# A Lender-Based Theory of Collateral*

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**Abstract**

We offer a novel explanation for collateral based on the notion that lenders make discretionary credit decisions that are too conservative. There is no borrower asymmetric information or moral hazard. Rather, the problem is that if lenders cannot extract the full surplus from the projects they finance (e.g., due to credit market competition), they may reject low-, but positive-NPV projects. Collateral provides lenders with additional protection in bad states, thus improving their payoffs from projects with a relatively high likelihood of bad states and thus precisely from those projects that are inefficiently rejected. Our model is consistent with existing empirical evidence and provides new empirical predictions.

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# 1 Introduction

Lending decisions are often discretionary.[1] After reviewing a borrower's request for credit, lenders use their judgement and expertise to decide whether to accept the borrower:

> "[F]ormalized interviews with bank managers indicate that loan officers located in the bank's branches enjoyed substantial autonomy when granting and pricing small business loans. The officers' own assessment of the development of the relationship with the firm, the skills and reputation of the firm's management, and the quality of the firm's business vision (i.e., "soft" information in Stein (2002)) played key roles in the lending decision. Though loan officers were required to "harden" their assessment internally by supplying key statistics and other relevant written information, much local discretion remained" (Degryse and Ongena (2003)).[2]

This paper argues that the discretionary nature of credit decisions provides a natural role for collateral. The argument proceeds in two steps: (i) in many instances, the lender's accept or reject decision will be biased, and (ii) collateral helps to alleviate this bias.

A bias in the lender's credit decision arises if—e.g., due to credit market competition—the lender must leave the borrower a share of the created surplus, implying that she cannot keep the full project cash flow for herself. As the lender provides the full investment outlay, this implies that she only accepts projects for which the expected project cash flow sufficiently exceeds the investment outlay. The lender thus rejects low-, but positive-NPV projects. Put simply, the lender is inefficiently conservative.[3]

The unique role of collateral is that it allows the lender to obtain a repayment in excess of the project's cash flow. Evidently, the lender cannot receive excess repayments in *all* states of nature. The question is thus how should collateral be used? We show that collateral should be used only in "bad" states of nature, i.e., in states where the project's cash flow is low.

[1] An important exception is fully automated credit scoring as used, e.g., in the credit card industry.

[2] See also Saunders and Allen (2002): "The credit decision is left to the local or branch lending officer or relationship manager. Implicitly, this person's expertise, subjective judgement, and his weighting of certain key factors are the most important determinants in the decision to grant credit".

[3] Inderst and Müller (2003) examine the implications of this inefficiency for the optimal security design. It is assumed that the borrower has no pledgeable assets; hence there is no role for collateral.

Giving the lender collateral in bad states—and leaving her less than the full project cash flow in good states—yields the greatest "bang for the buck": as bad states are relatively more likely under low-NPV projects, it improves the lender's payoff from precisely those projects that are inefficiently rejected. By contrast, giving the lender collateral in good states—and leaving her less than the full project cash flow in bad states—improves the lender's payoff primarily from high-NPV projects. But these are projects the lender would have accepted anyway. Hence, adding collateral in bad states maximizes the lender's incentives to accept low-, but positive-NPV projects, thus minimizing her conservative bias.

This is not the first paper to argue why collateral may be optimal. Previous work has argued that collateral can alleviate problems of borrower moral hazard and asymmetric information.[4] In asymmetric information models (e.g., Bester (1985); Chan and Kanatas (1987); Besanko and Thakor (1987a,b)), collateral is used as a sorting device. In equilibrium, low-risk borrowers pledge more collateral than high-risk borrowers, which is the opposite of what Berger and Udell (1995) and other empirical studies find.

In moral hazard models, collateral is used as an incentive device (e.g., Chan and Thakor (1987); Boot, Thakor, and Udell (1991); Boot and Thakor (1994)). Collateral improves the borrower's incentives to work hard. By working hard, he can reduce the probability of default, thus avoiding the loss of his collateral. According to this basic intuition, collateralized loans should have a lower default probability than unsecured loans, which the opposite of what Berger and Udell (1990) and other studies find.

This basic intuition may not hold if there are different borrower types. For instance, Boot, Thakor, and Udell (1991) show that if borrower quality and effort are substitutes, collateral may be associated with both a higher ex-ante borrower risk and a higher ex-post default risk, which is consistent with the empirical evidence.[5] However, if quality and effort are complements, collateral may be associated with both a lower ex-ante borrower risk and a lower default risk. It is thus probably fair to say that moral hazard does not necessarily (in the sense of a robust empirical prediction) produce the positive correlation between collateral and risk found in empirical studies. In our model, collateral is associated with both a higher ex-ante borrower risk

[4] See Coco (2000) for a literature survey.

[5] See also Chan and Thakor (1987) and the discussion in Section 5.

and a higher ex-post default risk.

Given that lender discretion is at the heart of our argument, we attempt to endogenize it. Consistent with the opening quote, we assume that the lender's decision to accept or reject the borrower is partly based on qualitative, "soft" information, e.g., information about the borrower's managerial quality. Naturally, the assessment of qualitative information is difficult—if not impossible—to verify vis-á-vis outsiders. (The lender could always argue that her impression of the borrower is bad even when it is good.)[6] Given that contracts cannot condition on the lender's credit risk assessment, her accept or reject decision is discretionary.

For the lender's decision to be truly discretionary, a further assumption is needed: not only must the lender's credit risk assessment be unobservable to outsiders, it must also be true that nobody can *replicate* it. As for competing lenders, we assume that they do not have access to the same soft information as the (original) lender. Insofar as soft information is valuable for the assessment of credit risk, this implies that the (original) lender has a better estimate of the project's default likelihood.[7] For instance, Brunner, Krahnen, and Weber (2000) argue that "private corporate ratings (internal ratings) reflect the core business of commercial banks, whose superior information as compared to an external assessment by the market allows a more precise estimate of the POD [probability of default]". To fix ideas, we may think of the borrower as a small business, the lender as a local small-business lender, and competing lenders as distant, "arm's-length" lenders or as a "competitive credit market".

As far as the borrower is concerned, we assume that what distinguishes borrowers from (professional) lenders is that—even if the borrower had access to the same information as the local lender—he would lack the expertise to replicate the local lender's credit risk assessment and thus her estimate of the project's success likelihood.[8]

[6] Stein (2002) argues that a distinguishing characteristic of soft information is its nonverifiability. Similarly, Brunner, Krahnen, and Weber (2000) argue that "internal ratings should therefore be seen as private information. Typically, banks do not inform their customers of the internal ratings or the implied PODs [probability of default], nor do they publicize the criteria and methods used in deriving them". Similarly, Treacey and Carey (2000) note that "[A]t banks, ratings are kept private ...".

[7] Grunert, Norden, and Weber (2002), analyzing credit file data from large German banks, find that the inclusion of soft information in internal credit ratings significantly improves the accuracy of predicting default.

[8] Manove, Padilla, and Pagano (2001) note: "As a result, banks are likely to be more knowledgeable about some aspects of project quality than many of the entrepreneurs they lend to [...] This is why banks are, and

In summary, neither the borrower (for lack of expertise) nor the competitive credit market (for lack of access to soft information) can observe or replicate the local lender's credit risk assessment. This has additional implications. First, it implies that both the optimal contract and the lender's credit decision are renegotiation-proof, for renegotiation between the local lender and the borrower takes place under asymmetric information. Second, competition between the local lender and the competitive credit market is imperfect.

While most of the existing arguments for collateral assume borrower moral hazard or asymmetric information, some papers, like ours, assume incentive problems on the part of the lender. Rajan and Winton (1995) examine the effect of collateral on the lender's ex-post monitoring incentives. Monitoring is valuable because it allows the lenders to claim additional collateral if the firm enters into financial distress. The issue is thus not whether financial claims should be collateralized ex ante, but whether lenders will try to collateralize their claims ex post. Manove, Padilla, and Pagano (2001) argue that collateral and screening are substitutes. Lenders either demand collateral *or* screen borrowers. By contrast, in our model collateral reinforces the efficiency of screening: if the local lender did not assess the borrower's credit risk prior to the credit decision, collateral would have no value.

The rest of this paper is organized as follows. Section 2 presents the model. Section 3 shows why the local lender has a conservative bias and why collateral alleviates this bias. Section 4 considers exogenous variations in the borrower's assets, project risk, project size, and the cost of borrowing from distant lenders. Section 5 summarizes the empirical implications of our model. Section 6 concludes. Appendix A extends our basic setting to a continuum of cash flows. All proofs are in Appendix B.

## 2 The Model

### Basic Setup

A firm ("the borrower") has an indivisible project requiring a fixed investment outlay $k > 0$.[9]

ought to be, in the project-evaluation business". Consistent with the notion that professional lenders are better at estimating default risk, Reid (1991) finds that bank-financed firms are more likely to survive than firms funded by family investors.

[9] Bester (1985), Chan and Kanatas (1985), Chan and Thakor (1987), Besanko and Thakor (1987a), Boot,

Financing can be either provided by a local lender or distant lenders. We frequently refer to the distant lenders as "competitive credit market".

To secure the loan, the borrower can pledge assets, e.g., business property, machines, or receivables due in the future. Denote the value of the borrower's assets by $w$. The project cash flow $x$ is verifiable and can be either high ($x = x_h$) or low ($x = x_l$). The two cash-flow model is the simplest framework to illustrate our main argument. In Appendix A, we show that it straightforwardly extends to a setting with a continuum of cash flows. We finally assume that $x_l + w < k$, which implies that the investment cannot be financed through a safe claim. The risk-free rate of interest is normalized to zero.

**The "Local Advantage"**

The difference between the local lender and the competitive credit market is that the local lender has better information about the project's success likelihood. Precisely, we assume that the competitive credit market has access to all verifiable (or "hard") information such as, e.g., financial data. Given this information, the probability that the project will be successful—i.e., that $x = x_h$—is $p \in (0, 1)$. The expected project cash flow is $\mu := px_h + (1 - p)x_l$. We assume that $\mu > k$, i.e., the project's NPV based on hard information information is positive.

Besides having access to hard information, the local lender has additionally access to qualitative (or "soft") information, e.g., information about managerial quality.[10] We assume that the local lender's credit risk assessment can be represented by a continuous variable $s \in [0, 1]$.[11] A high value of $s$ indicates that the project is of high quality, implying that it has a high probability of success. Let $p_s$ denote the project's success probability based on hard *and* soft information. Accordingly, we assume that $p_s$ is increasing in $s$, which implies that the expected project cash flow, $\mu_s := p_s x_h + (1 - p_s)x_l$, is also increasing in $s$. As the competitive credit market only observes $p$—but not $p_s$—consistency of beliefs implies that $p = \int_0^1 p_s f(s)ds$, where $f(s)$ is the

Thakor, and Udell (1991), Boot and Thakor (1994), and Rajan and Winton (1995) all assume a fixed loan size. A notable exception is Besanko and Thakor (1987b). Assuming a fixed loan size implies that the set of contracting variables reduces to two variables: collateral and interest rate.

[10] See our introductory remarks. Another reason why the local lender might have an advantage is due to her experience from having granted similar loans in the past (e.g., Boot and Thakor (2000)).

[11] This may be interpreted as the local lender's internal rating of the borrower. See Treacy and Carey (2000) for an overview of internal rating procedures at US banks.

density of $s$.

To formalize the idea that having access to soft information is beneficial, we assume that $\mu_1 > k$ and $\mu_0 < k$, i.e., the project's NPV taking into account hard and soft information is positive for high $s$ and negative for low $s$. Hence, the local lender is able to distinguish between positive- and negative-NPV projects. The competitive credit market, by contrast, only knows the project's NPV based on hard information, which is positive.

We are not the first to model credit market competition between a better informed lender and an arm's-length credit market: Sharpe (1990), Rajan (1992), and von Thadden (2004) all consider competition between a single lender with better information about a borrower and an uninformed credit market.[12]

A possible application of our setting is small-business lending. Petersen and Rajan (2002) (for the United States) and Degryse and Ongena (2003) (for Belgium) document that the median distance between banks and small-firm borrowers is four and 1.4 miles, respectively, suggesting that local lenders have indeed a competitive advantage. In the same spirit, Petersen and Rajan (1995) argue that "credit markets for small firms are local", and Guiso, Sapienza, and Zingales (2004) refer to "direct evidence of the informational disadvantage of distant lenders in Italy". Similarly, Berger, Klapper, and Udell (2001) find that small firms in Argentina are more apt to borrow from local than from foreign banks.[13] Finally, access to qualitative information appears to be particularly important in small-business lending: Treacy and Carey (2000) show that qualitative factors—while less important in internal ratings of loans to larger firms—are crucial in internal ratings of small- and medium-sized borrowers.

As far as the borrower is concerned, we assume that—even if he had access to the same information—he lacks the skills and expertise to replicate the local lender's credit risk assessment and probability estimate.[14] Hence, neither the competitive credit market (for lack of access to

[12] See also Dell'Ariccia and Marquez (2004) and Hauswald and Marquez (2003). Our asymmetric notion of credit market competition differs from Thakor (1996), who considers competition between multiple lenders who all observe the same (high) signal about a borrower after screening his project. (Lenders who observe a low signal optimally drop out of the competition.)

[13] Like our paper, Hauswald and Marquez (2003) and Almazan (2002) assume that lenders who are closer to a borrower have better information about the borrower than distant lenders.

[14] As the information is partly soft, the borrower cannot credibly communicate it to other lenders. For instance,

soft information) nor the borrower (for lack of expertise) can observe or replicate $s$ or $p_s$. As the local lender's credit decision is based on these variables, her decision is discretionary.

It is easy to show that—given our assumptions—the "standard solution" of having the local lender reveal her private information by choosing from a prespecified menu of contracts with different collateral levels and repayment requirements (i.e., interest rates) is *strictly* suboptimal in our model. Rather, the unique solution is to offer a single contract and accept or reject the borrower on the basis of this contract. This is consistent with the notion that "loan decisions for many types of retail loans are accept or reject decisions. All borrowers who are accepted are often charged the same rate of interest and by implication the same risk premium. [...] In the terminology of finance, retail customers are more likely to be sorted or rationed by loan quantity restrictions ratherthan by price or interest rate differences" (Saunders and Thomas (2001)).

**Sequence of Events**

There are three dates: $\tau = 0$, 1, and 2.

$\tau = 0$. To attract the borrower, the local lender and the competitive credit market make competiting offers.[15] The borrower then decides whether to visit the competitive credit market or the local lender.

$\tau = 1$. Lenders evaluate the borrower's project and decide whether to provide financing. We allow for both renegotiation and competition *after* the project evaluation. We consider renegotiation at the end of Section 3. As far as interim competition is concerned, we assume that the competitive credit market can observe whether the borrower has previously visited the local lender.[16] In practice, lenders typically check a borrower's credit history prior to making an offer. In many countries, including the United States, credit bureaus provide this information in the form of credit reports. Such credit reports commonly show whether and which other lenders have made similar inquiries in the past, including the date of the inquiry and the identity of the

he would always argue that his managerial skills are good.

[15]Is it a reasonable assumption that lenders make offers *before* evaluating a borrower's projects? At least in the context of small-business lending, this appears to be the case. At Chase Manhattan, a major small-business lender in the United States, applicants for small business loans are initially shown a pricing chart illustrating what interest rate they will get *if* their loan application is approved. The interest rate depends on verifiable data such as the prime rate, loan size, maturity, and collateral. A copy is available from the authors.

[16]Without this assumption, there exists no equilibrium in pure strategies. See Broecker (1990) for details.

inquirer (Jappelli and Pagano (2002)). To the extent that the competitive credit market has access to the borrower's credit report, it thus knows whether or not the borrower has previously visited the local lender.

$\tau = 2$. The project cash flow is realized and repayments are made.

**Financial Contracts**

A financial contract specifies repayments $t_l \leq x_l$ and $t_h \leq x_h$ out of the project's cash flow, an amount of collateral $C \leq w$, and repayments $c_l \leq C$ and $c_h \leq C$ out of the collateralized assets. The total repayment in the low and high cash-flow state is denoted by $R_l := t_l + c_l$ and $R_h := t_h + c_h$, respectively.[17]

## 3 Lender Conservativism and Collateral

We first derive some general properties of the optimal credit decision and financial contract. Subsequently, we solve for the (unique) equilibrium outcome in the credit market.

**Optimal Credit Decision: General Properties**

The first-best credit decision takes a simple form. Given that $\mu_s < k$ at low values of $s$ and $\mu_s > k$ at high values of $s$, and given that $\mu_s$ is increasing in $s$, there exists a unique first-best cutoff $s_{FB} \in (0,1)$ defined by $\mu_{s_{FB}} = k$ such that the project's NPV is negative if $s < s_{FB}$, zero if $s = s_{FB}$, and positive if $s > s_{FB}$. The first-best optimal credit decision is thus to grant credit if and only if $s \geq s_{FB}$, or equivalently if and only if $p_s \geq p_{s_{FB}}$, where

$$p_{s_{FB}} = \frac{k - x_l}{x_h - x_l}. \tag{1}$$

Consider next the local lender's privately optimal credit decision. The local lender accepts the project if and only if her conditional expected payoff

$$U_s(R_l, R_h) := p_s R_h + (1 - p_s) R_l$$

[17] This excludes the possibility that the local lender "buys" the borrower's project *before* evaluating it. Using a standard argument, we assume that such upfront payments would attract a large pool of fraudulent borrowers, or "fly-by-night operators", i.e., borrowers with no real project (see Rajan (1992)). This argument also rules out that the local lender pays a penalty to the borrower if he is rejected.

equals or exceeds her investment outlay $k$. It is immediate that we can exclude contracts under which the project is either accepted or rejected for *all* $s \in [0,1]$. Since $R_l \leq w + x_l < k$, this implies that $R_h > k > R_l$. Given that $p_s$ is increasing in $s$, this in turn implies that $U_s(R_l, R_h)$ is increasing in $s$, which in turn implies that the local lender grants credit if and only if $s \geq s^*(R_l, R_h)$, where $s^*(R_l, R_h) \in (0,1)$ is unique and defined by $U_{s^*}(R_l, R_h) = k$. Hence, the local lender's privately optimal decision also takes the form of a cutoff rule: grant credit if and only if the assessment of the borrower is sufficiently positive. As above, we can alternatively express the local lender's decision in terms of a critical success probability. Accordingly, the local lender grants credit if and only if $p_s \geq p_{s^*}$, where

$$p_{s^*} = \frac{k - R_l}{R_h - R_l} \tag{2}$$

This is summarized in the following lemma.

**Lemma 1.** *The first-best credit decision is to accept the borrower if and only if* $p_s \geq p_{s_{FB}}$, *where* $p_{s_{FB}}$ *is defined in* (1). *In contrast, the local lender's privately optimal credit decision is to accept the borrower if and only if* $p_s \geq p_{s^*}$, *where* $p_{s^*}$ *is defined in* (2).

**Optimal Financial Contract: General Properties**

We now derive our basic argument for why collateral is optimal. We proceed in two steps. Denote by $\overline{V} > 0$ the borrower's reservation utility—i.e., his expected utility from going to the competitive credit market—at $\tau = 0$. Taking $\overline{V}$ as given, we first derive the optimal contract maximizing the local lender's expected payoff.[18] In a second step, we solve for the unique credit market equilibrium, which pins down a unique equilibrium value of $\overline{V}$.

The local lender's problem is to choose a contract that maximizes her expected payoff

$$U(R_l, R_h) := \int_{s^*}^{1} [U_s(R_l, R_h) - k] f(s) ds \tag{3}$$

subject to the constraint that the borrower receives at least $\overline{V}$,

$$\int_{s^*}^{1} [\mu_s - U_s(R_l, R_h)] f(s) ds \geq \overline{V}, \tag{4}$$

and the constraint $U_{s^*}(R_l, R_h) = k$ characterizing the optimal credit decision at $\tau = 1$. Note that if the borrower is rejected, the project will not be financed. As we show below, this is indeed the unique equilibrium outcome in the credit market.

[18] To ensure that the lender's maximization program has a solution, we must assume that $\overline{V}$ is not too large.

By standard arguments, the borrower's participation constraint (4) binds, implying that the local lender receives any surplus in excess of $\overline{V}$. As the residual claimant, the local lender designs a contract inducing her to make as efficient as possible a credit decision at $\tau = 1$. As the following proposition shows, the optimal contract stipulates a positive amount of collateral.

**Proposition 1.** *The optimal financial contract stipulates a positive amount of collateral* $C \in (0, w]$ *such that the local lender receives* $R_l = x_l + C$ *in the low cash-flow state and* $R_h$ *in the high cash-flow state, where* $R_l < R_h < x_h$.

**Proof.** See Appendix.

By Proposition 1, collateral is used only in the low cash-flow state.[19] The intuition is as follows. As the local lender must leave the borrower an expected utility of $\overline{V} > 0$, she cannot keep the full project cash flow for herself. Without collateral, the local lender's expected payoff $U_s(R_l, R_h)$ is thus strictly less than the expected project cash flow $\mu_s$ for all $s \in [0, 1]$. In particular, we have that $U_{s_{FB}}(R_l, R_h) < \mu_{s_{FB}} = k$, i.e., the local lender does not break even at $s = s_{FB}$. As $U_s(R_l, R_h)$ is increasing in $s$, this implies that $s^*$ must be strictly higher than $s_{FB}$. Consequently, the local lender rejects projects with a low, but positive NPV.

Collateral allows the borrower to make repayments in excess of the project's cash flow. There are two alternatives: collateral can be added in the high cash-flow state ($R_h > x_h$) or in the low cash-flow state ($R_l > x_l$). (Evidently, having *both* $R_h > x_h$ *and* $R_l > x_l$ is not feasible.) The two alternatives differ in the way how they affect the local lender's expected payoff at different values of $s$. As $p_s$ is increasing in $s$, adding collateral in the low cash-flow state primarily improves the local lender's expected payoff from precisely those projects that—in the absence of collateral—are inefficiently rejected. By contrast, adding collateral in the high cash-flow state primarily improves the local lender's expected payoff from high-NPV projects, i.e., projects that she would accepted anyway. Hence, the optimal solution is to shift as much as possible of the local lender's payoff into the low cash-flow state and leave the borrower a positive payoff in the high cash-flow state to satisfy his participation constraint. This maximizes the local lender's expected payoff from low-NPV projects, thus minimizing her privately optimal cutoff $s^*$ and

[19] We stipulate that in case of indifference, repayments are first made out of the project's cash flow. The repayment in the high cash-flow state, $R_h$, is implicitly defined by the binding participation constraint (4) after inserting $R_l = x_l + C$ and the optimality condition $U_{s^*}(R_l, R_h) = k$ defining the cutoff $s^*$.

pushing it as far down as possible towards the first-best cutoff $s_{FB}$.

It proves convenient to rewrite the optimal repayment in the high cash-flow state $R_h$ in terms of an optimal interest rate $r$, where $R_h := (1+r)k$. As the risk-free rate of interest is zero, $r$ is also the required risk premium. By Proposition 1, the optimal contract then consists of two parameters: $r$ and $C$.

**Credit Market Equilibrium**

Up to this point, we have *assumed* that the borrower's outside option $\overline{V}$ is positive. As the following proposition shows, this is indeed true in equilibrium.

**Proposition 2.** *There exists a unique credit market equilibrium. At* $\tau = 0$, *the borrower goes to the local lender, which provides the borrower with an expected utility of* $\overline{V} = \mu - k > 0$. *At* $\tau = 1$, *borrowers who are accepted by the local lender stay with the local lender, while borrowers who are rejected cannot obtain financing elsewhere.*

**Proof.** See Appendix.

At $\tau = 0$, the maximum the competitive credit market can offer is $\mu - k > 0$, which is the project's NPV based on hard information.[20] Due the local lender's ability to screen out negative-NPV projects, the local lender can create additional surplus over and above what is created if the project is financed by the competitive credit market, however. Consequently, the local lender is able to outbid the competitive credit market. By standard arguments, the unique equilibrium is where the local lender exactly matches the competitive credit market's offer and the borrower goes to the local lender.

Our notion that the local lender can successfully outbid the competitive credit market is consistent with Petersen and Rajan's (1994) observation that 95 percent of the smallest firms in their sample borrow from a single bank. As the median distance between banks and small-business borrowers in the United States is (only) four miles (Petersen and Rajan (2002)), this will typically be a local lender.

[20]Like the local lender, the competitive credit market makes offers conditional on the project being approved. As the competitive credit market evaluates projects solely on the basis of hard information, and as the NPV based on this information is positive, the borrower rationally anticipates that the competitive credit market will approve his project, which provides him with an expected utility of $\overline{V} = \mu - k$.

While the competitive credit market cannot outbid the local lender, its existence nonetheless puts pressure on the local lender to leave the borrower a share of the surplus. This distorts the local lender's credit decision, which in turn provides a role for collateral. By contrast, if there was no competitive pressure—e.g., if the credit market is perfectly monopolistic—we would have $\overline{V} = 0$.[21] The local lender could then keep the full project cash flow, which yields the first best.

Consider next interim competition at $\tau = 1$. Given that the borrower is rejected with positive probability, the fact that his ex-ante expected utility equals $\overline{V} > 0$ implies that his expected utility *conditional on being accepted* must strictly exceed $\overline{V}$. The competitive credit market cannot offer more than $\overline{V}$, however.[22] Accordingly, not only does the competitive credit market lose against the local lender at $\tau = 0$, it also loses in the interim competition at $\tau = 1$. But if accepted borrowers stay with the local lender, the only borrowers who might potentially visit the competitive credit market are rejected borrowers. As we show in the Proof of Proposition 2 in the Appendix, such borrowers have a negative expected NPV, implying that the competitive credit market is better off not making any positive offer at $\tau = 1$.[23]

**Optimal Credit Decision and Financial Contract: Equilibrium Values**

Having shown that $\overline{V} = \mu - k > 0$, we can finally solve for the *optimal* level of collateral. There are two subcases. In one subcase, the borrower has insufficient pledgeable assets. In this case, the optimal contract requires that he pledges all his assets as collateral. In the other subcase, the borrower has sufficient pledgeable assets to attain the first best. In this case, there exists a unique first-best optimal collateral $C_{FB}$ and interest rate $r_{FB}$, where $C_{FB}$ and $r_{FB}$ are jointly determined by the borrower's binding participation constraint (4) with $\overline{V} = \mu - k$ and the optimality condition

$$p_{s_{FB}}(1 + r_{FB})k + (1 - p_{s_{FB}})(x_l + C_{FB}) = k, \tag{5}$$

[21] A perfectly monopolistic credit market alone does not ensure that $\overline{V} = 0$. It must additionally hold that the borrower has no bargaining power.

[22] Even an offer of $\overline{V}$ is sustainable only under the "most optimistic beliefs" that—in addition to all rejected borrowers—also all accepted borrowers will switch from the local lender to the competitive credit market. (Any offer by the credit market that attracts accepted borrower will inevitably attract all rejected borrowers.)

[23] See Rajan (1992) for a similar argument.

where $p_{s_{FB}}$ is defined in (1). Solving these two equations yields

$$C_{FB} = \frac{(k - x_l)(\mu - k)}{\int_{s_{FB}}^{1} (\mu_s - k) f(s) ds} \tag{6}$$

and

$$r_{FB} = \frac{1}{k} \left[ x_h - C_{FB} \frac{x_h - k}{k - x_l} \right] - 1. \tag{7}$$

The following proposition summarizes our results.

**Proposition 3.** *If the borrower has pledgeable assets $w \geq C_{FB}$, the first best can be attained with an optimal contract $(r_{FB}, C_{FB})$ defined in (6)-(7).*

*If $w < C_{FB}$ the local lender is inefficiently conservative and denies credit to low-, but positive-NPV projects. The optimal contract then stipulates that $C = w$, i.e., the borrower must pledge all his assets as collateral.*[24]

**Proof.** See Appendix.

The case where the first best can be attained shows that there is a limit to how "flat" the optimal repayment schedule may be. Even if the borrower has sufficient pledgeable assets, the repayment in the low cash-flow state is at most $x_l + C_{FB}$, which is strictly less than the corresponding repayment in the high cash-flow state $(1 + r_{FB})k$.[25]

### Renegotiations

If the borrower has insufficient pledgeable assets, the local lender's accept or reject decision is inefficient: if $s \in [s_{FB}, s^*)$, the local lender rejects the project even though its NPV is positive. This potentially creates scope for mutually beneficial renegotiations: rather than being denied credit, the borrower might propose a new, more favorable contract allowing the local lender to at least break even. This is precisely what would happen if $s$ was observable, in which case mutually beneficial renegotiations would eliminate all inefficiencies.

[24] See footnote 19 on how to compute the optimal interest rate $r = (R_h/k) - 1$ in the second-best case $w < C_{FB}$.

[25] Formally, the wedge between the first best repayment in the high and low cash-flow state is

$$(1 + r_{FB})k - (x_l + C_{FB}) = k - x_l - (x_h - k) \frac{\int_0^{s_{FB}} (\mu_s - k) f(s) ds}{\int_{s_{FB}}^{1} (\mu_s - k) f(s) ds},$$

which is strictly positive as $x_h > k > x_l$ and $\mu_s - k > 0$ for all $s > s_{FB}$ while $\mu_s - k < 0$ for all $s < s_{FB}$.

Given that only the local lender observes $s$, however, such mutually beneficial renegotiations must fail: anticipating that the borrower will propose a more favorable contract, the local lender has every incentive to claim that her assessment of the project is low—i.e., that $s \in [s_{FB}, s^*)$—even when it is actually high. From the borrower's perspective, this implies that he will not know if $s \in [s_{FB}, s^*)$, in which case he would be willing to offer a more favorable contract, or if $s \geq s^*$, in which case offering a more favorable contract merely constitutes a wealth transfer to the local lender. As we show below, the *expected* gain to the borrower from renegotiating the optimal contract is strictly negative.

We consider the following renegotiation game. After the local lender has evaluated the borrower's project, but before she makes a decision, a new contract can be offered. (As the local lender can always reverse her decision, it is actually irrelevant whether a decision has been made or not.) If the local lender proposes a new contract, the borrower must agree, while if the borrower proposes a new contract, the local lender must agree. The following proposition shows that the optimal contract defined in Propositions 1 and 3 is renegotiation-proof.

**Proposition 4.** *Regardless of who can propose a new contract in the renegotiation game, the optimal contract in Propositions 1 and 3 will not be renegotiated.*

**Proof.** See Appendix.

# 4 Comparative Analysis

When we refer to the borrower as having either sufficient or insuffient pledgeable assets, we are referring to the two subcases $w \geq C_{FB}$ and $w < C_{FB}$ in Proposition 3. The empirical implications of this section are summarized in Section 5.

### Variations in Pledgeable Assets

Proposition 3 shows that if $w < C_{FB}$ the local lender's accept or reject decision is inefficient. The following proposition completes this picture by showing that, in this case, an increase in the borrower's pledgeable assets lowers local lender's cutoff $s^*$, thus improving the credit likelihood $1 - F(s^*)$ and mitigating the inefficiency. Evidently, in the first-best case $w \geq C_{FB}$ an increase in pledgeable assets has no effect.

**Proposition 5.** *If the borrower has insufficient pledgeable assets, an increase in his pledgeable assets implies that he can post more collateral, thus improving the credit likelihood.*

**Proof.** See Appendix.

The probability that $x = x_l$ *conditional on the project being accepted* is

$$D := \int_{s^*}^{1} (1 - p_s) \frac{f(s)}{1 - F(s^*)} ds, \tag{8}$$

where $f(s)/[1 - F(s^*)]$ is the density of $s$ conditional on $s \geq s^*$. Hence, $D$ is the expected, or average, default probability of an accepted borrower. Given that the average quality of the pool of accepted borrowers increases with $s^*$, the expected default probability $D$ increases with the acceptance probability $1 - F(s^*)$. In conjunction with the above result that the acceptance probability increases with the posted collateral, this yields the following result.[26]

**Corollary 1.** *Conditional on the loan being accepted, the expected default probability of a loan is increasing in the posted collateral.*

The intuition is straightforward. The average quality of accepted borrowers under a lenient credit policy is lower than under a conservative credit policy. As collateral makes the local lender more lenient, the average quality of accepted borrowers decreases with the collateral.

### Variations in Observable Project Risk

Corollary 1 concerns ex-post default risk. While the true success probability $p_s$ is only observable by the local lender, knowing the collateral and whether or not the project was accepted allows outsides to draw inferences about $p_s$. This is what Corollary 1 is about.

This subsection concerns variations in *observable* ex-ante risk. Precisely, we consider a mean-preserving spread in the project's ex-ante cash-flow distribution and examine its effect on the required collateral, risk premium, and credit likelihood.

There are two subcases. If the borrower has sufficient pledgeable assets, the increase in risk can be fully absorbed by an increase in both the required collateral and the interest rate. As a consequence, the credit likelihood remains first-best optimal. If the borrower has insufficient

[26] By Proposition 3, only collateral values of $C \leq C_{FB}$ will be posted in equilibrium. For all $C < C_{FB}$, the acceptance probability $1 - F(s^*)$ is strictly increasing in $C$.

pledgeable assets, this is not possible. To protect herself against the increased likelihood of making a loss, the local lender becomes more conservative.

**Proposition 6.** *Consider a mean-preserving spread in the project's cash-flow distribution. If the borrower has sufficient pledgeable assets, the increase in project risk can be absorbed by increasing both the required collateral and the risk premium. The credit likelihood remains unaffected.*

*If the borrower is unable to pledge more collateral, the credit likelihood decreases.*

**Proof.** See Appendix.

### Variations in Project Size

To compare projects of different sizes, we multiply the investment outlay and all cash flows by a scaling factor $\alpha$. Incidentally, while a higher $\alpha$ implies a greater loan size, the first-best cutoff $s_{FB}$ is invariant with respect to $\alpha :$ given that $\mu_s := p_{s_{FB}} \alpha x_h + (1 - p_{s_{FB}}) \alpha x_l = \alpha k$, the scaling factor $\alpha$ cancels out. We have the following result.

**Proposition 7.** *Consider an increase in the project—and thus in the loan—size. If the borrower has sufficient pledgeable assets, the increase in loan size goes hand in hand with an increase in the required collateral. The credit likelihood remains unaffected.*

*If the borrower is unable to pledge more collateral, the credit likelihood decreases.*

**Proof.** See Appendix.

If the borrower has sufficient pledgeable assets, all that changes is the collateral, namely, from $C_{FB}$ to $\alpha C_{FB}$, which implies it increases with the loan size. By contrast, the risk premium remains unchanged: as the investment outlay is now $\alpha k$, the first-best repayment in the high cash-flow state is $(1 + r_{FB})\alpha k$. Hence, even if $r_{FB}$ is unchanged, the repayment in the high cash-flow state increases by a multiple $\alpha$. As both the investment size $\alpha k$ and the repayment in the low cash-flow state $\alpha x_l + \alpha C_{FB}$ increase by the same multiple, first-best optimality is preserved.

Intuitively, the local lender has more at stake when granting a bigger loan. In particular, her loss in the low cash-flow state is $\alpha k - (\alpha x_l + C)$, which is increasing in $\alpha$. One way to avoid becoming more conservative is to increase the collateral requirement by the same multiple $\alpha$. If this is not possible, e.g., because the borrower has already pledged all his assets, the local lender lowers the credit likelihood.

**Variations in the Cost of Borrowing from Distant Lenders**

Suppose borrowing from the competitive credit market comes at a cost of $\kappa$. One interpretation is that $\kappa$ reflects the cost of traveling to, or doing business with, distant lenders. Alternatively, $\kappa$ may reflect greater monitoring expenses by distant lenders. In the first case, $\kappa$ is directly incurred by the borrower. In the second case, $\kappa$ is initially incurred by the competitive credit market but will be ultimately passed on to the borrower. Either way, the borrower's expected utility from going to the competitive credit market decreases to $\overline{V} = \mu - k - \kappa$. We assume that $\kappa$ is sufficiently small, so that $\overline{V}$ remains positive.

Empirical studies (e.g., Petersen and Rajan (2002)) suggest that—primarily due to advances in information technology—the cost of borrowing from distant lenders has decreased over time, thus putting greater pressure on incumbent lenders. The following proposition examines the implications of a decrease in $\kappa$.

**Proposition 8.** *Consider a decrease in the cost of borrowing from distant lenders. If the borrower has sufficient pledgeable assets, the local lender responds to the increase in competitive pressure by distant lenders by requesting more collateral.*

*If the borrower is unable to pledge more collateral, the credit likelihood decreases.*

**Proof.** See Appendix.

The intuition follows directly from (5). A decrease in the cost of borrowing from distant lenders makes the credit market more competitive, thereby lowering the interest rate. If the borrower is unable to post more collateral, the local lender no longer breaks even at her original cutoff $s^*$, which implies this cutoff must increase. On the other hand, if the borrower has sufficient pledgeable assets, the increase in $s^*$ can be avoided by requesting more collateral.

## 5 Empirical Implications

This section summarizes the empirical implications of our model. The first implication follows directly from Proposition 5.

**Implication 1.** *Low net-worth firms should face a higher likelihood of being denied credit than high net-worth firms.*

Low net-worth firms are firms with few liquid or tangible assets, or highly indebted firms. Such firms possess relatively few pledgeable assets, implying that they will face a higher likelihood of being denied credit than high net-worth firms.[27]

The next implication concerns the relation between collateral and ex-post default risk.

**Implication 2.** *Controlling for ex-ante observable differences in borrower risk, the expected default probability of a loan should be increasing in the pledged collateral.*

Implication 2 is a restatement of Corollary 1, which argues that—holding ex-ante observable borrower risk constant—loans with a higher collateral are more likely to default ex post. The intuition is that collateral makes lenders more lenient in their accept decisions, which implies that it reduces the *average* quality within the pool of accepted borrowers.

Jiménez and Saurina (2004) and Jiménez, Salas, and Saurina (2004)—using data from over three million Spanish loans—test the relation between collateral and ex-post default risk. Consistent with Implication 2, they find that the presence of collateral significantly increases the default likelihood in the year after the loan has been granted. Similarly, Berger and Udell (1990)—using charge-offs and past dues as proxies for default risk—also find that collateral is positively correlated with the likelihood of default.

The next implication concerns differences in ex-ante observable borrower risk. It follows from Proposition 6.

**Implication 3.** *High-risk borrowers should post more collateral and pay a higher risk premium than low-risk borrowers.*

A general difficulty in testing Implication 3 is to find a good proxy for ex-ante borrower risk.[28] Empirical studies have used a number of different risk proxies. However, all of them appear to find a positive relation between borrower risk and collateral (e.g., Orgler (1970), Hester (1979), Leeth and Scott (1989), Berger and Udell (1995), Strahan (1999), Dennis, Nandy, and Sharpe (2000), Jiménez, Salas, and Saurina (2004), Gonas, Highfield, and Mullineaux (2004)).[29]

[27] Most theoretical models of collateral assume that borrowers have unlimited pledgeable net worth. An exception is Besanko and Thakor (1987a).

[28] See Boot, Thakor, and Udell (1991), who "wish to caution that any statements about observed cross-sectional relationships between borrower risk and collateral are sensitive to the risk measures employed."

[29] Orgler (1970) uses loan classifications by bank managers as a proxy; Hester (1979) uses various accounting

Implication 3 also suggests that collateral and risk premia should go hand in hand. Berger and Udell (1990) find strong evidence for a positive relation between collateral and risk premia. They interpet their finding as suggesting that high-risk borrowers post more collateral than low risk-borrowers. However, underlying this interpretation is the assumption that risk premia are positively correlated with borrower risk. Proposition 6 in this paper endogenizes this assumption. Other studies that find a positive relation between collateral and risk premia are Booth (1992), Angbazo, Mei, and Saunders (1998), Strahan (1999), and Dennis, Nandy, and Sharpe (2000), while Degryse and Ongena (2003) find a negative relation.

We next turn to the relation between loan size and collateral. Proposition 7 is based on a simple intuition: if the investment outlay increases, the local lender makes a bigger loss if the project cash flow is low. To protect herself against this bigger loss, she raises the collateral requirement.[30]

**Implication 4.** *An increase in loan size should result in an increase in the required collateral.*

The empirical evidence is both sparse and mixed. Boot, Thakor, and Udell (1991) find that larger loans are less likely to be secured, while Leeth and Scott (1989) and Jiménez, Salas, and Saurina (2004) find that larger loans are more likely to be secured.[31] However, all these studies consider the incidence of collateralization (yes/no), not the amount of collateral. We are not aware of any study examining how the amount of collateral varies with the loan size.

Proposition 8 considers a reduction in the cost of borrowing from distant lenders. It argues that in order to cushion the effects of an increase in competitive pressure by distant lenders, the

variables; Leeth and Scott (1989) use firm age; Berger and Udell (1995) use leverage and firm age; Strahan (1999) uses firm size, leverage, opaqueness (i.e., market-to-book ratio), and credit ratings; Dennis, Nandy, and Sharpe (2000) use Altman's Z-Score and leverage; Jiménez, Salas, and Saurina (2004) use firm age and whether or not the firm has an existing loan in default at the time when the new loan is granted; Gonas, Highfield, and Mullineaux (2004) use credit ratings.

[30] Note that—while the absolute amount of collateral increases proportionately with the loan size—the *fraction* of the loan that is secured is independent of the loan size in our model.

[31] Jiménez, Salas, and Saurina find that both the likelihood that the loan is secured at all and that it is secured on a 100 percent basis is increasing with the loan size, which is consistent with Implication 4. They also find that an increase in loan size increases the relative likelihood of 50 percent versus 100 percent collateralization. This latter finding is consistent with our model if borrowers can fully meet thecollateral requirement for smaller loans, but not necessarily the (much higher) collateral requirement for large loans (i.e., $C \leq w$ becomes binding).

local lender will raise the collateral requirement.

**Implication 5.** *An increase in credit market competition should result in an increase in the required collateral.*

The relation between credit market competition and collateral has received little attention in empirical research. To our knowledge, the only empirical study addressing this issue is a recent paper by Jiménez, Salas, and Saurina (2004). Consistent with Implication 5, they find a positive relation between collateral and competition (measured by the Herfindahl index).

Our final implication concerns situations in which the borrower cannot meet demands by the lender to post more collateral. In this case, the credit likelihood decreases.

**Implication 6.** *The likelihood of being denied credit should be increasing in the loan size, ex-ante borrower risk, and degree of credit market competition.*

It is difficult to find a good proxy for credit rationing. Petersen and Rajan (1994, 1995) argue that firms extending trade credit past the due date—thereby incurring high costs and penalties—are likely to suffer from credit rationing. Consistent with Implication 6, they find that the availability of credit to small- and medium-sized firms is lower in more competitive credit markets. Relatedly, Zarutskie (2004) finds that firms in less competitive credit markets are more likely both to receive debt and to receive debt more quickly. We are not aware of empirical studies examining the effects of borrower risk and loan size on credit rationing.

### Asymmetric Information and Moral Hazard

Let us briefly review some of the main implications from asymmetric information and moral hazard theories of collateral.

*Asymmetric Information*

(Pure) borrower asymmetric information models (e.g., Bester (1985), Chan and Kanatas (1987), Besanko and Thakor (1987a, b)) predict that high-risk borrowers post less collateral than low-risk borrowers, which is difficult to reconcile with the empirical evidence listed in connection with Implication 3.

*Moral Hazard*

The basic argument why collateral alleviates borrower moral hazard is both simple and intuitive. Collateral constitutes a bond that is forfeited in bad (i.e., low cash-flow) states. As bad states are more likely if the borrower shirks—and less likely if he works hard—posting collateral improves the borrower's incentives to work hard (e.g., Chan and Thakor (1987); Boot, Thakor, and Udell (1991); Boot and Thakor (1994)). In equilibrium, borrowers who can post more collateral work harder, which reduces the likelihood of bad states. Hence, the basic moral hazard intuition suggests a negative relation between collateral and ex-post default risk, which is the opposite of our Implication 2.

The above argument assumes that there is only one type of borrower quality. If there are many quality types, Boot, Thakor, and Udell (1991) show that it may be nontrivial to obtain a robust empirical prediction.[32] If borrower quality and effort are substitutes, collateral and risk (both in the ex-ante and ex-post sense) may be positively related, like in Implications 2 and 3 above.[33] However, if borrower quality and effort are complements, collateral and risk may be negatively related. Hence, the empirical prediction depends critically on whether high- or low-quality borrowers have a higher marginal productivity of effort. Given that it is difficult to argue which is true empirically, it is probably fair to say that Implications 2 and 3 do not necessarily (in the sense of a robust empirical prediction) follow from moral hazard arguments.[34]

## 6 Conclusion

This paper offers a novel, and we believe intuitive, theory of collateral based on the notion that lenders make discretionary credit decisions that are inefficiently conservative. Our theory is entirely lender-based and assumes no borrower moral hazard or private information. This implies, among other things, that it applies to owner-managed firms as well as firms in which ownership and management are separated.

The basic argument is simple. If there is credit market competition, or if the borrower has some bargaining power, the lender must leave the borrower a share of the created surplus,

[32] See also Chan and Thakor (1987).

[33] Boot, Thakor, and Udell (1991, Proposition 2) provide sufficient conditions ensuring that low-quality borrowers have both a higher ex-ante risk and a higher default risk than high-quality borrowers. If these conditions fail to hold, collateral and risk may be negatively related even if borrower quality and effort are susbtitutes, but positively related if they are complements.

[34] Boot, Thakor, and Udell (1991) also show that borrower moral hazard implies a *negative* relation between loan size and collateral, which is the opposite of our Implication 4.

implying that she cannot keep the full project cash flow for herself. As the lender provides the full investment outlay, this implies that she only accepts projects whose expected cash flow exceeds the investment outlay by a sufficient margin. As a consequence, the lender rejects low-, but positive NPV projects.

Collateral makes the lender more willing to accept the borrower's project, thus alleviating her conservative bias. In particular, adding collateral in low cash-flow states produces the greatest "bang for the buck": as low-NPV projects have a relatively high likelihood of low cash-flow states, adding collateral in these states improves the lender's payoff from precisely those projects that are inefficiently rejected.

At the heart of our model is a multitasking problem: the lender provides financing and assesses the borrower's credit risk. The fact that the lender provides financing prevents her from truthfully revealing the outcome of her credit risk assessment. The fact that the credit risk assessment cannot be contracted upon, in turn, distorts the lender's decision to provide financing. In our model, we take the fact that the lender performs both tasks as given and examine its implications for the optimal credit decision and collateral. In practice, banks *do* typically perform both tasks jointly. Especially for small- and medium-sized loans, the reason might be a simple cost argument. Given the relatively small margins on such loans, combining both activities under one roof saves on valuable fixed costs.[35].

# 7 Appendix A: Continuum of Cash Flows

This section shows that our argument for why collateral is optimal extends to a continuum of cash flows. Unlike the two cash-flow model in the main text, it not only shows *that* collateral is used in low cash-flow states, but also how precisely repayments out of the pledged assets are made as a function of the project's cash flow if cash flow is a continuous variable.

We assume that the project cash flow $x$ is distributed with atomless CDF $G_s(x)$ over the support $X := [0, \overline{x}]$, where $\overline{x} > 0$ can be either finite or infinite. The density $g_s(x)$ is everywhere

[35] "A \$50,000 loan yielding three percent over cost of funds provides only \$1,500 p.a. in gross revenues, before provisions of credit losses and allocation of overheads. This level of gross revenue can pay for very little of the loan officer's analytical and monitoring time" (Saunders and Thomas (2001)). The average loan size in Petersen and Rajan's (2002) sample of small-business loans is \$18,000.

continuous and positive. If $\overline{x}$ is infinite, we assume that $\mu_s := \int_X x g_s(x)dx$ exists for all $s \in [0, 1]$. We assume that $G_s(x)$ satisfies MLRP, which states that for any pair $(s, s') \in S$ with $s' > s$, the ratio $g_{s'}(x)/g_s(x)$ is strictly increasing in $x$ for all $x \in X$.

With a continuum of cash flows, a contract specifies a repayment schedule $t(x) \leq x$ out of the project's cash flow, an amount $C \leq w$ of collateral, and a repayment schedule $c(x) \leq C$ out of the pledged assets. It is convenient to write $R(x) := t(x) + c(x)$. We make the standard assumption that $R(x)$ is nondecreasing for all $x \in X$ (e.g., Innes (1990)). The local lender's and borrower's expected payoffs are $U_s(R) := \int_X R(x)g_s(x)dx$, $V_s(R) := \mu_s - U_s(R)$, $U(R) := \int_{s^*}^1 [U_s(R) - k] f(s)ds$ and $V(R) := \int_{s^*}^1 V_s(R)f(s)ds$, respectively. Analogous to the main text, the local lender's privately optimal cutoff is given by the optimality condition $U_{s^*(R)}(R) = k$. The local lender's program is to maximize $U(R)$ subject to the borrower's participation constraint $V(R) \geq \overline{V}$.

The following result extends Proposition 1 to the case with a continuum of cash flows.

**Proposition.** *With a continuum of cash flows, the optimal financial contract stipulates a repayment $R \in (0, \overline{x})$ and collateral $C \in (0, w]$ such that the local lender receives $R(x) = x + C$ if $x \leq R$ and $R(x) = R$ if $x > R$.*

As for repayments made out of the project's cash flow, we have that $t(x) = x$ for $x \leq R$ and $t(x) = R$ for $x > R$. Collateral is used as follows: if $x \leq R - C$, the local lender receives the full collateral, i.e., $c(x) = C$, if $R - C < x \leq R$, the local lender receives a fraction $c(x) = R - x$ of the collateralized assets (after liquidation), while if $x > R$, the local lender receives no repayment out of the collateralized assets because the project cash flow itself is sufficient to make the contractually required repayment.

To prove the proposition, suppose to the contrary that the optimal contract stipulated a repayment scheme $R(x)$ that is different from that in the proposition. We can then construct a new repayment scheme $\widetilde{R}(x) = \min\left\{x + \widetilde{C}, \widetilde{R}\right\}$ where $\widetilde{C} = w$, and where $\widetilde{R}$ satisfies

$$\int_{s^*(R)}^1 \left[\int_X z(x)g_s(x)dx\right] f(s)ds = 0, \tag{9}$$

with $z(x) := \widetilde{R}(x) - R(x)$. (Hence, holding the cutoff fixed at $s^*(R)$, expected payoffs remain constant.)[36] By construction of $\widetilde{R}(x)$, there exists a value $0 < \widetilde{x} < \overline{x}$ such that $z(x) \geq 0$ holds

[36] Existence and uniqueness of a value $\widetilde{R}$ solving (9) follows as i) the local lender's payoff is continuous and

for all $x < \widetilde{x}$ and $z(x) \leq 0$ holds for all $x > \widetilde{x}$, where the inequalities hold strictly on sets of positive measure.

**Claim 1.** $s^*(\widetilde{R}) < s^*(R)$.

**Proof.** By (9) and continuity of $g_s(x)$ in $s$, there exists a value $\widetilde{s}$ satisfying $s^*(R) < \widetilde{s} < 1$, where $\int_X z(x)g_{\widetilde{s}}(x)dx = 0$. From $\widetilde{s} > s^*(R)$ and MLRP it follows that $g_{s^*(R)}(x)/g_{\widetilde{s}}(x)$ is strictly decreasing in $x$ such that

$$\begin{aligned}\int_X z(x)g_{s^*(R)}(x)dx &= \int_{x\leq\widetilde{x}} z(x)g_{\widetilde{s}}(x)\frac{g_{s^*(R)}(x)}{g_{\widetilde{s}}(x)}dx + \int_{x>\widetilde{x}} z(x)g_{\widetilde{s}}(x)\frac{g_{s^*(R)}(x)}{g_{\widetilde{s}}(x)}dx \\ &> \frac{g_{s^*(R)}(\widetilde{x})}{g_{\widetilde{s}}(\widetilde{x})}\int_X z(x)g_{\widetilde{s}}(x)dx = 0.\end{aligned}$$

As $\int_X z(x)g_{s^*(R)}(x)dx > 0$ and $\int_X R(x)g_{s^*(R)}(x)dx = k$ from the definition of $s^*(R)$, we have that $\int_X \widetilde{R}(x)g_{s^*(R)}(x)dx > k$. As $U_s(\widetilde{R})$ is strictly increasing in $s$, we have that $s^*(\widetilde{R}) < s^*(R)$. **Q.E.D.**

Note that the new cutoff $s^*(\widetilde{R})$ may lie *below* $s_{FB}$. In this case, we can make the following adjustment:

**Claim 2.** *In case $s^*(\widetilde{R}) < s_{FB}$ for $\widetilde{C} = w$, we can adjust the new contract by decreasing $\widetilde{C}$ and increasing $\widetilde{R}$ such that* (9) *continues to hold, while $s^*(\widetilde{R}) = s_{FB}$.*

**Proof.** Take first some contract $(\widehat{R}, \widehat{C})$ such that $\widehat{R} > \widetilde{R}$ and $\widehat{C} < \widetilde{C}$ and (9) holds with $z(x) := \widehat{R}(x) - \widetilde{R}(x)$. From (9)—together with $\widehat{R} > \widetilde{R}$ and $\widehat{C} < \widetilde{C}$—it follows that there exists a value $0 < \widetilde{x} < \overline{x}$ such that $z(x) \geq 0$ holds for all $x > \widetilde{x}$ and $z(x) \leq 0$ holds for all $x < \widetilde{x}$, where the inequalities are strict on sets of positive measure. By the argument in Claim 1, this implies that $s^*(\widehat{R}) > s^*(\widetilde{R})$. As we decrease $\widehat{C}$ and adjust $\widehat{R}$ accordingly to satisfy (9), we have from the definition of $s^*$ and continuity of $g_s(x)$ that $s^*(\widehat{R})$ increases continuously. As $s^*(\widehat{R}) > s_{FB}$ at $\widehat{C} = 0$, the claim follows. **Q.E.D.**

We show next that the borrower is not worse off under the new contract $(\widetilde{R}, \widetilde{C})$.

**Claim 3.** $V(\widetilde{R}) \geq V(R)$.

**Proof.** We can distinguish between three cases.

**Case 1:** $s^*(R) = s_{FB}$. The assertion follows immediately from (9) and $s^*(R) = s^*(\widetilde{R})$.

strictly increasing in $\widetilde{R}$ for a given cutoff, and ii) the left-hand side of (9) is strictly positive at $\widetilde{R} = \overline{x}$ and strictly negative at $\widetilde{R} = 0$.

**Case 2:** $s^*(R) > s_{FB}$. In this case, construction of $\widetilde{R}(x)$ implies that $s_{FB} \leq s^*(\widetilde{R}) < s^*(R)$. It also follows from the construction of $\widetilde{R}(x)$ that the borrower's expected payoff remains unchanged if the loan is approved if and only if $s \geq s^*(R)$. Hence, $V(\widetilde{R}) \geq V(R)$ follows if $V_s(\widetilde{R}) \geq 0$ for all $s \in [s^*(\widetilde{R}), s^*(R)]$. To see that this is the case, note first that $V_{s^*(\widetilde{R})}(\widetilde{R}) \geq 0$ holds because of $U_{s^*(\widetilde{R})}(\widetilde{R}) = 0$ and $s_{FB} \leq s^*(\widetilde{R})$. It thus remains to show that $V_s(\widetilde{R})$ is nondecreasing in $s$. Partial integration yields

$$V_s(\widetilde{R}) = \int_{\widetilde{R}-\widetilde{C}}^{\overline{x}} [1 - G_s(x)]\, dx - \widetilde{C}. \tag{10}$$

MLRP implies that $G_s(x)$ is strictly decreasing in $s$ for all $0 < x < \overline{x}$. By (10), this implies that $V_s(\widetilde{R})$ is strictly increasing in $s$.

**Case 3:** $s^*(R) < s_{FB}$. In this case, construction of $\widetilde{R}(x)$ implies that $s^*(\widetilde{R}) = s_{FB}$. It remains to show that $V_s(\widetilde{R}) \leq 0$ for all $s \in [s^*(\widetilde{R}), s_{FB}]$. From $s^*(\widetilde{R}) = s_{FB}$—implying that $U_{s_{FB}}(\widetilde{R}) = 0$—it follows that $V_{s_{FB}}(\widetilde{R}) = 0$, while the argument in Case 2 implies that $V_s(\widetilde{R})$ is nondecreasing in $s$. Together, this implies that $V_s(\widetilde{R}) \leq 0$ for all $s \in [s^*(\widetilde{R}), s_{FB}]$. **Q.E.D.**

Summing up, we have constructed a new contract $(\widetilde{R}, \widetilde{C})$ with the following characteristics: i) $\widetilde{R}(x) = \min\left\{x + \widetilde{C}, \widetilde{R}\right\}$; ii) (9) is satisfied; iii) if $s^*(R) \geq s_{FB}$, it holds that $s_{FB} \leq s^*(\widetilde{R}) \leq s^*(R)$, where $s^*(\widetilde{R}) < s^*(R)$ if $s^*(R) > s_{FB}$; iv) if $s^*(R) < s_{FB}$, it holds that $s^*(R) < s^*(\widetilde{R}) = s_{FB}$; v) $V(\widetilde{R}) \geq V(R)$. The new contract is thus acceptable for the borrower and does not make the lender worse off. In fact, the lender is strictly better off if $s^*(\widetilde{R}) \neq s^*(R)$, which follows immediately from (9) and optimality of $s^*$. Finally, if the original contract implements the first-best cutoff, i.e., if $s^*(\widetilde{R}) = s^*(R) = s_{FB}$, the transfer made out of collateralized assets is strictly lower under the new contract, i.e., $\int_{s_{FB}}^1 \left[\int_X c(x) g_s(x) dx\right] f(s) ds > \int_{s_{FB}}^1 \left[\int_X \widetilde{c}(x) g_s(x) dx\right] f(s) ds$. **Q.E.D.**

# 8 Appendix B: Proofs

**Proof of Proposition 1.** Define

$$\begin{aligned} V_s(R_l, R_h) \ &: = (1 - p_s)(x_l - R_l) + p_s(x_h - R_h), \\ V(R_l, R_h) \ &: = \int_{s^*}^1 V_s(R_l, R_h) f(s) ds. \end{aligned}$$

Hence, $V(R_l, R_h)$ is the borrower's expected payoff at $\tau = 0$, while $V_s(R_l, R_h)$ is his expected payoff at $\tau = 1$ conditional on $s$. We show that there exist unique optimal values of $R_l$ and $R_h$, and that these values satisfy $x_l < R_l \leq x_l + w$ and $R_l < R_h < x_h$. The following observations are obvious: (i) if we increase $R_l$ while holding $R_h$ constant, $U(R_l, R_h)$ increases while $s^*$ decreases, (ii) if we increase $R_h$ while holding $R_l$ constant, $U(R_l, R_h)$ increases while $s^*$ decreases, and (iii) $s^*$ changes continuously with $R_l$ and $R_h$, implying that $V(R_l, R_h)$ and $U(R_l, R_h)$ also change continuously.

Next, suppose we increase $R_l$ and adjust $R_h$ so as to keep $s^*$ constant, i.e., we compare two contracts with $\widetilde{R}_l > R_l$, $\widetilde{R}_h < R_h$, and

$$p_{s^*} = \frac{k - R_l}{R_h - R_l} = \frac{k - \widetilde{R}_l}{\widetilde{R}_h - \widetilde{R}_l}, \tag{11}$$

where (11) is adapted from (2). Then it holds that $V(\widetilde{R}_l, \widetilde{R}_h) > V(R_l, R_h)$. This is immediate as $s^*$ stays constant, while strict monotonicity of $p_s$, $\widetilde{R}_l > R_l$, and $\widetilde{R}_h < R_h$ implies that $V_s(\widetilde{R}_l, \widetilde{R}_h) > V_s(R_l, R_h)$ for all $s > s^*$. Recall next that, by optimality, the local lender chooses a contract implementing the lowest feasible cutoff $s^* \geq s_{FB}$ while satisfying the borrower's participation constraint (4). To complete the proof of Proposition 1, we distinguish between the following cases.

**Case 1:** $s^* = s_{FB}$. If it is possible to achieve the first best, we argue first that this implies $R_l > x_l$. By (1) and (2) we have that $s^* = s_{FB}$ holds if

$$\frac{k - R_l}{R_h - R_l} = \frac{k - x_l}{x_h - x_l}. \tag{12}$$

Suppose first that $R_l = x_l$. In this case, (12) implies that $R_h = x_h$, which implies that $V(R_l, R_h) = 0$, violating (4). Suppose next, we decrease $R_l$ so that $R_l < x_l$ and adjust $R_h$ accordingly to satisfy $s^* = s_{FB}$. By our previous arguments, $V(R_l, R_h)$ then decreases to $V(R_l, R_h) < 0$, again violating (4). Hence, to satisfy (4) it must hold that $R_l > x$. Uniqueness of $(R_l, R_h)$ follows from the same argument. If $s^* = s_{FB}$ is feasible, we thus have shown that there is a unique optimal contract with $x_l < R_l \leq x_l + w$ and $R_l < R_h < x_h$.

**Case 2.** $s^* > s_{FB}$. We claim that in this case, optimality implies that $R_l = x_l + w$. Suppose that this was not true, i.e., that $s^* > s_{FB}$ and $R_l < x_l + w$. By our previous arguments, there then exists a contract with $\widetilde{R}_l > R_l$, $\widetilde{R}_h < R_h$, $s^*(\widetilde{R}_l, \widetilde{R}_h) = s^*(R_l, R_h) = s^*$, and

$V_s(\widetilde{R}_l, \widetilde{R}_h) > V_s(R_l, R_h)$ for all $s > s^*$. That is, under $(\widetilde{R}_l, \widetilde{R}_h)$ the borrower's participation constraint is relaxed. This in turn allows to adjust $(\widetilde{R}_l, \widetilde{R}_h)$ by increasing either one (or both) repayment until (4) binds again. Denote this contract by $(\widetilde{R}'_l, \widetilde{R}'_h)$. (Existence follows from the fact that all payoffs change continuously in $\widetilde{R}_l$ and $\widetilde{R}_h$.) For the contract $(\widetilde{R}'_l, \widetilde{R}'_h)$, we then have that $s^*(\widetilde{R}'_l, \widetilde{R}'_h) < s^*(R_l, R_h)$. As we can adjust the contract—and thereby $s^*$—continuously, we can choose $(\widetilde{R}'_l, \widetilde{R}'_h)$ such that $s^*(\widetilde{R}'_l, \widetilde{R}'_h) \geq s_{FB}$. As the new cutoff is more efficient and the borrower's participation constraint is satisfied with equality, the new contract must make the lender strictly better off, contradiction. Given that $R_l = x_l + w$, $R_h$ is also uniquely pinned down: it is the maximum feasible value at which (4) binds. (Existence follows again from continuity of all payoffs in $R_h$.)

**Case 3.** $s^* < s_{FB}$. By the arguments in Case 2, we can rule out Case 3 as it is always possible to obtain a higher and more efficient cutoff $s^*$. **Q.E.D**.

**Proof of Proposition 2.** In a slight change of notation, we abbreviate expected payoffs by $V_s(r, C)$, $V(r, C)$, $U_s(r, C)$, and $U(r, C)$ respectively. Given the arguments in the main text, it remains to show that rejected borrowers have a negative average NPV.

**Claim 1.** *If the local lender's offer attracts the borrower in $\tau = 0$, a rejected borrower can not obtain financing from the competitive credit market in $\tau = 1$ as his (expected) NPV is negative.*

**Proof.** We argue to a contradiction and suppose that there is an equilibrium in which the local lender's offer $(r, C)$ attracts the borrower, while a rejected borrower can still obtain financing by the competitive credit market in $\tau = 1$. To sustain this equilibrium, it must hold that

$$\int_0^{s^*} [\mu_s - k] \frac{f(s)}{1 - F(s^*)} ds \geq 0. \tag{13}$$

Given that the credit market is perfectly competitive, all of the surplus, i.e., the left-hand side of (13), accrues to the borrower. To attract the borrower in $\tau = 0$, the local lender's offer must thus satisfy

$$\int_0^{s^*} [\mu_s - k] f(s) ds + \int_{s^*}^1 V_s(r, C) f(s) ds \geq \mu - k = \int_0^1 [\mu_s - k] f(s) ds,$$

implying that

$$\int_{s^*}^1 V_s(r, C) f(s) ds \geq \int_{s^*}^1 [\mu_s - k] f(s) ds. \tag{14}$$

However, $k(1+r) > x_l + C$—which is necessary for the loan to be approved with positive probability—implies that $U_s(r, C) > 0$ for all $s > s^*$, contradicting (14). **Q.E.D.**

**Proof of Propositions 3 and 5.** We already know that $w = 0$ implies $s^* > s_{FB}$. Moreover, from the Proof of Proposition 1 we have that $s^* > s_{FB}$ implies $C = w$ whenever $w > 0$, while a marginal increase in $C = w$ allows to strictly reduce $s^*$. Finally, by definition, $s^* = s_{FB}$ is feasible if and only if $w \geq C_{FB}$, in which case it holds that $C = C_{FB}$. **Q.E.D.**

**Proof of Proposition 4.** We can restrict ourselves to the case where $s^* > s_{FB}$. We make use of the following result from the Proof of Proposition 1: If $(R_l, R_h)$ is the (unique) optimal contract and $(\widetilde{R}_l, \widetilde{R}_h)$ is some other contract with $V_s(\widetilde{R}_l, \widetilde{R}_h) \leq V_s(R_l, R_h)$ for some $s = \widetilde{s} < \overline{s}$, then it must hold for all $s > \widetilde{s}$ that $V_s(\widetilde{R}_l, \widetilde{R}_h) < V_s(R_l, R_h)$.

Suppose first the borrower would make a new offer $(\widetilde{R}_l, \widetilde{R}_h)$. For this offer to be profitable, it must hold that $s^*(\widetilde{R}_l, \widetilde{R}_h) \leq s^*(R_l, R_h)$, which—by the definition of $s^*$ and previous argument—implies that $U_s(\widetilde{R}_l, \widetilde{R}_h) > U_s(R_l, R_h)$ for all $s > s^*(\widetilde{R}_l, \widetilde{R}_h)$. Consequently, the borrower's expected payoff from the new offer is just $V(\widetilde{R}_l, \widetilde{R}_h)$. By optimality of $(R_l, R_h)$ and $s^*(\widetilde{R}_l, \widetilde{R}_h) \leq s^*(R_l, R_h)$, this implies that $V(\widetilde{R}_l, \widetilde{R}_h) < V(R_l, R_h) = \overline{V}$, which in turn implies that offering $(\widetilde{R}_l, \widetilde{R}_h)$ cannot be profitable for the borrower. The argument where the local lender makes a new offer, which results in a game of signaling, is analogous. **Q.E.D.**

**Proof of Proposition 6.** We consider a (marginal) shift in the borrower's riskiness. We denote the characteristics of the more risky borrower by $\widehat{x}_l$, $\widehat{x}_h$, and $\widehat{p}_s$ for all $s \in S$. Note that by the mean-preserving spread (MPS) we have $\widehat{x}_l \leq x_l$ and $\widehat{x}_h \geq x_h$, where at least one inequality must hold strictly. Also, the first-best cutoff$s_{FB}$ remains unchanged and we have $p_s = \frac{\mu_s - x_l}{x_h - x_l}$ and $\widehat{p}_s = \frac{\mu_s - \widehat{x}_l}{\widehat{x}_h - \widehat{x}_l}$. We denote the respective optimal contracts by $(r, C)$ and $(\widehat{r}, \widehat{C})$ and the respective cutoffs by $s^*$ and $\widehat{s}^*$.

We first consider the case where $w > C_{FB}$ such that following a (marginal) increase in riskiness we have $s^* = \widehat{s}^* = s_{FB}$. We show that $\widehat{C} > C$ and $\widehat{r} > r$. Note first that

$$p_s - \widehat{p}_s = \frac{\mu_s[(\widehat{x}_h - \widehat{x}_l) - (x_h - x_l)] + \widehat{x}_l x_h - x_l \widehat{x}_h}{(x_h - x_l)(\widehat{x}_h - \widehat{x}_l)}, \tag{15}$$

which by the strict monotonicity of $\mu_s$ and $\widehat{x}_h - \widehat{x}_l > x_h - x_l$ is strictly increasing in $s$. To ensure that $s^* = \widehat{s}^* = s_{FB}$, it must hold that

$$\widehat{p}_{s_{FB}} k(1+\widehat{r}) + (1-\widehat{p}_{s_{FB}})(\widehat{C}+\widehat{x}_l) = p_{s_{FB}} k(1+r) + (1-p_{s_{FB}})(C+x_l), \tag{16}$$

while from (4) we obtain the requirement

$$\begin{aligned} & \int_{s_{FB}}^{1} \left[\widehat{p}_s[\widehat{x}_h - k(1+\widehat{r})] - (1-\widehat{p}_s)\widehat{C}\right] f(s)ds \\ = & \int_{s_{FB}}^{1} \left[p_s[x_h - k(1+r)] - (1-p_s)C\right] f(s)ds. \end{aligned} \tag{17}$$

Dividing (17) by $[1-F(s_{FB})]$ and using the MPS, this transforms to the requirement

$$\begin{aligned} & \int_{s_{FB}}^{1} \left[\widehat{p}_s k(1+\widehat{r}) + (1-\widehat{p}_s)(\widehat{C}+\widehat{x}_l)\right] \frac{f(s)}{1-F(s_{FB})} ds \\ = & \int_{s_{FB}}^{1} \left[p_s k(1+r) + (1-\widehat{p}_s)(C+x_l)\right] \frac{f(s)}{1-F(s_{FB})} ds. \end{aligned} \tag{18}$$

As $p_s - \widehat{p}_s$ is strictly increasing from (15), (16) and (18) can only be jointly satisfied if

$$k(1+\widehat{r}) - \widehat{C} - \widehat{x}_l = k(1+r) - C - x_l. \tag{19}$$

Given that $p_{s_{FB}} > \widehat{p}_{s_{FB}}$ by the MPS, (19) and (16) jointly imply

$$\widehat{C} + \widehat{x}_l > C + x_l. \tag{20}$$

From (20) and $\widehat{x}_l \le x_l$ by the MPS, it then follows that $\widehat{C} > C$. Finally, (19) and (20) jointly imply that $k(1+\widehat{r}) > k(1+r)$ and thus that $\widehat{r} > r$.

Suppose next that $w < C_{FB}$ such that following a (marginal) increase in riskiness we have $s^* > s_{FB}$ and $\widehat{s}^* > s_{FB}$, implying $C = \widehat{C} = w$ from Propositions 1 and 3. We show that $\widehat{s}^* > s^*$. We argue to a contradiction and suppose that $\widehat{s}^* \le s^*$. Define next $\widetilde{r}$ such that a contract $(C = \widetilde{r}, w)$ for the less risky borrower leads to the cutoff $\widetilde{s}^* = \widehat{s}^*$, i.e.,

$$p_{\widehat{s}^*}[k(1+\widetilde{r}) - w - x_l] + (w+x_l) = \widehat{p}_{\widehat{s}^*}[k(1+\widehat{r}) - w - \widehat{x}_l] + (w+\widehat{x}_l). \tag{21}$$

We show now that under $(\widetilde{r}, w)$ the constraint (4) is relaxed, which by the MPS and $\widetilde{s}^* = \widehat{s}^*$ is indeed the case if

$$\begin{aligned} & \int_{s_{FB}}^{1} \left[\widehat{p}_s[k(1+\widehat{r}) - w - \widehat{x}_l] + (w+\widehat{x}_l)\right] \frac{f(s)}{1-F(s_{FB})} ds \\ < & \int_{s_{FB}}^{1} \left[p_s[k(1+\widetilde{r}) - w - x_l] + (w+x_l)\right] \frac{f(s)}{1-F(s_{FB})} ds. \end{aligned} \tag{22}$$

As $p_s - \widehat{p}_s$ is strictly increasing from (15), (22) holds from (21). Given that (4) is relaxed, the lender can further increase $\widetilde{r}$ and, thereby, decrease the respective cutoff $\widetilde{s}^*$ until (4) binds. (Such a contract exists from continuity of the borrower's payoff in $\widetilde{r}$.) This contradicts optimality of the original contract $(w, r)$ where $s^* > \widehat{s}^* > \widetilde{s}^*$.

Finally, we turn to the case where $w = C_{FB}$. After a MPS the new threshold in (6) satisfies $\widehat{C}_{FB} \geq C_{FB}$, which holds strictly if $\widehat{x}_l < x_l$. In this case, it follows immediately from our previous results that $C = \widehat{C} = w$ and $\widehat{s}^* > s^* = s_{FB}$. **Q.E.D.**

**Proof of Proposition 7.** Note first that the first-best cutoff does not depend on the size. Moreover, we have $s^* = s_{FB}$ if

$$(\alpha x_l + C) + p_{s_{FB}} [\alpha k(1 + r) - (\alpha x_l + C)] = \alpha k, \tag{23}$$

while a similar transformation and substitution of $\overline{V} = \alpha(\mu - k)$ yields from (4) that

$$\int_{s_{FB}}^{1} [p_s[\alpha x_h - \alpha k(1 + r) + C] - C] f(s) ds = \alpha(\mu - k). \tag{24}$$

Denote next for two projects of sizes $\alpha < \widehat{\alpha}$ the respective optimal contracts by $(r, C)$ and $(\widehat{r}, \widehat{C})$ and the respective cutoffs by $s^*$ and $\widehat{s}^*$. We suppose again first that $w > C_{FB}$ such that from continuity we have after a marginal change in size that $s^* = \widehat{s}^* = s_{FB}$. As $p_s$ is strictly increasing, (23)-(24) can only hold jointly for $\alpha$ and $\widehat{\alpha}$ in case $\widehat{r} = r$ and $\widehat{C}/C = \widehat{\alpha}/\alpha$.

Suppose next that $w < C_{FB}$, such that both $C = \widehat{C} = w$. We show that it must then hold that $s^* < \widehat{s}^*$. We argue to a contradiction and suppose that $\widehat{s}^* \leq s^*$. Choose next for the project of size $\alpha$ a value $\widetilde{r}$ together with $C = w$ such that the cutoff is equal to $\widehat{s}^*$, i.e.,

$$\begin{aligned} & (\alpha x_l + w) + p_{\widehat{s}^*} [\alpha k(1 + \widetilde{r}) - (\alpha x_l + w)] - \alpha k \\ = \; & (\widehat{\alpha} x_l + w) + p_{\widehat{s}^*} [\widehat{\alpha} k(1 + \widehat{r}) - (\widehat{\alpha} x_l + w)] - \widehat{\alpha} k. \end{aligned} \tag{25}$$

We show now that (25) implies that under $(\widetilde{r}, C)$ the constraint (4) is not binding for the smaller project, which yields a contradiction as we can further increase $\widetilde{r}$ and thereby ensure that the lender applies an even more efficient cutoff. This holds if

$$\int_{\widehat{s}^*}^{1} [\alpha \mu_s - (\alpha x_l + w) - p_s [\alpha k(1 + \widetilde{r}) - (\alpha x_l + w)]] f(s) ds > \alpha(\mu - k). \tag{26}$$

Substituting the binding constraint (4) for $\widehat{\alpha}$ and equation (25) reveals that (26) holds. In the final case where $w = C_{FB}$ we have from (6) that the threshold does not depend on the project's size, i.e., that $\widehat{C}_{FB} = C_{FB}$. By the previous arguments this implies $C = \widehat{C} = w$ and $s^* < \widehat{s}^*$ This completes the proof of Proposition 7. **Q.E.D.**

**Proof of Proposition 8.** With $\overline{V} = \mu - k - \kappa$ the threshold (6) changes to

$$C_{FB} := \frac{(k - x_l)(\mu - k - \kappa)}{\int_{s_{FB}}^{1} (\mu_s - k) f(s) ds}, \tag{27}$$

which is strictly decreasing in $\kappa$. We consider a marginal decrease in the additional lending costs $\kappa$. For $\kappa > \widehat{\kappa}$ denote the respective contracts by $(r, C)$ and $(\widehat{r}, \widehat{C})$ and the respective cutoffs by $s^*$ and $\widehat{s}^*$. In case $w > C_{FB}$ we have for a marginal reduction in costs that $s^* = \widehat{s}^* = s_{FB}$ while $C = C_{FB}$ and $\widehat{C} = \widehat{C}_{FB}$ with $\widehat{C}_{FB} < C_{FB}$. In case $w < C_{FB}$ we have that $C = \widehat{C} = w$ and $\widehat{s}^* > s^* > s_{FB}$. To see that $\widehat{s}^* > s^*$, note that offering $(r, C)$ if the competitive market has costs $\kappa$ leaves the borrower's participation constraint (4) slack. By previous arguments this allows to construct a new contract $(\widehat{r}', C)$ where (4) binds and the cutoff is strictly lower, contradicting the optimality of the original contract $(\widehat{r}, \widehat{C})$. **Q.E.D.**

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