Financial Innovation, Market Participation and Asset Prices*

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Abstract

This paper theoretically investigates the pricing effects of financial innovation in an economy with endogenous participation and heterogeneous income risks. The introduction of non-redundant assets can endogenously modify the participation set, reduce the covariance between dividends and participants' consumption and thus lead to lower risk premia. This mechanism is demonstrated in a tractable exchange economy with a finite number of macroeconomic factors. Agents can freely borrow and lend, but must pay a fixed entry cost to invest in risky assets. Security prices and the participation structure are jointly determined in equilibrium. The model is consistent with several features of financial markets over the past few decades: substantial financial innovation; a sharp increase in investor participation; improved risk management practices; a slight increase in interest rates; and a reduction in risk premia.

Keywords: Endogenous participation, Epstein-Zin utility, financial innovation, incomplete markets, multiple risk factors, spanning.

JEL Classification: D52, E44, G12.

1. Introduction

This paper theoretically investigates how financial innovation affects asset prices in an economy with endogenous participation and heterogeneous income risks. We show that the introduction of non-redundant securities can endogenously modify the participation set, decrease the covariance between dividends and participants' consumption, and thus lead to a lower market premium. In multifactor economies, financial innovation spreads across markets through the diversified portfolio choices of new entrants, and can have rich effects on the cross-section of expected returns.

Our approach builds on two stylized facts. First, participation in financial markets is costly. Corporate hedging requires the employment of experts able to effectively reduce the firm's risk exposure using existing financial assets. Investors have to sustain learning efforts, and expenses related to the opening and maintenance of accounts with an exchange or a brokerage firm. Statutory and government regulations often create costly barriers to the participation of institutional investors in some markets. Second, when some asset markets are initially missing, financial innovation affects risk-sharing and investment opportunities. For instance, options and futures can provide additional insurance against the price risk of commodities and financial assets.[1] Similarly, asset-backed securities allow lending institutions to reduce their risk exposure to various forms of debt contracts. For this reason, new securities affect individual incentives to participate in financial markets when trading is limited by entry costs.

We introduce a two-period economy with incomplete markets and endogenous participation. Agents receive heterogeneous random incomes determined by a finite number of macroeconomic risk factors. They can borrow or lend freely, but have to pay a fixed cost to invest in risky assets. Security prices and the participation structure are jointly determined in equilibrium. The model is computationally tractable and leads to a closed-form system of equilibrium equations in the CARA-normal case.

The introduction of non-redundant instruments encourages more investors to participate in financial markets for hedging and diversification purposes. This tends to reduce the precautionary demand for savings and thus increase the equilibrium interest rate. Under plausible conditions on the cross-sectional distribution of risk, the new entrants reduce the covariance between stock returns and the mean consumption of participants, leading to a lower market premium. The

[1] See for instance Ross (1976).

model is thus consistent with features that have characterized financial markets in the past few decades: substantial financial innovation, a sharp increase in investor participation, improvements in risk-management practices, a slight increase of real interest rates, and a reduction in the risk premium.[2]

Participation can also play an important role in spreading the effects of innovation across markets. When a factor becomes tradable, new agents decide to enter financial markets in order to manage their risk exposure. The new participants also trade other assets for diversification purposes, and can modify the risk premia of securities uncorrelated to the factor. Financial innovation can thus differentially affect distinct sectors of the economy and have a rich impact on the cross-section of expected returns. Furthermore, the price changes caused by the new entrants adversely affect a subset of existing participants and lead some of them to leave financial markets. The introduction of new assets can thus induce the simultaneous entry and exit of investors, giving rise to non-degenerate forms of participation turnover.[3]

The tractable model introduced in this paper emphasizes the role of the cross-sectional distribution of risk in an economy with endogenous participation. We mainly use this approach to analyze the effects of new securities on risk pricing, a topic that earlier research has mostly examined without consideration of participation. Changes in taxes or in entry costs represent other direct applications, which are discussed only briefly because of space constraints. Extensions to richer cost structures or heterogeneity in risk aversion would also preserve the tractability of the model and prove useful for empirical applications.

Section 2 introduces a simple asset pricing model with endogenous market participation. Section 3 demonstrates the pricing and participation effects of financial innovation in a one factor model of risk exposure. We consider in Section 4 an economy with multiple risk factors. Simple numerical simulations show that financial innovation can substantially reduce the equity premium, differentially spread across security markets, and either increase or decrease the interest rate. All proofs are given in the Appendices.

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[2]The recent decrease in the risk premium is reported in Blanchard (1993), Campbell and Shiller (2001), Cochrane (1997), Fama and French (2002) and Vuolteenaho (2000). Similarly, Barro and Sala-i-Martin (1990) and Honohan (2000) document a slight increase in real interest rates over the past three decades.

[3]Hurst, Luoh and Stafford (1998) and Vissing-Jørgensen (2002) report substantial rates of entry and exit in US stockownership.

1.1. Review of Previous Literature

This paper builds on two strands of the asset pricing literature that have essentially been developed separately. First, researchers have examined how limited investor participation affects the prices of a fixed set of securities. Second, the price impact of financial innovation has been examined both empirically and theoretically without consideration of participation. The novelty of this paper is to combine these two lines of research in a simple and tractable framework. We show that one of the consequences of financial innovation could be its effect on participation, which could induce a reduction in the risk premium. This potentially provides useful guidance for future empirical research.

Research on limited participation was pioneered by Mankiw and Zeldes (1991), who reported that only 28% of households owned stocks in 1984, and that only 47% of the households holding other liquid assets in excess of $\$100,000$ held any equity.[4] The fraction of households owning stocks increases with income and education, implying that there could be fixed information costs to participate in financial markets. The consumption of stockholders is also more highly correlated with the stock market than aggregate consumption. The distinction between stockholders and non-stockholders therefore helps explain the equity premium puzzle. The empirical validity of this mechanism is further confirmed by Vissing-Jørgensen (1997) and Attanasio, Banks and Tanner (1998). Poterba and Samwick (1995) and Vissing-Jørgensen (1997) also document the sharp increase of stock market participation in the United States since 1945.

These empirical findings have prompted the development of theoretical models that restrict participation exogenously. Saito (1995) and Basak and Cuoco (1998) thus consider two-asset exchange economies in which individual income is spanned by the risky security. They succeed in matching the historical risk premium with a reasonable degree of risk aversion at low participation levels. Heaton and Lucas (1999) extend their analysis by considering heterogeneous incomes with a common nonmarketable factor. In contrast to this earlier work, we consider multiple assets and factors, and endogenize the participation structure by considering fixed costs to trading in financial markets. The entry-cost approach has been widely used in finance to analyze issues such as portfolio choice (Campbell, Cocco, Gomes and Maenhout, 2001), volatility (Pagano, 1989; Allen and Gale, 1994*b*; Orosel, 1998), futures risk premia (Hirshleifer, 1988), market size (Allen and Gale, 1990;

[4] The structure of stockownership is further analyzed by Blume and Zeldes (1993) and Bertaut and Haliassos (1995).

Pagano, 1993), and the effect of social security reform on capital accumulation (Abel, 2001). We use this setup to analyze how financial innovation affects investor participation and asset prices.

The paper is also related to a line of research that examines the price impact of financial innovation without consideration of participation. Conrad (1989) and Detemple and Jorion (1990) find empirically that the introduction of new batches of options had a substantial price impact between 1973 and 1986. The effect is stronger for underlying stocks, but can also be observed for an industry index that excludes the optioned stock as well as for the market index. Similar empirical evidence is available for other countries and derivative markets (e.g. Jochum and Kodres, 1998). These empirical findings have prompted a rich theoretical literature. In the presence of informational asymmetries, the introduction of an option contract has been shown to affect the volatility of the underlying stock (e.g. Stein, 1987; Grossman, 1989). Another line of research focuses on the risk-sharing component of new derivatives (Detemple and Selden, 1991; Huang and Wang, 1997).

Although our model can be applied to a variety of settings, the most promising application may be the long-term effect of innovation on market participation and the equity premium. Intuition suggests that the price of a diversified portfolio of assets may be more influenced by risk-sharing than by information asymmetries. It is not straightforward, however, to capture the price impact of financial innovation in economies with *exogenous* participation. New risk sharing opportunities reduce precautionary savings and increase the interest rate, which tends to lower the price of all assets. Furthermore in standard mean-variance economies, financial innovation does not affect the relative price of risky assets relative to bonds (Oh, 1996). We will show that new assets modify the pricing of risk and occasionally reduce the interest rate in CARA-normal economies when participation is endogenized.

2. A Model of Endogenous Market Participation

We examine an exchange economy with two periods ($t = 0, 1$) and a single perishable good. The economy is stochastic, and all random variables are defined on a probability space $(\Omega, \mathcal{F}, \mathbb{P})$. During his life, each agent h receives an exogenous random endowment $e^h = \left(e_0^h, \tilde{e}^h\right)$, which corresponds for instance to a stochastic labor income. Investors have preferences over consumption streams $(c_0^h, \widetilde{c}^h)$, which are represented by a utility function $U^h(c_0^h, \widetilde{c}^h)$. We thus adopt the two-period setup that has widely been used in the financial innovation literature for its tractability (e.g. Allen and Gale 1994 *a*, *b*). We anticipate that our model provides useful insights on the properties of multiperiod economies with permanent shocks.[5]

This paper places no restriction on the set of agents H, which can be finite or infinite. To provide a uniform treatment, we endow the space H with a measure μ that satisfies $\mu(H) = 1$. This is equivalent to viewing each element of H as a type, and the measure μ as a probability distribution over all possible types.

At date $t = 0$, agents can exchange two types of real securities. First, they can trade a riskless asset costing $\pi_0 = 1/R$ in date $t = 0$ and delivering one unit of the good with certainty at date $t = 1$. Note that R is the gross interest rate. Second, there also exist J risky assets $(j = 1, ...J)$ with price π_j and random payoff $\tilde{a}_j$. We assume for simplicity that all assets are in zero net supply.[6] Investors can freely operate in the bond market but have to pay a fixed entry cost κ in order to invest in one or more risky assets. This assumption is consistent with complementarities of learning in trading activities, and the results of the paper easily generalize to more flexible specifications of the entry cost. Investors are price-takers both in their entry and portfolio decisions, and there are no constraints on short sales. Let π denote the vector of *risky* asset prices, and θ^h the vector of *risky* assets bought (or sold) by investor h. We also consider the dummy variable $1_{\{\theta^h \neq 0\}}$ equal to 1 if $\theta^h \neq 0$, and equal to 0 otherwise. The agent is subject to the budget constraints

$$\begin{aligned} &c_0^h + \theta_0^h/R + \pi \cdot \theta^h + \kappa 1_{\{\theta^h \neq 0\}} = e_0^h, \\ &\tilde{c}^h = \tilde{e}^h + \theta_0^h + \tilde{a} \cdot \theta^h. \end{aligned}$$

These equations are standard, except for the presence of the entry cost in the resource constraint at date 0. We determine the optimal choice $(c_0^h, \tilde{c}^h, \theta_0^h, \theta^h)$ by calculating the consumption-portfolio decision under entry and non-entry. Comparing the resulting utility levels yields the optimal participation decision.

Let $e_0 = \int_H e_0^h d\mu(h)$ and $\tilde{e} = \int_H \tilde{e}^h d\mu(h)$ denote the average income of the *entire population*.

[5] A large body of research shows that transitory shocks can be easily self-insured while persistent shocks have profound pricing and welfare implications in dynamic economies with incomplete markets (e.g. Bewley, 1977; Telmer, 1993; Constantinides and Duffie, 1996; Storesletten, Telmer and Yaron, 1996; Levine and Zame, 2002). The extension of our framework to the dynamic case, for instance following Calvet (2001), is a promising direction for future research.

[6] A positive supply of assets could be considered by redefining individual endowment as the sum of a labor income and an exogenous portfolio of securities. This is a standard convention in asset pricing theory, as discussed for instance in Magill and Quinzii (1996, ch. 3).

Definition. *A* general equilibrium with endogenous participation (GEEP) *consists of an interest rate R, a price vector π, and a collection of optimal plans* $\left(c_0^h, \tilde{c}^h, \theta_0^h, \theta^h\right)_{h \in H}$ *such that*

1. *The good market clears in every state:* $\int_H (c_0^h + \kappa 1_{\left\{\theta^h \neq 0\right\}}) d\mu(h) = e_0$, *and* $\int_H \tilde{c}^h(\omega)\, d\mu(h) = \tilde{e}(\omega)$ *for all* $\omega \in \Omega$.

2. *The asset markets clear:* $\int_H \theta^h d\mu(h) = 0$.

Under free participation ($\kappa = 0$), the definition coincides with the traditional concept of general equilibrium under incomplete markets (GEI). With positive entry costs, a GEEP equilibrium differs from a GEI through two different channels. First, agents endogenously make their participation decisions, and decide whether to pay the entry cost. Second, trading activities use some of society's resources and thus crowd out private consumption, as seen in the market clearing condition at date $t = 0$. This phenomenon, which we call *the displacement effect*, probably plays a minor role in actual economies. Extensions of our model could transfer a fraction of trading fees to certain consumers (such as exchange owners), or seek to provide a more detailed description of the financial industry.

The existence and constrained efficiency of equilibrium are shown in Appendix A. In order to analyze the effect of financial innovation on participation and prices, we now specialize to a tractable class of CARA-normal economies. Investors have identical utility of the Epstein-Zin type:

$$U(c_0, \widetilde{c}) = -e^{-\chi c_0} - \beta[\mathbb{E}\, e^{-\gamma \widetilde{c}}]^{\chi/\gamma},$$

where γ and χ are positive coefficients. The agent maximizes $-e^{-\chi c_0} - \beta e^{-\chi c_1}$ when she reallocates through time a deterministic income flow. On the other hand, atemporal risky choices only depend on $\mathbb{E} e^{-\gamma \widetilde{c}}$. When future consumption is normally distributed, we can rewrite the utility as

$$-e^{-\chi c_0} - \beta e^{-\chi[\mathbb{E}\widetilde{c} - \gamma Var(\widetilde{c})/2]}.$$

The specification corresponds to the standard expected utility when $\chi = \gamma$.

Individual endowments and the payoffs of risky assets are jointly normal. The securities generate a linear subspace in the set $L^2(\Omega)$ of square-integrable random variables. We assume without loss of generality that the risky assets are centered and mutually independent: $(\tilde{a}_1, .., \tilde{a}_J) \sim \mathcal{N}(0, I)$. Let A denote the span of the *risky* assets, and $A^\perp$ the subspace orthogonal to *all* securities (including the bond). Projections will play an important role in the discussion, and it will be convenient to denote by $\tilde{x}^V$ the projection of a random variable $\tilde{x}$ on a subspace V.

2.1. Individual Entry Decision

We solve the decision problem of an individual trader h by calculating the consumption - portfolio choice under entry and non-entry. Consider the tradable security $\tilde{m}^A \equiv -(R/\gamma)\sum_{j=1}^{J}\pi_j\tilde{a}_j$, which is determined by risk aversion and market prices. We show in Appendix B:

Theorem 1. *When participating in the risky asset market, the investor buys*

$$\theta_0^{h,p} = \frac{R}{1+R}\left\{e_0^h - \mathbb{E}\,\tilde{e}^h - \kappa - \pi\cdot\theta^{h,p} + \frac{\ln(R\beta)}{\chi} + \frac{\gamma}{2}\left[Var(\tilde{e}^{hA^{\perp}}) + Var(\tilde{m}^A)\right]\right\}$$

units of the bond, and $\theta_j^{h,p} = -Cov(\tilde{a}_j, \tilde{e}^h) - R\pi_j/\gamma$ *units of risky asset* j*. Consumption is then*

$$\tilde{c}^{h,p} = \mathbb{E}\,\tilde{e}^h + \theta_0^{h,p} + \tilde{m}^A + \tilde{e}^{hA^{\perp}} \tag{2.1}$$

in the second period.

We can infer from (2.1) that the investor exchanges the *marketable* component $\tilde{e}^{hA}$ of her income risk for the tradable portfolio $\tilde{m}^A$, which allows an optimal allocation of risk and return. Because markets are incomplete, she is also constrained to bear the undiversifiable income risk $\tilde{e}^{hA^{\perp}}$.

Investment in the riskless asset is the sum of two components, which correspond to intertemporal smoothing and the precautionary motive. First, the agent uses the riskless asset to reallocate her expected income stream between the two periods. Note that she compensates for any discrepancy between her subjective discount factor and the interest rate. Second, she saves more when future prospects are more uncertain. As will be seen in the next section, financial innovation affects this precautionary component by modifying the riskiness of the portfolio $\tilde{m}^A$ and by reducing the undiversifiable income risk $\tilde{e}^{hA^{\perp}}$.

The consumption of the non-participating investor is obtained from Theorem 1 by setting $A = \{0\}$ and $\kappa = 0$.

Proposition 1. *When not trading risky assets, the investor saves*

$$\theta_0^{h,n} = \frac{R}{1+R}\left[e_0^h - \mathbb{E}\,\tilde{e}^h + \frac{1}{\chi}\ln(R\beta) + \frac{\gamma}{2}Var\left(\tilde{e}^h\right)\right] \tag{2.2}$$

in the first period, and consumes $\tilde{c}^{h,n} = \tilde{e}^h + \theta_0^{h,n}$ *in the second.*

The non-participating agent bears all the endowment risk in her final consumption. The precautionary demand for the bond therefore depends on the whole variance of future income.

The investor makes her participation choice by comparing utility under entry and non-entry. In the CARA-normal case, this reduces to maximizing the certainty equivalent $\mathbb{E}\widetilde{c}^h - \gamma Var(\widetilde{c}^h)/2$. As shown in the appendix, the benefit of trading risky assets is $\gamma Var\left(\tilde{e}^{hA} - \tilde{m}^A\right)/2$, while the opportunity cost is κR. This leads to

Theorem 2. *The investor trades risky assets when*

$$\frac{\gamma}{2} Var\left(\tilde{e}^{hA} - \tilde{m}^A\right) > \kappa R, \tag{2.3}$$

and is indifferent between entry and non-entry if the relation holds as an equality.

Relation (2.3) has a simple geometric interpretation in $L^2(\Omega)$, which is illustrated in Figure 1. The agent trades risky assets if the distance between her income risk $\tilde{e}^{hA}$ and her optimal portfolio $\tilde{m}^A$ is larger than $\sqrt{2\kappa R/\gamma}$. The trader pays the entry fee only if her initial position is sufficiently different from the optimum, as is standard in decision problems with adjustment costs.

The theorem has a natural interpretation when all agents have a positive exposure to certain classes of risks. Participants with low exposure to marketable shocks buy the corresponding assets to earn a risk premium; these agents are called *speculators* or *investors*.[7] On the other hand, agents with a high risk exposure will hedge by selling the corresponding risky assets; these agents are called *hedgers* or *issuers*. The model thus closely matches the type of risk-sharing examined in the futures literature.

2.2. Equilibrium

Let $\mathcal{P} \subseteq H$ denote the set of participants in the risky asset markets. When the class of indifferent agents has measure zero, we can write

$$\mathcal{P} = \left\{h \in H : \gamma Var(\tilde{e}^{hA} - \widetilde{m}^A)/2 \geq \kappa R\right\}. \tag{2.4}$$

[7] Massa and Simonov (2002) document empirically the substantial impact of labor income risk on individual participation. In particular, households are less likely to invest in financial markets when their income has a higher correlation with a diversified stock portfolio. These findings are consistent with entry condition (2.3) since the participation benefit can be rewritten as $\gamma[-2Cov(\tilde{m}^A; \tilde{e}^h) + Var(\tilde{e}^{hA}) + Var(\tilde{m}^A)]/2$.

Market participants can have different income risk characteristics than the entire population. We will show in Sections 3 and 4 that this difference is a driving element of the model.[8] While $\tilde{e}$ denotes the mean income in the population, we define the average endowment *among participants* as

$$\tilde{e}^p = \int_{\mathcal{P}} \tilde{e}^h d\mu^p(h),$$

where μ^p is the conditional measure $\mu/\mu(\mathcal{P})$ if $\mu(\mathcal{P}) > 0$, and identically zero otherwise.

In equilibrium, the common consumption risk $\tilde{m}^A$ must coincide with the average tradable income risk of participants:

$$\tilde{m}^A = \tilde{e}^{pA}. \tag{2.5}$$

We also establish

Theorem 3. *In equilibrium, an asset $\tilde{a}$ is worth*

$$\pi(\tilde{a}) = [\mathbb{E}\,\tilde{a} - \gamma Cov(\tilde{e}^p, \tilde{a})]/R. \tag{2.6}$$

The interest rate satisfies

$$\ln R = \ln R_0 + \chi\mu(\mathcal{P}) \left[\kappa + \frac{\gamma}{2} \int_{\mathcal{P}} Var\left(\tilde{e}^{hA} - \tilde{e}^{pA}\right) d\mu^p(h)\right], \tag{2.7}$$

where $\ln R_0 = \ln(1/\beta) + \chi(\mathbb{E}\,\tilde{e} - e_0) - (\chi\gamma/2)\int_H Var\left(\tilde{e}^h\right) d\mu(h)$.

The participation set and asset prices are thus jointly determined by (2.4) – (2.7).

As in the standard CCAPM, an asset is valuable if it provides a good hedge against the consumption risk of participants. Since participation is endogenous in our setup, financial innovation can change the market endowment $\tilde{e}^p$, and therefore the *relative* price $\pi(\tilde{a})/R^{-1} = \mathbb{E}\,\tilde{a} - \gamma Cov(\tilde{e}^p, \tilde{a})$ of a risky asset relative to the bond. The possible effect of financial innovation on the risk premium crucially relies on the endogeneity of participation, and is one of the main properties of the model.[9]

[8] Heaton and Lucas (2000) show the empirical validity of this distinction.

[9] When the set of traders is fixed, an increase in the asset span has no effect on the relative price of a risky asset relative to the bond, as noted in Oh (1996).

The equilibrium interest rate R is influenced by the two economic effects that correspond to the last two terms of equation (2.7). First, the interest rate tends to be higher when more first period resources $\kappa\mu(\mathcal{P})$ are absorbed in the entry process. The second term of (2.7) corresponds to the precautionary motive. To illustrate this point, recall that the variance of individual consumption is $Var(\tilde{e}^{pA}) + Var(\tilde{e}^{hA^{\perp}})$ if an agent participates, and $Var(\tilde{e}^{hA}) + Var(\tilde{e}^{hA^{\perp}})$ otherwise. Entry reduces on average the variance of consumption by

$$\int_{\mathcal{P}} Var\left(\tilde{e}^{hA}\right) d\mu^p(h) - Var\left(\tilde{e}^{pA}\right) = \int_{\mathcal{P}} Var\left(\tilde{e}^{hA} - \tilde{e}^{pA}\right) d\mu^p(h). \tag{2.8}$$

This term is large when many agents participate or many hedging instruments are available. The financial markets then permit agents to greatly reduce their risk exposure, which dampens their precautionary motive, reduces the demand for the riskless asset, and leads to an increase in the equilibrium interest rate.[10]

The entry condition (2.3) suggests that a lower entry fee or improved spanning tends to encourage entry. For instance when the entry cost κ is infinite, no agent trades risky assets and the equilibrium interest rate equals R_0.[11] The equilibrium set of participants, however, may not increase monotonically with the financial structure. This is because the entry condition (2.3) depends on the endogenous variables $\tilde{e}^p$ and R. When new assets are added, a participating agent h may leave the market because the diversification benefit $\gamma Var(\tilde{e}^{hA} - \widetilde{e}^{pA})/2$ has dropped or the opportunity cost has increased. We will provide examples of such behaviors in Sections 3 and 4.

The effect of financial innovation on the interest rate can be predicted when $\tilde{e}^p$ remains constant.

Proposition 2. *Financial innovation leads to a higher interest rate when the mean endowment $\tilde{e}^p$ is unchanged.*

The proof has a straightforward intuition. Financial innovation and a decrease in the interest rate would both encourage entry and lead, by (2.7), to a higher interest

[10] This equation is thus consistent with the well-known effect that financial innovation increases the interest rate when the participation structure is exogenous (Weil, 1992; Elul, 1997; Calvet, 2001).

[11] More generally, let $\kappa_{\max}(A)$ denote the essential supremum of $(\gamma/2R_0)Var\left(\tilde{e}^{hA}\right)$ in the population. It is easy to show that when $\kappa \geq \kappa_{\max}(A)$, the economy has a unique equilibrium, in which no agent trades risky assets. On the other hand if $\kappa < \kappa_{\max}(A)$, any equilibrium has a non-negligible set of participants.

rate - a contradiction. Thus if the participants' average endowment does not vary, existing asset prices necessarily *decrease* with financial innovation. Changes in $\tilde{e}^p$ thus play a crucial role in determining the impact of financial innovation on asset prices. To better understand this role, we now introduce a factor model of risk exposure.

3. Economies with a Unique Risk Factor

We consider in this section a class of economies with a unique risk factor $\tilde{\varepsilon}$. The participation structure and interest rate are determined by the intersection of two curves, which respectively correspond to the entry condition and the market clearing of the bond. We derive the comparative statics of the economy and develop intuition on the risk premium that will be useful for the multifactor analysis of Section 4.

We specify the endowment of each investor h as

$$\tilde{e}^h = \mathbb{E}\,\tilde{e}^h + \varphi^h \tilde{\varepsilon}, \tag{3.1}$$

and call φ^h the individual loading.[12] The factor $\tilde{\varepsilon}$ is a macroeconomic shock that linearly affects all incomes. The model is tractable when the factor and the asset payoffs are jointly normal. Without loss of generality, we assume that $\tilde{\varepsilon}$ has a standard distribution $\mathcal{N}(0,1)$, and that the average loading $\bar{\varphi} = \int_{\mathbb{R}} \varphi d\mu(\varphi)$ in the population is non negative.

When financial markets are incomplete, existing securities span only partially the common shock. The projection of $\tilde{\varepsilon}$ on the asset span, $\tilde{\varepsilon}^A = \sum_{i=1}^{J} Cov(\tilde{\varepsilon}^A, \widetilde{a}_j)\widetilde{a}_j$, is called the tradable component of the factor. The corresponding variance

$$\alpha = Var(\tilde{\varepsilon}^A)$$

is a useful *index of market completeness*, which quantifies the fraction of the risk $\tilde{\varepsilon}$ that is directly insurable. Since $\tilde{\varepsilon}$ has unit variance, the completeness index α is contained between 0 and 1. The values $\alpha = 0$ and $\alpha = 1$ respectively correspond to the absence of risky assets ($A = \{0\}$) and the full marketability of the shock ($\tilde{\varepsilon} \in A$). Intermediate values of α arise when agents can only trade the bond and a risky asset imperfectly correlated with the aggregate shock. The portfolio $\tilde{\varepsilon}^A$ and the completeness index α have direct empirical interpretations. We can

[12] Purely idiosyncratic shocks are ruled out in this section for expositional simplicity.

calculate $\tilde{\varepsilon}^A$ by regressing the factor $\tilde{\varepsilon}$ on the asset payoffs. The corresponding determination coefficient R^2 is then an estimate of the completeness index α.

The one-factor model discussed in this section has a natural interpretation when the factor represents GDP or a market risk that is not directly tradable on organized exchanges (Roll, 1977; Athanasoulis and Shiller, 2000). New assets then help market participants hedge more closely the risk $\tilde{\varepsilon}$, and thus imply an increase in the completeness index α. Similarly, macroeconomic variables such as GDP are observed with measurement errors and lags. Improvements in national accounting can lead to more precise hedging instruments and a corresponding increase in α.

The distribution of the loading φ in the population is specified by a measure μ on the real line. To clarify the intuition, we assume that the measure μ has a continuous density $f(\varphi)$, whose support is the nonnegative interval $[0,\infty)$.[13] The parameters α and κ are also taken to be non-degenerate, in the sense that $\alpha > 0$ and $0 < \kappa < \infty$.[14]

We easily infer from Section 2 the equilibrium conditions. Let φ^p denote the average loading of participants:

$$\varphi^p = \int_{\mathcal{P}} \varphi d\mu^p(\varphi). \tag{3.2}$$

Market entrants have average income $\tilde{e}^p = \mathbb{E}\,\tilde{e}^p + \varphi^p(\tilde{\varepsilon}^A + \tilde{\varepsilon}^{A^\perp})$, and their individual consumption satisfies

$$\widetilde{c}^h = \mathbb{E}\,\widetilde{c}^h + \varphi^p\,\tilde{\varepsilon}^A + \tilde{e}^{hA^\perp}.$$

As seen in Theorem 1, the marketable consumption risk $\varphi^p\,\tilde{\varepsilon}^A$ is identical for all participants. Consider an asset $\widetilde{a}$, $\pi(\widetilde{a}) > 0$, that positively covaries with the factor. The endogenous loading φ^p controls the covariance between the security and individual consumption, $Cov(\widetilde{c}^h, \widetilde{a}) = \varphi^p Cov(\tilde{\varepsilon}, \widetilde{a})$, and therefore the pricing of risk. Let $\widetilde{R}_a = \widetilde{a}/\pi(\widetilde{a})$ denote the random (gross) return on the asset. By Theorem 3, the *relative risk premium* satisfies

$$\frac{\mathbb{E}\,\widetilde{R}_a - R}{R} = \frac{\gamma\varphi^p Cov(\widetilde{a}, \tilde{\varepsilon})}{\mathbb{E}\,\widetilde{a} - \gamma\varphi^p Cov(\widetilde{a}, \tilde{\varepsilon})}.$$

We will show that financial innovation can reduce the consumption loading φ^p and thus diminish the risk premium of preexisting securities.

[13] The theorems of this section are in fact proved for densities $f(\varphi)$ with *arbitrary* unbounded supports.

[14] Degenerate values of α and κ are discussed in Appendix B.

We now turn to the equations that determine the interest rate and the participation set. The equilibrium of the bond market implies

$$\ln R = \ln R_0 + \chi\mu(\mathcal{P}) \left[\kappa + \frac{\alpha\gamma}{2} Var_{\mathcal{P}}(\varphi)\right], \tag{3.3}$$

where $Var_{\mathcal{P}}(\varphi) = \int_{\mathcal{P}}(\varphi - \varphi^p)^2 d\mu^p(\varphi)$ denotes the variance of the participants' loadings. By Theorem 2, an agent enters if the diversification benefit $\alpha\gamma(\varphi - \varphi^p)^2/2$ is larger than the opportunity cost κR. As a result, the participation set

$$\mathcal{P} = (-\infty, \varphi^p - \Lambda] \cup [\varphi^p + \Lambda, +\infty) \tag{3.4}$$

is the union of two half-lines that are equidistant from φ^p by length

$$\Lambda = \sqrt{2\kappa R/(\alpha\gamma)}. \tag{3.5}$$

Agents $\varphi \geq \varphi^p + \Lambda$ are hedgers who trade risky assets to reduce their exposure. Conversely, agents with loadings $\varphi \leq \varphi^p - \Lambda$ are speculators who increase their consumption risk in order to earn a higher return. An equilibrium is thus a triplet (R, Λ, φ^p) satisfying equations (3.2) – (3.5).

The equilibrium calculation is simplified by the following observation. Equations (3.2) and (3.4) impose that φ^p is both the average loading and the center of symmetry of the participation set $\mathcal{P}$. In the Appendix, we show that this restriction implies

Property 1. *For any $\Lambda \geq 0$, there exists a unique $\varphi^p(\Lambda)$ satisfying equations* (3.2) *and* (3.4).

We can now define $\mathcal{P}_\Lambda$ as the participation set $(-\infty; \varphi^p(\Lambda) - \Lambda] \cup [\varphi^p(\Lambda) + \Lambda; +\infty)$ corresponding to a given length Λ. It is easy to show

Property 2. *The participation set $\mathcal{P}_\Lambda$ decreases with the length parameter: $\mathcal{P}_{\Lambda'} \subseteq \mathcal{P}_\Lambda$ for all $\Lambda \leq \Lambda'$.*

Since the sets $\{\mathcal{P}_\Lambda; \Lambda \geq 0\}$ are nested, the length parameter Λ provides a precise ordering of the participation structure. A high value of Λ corresponds to a small set $\mathcal{P}_\Lambda$ and thus a low participation rate $\mu(\mathcal{P}_\Lambda)$.

To develop intuition on the risk premium, consider the simpler model in which the interest rate R is *exogenous.* The formula $\Lambda = \sqrt{2\kappa R/(\alpha\gamma)}$ then expresses the participation parameter as a function of exogenous quantities only. A higher completeness index α reduces Λ and thus increases the participation set $\mathcal{P}_\Lambda$. The implied movement in the loading φ^p then controls changes in pricing of risk.

Property 3. *When the loading density verifies the skewness condition*

$$f(\varphi^p - \Lambda) > f(\varphi^p + \Lambda), \tag{3.6}$$

the average loading $\varphi^p(\Lambda)$ locally increases with Λ.

Figure 2 illustrates the mechanism underlying this key result. When Λ decreases, the skewness of the loading density implies that more agents enter to the left (speculators) than to the right (hedgers) of φ^p, which pushes down the average consumption loading φ^p. A majority of the new entrants seeks to buy the factor's marketable component $\tilde{\varepsilon}^A$, bid up its price, and thus drive down the risk premium. The fixed interest rate setup thus illustrates the role of the loading density $f(\varphi)$ on the comparative statics of asset prices.

The equilibrium analysis requires more care in the full-fledged model in which the interest rate is endogenous. Properties 1 and 2 imply that in the (Λ, R) plane, an equilibrium corresponds to the intersection of the two curves:

$$R_1(\Lambda) = \alpha\gamma\Lambda^2/(2\kappa), \tag{3.7}$$

$$R_2(\Lambda) = R_0 \exp\{\chi\mu(\mathcal{P}_\Lambda)\,[\kappa + \alpha\gamma(Var_{\mathcal{P}_\Lambda}\varphi)/2]\}. \tag{3.8}$$

The functions express respectively the entry decision and the equilibrium of the bond market. We observe that $R_1(\Lambda)$ is increasing and quadratic, while $R_2(\Lambda)$ monotonically decreases with Λ (by Property 2). This helps establish

Theorem 4. *There exists a unique equilibrium.*

Figure 3 illustrates the geometric determination of equilibrium, and helps to analyze the effect of financial innovation. An increase in α pushes up both curves in the figure, implying a higher interest rate and an ambiguous change in the participation parameter Λ.

Theorem 5. *The riskless rate R increases with financial innovation. As the completeness index α increases from 0 to 1, the set of participants $\mathcal{P}$ has two possible behaviors. It is either monotonically increasing; or there exists $\alpha^* \in (0,1)$ such that $\mathcal{P}$ increases on $[0, \alpha^*]$ and decreases on $[\alpha^*, 1]$.*

The two behaviors are illustrated in Figure 4. The ambiguous effect of financial innovation on market participation has a simple intuition. On one hand, a higher α increases the diversification benefit $\alpha\gamma(\varphi^h - \varphi^p)^2/2$ of trading risky assets and encourages entry. On the other hand, new assets reduce the precautionary motive and increase the interest rate, thus discouraging participation. In empirical settings, we expect that the favorable effect of improved diversification, which stems from risk aversion, will tend to dominate.

The change in participation depends on the sensitivity of curves R_1 and R_2 to the innovation parameter α. Let $\eta_{X,\alpha} = d \ln X / d \ln \alpha$ denote the elasticity of an endogenous quantity X. We infer from equation (3.7) that

$$\eta_{\Lambda,\alpha} = (\eta_{R,\alpha} - 1)/2.$$

Financial innovation increases the set of participants ($\eta_{\Lambda,\alpha} < 0$) if it only has a weak impact on the interest rate ($\eta_{R,\alpha} < 1$). In addition, we observe that the elasticity of $R_2(\Lambda)$ with respect to α increases with the dispersion of the participants' loadings $Var_{\mathcal{P}_\Lambda}\varphi$. When traders have very heterogeneous incomes, financial innovation allows agents to greatly reduce their average consumption risk, as shown by (2.8). As a result, new assets have a strong impact on the individual precautionary motive and the equilibrium interest rate. This explains why participation is non-monotonic in Figure 4 for the loading density with the highest variance. Furthermore since the participation sets $\mathcal{P}_\Lambda$ are nested, financial innovation cannot induce simultaneous entry and exit in the one-factor case.

The effect of innovation on the risk premium is easily examined. Consistent with Figure 2, we show

Proposition 3. *The relative risk premium locally decreases with financial innovation if* $\eta_{\Lambda,\alpha}\left[f(\varphi_p - \Lambda) - f(\varphi_p + \Lambda)\right] < 0.$

This *local* result is analogous to condition (3.6) derived in the exogenous interest rate case, but now controls for changes in the participation parameter Λ.

The one-factor model may help explain a number of features that have characterized financial markets in the past three decades. New financial instruments encouraged investors to participate in financial markets, which led to a reduction in the precautionary motive and in the covariance between stockholder consumption and the aggregate shock. These two effects in turn increased the interest rate and reduced the risk premium.[15] Note that this argument is consistent with earlier

[15] As shown in Appendix B, a higher entry cost implies a higher risk premium under condition

empirical findings. Mankiw and Zeldes (1991) thus show that the consumption of stockholders tends to be more correlated with the market than the consumption of non-stockholders. As financial innovation leads more people to enter the market, the risk premium falls. We leave the empirical exploration of this mechanism to further research.

In this section, financial innovation consisted of providing a better hedge against a common risk factor. In practice, however, households and firms face multiple sources of income shocks, and innovation often permits to hedge classes of risk that had been previously uninsurable. For this reason, we now examine a multifactor model of risk.

4. Multifactor Economies

We now consider an economy with a finite number of risk factors $(\tilde{\varepsilon}_1, .., \tilde{\varepsilon}_L)$, which correspond to macroeconomic or sectoral shocks affecting individual income. For instance, $\tilde{\varepsilon}_1$ could be an aggregate risk, and $\tilde{\varepsilon}_2, .., \tilde{\varepsilon}_L$ industry or firm-specific shocks. We specify the income of each investor h as

$$\tilde{e}^h = \mathbb{E}\,\tilde{e}^h + \sum_{\ell=1}^{L} \varphi_\ell^h \tilde{\varepsilon}_\ell, \tag{4.1}$$

and denote by $\varphi^h = (\varphi_1^h, ..., \varphi_L^h)$ the vector of individual loadings. The model is tractable when the risk factors and the asset payoffs are jointly normal. Without loss of generality, we normalize the factors to have unit variances and no mutual correlation: $(\tilde{\varepsilon}_1, .., \tilde{\varepsilon}_L) \sim \mathcal{N}(0, I)$. The distribution of factor loadings in the population is specified by a continuous density $f(\varphi)$ on $\mathbb{R}^L$.

The factors may not be fully tradable when financial markets are incomplete. As in the previous section, it is useful to consider their projections $\tilde{\varepsilon}_\ell^A = \sum_{j=1}^{J} Cov\left(\tilde{\varepsilon}_\ell, \tilde{a}_j\right) \tilde{a}_j$ on the asset span. We interpret $\tilde{\varepsilon}_\ell^A$ as the marketable component of factor ℓ, which can be estimated by regressing $\tilde{\varepsilon}_\ell$ on the asset payoffs. We conveniently stack the projected factors in a vector $\tilde{\varepsilon}^A = (\tilde{\varepsilon}_1^A, .., \tilde{\varepsilon}_L^A)$. The covariance matrix

$$\Sigma^A = Var\left(\tilde{\varepsilon}^A\right)$$

is a generalized index of market completeness, whose diagonal coefficients $\alpha_\ell = Var\left(\tilde{\varepsilon}_\ell^A\right)$ quantify the insurable fraction of each factor.

(3.6). Like models with exogenously restricted participation (e.g. Basak and Cuoco, 1998), our framework thus helps explain the equity premium puzzle.

We assume for simplicity that the projected factors are mutually uncorrelated: $Cov(\tilde{\varepsilon}_\ell^A, \tilde{\varepsilon}_k^A) = 0$ for all distinct ℓ and k. In the next subsections, this hypothesis will make it more striking that the improved marketability of factor ℓ affects the risk premium on an uncorrelated component $\tilde{\varepsilon}_k^A$. The covariance matrix is then diagonal:

$$\Sigma^A = \begin{bmatrix} \alpha_1 & & \\ & \ddots & \\ & & \alpha_L \end{bmatrix},$$

with coefficients $\alpha_\ell = Var\left(\tilde{\varepsilon}_\ell^A\right)$ contained between 0 and 1. We note that Σ^A is equal to zero when there are no assets, and to the identity matrix when markets are complete.

The equilibrium calculation follows directly from Section 2. By (4.1), the mean endowment of participants satisfies

$$\tilde{e}^p = \mathbb{E}\,\tilde{e}^p + \sum_{\ell=1}^{L} \varphi_\ell^p \tilde{\varepsilon}_\ell, \tag{4.2}$$

where φ_ℓ^p represents the traders' average exposure to factor ℓ. The equilibrium of financial markets implies the relations

$$\pi(\tilde{a}) = [\mathbb{E}\,\tilde{a} - \gamma Cov(\tilde{e}^p, \tilde{a})]/R \tag{4.3}$$

and

$$\ln R = \ln R_0 + \chi\mu(\mathcal{P}) \left[\kappa + \frac{\gamma}{2}\sum_{i=1}^{L} \alpha_i Var_{\mathcal{P}}(\varphi_i)\right], \tag{4.4}$$

where $\ln R_0 = \ln(1/\beta) + \chi(\mathbb{E}\widetilde{e} - e_0) - (\gamma\chi/2)\sum_{i=1}^{L}\mathbb{E}(\varphi_i^2)$. These equations suggest that when the utility coefficients γ and χ^{-1} are large, financial innovation generates both substantial variations in the pricing of risk and small movements in the interest rate.

The entry condition (2.3) implies the participation set

$$\mathcal{P} = \left\{\varphi : \frac{\gamma}{2}\sum_{\ell=1}^{L} \alpha_\ell(\varphi_\ell - \varphi_\ell^p)^2 \geq \kappa R\right\}. \tag{4.5}$$

When all the coefficients α_ℓ are strictly positive, the participants are located outside an ellipsoid centered at $\varphi^p = (\varphi_1^p, .., \varphi_L^p)$.[16] The lengths $\Lambda_\ell = \sqrt{2\kappa R/(\alpha_\ell\gamma)}$

[16] The participants are located outside a cylinder when some coefficients α_ℓ are equal to zero.

of the ellipsoid along each axis depend on the completeness index α_ℓ and the endogenous interest rate. We show in Appendix C:

Theorem 6. *There exists a unique equilibrium.*

The proof begins by establishing that the lengths $\Lambda = (\Lambda_1, .., \Lambda_L)$ define a unique participation set $\mathcal{P}_\Lambda$. In contrast to the one-factor case, however, $\mathcal{P}_\Lambda$ can move in more than one direction and thus need not be decreasing as a set in each component Λ_ℓ. The market clearing of the bond uniquely determines the interest rate R and the lengths $\Lambda_\ell = \sqrt{2\kappa R/(\alpha_\ell \gamma)}$. The proof also provides a simple algorithm for the numerical computation of equilibrium. We now examine the comparative statics of participation and asset prices with respect to financial innovation.

4.1. Financial Innovation and the Risk Premium

The one-factor model shows that financial innovation can reduce the risk premium of securities correlated with the aggregate shock. In a multifactor economy, the improved marketability of a given factor can also have pricing effects on uncorrelated sectors. Consider for instance an economy with two factors $\tilde{\varepsilon}_1$ and $\tilde{\varepsilon}_2$. The random variable $\widetilde{\varepsilon}_1$ is an *aggregate shock* to which all investors are positively exposed, while the *idiosyncratic* risk $\widetilde{\varepsilon}_2$ is purely distributional. Let $\tilde{a} = \mathbb{E}\,\widetilde{a} + \tilde{\varepsilon}_1^A$, $\pi(\tilde{a}) > 0$, denote an asset or *stock* that is only correlated with the aggregate factor.[17] By equation (4.3), the stock has relative risk premium

$$\frac{\mathbb{E}\,\widetilde{R}_a - R}{R} = \frac{\gamma \varphi_1^p \alpha_1}{\mathbb{E}\,\widetilde{a} - \gamma \varphi_1^p \alpha_1}. \tag{4.6}$$

Consider how the premium is affected by an increase in the completeness index α_2. If participation were exogenous, the consumption loading φ_1^p would be a constant parameter, and the improved spanning of the idiosyncratic shock $\tilde{\varepsilon}_2$ would not affect the risk premium (4.6). In our model, however, innovation can affect the consumption loading φ_1^p and the premium (4.6) even though the stock $\tilde{a}$ and the idiosyncratic risk $\tilde{\varepsilon}_2$ are statistically independent.

Consider for example an economy in which a single risky asset $\tilde{a}$ is initially traded ($\alpha_2 = 0$). By (4.5), non-participants have loadings φ_1 that are close to the

[17] While the assets are assumed to be in zero net supply, we easily reinterpret the model in terms of equity by viewing endowment as the sum of a labor income and an exogenous endowment of stocks.

market average: $|\varphi_1 - \varphi_1^p| \leq \sqrt{2\kappa R/(\alpha_1\gamma)}$. When the index α_2 increases, some of these agents become willing to pay the entry cost because their exposure φ_2 is sufficiently different from the average loading φ_2^p. The new participants also trade the stock $\tilde{a}$ to achieve an optimal level of diversification. The risk premium on $\tilde{a}$ thus declines if a majority of the new entrants have low exposure to the aggregate shock ($\varphi_1 < \varphi_1^p$) and increase the demand for the stock. We expect this logic to hold when the distribution of φ_1 is skewed towards the origin, consistent with the intuition developed in the one-factor case.

A simple simulation of the cross-sectoral effect is presented in Figure 5. We assume for simplicity that exposures to the aggregate and idiosyncratic risks are independent in the population. The cross-sectional loading density is then $f(\varphi_1, \varphi_2) = f_1(\varphi_1)f_2(\varphi_2)$. This hypothesis makes it perhaps more surprising that increased marketability of the idiosyncratic risk modifies the equity premium. We specify $f_2(\varphi_2)$ to be symmetric around zero, which implies that $\varphi_2^p = 0$ in equilibrium. We discuss the choice of parameters in Appendix C. The stock is an asset of the form $\widetilde{a} = x + \widetilde{\varepsilon}_1^A$. We select the weighting coefficient x to obtain a risk premium $\mathbb{E}\widetilde{R}_a - R$ equal to 7% before the introduction of new contracts ($\alpha_2 = 0$). In the absence of a futures market, the net interest rate R equals 1% and the standard deviation of the stock return is $[Var(\widetilde{R}_a)]^{1/2} = 15\%$, implying an initial Sharpe ratio of about 1/2.

When α_2 increases from 0 to 1, the risk premium on the stock declines from 7% to 4.5%. Providing insurance against the idiosyncratic shock substantially decreases the risk premium through changes in participation. Consistent with empirical evidence, the standard deviation of the stock return is almost constant at 15%.[18] We observe that most of the decline in the risk premium occurs when the hedging coefficient α_2 increases from 0 to 0.5. The value $\alpha_2 = 0.5$ also yields values for participation (60%) and the real net interest rate (2.5%) that are reasonable for the current US economy. This suggests that the cross-sectoral effects induced by financial innovation may be quantitatively significant.[19] We leave to further

[18] Campbell, Lettau, Malkiel and Xu (2001) document that the volatility of stock market indices have been stationary over the past century.

[19] An increase in α_2 reduces the idiosyncratic or background risk of all participants. When utilities are isoelastic (or more generally exhibit decreasing absolute risk aversion), the reduction in background risk increases the demand for the stock and thus further reduces the risk premium (Kimball, 1990; Gollier, 1999). We anticipate that this additional channel substantially amplifies the pricing effect of financial innovation in more general setups. Heaton and Lucas (1999, pp. 237-239) make a similar argument in their insightful paper. Their framework, however, only considers a unique risk factor and asset, and therefore does not permit the distinction between

research the full empirical assessment of this mechanism.

4.2. Differential Effects and Participation Turnover

We now explore three additional consequences of innovation in the multifactor model: differential changes in sectoral risk premia, simultaneous entry and exit, and a possible reduction of the interest rate.

The previous simulations assumed that the loading density $f_2(\varphi_2)$ is symmetric around zero. Participants insure the marketable component of the idiosyncratic shock at no cost, and an asset correlated only with $\widetilde{\varepsilon}_2$ yields no risk premium ($\varphi_2^p = 0$). We now examine an economy in which the loading density $f_2(\varphi_2)$ has a positive support and is skewed towards the origin. The risks $\widetilde{\varepsilon}_1$ and $\widetilde{\varepsilon}_2$ are then independent sources of aggregate uncertainty that yield positive and possibly distinct premia. Financial innovation can differentially affect asset prices across sectors, and thus have rich effects on the cross-section of expected returns.

The comparative statics analysis of Figure 2 easily extends to the two-factor case. We consider a financial structure with completeness indices α_1 and α_2, and assume for simplicity that interest rate R is exogenous. The ellipse delimiting the participation set is illustrated by a solid line in Figure 6A. It is centered at φ^p and has length $\Lambda_\ell = \sqrt{2\kappa R/(\alpha_\ell \gamma)}$ along each axis. Consider an increase in the second index from α_2 to α_2'. Since the interest is fixed, the limiting boundary in the new equilibrium has the same horizontal length Λ_1 but a shorter vertical length Λ_2'. We represent the intermediate ellipse centered at φ^p with lengths Λ_1 and Λ_2' in dotted lines. Agents in the shaded area have smaller average loadings than φ_1^p and φ_2^p, and thus tend to push the new equilibrium set towards the origin. Because these agents are more spread out vertically than horizontally, the induced movement in φ^p tends to be stronger along the vertical axis, i.e. in the direction of innovation. The increased marketability of the shock $\widetilde{\varepsilon}_2$ may thus predominantly influence the risk premium in the second sector.

The new set of participants is delimited by the ellipse centered at φ_{new}^p with lengths Λ_1 and Λ_2', as illustrated in Figure 6B. Financial innovation induces simultaneous entry and exit. Agents in the shaded area are initially out of the market. When the new asset is introduced, these agents face lower hedging costs and decide to participate. Agents in the dashed area, on the other hand, are

background risk and aggregate shock. We anticipate that their numerical results would be strengthened by the simultaneous reduction of trader exposure to the aggregate and idiosyncratic risks considered in this paper.

initially investing in financial assets. Lower risk premia reduce the profitability of their investments and result in their leaving the market. The possibility of simultaneous entry and exit is thus an attractive feature of the multifactor model, which is expected to be applicable in a variety of contexts.[20]

The differential effect is illustrated in Figure 7 on a numerical example. The marginal densities of the factor loadings are identical log-normals. The initial economy has hedging coefficients $\alpha_1 = \alpha_2 = \underline{\alpha}$. We assume that the interest rate is endogenous and consider two fixed assets $\widetilde{a}_\ell = x_\ell + \widetilde{\varepsilon}_\ell^A$ ($\ell = 1, 2$) with a risk premium of 7%. The symmetry of the economy imposes that $x_1 = x_2 = x$. As α_2 increases from $\underline{\alpha}$ to 1, both risk premia fall and the effect is stronger for the second asset. The results of the figure are almost unchanged when the net interest rate is exogenously set at 2%. The differential effect is an important feature of the multifactor economy. It distinguishes the introduction of sector-specific securities from changes affecting all security markets, such as a reduction in taxes or transaction costs. In future work, this property may prove useful in explaining empirical findings on the price impact of financial innovation.[21]

Multifactor economies also imply novel results for the comparative statics of the interest rate. As discussed in Section 2, the introduction of new assets increases risk-sharing opportunities and weakens the precautionary demand for savings. In models with exogenous participation, this leads to a higher equilibrium interest rate under many specifications, including CARA-normal (Weil, 1992; Elul, 1997; Calvet 2001). The Appendix establishes that when participation is endogenous,

Proposition 4. *The interest rate locally decreases with financial innovation in some multifactor economies.*

This result has a simple geometric intuition. When new assets are introduced, the movement of φ^p pushes the ellipse towards a region containing a large number of participants. In some economies, this effect is sufficiently strong to reduce overall participation and the interest rate.

5. Conclusion

This paper develops a tractable asset pricing model with incomplete markets and endogenous participation. Agents receive heterogeneous random incomes deter-

[20] Participation tunover is discussed in this subsection for expositional convenience. It also arises when the distribution of φ_2 is symmetric, as shown in the proof of Proposition 4.

[21] See Allen and Gale (1994*a*) for a review of this literature.

mined by a finite number of risk factors. They can borrow or lend freely, but must pay a fixed entry cost to invest in risky assets. Security prices and the participation set are jointly determined in equilibrium. The introduction of non-redundant assets encourages investors to participate in financial markets for hedging and diversification purposes. Under plausible conditions on the cross-sectional distribution of risk, the new entrants reduce the covariance between dividends and trader consumption, which induces a reduction in the risk premium.

This logic is easily demonstrated in a simple one-factor model. Financial innovation also has cross-sectoral effects in economies with multiple sources of risk. When a factor becomes tradable, new agents are drawn to the market in order to manage their risk exposure. Under complementarities of learning or increasing returns to trading activities, the new agents also trade in preexisting markets and can modify the risk premia of securities uncorrelated to the factor. This mechanism differentially affects distinct sectors of the economy and thus may have a rich impact on the cross-section of expected returns. Simultaneous entry and exit is another attractive feature of the multifactor model, which is expected to be applicable in a variety of contexts.

This paper suggests several directions for empirical research. Future work could assess the contribution of financial innovation to the decline of the equity premium in the past few decades. Participation changes may also help explain the pricing effects of new derivatives reported in the empirical literature. From a policy perspective, the mechanisms examined in this paper provide useful insights on current debates in public and international economics. When countries face fixed costs to financial integration, the model implies that the creation of new markets can have profound pricing, participation and welfare consequences. An extension of this work could investigate the political economy of the macro markets advocated by Shiller and others. Further research may also evaluate government policies affecting asset creation and participation costs, such as financial regulation, taxes, and social security reform.

6. Appendix A - Existence and Efficiency of Equilibrium

This Appendix establishes the existence and constrained efficiency of general equilibrium with endogenous participation.

We prove existence for a standard convex economy, in which the state space is finite $\Omega = \{1, .., S\}$ and individuals consume non-negative amounts in every state. Assume that the utility function U^h of every agent is continuous, strongly monotonic and strictly quasi-concave on the non-negative orthant $\mathbb{R}_+^{S+1}$. At prices where agents are indifferent between entry ($\theta^h \neq 0$) and non-entry ($\theta^h = 0$), individual demand consists of two distinct points, which may lead to discontinuities in aggregate demand. This difficulty can be solved by making the following convexifying hypothesis. There is a finite number of individual types $h = 1, .., H$, and a continuum of agents in each type. We can then show

Theorem A.1. *There exists a GEEP equilibrium.*

Under standard conditions (Aumann, 1966), this result extends to any economy with a continuum of agents.

As in the GEI case, equilibrium allocations are usually Pareto inefficient because the absence of certain markets induces incomplete risk-sharing. With two periods and a single good, however, GEI allocations are known to satisfy a limited or *constrained* form of efficiency. No social planner can improve the utility of all agents when income transfers are constrained to belong to the asset span. This limited form of efficiency easily generalizes to our setting by taking into account the entry fee.

Definition. *An allocation* $(c_0^h, \tilde{c}^h)_{h \in H}$ *is called* feasible *if and only if*

1. *For all* h*, there exists* $(\theta_0^h, \theta^h) \in \mathbb{R} \times \mathbb{R}^J$ *such that* $\tilde{c}^h = \tilde{e}^h + \theta_0^h + \tilde{a} \cdot \theta^h$
2. $\int (c_0^h + \kappa 1_{\{\theta^h \neq 0\}}) d\mu(h) = e_0$*, and* $\int \tilde{c}^h(\omega)\, d\mu(h) = \tilde{e}(\omega)$ *for all* $\omega \in \Omega$.

We can then introduce

Definition. *A feasible allocation* $(c_0^h, \tilde{c}^h)_{h \in H}$ *is called* constrained Pareto-efficient *if no other feasible allocation makes all agents strictly better off.*

We show that any equilibrium allocation is constrained Pareto-efficient.

Theorem A.2. *An equilibrium allocation is constrained Pareto-efficient.*

The theorem implies that the introduction of a new asset cannot make all agents worse off.

6.1. Proof of Theorem A.1

We base our argument on the existence proof provided by Hens (1991) for the standard GEI case.

Individual Excess Demand

Given $p_0 > 0$ and a vector (π_0, π) of asset prices, it is convenient to define $q = (p_0, \pi_0, \pi)$ and the budget set

$$\widehat{B}^h(q) = \left\{ (c_0, \theta_0, \theta) \,\middle|\, \begin{array}{c} p_0(c_0 + \kappa 1_{\{\theta^h \neq 0\}}) + \pi_0\theta_0 + \pi \cdot \theta \leq p_0 e_0^h \\ \tilde{e}^h + \theta_0 + \tilde{a} \cdot \theta \geq 0 \end{array} \right\}.$$

The no-arbitrage set

$$Q = \left\{ (p_0, \pi_0, \pi) \in \mathbb{R}_{++} \times \mathbb{R}^{J+1} \,\middle|\, \begin{array}{c} \text{there exists } \Lambda \in \mathbb{R}^S_{++} \text{ such that} \\ \pi_j = \Lambda . a_j \text{ for all } j = 0, .., J \end{array} \right\}.$$

is an open convex cone of $\mathbb{R}^{J+2}$, and it is useful to consider its closure

$$\overline{Q} = \left\{ (p_0, \pi_0, \pi) \in \mathbb{R}_{+} \times \mathbb{R}^{J+1} \,\middle|\, \begin{array}{c} \text{there exists } \Lambda \in \mathbb{R}^S_{+} \text{ such that} \\ \pi_j = \Lambda . a_j \text{ for all } j = 0, .., J \end{array} \right\}.$$

Given $q \in Q$, we can calculate the optimal excess demands $Z^{hp}(q) \equiv [c_0^{hp}(q) + \kappa - e_0^h, \theta_0^{hp}(q), \theta^{hp}(q)]$ and $Z^{hn}(q) \equiv [c_0^{hn}(q) - e_0^h, \theta_0^{hn}(q), 0]$ of a participating and non-participating agent of type h. Given a participation decision $d \in \{p, n\}$, the excess demand function $Z^{hd}(q)$ is continuous, homogeneous of degree 0, and satisfies Walras' law. We can then define the excess demand correspondence

$$Z^h(q) = \begin{cases} Z^{hp}(q) & \text{if } V^h\left[Z^{hp}(q)\right] > V^h\left[Z^{hn}(q)\right] \\ Z^{hn}(q) & \text{if } V^h\left[Z^{hp}(q)\right] < V^h\left[Z^{hn}(q)\right] \\ [Z^{hp}(q), Z^{hn}(q)] & \text{if } V^h\left[Z^{hp}(q)\right] = V^h\left[Z^{hn}(q)\right] \end{cases},$$

where $V^h(z)$ denotes the utility $U^h(c_0, \widetilde{e}^h + \theta_0 + \widetilde{a}.\theta)$ associated to a vector $z = [c_0 + \kappa 1_{\{\theta \neq 0\}} - e_0^h, \theta_0, \theta]$. We observe that $Z^h(q)$ is homogeneous of degree 0, upper hemi-continuous and satisfies Walras' law.

Consider a vector $\overline{q} \in \overline{Q} \backslash Q$, $\overline{q} \neq 0$, and a sequence $\{q^n\}_{n=1}^{\infty}$ of elements of Q converging to $\overline{q}$. We want to show that $\inf\{\|z\|\,; z \in Z^h(q^n)\} \to +\infty$. Proceed by contradiction and assume that there exists a bounded sequence $\{z^{n_k}\}_{k=0}^{\infty}$, $z^{n_k} \in Z^h(q^{n_k})$ for all k. The sequence $\{z^{n_k}\}_{k=0}^{\infty}$ has then a cluster point $\overline{z}$. Without loss of generality, it is convenient to henceforth neglect subsequence notation and directly assume that $z^n \to \overline{z}$. Given $x \in \widehat{B}^h(\overline{q})$, we know that x is the limit of a sequence $\{x^n\}$, $x^n \in \widehat{B}^h(q^n)$. Since $x^n \in \widehat{B}^h(q^n)$, we know that $V^h(x^n) \leq V^h(z^n)$ for all n. Letting n go to infinity, we infer that $V^h(x) \leq V^h(\overline{z})$ for all $x \in \widehat{B}^h(\overline{q})$, which is absurd. This establishes that $\inf\{\|z\|\,; z \in Z^h(q^n)\} \to \infty$ as $n \to \infty$. We can also consider the matrices $M = [a_0, .., a_J]$ and $N = \begin{bmatrix} 1 & \\ & M \end{bmatrix}$, and show by a similar argument that $\inf\{\|z\|\,; z \in NZ^h(q^n)\} \to \infty$ as $n \to \infty$. Moreover since consumption is non-negative, the set $NZ^h(q^n) \geq -e^h$ is bounded below.

Market Excess Demand

We now define the market excess demand

$$Z(q) \equiv \sum_{h=1}^{H} \mu(h) Z^h(q).$$

The correspondence $Z(q)$ is upper hemi-continuous, convex and compact-valued, homogeneous of degree 0 and satisfies Walras' law: $q.Z(q) \equiv 0$. Moreover consider an arbitrary vector $\widehat{q} \in Q$ and a sequence $\{q^n\}_{n=1}^{\infty}$ of elements of Q converging to a vector $\overline{q} \in \overline{Q} \backslash Q$, $\overline{q} \neq 0$. Since each $NZ^h(q^n)$ is bounded below, we infer that $NZ(q^n)$ is bounded below and $\inf\{\|z\|\,; z \in NZ(q^n)\} \to \infty$. The absence of arbitrage implies that $\widehat{q} = N^{\intercal}\widehat{\Lambda}$ for some $\widehat{\Lambda} \in \mathbb{R}_{++}^{S+1}$. Since $\inf\{\|z\|\,; z \in NZ(q^n)\} \to \infty$, we infer that $\widehat{q}.Z(q^n) = \widehat{\Lambda}.NZ(q^n) > 0$ for n large enough. We then conclude by standard arguments (Debreu, 1956; Grandmont, 1977; Hens, 1991) that there exists an equilibrium price.

6.2. Proof of Theorem A.2

Assume that there exists a feasible allocation $(d_0^h, \widetilde{d}^h)_{h \in H}$ such that $U^h(d_0^h, \widetilde{d}^h) > U^h(c_0^h, \tilde{c}^h)$ for all h. We know that for all h, there exists (η_0^h, η^h) such that $\widetilde{d}^h = \tilde{e}^h + \eta_0^h + \tilde{a} \cdot \eta^h$. Since $(d_0^h, \widetilde{d}^h)$ is strictly preferred to $(c_0^h, \tilde{c}^h)$, it must be that $d_0^h + \pi_0 \eta_0^h + \pi \cdot \eta^h + \kappa 1_{\{\eta^h \neq 0\}} > e_0^h$. We aggregate across consumers: $\int \eta_0^h d\mu = 0$, $\int \eta^h d\mu = 0$ and $\int (d_0^h + \kappa 1_{\{\eta^h \neq 0\}}) d\mu > e_0$, which contradicts feasibility.

7. Appendix B - CARA-Normal Economies (General Case and One Factor)

7.1. Proof of Theorems 1 and 2

The decision problem of a participant consists of maximizing

$$-e^{-\chi c_0} - \beta \left\{ \mathbb{E} e^{-\gamma\left[\tilde{e}^h + \tilde{a}.\theta + R(e_0^h - c_0 - \pi.\theta - \kappa)\right]} \right\}^{\chi/\gamma}$$

with respect to the *unconstrained* variables c_0 and θ. For any choice of these variables, the random consumption $\tilde{c}$ has a normal distribution with mean $\mathbb{E}\widetilde{e}^h + R(e_0^h - c_0 - \pi.\theta - \kappa)$ and variance $Var(\widetilde{e}^{hA^\perp}) + Var(\widetilde{e}^{hA} + \widetilde{a}.\theta)$. With the notation $u(c) = -e^{-\chi c}$, the objective function reduces to

$$u(c_0) + \beta u[\mathbb{E}\tilde{c} - \gamma Var(\tilde{c})/2],$$

or equivalently

$$u(c_0) + \beta u(D - Rc_0) \exp\left[\chi R(\,\pi.\theta + \kappa) + \chi\gamma Var(\widetilde{e}^{hA} + \widetilde{a}.\theta)/2\right],$$

where $D = \mathbb{E}\widetilde{e}^h + Re_0^h - \gamma Var(\widetilde{e}^{hA^\perp})/2$ is exogenous to the agent.

The utility maximization problem is decomposed in two steps. First, the optimal portfolio θ^h minimizes the quadratic function

$$R\,(\pi.\theta + \kappa) + \gamma Var(\widetilde{e}^{hA} + \widetilde{a}.\theta)/2.$$

The first order condition implies that $\theta_j^{h,p} = -Cov(\tilde{a}_j, \tilde{e}^h) - R\pi_j/\gamma$. The optimal portfolio has random payoff $\widetilde{a}.\theta^{h,p} = -\tilde{e}^{hA} + \tilde{m}^A$, where $\tilde{m}^A = -(R/\gamma)\sum_{j=1}^J \pi_j \tilde{a}_j$.

Second, the initial consumption c_0 is chosen to maximize

$$u(c_0) + \beta u(D - Rc_0) \exp\left[\chi R\,(\pi.\theta^{h,p} + \kappa) + \chi\gamma Var(\widetilde{m}^A)/2\right]. \tag{7.1}$$

The first order condition $u'(c_0) = \beta R u'(D - Rc_0) \exp\left[\chi R\,(\pi.\theta^{h,p} + \kappa) + \chi\gamma Var(\widetilde{m}^A)/2\right]$ can be rewritten $c_0 = -\ln(\beta R)/\chi + D - Rc_0 - [R(\pi.\theta^h + \kappa) + \gamma Var(\widetilde{m}^A)/2]$, which implies

$$c_0^{h,p} = \frac{1}{1+R}\left\{ R(e_0^h - \kappa - \pi.\theta^{h,p}) + \mathbb{E}\,\tilde{e}^h - \frac{1}{\chi}\ln(R\beta) - \frac{\gamma}{2} Var(\widetilde{c}^h) \right\}.$$

We then deduce $\theta_0^{h,p}$ from the budget constraint.

Similarly, a non-participant maximizes the function

$$u(c_0) + \beta u(D - Rc_0) \exp\left[\chi\gamma Var(\widetilde{e}^{hA})/2\right]. \tag{7.2}$$

Comparing the functional forms (7.1) and (7.2), we infer that participation is optimal if

$$\gamma Var(\widetilde{e}^{hA})/2 \geq R\,(\pi.\theta^{h,p} + \kappa) + \gamma Var(\widetilde{m}^{A})/2.$$

This is equivalent to $\gamma Var(\widetilde{e}^{hA} - \widetilde{m}^{A})/2 \geq \kappa R$.

7.2. Proof of Theorem 3

We obtain the price of risky assets by averaging $\theta_j^{h,p}$ across participating agents. The mean demand $\int_H \theta_0^h d\mu(h)$ for the riskless asset is

$$\frac{R}{1+R}\begin{bmatrix} e_0 - \mathbb{E}\tilde{e} + \chi^{-1}\ln(R\beta) + \frac{\gamma}{2}\int_H Var\left(\tilde{e}^h\right)d\mu(h) \\ -\kappa\mu(\mathcal{P}) + \frac{\gamma}{2}\mu(\mathcal{P})Var\left(\tilde{e}^{pA}\right) - \frac{\gamma}{2}\int_{\mathcal{P}} Var\left(\tilde{e}^{hA}\right)d\mu(h) \end{bmatrix}.$$

In equilibrium, mean demand is zero and the interest rate therefore satisfies (2.7).

7.3. Proof of Proposition 2

Financial innovation increases the assets span to $A' \supset A$, $A' \neq A$. The space A' can be decomposed in two orthogonal subspaces A and $B = A^{\perp} \cap A'$. By definition, R' and $\mathcal{P}'$ solve the system

$$\begin{cases} \mathcal{P}' = \left\{h : \frac{\gamma}{2}\left[Var\left(\tilde{e}^{hA} - \tilde{e}^{A}\right) + Var\left(\tilde{e}^{hB} - \tilde{e}^{B}\right)\right] \geqslant R'\kappa\right\} \\ \ln R' = \ln R_0 + \chi\mu\left(\mathcal{P}'\right)\kappa + \frac{\chi\gamma}{2}\int_{\mathcal{P}'}\left[Var\left(\tilde{e}^{hA} - \tilde{e}^{A}\right) + Var\left(\tilde{e}^{hB} - \tilde{e}^{B}\right)\right]d\mu(h). \end{cases}$$

Assume that $R' < R$. The first equation implies $\mathcal{P} \subseteq \mathcal{P}'$, and we infer from the second equation that $R' \geq R$, a contradiction.

7.4. Degenerate Cases of the One-Factor Economy

We begin by analyzing the special cases $\alpha = 0$ and/or $\kappa = 0$. When assets have no correlation with the risk factor ($\alpha = 0$), the participation set is empty under costly entry, and indeterminate under free entry. In either case, the risk premium is zero and the interest rate is uniquely determined: $R = R_0$. When the completeness index is positive ($\alpha > 0$) and the entry cost is positive and finite, we infer from Proposition 1 and Assumption 3 that the set of participants and non-participants both have a positive measure: $0 < \mu(\mathcal{P}) < 1$,[22] implying $R > R_0$ in

[22] If everyone participates, $\varphi^p = \overline{\varphi}$ and $\alpha\gamma(\varphi^h - \overline{\varphi})^2/2 \geq \kappa R$ for almost every agent h, which leads to a contradiction since the density f is strictly positive on every neighborhood of $\overline{\varphi}$.

any equilibrium. Finally, there are no participants ($\mathcal{P} = \emptyset$) when the entry cost is infinite.

7.5. Proof of Properties 1-3

Consider the function

$$G(\varphi_p, \Lambda) = \int_{-\infty}^{\varphi_p - \Lambda} (\varphi - \varphi_p) d\mu + \int_{\varphi_p + \Lambda}^{+\infty} (\varphi - \varphi_p) d\mu$$

with domain $\mathbb{R} \times [0, +\infty)$. For every fixed $\Lambda \geq 0$, the partial function $G_\Lambda(\varphi_p) = G(\varphi_p, \Lambda)$ is continuous, strictly decreasing, and satisfies $\lim_{\varphi_p \to -\infty} G_\Lambda(\varphi_p) = +\infty$, $\lim_{\varphi_p \to +\infty} G_\Lambda(\varphi_p) = -\infty$. The equation $G_\Lambda(\varphi_p) = 0$ has therefore a unique solution, which is denoted by $\varphi_p(\Lambda)$. It is then convenient to define the set $\mathcal{P}_\Lambda = \{\varphi : |\varphi - \varphi^p(\Lambda)| \geq \Lambda\}$.

We infer from the Implicit Function Theorem that the function $\varphi_p(\Lambda)$ is differentiable. Let $\Delta(\Lambda) = f[\varphi^p(\Lambda) + \Lambda] - f[\varphi^p(\Lambda) - \Lambda]$ and $\nabla(\Lambda) = f[\varphi^p(\Lambda) + \Lambda] + f[\varphi^p(\Lambda) - \Lambda]$. We observe that $\partial G / \partial \varphi_p = -\Lambda \nabla - \mu(\mathcal{P}) < 0$, $\partial G / \partial \Lambda = -\Lambda \Delta$, and therefore

$$\frac{d\varphi_p}{d\Lambda} = -\frac{\Lambda \, \Delta(\Lambda)}{\Lambda \, \nabla(\Lambda) + \mu(\mathcal{P}_\Lambda)}.$$

The sign of $d\varphi_p / d\Lambda$ thus depends on the value of the density f at the endpoints $\varphi^p - \Lambda$ and $\varphi^p + \Lambda$. Since $\left| d\varphi_p / d\Lambda \right| \leq \Lambda \nabla / [\Lambda \nabla + \mu(\mathcal{P}_\Lambda)] \leq 1$, the functions $\varphi^p(\Lambda) - \Lambda$ and $\varphi^p(\Lambda) + \Lambda$ are respectively decreasing and increasing in Λ. We conclude that the set $\mathcal{P}_\Lambda$ is (weakly) decreasing in Λ.

7.6. Proof of Theorem 4

The existence of equilibrium was established in Appendix A for a standard economy with a finite state space and non-negative consumption sets. We now prove that in the one-factor CARA-normal case, equilibrium exists and is unique.

Consider the functions $H_0(\Lambda) = \mu(\mathcal{P}_\Lambda)$ and $H_1(\Lambda) = \mu(\mathcal{P}_\Lambda)(Var_{\mathcal{P}_\Lambda} \varphi)$. The monotonicity of $\mathcal{P}_\Lambda$ implies that $H_0(\Lambda)$ is decreasing in Λ. Similarly, the function

$$H_1(\Lambda) = \int_{-\infty}^{\varphi^p - \Lambda} (\varphi - \varphi_p)^2 f(\varphi) d\varphi + \int_{\varphi^p + \Lambda}^{+\infty} (\varphi - \varphi_p)^2 f(\varphi) d\varphi$$

has derivative $\Lambda^2 \left[f(\varphi_p - \Lambda) \frac{d(\varphi_p - \Lambda)}{d\Lambda} - f(\varphi_p + \Lambda) \frac{d(\varphi_p + \Lambda)}{d\Lambda} \right] + \int_{\mathcal{P}} 2(\varphi - \varphi_p) f(\varphi) d\varphi$, or

$$\frac{dH_1}{d\Lambda} = \Lambda^2 \frac{dH_0}{d\Lambda} < 0.$$

It is thus decreasing in Λ.

In equilibrium, R and Λ are determined by the system (3.7) − (3.8). We observe that R_1 is strictly increasing, R_2 is decreasing, $R_2(0) > R_0 > R_1(0) = 0$, and $R_1(+\infty) = +\infty$. The difference function $R_1(\Lambda) - R_2(\Lambda)$ is therefore strictly increasing and maps $[0, +\infty)$ onto $[-R_2(0), +\infty)$. There thus exists a unique equilibrium.

7.7. Proof of Theorem 5

The equilibrium (R, Λ) is determined by the system

$$\begin{cases} \kappa R - \alpha\gamma\Lambda^2/2 = 0, \\ \ln R - \ln R_0 - \kappa\chi H_0(\Lambda) - \alpha\chi\gamma H_1(\Lambda)/2 = 0. \end{cases}$$

The corresponding Jacobian matrix is

$$J = \begin{pmatrix} \kappa & -\alpha\gamma\Lambda \\ R^{-1} & J_{22} \end{pmatrix} \tag{7.3}$$

where $J_{22} = -\kappa\chi H_0'(\Lambda) - \alpha\chi\gamma H_1'(\Lambda)/2 = -\chi\kappa(1+R)H_0'(\Lambda) > 0$. We infer that $\det J > 0$.

We now infer from Cramer's rule the effect of financial innovation on the interest rate:

$$\frac{dR}{d\alpha} = -\frac{1}{\det J}\begin{vmatrix} -\gamma\Lambda^2/2 & -\alpha\gamma\Lambda \\ -\chi\gamma H_1(\Lambda)/2 & J_{22} \end{vmatrix} > 0. \tag{7.4}$$

Financial innovation therefore increases the interest rate. We similarly infer

$$\begin{aligned} \frac{d\Lambda}{d\alpha} &= -\frac{1}{\det J}\begin{vmatrix} \kappa & -\gamma\Lambda^2/2 \\ R^{-1} & -\chi\gamma H_1(\Lambda)/2 \end{vmatrix} \\ &= -(\kappa/\alpha)\left[1 - \alpha\chi\gamma\mu(\mathcal{P})(Var_{\mathcal{P}}\varphi)/2\right] / \det J, \end{aligned} \tag{7.5}$$

which has an ambiguous sign. The global behavior of Λ is established by a single crossing argument. We know that $\Lambda'(\alpha)$ has the same sign as $\alpha\chi\gamma H_1[\Lambda(\alpha)] - 1 \equiv G(\alpha) - 1$. Since $G(0) = 0$, the function $\Lambda(\alpha)$ is decreasing on a neighborhood of $\alpha = 0$. We observe that

$$G'(\alpha) = \chi\gamma H_1[\Lambda(\alpha)] + \alpha\chi\gamma\Lambda'(\alpha)H_1'[\Lambda(\alpha)].$$

Thus if α satisfies $G(\alpha) = 1$, we know that $\Lambda'(\alpha) = 0$ and $G'(\alpha) = \chi\gamma H_1[\Lambda(\alpha)] > 0$. The equation $G(\alpha) = 1$ has thus at most one solution on $(0, 1]$.

7.8. Proof of Proposition 3

The chain rule implies that

$$\frac{d\varphi^p}{d\alpha} = \frac{d\varphi^p}{d\Lambda}\frac{d\Lambda}{d\alpha}$$

has the same sign as $\eta_{\Lambda,\alpha}[f(\varphi^p + \Lambda) - f(\varphi^p - \Lambda)]$.

7.9. Effect of the Entry Fee

We can similarly analyze the effect of the transaction cost κ. We note that

$$\frac{dR}{d\kappa} = -\frac{1}{\det J}\begin{vmatrix} R & -\alpha\gamma\Lambda \\ -\chi H_0(\Lambda) & J_{22} \end{vmatrix}$$

has the sign of $\alpha\chi\gamma\Lambda\mu(\mathcal{P}_\Lambda) - RJ_{22}$, while

$$\frac{d\Lambda}{d\kappa} = -\frac{1}{\det J}\begin{vmatrix} \kappa & R \\ R^{-1} & -\chi H_0(\Lambda) \end{vmatrix} > 0.$$

This implies that the mass of participants decreases with the transaction cost κ. Finally, $d\varphi^p/d\kappa$ has the sign of $f(\varphi^p + \Lambda) - f(\varphi^p - \Lambda)$.

8. Appendix C - Multifactor Economies

8.1. Proof of Theorem 6

For every $\varphi^p \in \mathbb{R}^L$ and $\Lambda = (\Lambda_1, .., \Lambda_L) \in \mathbb{R}^L_{++}$, consider the set

$$\mathcal{P}(\varphi^p, \Lambda) = \left\{ \varphi : \sum_{i=1}^{L} \left(\frac{\varphi_i - \varphi_i^p}{\Lambda_i} \right)^2 \geq 1 \right\}. \tag{8.1}$$

The boundary of this set is an ellipsoid. An equilibrium consists of φ^p, Λ, and R satisfying

$$\int_{\mathcal{P}(\varphi^p,\Lambda)} (\varphi - \varphi^p) d\mu(\varphi) = 0, \tag{8.2}$$

$$\Lambda_\ell = \sqrt{2\kappa R/\alpha_\ell \gamma}, (1 \leq \ell \leq L),$$

and the market clearing condition

$$\ln R = \ln R_0 + \chi \int_{\mathcal{P}(\varphi^p,\Lambda)} \left[\kappa + \frac{\gamma}{2} \sum_{i=1}^{L} \alpha_i (\varphi_i - \varphi_i^p)^2 \right] d\mu(\varphi). \tag{8.3}$$

The analysis is simplified by

Fact C.1. *For any* $\Lambda \in \mathbb{R}^L_{++}$, *the equation* $\int_{\mathcal{P}(\varphi^p;\Lambda)}(\varphi - \varphi^p)d\mu(\varphi) = 0$ *has a unique solution* $\varphi^p \in \mathbb{R}^L$.

Proof. The equation can be conveniently rewritten as a convex optimization problem. More specifically, consider

$$k(\varphi;\varphi^p,\Lambda) = \left[\sum_{i=1}^{L}\left(\frac{\varphi_i - \varphi_i^p}{\Lambda_i}\right)^2 - 1\right] 1_{\mathcal{P}(\varphi^p,\Lambda)}(\varphi),$$

where $1_{\mathcal{P}(\varphi^p,\Lambda)}$ denotes the indicator function of $\mathcal{P}(\varphi^p,\Lambda)$. Since $k(\varphi;\varphi^p,\Lambda)$ is convex in φ^p and the measure μ has an unbounded support, the function

$$K(\varphi^p,\Lambda) = \frac{1}{2}\int_{\mathbb{R}^L} k(\varphi;\varphi^p,\Lambda)d\mu(\varphi) = \frac{1}{2}\int_{\mathcal{P}(\varphi^p,\Lambda)}\left[\sum_{i=1}^{L}\left(\frac{\varphi_i - \varphi_i^p}{\Lambda_i}\right)^2 - 1\right] d\mu(\varphi).$$

is *strictly convex* in φ^p. A vector φ^p thus minimizes $K(\varphi^p,\Lambda)$ on $\mathbb{R}^L$ if and only if $\partial K/\partial\varphi^p(\varphi^p,\Lambda) = 0$, which coincides with (8.2). It is therefore equivalent for a vector φ^p to minimize $K(\varphi^p,\Lambda)$ or to be the center of mass of $\mathcal{P}(\varphi^p,\Lambda)$. This observation is very useful for the numerical calculation of equilibrium. From a theoretical standpoint, note that the function $K(\varphi^p,\Lambda)$ is strictly convex on $\mathbb{R}^L$ and diverges to $+\infty$ as $\|\varphi^p\| \to +\infty$. This implies that the function $K(\varphi^p,\Lambda)$ reaches a minimum at a *unique* point φ^p. ■

Let φ^p_Λ denote the unique solution to (8.2), and $\mathcal{P}_\Lambda$ the corresponding participation set. Fact C.1 allows us to rewrite the equilibrium system as an equation of a unique variable, the interest rate R. For every $R > 0$, consider the lengths $\Lambda_\ell(R) = \sqrt{2\kappa R/(\alpha_\ell\gamma)}$, $(1 \le \ell \le L)$, and the vector $\Lambda(R) = [\Lambda_1(R), ..., \Lambda_L(R)]$. It is then natural to define the continuous functions $\varphi^p_{\Lambda(R)}$ and $\mathcal{P}_{\Lambda(R)}$, which will be henceforth denoted $\varphi^p(R)$ and $\mathcal{P}(R)$ for simplicity. We also consider the function

$$z(R) = \ln R_0 + \chi\int_{\mathcal{P}(R)}\left\{\kappa + \frac{\gamma}{2}\sum_{i=1}^{L}\alpha_i\left[\varphi_i - \varphi_i^p(R)\right]^2\right\} d\mu(\varphi). \qquad (8.4)$$

The market clearing of the bond imposes that

$$z(R) = \ln R.$$

An equilibrium exists and is unique when the function $z(R)$ is (weakly) decreasing. We can indeed establish

Fact C.2. *The function $z(R)$ is decreasing in R.*

Proof. We show this property by differentiating $z(R)$ with respect to the interest rate. Note that on the boundary of $\mathcal{P}(R)$, the integrand $\kappa+\frac{\gamma}{2}\sum_{i=1}^{L}\alpha_i\left[\varphi_i-\varphi_i^p(R)\right]^2$ takes the constant value $\kappa(1+R)$. The chain rule therefore implies

$$z'(R)=\chi\kappa(1+R)\frac{d\mu[\mathcal{P}(R)]}{dR}-\chi\gamma\int_{\mathcal{P}(R)}\left\{\sum_{i=1}^{L}\alpha_i\left[\varphi_i-\varphi_i^p(R)\right]\frac{d\varphi_i^p(R)}{dR}\right\}d\mu(\varphi). \tag{8.5}$$

The second term is zero because φ^p is the center of mass. Thus,

$$z'(R)=\chi\kappa(1+R)\frac{d\mu[\mathcal{P}(R)]}{dR}. \tag{8.6}$$

This expression is non-positive by Fact C.3 below. ■

Fact C.3. *The mass of participants $\mu[\mathcal{P}(R)]$ is a decreasing function of R.*

Proof. The discussion proceeds in two steps. We first show that the property holds when indifferent agents are located on a sphere. We then extend the result to arbitrary ellipsoids.

Consider economies such that $\alpha_\ell=1$ for all ℓ. The boundary of a participation set $\mathcal{P}(R)$ is a sphere, which is denoted $S(R)$. Given two positive numbers R and δ, $\delta<R$, we seek to show that

$$\mu[\mathcal{P}(R)]\leq\mu[\mathcal{P}(R-\delta)]. \tag{8.7}$$

The inequality is trivially satisfied when $\mathcal{P}(R)\subseteq\mathcal{P}(R-\delta)$. We now focus on the case $\mathcal{P}(R)\subsetneq\mathcal{P}(R-\delta)$. Since the indifference sets $S(R)$ and $S(R-\delta)$ are spheres, their intersection is contained in a hyperplane H:

$$S(R)\cap S(R-\delta)\subset H.$$

Without loss of generality, we choose the axes so that the hyperplane H is described by the equation $\varphi_1=0$, and the center of gravity $\varphi^p(R)=(x,0\ldots0)$ has a positive first coordinate x. It is straightforward to show that $\varphi^p(R-\delta)$ has coordinates $(y, 0 \dots 0)$, where $y < x$.[23] We denote by $\mathcal{P}_- = \mathcal{P}(R) \backslash \mathcal{P}(R-\delta)$ the set of participants lost in moving from R to $R-\delta$, by $\mathcal{P}_+ = \mathcal{P}(R-\delta) \backslash \mathcal{P}(R)$ the set of gained participants, and by $\mathcal{P}_C = \mathcal{P}(R) \cap \mathcal{P}(R-\delta)$ the common intersection. Figure C1 illustrates these definitions. The subset $\mathcal{P}_-$ is contained in the half-space $\varphi_1 < 0$, and the subset $\mathcal{P}_+$ in the half-space $\varphi_1 > 0$. Since $\mathcal{P}(R) = \mathcal{P}_- \cup \mathcal{P}_C$ and $\mathcal{P}(R-\delta) = \mathcal{P}_+ \cup \mathcal{P}_C$, we infer that

$$\begin{aligned} \int_{\mathcal{P}_-} \varphi_1 d\mu(\varphi) + \int_{\mathcal{P}_C} \varphi_1 d\mu(\varphi) &= x \, \mu(\mathcal{P}_- \cup \mathcal{P}_C), \\ \int_{\mathcal{P}_+} \varphi_1 d\mu(\varphi) + \int_{\mathcal{P}_C} \varphi_1 d\mu(\varphi) &= y \, \mu(\mathcal{P}_+ \cup \mathcal{P}_C). \end{aligned}$$

Substracting these equalities implies

$$\int_{\mathcal{P}_+} \varphi_1 d\mu(\varphi) - \int_{\mathcal{P}_-} \varphi_1 d\mu(\varphi) = y \, \mu(\mathcal{P}_+ \cup \mathcal{P}_C) - x \, \mu(\mathcal{P}_- \cup \mathcal{P}_C).$$

The left-hand side of the equation is positive because $\mathcal{P}_+$ is contained in the half-space $\varphi_1 > 0$ and $\mathcal{P}_-$ is contained in the half-space $\varphi_1 < 0$. This implies the inequality: $y \, \mu(\mathcal{P}_+ \cup \mathcal{P}_C) \geq x \, \mu(\mathcal{P}_- \cup \mathcal{P}_C)$. Since $x > y$, we infer that

$$x \, \mu(\mathcal{P}_+ \cup \mathcal{P}_C) \geq y \, \mu(\mathcal{P}_+ \cup \mathcal{P}_C) \geq x \, \mu(\mathcal{P}_- \cup \mathcal{P}_C),$$

and conclude that inequality (8.7) holds in the spherical case.

When the coefficients α_ℓ are arbitrary, a linear change of variables allows us to return to the spherical case we just examined. Thus, consider the linear rescaling $\varphi_\ell^* = \Phi_\ell(\varphi) = \varphi_\ell \sqrt{\alpha_\ell}$, and the corresponding measure $\mu^* = \mu \circ \Phi^{-1}$. Note that this transformation does not involve a particular choice of R. For every $R > 0$, the rescaled set $\mathcal{P}^*(R) = \Phi[\mathcal{P}(R)]$ has a spherical boundary centered around $\varphi^{*p}(R) = \Phi[\varphi^p(R)]$:

$$\mathcal{P}^*(R) = \left\{ \varphi : \frac{\gamma}{2} \sum_{\ell=1}^{L} [\varphi_\ell^* - \varphi_\ell^{*p}(R)]^2 \geq \kappa R \right\}.$$

Furthermore, the condition $\int_{\mathcal{P}(R)} [\varphi - \varphi^p(R)] d\mu(\varphi) = 0$ implies that $\varphi^{*p}(R)$ is the center of gravity of $\mathcal{P}^*(R)$. We then conclude from the previous paragraph that the function $\mu[\mathcal{P}(R)] = \mu^*[\mathcal{P}^*(R)]$ is decreasing in R. ■

[23] The condition $\mathcal{P}(R) \subsetneq \mathcal{P}(R-\varepsilon)$ implies that $y - \Lambda_1(R-\varepsilon) < x - \Lambda_1(R)$ and thus $y < x + \Lambda_1(R-\varepsilon) - \Lambda_1(R) < x$.

8.2. Numerical Simulation

This subsection presents the parameterization used in the example of Figure 5. We begin by introducing a microeconomic structure that relates the loading density $f_1(\varphi_1)$ to aggregate volatility and the distribution of income. The random endowment of an agent is specified as

$$\widetilde{e}^h = e_0^h(1 + \sigma_1\widetilde{\varepsilon}_1) + \varphi_2^h\widetilde{\varepsilon}_2.$$

The individual loading $\varphi_1^h = \sigma_1 e_0^h > 0$ is therefore proportional to expected income.[24] The aggregate endowment in period 1 satisfies

$$\widetilde{e} = e_0(1 + \sigma_1\widetilde{\varepsilon}_1).$$

Without loss of generality, mean income is normalized to unity: $e_0 = 1$.

We specify the cross-sectional distribution of income to be lognormal: $\ln e_0^h \sim \mathcal{N}(\mu_z, \sigma_z^2)$. Since mean income is normalized to 1, the parameters μ_z and σ_z^2 satisfy the restriction $\mu_z + \sigma_z^2/2 = 0$. We choose $\mu_z = -0.25$, which corresponds to a reasonable Gini coefficient of 0.4. The standard deviation of aggregate income growth σ_1 is set at 0.04. Since $\varphi_1^h = \sigma_1 e_0^h$, the loading density $f_1(\varphi_1)$ is now fully specified.

The loading density $f_2(\varphi_2)$ is assumed to be a centered Gaussian $\mathcal{N}(0, \sigma_2^2)$ with standard deviation $\sigma_2 = 0.10$. The discount factor is $\beta = 0.96$. Since $e_0 = 1$, the utility coefficients γ and χ^{-1} coincide with relative risk aversion and the elasticity of intertemporal substitution at the mean endowment point. We choose $\gamma = 10$ and $\chi^{-1} = 2$. The quantity κ is the fraction of mean income used in the entry process, and is set equal to $\kappa = 0.8\%$.

The aggregate shock is partially tradable. We choose the corresponding completeness index to be $\alpha_1 = 0.5$, which is roughly consistent with the correlation between the NYSE value-weighted stock return and the permanent aggregate labor income shock reported in Campbell, Cocco, Gomes and Maenhout (2001). The stock is a traded asset of the form $\widetilde{a} = x + \widetilde{\varepsilon}_1^A$. We select the weighting coefficient x to obtain a risk premium $\mathbb{E}\widetilde{R}_a - R$ equal to 7% before the introduction of new contracts ($\alpha_2 = 0$). The net interest rate R is equal to 1% and the standard deviation of the stock return is $[Var(\widetilde{R}_a)]^{1/2} = 15\%$, implying a Sharpe ratio of about 1/2. These numbers are roughly consistent with historical data (e.g. Mehra and Prescott, 1985; Campbell, Lettau, Malkiel and Xu, 2001).

[24] We assume for simplicity that there is no expected growth between the two periods.

8.3. Proof of Proposition 4

We provide an example in an economy with two uncorrelated factors $(\varepsilon_1, \varepsilon_2)$ and a finite number of types. Letting $\delta = 0.01$, we consider $\varphi^A = (-2, 0)$, $\varphi^B = (1+\delta, 0)$, $\varphi^C = (2, 0)$, $\varphi^- = (0, -1+\delta)$ and $\varphi^+ = (0, 1-\delta)$, with respective weights $m^A = m^B = 1/5$, $m^C = 1/10$, $m^+ = m^- = 1/4$. The other parameters of the economy are $\gamma = \chi = 0.7$, $\kappa = 0.3$, $e_0 = \mathbb{E}\widetilde{e} = 1$, $\alpha_2 = 0.9$.

A straightforward extension of Theorem 7 implies that a unique equilibrium exists for any given value of α_1. When $\alpha_1 = 0.55$, we check that the participation set contains all the agents of type $A, C, +$ and $-$. The fraction of participants is $4/5$ and the net rate is approximately 7.9%.

On the other hand when $\alpha_1 = 0.9$, the participation set contains all the agents of type A, B, C. The participation rate has now fallen to $1/2$ and the net interest rate is now approximately 5.7%.

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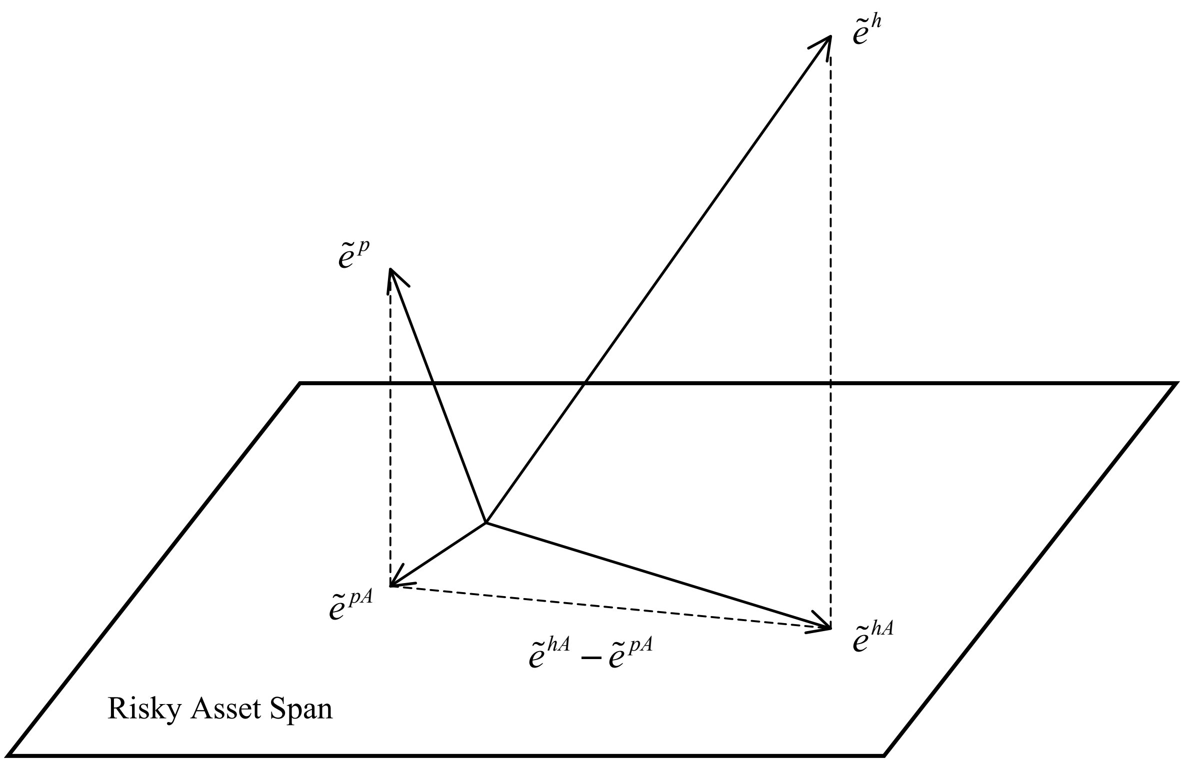

Figure 1: **Geometry of the Entry Condition**

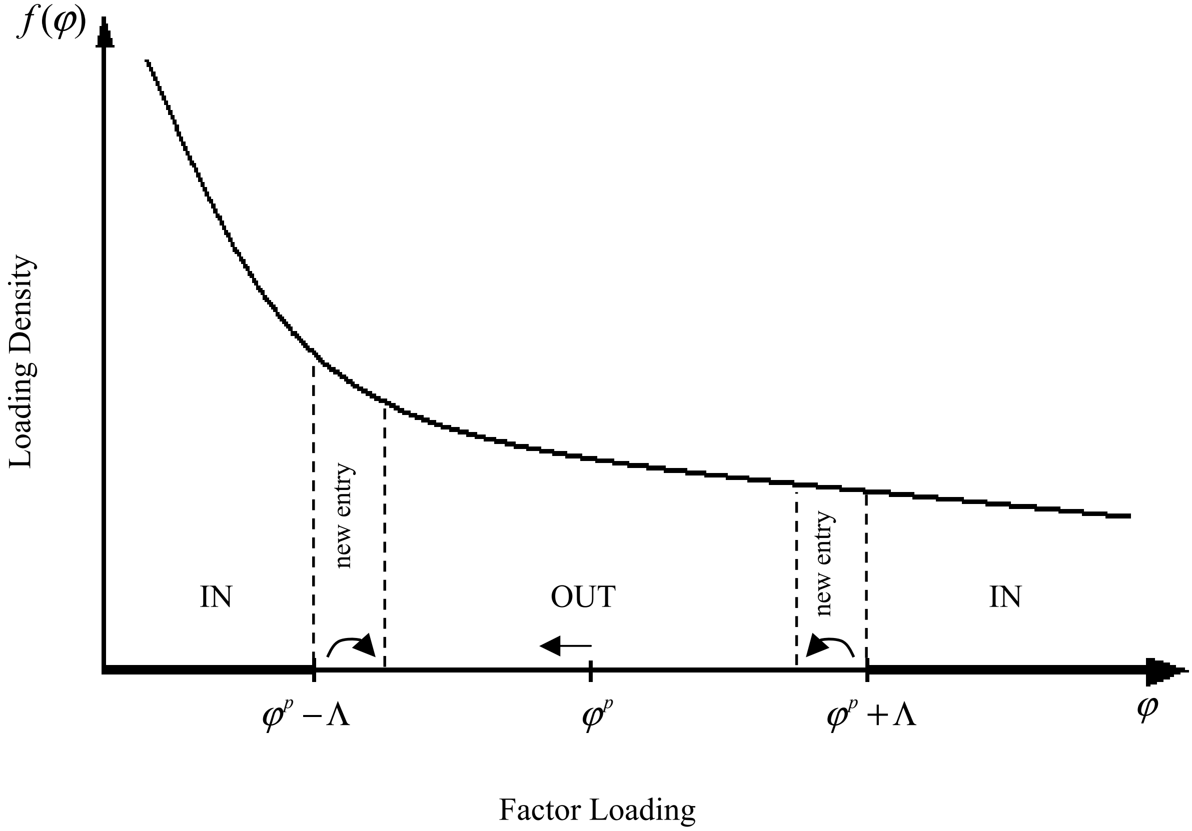

Figure 2: **Effect on Participation of a Decrease in Λ**

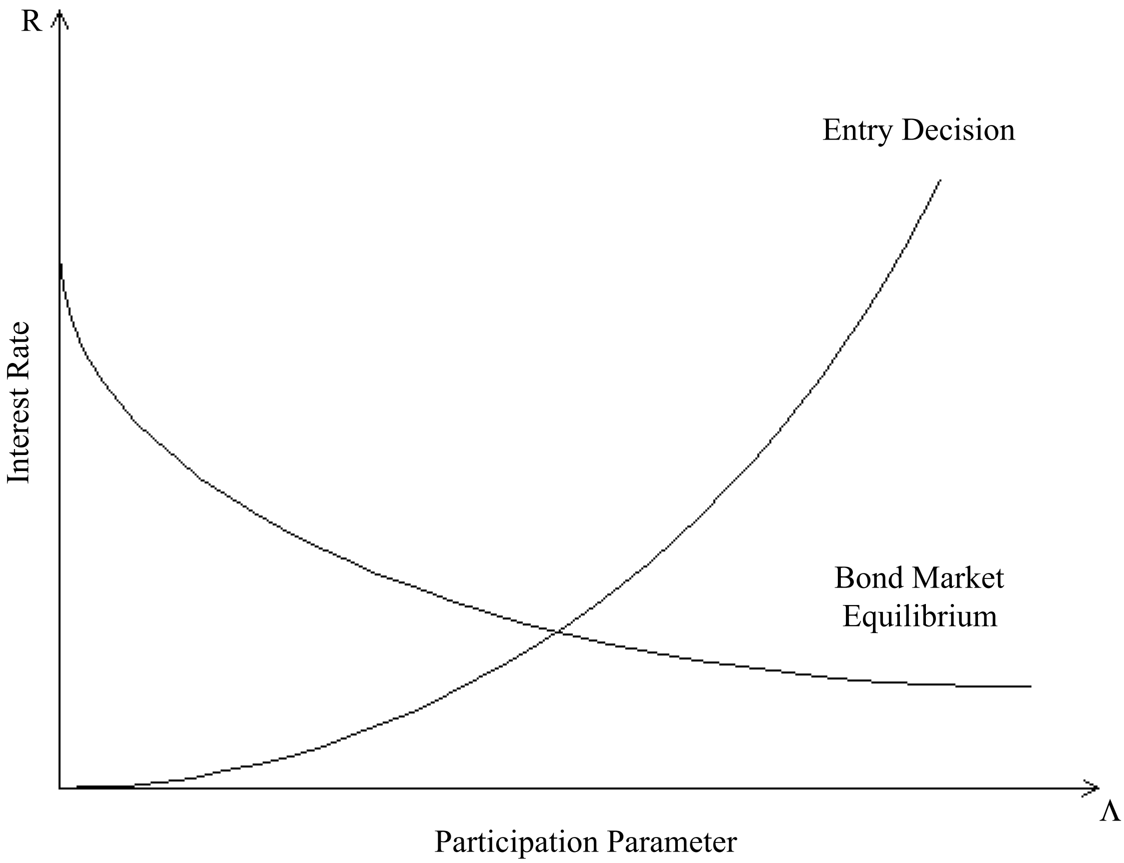

Figure 3: **Equilibrium of the One-Factor Economy**

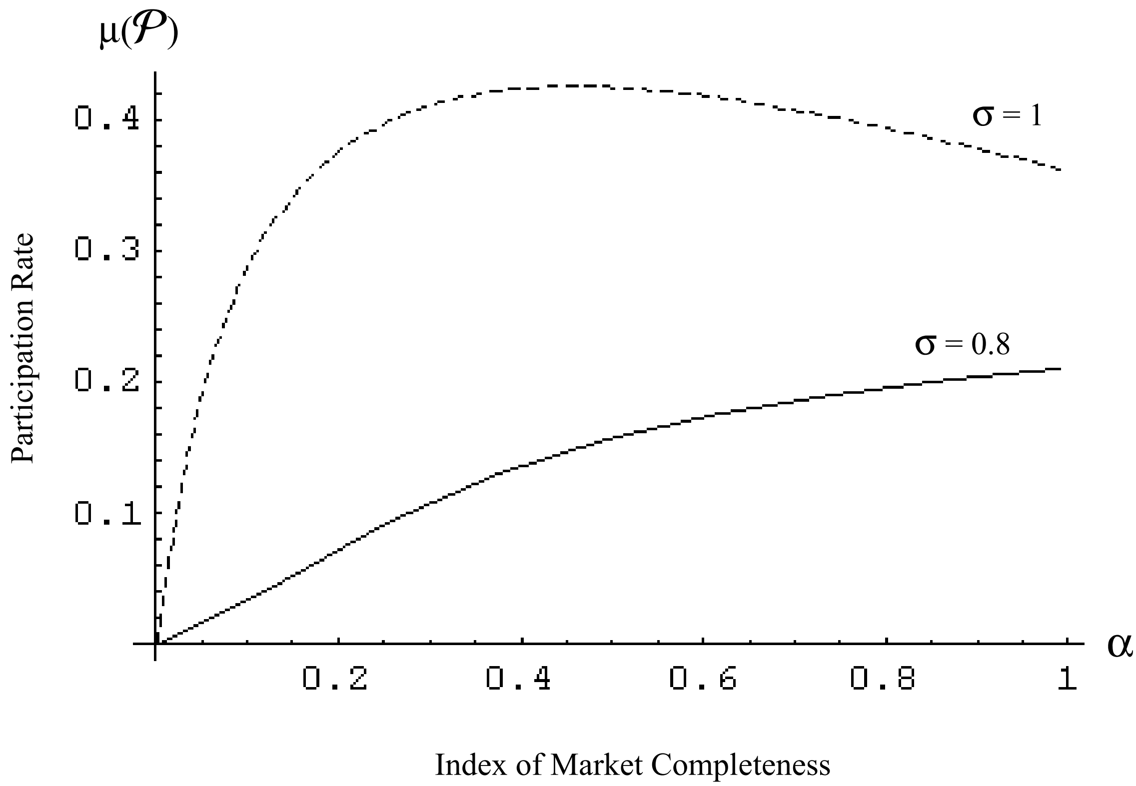

Figure 4: **Effect of Financial Innovation on Market Participation.** The cross-sectional loading distribution is log-normal: $\ln(\varphi) \sim \mathcal{N}(0, \sigma^2)$. The solid curve corresponds to $\sigma = 0.8$, and the dashed curve to $\sigma = 1$. The other parameters of the economy are: $\gamma = \chi = 1$, $\kappa = 1$ and $\beta = 1$.

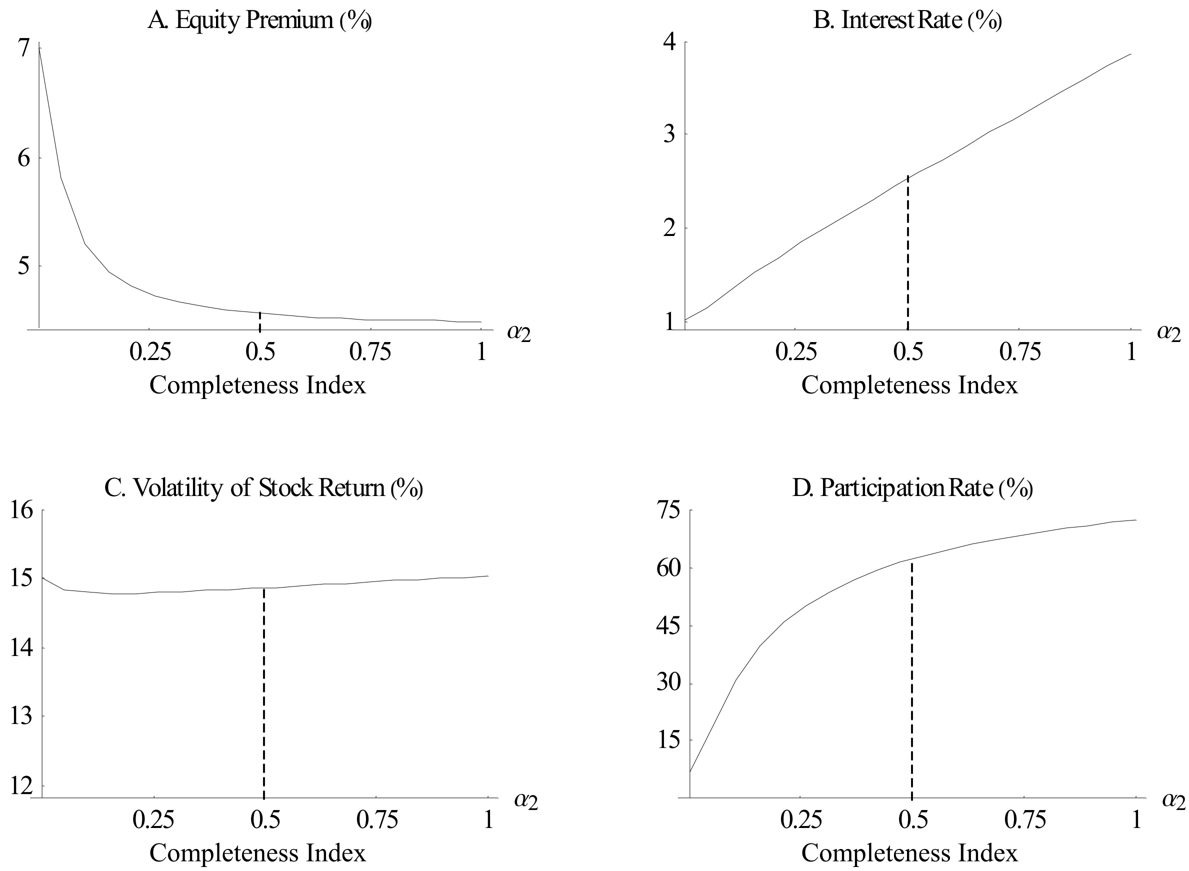

Figure 5: **Comparative Statics in a Two-Factor Economy.** Individual labor income is exposed to an aggregate shock ε_1 and an idiosyncratic risk ε_2. The aggregate shock is partially insurable ($\alpha_1 = 0.5$). The idiosyncratic risk is uncorrelated to the existing asset when $\alpha_2 = 0$ and is fully insurable when $\alpha_2 = 1$. The other parameters of the economy are $\beta = 0.96$, $\gamma = 10$, $\chi = 0.5$, $\kappa = 0.8\%$, $\sigma_1 = 4\%$ and $\sigma_2 = 10\%$.

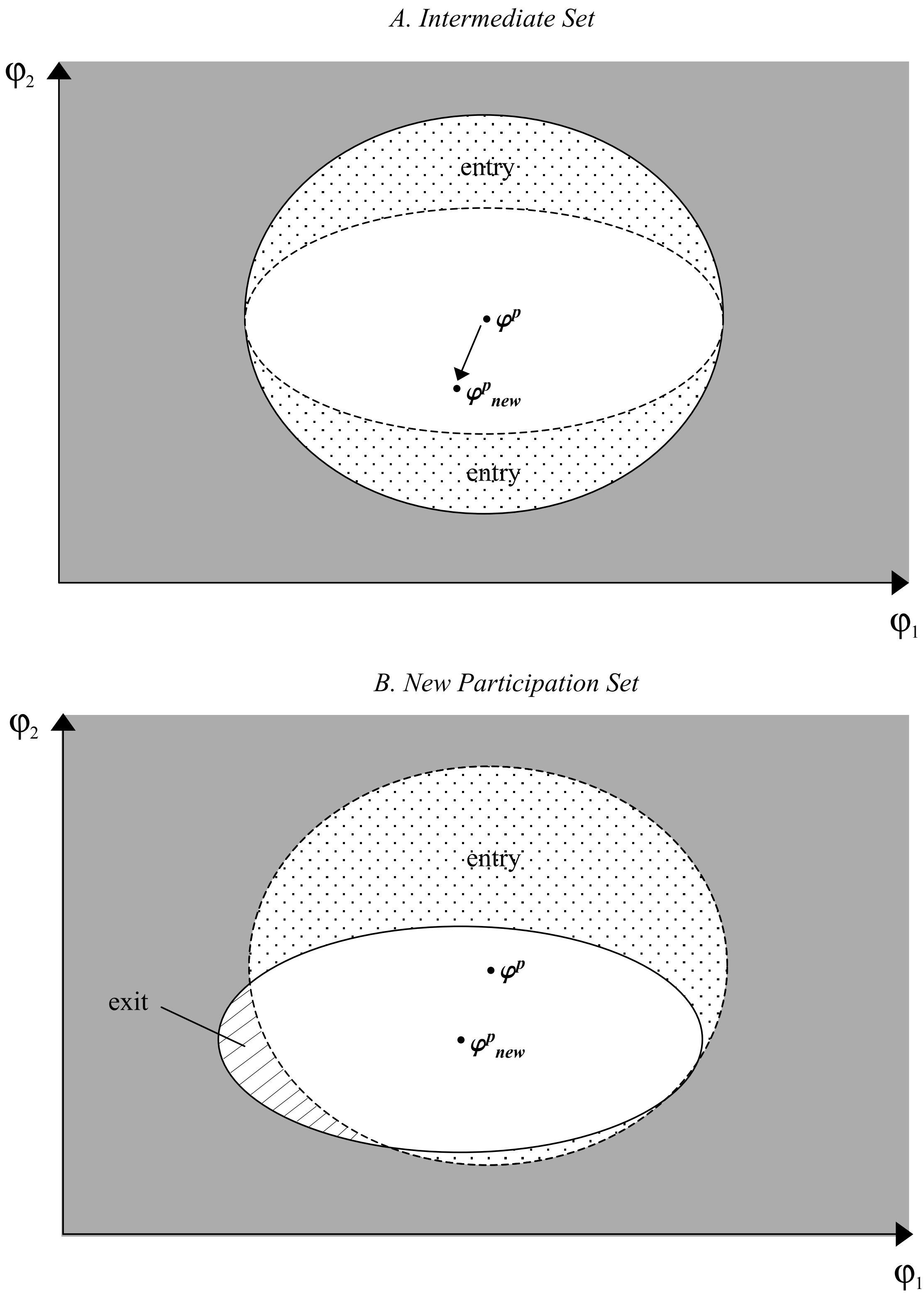

Figure 6: **Effect of an Increase in α_2 on Participation.** Initial participants are located outside the large ellipse of Panel A. When α_2 increases, the boundary shrinks vertically (small ellipse), and new entrants move the average loadings from φ^p to φ^p_{new}. The new participation set is delimited by the small ellipse of Panel B. The dotted area contains the new entrants, and the dashed area the agents who left the markets.

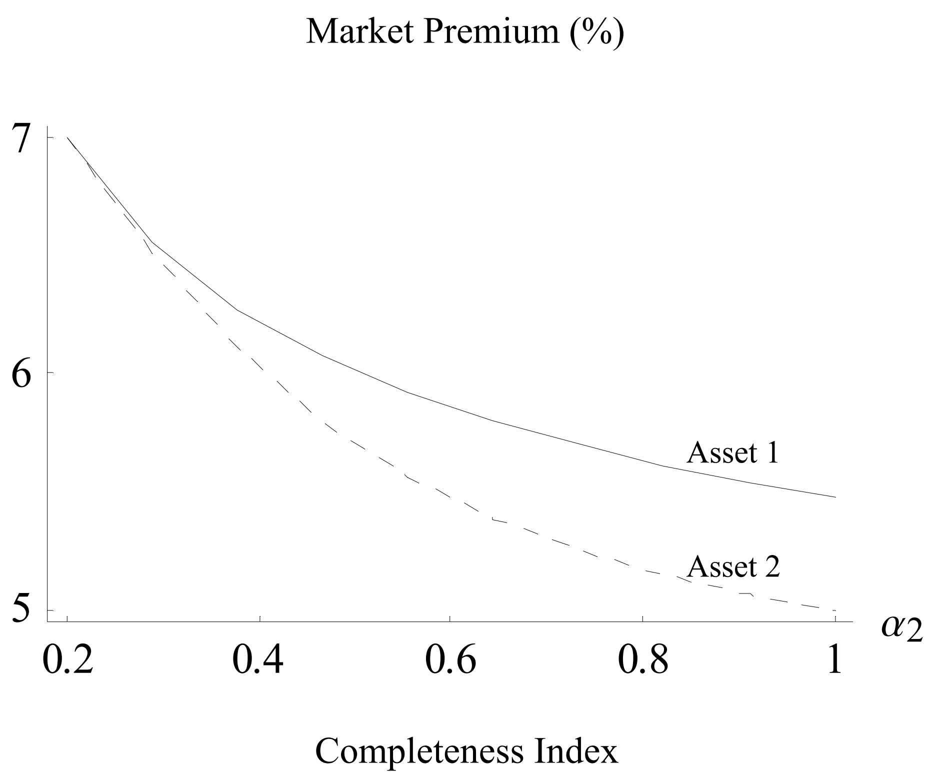

Figure 7: **Differential Effects of Financial Innovation.** The cross-sectional loading density is the product: $f(\varphi_1, \varphi_2) = g(\varphi_1)\, g(\varphi_2)$, where the function g is the density of a log-normal variable Z: $\ln Z \sim \mathcal{N}(-3.5, 1)$. The other parameters of the economy are: $\alpha_1 = 0.2$, $\gamma = 10$, $\chi = 0.5$, $\beta = 0.96$ and $\kappa = 0.8\%$.

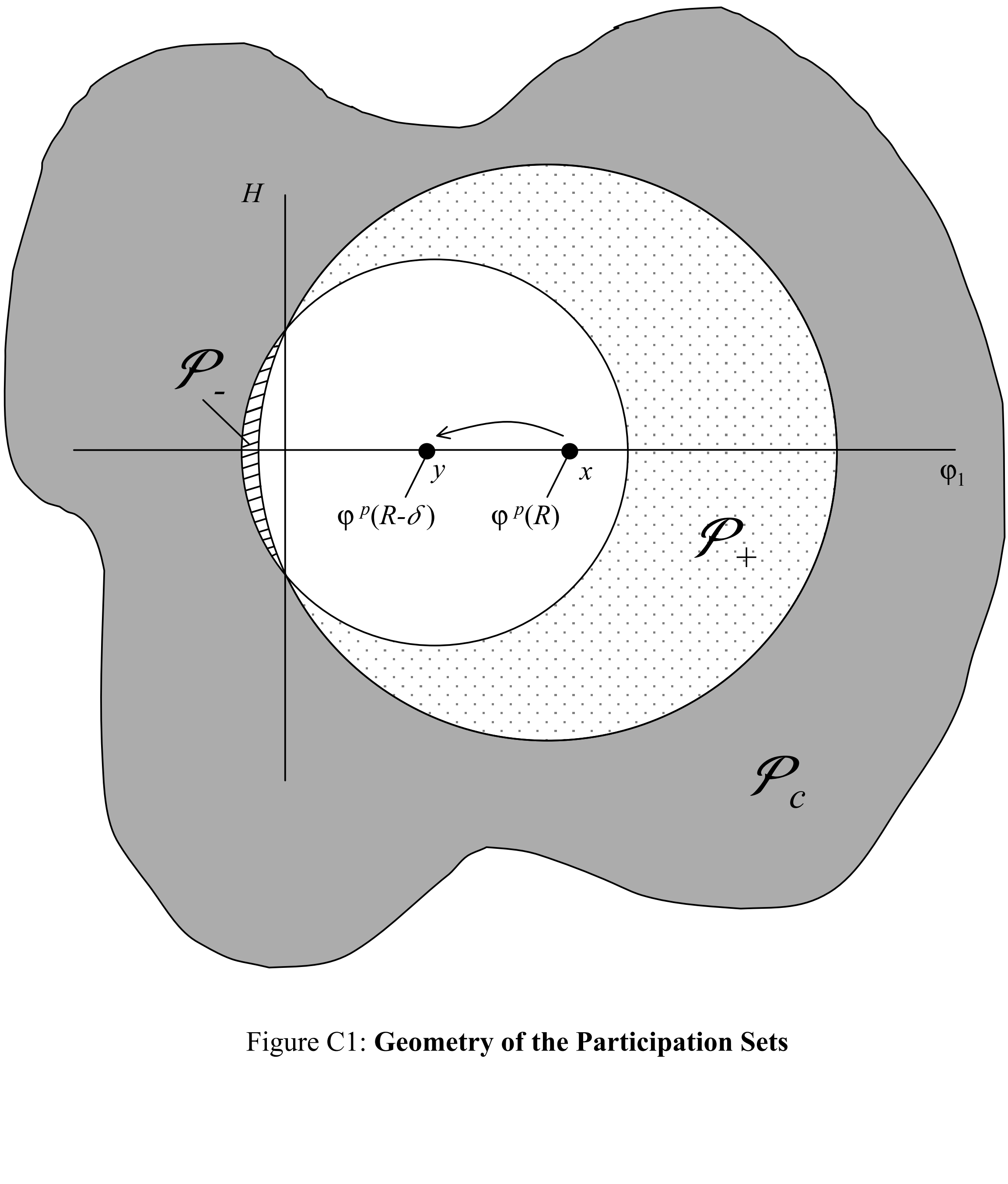

Figure C1: **Geometry of the Participation Sets**