Credit Constraints and Investment-Cash Flow Sensitivities

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Abstract

This paper analyzes the investment behavior of firms under a quantity constraint on the amount of external funds which can be raised at a given cost (credit constraints). In this world, investment-cash flow sensitivities decrease in the degree of credit constraints, until a firm becomes effectively unconstrained. This generates a "U-shaped" curve for the relationship between sensitivities and credit constraints. From an empirical perspective, the good news is that we suggest a theoretically consistent way to identify the impact of financial constraints on investment behavior, at least under the condition that financial constraints affect primarily the quantity of credit available to firms. The bad news is that our prediction is in a sense the opposite as the one explored in previous empirical literature.

1 Introduction

There is a large finance and macroeconomics literature[1], starting with Fazzari, Hubbard and Petersen (1988), which looks for evidence of financial constraints by examining the sensitivity of investment to changes in cash flow.

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[1] Gilchrist and Himmelberg, 1995, Hoshi et.al, 1991, Hubbard and Kashyap, 1992, Hubbard et.al, 1995, Kashyap et.al, 1994, Lamont, 1997, Oliner and Rudesbusch, 1992, Schaller, 1993 and Whited, 1992 are only some of the other references on this large literature. See Bernanke, Gertler and Gilchrist, 1996 and Hubbard 1998 for comprehensive surveys.

The basic idea behind these empirical exercises is that the sensitivity of investment to cash flow should be higher for firms which are more financially constrained (the *monotonicity hypothesis*). Therefore, we should be able to identify the presence of financial constraints by looking at cross-sectional differences in investment-cash flow sensitivities. For example, Hoshi, Kashyap and Scharfstein (1991) classify Japanese firms according to membership in a Japanese industrial group (Keiretsu), and find that Keiretsu firms have lower investment-cash flow sensitivities. They interpret the result as evidence that Keiretsu firms are less financially constrained than the other firms in the Japanese economy.

The validity of such empirical exercises have been criticized by Kaplan and Zingales (1997). The basic problem is that the monotonicity hypothesis is not a necessary implication of a firm's optimal investment decisions under financial constraints, as they argue. Therefore, we cannot conclude that cross-sectional differences in investment-cash flow sensitivities are evidence of financial constraints. For example, the results in Hoshi, Kashyap and Scharfstein (1991), cannot be interpreted as evidence that Keiretsu firms are more financially constrained than non-Keiretsu ones. Moreover, there has been recent empirical work where the authors (Kaplan and Zingales (1997), and Cleary (1999), more specifically) argue that, in their particular samples, more constrained firms are actually less sensitive to changes in cash flow.

These recent papers have created a debate in the literature, particularly between Fazzari, Hubbard and Petersen (2000), and Kaplan and Zingales (2000). On the theoretical front, FHP attempt to provide conditions under which investment-cash flow sensitivities are indeed monotonic in financial constraints. However, KZ show that the FHP condition is still not sufficient to ensure that investment-cash flow sensitivities should bear any precise relationship at all with financial constraints. In our view, the conclusions to derive from this debate are mostly negative. Although we know that financial constraints should influence investment-cash flow sensitivities, we cannot be sure about the precise theoretical relationship. Thus, cross-sectional differences in investment-cash flow sensitivities cannot be used per se as evidence of financial constraints.

The current paper brings several contributions to this recent debate. Our most important contribution is to suggest a specific scenario where we can make precise inferences about the nature of the relationship between financial constraints and investment-cash flow sensitivities. This happens when financial constraints translate into a quantity constraint on the amount of external funds which can be raised at a given cost (credit constraint). The credit constraint is endogenous, in the sense that the credit limit depends on the value of the firm's assets. In this world less constrained firms are the ones which can borrow a higher fraction of the value of their assets.

In such a world, we get precise implications for investment - cash flow sensitivities. However, the implication is *not* that investment-cash flow sensitivities increase in the degree of financial constraints. The implication of the model is that sensitivities should *decrease* with financial constraints, as long as firms are not entirely unconstrained. We get therefore a "U-shaped" curve for the relationship between sensitivities and the measure of financial constraints.

The intuition for this result is simple. A change in cash flow (ΔW) has a *direct* effect on the investment of financially constrained firms. Constrained firms invest all their funds, and therefore the impact of this direct effect is the same for all constrained firms (and equal to ΔW). However, there is also an *indirect* effect which is due to the endogenous change in borrowing capacity. For any change in investment ΔI, borrowing capacity changes by a certain fraction for all firms. However, this change in borrowing capacity will be higher for the firms which can borrow a higher fraction of the value of their assets (the less constrained ones). In other words, not only can a less constrained firm borrow more, but its debt capacity is also more sensitive to a change in cash flow. It is this indirect amplification effect which drives the differences in investment-cash flow sensitivities in our model. Naturally, if borrowing capacity is so large that firms are unconstrained, sensitivities go back to zero. This gives us the U-shaped relationship between financial constraints and investment-cash flow sensitivities.

From an empirical perspective, our analysis brings both positive and negative contributions. The positive one is that we suggest a theoretically consistent way to identify the impact of financial constraints on investment behavior, at least under certain conditions. If if financial constraints affect primarily credit constraints on firms, investment-cash flow sensitivities are useful measures of financial constraints. The bad news is that our prediction about the relationship between financial constraints and sensitivities is *the opposite* as the one explored in previous empirical literature. Thus, our results cannot be used to rescue the investment-cash flow literature from the Kaplan and Zingales critique.

One paper which directly uses the empirical approach suggested by this paper (although in a slightly different context), is Almeida (2000). One good example of the particular financing and investment decisions described here is on housing finance contracts. The availability of mortgage credit to households is usually limited to a specific fraction of the value of the house being purchased (the maximum loan-to-value, or LTV ratio), which is used as collateral. That is, credit rationing seems to be a crucial feature of such contracts. Furthermore, housing finance development differs widely across the world, and this has a direct effect on observed maximum LTV ratios in different countries. Almeida (2000) looks at the international data, and shows that house prices are more sensitive to shocks which affect household income, in countries where household finance is more developed. This is evidence that relaxation of credit constraints does tend to increase the extent to which investment and prices respond to shocks to net worth.

Our results also suggest interesting policy implications. Basically, they suggest that financial development may lead to higher fluctuations in investment (and prices). Even if financial development is desirable for other reasons, the potential associated increase in the extent of fluctuations could become an explicit policy concern. This is in stark contrast to the policy implications which arise from papers like Fazzari, Hubbard and Petersen (1988), which in a broad sense imply that financial development should reduce the extent of fluctuations in investment.

An important question raised by our results is why are they different than the ambiguous results which obtain in KZ. A key assumption in KZ is that firms can always raise external funds if they pay the right price. Financial constraints in this world translate entirely into higher costs of external funds (and costs which are more sensitive to changes in the amount of external funds). However, there is no credit rationing. In other words, while KZ focus on the effects of capital market imperfections on the *cost* of external funds, our focus is on *quantities,* and liquidity constraints.

In KZ, investment-cash flow sensitivities depend on how financial constraints affect the slopes of marginal costs of external funds, and the slope of the marginal productivity of investment. The reason why our results are so strong is because *none* of the effects which drive sensitivities in the Kaplan and Zingales model matter in our model. First of all, the slope of marginal costs of external funds is the same, *in equilibrium*, for all firms. In equilibrium, all constrained firms are at the point at which the supply of capital becomes inelastic (since they are credit-constrained). Therefore, the effects related to changes in the slope of marginal costs do not matter. Furthermore, in any constrained equilibrium, the slope of the capital demand curve does not matter, since it is not equal on the margin to the slope of the capital supply curve (which is equal to the slope of marginal costs of external funds). Thus, this slope does not influence investment-cash flow sensitivities. On the other hand, the indirect amplification effect described above is present whenever there are liquidity constraints, and when borrowing capacity is endogenous.

This suggests that it is crucial to determine if financial constraints affect primarily credit constraints on firms (that is, the availability of finance at a given cost), or the cost of external finance to different firms. We explore this idea further, by considering a scenario where both credit constraints, and deadweight costs of external finance influence investment and financing decisions. The world we describe is a world in which there is a "pecking order" in the use of external finance. Firms exhaust their collateralized debt capacity first (rationed funds), because it is the cheapest way they can raise external funds. Then, they raise the balance at higher costs. Both the amount they can raise at the cheaper price, and the marginal costs of increasing funds above this limit, are affected by the degree of financial constraints.

It is no longer the case that investment-cash flow sensitivities have the "U-shape" described above because we bring back the effects of changes in the cost of non-rationed funds into the picture. The flip side from this condition is also true. Once we introduce credit constraints in a model such as Kaplan and Zingales', we automatically get an effect which pushes towards higher sensitivities for less constrained firms. This makes it even harder to obtain conditions under which the monotonicity hypothesis should hold.

We start in section 2, by introducing a model where there are credit constraints on firms. We derive implications for investment-cash flow sensitivities. We also compare the implications to those of previous literature. In section 3 we introduce a general model where both credit constraints and changes in the marginal costs of external funds are important. The empirical and policy implications of the theoretical analysis are discussed in detail in section 4. Section 5 concludes.

2 The basic model[2]

Assume that firms have production technologies $f(I)$, which produce output from an amount of physical investment equal to I. However, following Hart and Moore (1994), production only occurs if managers input their human capital into production. Also, human capital is inalienable, in the sense that managers cannot commit to input their human capital ex-ante. Depending on the amount of debt they have outstanding, entrepreneurs may then decide to renege on their debt and renegotiate with their creditors (the lenders). The contractual outcome in this framework is that lenders will only lend up to the value of the firm's collateralizable assets. If V is the value of the firm's collateralizable assets, the borrowing constraint is:

$$B \leq V$$

where B is the amount of collateralized debt raised by firms. In order to derive V, assume that the physical goods invested by the firm have a current price[3] equal to 1 (a normalization), and a price next period (the period when output is produced) equal to q_1.

We also assume that liquidation of the firm's physical assets I by lenders entails transaction costs that are proportional to the value of the physical collateral held by entrepreneurs. More specifically, if the firm's assets I are seized by lenders, a fraction $\tau \in (0, 1)$ of the proceeds $q_1 I$ is lost. An increase in τ will then decrease the liquidity of collateral[4]. Thus, the total liquidation value of the assets is $(1 - \tau)q_1 I$.

The parameter τ is a simple way to measure the degree of capital market imperfections in this world. Firms with low τ are able to borrow more because they have assets which are (potentially) worth more for outside creditors. The

[2] The model we describe here draws heavily on Kiyotaki and Moore (1997), and on Almeida (1999).

[3] The current period should be interpreted as the period when investment is made.

[4] The parameter τ is a function of factors such as the tangibility of the firm's physical assets, and the legal environment that dictates the relations between debtors and creditors. Myers and Rajan (1998) parametrize the liquidity firm's assets in a similar way.

borrowing constraint will then be[5]:

$$B \leq (1-\tau) q_1 I$$

Managers will then choose investment and debt in order to maximize the value of their equity on the firm (there is no outside equity in this model). If we assume that the discount rate is equal to 1, this implies the following program:

$$\begin{aligned} & \max \ (c_0 + c_1) \ s.t. \\ c_0 &= W - I + B \geq 0 \\ c_1 &= f(I) + q_1 I - B \\ B &\leq (1-\tau) q_1 I \end{aligned}$$

Notice that, if there was no borrowing constraint on firms (third constraint not binding), this problem would reduce to:

$$\max_I \ f(I) + q_1 I - I$$

if we let:

$$F(I) = f(I) + q_1 I$$

The interpretation is that, in the present model, firms can invest at the opportunity cost of internal funds, as long as the amount they have to borrow does not exceed a certain amount given by the value of their collateralizable assets. Financial market imperfections affect not the cost of external funds, but the quantity that can be raised at a given price.

The solution to the firm's optimal investment depends on whether the borrowing constraint is binding or not. If the borrowing constraint is not binding, we obtain the efficient level of investment:

$$F'(I_{FB}) = 1$$

[5] We are assuming that firms cannot pledge future cash flows directly to creditors. In terms of the Hart and Moore model, we are implicitly assuming that firm managers can make a take-it-or-leave-it offer to outside creditors, if they are both bargaining for current cash flows. Nothing in the analysis will change if we assume that creditors get a fixed fraction of the proceeds in the bargaining process, as long as this fraction is not correlated with the degree of capital market imperfections. See Almeida (1999) for an analysis of the case where the degree of capital market imperfections also affects the fraction of cash flows that can be pledged.

If the amount of internal funds firms have is high enough, then the efficient level of investment will obtain. The same is true if τ is low enough. All we need is that the amount of internal funds plus the amount that can be raised in the market is enough to finance the efficient level of investment, that is:

$$W + (1-\tau)q_1 I^{FB} \geq I^{FB}$$

For a given W, this equation determines the minimum value of τ which will lead to a constrained solution, and vice-versa. Investment will be constrained as long as:

$$\begin{aligned} \tau &\geq \tau_{\min} = 1 - \frac{I^{FB} - W}{q_1 I^{FB}} \\ W &\leq W_{\max} = (1 - q_1 + \tau q_1) I_{FB} \end{aligned}$$

Thus, the level of investment will be given by:

$$\begin{aligned} I(\tau, W) &= \frac{W}{(1 - q_1 + \tau q_1)}, \text{ if } \tau \geq \tau_{\min} \text{ and } W \leq W_{\max} \\ &= I_{FB,} \text{ otherwise} \end{aligned} \tag{1}$$

Figure 1 depicts the determination of the optimal investment level in this model. Notice that everything works as if the firm had an infinitely elastic supply of funds (at the right cost) until the equilibrium investment level $I(\tau, W)$, but a completely inelastic supply of funds after that. Constrained equilibrium investment is determined at the point where the function $\frac{W}{I} + (1-\tau)q_1$ is equal to the opportunity cost of investment.

The investment cash-flow sensitivity in this model is given by:

$$\begin{aligned} \frac{\partial I}{\partial W}(W, k) &= \frac{1}{(1 - q_1 + \tau q_1)} \text{ if } \tau \geq \tau_{\min} \text{ and } W \leq W_{\max} \\ &= 0, \text{ otherwise} \end{aligned} \tag{2}$$

Thus, as long as firms are financially constrained, the investment cash-flow sensitivity will in fact be *higher* for less constrained firms (low τ firms). We depict the investment-cash flow sensitivity predicted by this model in figure 2. The model delivers an “U-shaped” investment cash flow sensitivity. This is consistent with the non-monotonicity pointed out by Kaplan and Zingales. The crucial difference is that, contrary to their “anything goes” result, our model predicts a precise relationship between financial constraints and investment-cash flow sensitivities. On the other hand, this relationship is the opposite as the one postulated by Fazzari, Hubbard and Petersen (1988, 2000) and others. A progressive relaxation of financial constraints should actually increase investment-cash flow sensitivities, as long as firms do not become entirely unconstrained.

The intuition for this result is simple once we consider it in the context of the discussion above. Consider figure 3, which compares the impact of a positive change in cash flow for two firms which differ only according to the degree of capital market imperfections (measured by τ).

The change in the availability of internal funds (ΔW) has a *direct* effect on constrained investment, which is the same for both firms (and equal to ΔW). However, there is also an *indirect* effect which is due to the endogenous increase in borrowing capacity for both firms. For any increase in investment ΔI, borrowing capacity increases by $(1-\tau)q_1\Delta I$ for both firms. Therefore, this increase in borrowing capacity will be higher for the firm with low τ, the less constrained firm, whose debt capacity is more sensitive to a change in cash flow. It is this indirect amplification effect which drives the difference in investment-cash flow sensitivities in our model.

2.1 Credit constraints versus price effects. Why is our result different?

This raises a natural question. Why is our result so different than the ones stressed in the recent theoretical literature about financial constraints and investment ? As we will show here, the main reason for that is our focus on quantities, vis-a-vis the focus on costs used in previous literature.

Let us first summarize the main results in Fazzari, Hubbard and Petersen (2000), and Kaplan and Zingales (1997, 2000). In the context of the discussion, it will be easy to understand what is special about our results.

2.1.1 The current theoretical debate

Kaplan and Zingales (1997) argue that there is nothing we can say a priori about financial constraints and investment-cash flow sensitivities, apart from the obvious result that unconstrained firms have zero, and constrained firms have positive, sensitivities.

The essence of their model (which we call KZ model from now on), can be summarized in figures 4 and 5. Firms have an amount of internal funds equal to W, and a production technology $F(I)$ which satisfies ordinary assumptions. Any amount of external funds raised by firms entails deadweight costs equal to $C(I-W,k)$. The parameter k can be interpreted as a measure of a firm's wedge between internal and external costs of funds. It is the counterpart of the parameter τ above. Any amount of external funds that firms raise generates (because of information or agency problems) deadweight cost equal to $C(I-W,k)$, which will be higher the higher is the parameter k.

Given this set up, the equilibrium amount of investment (depicted in figure 4) is given by:

$$F^{'}(I) = 1 + C_E[I - W, k] \tag{3}$$

Kaplan and Zingales also measure the sensitivity of investment to cash flow as the derivative $\frac{\partial I}{\partial W}(W,k)$. The empirical approach pioneered by Fazzari, Hubbard and Petersen (1988) consists of measuring the degree of financial constraints faced by difference firms as differences in W or k, and then looking at the cross-sectional differences in $\frac{\partial I}{\partial W}$. As depicted in figure 5 (the counterpart of figure 3 above), this consists of comparing the impact of similar changes in W on the investment of two different firms (indexed by k in the figure).

The problem pointed out by Kaplan and Zingales is that neither $\frac{\partial^2 I}{\partial W \partial k}$, nor $\frac{\partial^2 I}{\partial W^2}$ have a well defined sign. The most we can say is that an unconstrained firm (W very high[6], or k equal to zero) has sensitivity equal to zero, while a constrained firm (a firm which faces positive $C(.)$) has positive $\frac{\partial I}{\partial W}$, at least if $C_{EE}(.)$ is greater than zero.

This result generated a debate between Fazzari Hubbard and Petersen (FHP 2000), and Kaplan and Zingales (KZ 2000). FHP (2000) also start from the KZ model, and basically argue that the relevant source of firm heterogeneity in the empirical studies is the slope of the marginal cost curve C_E, and that once we take that into account the ambiguity stressed by Kaplan

[6] More precisely, the condition is that W is higher than the unconstrained investment level, which can be defined by:

$$F'(I_{FB}) = 1$$

and Zingales disappears. In terms of the KZ model, this is equivalent to the assumption that:

$$C_{EEk}[I - W, k] > 0 \;\forall\; (I - W) \tag{4}$$

that is, firms which face more severe capital market imperfections (high k firms) find it more expensive on the margin to increase the amount of external finance they raise, starting from any initial amount of external funds. In other words, additional financing has a larger impact on the marginal cost schedule if firms are more financially constrained, as depicted in figures 4 and 5.

FHP (2000) argue that condition 4 is sufficient to yield $\frac{\partial I}{\partial W \partial k} > 0$. Unfortunately, this is not the case, as shown in KZ (2000). If the FHP condition 4 holds, firms with higher k will find it more expensive to borrow funds in order to smooth out the fluctuation in cash flow. This will push in the direction of a higher decrease in investment for the more constrained firm. On the other hand, as FHP point out themselves, C_E tends to be convex, since marginal agency costs of debt tend to increase with leverage[7]. But if $C_{EEE} > 0$, then less constrained firms (low k firms) are pushed into a range where marginal costs of external finance are more sensitive to further changes in external funds. This is because such firms always invest and borrow more in equilibrium, as compared to more constrained firms. The negative shock to cash flow will then have a larger impact for this group of firms.

If the marginal productivity of investment is not linear, then we can have a similar effect associated with the slope of the demand for funds. If $F^{'''}(.)$ < 0, then less constrained firms (those which have higher investment) will be in a range where a similar change in the supply of funds will have a higher effect on equilibrium investment, since the slope of the demand for investment has a more negative slope for these firms.

In other words, given that we cannot expect both the marginal cost C_E and the marginal productivity of investment $F'(I)$ to be always linear, investment-cash flow sensitivities and financial constraints can bear almost any shape. In our view, this is the main conclusion to take from this debate.

2.1.2 Why is our result different ?

A key assumption which is common to Kaplan and Zingales and Fazzari, Hubbard and Petersen is that firms which face agency or information problems

[7]See also Hubbard, Kashyap and Whited (1995).

cannot raise *any* amount of external funds at the "right" cost (the opportunity cost of internal funds). Financial constraints in this world translate entirely into higher costs of external funds (and costs which are more sensitive to changes in the amount of external funds).

On the other hand, in the model we described in section 2, financial constraints translate into a *quantity constraint* on the amount of funds which can be raised at a given cost. However, the extent to which different firms can do this is limited by a borrowing constraint. This is the key reason why we are able to derive precise empirical implications from our model.

Notice that, in the world we describe in section 2, *none* of the effects which drive sensitivities in the Kaplan and Zingales model matter. First of all, notice in figure 3 that the slope of the capital supply curve is the same, *in equilibrium*, for both firms. In equilibrium, all constrained firms are at the point at which the supply of capital becomes inelastic. Therefore, the effects related to changes in the slope of the supply of capital do not matter. Furthermore, in any constrained equilibrium, the slope of the capital demand curve does not matter, since it is not equal on the margin to the slope of the supply curve (again, see figure 3). Thus, the third derivative of the production function does not influence investment-cash flow sensitivities.

Thus, investment-cash flow sensitivities are entirely driven by the fact that the change in borrowing capacity associated with a change in cash flow will be higher for the firm with low τ, the less constrained firm (our indirect amplification effect). And this effect operates in the opposite direction as the direction assumed in most of the empirical literature. Less constrained firms should in fact be more sensitive to changes in cash flow.

3 Credit constraints and price effects together - the general case

In a sense, both Kaplan and Zingales and the model in section 2 describe two extreme worlds. If all borrowing is subject to deadweight costs associated with capital market imperfections, the Kaplan and Zingales model applies. In general, implications for investment-cash flow sensitivities are ambiguous in this world. If firms can raise different amounts of external finance at the right cost, then the model in section 2 suggests that the function relating investment-cash flow sensitivities and capital market imperfections has a well defined U-shape. As financial constraints are progressively relaxed, investment-cash flow sensitivities increase unambiguously until firms become unconstrained. Then, sensitivities decrease to zero.

It is clearly desirable from a theoretical perspective to bring together the effects of borrowing constraints, and the effects of deadweight costs of external finance. In this section, we describe a simple model which nests KZ and the model in section 2 as special cases.

Not surprisingly, the implications of this model for investment-cash flow sensitivities will depend on all the effects described above for the two models separately. Furthermore, if the parameters τ and k are positively correlated (that is, firms which have high debt capacity also face lower marginal costs of external finance), a novel interaction term arises. Let us turn now to the model.

3.1 A general model

The following model brings together the features of KZ and the model in section 2. The basic idea in the model is that firms can raise a certain amount of collateralized debt at the same opportunity costs of internal funds. Debt capacity is then measured as the liquidity of the firm's assets, as in section 2. The novel feature here is that we allow firms to raise external finance in excess of the value of their collateral. As in KZ, this will entail deadweight costs which will be captured by a certain function $C(.)$.

More formally, if we let D be the total amount of external finance raised by firms, we have:

$$D = B + E$$

where B is the amount of collateralized debt, and E is the amount of uncollateralized external finance raised by firms. As in KZ, deadweight costs will be given by $C(E, k)$. As in section 2, there is a constraint on the total amount of collateralized debt firms can raise:

$$B \leq (1 - \tau) q_1 I$$

In this world, entrepreneurs will solve the following problem:

$$\begin{aligned} & \max \ (c_0 + c_1) \ s.t. && (5) \\ c_0 &= W - I + B + E \geq 0 \\ c_1 &= f(I) + q_1 I - B - (E + C(E,k)) \\ B &\leq (1-\tau) q_1 I \end{aligned}$$

It is easy to see that, if $\tau = 1$ we are back to the Kaplan and Zingales model. If there is no uncollateralized borrowing ($E = 0$), we are back to the model described in section 2. Thus, this model nests both KZ and the model in section 2.

As in the previous section, the unconstrained investment level will obtain as log as:

$$W + (1-\tau) q_1 I^{FB} \geq I^{FB}$$

Otherwise, firms exhaust their collateralizable debt capacity and will also raise uncollateralized external finance. In such a case, program 5 can be rewritten as:

$$\max_I \ F(I) - I - C[q(\tau) I - W, k]$$

where $F(I) = f(I) + q_1 I$ and:

$$q(\tau) = 1 - (1-\tau) q_1$$

Notice that $(1 - q_1) \leq q(\tau) \leq 1$, and $q^{'}(\tau) > 0$. As long as $\tau < 1$, firms can borrow $(1-\tau)q_1$ times the investment level, and thus they can invest more than W, without generating any deadweight costs of external finance.

The first order condition is:

$$F^{'}(I) = 1 + q(\tau) C_E[q(\tau) I - W, k]$$

Figure 6 depicts the optimal investment level in this model. The key feature to notice here is that, if $\tau < 1$ (so $q(\tau) < 1$), the slope of the capital supply function becomes less steep, irrespective of k. This is because an increase in uncollateralized external finance which is channeled into investment will also enable the firm to raise more collateralized debt. Thus, the marginal costs of uncollateralized funds are effectively lower than in the KZ model.

Now, investment-cash flow sensitivity is given by:

$$\begin{aligned}\frac{\partial I}{\partial W} &= -\frac{C_{EE}[q(\tau)I-W,k]}{F"(I)-q(\tau)^2 C_{EE}[q(\tau)I-W,k]},\ \text{if } I<I^{FB} \\ &= 0,\ \text{if } I=I^{FB}\end{aligned}$$

The implications for investment-cash flow sensitivity will depend on the correlation between the parameters τ and k. The most reasonable case is one in which they are positively correlated. This means that firms which have high debt capacity also have lower deadweight costs of uncollateralized funds. This can be formalized by assuming that $\tau = k$. The impact of the degree of capital market imperfections on investment-cash flow sensitivities can then be measured by the derivative $\frac{\partial I}{\partial W \partial \tau}$, which can be shown to have the same sign as the expression:

$$-2q(\tau)q^{'}(\tau)-F^{''}C_{EEE}q^{'}(\tau)I-F^{''}C_{EE\tau}-\frac{\partial I}{\partial \tau}[F^{''}C_{EEE}q(\tau)I-C_{EE}F^{'''}]$$

The last three terms are the ones discussed in KZ (2000), and FHP (2000). If $C_{EE\tau}$, additional uncollateralized financing has a larger impact on the marginal cost schedule if firms face stronger capital market imperfections. A shock to cash flow will then have a larger impact on such firms. If $C_{EEE} > 0$, then less constrained (low τ) firms are pushed into a range where marginal costs of external finance are more sensitive to further changes in external funds. The negative shock to cash flow will then have a larger impact for this group of firms. Finally, if $F^{'''}(.)$ is higher (lower) than zero, then more (less) constrained firms will be in a range where a similar change in the supply of funds will have a higher effect on equilibrium investment.

The first term (always negative) is the one emphasized in section 2. If firms have high (collateralized) debt capacity (low τ), then the change in borrowing capacity induced by ΔW will be higher for such firms. This indirect amplification effect always pushes in the direction of higher investment-cash flow sensitivities for less constrained firms.

Finally, the second term arises from the interaction between collateralized and uncollateralized borrowing. Let us assume that $C_{EEE} > 0$, which is the most reasonable case as we saw above. This means that the second term is positive, pushing towards higher sensitivities for more constrained firms. This is due to the effect of changes in τ in the capital supply curve. As we saw in figure 6, a decrease in the liquidity of assets (increase in τ) tends to shift the capital supply curve up and to the left. This moves a constrained firm to a range the supply curve is steeper, if $C_{EEE} > 0$. Thus, marginal costs of uncollateralized finance are more sensitive to further changes in external funds for such firms.

Thus, when we consider both collateralized and uncollateralized borrowing together in the same model, we are basically back to the ambiguous world of the Kaplan and Zingales model. Not only do we bring back the ambiguous effects present in the KZ model, but we also add another effect which arises from the interaction between collateralized and uncollateralized borrowing.

4 Empirical and policy implications

In general, the function relating investment-cash flow sensitivities to the degree of financial constraints can have any shape, as shown in the previous section. However, our analysis also points to some special cases when it will be possible to make specific predictions about sensitivities.

4.1 Credit constraints and investment-cash flow sensitivities

The most clear case is when firms are c*redit constrained,* and differences in capital market imperfections change the degree of rationing. In this case, we get a U-shaped relationship between financial constraints and investment-cash flow sensitivities. As financial constraints are progressively relaxed, sensitivities always increase. If firms become unconstrained, sensitivities decrease to zero.

Two important empirical properties of the investment-cash flow sensitivity which come out of this model (see equation 2 above) are as follows. First, sensitivities do not depend directly on the availability of internal funds. Internal funds will only influence whether a particular firm is constrained or not. Thus, we can test the prediction that sensitivities are increasing in the degree of financial constraints, across a group of firms which are a priori classified as financially constrained, without controlling for variables like cash stocks. Second, sensitivities are not affected by the endogeneity of financial policy, that is, by the effect of capital market imperfections on the level of investment and external finance. The U-shape implication is therefore robust to heterogeneity in the availability of internal funds, and to the endogeneity of financial policy.

This analysis suggests a theoretically consistent way to identify the impact of financial constraints on investment behavior, at least under certain conditions. Basically, all we need is to isolate a situation when financial constraints affect primarily credit constraints on firms. If this is the case, then investment-cash flow sensitivities are indeed empirically useful measures of financial constraints.

From the perspective of previous empirical literature, the bad news is that our prediction about the relationship between financial constraints and sensitivities is *the opposite* as the one explored in previous empirical literature. Thus, our results cannot be used to rescue the investment-cash flow literature from the Kaplan and Zingales critique.

The empirical approach suggested by the current paper has already been shown to be useful, although in a slightly different context. One good example of the particular financing and investment decisions described in the model of section 2 is on housing finance contracts. The availability of mortgage credit to households is usually limited to a specific fraction of the value of the house being purchased (the maximum loan-to-value ratio), which is used as collateral. That is, credit rationing seems to be a crucial feature of such contracts. Furthermore, housing finance development differs widely across the world, and this has a direct effect on observed maximum LTV ratios in different countries.

Almeida (2000) builds a model which suggests that housing demand and house prices should be more sensitive to shocks which affect household income, in countries where household finance is more developed, as long as financial development is not so high that households become unconstrained (the U-shape above). This result arises precisely from the amplification effects described here in section 2. He also tests this prediction using international data on house prices, and obtains a result which is consistent with the empirical prediction above. This is strong evidence that relaxation of credit constraints tends to increase the extent to which investment and prices respond to shocks to net worth. Thus, in these circumstances financial constraints are important, but in a different way than the one suggested by previous literature.

The difference in empirical predictions has an interesting policy counterpart. The process of financial development should be correlated with the relaxation of credit constraints, since it tends to increases the amount of external finance agents can raise at the right cost. Thus, our paper suggests that financial development could lead to higher fluctuations in investment (and prices). Even if financial development is desirable for other reasons, the potential associated increase in the extent of fluctuations could become an explicit policy concern.

4.2 Implications from the general model

The analysis in section 3 shows that, if we bring together quantity constraints and effects of financial constraints on the cost of external funds, we will again derive ambiguous implications for investment-cash flow sensitivities. The world described in section 3 is a world in which there is a "pecking order" in the use of external finance. Firms exhaust their collateralized debt capacity first, because it is the cheapest way they can raise external funds. Then, they raise the balance at higher costs. Both the amount they can raise at the cheaper price, and the marginal costs of increasing funds above this limit, are affected by the degree of financial constraints.

It is no longer the case that investment-cash flow sensitivities have the U-shape described in section 2, because we bring back the effects of changes in the cost of non-rationed funds into the picture. This is true even if marginal costs and the marginal productivity of investment are linear, or if we control for the endogeneity of financial policy.

In the linear case, investment-cash flow sensitivities will depend on the trade-off between the effect of capital market imperfections on the slope of marginal costs (the FHP effect), and the effect of imperfections on the amplification effect emphasized by Almeida. This will still be the case, even if we control for the endogeneity of financial policy.

Furthermore, if marginal costs of non-rationed funds are not linear, we bring back the ambiguous effects emphasized by KZ (which we can take care of by controlling for the endogeneity of financial policy), and an extra interaction effect (which we cannot handle). As shown above, a tightening in credit constraints shifts the capital supply curve up and to the left. This moves a constrained firm to a range the supply curve is steeper. Thus, marginal costs of non-rationed finance are more sensitive to further changes in external funds for such firms.

This suggests that it is crucial to determine if financial constraints affect primarily credit constraints on firms (that is, the availability of finance at a given cost), or the cost of external finance to different firms. Most of the previous literature has not attempted to compare the relative importance of differential costs and liquidity constraints. One exception is Japelli and Pagano (1989), in the context of household finance. They argue that the wedge between the borrowing rate in the mortgage market, and an appropriate lending rate does not appear to be a viable explanation of the cross-country differences in the financial liabilities of households. Differences among the interest rate wedges seem negligible, and there is no clear relation between lending volumes and wedges. On the other hand, cross-country differences in liquidity constraints on households seem to have a very strong effect on household balance sheets. This is consistent with the focus on quantities that we propose in this paper.

5 Conclusions and Extensions

The main point of this paper is that, when financial constraints translate into a quantity constraint on the amount of external funds which can be raised at a given cost (credit constraint), we can get precise implications for investment-cash flow sensitivities. However, the implication is *not* that investment-cash flow sensitivities increase in the degree of financial constraints. The implication of the model is that sensitivities should *decrease* with financial constraints, as long as firms are not entirely unconstrained. We get therefore a "U-shaped" curve for the relationship between sensitivities and the measure of financial constraints.

From an empirical perspective, our analysis brings both positive and negative contributions. The positive one is that we suggest a theoretically consistent way to identify the impact of financial constraints on investment behavior, at least under certain conditions. If if financial constraints affect primarily credit constraints on firms, investment-cash flow sensitivities are useful measures of financial constraints. The bad news is that our prediction about the relationship between financial constraints and sensitivities is *the opposite* as the one explored in previous empirical literature. Thus, our results cannot be used to rescue the investment-cash flow literature from the Kaplan and Zingales critique.

Existing empirical evidence (Almeida, 2000) already indicates that the effects we emphasize, and the approach we propose are relevant and useful. However, this evidence is for housing markets and housing finance. The natural extension is empirical work in the context of firm investment as well.

Our results suggest that it is crucial to determine if financial constraints affect primarily credit constraints on firms (that is, the availability of finance at a given cost), or the cost of external finance to different firms. On the other hand, more work is clearly warranted on the issue of joint effects of financial constraints. The model we worked with (just like KZ) is not derived from first principles, unlike the model in section 2. Perhaps a more precisely specified model can yield tighter predictions.

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Figure 1- Equilibrium investment

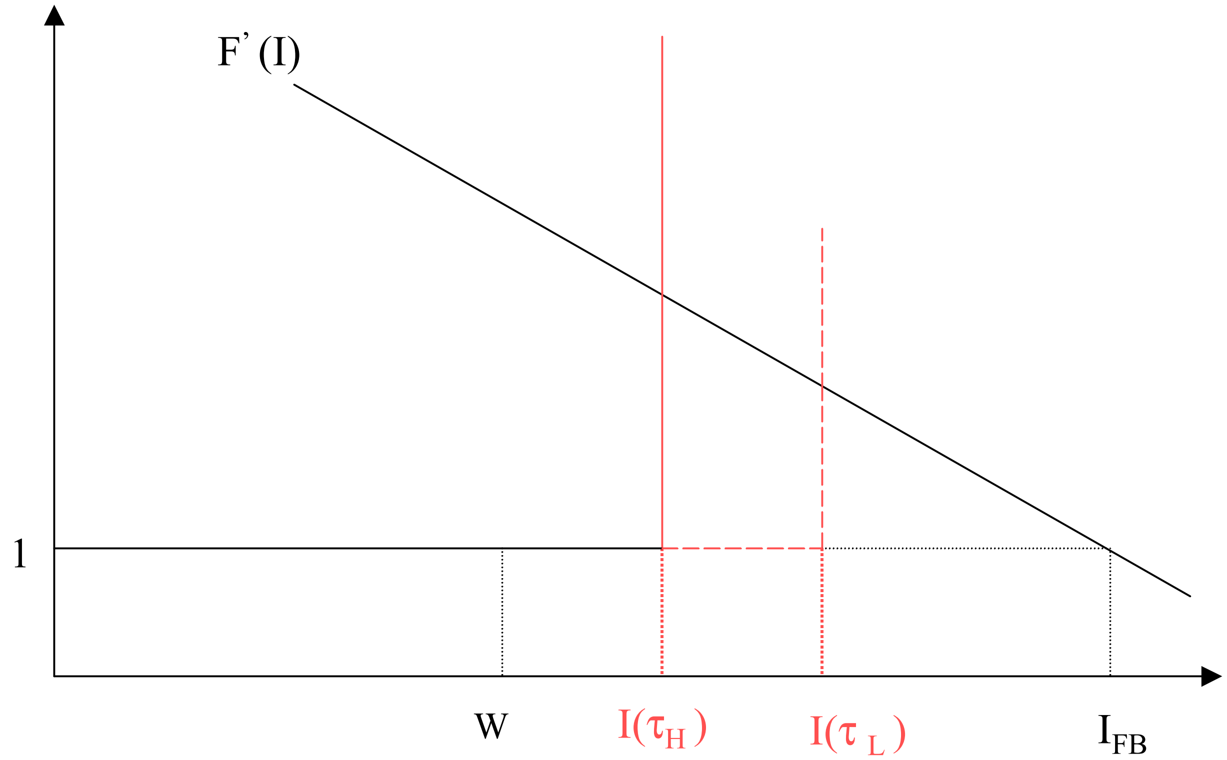

Figure 2 - Investment-cash flow sensitivity and financial constraints

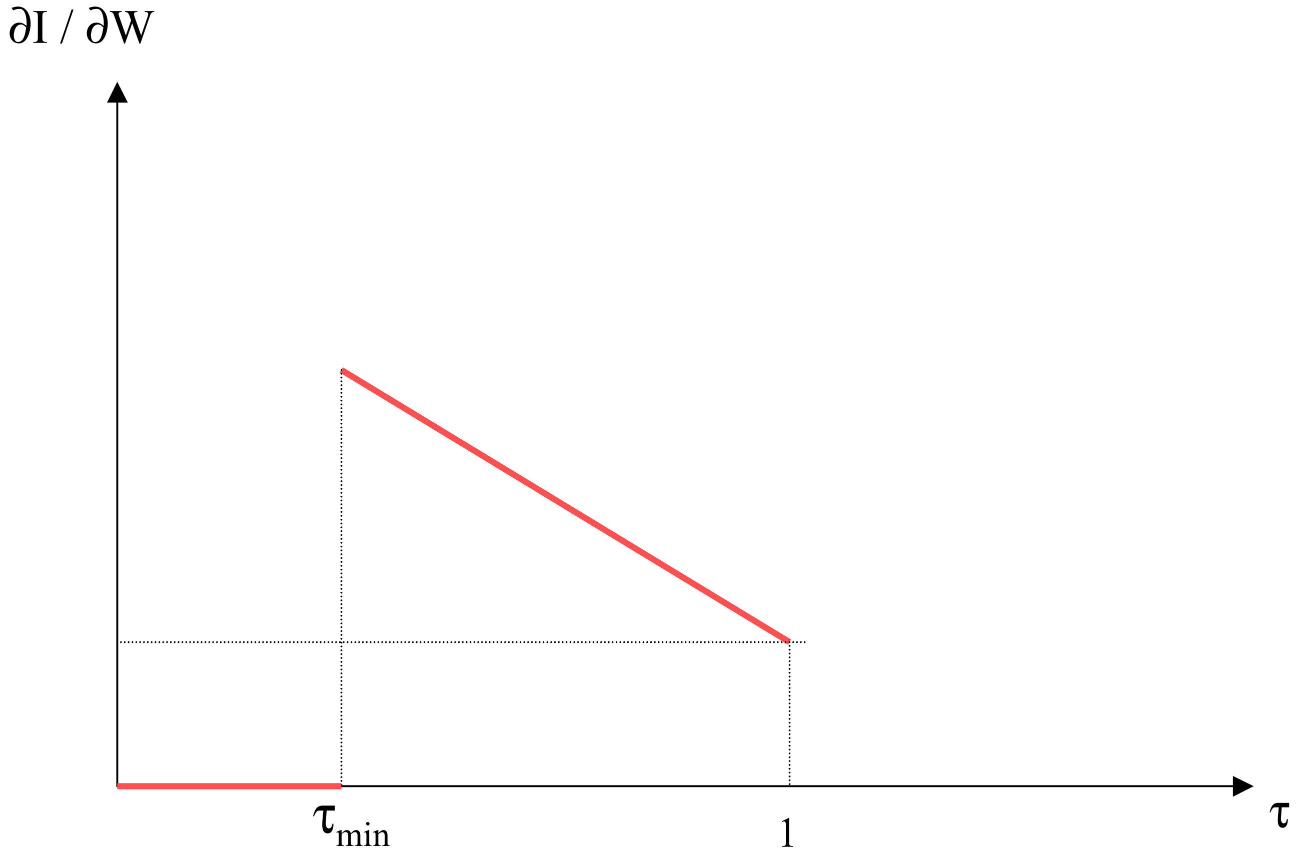

Figure 3 -Negative cash-flow shock

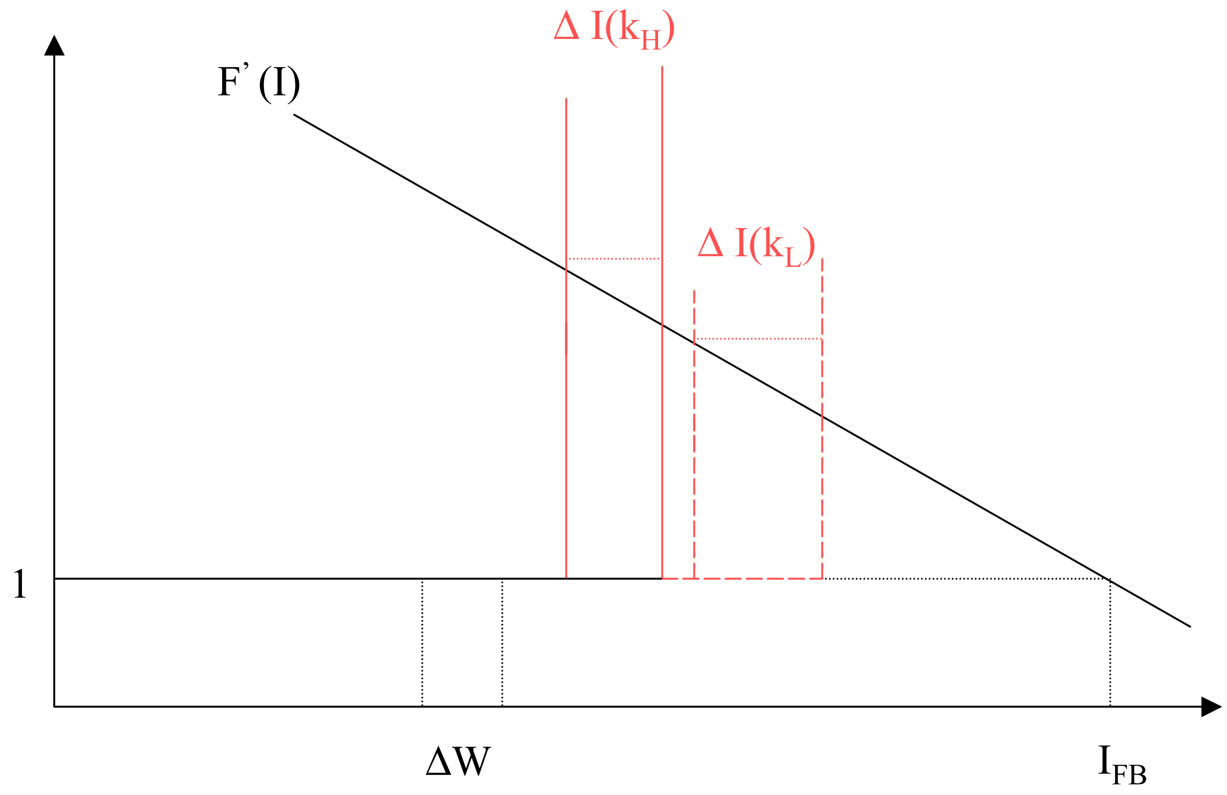

Figure 4 - Equilibrium investment in the KZ model

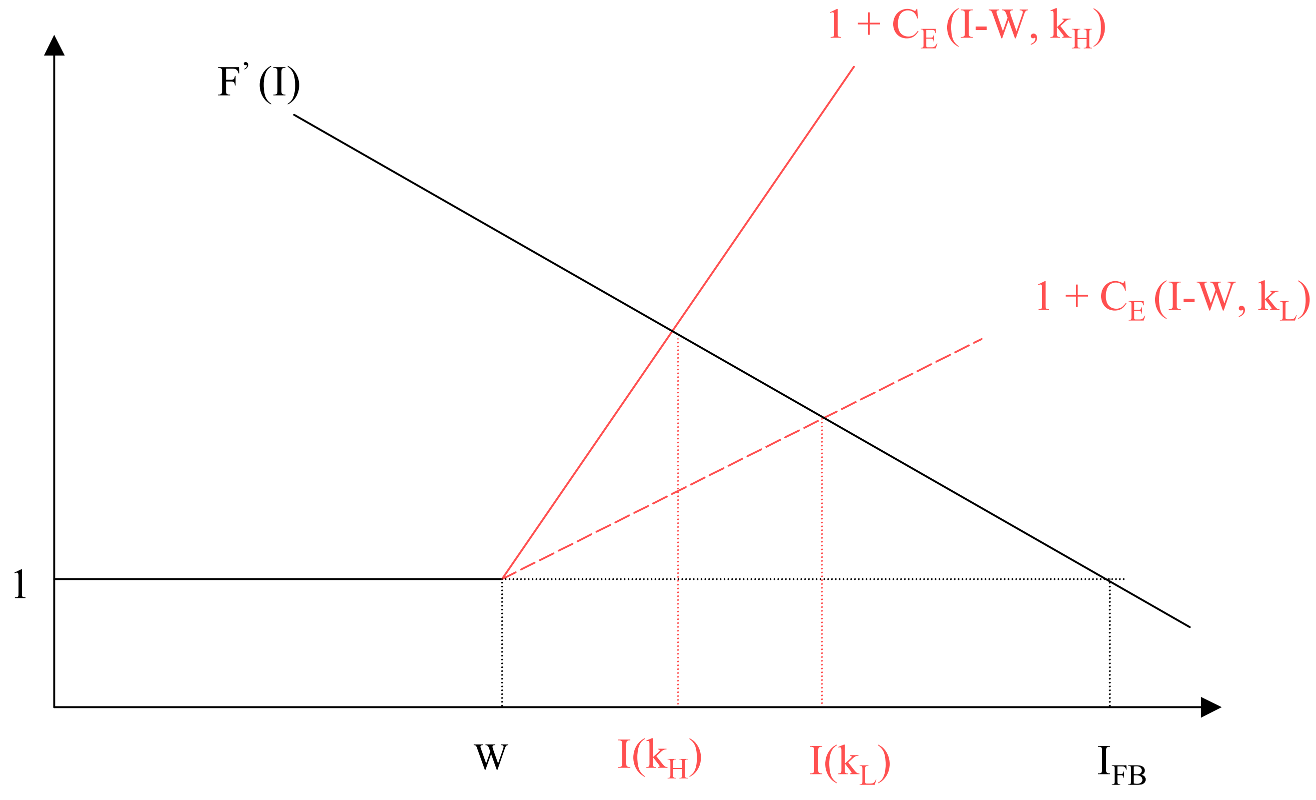

Figure 5 - Negative cash-flow shock in the KZ model

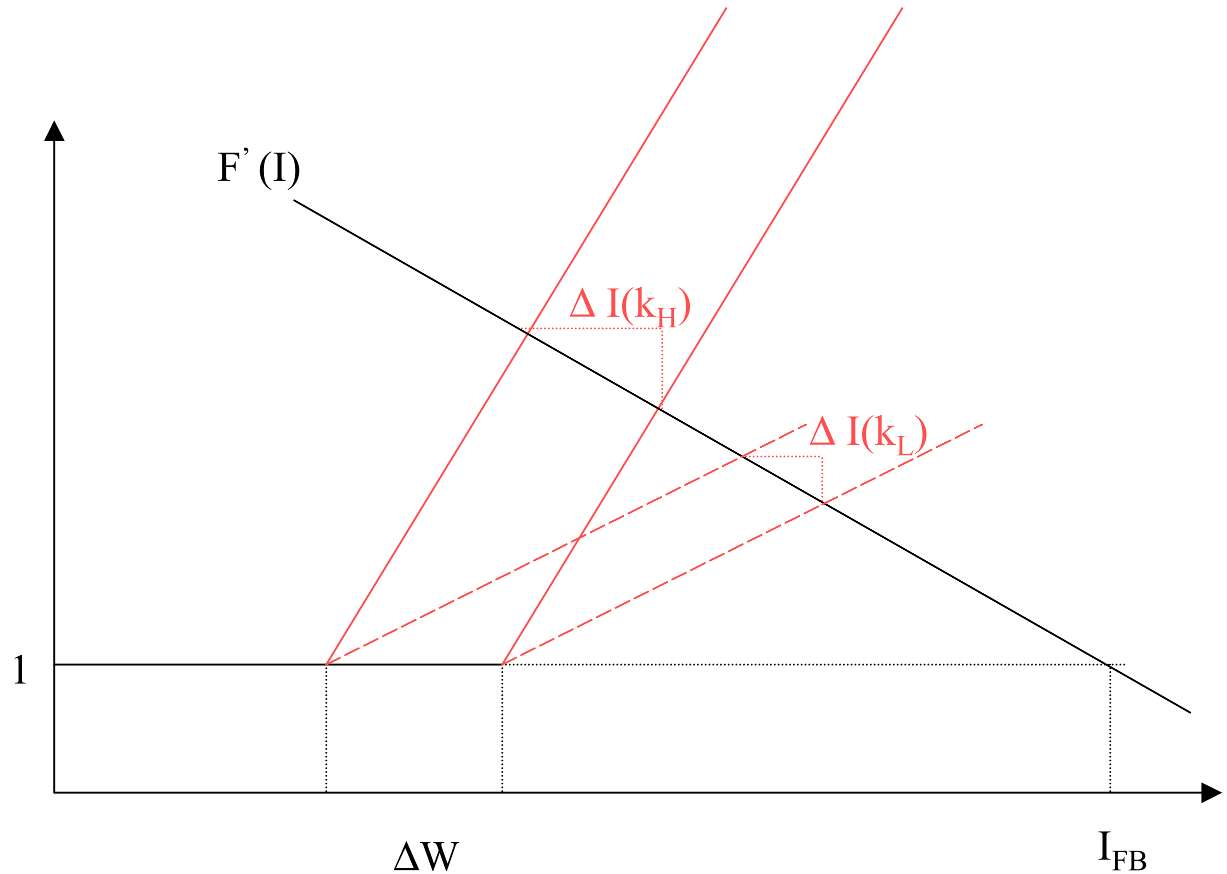

Figure 6 - Equilibrium investment in general case