrepreneurs maximize their payoff subject to the participation constraint of the investor.

Capital market at date t_1 At date t_1 the type of the project is realized and liquidation occurs according to the specified probabilities in the contract. After liquidation occurs, date t_1 investors (investors that stored capital from date t_0 to date t_1, entrepreneurs, and date t_0 investors of liquidated firms) allocate their capital to continuing projects and to the general technology. The total amount of capital in the hands of date t_1 investors is $K + T$, where T is the capital released from liquidated projects. That is, $T = \int_{j \notin C} dj$ with C being the set of projects that continue.

Each continuing project j announces P^j, the amount it pays at date t_2 for the first unit of capital.[9] Projects have no use for additional units of capital and thus offer to pay zero for those units. Next, date t_1 investors allocate their capital to either the projects or to the general technology. An allocation in the capital market can be described by r^j, the probability that project $j \in C$ gets capital and ω, the amount allocated to the general technology. An equilibrium allocation must be consistent with date t_1 investor maximization and satisfy the market clearing condition

$$\omega^* + \int_{j \in C} r^j dj = K + T. \tag{1}$$

To maximize their payoff, date t_1 investors start allocating capital to the technologies offering the highest return and then allocate capital to technologies in decreasing order of the offered return. When several projects offer the same return but there is not enough capital to

[8]The assumption that entrepreneurs have all the bargaining power can be justified by the fact that there is excess supply of capital at date t_0.

[9]This announcement depends on s, the realized productivity of the project. However, to lighten notation we suppress it.

allocate one unit to each project, any allocation rule is consistent with investor maximization since investors are indifferent as to which technology receives capital. After describing limited pledgeability, we specify an intuitively appealing rule for this case.

Limited pledgeability of cash flows We consider an imperfection at the firm level: firms cannot pledge to outside investors the entire cash flow generated by projects. This limited pledgeability assumption applies only to the date t_2 cash flows but not to the date t_1 liquidation proceeds. That is, when liquidation takes place at date t_1, the unit recovered is fully pledgeable:

$$D^j_{1s} \leq 1. \tag{2}$$

However, the returns at date t_2 are not. In particular, we assume that only a fraction λ of the returns of the second unit invested is pledgeable.[10] This imposes the additional constraint that

$$D^j_{2s} \leq \lambda Y_s. \tag{3}$$

Limited pledgeability also has implications for the price a firm can offer in the date t_1 reallocation market. When its realized productivity is s, a continuing project j can offer at most

$$P^j \leq \overline{P}^j_s \equiv \lambda Y_s - D^j_{2s}. \tag{4}$$

The limited pledgeability assumption can be justified as being a consequence of lack of institutional development, as shown in Shleifer and Wolfenzon (2002). In their model, insiders can expropriate outside investors, but expropriation has costs that limit the optimal amount of expropriation that the insider undertakes. Higher levels of protection of outside investors (i.e., higher costs of expropriation) lead to lower expropriation and consequently higher pledgeability. To justify the full pledgeability of cash flows in the case of liquidation, we could make the natural assumption that liquidation proceeds are easier to verify than project returns.

Limited pledgeability also arises in other contracting frameworks. For example, limited pledgeability is a consequence of the inalienability of human capital (Hart and Moore 1994). Entrepreneurs cannot contractually commit never to leave the firm. This leaves open the possibility that an entrepreneur could use the threat of withdrawing his human capital to renegotiate

[10] The assumption that the cash flows from the first unit are not pledgeable is made only for simplicity. It will become clear our main results do not hinge on this assumption (see footnote **??**)

the agreed upon payments. If the entrepreneur's human capital is essential to the project, he will get a fraction of the date t_2 cash flows.[11] A natural assumption is that the entrepreneur's human capital is not needed to liquidate the firm. This justifies the fact that the entire liquidation proceeds are pledgeable, but only a fraction of the date t_2 cash flows are. Limited pledgeability is also an implication of the Holmstrom and Tirole (1997) model of moral hazard in project choice. In that model project choice cannot be specified contractually. As a result, investors must leave a high enough fraction of the payoff to entrepreneurs to induce them to choose the project with low private benefits but high potential profitability. Since liquidation in our framework can be specified contractually, there is no need to leave anything to the entrepreneur when liquidation occurs. Again, this justifies the full pledgeability of liquidation proceeds. Moreover, institutional development can also affect pledgeability in these alternative contracting frameworks. For example, in the Holmstrom and Tirole framework we could model institutional development as facilitating monitoring activities that reduce the size of private benefits that insiders can extract.

Finally, we describe the allocation rule when several projects offer the same return, P, but there is not sufficient capital to allocate one unit to each project. As we discussed above, any allocation rule would be consistent with the maximization behavior of date t_1 investors. We assume a rule that favors firms with $\overline{P}_s^j > P$ over those with $\overline{P}_s^j = P$: capital is allocated first to the former set of firms and only when each of them receives one unit does the other set of firms receive capital. This rule reflects the fact that firms with $\overline{P}_s^j > P$ can always outbid firms with $\overline{P}_s^j = P$ by offering to pay slightly above P if the allocation rule does not favor them.[12] Thus, the allocation rule is a function r of both P^j's and $\overline{P}^j$'s.

[11] See Diamond and Rajan (2000, 2001) for another recent model which hinges on limited pledgeability of cash flows arising from the Hart and Moore (1994) framework.

[12] This assumption is weak since, with the addition of a mild condition, all allocation rules yield similar equilibria. The condition is that prices can be quoted in very small but discrete intervals $\delta > 0$. Suppose, for instance, that the allocation rule does not favor firms with $\overline{P}_s^j > P$. In this case, firms with $\overline{P}_s^j > P$ would offer $N\delta$, with N being the lowest integer such that $N\delta > P$ while firms with $\overline{P}_s^j = P$ would offer $(N-1)\delta$. Thus, firms with $\overline{P}_s^j > P$ would be guaranteed one unit and those with $\overline{P}_s^j = P$ would receive capital only after the demand from the first set of firms has been satisfied. The rule we propose delivers a similar equilibrium, but is algebraically simpler since it does not involve dealing with the additional constant δ.

2.2 Illustration of the main effect

In this section we present an informal argument to illustrate the main mechanism of the model. In the next section we formally characterize the equilibrium.

It is useful to think about the date t_1 reallocation market in terms of demand and supply of funds.[13] On the supply side, there is a total of $K + T$ to be allocated with T being the capital from liquidated projects. This amount is determined by the contract that entrepreneurs enter at date t_0 to raise the necessary capital Z. Since continuation of low productivity projects generates no cash, these projects are always liquidated. The liquidation proceeds are supplied to the market since the entrepreneur has no use for them.[14] As a result, aggregate liquidation proceeds are at least p_L ($T \geq p_L$). Figure 2 depicts the demand and supply schedules for two different supply scenarios. The supply of capital (vertical line) in panel A is $K + p_L$ while total supply in panel B is higher.

The demand for capital arises from good and medium projects and from the general technology. To raise capital at date t_1, firms pledge part of the project's date t_2 cash flows. Due to limited pledgeability, firms with high (medium) productivity projects can offer a maximum of λY_H (λY_M) per unit of capital (for the sake of the argument we assume that $D_{2H} = D_{2M} = 0$) and the general technology offers $x(\omega)$. The aggregate demand schedule (the downward sloping lines in panels A and B) shows the capital demanded for different levels of market return. If we assume that $x(\omega) > \lambda Y_H$ for low ω, as we do in Figure 2, some capital must be allocated to the general technology before the projects get additional capital (the general technology can initially offer a higher return than good and medium projects). As additional capital is allocated to the general technology, the return it offers (the required return on capital) decreases. When this return reaches λY_H, good firms can start attracting capital. Additional capital is allocated to good firms until they all get one unit. This explains the flat region of the aggregate demand. The rest of the demand schedule is derived in a similar way.

[13] In this supply and demand framework the existence of an equilibrium is not guaranteed. In the formal characterization of the equilibria, we model the date t_1 capital market using the non-cooperative game described above to guarantee the existence of an equilibrium. However, the intuition for both cases is very similar.

[14] Section **??** shows that when firms are composed of more than one project, it is possible to have $T < p_L$ because some of the liquidation proceeds are allocated internally rather than being supplied to the market.

In panel A, the market equilibrium rate is $R^* = x(K + p_L)$. In this case, date t_1 investors allocate their capital to the general technology. Even though high productivity firms are the best users of capital, they cannot attract any capital because the return in the market is higher than the maximum return they can credibly offer ($\lambda Y_H < R^*$).

Panel A assumes that the supply of funds is $K + p_L$. That is, only low productivity projects are liquidated. Notice that since good firms do not attract new capital in equilibrium, it would be socially optimal to liquidate medium productivity projects and supply their capital to the good projects. However, this reallocation does not happen voluntarily. Firms with medium productivity projects generate cash flows of Y_M if they continue. If these projects are liquidated and the unit recovered is invested in the market, they receive $R^* = x(K+p_L)$. Since $x(K+p_L) < Y_M$, firms with medium productivity projects do not liquidate but rather continue.

In sum, limited pledgeability distorts capital allocation in two ways. First, good firms cannot attract the capital that is available for allocation. And second, given that good firms cannot pledge enough cash, firms with medium productivity projects choose not to liquidate and supply their capital to the good firms.

How can firms with medium productivity projects be forced to liquidate their projects? As we argue in detail in the next section, this is precisely the role of external finance at date t_0. If the initial financing requirement is high enough, the only way firms are able to repay initial investors is by having some liquidation at date t_1. Moreover, it is less costly to liquidate projects of medium productivity than projects of high productivity. Thus, if liquidation is necessary for initial financing, the optimal contract will require medium projects to be liquidated first.

This situation is depicted in Panel B. In this panel the supply of capital is higher than in panel A ($K + T > K + p_L$) because of increased date t_0 financing requirements. In this case, the additional capital drives down the market return on capital so that good firms attract some funds. In other words, some of the capital from the liquidated medium projects finds its way to the good projects, potentially improving the aggregate payoff.

2.3 Characterization of the equilibrium

We solve the model backwards. Since at date t_2 no decisions are taken, we start by describing the equilibrium of the date t_1 reallocation market for any possible subgame. After liquidation has occurred, a subgame is defined by any set C of projects that continue and any arbitrary contract for each of these projects. For a continuing project j with realized productivity s, the relevant part of the contract is $\overline{P}_s^j = \lambda Y_s - D_{2s}^j$, the maximum that project j can offer. In what follows we drop the subscript s.

Lemma 1 *Given any set of projects C that continue and any contract for each $j \in C$, the unique equilibrium in the date t_1 reallocation market is as follows*

- *Define R^* by:*

$$x^{-1}(R^*) \quad + \int\limits_{\{j \in C | \overline{P}^j > R^*\}} dj \;\leq\; K + T \;\leq\; x^{-1}(R^*) \quad + \int\limits_{\{j \in C | \overline{P}^j \geq R^*\}} dj \tag{5}$$

  where x^{-1} is the inverse of x, and $x^{-1}(R) = 0$ for $R \geq x(0)$.

- *The capital allocated to the general technology is $\omega^* = x^{-1}(R^*) > 0$ and the allocation function r satisfies*
  - $r(P^j, \overline{P}^j) = 1$ *if* $P^j > R^*$,
  - $r(P^j, \overline{P}^j) = 0$ *if* $P^j < R^*$,
  - $r(P^j, \overline{P}^j) = 1$ *if* $P^j = R^*$ *and* $\overline{P}^j > R^*$ *and* $r(R^*, R^*) \in [0, 1]$ *is chosen to satisfy the market clearing condition in equation* **??**.[15]

- *Entrepreneur j's announcement is: $P^j = R^*$ for $\overline{P}^j \geq R^*$ and $P^j \in [0, \overline{P}^j]$ otherwise.*

Date t_1 investors allocate capital to maximize their payoff. They start allocating capital to the technologies offering the highest return and then allocate capital to technologies in decreasing order of the offered return. As a result of this maximization behavior, there is a cutoff value R^* such that all technologies offering more than R^* get capital for sure and technologies offering

[15] We do not need to define $r(R^*, \overline{P}^j)$ with $R^* > \overline{P}^j$ because R^* is not a feasible announcement in this case.

less than R^* do not get capital. For projects, this rule implies that $r = 1$ if $P^j > R^*$ and $r = 0$ if $P^j < R^*$. For the general technology this rule implies that ω^* satisfies $x(\omega^*) = R^*$. For projects offering exactly R^*, investor maximization does not impose any constraint on how capital is allocated among them. In case there is not sufficient capital to allocate one unit to each of them, we use the rule specified above that gives priority to projects with $\overline{P}^j > R^*$. The definition of R^* guarantees that the market clears and that $r(R^*, R^*) \in [0, 1]$.

When they are able to do so, entrepreneurs benefit from raising capital because, even when they offer investors the maximum return possible, they keep a fraction $(1 - \lambda)Y_s > 0$ of the cash flows. An entrepreneur with $\overline{P}^j > R^*$ receives capital for sure for any announcement $P^j \geq R^*$. Thus, it is optimal for him to offer only R^*. Entrepreneurs with $\overline{P}^j = R^*$ get capital with probability $r(R^*, R^*) \in [0, 1]$ when they announce R^*, do not get capital for a lower announcement and, due to limited pledgeability, cannot offer more than R^*. Thus, the best they can do is to offer R^*. Finally, entrepreneurs with $\overline{P}^j < R^*$ cannot raise capital with any feasible announcement and so are indifferent with regard to all announcements.

We now solve for the contracts offered at date t_0. We focus on symmetric equilibria in which all entrepreneurs offer the same contract. We assume that all entrepreneurs offer contract $\{q_s, D_{1s}, D_{2s}\}_{s=L,H,M}$ and use Lemma **??** to find the equilibrium return and allocation rule (R^* and r) generated by the proposed contract. For these R^* and r we find the best possible deviation $\{q_s^j, D_{1s}^j, D_{2s}^j\}_{s=L,M,H}$ by entrepreneur j and check that $\{q_s^j, D_{1s}^j, D_{2s}^j\}_{s=L,M,H} = \{q_s, D_{1s}, D_{2s}\}_{s=L,H,M}$. That is, that entrepreneur j has no incentives to deviate.

The best possible deviation by entrepreneur j, given R^* and r, solves the following problem. Since projects are of infinitesimal size, entrepreneur j's contract does not affect the return R^* or the function r. We let $r_s^j \equiv r(P_s^j, \overline{P}_s^j)$ where the announcement P_s^j and the function r are given in Lemma **??**. Since we assume that entrepreneurs have all the bargaining power at date t_0, entrepreneur j's optimal contract $\{q_s^j, D_{1s}^j, D_{2s}^j\}_{s=L,M,H}$ maximizes his payoff, U_E^j,

$$\textstyle\sum_s p_s \left\{ q_s^j (1 - D_{1s}^j) R^* + (1 - q_s^j)[Y_s + r_s^j (Y_s - D_{2s}^j - R^*)] \right\} \tag{6}$$

subject to the date t_0 investor's payoff, U_I^j being at least his reservation payoff,

$$\textstyle\sum_s p_s \left\{ q_s^j D_{1s}^j R^* + (1 - q_s^j) r_s^j D_{2s}^j \right\} \geq Z R^*. \tag{7}$$

When productivity is s, the project is liquidated with probability q_s^j. In this case the investor receives D_{1s}^j and the entrepreneur receives $1-D_{1s}^j$. They invest these amounts in the reallocation market and earn a return of R^*. This explains the first term in equations **??** and **??**. If the project is not liquidated, the firm always generates cash flows of Y_s. In addition, the firm receives additional capital with probability r_s^j, generates Y_s from that unit of capital and pays D_{2s}^j to the date t_0 investors and R^* to the date t_1 investor. D_{2s}^j is paid only when additional capital is allocated to the firm since the payoff from the first unit of capital is not pledgeable (by assumption). This explains the second term in equations **??** and **??**. The reservation payoff of the date t_0 investor is ZR^* since this investor can alternatively store his capital from date t_0 to date t_1 and then invest it in the reallocation market at a return of R^*.

We characterize the equilibrium contracts as a function of the degree of limited pledgeability λ and the financing requirement Z. In the text, we focus on the most interesting cases, in which the good projects are capital constrained (receive capital with probability less than 1).[16] The remaining cases are described in the appendix. Since we consider symmetric equilibria, the variable $\overline{P}^j$ can take only three values depending on the realized productivity of project s ($\overline{P}_s \equiv \lambda Y_s - D_{2s}$). The probability of receiving capital depends only on $\overline{P}^j$, thus it can also take only three values depending on the realized productivity of the project. We let these probabilities be r_H, r_M and r_L.

Lemma 2 *There is a function $\lambda_1(Z)$ such that the equilibrium contracts offered at date t_0 are:*

a. For $Z \leq p_L$, $(q_L, D_{1L}, D_{2L}) = (1, Z/p_L, 0)$; $(q_M, D_{1M}, D_{2M}) = (0,0,0)$; $(q_H, D_{1H}, D_{2H}) = (0,0,0)$. That is, entrepreneurs liquidate their projects for sure when productivity is low and do not liquidate their projects when productivity is medium or high. In this case, $T = p_L$.

b. For $p_L < Z \leq p_L + p_M$ and $\lambda \leq \lambda_1$, $(q_L, D_{1L}, D_{2L}) = (1,1,0)$, $(q_M, D_{1M}, D_{2M}) = (\frac{Z-p_L}{p_M}, 1, 0)$; $(q_H, D_{1H}, D_{2H}) = (0,1,0)$. That is, entrepreneurs liquidate their projects for sure when productivity is low and also liquidate their projects with some probability when productivity is medium. They never liquidate high productivity projects. In this case,

[16] We also assume an upper bound for the financing requirement, $Z \leq p_L + p_M$. We discuss later what happens for higher Z.

$T = Z$.

By market clearing, ω^* must satisfy $0 < \omega^* \leq 1 + K$. It follows that $x(1+K) \leq R^* < x(0)$ and, by assumptions **??** and **??**:[17]

$$Y_L \equiv 0 < R^* < Y_M < Y_H. \tag{8}$$

Clearly, the entrepreneur's problem is equivalent to maximizing

$$U_E^j + U_I^j = \sum_s p_s \left\{ q_s^j R^* + (1 - q_s^j)[Y_s + r_s^j (Y_s - R^*)] \right\}$$

subject to $U_I^j = ZR^*$. This implies that the entrepreneur benefits from liquidating a low productivity project, but loses if he liquidates either the medium or the high productivity project. If the capital requirements are low, $Z \leq p_L$, then liquidation of the low productivity project is sufficient to satisfy the participation constraint of the initial investor. This explains part a.[18]

However, if the capital requirements are high enough ($Z > p_L$), liquidation of medium or high productivity firms might be the only way to pay back the initial investor. Of course, a more efficient way to compensate the initial investor is by paying him out of the date t_2 payoff ($D_{2s} > 0$) since this avoids liquidation of H or M projects. However, this is not always possible. When pledgeability is very low, firms cannot get capital in the reallocation market and, since only the returns of the second unit are pledgeable, it is impossible to offer a fraction of the date t_2 payoffs. But the problem is more severe. The equilibrium has $D_{2H} = 0$, even when type H projects get capital in the reallocation market, as long as they get it with probability less than 1. The lower is D_{2H} (i.e., the higher is $\overline{P}_H$), the more likely a project is to raise capital since it can offer a higher return in the reallocation market (see Lemma **??**). If all the projects were offering $D_{2H} > 0$ and getting capital with probability less than one, a single firm would be better off deviating and offering a contract with a slightly lower D_{2H} (i.e., a slightly higher $\overline{P}_H$) so as to be able to outbid all the other firms. Thus when good firms are constrained ($r_H < 1$) or equivalently when $\lambda \leq \lambda_1$, competition for capital at date t_1 drives D_{2H} to 0. By the same logic, $D_{2M} = 0$ in this range.

[17] An important inequality in what follows is $R^* < Y_M$. As we explain in the text, this inequality follows from the market clearing condition and assumption **??**. However, this assumption is not necessary for the results. The reason is that R^* is increasing with λ (the simple demand and supply framework should convince the reader of this). Thus, for low enough λ, we would still have $R^* < Y_M$.

[18] Notice that part a) does not depend on λ being lower than λ_1.

Given that D_{2H} and D_{2M} are 0 in the range of λ considered, initial investors must be paid out entirely out of date t_1 liquidation proceeds. Liquidation of medium and high productivity firms reduce the combined payoff. Thus, when liquidation occurs, the entire liquidation proceeds should be given to the initial investors as this reduces the probability of liquidation. Since the medium project is less productive, it is better to start liquidating it first. It is only when liquidation of the low and medium productivity projects is not sufficient to satisfy the investor participation constraint that the good projects start being liquidated. When $Z \leq p_L + p_M$, liquidation of the low and medium productivity projects is sufficient to pay back the investor.[19] Notice that q_M is increasing in Z. The higher the financing requirement, the more frequent the medium project needs to be liquidated.

All the other cases are described in the appendix. For $\lambda > \lambda_1$, good projects can pledge a sufficiently high return to receive capital with probability 1. Since good projects are no longer capital constrained, there is no benefit in decreasing D_{2H} to outbid other firms. Thus, it becomes possible to sustain an equilibrium with $D_{2H} > 0$. In fact it is optimal to have as large a D_{2H} as possible since by paying the investor at date t_2, the entrepreneur avoids costly liquidation. For even higher levels of λ, it is possible to sustain equilibria with $D_{2M} > 0$. Finally, when the degree of pledgeability is very large, the entrepreneur is able to pay the investor out of date t_2 proceeds only.

Lemma **??** describes the equilibrium contract for each pair (λ, Z). Using these contracts in Lemma **??**, we obtain the equilibrium allocation r_H and r_M, and the equilibrium return in the reallocation market R^*. Since for parts (a) and (b) of Lemma **??**, $D_{2s} = 0$, the contracts in Lemma **??** imply that $T = p_L$ in case (a) and $T = Z$ in case (b).

This is precisely the cases we showed in Figure 2. Panel A corresponds to case (a) of Lemma **??** with $\lambda Y_H < R^* = x(K + p_L)$. In this case all the capital is allocated to the general technology since good projects cannot offer a sufficiently high return to attract capital. Thus, for low levels of pledgeability, good projects do not get all the capital they demand and, at the same time, medium projects are continued and their capital is not supplied to high productivity projects.

[19] When $Z > p_L + p_M$, some liquidation of the good project is necessary. In this case, we would have full liquidation of medium projects ($q_M = 1$) and partial liquidation of good projects ($q_H = (Z - p_L - p_M)/q_M$) in order to satisfy the financing requirement.

That is, low pledgeability leads to a sub-optimal level of external market activity. Panel B corresponds to case (b) Lemma **??** with $x(K+Z) \leq \lambda Y_H < x(K+Z-p_H)$. In this case, $\lambda Y_H = R^*$ and good firms are able to attract some capital. However, Z is not large enough to allow all good firms to raise one unit of capital.

Finally, it is important to note that the potential benefits of an increase in external finance are an externality to other firms. The firm that seeks outside finance does not benefit from it, because the market does not compensate this firm for the capital that it releases to the market. In this model, firms seek external finance only when they do not have sufficient funds to invest. This effect can be seen in the entrepreneur's maximization problem. Everything else constant, his payoff is decreasing in q_M. Therefore, introducing an outside investor that requires liquidation when the firm is of medium productivity is sub-optimal for the firm.

2.4 Comparative Statics in λ and Z

We now analyze what happens to the aggregate payoff when pledgeability and the financing requirements change. In the following proposition, the threshold λ_1 is the same as in Lemma **??**.

Proposition 1 *The aggregate payoff*

- *is non-decreasing in the level of pledgeability, λ,*
- *for $Z < p_L$, it is unaffected by the financing requirement Z,and*
- *for $p_L \leq Z < p_L + p_M$, there exists a $\lambda_0 < \lambda_1$ such that the aggregate payoff decreases with Z for low λ ($\lambda < \lambda_0$), increases with Z for intermediate levels of λ ($\lambda_0 \leq \lambda < \lambda_1$), and then decreases with (or is unaffected by) Z for high λ ($\lambda \geq \lambda_1$).*

Better pledgeability allows projects to raise capital more easily in the reallocation market. Since this capital would have ended up invested in the general technology, an increase in pledgeability leads to higher aggregate output. This result is consistent with Wurgler's (2000) finding that better investor protection leads to a more efficient allocation of capital.

For $Z < p_L$, only bad projects are liquidated in equilibrium. In this range an increase in the financing requirement does not affect q_M or q_H. Since the total amount of capital from liquidation proceeds remains unchanged, aggregate output is not affected.

For $Z \geq p_L$, increases in the initial financing requirement make it necessary to liquidate medium productivity projects more often. In case b) of Lemma **??**, the aggregate amount of liquidation is $T = Z$. Since the return of the medium project is higher than that of the general technology, this liquidation creates a social inefficiency when the released capital ends up invested in the general technology. This happens when pledgeability is low ($\lambda < \lambda_0$) since good firms are not able to offer a sufficiently high return to attract the released capital. If pledgeability is higher ($\lambda > \lambda_0$), good firms are able to attract some of the released capital, increasing the aggregate payoff. At pledgeability levels above λ_1, all good firms receive one unit in the reallocation market and so an increase in liquidation of medium projects cannot raise aggregate output.[20] [21]

The reason higher financing requirements improve the reallocation of capital is that, because of limited pledgeability, medium firms do not voluntarily liquidate their projects to invest in high productivity firms unless forced by outside investors. Outside investors require liquidation as this is the only way they can get their money back.[22] Even though this forced liquidation by outside investors is privately inefficient from the perspective of a medium firm, it is beneficial

[20]A similar result holds when $Z > p_L + p_M$. In such a case, either good firms are liquidated in equilibrium (for lower λ), or they are not capital constrained (for higher λ). In order to see why, notice that if good firms are capital constrained it must be that $D_{2H} = 0$ in equilibrium, leaving no chance that initial investors will get their money back without liquidation. But if $Z > p_L + p_M$, liquidation of bad and good firms is not enough, so good firms must also be liquidated. Thus, if all good firms continue they cannot be capital constrained, leaving no room for external finance to improve the allocation of capital.

[21]The fact that external finance cannot be beneficial when the pledgeability parameter is higher than λ_1 is driven by the fact that our model only has three types of technologies. When all the good firms receive one unit of capital, further reallocation is not possible. With a continuum of productivities, external finance would always allow capital to flow from lower to higher productivity firms. The potential benefit would only disappear when all firms receive the efficient amount of capital.

[22]In the model we assume for simplicity that the cash-flows from the first unit cannot be pledged. Since medium firms receive no additional capital, it is clear why an outside investor prefers to liquidate a medium firm rather than to allow such a firm to continue until the final date. However, this result would hold more generally if we allowed the first unit to be pledged. In order to see this, suppose that the same fraction λ of both units could be pledged. In all possible equilibria at which good firms are capital constrained, the return on capital x would be

for the economy because the additional capital supplied to the external market finds its way to a better user.

Proposition ?? makes it clear that external finance can only be beneficial when capital leaving mediocre projects actually reaches high productivity projects. Thus, when pledgeability is very low, the increase in liquidation of mediocre projects cannot improve the allocation of capital because the external capital market can never materialize. Conversely, if pledgeability is large enough, there is no need for external finance since good projects will attract capital anyway. On the other hand, for intermediate levels of pledgeability, the reallocation market has the potential to work well, but is constrained by the availability of funds to high productivity firms. For this range of the pledgeability parameter, the increase in liquidation brought about by external finance increases the efficiency of capital allocation.

These results are related to the literature on the benefits and costs of liquidation in an equilibrium context. The 'liquidationist' view (Schumpeter, 1942, and De Long, 1990) is consistent with our model when pledgeability is in a medium range ($\lambda_0 \leq \lambda < \lambda_1$). In this case, the liquidation of existing firms may be healthy for the economy because the released capital can be used more productively for new projects. However, as in Caballero and Hammour (1996, 1998, 1999), liquidation might also be excessive if the released factors end up being unemployed. In our model, this is the case when pledgeability is low ($\lambda < \lambda_0$). Thus, both views are consistent with our model for different degrees of institutional development.

Our results suggest that an increase in the external financing of firms in a given country can improve measures of the efficiency of capital allocation (such as the measure suggested by Wurgler, 2000). This result helps explain why countries where firms rely heavily on external finance are also countries with high productivity and growth. The traditional explanation for this correlation is that external finance allows firms to undertake investments over and above the investments that could be financed with internal funds (Demirguc-Kunt and Maksimovic, 1998). The mechanism in our model suggests an alternative and complementary explanation: an

given by:

$$x(\omega) \geq \lambda Y_H > \lambda Y_M$$

Thus, since $x(\omega) > \lambda Y_M$ it would still be true that outside investors strictly prefer to liquidate a medium firm that cannot get additional capital in the external market.

increase in external finance improves the flow of funds from lower to higher productivity projects, because of the equilibrium effect of higher liquidation. Countries with large external capital markets have high productivity and growth precisely because they do better at reallocating capital across firms and projects in the economy.

3 Internal capital markets and the equilibrium allocation of capital

The last section shows how limited pledgeability distorts the capital allocation decision. High productivity projects cannot get capital, while at the same time mediocre projects do not release their capital to the external market. An increase in external financing requirements can mitigate this misallocation of resources, but such an increase will not come about voluntarily because it is privately inefficient for a firm in isolation.

One possible response of the economic system to these intrinsic limitations of external capital markets is the creation of conglomerates. If conglomerates can reallocate capital internally from bad to good projects, they can potentially improve the flow of capital in an economy. In this section we extend the model of the previous section to analyze the effect of the degree of conglomeration (the fraction of projects in conglomerates) on the efficiency of capital allocation. Surprisingly, we find that an increase in conglomeration can be detrimental to the efficiency of capital allocation even when conglomerates are assumed to allocate capital to their best projects. This is because conglomerates, by reallocating capital among their divisions, reduce the supply of capital to the external market.

Whether internal capital markets are efficient is still an open question (see Gertner, Scharfstein and Stein, 1994, Stein 1997, Fluck and Lynch, 1999, Maksimovic and Phillips, 2001, 2002, and Matsusaka and Nanda, 2002 for arguments and evidence on efficient internal capital markets, and Shin and Stulz, 1998, Rajan, Servaes and Zingales, 2000 and Scharfstein and Stein, 2000 for evidence and arguments to the contrary).[23] In fact, whether internal allocation of capital in a conglomerate is privately efficient is not crucial for our arguments. Our assumption of efficient internal reallocation only makes it more difficult to find a cost of conglomeration. As we explain

[23] For a survey of this literature, see Stein (2001).

below, the feature of internal capital markets that drives our results is that conglomerates have a preference for internal reallocation that is not always aligned with social optimality. In our model, this preference is a direct consequence of limited pledgeability.

3.1 Introducing conglomerates and internal capital reallocation

We consider a simple extension of the model presented above. To focus on the conglomeration effect, we abstract from initial financing and assume that at date t_0 projects are already set up (they all have one unit of capital invested, with no external claims). In the model, a conglomerate is an organization with multiple projects in which a central authority ('headquarters') has control rights over the allocation of capital. Other types of organizations, such as business groups, also fit our definition of a conglomerate. We assume that there are no agency problems inside the conglomerate.

We let c be the fraction of the projects that are part of conglomerates. The remaining fraction of the projects are in single project firms ('stand alone' projects). We refer to c as the degree of conglomeration in the economy. Further, we assume that each conglomerate is composed of two projects. Thus, there are $1 - c$ stand alone projects and $c/2$ conglomerates. After the type of the projects is realized, there are six different types of conglomerates that we denote by (s, t) where $s, t = H, M, L$ refers to the productivity of the projects in the conglomerate.

In the date t_1 reallocation market, stand alone projects announce the price they pay for the first unit of capital (identical to the previous case). Conglomerates announce the price they pay for the first unit and second unit. After the announcements, date t_1 investors allocate their capital.

After the date t_1 reallocation market clears, conglomerates allocate their capital to their different projects. If a conglomerate raises additional capital, it decides which project to allocate it to. If it does not raise additional capital, a conglomerate can still take existing capital away from one project and reallocate it to the other.

Regarding pledgeability, we maintain the assumption that the maximum an investor can get from the date t_2 cash flows of a project of type s is a fraction λ of the returns generated by the additional units invested. Since there are no agency problems inside the conglomerate, a conglomerate of type (s,t) that raises one unit of capital allocates it to its best unit. Therefore, in the reallocation market, this conglomerate can credibly pledge up to $\lambda \max\{Y_s, Y_t\}$ for the first unit. This conglomerate can pledge up to $\lambda(Y_s + Y_t)/2$ per unit of capital when it raises two units.

As in the previous section, we assume that when there are more offers at a price of P and not enough capital for all of them, the allocation rule favors those conglomerates or firms that could potentially offer more.

3.2 The main mechanism

In this section we explain the main mechanism driving the results. The next two sections formalize this intuition. The most important result is that an increase in conglomeration may decrease the efficiency of the economy-wide capital allocation even when conglomerates allocate capital to their best projects.

We show that some conglomerates use only internal markets and do not participate in the external capital market. Thus, an increase in the degree of conglomeration introduces a trade-off. It reduces the number of transactions that take place in the external capital market, and increases the number that take place in the various internal capital markets. Whether increasing the degree of conglomeration is socially beneficial depends on which of these two markets (external or internal) does a better job of allocating capital.

It would seem that, since conglomerates allocate capital to their best units whereas external capital markets are plagued by pledgeability issues, it is always optimal to increase the degree of conglomeration. However, this is not always the case. Conglomeration might hurt the efficiency of capital allocation because the best project available to a conglomerate might not be the best project available to the economy as a whole. In this case, increasing the number of conglomerates exacerbates this capital allocation distortion. Conglomerate (M, L) always opts out of the market and transfers capital from a project of type L to a project of type M. This conglomerate prefers this internal allocation over supplying its funds to the market regardless of whether there are projects of type H in the market that do not receive capital. This is because, due to limited pledgeability, a project of type H cannot offer a sufficiently large fraction of the returns to make it worthwhile for the conglomerate to supply its capital (we have shown that $R^* < Y_M$). If instead these two projects were stand alone projects, the firm with a project of type L would have no better alternative than to supply its capital to the market. If the degree of pledgeability is not too low, this unit of capital will find its way to a project of type H. Therefore, as we show more precisely in the next two sections, it is possible that an increase in conglomeration will decrease the efficiency of economy-wide capital allocation.

3.3 Characterization of the equilibrium

We solve the model backwards. First, we analyze a conglomerate's decision to allocate capital after the reallocation market of date t_1 has cleared. Since there are no agency conflicts inside the conglomerate, headquarters allocate capital to maximize the conglomerate's payoff. The allocation rule inside the conglomerate is then as follows. A conglomerate that enters this stage with two additional units of capital allocates one to each project. A conglomerate that enters with only one unit allocates it to its higher productivity project. Finally, a conglomerate that enters with no additional capital transfers one unit of capital from its existing lower productivity project to its higher productivity project.

We now solve for the date t_1 equilibrium announcements and allocation, using the results from Section **??**. In Lemma **??**, the maximum a project j can offer in the reallocation market, $\overline{P}^j$, is an important determinant of the equilibrium. This case is similar. A stand alone project j with productivity s has $\overline{P}^j = \lambda Y_s$ (there are no contracts at date t_0 so $D^j_{2s} = 0$). If project j is in a conglomerate, it is the conglomerate and not the individual project that promises cash flows to investors. However, with a suitable definition of $\overline{P}^j$ that incorporates the information that project j is in a conglomerate, we are able to solve the equilibrium as if all projects were stand alones. This simplifies the analysis since we can directly use the results of Section **??**.

A conglomerate allocates the first unit of additional capital to its higher productivity project. Thus, for project j of type s that is in a conglomerate (s,t) with $Y_s > Y_t$, we define $\overline{P}^j = \lambda Y_s$. The same conglomerate (s,t) can offer a maximum of $\lambda(Y_s + Y_t)/2$ per unit when it raises two units of capital. Therefore for project j of type s in a conglomerate (s,t) with $Y_s \leq Y_t$, we define $\overline{P}^j = \lambda(Y_s + Y_t)/2$.

We apply this rule to all types of conglomerates and stand alones. As explained in Section **??**, $Y_L < R^* < Y_M < Y_H$. Thus, conglomerates of type (H, H), (M, M) and (H, M), and stand alone projects of type H and M do not liquidate, but rather demand funds from the market. Given the above discussion, a project j of productivity s in a conglomerate (s, s) has $\overline{P}^j = \lambda Y_s$ and project j of type H in a conglomerate (H, M) has $\overline{P}^j = \lambda Y_H$. That is, these projects can offer the same return as a stand alone of the same productivity, and thus, in equilibrium, receive capital with the same probability. However, project j of productivity M in a conglomerate (H, M) has $\overline{P}^j = \lambda(Y_H + Y_M)/2 > \lambda Y_M$. This project can offer more than a stand alone of type M since the conglomerate can use some of the cash flows from the type H project to raise capital for the type M project.[24] As a result, project M in a conglomerate (H, M) receives capital before any other project of type M.

A key difference between this model and that described in Section **??** is that not all firms participate in the market. In particular conglomerates (H, L) and (M, L) are (weakly) better off allocating capital internally than participating in the external capital market. This is because the best they can do in the market is to supply the unit of capital in their low productivity project and raise one unit for their higher productivity one. But, they do not need the market to accomplish this transaction. Furthermore, if the probability of raising capital in the external market is less than one, they strictly prefer internal reallocation.

Since $Y_L = 0 < R^*$, conglomerates (L, L) and all the stand alone projects L liquidate and supply their capital to the market. There are a number $(1 - c)p_L$ of stand alone projects of productivity L and a number $\frac{c}{2}(p_L)^2$ of conglomerates (L, L), each with two units of capital. The additional supply to the external capital market is then

$$T = (1 - c)p_L + cp_L^2.$$

When the degree is conglomeration is positive, $c > 0$, the additional capital supplied to the market from existing firms is lower than p_L $(T < p_L)$. Recall that in Section **??**, the amount of additional capital in the external market from existing projects was at least p_L since all projects with productivity L supplied their capital to the market. With external finance it was possible

[24]The conglomerate is thus relaxing the project M's external financing constraint. This effect is what Stein (2001) calls the "more money" effect of conglomeration. Fluck and Lynch (1999) and Inderst and Mueller (2002) also have models that feature a similar "more money" effect.

that $T > p_L$. With a positive degree of conglomeration, the opposite occurs. Conglomerates (H, L) and (M, L) opt out of the market because they prefer to allocate capital internally. As a result, not all the capital in type L projects is supplied to the external market. The higher the degree of conglomeration, the higher the number of projects that opt out of the market, and the smaller the amount of capital supplied.

Based on the discussion above, $\overline{P}^j$ takes only three values: λY_H, $\lambda(Y_H + Y_M)/2$, and λY_M. Projects with the same $\overline{P}^j$ offer the same price in the reallocation market and hence receive capital with the same probability. We define r_H, r_{HM}, and r_M as the probabilities that projects with $\overline{P}^j$ of λY_H, $\lambda(Y_H + Y_M)/2$, and λY_M receive capital, respectively. Applying Lemma **??**, with C being the set of projects that continue and participate in the market, the equilibrium r_H, r_{HM}, and r_M can be obtained (these expressions are derived in the appendix).

3.4 Aggregate payoff and conglomeration

The aggregate payoff in the economy, Π, can be written as:

$$\Pi = \omega x(\omega) + (1-c)\left[p_H(Y_H + r_H Y_H) + p_M(Y_M + r_M Y_M)\right] + \frac{c}{2}\left[\begin{array}{c} p_H^2(2)(Y_H + r_H Y_H) + p_M^2(2)(Y_M + r_M Y_M) + 2p_H p_M(2Y_H + r_H Y_M + r_{HM} Y_M) \\ +2p_H p_L(2Y_H) + 2p_M p_L(2Y_M) \end{array}\right]$$

where:

$$\omega = K + (1-c)p_L + cp_L^2 - r_H(p_H - cp_L p_H) - r_{HM}(cp_H p_M) - r_M((1-c)p_M + cp_M^2)$$

The first term is the cash flow generated by the general technology. The second term is the cash flow generated by the stand alone projects. Each stand alone project of type s generates Y_s, and with probability r_s receives one additional unit of capital that generates an additional Y_s. The third term is the cash flow generated by conglomerates. Each term inside the bracket is the cash flow generated by a type of conglomerate. The first term is the cash flow generated by conglomerates of type (H, H). Out of a number $c/2$ of conglomerates, a fraction p_H^2 are of type (H, H). Since, as explained above, each of its two projects receive capital with probability r_H, it generates $2(Y_H + r_H Y_H)$. The next term is the cash flow generated by the (M, M) conglomerate and is derived similarly. The third term inside the brackets is the cash flow generated by the fraction $2p_M p_H$ of conglomerates of type (H, M). When pledgeability is very low, a conglomerate (H, M) receives no capital. In this case, it transfers its existing unit in the type M project to the type H project and generates cash flows of $2Y_H$. For higher levels of pledgeability, this conglomerate receives one unit of capital with probability r_H. When it receives this unit, it allocates it to the H project and keeps the existing unit in the type M project to generate $2Y_H + Y_M$. Finally, for even higher levels of pledgeability, the conglomerate receives one unit for sure ($r_H = 1$) and receives a second unit with probability r_{HM}. When it receives two units, it generates $2Y_H + 2Y_M$. The expected cash flow generated by this type of conglomerate is then $2Y_H + r_H Y_M + r_{HM} Y_M$. Finally, the last two terms inside the brackets are the cash flows generated by conglomerates (H, L) and (M, L). These conglomerates never participate in the market. They simply transfer the existing unit of capital from their type L project to their higher productivity project.

Since we have derived expressions for r_H, r_{HM}, and r_M, we can find the effect of the degree of conglomeration in the aggregate payoff by differentiating Π with respect to c.

Proposition 2 *There are functions $\widehat{\lambda}_1(c) < \widehat{\lambda}_2(c)$ such that:*[25]

a. For $\lambda < \widehat{\lambda}_1(c)$, the aggregate payoff is increasing in the degree of conglomeration,

b. For $\widehat{\lambda}_1(c) < \lambda < \widehat{\lambda}_2(c)$, the aggregate payoff is decreasing in the degree of conglomeration, and

c. For $\lambda > \widehat{\lambda}_2(c)$, the aggregate payoff is non-decreasing in the degree of conglomeration.

For low levels of pledgeability, the external market does a poor job of allocating capital. Since conglomerates allocate capital to their best units irrespective of the level of pledgeability, a higher degree of conglomeration increases the number of efficient capital transfers that take place in the economy, thereby increasing the aggregate payoff. This explains part (a).

In region (b) the market does a better job of allocating capital than in region (a). In particular, the functions $\widehat{\lambda}_1$ and $\widehat{\lambda}_2$ are chosen such that, in this region, r_H is large but less

[25]This derivative does not exist at $\lambda = \widehat{\lambda}_1$ and $\lambda = \widehat{\lambda}_2$.

than 1. The reason why conglomeration is detrimental to the aggregate payoff is that the best project available to a conglomerate might not be the best project available to the economy as a whole. Therefore, increasing the number of conglomerates exacerbates the existing distortion in capital allocation. In particular, conglomerates of type (M, L) always opt out of the market. These conglomerates allocate the capital from their type L project to their type M project. However, if instead these projects were stand alone projects, the capital in the type L project would be supplied to the market. Since in this range of pledgeability the external capital market works relatively well, these units of capital would find their way to type H projects, thereby increasing the aggregate payoff. This reasoning does not apply to case (a). In this case, due to low pledgeability, external capital markets do a poor job of allocating capital and therefore, if the conglomerates were separated, the unit released would not find its way to a good project.

As in Section **??**, the bias towards internal investment is due to limited pledgeability. A conglomerate of type (M, L) does not supply capital to the market but rather allocates it internally (even when there is unsatisfied demand from projects of type H), since projects of type H cannot offer a sufficiently high return. In this range, the maximum return a type H project can offer is lower than the cash flows a project of type M generates (i.e., $\lambda Y_H < Y_M$) and thus it is privately optimal for the conglomerate to allocate internally whereas it is socially optimal to supply its capital to the market.

Finally, for higher levels of pledgeability, increasing conglomeration is never detrimental to the aggregate payoff. This is because for high levels of pledgeability, all the projects of type H receive capital in the external capital market ($r_H = 1$). As a result, transfers that take place inside the (M, L) conglomerates are no longer suboptimal since the best project in need of capital for the economy as a whole is now project M.

In sum, for intermediate levels of pledgeability, an increase in conglomeration worsens capital allocation by further reducing external market activity.

3.5 Business groups and the allocation of capital

Although we did not explicitly discuss it, in our model firms have private incentives to conglomerate since, by creating an internal capital market, project-owners increase the probability of receiving capital. Furthermore, this incentive is higher when the external market works poorly, or equivalently, when pledgeability is low. This result is consistent with the evidence on business groups. These organizations, which are prevalent in developing countries where investor protection is typically poor, have internal capital markets (Leff 1976, Khanna and Palepu 1997). Claessens, Djankov, Fan, and Lang (2000) find that 68 percent of listed firms in 9 Asian economies belong to business groups. The listed companies of a single group in India, the Tata group, accounts for approximately 8% of the country's public companies (Khanna and Palepu, 1997).[26] [27]

The novel implications of our model regard the effect of conglomeration on the efficiency of economy-wide capital allocation. Conglomerates impose a negative externality to other firms because internal reallocation dries up the external market and makes it more difficult for good projects to raise funds. The large size of business groups suggest that their potential for reducing the depth of the external capital market can be a first order effect.

Even if we assume that conglomerates allocate capital to their best units, we show that this negative externality can be sufficiently strong to outweigh the benefits of efficient internal capital markets. This is the case when investor protection is in an intermediate range. In this range, the external capital market has the potential to work well when sufficient capital is supplied. Therefore, an increase in conglomeration, by drying up the external market, can have a large negative effect on capital allocation. However, when investor protection is low, regardless of the amount of capital supplied, the external market never materializes. As a result, the degree of conglomeration has no impact on the efficiency of external reallocation. In this range, the benefits of efficient internal capital markets dominate the negative externality.

These results shed light on a recent debate about whether business groups should be dismantled (Khanna and Palepu 1999). An argument against dismantling business groups is that their internal capital markets provide a substitute for the lack of external capital markets. The

[26] Hubbard and Palia (1999) argue that even in the U.S. the merger wave of the 1960s was motivated by a desire to overcome the absence of well-developed capital markets at that time. However, it should be mentioned that Matsusaka (1993) offers a technological interpretation for the merger wave of the 1960s (managerial synergies).

[27] Stein (1997) also conjectures that in countries where external markets are underdeveloped, conglomerates are likely to be socially optimal because it is more important to ensure that the limited funds available are efficiently reallocated across projects.

results in this section show that, even when conglomerates allocate capital to their best units, their presence can hurt economy wide capital allocation by reducing external market activity. This result might help explain why governments may need to take an active role in dismantling conglomerates. Individual conglomerates have no incentives to dismantle since they would not be properly compensated by the capital they provide to the external market.

We should note that other models also have the implication that, as the financing-related benefits of conglomeration decrease, costs of conglomeration such as less effective monitoring (Stein, 1997), coordination costs (Fluck and Lynch, 1999) and free cash-flow problems (Matsusaka and Nanda, 2002) make conglomerates less desirable. However, our model is novel in two ways.

First, most of the literature focuses on conglomerates in isolation and thus is unable to analyze the effect of conglomeration on economy-wide capital allocation.[28] More importantly, an implication of our main result is that it is not possible to extrapolate results about the efficiency of capital allocation from models of conglomerates in isolation. In our set up conglomerates are assumed to allocate capital efficiently. Yet, for intermediate levels of pledgeability, increases in conglomeration actually hurt economy-wide capital allocation.

Second, our model predicts that the (social) costs of conglomeration are the highest when the underlying imperfection (cash-flow pledgeability) is at an *intermediate* range. Other models, such as Stein (1997) and Fluck and Lynch (1999), imply a monotonic relationship between the underlying imperfection and the benefits of conglomeration. The novel empirical implication is that conglomeration is more costly for countries whose institutions are in a process of development, and not necessarily for countries with very well developed capital markets such as the US in the 1990s.

[28]Maksimovic and Phillips (2001, 2002) are an exception. They analyze the allocation of resources by conglomerates in an equilibrium context. However, their model is based on differences in managerial and organizational abilities (as in Matsusaka, 2001), with no role for financial market imperfections.

4 Conclusion

We analyze the allocation of capital to investment projects in an equilibrium context. This allocation is affected by an imperfection at the firm level: due to poor institutional development and investor protection, project-owners can only pledge a fraction of the returns generated by the additional investment. As a result, the productivity of projects is higher than the return project-owners can pledge to outsiders. This wedge distorts the flow of capital across projects and firms in the economy. Firms may not be able to raise finance for their good projects. At the same time, rather than supply their capital to the market, other firms fail to liquidate mediocre projects. Moreover, conglomerates allocate additional resources to their mediocre projects. That is, lack of institutional development reduces external market activity by biasing firms' decisions towards internal investment.

Stimulating external market activity can improve the allocation of capital. In this paper, we study two scenarios: the need for external finance and deconglomeration. External finance increases the supply of capital in the reallocation market because external investors liquidate projects too often. Conglomeration affects the supply of capital because of the possibility of internal reallocation.

The benefits of external finance and deconglomeration come about because they increase activity in the external capital market, and improve its allocative role. Interestingly, these benefits occur even though external finance and deconglomeration are always privately inefficient. We show in the paper that whether these social benefits dominate private costs depends on the degree of pledgeability. When pledgeability is too low, the costs dominate the benefits. Pledgeability is one variable which will be related to the efficiency of reallocation, but in general any variables which affect reallocation will have a similar effect. This suggests the following conjectures.

The ease of reallocating capital is correlated with the flexibility of the market for corporate assets (mergers, divestitures, etc.). In countries where these markets are large, active and relatively flexible, it is more likely that external finance and deconglomeration will have the kinds of benefits that we are describing here. Having more external finance, for example, may force firms to either acquire more assets or to divest, and this may lead to an improvement in the allocation of resources.

The increase in activity in external capital markets that generates our results can also be generated by "short-term" outside investors. Firms would never voluntarily choose to be short-termist, since this would bias decisions against long term projects. However, short-termism might be socially beneficial because it might force firms to liquidate investments instead of betting on a long term project. This might improve the efficiency of the reallocation market because it increases the supply of funds to be reallocated, and may in fact lead to a higher social payoff.

Our results also have implications for the literature on the boundaries of the firm. Bolton and Scharfstein (1998) and Stein (2001) have recently argued for a "capital-allocation-centric" view of the theory of the firm. Their argument is that a collection of assets (such as our projects) should reside under a single roof if internal capital markets do a better job of allocating capital to these projects than does the external capital market. Our model suggests that such privately optimal boundaries might not be socially efficient once we embed assets and firms in an equilibrium model of capital allocation.

On a more abstract level, our paper shows the need to model financial imperfections in an equilibrium framework in order to derive positive and normative implications about overall economic efficiency because the intuition obtained from partial equilibrium models does not always apply directly. Our results that the need for external finance might be beneficial and that the presence of efficient conglomerates might be detrimental to capital allocation go against the intuition obtained from partial equilibrium models.

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5 Appendix

Proof of Lemma **??**

Suppose any cutoff R^*. For $\overline{P}^j \geq R^*$, we can rule out all announcements except $P^j = R^*$. First, if $\overline{P}^j > R^*$, then $P^j > R^*$ is not an equilibrium announcement because the entrepreneur is better off offering $P^j - \epsilon > R^*$ and still receiving one unit of capital (recall that the allocation rule is $r = 1$ if $P^j > R^*$). For $\overline{P}^j = R^*$, $P^j > R^*$ is not feasible. Second, $P^j < R^*$ is not an equilibrium announcement either because the entrepreneur gets no capital for sure whereas by offering $P^j = R^*$ he gets capital with positive probability (in general). For $\overline{P}^j \leq R^*$, the announcement is irrelevant. This proves that if the equilibrium exists, it is unique.

The only way that $P^j = R^*$ for firms with $\overline{P}^j \geq R^*$ is an equilibrium is if $r^j = 1$ for $\overline{P}^j > R^*$. If this were not true, i.e., $r^j < 1$ for $\overline{P}^j > R^*$, then any firm with $\overline{P}^j > R^*$will benefit by deviating from $P^j = R^*$ to $P^j = R^* + \epsilon$ with $\epsilon > 0$ very small. This deviation increases the probability of getting capital from r to 1 and only increases the offer price by ϵ. Since ϵ can be made arbitrarily small, this is a profitable deviation.

Finally, equation **??** guarantees that R^* is set so as to satisfy $r^j = 1$ for $\overline{P}^j > R^*$ and market clearing.

We state the following Lemma that covers all cases for $Z < p_L + p_M$. The version of Lemma **??** described in the main text is a special case.

Lemma 2 *There are functions $\lambda_k(Z)$ for $k = 1, 2, 3, 4, 5$ such that equilibrium contracts offered by entrepreneurs at date t_0 are:*

a. *For $Z \leq p_L$, $(q_L, D_{1L}, D_{2L}) = (1, Z/p_L, 0)$; $(q_M, D_{1M}, D_{2M}) = (0, 0, 0)$; $(q_H, D_{1H}, D_{2H}) = (0, 0, 0)$*
b. *For $p_L < Z \leq p_L + p_H \frac{Y_H}{Y_M} - p_H$, $(q_L, D_{1L}, D_{2L}) = (1, 1, 0)$, $D_{1M} = D_{1H} = 1$, $q_H = 0$ and*
   1. *For $\lambda \leq \lambda_1$, $q_M = \frac{Z - p_L}{p_M}$, $D_{2M} = D_{2H} = 0$*
   2. *For $\lambda_1 < \lambda \leq \lambda_2$, $q_M = \frac{Z - \frac{p_H D_{2H}}{\widehat{R}} - p_L}{p_M}$, $D_{2M} = 0$, and $D_{2H} = \lambda Y_H - \widehat{R}$, where $\widehat{R}$ is defined by $x(K + Z - \frac{\lambda Y_H}{\widehat{R}} p_H) = \widehat{R}$*
   3. *For $\lambda_2 < \lambda$, $q_M = 0$, $D_{2M} = 0$, $D_{2H} = \frac{\widehat{R}(Z - p_L)}{p_H}$, where*
$$\widehat{R} = \begin{cases} x(K + p_L - p_H) & \text{if } \lambda Y_M < x(K + p_L - p_H) \\ \lambda Y_M & \text{if } x(K + p_L - p_H) \leq \lambda Y_M \leq x(K + p_L - p_H - p_M) \\ x(K + p_L - p_H - p_M) & \text{if } \lambda Y_M > x(K + p_L - p_H - p_M) \end{cases}$$
c. *For $p_L + p_H \frac{Y_H}{Y_M} < Z \leq p_L + p_M$, $(q_L, D_{1L}, D_{2L}) = (1, 1, 0)$, $D_{1M} = D_{1H} = 1$, $q_H = 0$ and*
   1. *For $\lambda \leq \lambda_1$, the contract is identical to that in case b.1*
   2. *For $\lambda_1 < \lambda \leq \lambda_3$, the contract is identical to that in case b.2*
   3. *For $\lambda_3 < \lambda \leq \lambda_4$, $q_M = \frac{Z - \frac{p_H D_{2H}}{\widehat{R}} - p_L}{p_M}$, $D_{2M} = 0$, and $D_{2H} = \lambda Y_H - \widehat{R}$, where $\widehat{R} = \lambda Y_M$*
   4. *For $\lambda_4 < \lambda \leq \lambda_5$, $q_M = \frac{Z - \frac{\lambda}{\widehat{R}}(p_H Y_H + p_M Y_M) - p_L + p_M + p_H}{p_M(2 - \frac{\lambda}{\widehat{R}} Y_M)}$, $D_{2M} = \lambda Y_M - \widehat{R}$, and $D_{2H} = \lambda Y_H - \widehat{R}$, where $\widehat{R}$ satisfies $x(K + 2(p_L + p_M q_M(\widehat{R})) - 1) = \widehat{R}$*
   5. *For $\lambda > \lambda_5$, $q_M = 0$, and D_{2M} and D_{2H} can be anything that satisfies $p_M D_{2M} + p_H D_{2H} + p_L \widehat{R} = Z\widehat{R}$ and $D_{2s} \leq \lambda Y_s - \widehat{R}$ where $\widehat{R}$ is defined by $\widehat{R} = x(K + p_L - p_H - p_M)$*

Proof

In the proof, as in the text, variables with superscript j denote the optimal level from the maximization problem (equation **??** subject to equation **??**), whereas variables with no superscript j denote proposed equilibrium variables. We rewrite the maximization problem for ease of reference. Entrepreneur j maximized his payoff

$$U_E^j = \textstyle\sum_s p_s \left\{ q_s^j (1 - D_{1s}^j) R^* + (1 - q_s^j)[Y_s + r_s^j (Y_s - D_{2s}^j - R^*)] \right\} \tag{9}$$

subject to the participation constraint of the date t_0 investor

$$U_I^j = \sum_s p_s \left\{ q_s^j D_{1s}^j R^* + (1 - q_s^j) r_s^j D_{2s}^j \right\} \geq ZR^* \tag{10}$$

We define $\lambda_1 \equiv x(K + Z - p_H)/Y_H$, $\lambda_2 \equiv \frac{x(K+p_L-p_H)}{Y_H} \quad 1 + \frac{Z-p_L}{p_H} \quad$, $\lambda_3 \equiv \frac{x(K+Z-p_H \frac{Y_H}{Y_M})}{Y_M}$, $\lambda_4 \equiv x(K + 2(Z - \frac{p_H Y_H}{Y_M} + p_H) - 1)/Y_M$ and $\lambda_5 \equiv \quad \frac{Z-p_L+p_H+p_M}{p_H Y_H + p_M Y_M} \quad x(K + p_L - p_M - p_H)$.

Step 1: $\frac{\partial U_E^j}{\partial q_L^j} \geq 0$ (with equality only when $D_{1L}^j = 1$), $\frac{\partial U_E^j}{\partial q_s^j} < 0$ (for $s = H, M$) and U_E^j is weakly decreasing in D_{2s}^j (for $s = H, M$, and it is strictly decreasing when $q_s^j < 1$ and $r_s^j > 0$)

$\overline{P}_L^j = 0$ by equations **??** and **??**. Thus $\overline{P}_L^j < R^*$ and, by Lemma **??**, $r_L^j = 0$. Now, $\frac{\partial U_E^j}{\partial q_L^j} = (1 - D_{1L}^j)R^* \geq 0$. Also, for $s = H, M$, $Y_s > R^*$ and thus $\frac{\partial U_E^j}{\partial q_s^j} = (1 - D_{1s}^j)R^* - [Y_s + r_s^j(Y_s - D_{2s}^j - R^*)] < 0$. Finally, an increase in D_{2s}^j for $s = H, M$ has two effects on U_E^j. The direct effect weakly decreases U_E^j. The indirect effect works by weakly decreasing r_s^j (because $\overline{P}_s^j = \lambda Y_s - D_{2s}^j$ decreases and, by Lemma **??**, r_s^j weakly decreases), which in turn weakly decreases U_E^j.

Step 2 (Proof of part a): If $Z \leq p_L$ then $(q_L^j, D_{1L}^j, D_{2L}^j) = (1, Z/p_L, 0)$ and $(q_M^j, D_{1M}^j, D_{2M}^j) = (q_H^j, D_{1H}^j, D_{2H}^j) = (0, 0, 0)$

From step 1, every entrepreneur maximizes the objective function by setting $q_L^j = 1$, $q_M^j = q_H^j = 0$ and $D_{2M}^j = D_{2H}^j = 0$. Since $q_M^j = q_H^j = 0$, D_{1M}^j and D_{1H}^j are irrelevant (we set them to zero). Also, since $q_L^j = 1$, D_{2L}^j is irrelevant (we set it to zero). Finally since $q_L^j = 1$, $D_{1L}^j = Z/p_L$ the participation constraint is satisfied. Therefore, the above is the optimal contract for every entrepreneur j.

We now consider the cases where $p_L < Z \leq p_L + p_M$.

Step 3: For $Z > p_L$, then $(q_L^j,\ D_{1L}^j, D_{2L}^j) = (1, 1, 0)$ and $D_{1M}^j = D_{1H}^j = 1$.

Since $Z > p_L$, then by the participation constraint, it must be that $\max\{q_M^j D_{1M}^j, q_H^j D_{1H}^j, (1-q_M^j) r_M^j D_{2M}^j, (1-q_H^j) r_H^j D_{2H}^j\} > 0$. First, we show that $q_L^j = 1$. Suppose not, i.e., $q_L^j < 1$. If $D_{1L}^j < 1$ then an increase in q_L^j increases U_E^j while not violating the participation constraint (U_I^j is also increasing in q_L^j). Contradiction. If $D_{1L}^j = 1$ an increase in q_L^j increases U_I^j, relaxing the participation constraint ($U_I^j > ZR^*$). Since $Z > p_L$, at least one of the following holds: $q_s^j D_{1s}^j > 0$ ($s = H, M$) or $(1 - q_s^j) r_s^j D_{2s}^j > 0$ ($s = H, M$). If the first case holds, then a slight decrease in q_s^j increases U_E^j without violating the constraint. Contradiction. If the second case holds, then a slight decrease in D_{2s}^j increases U_E^j (since it must be that $q_s^j < 1$ and $r_s^j > 0$) without violating the constraint. Contradiction.

Now we prove that $D_{1L}^j = 1$. Suppose not, i.e., $D_{1L}^j < 1$. Consider an increase in D_{1L}^j to $D_{1L}^j + \epsilon$ with $\epsilon > 0$. Again, since $Z > p_L$, at least one of the following holds: $q_s^j D_{1s}^j > 0$ ($s = H, M$) or $(1 - q_s^j) r_s^j D_{2s}^j > 0$ ($s = H, M$). Suppose the first case holds. Since $q_s^j > 0$, it must be that U_I^j is increasing in q_s^j or $D_{1s}^j R^* > r_s^j D_{2s}^j$ (otherwise setting $q_s^j = 0$ would increase the objective function and relax the constraint). Entrepreneur j can decrease q_s^j to $q_s^j - \delta$ where $\delta = \epsilon \frac{p_L R^*}{p_s (D_{1s}^j R^* - r_s^j D_{2s}^j)}$ is chosen so as to leave the participation constraint unchanged. Note that

$\delta > 0$ since $D^j_{1s}R^* > r^j_s D^j_{2s}$. The change in the objective function is given by: $\delta p_s(Y_s + r^j_s(Y_s - R^*) - R^*) > 0$. Contradiction. If the second case holds, a similar procedure shows a contradiction.

Next we show that $D^j_{1s} = 1$ for $s = M, H$. First, if $q^j_s = 0$ then D^j_{1s} is irrelevant (we set it to 1). Suppose that $q^j_s > 0$ and $D^j_{1s} < 1$. As before, since $q^j_s > 0$, $D^j_{1s}R^* > r^j_s D^j_{2s}$. An entrepreneur can increase D^j_{1s} to $D^j_{1s} + \epsilon$ and decrease q^j_s to $q^j_s - \delta$ where $\delta = \epsilon \frac{q^j_s R^*}{D^j_{1s}R^* - r^j_s D^j_{2s} + \epsilon R^*} > 0$ is chosen so as to leave the participation constraint unchanged. The change in the objective function is $\delta p_s(Y_s + r^j_s(Y_s - R^*) - R^*) > 0$. Contradiction.

We only need to find q^j_M, q^j_H, D^j_{2M} and D^j_{2H} to complete the characterization of the contract. We first show a preliminary and intuitive result that firms of type M are liquidated before firms of type H:

Step 4: If $r^j_M D^j_{2M} \leq r^j_H D^j_{2H}$ and $r^j_M \leq r^j_H$ then $q^j_H > 0$ implies $q^j_M = 1$.[29] This, together with $Z \leq p_L + p_M$, implies that $q_H = 0$.

Suppose not, i.e., $r^j_M D^j_{2M} \leq r^j_H D^j_{2H}$, $r^j_M \leq r^j_H$, $q^j_H > 0$ and $q^j_M < 1$. Entrepreneur j can deviate by increasing q^j_M to $q^j_M = q^j_M + \epsilon$ and decreasing q^j_H to $q^j_H = q^j_H - \delta$ with $\delta = \epsilon \frac{p_M(R^* - r^j_M D_{2M})}{p_H(R^* - r^j_H D_{2H})}$. The choice of δ guarantees that the participation constraint is unchanged. $r^j_M D^j_{2M} \leq r^j_H D^j_{2H}$ implies that $p_H\delta \geq p_M\epsilon$. The change in the objective function is given by $p_H\delta(1 + r^j_H)(Y_H - R^*) - p_M\epsilon(1 + r^j_M)(Y_M - R^*) > 0$. The inequality follows from $Y_H - R^* > Y_M - R^* > 0$ (by equation **??**), $r_H \geq r_M$ and $p_H\delta \geq p_M\epsilon$. Contradiction. This step implies that firm of type M is liquidated first and only then is a firm of type H liquidated. Since $Z \leq p_L + p_M$, then $q_H = 0$.

In what follows, the conditions of step 4 always hold so $q^j_H = 0$. With all these preliminary results we are ready to prove the different cases of the Lemma. But first, we describe the equilibrium R^*, r_H and r_M. The following table is derived assuming that $\overline{P}_H \geq \overline{P}_M$ (which will always be the case) and uses equations **??** and **??**.

Case	Range	R^*	r_H	r_M
1	$x(K+T) > \overline{P}_H$	$x(K+T)$	0	0
2	$x(K+T) \leq \overline{P}_H \leq x(K+T-p_H)$	$\overline{P}_H$	$\frac{K+T-x^{-1}(\overline{P}_H)}{p_H}$	0
3	$\overline{P}_H > x(K+T-p_H)$ and $\overline{P}_M < x(K+T-p_H)$	$x(K+T-p_H)$	1	0
4	$x(K+T-p_H) \leq \overline{P}_M \leq x(K+T-p_H-p_M)$	$\overline{P}_M$	1	$\frac{K+T-p_H-x^{-1}(\overline{P}_M)}{p_M}$
5	$\overline{P}_M > x(K+T-p_H-p_M)$	$x(K+T-p_H-p_M)$	1	1

Table 1: R^*, r_H and r_M as a function of T, $\overline{P}_H$ and $\overline{P}_M$.

Step 5 (proof of part b.1 and c.1): For $p_L < Z \leq p_L + p_M$ and $\lambda \leq \lambda_1$ the equilibrium contract has $(q_M, D_{2M}) = (\frac{Z-p_L}{p_M}, 0)$ and $(q_H, D_{2H}) = (0, 0)$.

[29]It can be shown that in any equilirbium $r_M D_{2M} \leq r_H D_{2H}$ and $r^j_M \leq r^j_H$. Thus, $q_H > 0$ always implies $q_M = 1$. However, for the rest of the proof, this less general result is sufficient.

First, we find the equilibrium given the contracts. Assuming all entrepreneurs offers the same contract, total liquidation is $T = p_L + q_M p_M + q_H p_H = Z$. Also $\overline{P}_H = \lambda Y_H$ and $\overline{P}_M = \lambda Y_M$. Now $\lambda \leq \lambda_1$ is equivalent to $\lambda Y_H \leq x(K + Z - p_H)$. By Table 1, $R^* \geq \lambda Y_H$.

Second, assuming every entrepreneur is offering the proposed contract we solve for entrepreneur's j optimal deviation (by maximizing equation **??** subject to equation **??**). Setting $D^j_{2H} > 0$ is not optimal. It guarantees that $r^j_H = 0$ and so it does not increase the payoff nor does it relax the participation constraint. Similarly, $D^j_{2M} > 0$ is not optimal. Finally, by the participation constraint $q^j_M = \frac{Z-p_L}{p_M}$. Since $(q^j_s, D^j_{2s})_{s=H,M} = (q_s, D_{2s})_{s=H,M}$, we have, in fact, an equilibrium.

Step 6 (proof of part b.2 and c.2): For $\lambda_1 < \lambda \leq \min\{\lambda_2, \lambda_3\}$ (this region is non-empty), every entrepreneur sets $(q_M, D_{2M}) = (\frac{Z - \frac{p_H D_{2H}}{\widehat{R}} - p_L}{p_M}, 0)$ and $(q_H, D_{2H}) = (0, \lambda Y_H - \widehat{R})$, where the market return $\widehat{R}$ is the solution to $x(K + Z - \frac{\lambda Y_H}{\widehat{R}} p_H) = \widehat{R}$.

First, we compute the equilibrium R^* given the contracts. Total liquidation is given by $T = p_L + p_M q_M = Z - \frac{p_H D_{2H}}{\widehat{R}} = Z - \frac{p_H \lambda Y_H}{\widehat{R}} + p_H$. Since $\overline{P}_H = \lambda Y_H - D_{2H} = \widehat{R} = x(K + Z - \frac{\lambda Y_H}{\widehat{R}} p_H) = x(K + T - p_H)$,this case always falls under case 2 of Table 1 . Thus $r_M = 0$ and $r_H = \frac{K+T-x^{-1}(\overline{P}_H)}{p_H} = \frac{K+T-(K+T-p_H)}{p_H} = 1$. Also, according to Table 1, $R^* = \overline{P}_H = \widehat{R}$. The other two variables of interest can be written now as

$$D_{2H} = \lambda Y_H - R^* \tag{11}$$

and,

$$T = Z - \frac{p_H D_{2H}}{R^*} \tag{12}$$

To analyze the properties of R^*, we define

$$F(R, \lambda) = x(K + Z - \frac{\lambda Y_H}{R} p_H) - R. \tag{13}$$

For any λ, the equilibrium $R^*(\lambda)$ satisfies $F(R^*(\lambda), \lambda) = 0$. Next, by the implicit function theorem, $\frac{\partial R^*}{\partial \lambda} = -\frac{F_\lambda}{F_R} > 0$. This implies, by equation **??** that $\frac{\partial T}{\partial \lambda} < 0$ and consequently $\frac{\partial q_M}{\partial \lambda} < 0$. Since $\frac{\partial R^*}{\partial \lambda} > 0$ and $\frac{\partial T}{\partial \lambda} < 0$, by equation **??**, it must be that $\frac{\partial D_{2H}}{\partial \lambda} > 0$.

Since Table 1 assumes that $\overline{P}_H \geq \overline{P}_M$, we need to show that this is true. Since $\overline{P}_H = R^*$ and $D_{2M} = 0$, it suffices to show that $R^* \geq \lambda Y_M$. The function $F(\lambda Y_M, \lambda) = x(K + Z - \frac{Y_H}{Y_M} p_H) - \lambda Y_M$ is strictly decreasing in λ and $F(\lambda_3 Y_M, \lambda_3) = 0$. Therefore $F(\lambda Y_M, \lambda) > 0$ for $\lambda < \lambda_3$. But since $F_R < 0$,it must be that $R^*(\lambda) > \lambda Y_M$ for all $\lambda < \lambda_3$.

Now, we prove that contract is feasible, i.e., that $0 \leq q_M \leq 1$ and $D_{2H} \geq 0$. First, we consider q_M. Since $F(x(K + p_L - p_H), \lambda_2) = 0$, then $R^*(\lambda_2) = x(K + p_L - p_H)$. Plugging the value of λ_2 and that of R^* in equations **??** and **??**, leads to $T = p_L$ or $q_M = 0$. Since $\frac{\partial q_M}{\partial \lambda} < 0$, then for $\lambda \leq \lambda_2$, $q_M > 0$. The result that $q_M \leq 1$ follows from $Z \leq p_L + p_M$. In the range considered $D_{2H} \geq 0$. Since $R^*(\lambda_1) = \lambda_1 Y_H$ then at $\lambda = \lambda_1$, $D_{2H} = 0$. $D_{2H} \geq 0$ follows from the result shown above that $\frac{\partial D_{2H}}{\partial \lambda} > 0$.

Finally, given the equilibrium, we solve for entrepreneur j's optimal deviation $(q^j_s, D^j_{2s})_{s=H,M}$. Since $R^* > \lambda Y_M$ for the range considered then $r^j_M = 0$ regardless of D^j_{2M}. We set $D^j_{2M} = 0$. By setting $D^j_{2H} = \lambda Y_H - R^*$, the entrepreneur raises one unit with probability 1 ($r_H^j = 1$). The entrepreneur does not gain by lowering D_{2H}^j since doing so leaves r_H^j unchanged and the entrepreneur needs to raise q_M^j to compensate the date t_0 investor. Also, the entrepreneur does not gain by increasing D_{2H}^j since, in that case, $\overline{P}_H^j < R^*$ and $r_H^j = 0$. This would decrease his payoff and would require more liquidation to compensate the date t_0 investor as well. From the participation constraint (it holds with equality) it follows that $q_M^j = \frac{Z - \frac{p_H D_{2H}}{\widehat{R}} - p_L}{p_M}$. Since $(q_s^j, D_{2s}^j)_{s=H,M} = (q_s, D_{2s})_{s=H,M}$, we have, in fact, an equilibrium.

Step 7 (proof of part b.3): If $p_L < Z \leq p_L + p_H \frac{Y_H}{Y_M} - p_H$, then for $\lambda > \lambda_2$, every entrepreneur sets $q_M = 0$,

$$D_{2M} = 0,\ D_{2H} = \frac{\widehat{R}(Z - p_L)}{p_H}, \text{ where } \widehat{R} = \begin{cases} x(K + p_L - p_H) & \text{if } \lambda Y_M < x(K + p_L - p_H) \\ \lambda Y_M & \text{if } x(K + p_L - p_H) \leq \lambda Y_M \leq \\ & \leq x(K + p_L - p_H - p_M) \\ x(K + p_L - p_H - p_M) & \text{if } \lambda Y_M > x(K + p_L - p_H - p_M) \end{cases}$$

First, we compute the equilibrium. Since $q_M = 0$ then $T = p_L$. Now, we show that $\overline{P}_H \geq x(K + p_L - p_H)$ for all λ covered in case b.3. Consider $V = \overline{P}_H - x(K + p_L - p_H) = \lambda Y_H - \frac{\widehat{R}(Z - p_L)}{p_H} - x(K + p_L - p_H)$. Now, $\widehat{R}(\lambda_2) = x(K + p_L - p_H)$. Because if $Z < p_L + p_H \frac{Y_H}{Y_M} - p_H$ (or $1 + \frac{Z - p_L}{p_H} < \frac{Y_H}{Y_M}$) then $\lambda_2 Y_M = \frac{x(K + p_L - p_H)}{Y_H}\ \ 1 + \frac{Z - p_L}{p_H}\ \ Y_M < x(K + p_L - p_H)$ and so $\widehat{R}(\lambda_2) = x(K + p_L - p_H)$, and if $Z = p_L + p_H \frac{Y_H}{Y_M} - p_H$ (or $1 + \frac{Z - p_L}{p_H} = \frac{Y_H}{Y_M}$) then $\lambda_2 Y_M = \frac{x(K + p_L - p_H)}{Y_H}\ \ 1 + \frac{Z - p_L}{p_H}\ \ Y_M = x(K + p_L - p_H)$ and again $\widehat{R}(\lambda_2) = \lambda Y_M = (K + p_L - p_H)$. Function V evaluated at λ_2 is $\lambda_2 Y_H - \frac{x(K + p_L - p_H)(Z - p_L)}{p_H} - x(K + p_L - p_H) = x(K + p_L - p_H)\ \ 1 + \frac{Z - p_L}{p_H}\ \ - x(K + p_L - p_H)\ \ 1 + \frac{Z - p_L}{p_H}\ \ = 0$. Now $\frac{\partial V}{\partial \lambda} = Y_H - \frac{\partial \widehat{R}}{\partial \lambda} \frac{(Z - p_L)}{p_H} \geq Y_H - \frac{\partial \widehat{R}}{\partial \lambda}\ \ \frac{Y_H}{Y_M} - 1\ \ \geq Y_H - Y_M\ \ \frac{Y_H}{Y_M} - 1\ \ = 1$, where the first inequality follows from the definition of case b.3 and the second inequality follows because $\frac{\partial \widehat{R}}{\partial \lambda} \leq Y_M$. Thus, in all the range of λ considered $\overline{P}_H \geq x(K + p_L - p_H)$. Since $\overline{P}_H \geq x(K + p_L - p_H)$, $\overline{P}_M = \lambda Y_M$, and $T = p_L$ then comparing the definition of $\widehat{R}$ with R^* from cases 3,4, and 5 of Table 1, leads to $R^* = \widehat{R}$. In addition, in this case (b.3), $r_H = 1$.

Second, an entrepreneur has no incentives to deviate since this contract allows him not to liquidate the type M project. Also by setting $D_{2M} = 0$, the entrepreneur maximizes his probability of receiving capital when his project is of type M.

Step 8 (proof of part c.3): If $Z - p_H \frac{Y_H}{Y_M} > p_L - p_H$, then for $\lambda_3 \leq \lambda \leq \lambda_4$ (this region is non-empty), every entrepreneur sets $(q_M, D_{2M}) = (\frac{Z - \frac{p_H D_{2H}}{\widehat{R}} - p_L}{p_M}, 0)$ and $(q_H, D_{2H}) = (0, \lambda Y_H - \widehat{R})$, where $\widehat{R} = \lambda Y_M$.

Again, we first compute the equilibrium. The total amount of liquidation is $T = p_L + p_M q_M = Z - \frac{p_H D_{2H}}{\widehat{R}} = Z - \frac{p_H Y_H}{Y_M} + p_H$. Since $\lambda \geq \lambda_3$, then $\overline{P}_M = \lambda Y_M \geq x(K + Z - p_H \frac{Y_H}{Y_M}) = x(K + T - p_H)$ and so this case falls under case 4 of Table 1. Since we are in case 4, $R^* = \lambda Y_M$, $r_H = 1$ and $r_M = \frac{K + T - p_H - x^{-1}(\lambda Y_M)}{p_M}$. Note that r_M is strictly increasing in λ. At $\lambda = \lambda_3$, $r_M = 0$ and at $\lambda = \lambda_4$, $r_M = 1$.

We need to check whether the contract is feasible. We show that $D_{2H} \geq 0$ and $0 \leq q_M \leq 1$. First, $D_{2H} = \lambda Y_H - \lambda Y_M > 0$. Now, $q_M = \frac{Z - \frac{p_H Y_H}{Y_M} + p_H - p_L}{p_M} > 0$ by the definition of this step ($Z - p_H \frac{Y_H}{Y_M} > p_L - p_H$). Also, since $Z - \frac{p_H Y_H}{Y_M} + p_H - p_L \leq Z - p_L \leq p_M$ (the last inequality follows from our assumption that $Z \leq p_L + p_M$) then $q_M < 1$.

Finally, given the equilibrium, we solve for entrepreneur j's optimal deviation $(q_s^j, D_{2s}^j)_{s=H,M}$. Just as in step

6, there is no profitable deviation from $D^j_{2H} = \lambda Y_H - R^*$. Also for $D^j_{2M} = 0$, $r^j_M \in [0,1]$ whereas any deviation to $D^j_{2M} > 0$, would lead to $r^j_M = 0$ (since $\overline{P}^j_M < \lambda Y_M = R^*$) and is therefore not profitable. Given these values of D^j_{2H} and D^j_{2M}, q^j_M satisfies the participation constraint with equality. Finally, since $(q^j_s, D^j_{2s})_{s=H,M} = (q_s, D_{2s})_{s=H,M}$, we have, in fact, an equilibrium.

Step 9 (proof of part b.4): If $Z - p_H \frac{Y_H}{Y_M} > p_L - p_H$, then for $\lambda_4 < \lambda < \lambda_5$, every entrepreneur sets $(q_M, D_{2M}) = (\frac{Z - \frac{\lambda}{\widehat{R}}(p_H Y_H + p_M Y_M) - p_L + p_M + p_H}{p_M(2 - \frac{\lambda}{\widehat{R}} Y_M)}, \lambda Y_M - \widehat{R})$ and $(q_H, D_{2H}) = (0, \lambda Y_H - \widehat{R})$, where $\widehat{R}$ satisfies $x(K + 2(p_L + p_M q_M(\widehat{R})) - 1) = \widehat{R}$.

The proof of this step is almost identical to step 6. So we do not repeat is here. The equilibrium $R^* = \widehat{R}$. In this case, $r_H = r_M = 1$. Also, $q_M > 0$ for $\lambda < \lambda_5$ and $q_M = 0$ for $\lambda = \lambda_5$.

Step 10 (proof of part b.5): If $Z - p_H \frac{Y_H}{Y_M} > p_L - p_H$ then for $\lambda > \lambda_5$, $q_M = 0$, and D_{2M} and D_{2H} can be anything that satisfies $p_M D_{2M} + p_H D_{2H} + p_L \widehat{R} = Z\widehat{R}$ and $D_{2s} \leq \lambda Y_s - \widehat{R}$ for $s = H, M$ where $\widehat{R}$ is defined by $\widehat{R} = x(K + p_L - p_H - p_M)$.

First, we find the equilibrium. Since $q_M = 0$ then $T = p_L$. The definition of D_{2s} guarantees that $\overline{P}_s \geq \widehat{R}$. Then, since $\overline{P}_s = \widehat{R} = x(K + p_L - p_H - p_M)$ we are in case 5 of Table 1. From the Table, $R^* = x(K + p_L - p_H - p_M) = \widehat{R}$. To check whether the contract is feasible we need to show that there are D_{2H} and D_{2M} that satisfy $0 \leq D_{2s} \leq \lambda Y_s - \widehat{R}$ for $s = H$, M and $p_M D_{2M} + p_H D_{2H} + p_L \widehat{R} = Z\widehat{R}$. First, $\lambda Y_M - \widehat{R} > \lambda_5 Y_M - \widehat{R} = \frac{\widehat{R}}{p_H Y_H + p_M Y_M}[Z Y_M + Y_M(-p_L + p_H + p_M) - p_H Y_H - p_M Y_M] > 0$, where the first equality follows from the definition of λ_5 and the second from the condition that $Z - p_H \frac{Y_H}{Y_M} > p_L - p_H$. Since $\lambda Y_H - \widehat{R} > \lambda Y_M - \widehat{R} > 0$, it is possible to have $D_{2s} > 0$ for $s = H, M$. The minimum value that $p_M D_{2M} + p_H D_{2H} + p_L \widehat{R}$ takes is $p_L \widehat{R} < Z\widehat{R}$ when $D_{2H} = D_{2M} = 0$. The maximum value is $p_M(\lambda Y_M - \widehat{R}) - p_H(\lambda Y_H - \widehat{R}) + p_L \widehat{R} > \lambda_5(p_M Y_M + p_H Y_H) - \widehat{R}(p_L - p_H - p_M) = \widehat{R}(Z - p_L + p_H + p_M) - \widehat{R}(p_L - p_H - p_M) = \widehat{R}Z$. Thus there are non-negative values for D_{2H} and D_{2M} such that $p_M D_{2M} + p_H D_{2H} + p_L \widehat{R} = Z\widehat{R}$ and $D_{2s} \leq \lambda Y_s - \widehat{R}$ for $s = H, M$.

With the proposed contract, an entrepreneur does not liquidate when the project is of type M or H. In addition, the entrepreneur always raises capital for sure with these two types of projects. Since $R^* = \widehat{R}$, the contract satisfies the investor's participation constraint. Thus an entrepreneur cannot do better.

Proof of Proposition **??**

For $Z < p_L + p_M$, $q_H = 0$ and therefore aggregate payoff can be written as:

$$\Pi = p_H(1 + r_H)Y_H + p_M(1 - q_M)(1 + r_M)Y_M + \omega x(\omega),$$

where $\omega = K + p_L + p_M q_M - p_H r_H - p_M(1 - q_M) r_M$.

In region (a), $q_M = q_H = 0$ (so $T = p_L$), and $D_{2H} = D_{2M} = 0$ so $\overline{P}_H = \lambda Y_H$ and $\overline{P}_M = \lambda Y_M$. Using $\overline{P}_H$, $\overline{P}_M$ and $T = p_L$ in Table 1, it can be readily seen that r_H and r_M are weakly increasing in λ. Since $\frac{d}{d\omega}(\omega x(\omega)) = x(\omega) + \omega x'(\omega) < Y_M < Y_H$, then $\frac{\partial \Pi}{\partial r_H} = p_H[Y_H - \frac{d}{d\omega}(\omega x(\omega))] > 0$ and $\frac{\partial \Pi}{\partial r_M} = p_M(1 - q_M)[Y_M - \frac{d}{d\omega}(\omega x(\omega))] > 0$. Therefore, in this region, $\frac{d\Pi}{d\lambda} \geq 0$. Finally, in this region, the equilibrium is independent of Z, and so $\frac{d\Pi}{dZ} = 0$.

In region b.1 and c.1, $q_H = 0$, $q_M = (Z - p_L)/p_M$ (so $T = Z$) $\overline{P}_H = \lambda Y_H$, $\overline{P}_M = \lambda Y_M$. Letting $\lambda_0 = x(K+Z)/Y_H < \lambda_1$and using Table 1 we obtain that $r_H = \begin{cases} 0 & \text{if } \lambda < \lambda_0 \\ [K+Z-x^{-1}(\lambda Y_H)]/p_H & \text{if } \lambda_0 \leq \lambda \leq \lambda_1 \end{cases}$, and $r_M = 0$. As before, r_H is weakly increasing in λ and so $\frac{d\Pi}{d\lambda} \geq 0$. For $\lambda < \lambda_0$, $\frac{d\Pi}{dZ} = -Y_M + \frac{d}{d\omega}(\omega x(\omega)) < 0$. For $\lambda_0 \leq \lambda \leq \lambda_1$, $\frac{d\Pi}{dZ} = Y_H - Y_M > 0$.

Now we analyze region b.2 and c.2. In the proof of Lemma **??**, we showed that, in this region $r_M = 0$, $r_H = 1$ and $\frac{\partial q_M}{\partial \lambda} < 0$. Inside this region, an increase in λ or Z does not affect the equilibrium allocation r_H or r_M but it does affect the amount of liquidation q_M. Since $\frac{\partial q_M}{\partial \lambda} < 0$, then $\frac{d\Pi}{d\lambda} = p_M[-Y_M + \frac{d}{d\omega}(\omega x(\omega))]\frac{\partial q_M}{\partial \lambda} > 0$. To obtain the effect of Z on q_M, consider first the effect of Z on the equilibrium return R^*. Recall that, in this region, $F(R, \lambda, Z) = x(K + Z - \frac{\lambda Y_H}{R} p_H) - R$ and R^* is such that $F(R^*, \lambda, Z) = 0$. By the implicit function theorem, $\frac{\partial R^*}{\partial Z} = -\frac{F_Z}{F_R} < 0$. This implies that $R^* = x(K+T-p_H)$ so that it must be the case that $\frac{\partial T}{\partial Z} > 0$. Since $q_M = (T - p_L)/p_M$ then $\frac{\partial q_M}{\partial Z} > 0$ and so $\frac{d\Pi}{dZ} < 0$.

In region b.3, $q_M = q_H = 0$ and so $T = p_L$. Using the fact that $T = p_L$, $\overline{P}_M = \lambda Y_M$ and $\overline{P}_H \geq X(K+p_L-p_H)$ (see proof of Lemma **??**) the allocation r_H and r_M can be derived. In case b.3, $r_H = 1$ and r_M is (weakly) increasing in λ and are not affected by Z. Since r_H and r_M are not affected by Z, $\frac{d\Pi}{dZ} = 0$. Also, higher levels of λ leads to (weakly) higher r_M and so $\frac{d\Pi}{d\lambda} \geq 0$.

In region c.3, q_M does not change with λ (see proof of Lemma **??**), $r_H = 1$ and r_M is increasing in λ and so $\frac{d\Pi}{d\lambda} > 0$. In this region $\frac{\partial q_M}{\partial Z} = \frac{\partial r_M}{\partial Z} = \frac{1}{p_M}$ and $\frac{d\Pi}{dZ} = -(q_M + r_M)(Y_M - \frac{d}{d\omega}(\omega x(\omega))) < 0$.

Region c.4 is similar to b.2 and region c.5 is similar to c.3.

<u>Proof of Proposition **??**</u>

Direct application of Lemma **??**, with C being the set of projects that are not liquidated and that belong to conglomerates that do not opt out of the external capital market, leads to:

$$r_H = \begin{cases} 0 & \text{if } \lambda Y_H < x(K+T) \\ \frac{K+T-x^{-1}(\lambda Y_H)}{p_H - cp_L p_H} & \text{if } x(K+T) \leq \lambda Y_H < x(K+T-p_H+cp_L p_H) \\ 1 & \text{if } \lambda Y_H \geq x(K+T-p_H+cp_L p_H) \end{cases}$$

$$r_{HM} = \begin{cases} 0 & \text{if } \lambda Y_{HM} < x(K+T-p_H+cp_L p_H) \\ \frac{K+T-p_H+cp_L p_H - x^{-1}(\lambda Y_{HM})}{cp_H p_M} & \begin{array}{l}\text{if } x(K+T-p_H+cp_L p_H) \leq \lambda Y_{HM} < \\ < x(K+T-p_H+cp_L p_H - cp_M p_H)\end{array} \\ 1 & \text{if } \lambda Y_{HM} \geq x(K+T-p_H+cp_L p_H - cp_M p_H) \end{cases}$$

$$r_M = \begin{cases} 0 & \text{if } \lambda Y_M < x(K+T-p_H+cp_L p_H - cp_M p_H) \\ \frac{K+T--p_H+cp_L p_H - cp_H p_M - x^{-1}(\lambda Y_M)}{(1-c)p_M + cp_M^2} & \begin{array}{l}\text{if } x(K+T-p_H+cp_L p_H - cp_M p_H) \leq \lambda Y_M < \\ < x(K+T-p_H+cp_L p_H - cp_M p_H - (1-c)p_M - cp_M^2)\end{array} \\ 1 & \text{if } \lambda Y_M \geq x(K+T-p_H+cp_L p_H - cp_M p_H - (1-c)p_M - cp_M^2) \end{cases}$$

where $Y_{HM} \equiv (Y_H + Y_M)/2$. The definition of r_H, r_{HM} and r_M motivate the definition of the following functions of λ : $\widetilde{\lambda}_1 = x(K+T)/Y_H$, $\widetilde{\lambda}_2 = x(K+T-p_H+cp_L p_H)/Y_H$, $\widetilde{\lambda}_3 = x(K+T-p_H+cp_L p_H)/Y_{HM}$,

$\widetilde{\lambda}_4 = x(K+T-p_H+cp_Lp_H-cp_Mp_H)/Y_{HM}$, $\widetilde{\lambda}_5 = x(K+T-p_H+cp_Lp_H-cp_Mp_H)/Y_M$ and $\widetilde{\lambda}_6 = x(K+T-p_H+cp_Lp_H-cp_Mp_H-(1-c)p_M-cp_M^2)/Y_M$. For any c, the value of these functions satisfy $\widetilde{\lambda}_1 < \widetilde{\lambda}_2 < \cdots < \widetilde{\lambda}_6$.

To lighten notation we let $r_t' \equiv \frac{\partial r_t}{\partial c}$ for $t = H$, HM, and M and $f(\omega) = \omega x(\omega)$. We analyze $\frac{d\Pi}{dc}$ in all the regions defined by $\widetilde{\lambda}_k$ $k = 1, 2 \ldots 6$. We reproduce the expression for Π from the text:

$$\Pi = f(\omega) + (1-c)\left[p_H(Y_H + r_HY_H) + p_M(Y_M + r_MY_M)\right] +$$
$$\frac{c}{2}\left[\begin{array}{c} p_H^2(2)(Y_H + r_HY_H) + p_M^2(2)(Y_M + r_MY_M) + 2p_Hp_M(2Y_H + r_HY_M + r_{HM}Y_M) \\ +2p_Hp_L(2Y_H) + 2p_Mp_L(2Y_M) \end{array}\right]$$

where $\omega = K + (1-c)p_L + cp_L^2 - r_H(p_H - cp_Lp_H) - r_{HM}(cp_Hp_M) - r_M((1-c)p_M + cp_M^2)$.

We denote by $\frac{\partial \Pi}{\partial c}$ the derivative of Π wrt c assuming r_H, r_{HM} and r_M are constants:

$$\frac{\partial \Pi}{\partial c} = p_Lp_H(1-r_H)(Y_H - f'(\omega)) + p_Hp_M(1-r_H)(Y_H - Y_M) + \tag{14}$$
$$+ p_M(1-p_M)(1-r_M)(Y_M - f'(\omega)) + p_Hp_M(r_{HM}-1)(Y_M - f'(\omega))$$

For $\lambda < \widetilde{\lambda}_1$, $r_H = r_{HM} = r_M = 0$ and since $r_H' = r_{HM}' = r_M' = 0$ we can use equation **??** to obtain $\frac{d\Pi}{dc} = \frac{\partial \Pi}{\partial c} = p_Lp_H(Y_H - f'(\omega)) + p_Hp_M(Y_H - Y_M) + p_Mp_L(Y_M - f'(\omega)) > 0$ since $f'(\omega) = x(\omega) + \omega x'(\omega) \leq x(\omega) < Y_M < Y_H$.

For $\widetilde{\lambda}_1 < \lambda < \widetilde{\lambda}_2$, $r_H \in (0,1)$, $r_{HM} = 0$ and $r_M = 0$ also $r_H' \neq 0$ and $r_{HM}' = r_M' = 0$. Also in this region, simple algebra leads to $\frac{\partial \omega}{\partial c} = 0$. Thus, $\frac{d\Pi}{dc} = p_Lp_H(1-r_H)Y_H + p_Hp_M(1-r_H)(Y_H - Y_M) + p_Mp_LY_M + r_H'[(1-c)p_HY_H + cp_H^2Y_H + cp_Hp_MY_M]$ and $r_H' = \frac{-p_L + p_L^2 + r_Hp_Hp_L}{p_H - cp_Lp_H}$. In the limit as $\lambda \to \widetilde{\lambda}_2$, $r_H \to 1$ and $\frac{d\Pi}{dc} = p_Mp_LY_M - p_Mp_L[\gamma Y_H + (1-\gamma)Y_M] < 0$ where $\gamma = (1 - c + cp_H)/(1 - cp_L)$. Also $\frac{\partial}{\partial \lambda}\frac{d\Pi}{dc} = \frac{\partial}{\partial r_H}\frac{d\Pi}{dc}\frac{\partial r_H}{\partial \lambda}$. Since $x()$ is a decreasing function $\frac{\partial r_H}{\partial \lambda}$ is increasing. And since $\frac{\partial}{\partial r_H}\frac{d\Pi}{dc} = -p_Hp_LY_H - p_Hp_M(Y_H - Y_M) + p_Hp_L[\gamma Y_H + (1-\gamma)Y_M] < 0$ then $\frac{\partial}{\partial \lambda}\frac{d\Pi}{dc} < 0$.

If $\frac{d\Pi}{dc}_{\widetilde{\lambda}_1} \leq 0$ then we define $\widehat{\lambda}_1 = \widetilde{\lambda}_1$. If $\frac{d\Pi}{dc}_{\widetilde{\lambda}_1} > 0$ then since $\frac{\partial}{\partial \lambda}\frac{d\Pi}{dc} < 0$ and $\frac{d\Pi}{dc}_{\widetilde{\lambda}_2} < 0$, there is a $\lambda^* \in (\widetilde{\lambda}_1, \widetilde{\lambda}_2)$ such that $\frac{d\Pi}{dc}|_{\lambda^*} = 0$. In this case we define $\widehat{\lambda}_1 = \lambda^*$. We also define $\widehat{\lambda}_2 = \widetilde{\lambda}_2$. So far we have shown that to the left of $\widehat{\lambda}_1$, $\frac{d\Pi}{dc}$ is positive, and from $\widehat{\lambda}_1$ to $\widehat{\lambda}_2$, $\frac{d\Pi}{dc}$ is negative. Left to prove is that to the right of $\widehat{\lambda}_2$, $\frac{d\Pi}{dc}$ is non-negative.

For $\widetilde{\lambda}_2 < \lambda < \widetilde{\lambda}_3$, $r_H = 1$, $r_{HM} = r_M = 0$ and $r_H' = r_{HM}' = r_M' = 0$. Using equation **??**, $\frac{d\Pi}{dc} = \frac{\partial \Pi}{\partial c} = p_Mp_L(Y_M - f'(\omega)) > 0$.

For $\widetilde{\lambda}_3 < \lambda < \widetilde{\lambda}_4$, $r_H = 1$, $r_{HM} \in (0,1)$, $r_M = 0$ and $r_{HM}' \neq 0$ and $r_H' = r_M' = 0$. Also in this region, simple algebra leads to $\frac{\partial \omega}{\partial c} = 0$. Thus, $\frac{d\Pi}{dc} = p_Mp_LY_M + p_Hp_Mr_{HM}Y_M + cp_Hp_Mr_{HM}'Y_M$ and $r_{HM}' = \frac{-p_L + p_L^2 - +p_Hp_L + r_{HM}p_Hp_M}{cp_Hp_M}$. Substituting the value of r_{HM}' and simplifying leads to $\frac{d\Pi}{dc} = 0$.

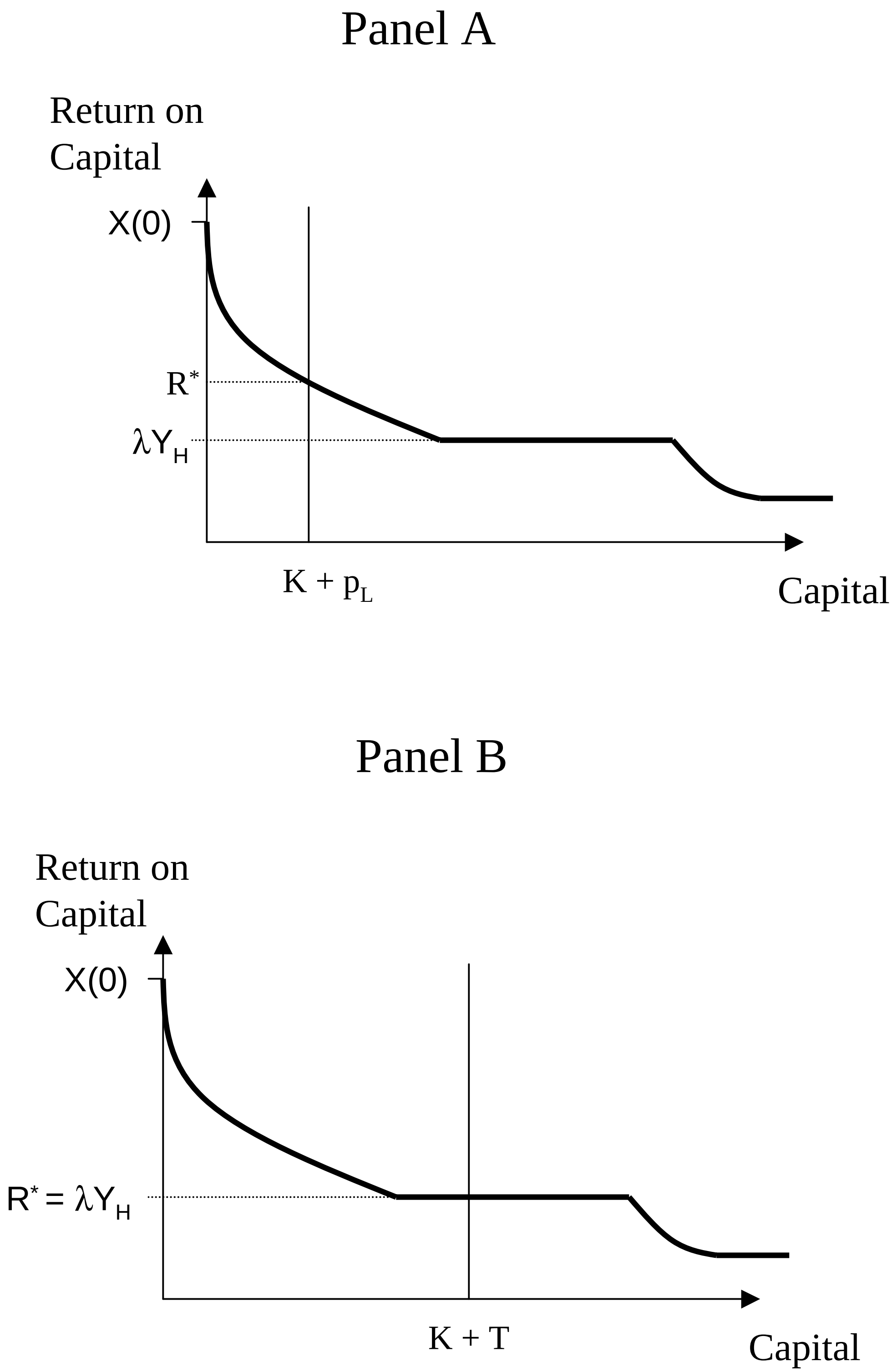
For $\widetilde{\lambda}_4 < \lambda < \widetilde{\lambda}_5$, $r_H = 1$, $r_{HM} = 1$, $r_M = 0$ and $r_H' = r_{HM}' = r_M' = 0$. Using equation **??**, $\frac{d\Pi}{dc} = \frac{\partial \Pi}{\partial c} = p_M(1-p_M)(Y_M - f'(\omega)) > 0$

For $\widetilde{\lambda}_5 < \lambda < \widetilde{\lambda}_6$, $r_H = 1$, $r_{HM} = 1$, $r_M \in (0,1)$ and $r_M' \neq 0$ and $r_{HM}' = r_M' = 0$. Also in this region, simple algebra leads to $\frac{\partial \omega}{\partial c} = 0$. $\frac{d\Pi}{dc} = p_Mp_LY_M + p_Hp_MY_M - p_Mr_MY_M + p_M^2r_MY_M + r_M'((1-c)p_MY_M + cp_M^2Y_M)$ and $r_M' = \frac{-p_L + p_L^2 + p_Lp_H - p_Hp_M - r_M(-p_M + p_M^2)}{(1-c)p_M + cp_M^2}$. Substituting the value of r_M' and simplifying leads to $\frac{d\Pi}{dc} = 0$.

Finally, for $\lambda > \widetilde{\lambda}_6, r_H = 1$, $r_{HM} = 1$, $r_M = 1$ and $r'_H = r'_{HM} = r'_M = 0$. Using equation **??**, $\frac{d\Pi}{dc} = \frac{\partial \Pi}{\partial c} = 0$.

Thus, to the right of $\widehat{\lambda}_2$, $\frac{d\Pi}{dc}$ is non-negative.

Figure 1


Panel A
Return on Capital
X(0)
R*
λY_H
K + p_L
Capital
Panel B
Return on Capital
X(0)
R* = λY_H
K + T
Capital