NYU STERN

Salomon Center for the Study of Financial Institutions

Working Paper Series ASSET MANAGEMENT Research Group

TURNING OVER TURNOVER

K.J. Martijn Cremers
Jianping Mei

SC-AM-03-11

Turning Over Turnover

K.J. Martijn Cremers
International Center for Finance
Yale School of Management
135 Prospect Street
New Haven, CT 06520
(203) 436-0649
martijn.cremers@yale.edu

Jianping Mei
Department of Finance
Stern School of Business
New York University
44 West 4th Street
New York, NY 10012-1126
(212) 998-0354
jmei@stern.nyu.edu

September 2003

This paper originally was titled “A New Approach to the Duo-factor-model of Return and Volume”. We would like to thank Andrew Ang, KC Chan, Kalok Chan, Lewis Chan, Robert Engel, Larry Harris, Joel Hasbrouck, Anthony Lynch, Gideon Saar, Jeff Wurgler, Wei Xiong, and Yexiao Xu, as well as seminar participants at NYU, HKUST, Rutgers University, and the University of Southern Califonia for helpful discussions. We also thank Andrew Lo and Jiang Wang, who graciously provided us with their MiniCRSP data manual.

Turning Over Turnover

Abstract

The methodology of Bai and Ng (2002, 2003) for decomposing large panel data into systematic and idiosyncratic components is applied to both returns and turnover. Combining this with a GLS-based principal components approach, we demonstrate that their procedure works well for both returns and turnover despite the presence of severe heteroscedasticity and non-stationarity in turnover of individual stocks. We then test Lo and Wang's (2000) theoretical model's restriction that returns and turnover should have the same number of systematic factors. This is strongly rejected by the data, suggesting stock price and trading volume may not be *compatible* under the existing multi-factor asset pricing-trading framework. We also demonstrate that several commonly used turnover measures may understate the price impact of stock trading.

I Introduction

There is increasing interest in improving the understanding of turnover since its emergence as an important proxy for investor overconfidence. Extending this, Lo and Wang (2000, 2003) have developed a multi-factor model for turnover based on asset pricing models. Their model gives rise to a decomposition of turnover into systematic and idiosyncratic components, just like the usual return-decomposition. This has several interesting applications in finance.[1]

First, a better understanding of turnover can help provide more powerful asset pricing tests. Lo and Wang (2003) demonstrate that one can form a unique hedging portfolio by constructing turnover factors, which provides additional tests of an asset pricing model. Our paper shows how to estimate these factors using principal components, including a consistent estimate of the number of factors.

Second, decomposition of firm-level turnover into systematic and idiosyncratic components is useful for a large number of research questions. Adjusting turnover for firm-fixed effects is typically dealt with by de-trending total turnover (see, for example, Chen, Hong and Stein (2001). In this paper, we propose the systematic-idiosyncratic decomposition of turnover as an alternative approach, and provide a detailed comparison with de-trending.

Third, numerous studies have found common (that is, systematic) components in liquidity (For example: Chordia, Roll, and Subrahmanyam (1998), and Hasbrouck and Seppi (2001)). Our decomposition directly measures how much trading is driven by systematic factors and how much is due to firm-specific causes. If turnover is a proxy for information trading, we can relate idiosyncratic turnover to firm-specific news and obtain a proxy for the degree of information asymmetry across stocks. This allows us to evaluate the impact of the risk of information asymmetry and of price discovery on asset pricing.

Despite these interesting applications, studies of turnover have largely been limited to portfolios or to a small number of individual stocks. This may be due in part to

---

[1] For theoretical models of turnover see, among others, Daniel, Hirshleifer, and Subrahmanyam (1998), Hong and Stein (2003), and Scheinkman and Xiong (2003). For empirical studies see Chen, Hong, and Stein (2002), Odean (1998), Ofek and Richardson (2002), and Mei, Scheinkman, and Xiong (2003).

the difficulty of implementing conventional multi-factor estimation procedures, which results from the severe heteroscedasticity and non-stationarity found in turnover data (see Lo and Wang (2000). However, applying procedures developed by Bai and Ng (2000, 2003), we are able to consistently estimate the turnover factor model and test for non-stationarity. We also can provide a close examination of turnover by "turning over" a large panel of individual stocks.

Our study undertakes to make a number of contributions to the turnover and factor model literature.

First, we demonstrate that for estimating the required number of factors, the Bai and Ng (2002) statistics work well for returns, but not for raw turnover. Instead, we introduce a modified GLS-like procedure that we show to be effective and simple to implement. We also show how to use the Bai and Ng (2003) method to test for non-stationarity in both systematic as well firm-specific turnover components.

Second, we provide a new test of the theoretical work by Lo and Wang (2000). In particular, our empirical study uses data from a large panel of individual stocks rather than the beta-sorted portfolios they used. By exploiting the advantage of a large cross-section of individual stocks, we get around the non-stationarity issue in turnover. As our empirical work shows, the number of systematic factors in return and turnover changes dramatically when individual stocks are used instead of beta-sorted portfolios.[2]

More specifically, we test Lo and Wang's (2000) theoretical model's restriction that returns and turnover should have the same number of systematic factors. This is strongly rejected by the data, suggesting stock price and trading volume may not be *compatible* under the existing multi-factor asset pricing-trading framework.

Third, our study complements recent studies in the market microstructure literature on the common variation in liquidity or trading volume (this issue was highlighted by the LTCM debacle, when there appeared to be a world-wide "flight-to quality" and a significant drop in trading volume across many assets). Chordia, Roll, and

---

[2] Berk (2000) shows a significant drop in statistical power in asset pricing tests using firm-characteristics sorted portfolios. Also, Brennan, Chordia, and Subrahmanyam (1998) report "… inferences are extremely sensitive to the sorting criteria used for portfolio formation, so that results based on regressions using portfolio returns should be interpreted with caution."

Subrahmanyam (2000) explore cross-sectional interactions in liquidity measures using quote data. They use the market portfolio to analyze the commonality in liquidity. Hasbrouck and Seppi (2001) use a multi-factor model to characterize relationships involving returns and order flows by using the Dow Jones Industrial Average of 30 actively traded firms. These studies all use high-frequency data rather than the weekly data used in our study.

Fourth, we provide average weekly liquidity estimates similar to Pastor and Stambaugh (2003). Using idiosyncratic turnover estimated from a multi-factor turnover model, we demonstrate that several commonly used turnover measures may significantly understate the price impact of stock trading.

The paper is organized as follows. Section II introduces an approximate multi-factor model for turnover. We then present a consistent statistic developed by Bai and Ng (2002) to determine the number of factors in the factor model and discuss how the PANIC framework of Bai and Ng (2003) can be employed in testing for non-stationarity in turnover data. In section III, we provide a description of the data set, followed by some evidence on the presence of severe heteroscedasticity and non-stationarity in turnover data. Then we discuss several statistical procedures to deal with these problems and provide a decomposition of turnover into systematic and idiosyncratic components. Monte Carlo Simulations are used to confirm our results. Section IV briefly describes the duo-factor-model of Lo and Wang and provides an explicit test of their theoretical results that the number of factors is the same for excess returns and turnover. We then show that several commonly used turnover measures may understate the price impact of stock trading. Section V concludes.

II Methodology for Decomposing Turnover

A. A Multi-factor Turnover Model

Lo and Wang (2000, 2003) provide a multi-factor model for turnover.

$$\tau_{jt} = \tau_j + \delta_{j1} g_{1t} + \ldots + \delta_{jK} g_{Kt} + \xi_{jt} \tag{1}$$

where δ_{jk} is the exposure of firm j to economy–wide trading shocks g_{kt} and τ_j is a constant. Using terms common for discussing returns, we call δ_{jk} turnover betas. ξ_{jt} has mean zero and is assumed to be orthogonal to g_{kt}. In addition, we assume ξ_{jt} satisfies the regularity conditions as given in the appendix.

More concisely, we can write (1) as:

$$\tau_{j,t} - \tau_j = D_j' G_t + e_{j,t} \quad j = 1,\ldots,N; \;\; t = 1,\ldots,T \tag{2}$$

B. The Bai and Ng (2002) Statistic

We first estimate the common factors in (1) using the asymptotic principal component method of Connor and Korajczyk (1988). Because the true number of factors K is unknown, we start with an arbitrary number k_{max} ($k_{max} < \min(N,T)$). We estimate the k systematic factors and factor loadings that solve the following optimization problem:

$$V(k) = \min_{D^k, G^k} T^{-1} N^{-1} \sum_{t=1}^{T} \sum_{j=1}^{N} (\tau_{jt} - D_j^k G_t^k)^2 \tag{3}$$

where G^k denotes the k-vector of systematic factors and D_j^k denotes k-vector of factor loadings for firm j.

To determine the number of factors, Bai and Ng propose the following information criterion (IC):

$$\hat{K} = argmin_{\,0<k<kmax}\, IC_1(k), \tag{4}$$

where $IC(k)$ equals the measure of the goodness-of-fit $V(k)$ as used in (3) plus a second term that serves as an adjustment for the increase in the degrees of freedom that result from increasing k:

$$IC(k) = \log\left\{V\left(k, \hat{G}^k\right)\right\} + k \cdot \left(\frac{N+T}{NT}\right) ln\left(\frac{NT}{N+T}\right). \tag{5}$$

Bai and Ng show that $\hat{K}$, the value of k that minimizes the *IC(k)* statistic in (5), is a consistent estimate for the number of factors in the factor model.[3]

Intuitively, the estimation procedure treats the determination of the number of factors as a model selection problem. As a result, the selection criteria depend on the usual trade-off between goodness-of-fit and parsimony (or model size). The difference here is that we not only take the sample size in both the cross-section and the time-series dimensions into consideration, but also the fact that the factors are not observed.

There are several distinctive advantages of the Bai and Ng approach compared to the methodology of Connor and Korajczyk (1993). First, Bai and Ng do not impose any restrictions between N and T, allowing for both large N and large T. Second, the results hold under heteroscedasticity in *both* the time and the cross-section dimensions. Third, the results also hold under *both* weak serial dependence and cross-section dependence. In addition, the model selection procedure is easy to implement. The conditions under which the consistency of $\hat{K}$ holds are given in the appendix.[4]

C. The Bai and Ng (2001) PANIC Test for Non-stationarity

Bai and Ng (2001) develop a methodology to detect whether there is non-stationarity in the systematic or idiosyncratic components, or both. They make use of the factor structure of a large panel data set, crucially showing that the components can be consistently estimated using the panel even in cases where individual (non-stationary) series would produce spurious regressions. In particular, they show that common stochastic trends can be consistently estimated by the principal components method, regardless of whether the idiosyncratic series contain unit roots. Similarly, their proposed unit root test of the idiosyncratic series is valid regardless of whether any of the systematic factors contain a unit root.

---

[3] They also proposed two other asymptotically equivalent statistics. Our empirical study finds these give similar results in our balanced panel, but the IC criterion has the best simulation results. Results are available on request.

[4] Jones (2001) introduces a heteroscedastic factor analysis (HFA) approach to extract factors but he does not provide a test on the number of factors, as Connor and Korajczyk (1993) do.

Using their approach, we start by testing whether the systematic turnover factors or the individual-firm turnover series contain a unit root. Differenced systematic factors are estimated using the differenced, standardized turnover panel, after which these factors are transformed into levels. Bai and Ng show that the standard Dickey-Fuller (1979) test statistics for testing for a unit root – with either a constant only or with a constant plus a linear time trend – in these systematic factors or in decomposed idiosyncratic turnover have the same limiting distribution as the regular test statistics for observed data series, as derived in Fuller (1976). As a result, the 5% asymptotic critical value of the Dickey-Fuller unit root test of -2.86 applies.

III. Dealing with Severe Heteroscedasticity and Non-stationarity in Turnover

A. Data Description

Following Lo and Wang (2000), we use the CRSP Daily File to construct weekly turnover series for individual NYSE and AMEX stocks from July 1967 to December 2001. The choice of a weekly horizon makes our results comparable to Lo and Wang and is a compromise between maximizing sample size while minimizing the day-to-day volume and return fluctuations that have less direct economic relevance.[5]

Because our focus is the implications of portfolio theory for volume behavior, we limit our attention to ordinary common shares on the NYSE and AMEX (CRSP share codes 10 and 11 only), omitting ADRs, REITs, closed-end funds, and others whose turnover may be difficult to interpret. We also omit NASDAQ stocks because of market structure differences relative to the NYSE and AMEX. Like Lo and Wang, we discard firms that have no or problematic turnover data.

Panel A of Table 1 presents various summary statistics of our sample, including the number of securities in each sample, the number of securities with no missing observations in turnover, as well as number of firms with two types of problematic data. The first type includes firms that have constant turnover in the time period. The second are those firms that have likely data entry problems as evidenced by an unusual large

[5] In addition to the weekly data, we conducted a parallel study of turnover by using monthly data. The results, briefly discussed in the next section, are similar.

standard deviation (specifically, 10 times the average standard deviation. See also the discussion on the so-called Z-flag in Lo and Wang. As they argue, such large standard deviations probably indicate data errors).

Table 1 Panel B reports the distributional characteristics of return and turnover volatility. Excess returns tend to be more volatile than turnover for individual stocks. This result is similar to what Lo and Wang obtained for portfolios. A close examination of the cross-sectional variation in both return and turnover indicates that the turnover distribution is significantly more extreme. After scaling each firm-level series by its standard deviation, turnover generally displays much larger skewness as well as kurtosis. The only exception is 1987-91, when the October 1987 market crash gave rise to much higher skewness and kurtosis for return volatility.

To develop a sense for cross-sectional differences in turnover, Figure 1 provides a graphic depiction of turnover for value-weighted size-sorted decile portfolios. For simplicity, we report only those for the first (smallest market capitalization) and tenth (largest) decile portfolios. There are several interesting patterns.

First, turnover for small stocks increased after 1975 when fixed commissions were abolished, although it took some time to regain and stay above the levels prior to the 1973-74 bear market.

Second, the turnover of the tenth decile portfolio, which consists of the largest stocks, rises sharply during the mid-1960s, then falls suddenly in the late of 1960s and remains relatively low in the remaining sample periods.

Third, turnovers for small stocks seem to display a strong presence of non-stationarity. In particular, there appears to be a strong trend component in turnover, which we will examine in great detail in section IV. However, by performing several (augmented) Dickey-Fuller tests on individual stock turnover, we find no evidence of unit roots among any of the stocks in our sample for all of the seven time periods. That is, for all of the firm-level turnover time series, we could reject a unit root with and without a time trend present. Further, unit root tests on decomposed turnover confirm that neither the systematic factors nor any of the idiosyncratic turnover components have a unit root in any time period. As a result, we will not first-difference the data (First-differencing the turnover data tends to increase noise in the idiosyncratic term when there is no unit root in turnover, and as a result can significantly increase estimation error).

The strong presence of both heteroscedasticity and time trends in turnover considerably affect the estimation of the number of systematic turnover factors.

Table 2 presents estimates of the number of factors for each sample period using raw and standardized turnover, as well as their detrended series. The standardization is conducted by first de-meaning and then normalizing each individual stock turnover series by its standard deviation over the sample period. The number of factors reported here corresponds to the number that minimizes the information criteria (IC) statistic developed by Bai and Ng. Specifically, in order to determine the number of systematic factors in turnover, we compute a goodness-of-fit statistic, IC, conditional on a wide range of included numbers of factors. For example, comparing $IC(k)$ for $k = 1,2,\ldots,20$ indicates that $k = 5$ provides the minimum $IC(k)$ for standardized turnover for 1967-71. This indicates that there may be five systematic factors for standardized turnover during the first sample period.

As can be seen from Table 2, the estimates of the number of factors for turnover are very sensitive to the standardization of the data, and somewhat sensitive to detrending. There seem to be an extraordinary large number of factors in raw turnover data. For example, there may be 16 systematic factors during 1967-71. In addition, detrending should not reduce the number of factors by more than one factor, but the results for raw turnover suggest otherwise.

In marked contrast, the estimates of the number of factors for excess returns are robust to standardization. Since standardization should not change the number of factors found, we conclude that, due to the presence of severe heteroscedasticity in turnover, the Bai-Ng statistics do not work well for raw turnover.

B. A panel approach to Trend in Turnover and a GLS solution to Severe Heteroscedasticity

The main reason for the failure of the Bai and Ng (2002) procedure is the presence of severe heteroscedasticity as documented in raw turnover. Using raw turnover essentially gives the stocks with enormous swings in turnover a lot more weight in the sum-ofsquared residuals in equation (3). This is mitigated by the standardization of raw turnover. This in effect amounts to using generalized least squares (GLS) rather than OLS in the turnover regression of equation (2).

As Table 2 shows, the use of standardized turnover leads to a large drop in the number of factors. For example, for 1967-71 the estimated number of factors drops from 16 for raw turnover to 5 for standardized turnover. Detrending standardized turnover does not result in a similar sharp drop in the number of systematic factors. In the next section, we will use Monte Carlo simulations to demonstrate that the principle components approach of Connor and Korajczyk, combined with the Bai-Ng statistics, has good small-sample properties for standardized turnover and for raw or standardized returns, but not for raw turnover.

The presence of possible time trends in turnover could affect the estimation of turnover factors. This is because, in the presence of a time trend, the time series variance-covariance matrix for turnover among stocks or portfolios, $var(\tau_i,\tau_j)$, is not well defined. As a result, in this case the conventional principal components approach based on $var(\tau_i,\tau_j)$ may not obtain consistent factor estimates. This is an issue Lo and Wang (2000) did not address. We get around this by taking advantage of the large cross-section of individual stocks. Rather than using the variance-covariance matrix of turnover *among portfolios,* we rely on the variance-covariance matrix of turnover *over different time periods*. In other words, we apply a principal-component approach to $var(\tau_t,\tau_s)$, where

$$\mathrm{Var}(\tau_t,\tau_\mathrm{s}) = N^{-1}\sum_{i=1}^{N}(\tau_{\mathrm{it}}-\bar{\tau}_t)(\tau_{\mathrm{is}}-\bar{\tau}_s), \text{ and } \bar{\tau}_s = N^{-1}\sum_{i=1}^{N}\tau_{\mathrm{it}} .$$

$Var(\tau_t,\tau_s)$ is well defined for any give time period t and s as long as the cross-sectional mean and variance for turnover exist. Intuitively, $var(\tau_t,\tau_s)$ depends on N-consistency rather than T-consistency. This implies τ_{jt} could have serial correlation, as well as time-varying mean and volatility. The factor estimates could still be consistent as long as the data satisfy some necessary moment conditions (for details, see Bai and Ng (2002), and the appendix).

C. The Number of Factors in Turnover

Table 3 provides the results of the test of the number of factors in standardized turnover. We report the incremental proportions of the explained variation (that is, the average R^2) from regressing the individual firm turnover series on 1 to 10 turnover factors. The first principal component of turnover typically explains between 6.5% and 15.0% of the variation of the standardized turnover. This is quite different from Lo and Wang (2000), who use turnover from broadly diversified portfolios and find their first principal component typically explains over 70% (and sometimes as high as 86%) of the variation in turnover. Further examination of our results suggests that the fourth and fifth components still explain a fair amount of turnover variation. For example, the fifth component explains 1.95% of variation for 1967-71.

The IC procedure selects a five-factor model for the first sample period, which is also reported in Table 3. It is reassuring to see that the number of factors identified by the IC statistic closely corresponds with the eigenvalues of the principal components. The eigenvalues of the statistically significant turnover factors typically exceed 1.95%.

Our result of four or five factors in turnover (as also reported in table 2) is quite different from the results reported in Lo and Wang, who find only one or two significant systematic factors, although without formally testing for the number of factors. This difference seems due mainly to the fact they use factors extracted from 10 beta-sorted portfolios, while we use a large cross-section of individual stocks. As pointed out by Shukla and Trzcinka (1990), beta-sorted portfolios tend to mask some cross-section differences in betas to systematic factors. As a result, the principal-components approach based on beta-sorted portfolios is likely to be biased towards finding a smaller number of factors.

While our procedure does not specifically identify what the factors are exactly, it does provide some guidance for equilibrium model construction. Our results suggest that, while the two-factor model of Lo and Wang (2003), which consists of a market factor and a hedging factor, provides a reasonable description of portfolio turnover, they still leave out a few systematic factors. This may help explain why their model does not fully capture the cross-section of average turnover.

Table 3 also reports the average R^2 of regressing individual stock turnover on the selected systematic turnover factors for each sample period. For example, for 1967-71 a five-factor model explains on average about 25.5% of variation in turnover of individual stocks, where the R^2 has a cross-sectional standard deviation of 12.9%. Comparing Table 3 with Table 7 for returns, we find that turnover factors are just as important for explaining the time variation of turnover across individual stocks as return factors are for individual stock returns.

In comparison to empirical results about trading volume found in market microstructure studies by Chordia, Roll and Subrahmanyam (2000), we find a *stronger* presence of commonality in turnover for most sample periods. Chordia et al use transaction data from a sample of 1,169 stocks in 1992. They examine the common movement in market depth using value- and equal- weight indices, and find the mean R^2 to be less than 2%. Hasbrouck and Seppi (2001) use order flow data from the 30 Dow stocks during 1994 to study the common factors in stock prices and liquidity. They find the first three common factors explain about 20% of the variation in order flows. However, they do not provide an explicit test for the number of factors in their factor model.

Because trading volume determines transaction costs in the stock market, our results imply that trading volume may have a systematic impact on after-cost returns. Therefore, liquidity risk associated with trading volume could be a systematic risk factor that is priced. This is consistent with the empirical results of Amihud (2001) and Pastor and Stambaugh (2003), who find that liquidity is an important risk factor in financial markets.

D. Monte Carlo Simulations

While Bai and Ng did a simulation study on the small-sample properties of their IC statistics, they used a general data generating processes (DGP) that is not calibrated to typical stock return and turnover data. In this section, we provide a simulation study to demonstrate that the IC estimates have good small-sample properties for standardized turnover. The DGP used follows Jones (2001) and is designed to mimic the actual data as closely as possible. Thus, rather than simulating factors under some arbitrary assumption, bootstrapped samples of factor and beta estimates from the actual data are used.

Specifically, conditional on a model with K systematic factors and given estimates of the $T \times K$ matrix $\mathbf{G}$ of factor realizations, we sample (with replacement) T rows of $\mathbf{G}$ to use as the true factors in the simulations. Let $\mathbf{G}_i$ denote the ith bootstrap draw of the factor matrix. The factor betas assumed in the DGP are bootstrapped samples of the least-squares estimates of the betas from the actual data. Denoting $\mathbf{D}$ as the $N \times K$ matrix of OLS estimates of the factor betas from the real data, we follow Jones by drawing with replacement N rows of the $\mathbf{D}$ matrix to use as the true betas in the simulations. We then draw the corresponding elements of the $N \times N$ diagonal matrix $\mathbf{\Sigma}$, whose (j,j) element is the unconditional sample variance of the residual of stock j. We denote $\mathbf{D}_i$ the ith bootstrap draw of the $\mathbf{D}$ matrix and $\mathbf{\Sigma}_i$ the corresponding draw of $\mathbf{\Sigma}$. Finally, the $N \times T$ matrix of simulated turnover Γ_i is generated by

$$\Gamma_i = D_i G_i + \Pi_i * E_i \tag{6}$$

where Π_i is the Cholesky-decomposition factor of $\mathbf{\Sigma}_i$ and $\mathbf{E}_i$ is an $N \times T$ matrix of i.i.d. standard normally distributed residuals. As a result, the systematic factors will (on average across bootstraps) explain the same amount of variation in the simulated turnover series as the actual factors do for the actual data.

Table 4 presents the frequency of the number of factors estimated for standardized turnover data over 500 simulations. Conditional on the number of factors found in Table 3, each simulation involves the draw of a set of $N \times T$ individual turnover data for the corresponding sample period. For example, for 1997-2001 this involves 500 draws of a 1385×252 panel of firm-level turnover.

As the first row of the table shows, if the true number of factors is 5, the IC criterion finds the right number of factors in 98.6% of the simulations, using parameters calibrated to resemble the data in the 1967-71 sample period. The mean of the estimated number of factors equals 4.99. It is worth noting that the accuracy of the IC approach depends on N and T: as we increase the number of companies used in the sample or the length of the sample period, accuracy tends to improve.

To illustrate the importance of standardizing turnover when estimating the number of required factors, we compare these small-sample properties to those for raw turnover.

Table 5 presents the frequency on the number of factors estimated using raw turnover. As the first row shows, if the true number of factors is 16, the IC criterion finds the right number of factors only in 1% of the simulations. The mean of the estimated number of factors is 13.4, showing a large downward bias compared to the true number. Thus, we conclude that, despite the presence of severe heteroscedasticity and the presence of time trends in turnover, the Bai and Ng (2002) procedure has excellent small-sample properties for standardized turnover, but not for raw turnover.

E. Understanding The Time Series Properties of Turnover Components

An important question in the study of turnover is whether there is a systematic time trend. Some evidence is in table 2, which presents the number of systematic turnover factors for both the standardized data as well as the first detrended and then standardized data. If there is indeed a time trend in turnover, we expect the number of factors to be affected by detrending. Detrending could reduce the number of systematic factors by one if one of the systematic factors is a pure time trend. This is the case in four of seven time periods; in the other three, the number of factors is not affected by detrending.

Next, we estimate whether there is a time trend in the factors extracted from the standardized (but not detrended) panel by regressing each factor on a constant, the lagged factor, and a time trend.

Table 6 panel A presents the results: in all time periods the majority of the systematic factors have a statistically significant time trend, with different factors having opposite signs for the same time period. These regressions include the lagged factor itself to ensure that the time trends are not merely an artifact of the large first-order autocorrelation.

In Table 6 panel B the pervasiveness of time trends in turnover is made clear. When regressing raw turnover on a constant and a time trend for each firm separately, we find a statistically significant time trend in 47% (in 1982-86) to 69% (in 1991-96) of firms.

Finally, taking out the systematic factors effectively removes about all occurrences of time trends. When regressing each firm's idiosyncratic turnover on a constant and a time trend, in five of seven time periods all firms have statistically insignificant trend coefficients. In the other two periods only 10% and 7% of firms show evidence of a statistically significant time trend in idiosyncratic turnover, using the same regression.

This allows us to get around a trend-related complication in turnover data, which is the presence of strong autocorrelation. Lo and Wang (2000) show that both the weekly equally weighted and value-weighted turnover indices display strong positive autocorrelation after linear, log-linear, linear-quadratic, and seasonal detrending. For example, the 10th autocorrelation for the value-weighted index remains at a high 55.8% after a seasonal detrending using the Gallant, Rossi, and Tauchen (GRT) method. Lo and Wang also show that detrending using moving average, first differencing, or kernel regressions all introduce large negative autocorrelations at various lag length.

Figure 2a displays the cross-sectional average of autocorrelation for raw turnover of individual stocks, from ρ_1 to ρ_{10}. For comparison, we also report the autocorrelation of (raw and GRT-detrended) turnover of the equally weighted index by Lo and Wang (they show that results for linear and log-linear detrending were similar but less successful than GRT detrending in removing persistence). The average autocorrelations of raw turnover of individual stocks display persistence similar to that of the index, but much smaller in magnitude. Linear detrending removes some of the autocorrelation, but significant autocorrelation remains (10%) even after the 7th lag. Furthermore, two popular approaches of removing market turnover (using 'excess turnover' or 'idiosyncratic (VW) turnover' computed by fitting a market model using VW turnover as the market turnover factor, as suggested by Lo and Wang) help little and may actually worsen the autocorrelation patterns of individual turnover series.

Removing the systematic components, however, significantly reduces autocorrelation in the idiosyncratic turnover series. For example, the autocorrelation drops to 5% after the 5th lag. There are similar results when we examine the average of the absolute autocorrelation, given in Figure 2b (the same results for autocorrelation also hold for other time periods as well, and are available on request). There is little difference between average autocorrelation and average absolute autocorrelation because autocorrelations are mostly positive for turnover. Thus, shocks to firm level (i.e., idiosyncratic) turnover die out in four weeks or so, much *faster* than what is suggested by the strong persistence at the index level.

It is worth noting that Jones (2001) introduces a heteroscedastic factor analysis (HFA) for extracting factors that allows for time-varying volatility in returns. While his simulation shows that an HFA may sometimes improve the accuracy of factor estimates, the methodology depends on the strong assumption that the idiosyncratic terms are uncorrelated over time. As shown in Figure 2, this assumption is seriously violated for turnover data. While the principle components approach of Connor and Korajczyk (1993) may not be as accurate as HFA in small samples, Bai and Ng have shown it is nonetheless consistent in the presence of autocorrelation and heteroscedasticity. The simulation results presented in this paper also show that the Connor-Korajczyk approach is quite accurate in estimating the number of factors in turnover. In our return application we also used this approach, as there is also strong evidence of return autocorrelation at the firm level, which has contributed to profits in momentum trading strategies (see, for example, Conrad, Hammed, and Niden (1994)).[6]

IV. Testing the Duo Factor Model of Lo and Wang

Although turnover data has long been available, researchers in finance have concentrated on "asset pricing" while paying scant attention to "asset quantity". In their seminal paper, Lo and Wang (2000) attempt to address this imbalance by deriving theoretically the relationship between return and turnover. Lo and Wang (2003) further establish a theoretical link by modeling heterogeneous investors who hedge market risk and changing market conditions by trading a market portfolio and a hedging portfolio. Using weekly data from various portfolios, they tried to empirically identify the hedging portfolio using volume data. They found that the return of the hedging portfolio does seem to provide the best predictor of future market returns, but the model is less successful in determining the cross-section of asset returns. Below, we provide a new test of Lo and Wang (2000) and shed new light on the results of Lo and Wang (2003).

A. The Duo Factor Model of Lo and Wang (2000)

[6] It would be interesting to compare the accuracy of extracted factors of the HFA and the Connor-Korajczyk approaches while allowing for serial correlation in idiosyncratic return and turnover. However,

Following Lo and Wang, we assume that returns are generated by the following approximate K'-factor model:[7]

$$R_{jt} = E_t(R_{jt}) + f_{1t}\beta_{j1} + \ldots + f_{K't}\beta_{jK'} + e_{jt} \quad j = 1,\ldots,N;\ t = 1,\ldots,T. \qquad (7)$$

where $f_t' = (f_{1t},\ldots,f_{K't})$ is a vector of unobservable pervasive shocks, $(\beta_{j1},\ldots,\beta_{jK'})$ is a vector of factor loadings that are constant over the sample period, and e_{jt} represents an idiosyncratic risk specific to asset j at time t. We also assume e_{jt} has mean zero and is orthogonal to f_{kt}. As discussed in Chamberlain (1983), the above economy implies the following linear pricing relationship if there exist K well-diversified portfolios:[8]

$$E_t(R_{jt}) = r_{ft} + \lambda_{1t}\beta_{j1} + \ldots + \lambda_{K't}\beta_{jK'} \qquad (8)$$

where $(\lambda_{1t},\ldots,\lambda_{K't})$ is a vector of risk premiums corresponding to the pervasive shocks $(f_{1t},\ldots,f_{K't})$, and r_{ft} is the return on a riskless asset.

Under the presence of K' well-diversified portfolios, Chamberlain (1983) shows that the above asset-pricing model satisfies K-fund separation. Under the assumptions that these K' portfolios are constant over time and the amount of trading in them is small for all investors, Lo and Wang derive the proposition that the turnover of each stock has an approximate K'-factor structure like equation (7).

In particular, they derive an easily testable hypothesis about the duo-factor model (equations (1) and (7)) that the two models should have exactly the same number of factors, that is $K = K'$. The reasoning is that in equilibrium well-diversified investors hold the K' separating funds and just trade them to hedge the systematic risk mimicked by the K' factor portfolios. As a result, systematic turnover reflects the trading in these K'-funds in the market. Therefore, turnover also has a K'-factor structure, just like excess returns.

---

that is beyond the scope of this paper.

[7] To avoid confusion with the K-factor turnover model, we will use K' to indicate the number of factors in the return model.

[8] Connor (1984) derived the same result under the condition that the supplies of the assets are well diversified. To derive the consistency result of the Bai-Ng statistic for the number of factors in the return model, some additional regularity conditions are imposed. These are provided in the appendix.

Although Lo and Wang do not formally address the issue of idiosyncratic turnover, one way to justify its existence is the presence of noise traders in the economy who hold non-diversified portfolios. They trade on either information or speculation but their trades affect neither asset pricing nor the systematic turnover. In this case, by identifying idiosyncratic turnover at the firm level, one may learn about trading related to firm-specific information as well as firm-specific speculation (see, for example, Michaely and Vila (1996) on trading volume in the presence of private valuation).

B. Understanding Returns

Table 7 provides the results of the test of the number of factors in excess returns. The firms used in the return sample are the same as those used in the turnover sample. To fill the missing observation in the return sample we use a standard EM algorithm. Similar to turnover, the first principal component of returns typically explains between 11% and 26% of the variation of excess returns, while the second and third components each explain about 2%. For example, for 1967-71 a two-factor model explains on average 23.4% (21.5% by the first factor) of the variation in weekly excess returns of individual stocks, with a standard deviation of R^2 of 8.5%. This is quite different from Lo and Wang, who use returns from broadly diversified portfolios. Their first principal component typically explains over 70% of the variation in the portfolio returns.

Table 7 documents a significant drop in average R^2 for excess returns for the last two sample periods, suggesting a significant increase in contribution of idiosyncratic risk to total return variation. This result is consistent with the result of Campbell, Lettau, Malkiel, and Xu (2001), who find a noticeable increase in firm-level volatility relative to market volatility in recent years. However, our results for excess returns suggest that they may underestimate the importance of systematic factors in returns, as the R^2 obtained through their market model appears to be substantially below the average R^2 found in our study under a multi-factor model. The contribution of the market to total volatility was 13.4% during the 1988-97 period covered by Campbell et al. In contrast, the average R^2 in our study was 28.5% for 1987-91 and 14.3% for 1992-96. Besides the different time periods, differences in weighting methods - Campbell et al use value-weighting, while we use equal weighting in Table 2 - may account for some of the difference in results.

Table 8 provides the results of a simulation study, which demonstrate that the IC estimates have good small-sample properties for raw as well standardized returns. The data generating processes are the same as previously described for turnover, and are designed to mimic the actual data as closely as possible. As Panel A shows, if the true number of factors is two, the IC criterion finds the right number of factors in 91% of the simulations, using parameters calibrated to resemble standardized returns in the 1967-71 time period. The mean of the estimated number of factors is 1.91, which shows a slight downward bias compared to the true number.

Table 8 panel B presents the frequency on the number of factors estimated using raw returns. For example, for the first time period, if the true number of factors is two, the IC criterion finds the right number of factors in 99% of the simulations. As a result, we conclude that the Bai and Ng (2002) procedure of estimating the number of systematic factors works well for raw as well as standardized returns.

The discovery of only two or three pervasive factors in the economy has interesting implications for unconditional asset pricing tests. Studies by Fama and French (1993), Jegadeesh and Titman (1993), and Pastor and Stambaugh (2003) report the presence of market, size, book-to-market, momentum, and liquidity factors. While our study has found only two or three systematic return factors, the discrepancy could come from our exclusion of NASDAQ stocks. It is also possible that some of these factor premiums might actually come from mispricing. However, further investigation of such issues is beyond the scope of this paper.

C. Is There Compatibility of Price and Trading Volume in US Stock Data?

Since Tables 3 and 7 document some apparent differences in the number of return and turnover factors, we will now formerly examine the Lo-Wang duo-factor model's hypothesis that the number of return and turnover factors should be equal. Table 9 presents the Type I and Type II error estimates of a formal test. The error estimates are based on 500 simulations for each time period, where each simulation involves the draw of a set of $N \times T$ individual return and turnover data.

For the type I error estimates, we set the true numbers of return and turnover factors equal to three. This is chosen because it is the highest number of return factors found in the seven studied time periods. For type II we set the true numbers of return and turnover factors equal to those found in the data. Therefore, the true difference is *K-K'*, which equals the difference of the numbers of return and turnover factors as reported in Tables 3 and 7. To maintain the correlation found in the data between excess return and turnover, an elaborate sampling scheme is used to mimic the actual data as closely as possible. (See Appendix II for details).

Overall, our simulation study indicates a clear and unambiguous rejection of the null hypothesis that there are same numbers of systematic factors in returns and turnover in all time periods. In all seven time periods, if the simulated number of factors are the same for return and turnover, then the probability that the IC criterion finds a difference equal to those as estimated in the actual data is 0%.

Table 9 Panel B show that the IC criterion has almost no Type II errors conditional on the actual number of factors found in the data. The probability of accepting the null of same factors while it is not true is zero for all time periods.

The rejection of the "same number of factors" restriction is not surprising, as the turnover factor model was derived based on K-fund separation, implying common mimicking factor portfolios held by all investors. To the extent that investors use private information to speculate on small or internet stocks, this could lead to a violation of K-fund separation and thus to the violation of the turnover factor model. For example, Llorente, Michaely, Saar, and Wang (2002) find that small firms tend to have high trading volume associated with asymmetric information.

Another possible explanation could be sample selection. Since our sample excludes bonds and NASDAQ stocks, our return sample may not be able to reflect all systematic risks in the economy. For example, Fama and French (1993) find that, with stocks, only three factors seem sufficient to explain their cross-section, but five are needed when bonds are included in asset pricing studies. To the extent that new technology and changing interest rates may have a systematic impact on the return of assets outside our sample, investors may need to rebalance their position on all assets. As a result, we may observe systematic changes in turnover but fail to detect their impact on returns in our sample.

Although our study cannot provide a definite test of the duo-factor model, the tests on the number of factors and non-stationarity outlined here provide useful tools for the analysis of return and turnover. For example, the test on the number of factors may help us understand that the failure of the two-factor model of Lo and Wang (2003) to explain the cross-section of returns could be due to missing factors.

D. Measuring the Price Impact of Stock Trading

Our results indicate that a one-factor model for turnover cannot capture the commonality for the time-series and cross-sectional variation in turnover. This calls into question the common practice of estimating "abnormal" volume by using an event-study style "market model". (See for example Brennan, Chordia, and Subrahmanyam (1997), Stickel and Verrecchia (1994), Tkac (1996), and Llorente, Michaely, Saar, and Wang (2002).)

Here, we demonstrate that failing to fully decompose turnover may lead to an underestimation of the price impact of stock trading. Following Pastor and Stambaugh (2003), we measure the price impact of stock trading by running the following regression,[9]

$$r^e_{i,t+1} = \theta_i + \phi_i r^e_{i,t} + \gamma_i sign(r^e_{i,t}) \tau^e_{i,t} + \varepsilon_{i,t+1}, \tag{9}$$

where $r^e_{i,t}$ is the (excess or otherwise) return on stock *i* and $\tau^e_{i,t}$ is a measure of the firm-specific turnover for stock *i*. Here, γ_i measures the price impact of order flow for stock *i*, constructed by using volume signed by the contemporaneous return.

Regression (9) estimates the average effect that week *t* trading has on the return in week *t+1*. Campbell, Grossman, and Wang (1993) show that a less liquid market would have a more negative γ_i due to the larger return reversal resulting from the larger price impact of trading. Intuitively, the measure of trading in (9) can be interpreted as signed

[9] We have also followed Pastor and Stambaugh (2003) by using $r_{i,t}$ rather than $r^e_{i,t}$ as the first regressor in equation (10). The results are similar.

order flow, and greater liquidity is interpreted as a weaker tendency of trading in the direction of returns in t to be followed by opposite price changes in $t+1$.

Llorente, Michaely, Saar, and Wang (2001) show that this volume-return relation may also reflect firm-level information asymmetry. While γ_i generally tends to be negative, they demonstrate it could be positive if the price impact of information trading dominates the liquidity effect. Therefore, market participants often pay close attention to the volume of trading to help distinguish portfolio-rebalancing trades from speculative trades based on private information. Following Pastor and Stambough, we simply call γ_i a measure of liquidity, where a larger (less negative) γ_i means less liquidity or a smaller price impact of trading.

To gauge the impact of different return and turnover measures on γ_i estimates, we set $r^e_{i,t}$ equal to either the excess return over the market, $r^e_{i,t} = r_{i,t} - r_{m,t}$, or to the idiosyncratic return in the multi-factor model of (7), e_{jt}. We also use five different measures of firm turnover:

(A) $\tau^e_{i,t}$ is the de-meaned raw turnover for stock i during week t,
(B) $\tau^e_{i,t} = \tau_{i,t} - \tau^{vw}_{mt}$ is turnover in excess of the value-weighted market turnover,
(C) $\tau^e_{i,t}$ is detrended turnover for stock i,
(D) $\tau^e_{i,t}$ (idio (VW)) is the residual turnover in the one-factor model of Lo and Wang (2002), with value-weighted turnover used as the market factor.
(E) $\tau^e_{i,t} = \zeta_{i,t}$ is the idiosyncratic turnover in the multi-factor model of (1).

Table 10 Panel A presents the cross-sectional average of the estimates of γ_i for the seven sample periods using excess return over the market, $r^e_{i,t} = r_{i,t} - r_{m,t}$, and five different measures of turnover. It also provides the t-tests for the hypotheses that the mean of γ_i under specifications A, B, C or D equal the mean of γ_i under E.

For idiosyncratic turnover (E), we can see that the average price impact was the smallest during 1967-71, then liquidity dropped sharply in 1972-76, but it has been improving ever since. For example, a 10% increase in weekly signed turnover would, on average, cause a 7.43% reversal in excess return $r^e_{i,t}$ over 1972-76 but only 1.38% over 1997-2001. On the other hand, a 10% increase in weekly signed turnover, would on average cause a 3.09% reversal in idiosyncratic return e_{jt} over 1972-76, but only 0.64% over 1997-2001.

If we compare the average estimates of γ_i based on E with A, B, C, and D, we find that all four alternative measures tend to provide less negative (or smaller in absolute value) estimates of γ_i, suggesting a smaller price impact. For example, the average γ_i was –0.591 under excess turnover B compared to –0.743 for idiosyncratic turnover (E) for 1971-76. The difference has a significant t-stat of –3.28. This suggests that, using market turnover to compute excess turnover may understate the price impact if one is interested in *firm-specific* trading volume. The same holds for using the residual from a market factor model (D). The reason for the smaller price impact is the fact that all four turnover measures contain portions of systematic turnover. It is not surprising that the price impact would be smaller if some trading were systematic, presumably because of risk sharing. The results for using idiosyncratic returns given in Panel B were similar, but they generally tend to be smaller in absolute value.

Therefore, we conclude that several commonly used turnover measures may significantly understate the price impact of stock trading. As a result, a multi-factor model is needed in estimating "abnormal" trading volume as well as its impact on asset prices.

O'Hara (2003) argues that microstructure effects such as liquidity and price discovery risk are important for asset pricing. While numerous studies have examined the impact of liquidity effects on asset pricing, few have examined the risks of price discovery. This is because it is hard to disentangle the two. However, as Easley et al (2002) and O'Hara (2003) pointed out, the risk of price discovery is closely related to asymmetric information.

To gauge the extent of information asymmetry among investors, it is helpful to separate trading related to public information from that related to private information. While the turnover decomposition developed in this paper only allows us to separate trading into systematic and idiosyncratic components, it is conceivable that idiosyncratic turnover is related to firm-specific risks that are largely driven by information asymmetry (For example, O'Hara (2003) shows idiosyncratic trading among informed and uninformed investors is driven by the presence of informed investors and whether price is fully revealing (p 1347, equation (10)). Therefore, if idiosyncratic turnover is closely related to private information trading, our decomposition could provide a useful measure of the difference in information asymmetry across firms.

E. Summary of Empirical Results on Monthly Data

In addition to weekly data, we looked at return and turnover using monthly data for NYSE and AMEX stocks. Here we briefly summarize our results.

First, the Bai-Ng statistics are quite robust and consistent in estimating the number of factors in monthly data for the balanced panel. We find four or five systematic factors driving firm turnover and, on average, 36.5% of firm turnover is determined by common turnover factors. Second, we find there are two or three systematic factors driving excess returns, so we again reject the restriction of Lo and Wang that excess return and turnover should have the same number factors in the duo-factor model. Third, idiosyncratic risk on average explains 32% of idiosyncratic turnover (detailed results available on request). This supports O'Hara (2003) that private-information risk can affect stock-specific trading among informed and uninformed investors. It suggests that there is an "inextricable link" between trading activity and return volatility at the firm level.

V. Conclusion

This paper looks at two statistical procedures developed by Bai and Ng (2002, 2003) to estimate an approximate factor model for turnover and test for non-stationarity. We document the presence of severe heteroscedasticity and non-stationarity in turnover data of individual stocks. We find the Bai and Ng (2002) information criteria works well for raw returns but not for raw turnover for estimating the required number of systematic factors. However, a modified GLS-type approach of standardizing turnover is effective in dealing with the problems in turnover data. The approach is also robust to the presence of correlation and heteroscedasticity at both time and cross-section dimensions.

Using this approach, we provide a new test of the duo-factor model developed by Lo and Wang (2002) on return and trading volume. An important element of our methodology is the use of data from individual stocks rather than from beta-sorted portfolios. In particular, by exploiting the advantage of a large cross-section of individual stocks we are able to get around the non-stationarity problems inherent in dealing with turnover data.

Based on a balanced panel of return and turnover data from NYSE and AMEX stocks, we find several results. First, systematic turnover factors are quite useful in explaining the variation of turnover for large panel data set. There are four or five systematic factors driving stock turnovers. These common factors explain 15% to 26% of trading volume. Second, we reject the restriction of Lo and Wang's theoretical model that excess returns and turnovers have the same number factors. This implies that stock price and trading volume are incompatible under the existing standard multi-factor asset pricing-trading framework. Fourth, we show that several commonly used turnover measures may significantly understate the price impact of stock trading.

There are several issues that remain to be examined. If the duo-factor model provides a parsimonious description of weekly data, it is interesting to know whether it works equally well on higher-frequency data. Second, our decomposition can provide firm-specific parts of turnover related to price momentum. It will be interesting to see if the firm-specific turnover can be used to identify different firm-specific stages of momentum-value cycles, as in Lee and Swaminathan (2000). Finally, if firm-level asymmetric information drives idiosyncratic volume and risk, then by using the return and turnover decomposition developed in this article we may obtain a proxy for measuring the degree of information asymmetry across stocks and thus be able to evaluate the impact of price discovery risk on asset pricing (see Llorente, Michaely, Saar, and Wang (2002) and O'Hara (2003)).

Appendix I

To derive the consistency result of the statistic for the number of factors in the APT model of (3), Bai and Ng (2002) introduce the following assumptions:

Assumption A: Factors

$E\|F_t\|^4 < \infty$ and $T^{-1}\sum_{t-1}^{T} F_t F_t^{'} \to \sum_F$ as $T \to \infty$ for some positive definite matrix $\sum_F$.

Assumption B: Factor Loadings

$\|\beta_i\| \leq \overline{\beta} < \infty$, and $\|B^{'} B / N - D\| \to 0$ as $N \to \infty$ for some KxK positive definite matrix D.

Assumption C: Time and Cross Section Dependence and heteroscedasticity

There exists a positive constant $M < \infty$, such that for all *N* and *T,*

1. $E(e_{it}) = 0 \, E|e_{it}|^8 \leq M$;
2. $E(e'_s e_t / N) = E(N^{-1}\sum_{i-1}^{N} e_{is} e_{it}) = \gamma_N(s,t), |\gamma_N(s,s)| \leq M$ for all s, and
$$T^{-1}\sum_{s=1}^{T}\sum_{t=1}^{T} |\gamma_N(s,t),| \leq M;$$
3. $E(e_{it} e_{jt}) = \tau_{ij,t}$ with $|\tau_{ij,t}| \leq |\tau_{ij}|$ for some τ_{ij} and for all t. In addition,
$$N^{-1}\sum_{i=1}^{N}\sum_{j=1}^{N} |\tau_{ij}| \leq M;$$
4. $E(e_{it} e_{js}) = \tau_{ij,ts}$ and $(NT)^{-1}\sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{T=1}^{T}\sum_{s=1}^{T} \|\tau_{ij,ts}| \leq M$;
5. For every (t,s), $E\left|N^{-1/2}\sum_{t=1}^{N}\left[e_{is}e_{it} - E(e_{is}e_{it})\right]\right|^4 \leq M$.

Assumption D: Weak dependence between factors and idiosyncratic errors

$$E\left(\frac{1}{N}\sum_{i=1}^{N}\left\|\frac{1}{\sqrt{T}}\sum_{t=1}^{T}F_t\ e_{it}\right\|^2\right)\leq M\ .$$

Assumption A and B are fairly standard for factor models and their ensure that each factor would have a bounded and non-trivial contribution to the variance of asset returns (or turnover). While we only consider non-random factor loadings here, the results still hold when B is random, provided they are independent of the factors and idiosyncratic errors. Assumption C allows for limited time series and cross section dependence in the idiosyncratic risks. Heteroscedasticity in both the time and cross section dimensions are also allowed. Therefore, our model is more general than a strict factor model of Ross (1976) that assumes no correlation across e_{it}. BN has shown that the above assumption C is consistent with the approximate factor model of Chamberlain and Chamberlain and Rothchild (1983) in the sense that it ensures that the largest eigenvalue of the N x N covariance matrix for the idiosyncratic risks must be bounded. While Chamberlain and Rothchild did not make any explicit assumption about the time series behavior of the factor, BN allows for serial correlation and heteroscedasticity. They have shown that Assumption C3 maintains the condition that the largest eigenvalue of the covariance matrix for the idiosyncratic risks will be bounded, thus their results is consistent with the approximate factor pricing model of Chamberlain and Rothchild. Here our discussion focus on the return factor model of (3), but the same assumptions A-D should also apply to the turnover factor model of (4) for estimating the number of factors.

Appendix II

Here we briefly discuss the simulation procedures used to test Lo and Wang (2000, LW).

Given estimates of the *T×K'* matrix **F** of factor realizations, we sample (with replacement) *T* rows of **F** to use as the true factors in the simulations. Let $\mathbf{F}_i$ denote the *ith* bootstrap draw of the factor matrix. The factor betas assumed in the DGP are bootstrap samples of the least squares estimates of the betas from the actual data and we assume then to be constant over time. Denoting **B** to be the $N \times K'$ matrix of OLS estimates of the factor betas from real data, we follow Jones (2001) by drawing with replacement N rows of the **B** matrix to use as the true betas in the simulations. We then draw the corresponding elements of the $N \times N$ diagonal matrix $\mathbf{\Omega}$, whose (j, j) element is the unconditional sample variance of the residual of stock j. We denote $\mathbf{B}_i$ to be the *ith* bootstrap draw of the beta matrix and $\mathbf{\Omega}_i$ the corresponding draw of $\mathbf{\Omega}$. As a result, the $N \times T$ matrix of simulated excess returns $\mathbf{R}_i$ will then be generated by the equation

$$\mathbf{R}_i = \mathbf{B}_i \mathbf{F}_i + \mathbf{\Psi}_i * \mathbf{E}_i \tag{9}$$

where $\mathbf{\Psi}_i$ is the Cholesky-decomposition factor of $\mathbf{\Omega}_i$ and $\mathbf{E}_i$ is an $N \times T$ matrix of independent standard normals. Here, we assume all alphas to be zero.

Similarly, given estimates of the $T \times K$ matrix **G** of factor realizations for normalized turnover, we draw T rows of **G** to use as the true factors in the simulations, maintaining the same order as returns. Let $\mathbf{G}_i$ denote the *ith* bootstrap draw of the factor matrix. The factor betas assumed in the DGP are the bootstrap samples of the least squares estimates of the turnover betas from the actual data, which are assumed to be constant over time. Denoting **D** to be the $N \times K$ matrix of OLS estimates of the turnover betas from real data, we draw the same N rows as returns of the **D** matrix that we use as the true betas in the simulations. We then draw the corresponding elements of the $N \times N$ diagonal matrix $\mathbf{\Sigma}$, whose (j, j) element is the unconditional sample variance of the residual turnover of stock j.

To maintain the correlation found in the data between residual excess return and residual turnover, we simulate residual turnover by the following equation,

$$\xi_{jt} = \omega_i e_{j,t} + \mu_{jt}, \tag{10}$$

where ω_i is a scaling coefficient to make the correlation between ξ_{jt} and $e_{j,t}$ to be ρ_j and μ_{jt} is independent standard normal. Here, ρ_j is the sample correlation between residual excess return and residual turnover for stock j. We then further scale ξ_{jt} so that its variance equal to the *jth* diagonal element of $\tilde{\Sigma}$. As a result, the $N \times T$ matrix of simulated turnover Γ_i will then be generated by the equation

$$\Gamma_i = \mathbf{D}_i \mathbf{G}_i + \mathrm{H}_i \tag{11}$$

where H_i is the *ith* draw of the NxT matrix whose elements are ξ_{jt}.

References

Amihud, Yakov. "Illiquidity And Stock Returns: Cross-Section and Time-Series Effects." Journal of Financial Markets, 31-56.

Andersen, T., 1996, "Return Volatility and Trading Volume: An Information Interpretation;' *Journal of Finance,* 51, 169-204.

Bai, J. and S. Ng, 2002, Determining The Number Of Factors In Approximate Factor Models, Econometrica, 191-222.

Bai, J. and S. Ng, 2003, Determining A PANIC Approach to Unit Root in Panel Data, Econometrica, forthcoming

Berk, J. 2000, Sorting Out Sorts, *Journal of Finance,* 55, 407-427.

Brennan, M., T. Chordia, and A. Subrahmanyam, 1997, "Alternative Factor Specifications, Security Characteristics and the Cross-Section of Expected Stock Returns," *Journal of Financial Economics*.

Chordia, T., R. Roll and A. Subrahmanyam, 2000, Commonality in Liquidity, *Journal of Financial Economics* 56, 3-28.

Chordia, T., R. Roll and A. Subrahmanyam, 2001, Market Liquidity and Trading Activity, *Journal of Finance,* 56, 501-530.

Chamberlain, G., 1983, "Funds, Factors, and Diversification in Arbitrage Pricing Models," *Econometrica,* 51, 1305-1323.

Chamberlain, G., and M. Rothschild, 1983, "Arbitrage, Factor Structure, and Mean-Variance Analysis on Large Asset Markets," *Econometrica,* 51, 1281-1304.

Chen, J. and H. Hong, and J. Stein, 2001, "Forecasting Crashes: Trading Volume, Past Returns and Conditional Skewness in Stock Prices", Journal of Financial Economics, September.

Connor, G. and R. Korajczyk, 1988, Risk and Return in Equilibrium APT: Application of A new Methodology, *Journal of Financial Economics* 21, 255-289.

Connor, G. and R. Korajczyk, 1993, A Test for the Number of Factors in an Approximate Factor Model, *Journal of Finance*, Vol. 48, No. 4., 1263-1291.

Conrad, J., A. Hammed, and C. Niden, 1994, Volume and Autocovariance in Short-horizon and individual stock returns, *Journal of Finance,* 49, 1305-1329.

Easley, David, Soeren Hvidkjaer, Maureen O'Hara, 2002, Is Information Risk a Determinant of Asset Returns?, *Journal of Finance, 57, 2185-2222.*

Fama, F. and K. French, 1996, "Multifactor Explanations of Asset Pricing Anomalies," *Journal of Finance*.

Gallant, R., P. Rossi, and G. Tauchen, 1992, "Stock Prices and Volume," *Review of Financial Studies,* 5, 199-242.

Gervais, S., R. Kaniel, D. Mingelgrin, 2001, The High Volume Return Premium, *Journal of Finance*, 56, 877-920.

Hasbrouck, J., D Seppi, 2001, Common Factors in Prices, Order Flows and Liquidity, *Journal of Financial Economics*, 59, 2, 383-411.

Hiernstra, C., and J. Jones, 1994, "Testing for Linear and Nonlinear Granger Causality in the Stock Price-Volume Relation," *Journal of Finance,* 49, 1639-1664.

Hong, H. and J. Stein, 2003, "Differences of Opinion, Short-Sales Constraints and Market Crashes", Review of Financial Studies, 487-526

Jones, C., 2001, ``Extracting Factors from Heteroskedastic Asset Returns," Journal of Financial Economics, Volume 62, Issue 2, November 2001.

Karpoff, *J.,* 1987, "The Relation between Price Changes and Trading Volume: A Survey," *Journal Financial and Quantitative Analysis,* 22, 109-126.

Lamont, O., 1998, Earnings and Expected Returns, *Journal of Finance* 53, 1563-1587.

Lamoureux, C., and W. Lastrapes, 1990, "Heteroskedasticity in Stock Return Data: Volume vs. GARCH Effects;' *Journal of Finance,* 45, 487-498.

Lee and Swaminathan (2000), Price Momentum and Trading Volume, *Journal of Finance,* 2017-2069.

Llorente, G., R. Michaely, G. Saar, and J. Wang, 2002, "Dynamic Volume-Return Relation of Individual Stocks", *Review of Financial Studies*, 1005-1049.

Lo, A., and J. Wang, 2001, "Trading Volume: Implications of an Intertemporal Capital Asset-Pricing Model," working paper, MIT.

Lo, A., and J. Wang, 2000, Trading Volume: Definitions, Data Analysis, and Implications of Portfolio Theory, *Review of Financial Studies* 13, 257-300.

Michaely, R., and J. Vila, 1996, "Trading Volume with Private Valuation: Evidence from the Ex-Dividend Day," *Review of Financial Studies,* 9, 471-509.

O'Hara, Maureen, 2003, Presidential Address: Liquidity and Price Discovery, *Journal of Finance, 58, 1335-1355.*

Pastor, L. and R. Stambaugh, 2003, Liquidity Risk and Expected Stock Returns, Journal of Political Economy, 642-685.

Stickel, *S.,* 1991, "The Ex-Dividend Day Behavior of Nonconvertible Preferred Stock Returns and Trading Volume;' *Journal of Finance and Quantitative Analysis,* 26, 45-61.

Tauchen, G., and M. Pitts, 1983, "The Price Variability-Volume Relationship on Speculative Markets," *Econometrica,* 51, 485-506.

Tkac, P., 1996, "A Trading Volume Benchmark: Theory and Evidence," working paper, University of Notre Dame.

Wang, J., 1994, "A Model of Competitive Stock Trading Volume;' Journal of Political *Economy,* 102, 127~168.

Xu, Y., 2001, "Extracting Factors with Maximum Explanatory Power", Working Paper, School of Management, The University of Texas at Dallas.

Table 1: Summary Statistics

Panel A: Number of Stocks on NYSE&AMEX during each time periods, Number of firms with no missing observation in turnover, Number of firms with problem data in turnover (those with CRSP Z flag), and number of firms used in the sample. Common shares are selected from CRSP share using codes 10 and 11.

Dates	Number of firms in NYSE&AMEX	Firms with No Missing data	Firms with Problem data	Number of firms Used in Sample	Mean Weekly Turnover (%)
1967-1971	2510	1592	6	1586	0.89
1972-1976	2527	1912	0	1912	0.53
1977-1981	2288	1753	0	1753	0.72
1982-1986	2141	1514	0	1514	1.00
1987-1991	1977	1400	1	1399	1.04
1992-1996	2249	1530	2	1528	1.06
1997-2001	2502	1391	6	1385	1.43

Panel B: Cross-sectional Distribution of return and turnover volatility of NYSE and AMEX common shares for January 1967 to December 2001. Return and Turnover are measured in percentages. Volatility is measured as time series standard deviation of return/turnover over the sample period.

	Turnover Volatility				**Excess Return Volatility**			
Dates	Mean	S.D.	Skewness	Kurtosis	Mean	S.D.	Skewness	Kurtosis
1967-1971	0.89	0.91	2.71	15.81	5.50	1.88	0.66	3.05
1972-1976	0.53	0.49	3.25	18.14	5.58	1.63	0.63	3.17
1977-1981	0.72	0.59	3.09	20.33	4.76	1.35	0.62	3.17
1982-1986	1.00	0.70	2.18	9.98	4.71	1.30	0.97	5.75
1987-1991	1.04	0.78	1.96	8.77	4.77	1.58	2.01	16.87
1992-1996	1.06	0.89	2.80	15.56	4.07	1.56	1.37	5.54
1997-2001	1.43	1.16	2.29	10.67	6.12	2.38	1.31	5.54

Table 2: Impact Of Heteroscedasticity And Nonstationarity On Estimates Of Number Of Factors For Turnover

We use turnover for NYSE and AMEX ordinary common shares from January 1967 to December 2001. Common shares are selected from CRSP share using codes 10 and 11. We use raw as well as standardized return and turnover.

	Turnover				*Excess Returns*	
Time Period	Raw Level	Raw detrended	Standardized Level	Standardized + detrended	Raw	Standardized
1967-1971	16	16	5	4	2.0	2.0
1972-1976	16	15	4	4	2.0	2.0
1977-1981	11	10	5	4	2.0	2.0
1982-1986	8	8	4	3	2.0	2.0
1987-1991	11	8	3	3	2.0	2.0
1992-1996	12	10	4	3	2.0	2.0
1997-2001	12	11	4	4	3.0	3.0

Table 3: Test of number of factors in turnover

Incremental R^2, θk , k = 1,…,10 of the covariance matrix of weekly turnover of NYSE and AMEX ordinary common shares for seven subperiods from July 1967 to December 2001. We also report the number of factors selected by the IC criterion and cross-sectional average R^2 for the selected factor model for each sample periods.

	Standardized Turnover												
Period	θ1	θ2	θ3	θ4	θ5	θ6	θ7	θ8	θ9	θ10	# factors	average R^2	STD R^2
1	11.14%	5.71%	4.45%	2.23%	1.95%	1.64%	1.47%	1.31%	1.27%	1.16%	**5**	25.47%	12.93%
2	15.03%	7.30%	2.25%	2.15%	1.69%	1.66%	1.42%	1.30%	1.23%	1.15%	**4**	26.74%	14.18%
3	11.48%	4.29%	3.22%	2.18%	1.95%	1.58%	1.47%	1.39%	1.23%	1.14%	**5**	23.13%	12.32%
4	10.73%	5.59%	2.39%	2.28%	1.68%	1.41%	1.26%	1.19%	1.10%	1.09%	**4**	21.00%	11.26%
5	11.45%	4.07%	2.89%	1.90%	1.80%	1.45%	1.20%	1.15%	1.08%	1.05%	**3**	18.41%	13.01%
6	6.54%	3.90%	2.81%	2.22%	1.83%	1.55%	1.41%	1.29%	1.17%	1.10%	**4**	15.47%	10.35%
7	10.79%	4.00%	3.13%	2.47%	1.92%	1.64%	1.50%	1.29%	1.22%	1.14%	**4**	20.39%	14.02%

Table 4: Simulation Test for the number of factors extracted for standardized turnover Using IC Criterion

The table presents the frequency on the number of factors extracted from turnover data over 500 simulations. Each simulation involves the draw of a set of *NxT* individual turnover data.

Standardized Turnover	Frequency (%) found in 500 simulation studies								
	True K	1	2	3	4	5	6	Mean K	Std K
1967-1971	**5.0**	0.0	0.0	0.0	1.4	98.6	0.0	4.99	0.12
1972-1976	**4.0**	0.0	0.0	0.0	100.0	0.0	0.0	4.00	0.00
1977-1981	**5.0**	0.0	0.0	0.0	7.4	92.6	0.0	4.93	0.26
1982-1986	**4.0**	0.0	0.0	2.2	97.8	0.0	0.0	3.98	0.15
1987-1991	**3.0**	0.0	0.0	100.0	0.0	0.0	0.0	3.00	0.00
1992-1996	**4.0**	0.0	0.0	1.0	99.0	0.0	0.0	3.99	0.10
1997-2001	**4.0**	0.0	0.0	0.0	100.0	0.0	0.0	4.00	0.00

Table 5. Simulation Test For The Number Of Factors Extracted Using Raw Turnover

The table presents the frequency on the number of factors extracted from turnover data over 500 simulations. Each simulation involves the draw of a set of *NxT* individual turnover data.

RAW Turnover		Frequency (%) found in 500 simulation studies													
	True K	5	6	7	8	9	10	11	12	13	14	15	16	Mean K	Std K
1967-1971	**16**	0	0	0	0	0	1	3.6	13	30.6	35.4	13.6	2.8	13.48	1.13
1972-1976	**16**	0	0	0	0.4	1.2	5.4	14	28.8	27.6	15.2	5.8	1.6	12.52	1.39
1977-1981	**11**	0	0	0.8	7.6	30.2	44.6	16.8	0	0	0	0	0	9.69	0.87
1982-1986	**8**	0.4	11.4	49.4	38.8	0	0	0	0	0	0	0	0	7.27	0.67
1987-1991	**11**	0.2	2.4	13	28.6	33.2	18.4	4.2	0	0	0	0	0	8.64	1.13
1992-1996	**12**	0	0.6	4.6	17.6	29.6	27.6	17	3	0	0	0	0	9.42	1.21
1997-2001	**12**	0	0	1.2	6.6	24.8	37.4	24.8	5.2	0	0	0	0	9.94	1.04

Table 6: Time Trend Test for Turnover Components

Panel A: Test of trend in Systematic Turnover

Period	Trend coefficients on the systematic turnover factors				
Period	T-statistics for the trend coefficients				
1	1.59	7.52	-5.97	-2.74	0.68
2	-1.87	-5.23	-6.43	5.89	--
3	3.08	3.58	-3.16	3.63	4.73
4	1.68	4.32	8.28	-2.90	--
5	-3.17	-7.81	5.07	--	--
6	-10.99	-5.35	7.69	1.80	--
7	-9.30	2.05	-4.94	5.75	--

Note: The systematic factors are computed using the turnover data in levels directly. All trend coefficients are estimated in a regression that ALSO included the once lagged series and a constant. We have multiplied the trend coefficient by 1,000.

Panel B: Test of trend in Raw as well as Idiosyncratic Turnover

Period	Obs	% with trend in raw τ_{it}	% with trend in idio. τ_{it}
1	1586	60.21%	0.00%
2	1912	57.74%	10.46%
3	1753	49.46%	0.00%
4	1514	47.42%	0.00%
5	1399	56.25%	7.15%
6	1528	68.78%	0.00%
7	1385	64.40%	0.00%

Note:

1) "% with trend in raw τ_{it}" stands for percentage of the raw turnover series with statistically significant trend coefficients if regressed on a constant and a time trend.
2) "% with trend in idio. τ_{it}" stands for percentage of the idiosyncratic turnover series with statistically significant trend coefficients.

Table 7: Test of number of factors in excess return data

Incremental R^2, θk , k = 1,…, 10 of the covariance matrix of weekly returns of NYSE and AMEX common shares in percentages for seven subperiods from July 1967 to December 2001. We also report the number of factors selected by the IC criterion and cross-sectional average R^2 for selected model for each sample periods.

	Excess Returns												
Period	θ1	θ2	θ3	θ4	θ5	θ6	θ7	θ8	θ9	θ10	# factors	average R^2	stdev R^2
1	21.47%	1.90%	1.66%	1.07%	1.08%	0.91%	0.82%	0.78%	0.74%	0.72%	**2**	23.36%	8.52%
2	22.33%	2.80%	1.68%	1.26%	1.03%	0.99%	0.91%	0.87%	0.87%	0.83%	**2**	25.13%	10.10%
3	19.62%	2.75%	1.74%	1.58%	1.02%	0.81%	0.75%	0.75%	0.72%	0.70%	**2**	22.37%	11.00%
4	17.68%	2.71%	1.90%	1.35%	0.99%	0.94%	0.83%	0.82%	0.77%	0.74%	**2**	20.39%	11.15%
5	25.73%	2.79%	1.57%	1.32%	1.15%	1.02%	0.91%	0.88%	0.83%	0.78%	**2**	28.52%	14.77%
6	10.92%	3.36%	1.76%	1.44%	1.28%	1.21%	1.02%	0.95%	0.87%	0.86%	**2**	14.28%	11.58%
7	13.32%	3.50%	2.54%	1.63%	1.52%	1.33%	1.05%	1.06%	0.91%	0.90%	**3**	19.36%	13.24%

Table 8. Simulation Test For The Number Of Factors Extracted Using IC Criterion
The table presents the frequency on the number of factors extracted from return data over 500 simulations. Each simulation involves the draw of a set of *NxT* individual return data.

A. Test for Standardized Returns

Standardized Return		Frequency (%) found in 500 simulation studies						
	True K'	1	2	3	4	5	Mean K'	Std K'
1967-1971	**2.0**	9.0	91.0	0.0	0.0	0.0	1.91	0.29
1972-1976	**2.0**	0.6	99.4	0.0	0.0	0.0	1.99	0.08
1977-1981	**2.0**	0.6	99.4	0.0	0.0	0.0	1.99	0.08
1982-1986	**2.0**	0.0	100.0	0.0	0.0	0.0	2.00	0.00
1987-1991	**2.0**	0.0	100.0	0.0	0.0	0.0	2.00	0.00
1992-1996	**2.0**	0.0	100.0	0.0	0.0	0.0	2.00	0.00
1997-2001	**3.0**	0.0	0.0	100.0	0.0	0.0	3.00	0.00

B. Test for Raw Returns

RAW Return		Frequency (%) found in 500 simulation studies						
	True K'	1	2	3	4	5	Mean K'	Std K'
1967-1971	**2.0**	1	99	0	0	0	1.99	0.10
1972-1976	**2.0**	0	100	0	0	0	2.00	0.00
1977-1981	**2.0**	0.4	99.6	0	0	0	2.00	0.06
1982-1986	**2.0**	0	100	0	0	0	2.00	0.00
1987-1991	**2.0**	13.2	86.8	0	0	0	1.87	0.34
1992-1996	**2.0**	15.6	84.4	0	0	0	1.84	0.36
1997-2001	**3.0**	0	0.6	99.4	0	0	2.99	0.08

Table 9: Simulation Results on the Factor Number Difference Using PC Criterion
The table presents the Type I and Type II Error Estimates for test on the difference between the number of return factors and the number of turnover factors based on 500 simulations for each time period. Each simulation involves the draw of a set of *NxT* individual return and turnover data. The bold-faced numbers give the probability of error.

Panel A: Type I Error Estimates based on 500 Simulation for Each Time Period

Time period	Specified K-K'	Frequency (%) Found					
		-3	-2	-1	0	1	2
1	0	**0**	0	9	91	0	0
2	0	0	**0**	0.6	99.4	0	0
3	0	**0**	0	0.6	99.4	0	0
4	0	0	**0**	0	100	0	0
5	0	0	0	**0**	100	0	0
6	0	0	**0**	0	100	0	0
7	0	0	0	**0**	100	0	0

Panel B: Type II Error Estimates based on 500 Simulation for Each Time Period

Time period	Specified K-K'	Frequency (%) Found					
		-4	-3	-2	-1	0	2
1	-3	10.4	88.2	1.4	0.0	**0**	0
2	-2	0.0	1.0	99.0	0.0	**0**	0
3	-3	0.2	92.4	7.4	0.0	**0**	0
4	-2	0.0	0.0	97.8	2.2	**0**	0
5	-1	0.0	0.0	0.0	100.0	**0**	0
6	-2	0.0	0.0	99.0	1.0	**0**	0
7	-1	0.0	0.0	0.0	100.0	**0**	0

Table 10. Average Estimates Of Price Impact Using Five Different Turnover Measures of Turnover

This table provides the cross-sectional average of the γ_i estimates for the seven sample periods using two different measures of returns and five different measures of turnover. The t-stats in the parentheses provide the tests of the hypothesis that the mean of γ_i under specification A, B, C, D equal to the mean of γ_i under specification E.

$$r^e_{i,t+1} = \theta_i + \phi_i r^e_{i,t} + \gamma_i sign(r^e_{i,t})\tau^e_{i,t} + \varepsilon_{i,t+1}$$

Panel A: $r^e_{i,t+1} = r_{i,t+1} - r_{m,t+1}$

	A	**B**	**C**	**D**	**E**
Turnover	**Raw**	**Excess**	**Detrended**	**Idio.(VW)**	**Idio.**
1	-0.023	-0.105	-0.006	-0.014	**-0.012**
	(0.75)	(2.81)	(-0.52)	(0.16)	
2	-0.739	-0.591	-0.792	-0.760	**-0.743**
	(-0.16)	(-3.28)	(2.35)	(0.79)	
3	-0.232	-0.215	-0.271	-0.235	**-0.415**
	(-14.87)	(-10.06)	(-13.23)	(-14.6)	
4	-0.146	-0.165	-0.137	-0.155	**-0.173**
	(-2.92)	(-0.57)	(-4.28)	(-1.97)	
5	-0.139	-0.180	-0.145	-0.141	**-0.197**
	(-5.60)	(-0.77)	(-5.80)	(-5.43)	
6	-0.073	-0.092	-0.078	-0.075	**-0.100**
	(-3.30)	(-0.39)	(-3.59)	(-3.16)	
7	-0.082	-0.116	-0.081	-0.085	**-0.138**
	(-5.57)	(-1.61)	(-6.70)	(-5.24)	

Panel B: $r^{e}_{i,t+1} = e_{i,t+1} = r_{i,t+1} - \sum B_{ik}(\lambda_{kt+1} + f_{kt+1})$

	A	B	C	D	E
Turnover	**Raw**	**Excess**	**Detrended**	**Idio.(VW)**	**Idio.**
1	-0.023	-0.076	-0.022	-0.035	**-0.052**
	(-1.89)	(0.87)	(-2.23)	(-1.13)	
2	-0.217	-0.231	-0.234	-0.226	**-0.309**
	(-4.67)	(-2.55)	(-4.28)	(-4.36)	
3	-0.087	-0.123	-0.100	-0.087	**-0.164**
	(-7.42)	(-2.43)	(-7.04)	(-7.52)	
4	-0.059	-0.098	-0.062	-0.066	**-0.092**
	(-4.13)	(0.54)	(-4.00)	(-3.35)	
5	-0.044	-0.071	-0.058	-0.047	**-0.081**
	(-4.09)	(-0.51)	(-3.15)	(-3.90)	
6	-0.018	-0.024	-0.027	-0.020	**-0.039**
	(-2.70)	(-0.85)	(-2.14)	(-2.52)	
7	-0.030	-0.056	-0.036	-0.032	**-0.064**
	(-4.11)	(-0.65)	(-4.17)	(-3.81)	

Note: The five different measures of firm turnover as defined as follows:

(A) $\tau^{e}_{i,t}$ is the demeaned raw turnover for stock *i* during week t,

(B) $\tau^{e}_{i,t} = \tau_{i,t} - \tau^{vw}_{mt}$ is turnover in excess of the value-weighted market turnover,

(C) $\tau^{e}_{i,t}$ is detrended turnover for stock i,

(D) $\tau^{e}_{i,t}$(idio.(VW)) is the residual turnover in one-factor model of Lo and Wang (2002). We use the value-weighted turnover as the market factor.

(E) $\tau^{e}_{i,t} = \xi_{i,t}$ is the idiosyncratic turnover in the multi-factor model of (1).

Figure 1: Raw Turnover for Smallest and Largest Decile Portfolios (Value Weighted)

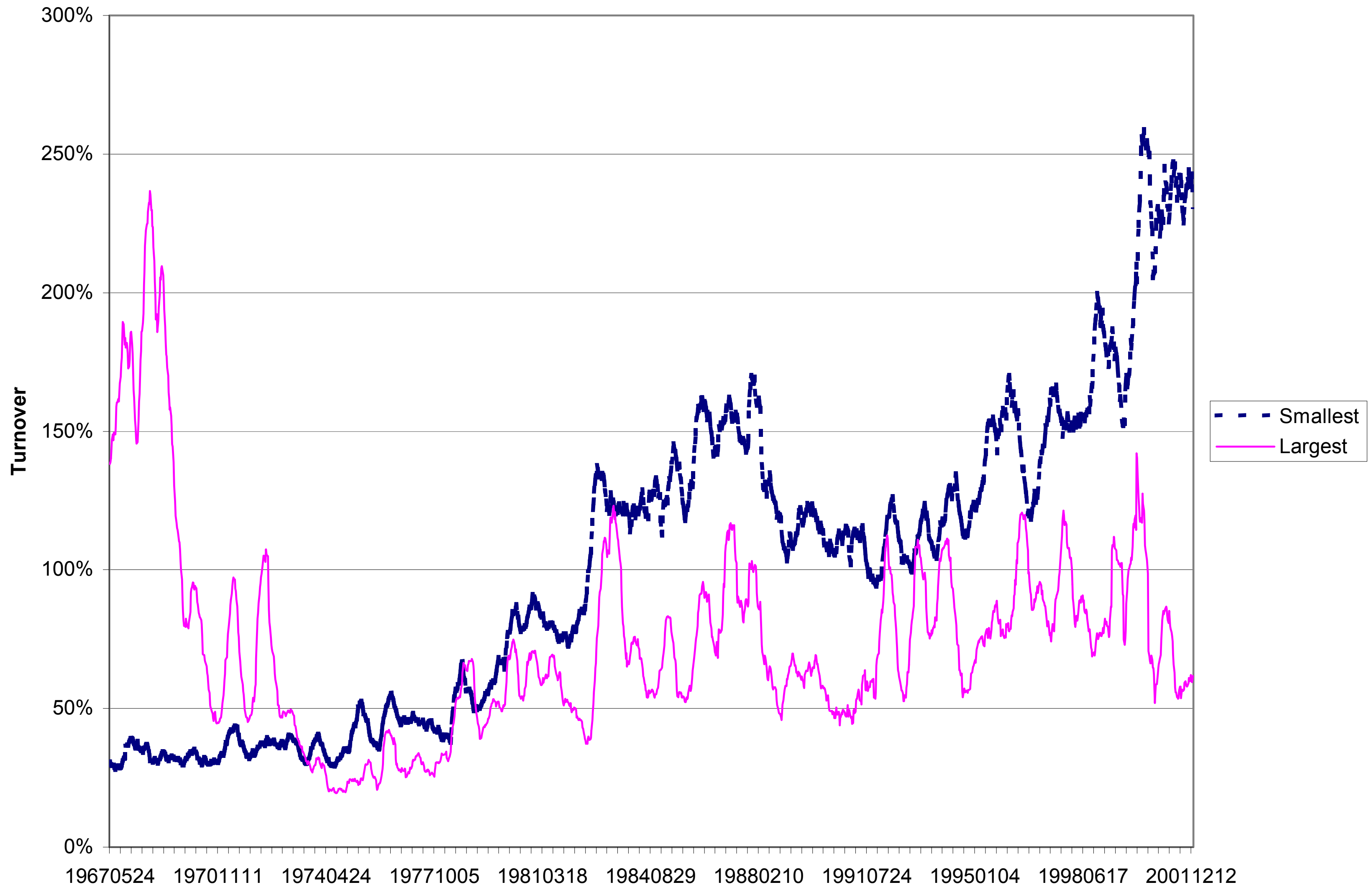

Figure 2a: Average Autocorrelations for EW Turnover Index (Raw and GRT detrended) and Individual Turnovers (Raw, Detrended, Idiosyncratic (VW), and Idiosyncratic(multi-factor))

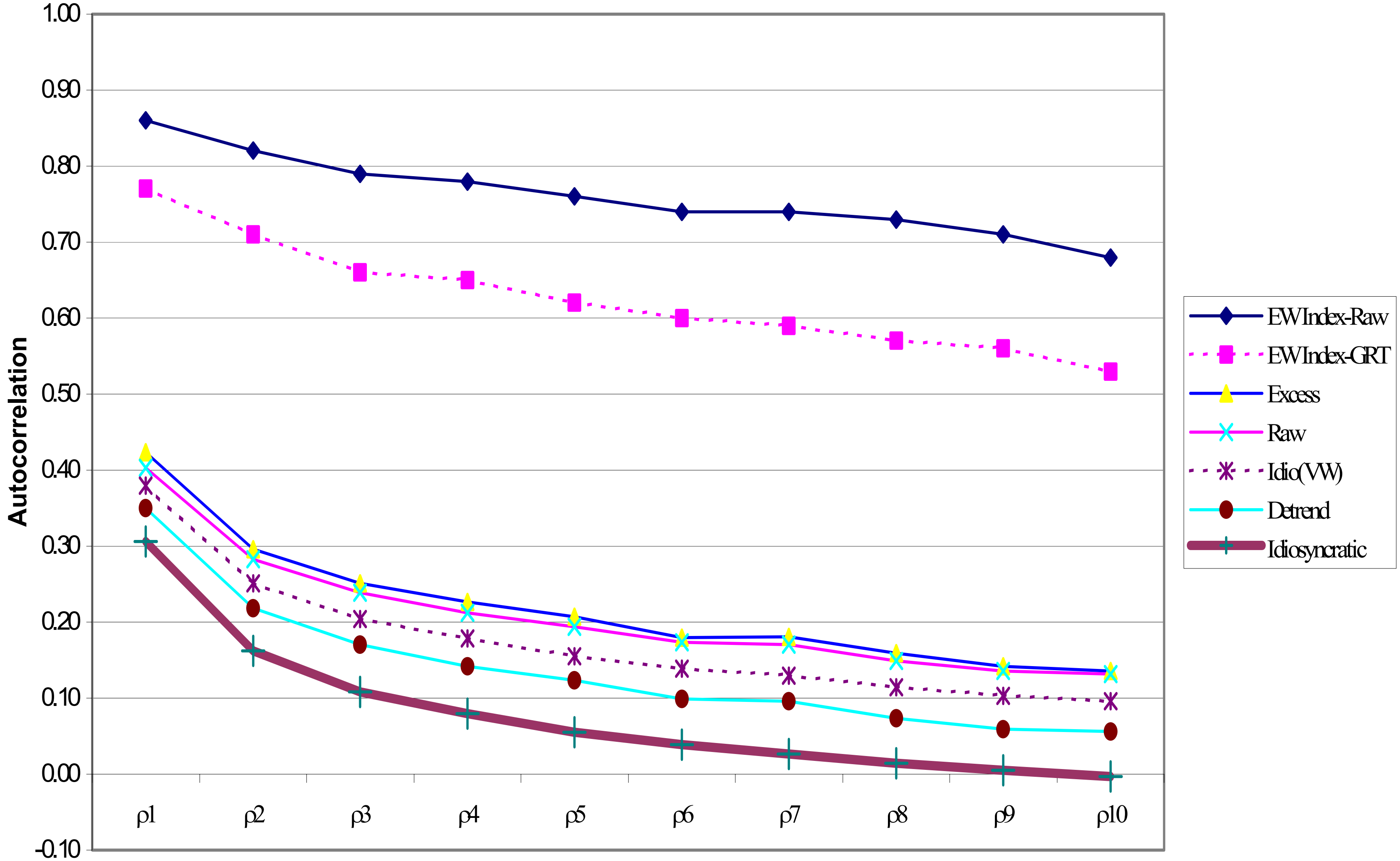

Figure 2b: Average Absolute Autocorrelations for Raw, Detrended, and Idiosyncratic Turnover (1997-2001)