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Default Recovery Rates in Credit Risk Modeling: A Review of the Literature and Empirical Evidence

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Abstract

Evidence from many countries in recent years suggests that collateral values and recovery rates on corporate defaults can be volatile and, moreover, that they tend to go down just when the number of defaults goes up in economic downturns. This link between recovery rates and default rates has traditionally been neglected by credit risk models, as most of them focused on default risk and adopted static loss assumptions, treating the recovery rate either as a constant parameter or as a stochastic variable independent from the probability of default. This traditional focus on default analysis has been partly reversed by the recent significant increase in the number of studies dedicated to the subject of recovery rate estimation and the relationship between default and recovery rates. This paper presents a detailed review of the way credit risk models, developed during the last thirty years, treat the recovery rate and, more specifically, its relationship with the probability of default of an obligor. Recent empirical evidence concerning this issue is also presented and discussed.

Keywords: credit rating, credit risk, recovery rate, default rate

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1. Introduction

Three main variables affect the credit risk of a financial asset: (i) the probability of default (PD), (ii) the "loss given default" (LGD), which is equal to one minus the recovery rate in the event of default (RR), and (iii) the exposure at default (EAD). While significant attention has been devoted by the credit risk literature on the estimation of the first component (PD), much less attention has been dedicated to the estimation of RR and to the relationship between PD and RR. This is mainly the consequence of two related factors. First, credit pricing models and risk management applications tend to focus on the systematic risk components of credit risk, as these are the only ones that attract risk-premia. Second, credit risk models traditionally assumed RR to be dependent on individual features (e.g. collateral or seniority) that do not respond to systematic factors, and to be independent of PD.

This traditional focus on default analysis has been partly reversed by the recent increase in the number of studies dedicated to the subject of RR estimation and the relationship between the PD and RR (Fridson, Garman and Okashima [2000], Gupton, Gates and Carty [2000], Jokivuolle and Peura [2003], Altman, Brady, Resti and Sironi [2001 and 2004], Frye [2000a, 2000b and 2000c], and Jarrow [2001]). This is partly the consequence of the parallel increase in default rates and decrease of recovery rates registered during the 1999-2002 period. More generally, evidence from many countries in recent years suggests that collateral values and recovery rates can be volatile and, moreover, they tend to go down just when the number of defaults goes up in economic downturns (Schleifer and Vishny [1992], Altman [2001], Hamilton, Gupton and Berthault [2001]).

This paper presents a detailed review of the way credit risk models developed during the last thirty years have treated the recovery rate and, more specifically, its relationship with the probability of default of an obligor. These models can be divided into two main categories: (a) credit pricing models, and (b) portfolio credit value-at-risk (VaR) models. Credit pricing models can in turn be divided into three main approaches: (i) "first generation" structural-form models, (ii) "second generation" structural-form models, and (iii) reduced-form models. These three different approaches together with their basic assumptions, advantages, drawbacks and empirical performance are reviewed in sections 2, 3 and 4. Credit VaR models are then examined in section 5. The more recent studies explicitly modeling and empirically investigating the relationship between PD and RR are reviewed in section 6. Section 7 presents some recent empirical evidence on recovery rates on both defaulted bonds and loans and also on the relationship between default and recovery rates. Section 8 concludes.

2. First generation structural-form models: the Merton approach

The first category of credit risk models are the ones based on the original framework developed by Merton (1974) using the principles of option pricing (Black and Scholes, 1973). In such a framework, the default process of a company is driven by the value of the company's assets and the risk of a firm's default is therefore explicitly linked to the variability of the firm's asset value. The basic intuition behind the Merton model is relatively simple: default occurs when the value of a firm's assets (the market value of the firm) is lower than that of its liabilities. The payment to the debtholders at the maturity of the debt is therefore the smaller of two quantities: the face value of the debt or the market value of the firm's assets. Assuming that the company's debt is entirely represented by a zero-coupon bond, if the value of the firm at maturity is greater than the face value of the bond, then the bondholder gets back the face value of the bond. However, if the value of the firm is less than the face value of the bond, the shareholders get nothing and the bondholder gets back the market value of the firm. The payoff at maturity to the bondholder is therefore equivalent to the face value of the bond minus a put option on the value of the firm, with a strike price equal to the face value of the bond and a maturity equal to the maturity of the bond. Following this basic intuition, Merton derived an explicit formula for risky bonds which can be used both to estimate the PD of a firm and to estimate the yield differential between a risky bond and a default-free bond.

In addition to Merton (1974), first generation structural-form models include Black and Cox (1976), Geske (1977), and Vasicek (1984). Each of these models tries to refine the original Merton framework by removing one or more of the unrealistic assumptions. Black and Cox (1976) introduce the possibility of more complex capital structures, with subordinated debt; Geske (1977) introduces interest-paying debt;

Vasicek (1984) introduces the distinction between short and long term liabilities which now represents a distinctive feature of the KMV model[1].

Under these models all the relevant credit risk elements, including default and recovery at default, are a function of the structural characteristics of the firm: asset volatility (business risk) and leverage (financial risk). The RR is therefore an endogenous variable, as the creditors' payoff is a function of the residual value of the defaulted company's assets. More precisely, under the Merton's theoretical framework, PD and RR tend to be inversely related (see Appendix A for a simulation exercise on this relationship). If, for example, the firm's value increases, then its PD tends to decrease while the expected RR at default increases (ceteris paribus). On the other side, if the firm's debt increases, its PD increases while the expected RR at default decreases. Finally, if the firm's asset volatility increases, its PD increases while the expected RR at default decreases, since the possible asset values can be quite low relative to liability levels.

Although the line of research that followed the Merton approach has proven very useful in addressing the qualitatively important aspects of pricing credit risks, it has been less successful in practical applications[2]. This lack of success has been attributed to different reasons. First, under Merton's model the firm defaults only at maturity of the debt, a scenario that is at odds with reality. Second, for the model to be used in valuing default-risky debts of a firm with more than one class of debt in its capital structure (complex capital structures), the priority/seniority structures of various debts have to be specified. Also, this framework assumes that the absolute-priority rules are actually adhered to upon default in that debts are paid off in the order of their seniority. However, empirical evidence in Franks and Torous (1994) indicates that the absolute-priority rules are often violated. Moreover, the use of a lognormal distribution in the basic Merton model (instead of a more fat tailed distribution) tends to overstate recovery rates in the event of default.

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[1] In the KMV model, default occurs when the firm's asset value goes below a threshold represented by the sum of the total amount of short term liabilities and half of the amount of long term liabilities.

[2] The standard reference is Jones, Mason and Rosenfeld (1984), who find that, even for firms with very simple capital structures, a Merton-type model is unable to price investment-grade corporate bonds better than a naive model that assumes no risk of default.

3. Second-generation structural-form models

In response to such difficulties, an alternative approach has been developed which still adopts the original Merton framework as far as the default process is concerned but, at the same time, removes one of the unrealistic assumptions of the Merton model, namely, that default can occur only at maturity of the debt when the firm's assets are no longer sufficient to cover debt obligations. Instead, it is assumed that default may occur any time between the issuance and maturity of the debt and that default is triggered when the value of the firm's assets reaches a lower threshold level[3]. These models include Kim, Ramaswamy and Sundaresan (1993), Hull and White (1995), Nielsen, Saà-Requejo, Santa Clara (1993), Longstaff and Schwartz (1995) and others.

Under these models the RR in the event of default is exogenous and independent from the firm's asset value. It is generally defined as a fixed ratio of the outstanding debt value and is therefore independent from the PD. For example, Longstaff and Schwartz (1995) argue that, by looking at the history of defaults and the recovery ratios for various classes of debt of comparable firms, one can form a reliable estimate of the RR. In their model, they allow for a stochastic term structure of interest rates and for some correlation between defaults and interest rates. They find that this correlation between default risk and the interest rate has a significant effect on the properties of the credit spread[4]. This approach simplifies the first class of models by both exogenously specifying the cash flows to risky debt in the event of bankruptcy and simplifying the bankruptcy process. The latter occurs when the value of the firm's underlying assets hits some exogenously specified boundary.

Despite these improvements with respect to the original Merton's framework, second generation structural-form models still suffer from three main drawbacks, which represent the main reasons behind their relatively poor empirical performance[5]. First, they still require estimates for the parameters of the firm's asset value, which is nonobservable. Indeed, unlike the stock price in the Black and Scholes formula for

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[3] One of the earliest studies based on this framework is Black and Cox (1976). However, this is not included in the second-generation models in terms of the treatment of the recovery rate.
[4] Using Moody's corporate bond yield data, they find that credit spreads are negatively related to interest rates and that durations of risky bonds depend on the correlation with interest rates.
[5] See Eom, Helwege and Huang (2001) for an empirical analysis of structural-form models.

valuing equity options, the current market value of a firm is not easily observable. Second, structural-form models cannot incorporate credit-rating changes that occur quite frequently for default-risky corporate debts. Most corporate bonds undergo credit downgrades before they actually default. As a consequence, any credit risk model should take into account the uncertainty associated with credit rating changes as well as the uncertainty concerning default. Finally, most structural-form models assume that the value of the firm is continuous in time. As a result, the time of default can be predicted just before it happens and hence, as argued by Duffie and Lando (2000), there are no "sudden surprises". In other words, without recurring to a "jump process", the PD of a firm is known with certainty.

4. Reduced-form models

The attempt to overcome the above mentioned shortcomings of structural-form models gave rise to reduced-form models. These include Litterman and Iben (1991), Madan and Unal (1995), Jarrow and Turnbull (1995), Jarrow, Lando and Turnbull (1997), Lando (1998), Duffie and Singleton (1999), and Duffie (1998). Unlike structural-form models, reduced-form models do not condition default on the value of the firm, and parameters related to the firm's value need not be estimated to implement them. In addition to that, reduced-form models introduce separate explicit assumptions on the dynamic of both PD and RR. These variables are modeled independently from the structural features of the firm, its asset volatility and leverage. Generally speaking, reduced-form models assume an exogenous RR that is independent from the PD. More specifically, reduced-form models take as primitives the behavior of default-free interest rates, the RR of defaultable bonds at default, as well as a stochastic process for default intensity. At each instant, there is some probability that a firm defaults on its obligations. Both this probability and the RR in the event of default may vary stochastically through time. Those stochastic processes determine the price of credit risk. Although these processes are not formally linked to the firm's asset value, there is presumably some underlying relation. Thus Duffie and Singleton (1999) describe these alternative approaches as a reduced-form models.

Reduced-form models fundamentally differ from typical structural-form models in the degree of predictability of the default as they can accomodate for defaults that are sudden surprises. A typical reduced-form model assumes that an exogenous random variable drives default and that the probability of default over any time interval is nonzero. Default occurs when the random variable undergoes a discrete shift in its level. These models treat defaults as unpredictable Poisson events. The time at which the discrete shift will occur cannot be foretold on the basis of information available today.

Reduced-form models somewhat differ by the manner in which the RR is parameterized. For example, Jarrow and Turnbull (1995) assumed that, at default, a bond would have a market value equal to an exogenously specified fraction of an otherwise equivalent default-free bond. Duffie and Singleton (1999) followed with a model that, when market value at default (i.e. RR) is exogenously specified, allows for closed-form solutions for the term-structure of credit spreads. Their model also allows for a random RR that depends on the pre-default value of the bond. While this model assumes an exogenous process for the expected loss at default, meaning that the RR does not depend on the value of the defaultable claim, it allows for correlation between the default hazard-rate process and RR. Indeed, in this model, the behavior of both PD and RR may be allowed to depend on firm-specific or macroeconomic variables and therefore to be correlated.

Other models assume that bonds of the same issuer, seniority, and face value have the same RR at default, regardless of the remaining maturity. For example, Duffie (1998) assumes that, at default, the holder of a bond of given face value receives a fixed payment, irrespective of the coupon level or maturity, and the same fraction of face value as any other bond of the same seniority. This allows him to use recovery parameters based on statistics provided by rating agencies such as Moody’s. Jarrow, Lando and Turnbull (1997) also allow for different debt seniorities to translate into different RRs for a given firm. Both Lando (1998) and Jarrow, Lando and Turnbull (1997) use transition matrices (historical probabilities of credit rating changes) to price defaultable bonds.

Empirical evidence concerning reduced-form models is rather limited. Using the Duffie and Singleton (1999) framework, Duffee (1999) finds that these models have difficulty in explaining the observed term structure of credit spreads across firms of different credit risk qualities. In particular, such models have difficulty generating both relatively flat yield spreads when firms have low credit risk and steeper yield spreads when firms have higher credit risk.

A recent attempt to combine the advantages of structural-form models – a clear economic mechanism behind the default process - and the ones of reduced-form models – unpredictability of default - can be found in Zhou (2001). This is done by modeling the evolution of firm value as a jump-diffusion process. This model links RRs to the firm value at default so that the variation in RRs is endogenously generated and the correlation between RRs and credit ratings reported in Altman (1989) and Gupton, Gates and Carty (2000) is justified (see also Table 4 below).

5. Credit Value-at-Risk Models

During the second part of the Nineties, banks and consultants started developing credit risk models aimed at measuring the potential loss, with a predetermined confidence level, that a portfolio of credit exposures could suffer within a specified time horizon (generally one year). These vaue-at-risk (VaR) models include J.P. Morgan's *CreditMetrics®* (Gupton, Finger and Bhatia [1997]), *Credit Suisse Financial Products' CreditRisk$^+$®* (1997), McKinsey's *CreditPortfolioView®* (Wilson, 1998), KMV's *CreditPortfolioManager®*, and Kamakura's

Credit VaR models can be gathered in two main categories: 1) Default Mode models (DM) and 2) Mark-to-Market (MTM) models. In the former, credit risk is identified with default risk and a binomial approach is adopted. Therefore, only two possible events are taken into account: default and survival. The latter includes all possible changes of the borrower creditworthiness, technically called "credit migrations". In DM models, credit losses only arise when a default occurs. On the other hand, MTM models are multinomial, in that losses arise also when credit migrations occur. The two approaches basically differ for the amount of data necessary to feed them: limited in the case of default mode models, much wider in the case of mark-to-market ones.

The main output of a credit risk model is the probability density function (PDF) of the future losses on a credit portfolio. From the analysis of such loss distribution, a financial institution can estimate both the expected loss and the unexpected loss on its credit portfolio. The expected loss equals the (unconditional) mean of the loss distribution; it represents the amount the bank can expect to lose within a specific period of time (usually one year). On the other side, the unexpected loss represents the "deviation" from expected loss and measures the actual portfolio risk. This can in turn be measured as the standard deviation of the loss distribution. Such measure is relevant only in the case of a normal distribution and is therefore hardly useful for credit risk measurement: indeed, the distribution of credit losses is usually highly asymmetrical and fat-tailed. This implies that the probability of large losses is higher than the one associated with a normal distribution. Financial institutions typically apply credit risk models to evaluate the "economic capital" necessary to face the risk associated with their credit portfolios. In such a framework, provisions for credit losses should cover expected losses[6], while economic capital is seen as a cushion for unexpected losses. This distinction is currently (in late 2003) the cause for a further delay in the "" of Basel II.

Credit VaR models can largely be seen as reduced-form models, where the RR is typically taken as an exogenous constant parameter or a stochastic variable independent from PD. Some of these models, such as *CreditMetrics®, CreditPortfolioView®* and *CreditPortfolioManager®*, treat the RR in the event of default as a stochastic variable – generally modeled through a beta distribution - independent from the PD. Others, such as *CreditRisk⁺®,* treat it as a constant parameter that must be specified as an input for each single credit exposure. While a comprehensive analysis of these models goes beyond the aim of this review[7], it is important to highlight that all credit VaR models treat RR and PD as two independent variables.

6. The latest contributions on the PD-RR relationship

During the last three years, new approaches explicitly modeling and empirically investigating the relationship between PD and RR have been developed. These models include Frye (2000a and 2000b), Jarrow (2001), Hu and Perraudin (2002), Jokivuolle and Peura (2003), Carey and Gordy (2003), Bakshi et al. (2001), Altman, Brady, Resti and Sironi (2001 and 2004), and Acharya, Bharath and Srinivasan (2003).

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[6] As discussed in Jones and Mingo (1998), reserves are used to cover expected losses..
[7] For a comprehensive analysis of these models, see Crouhy, Galai and Mark (2000) and Gordy (2000).

The model proposed by Frye (2000a and 2000b) draws from the conditional approach suggested by Finger (1999) and Gordy (2000). In these models, defaults are driven by a single systematic factor – the state of the economy - rather than by a multitude of correlation parameters. These models are based on the assumption that the same economic conditions that cause defaults to rise might cause RRs to decline, i.e. that the distribution of recovery is different in high-default periods from low-default ones. In Frye's model, both PD and RR depend on the state of the systematic factor. The correlation between these two variables therefore derives from their mutual dependence on the systematic factor.

The intuition behind Frye's theoretical model is relatively simple: if a borrower defaults on a loan, a bank's recovery may depend on the value of the loan collateral. The value of the collateral, like the value of other assets, depends on economic conditions. If the economy experiences a recession, RRs may decrease just as default rates tend to increase. This gives rise to a negative correlation between default rates and RRs.

While the model originally developed by Frye (2000a) implied recovery to be taken from an equation that determines collateral, Frye (2000b) modeled recovery directly. This allowed him to empirically test his model using data on defaults and recoveries from U.S. corporate bond data. More precisely, data from Moody's Default Risk Service database for the 1982-1997 period were used for the empirical analysis[8]. Results show a strong negative correlation between default rates and RRs for corporate bonds. This evidence is consistent with the most recent U.S. bond market data, indicating a simultaneous increase in default rates and LGDs for the 1999-2002 period[9]. Frye's (2000b and 2000c) empirical analysis allows him to conclude that in a severe economic downturn bond recoveries might decline 20-25 percentage points from their normal-year average. Loan recoveries may decline by a similar amount, but from a higher level.

Jarrow (2001) presents a new methodology for estimating RRs and PDs implicit in both debt and equity prices. As in Frye (2000a and 2000b), RRs and PDs are

[8] Data for the 1970-1981 period have been eliminated from the sample period because of the low number of default prices available for the computation of yearly recovery rates.

correlated and depend on the state of the macroeconomy. However, Jarrow's methodology explicitly incorporates equity prices in the estimation procedure, allowing the separate identification of RRs and PDs and the use of an expanded and relevant dataset. In addition to that, the methodology explicitly incorporates a liquidity premium in the estimation procedure, which is considered essential in light of the high variability in the yield spreads between risky debt and U.S. Treasury securities.

Using four different datasets (Moody's Default Risk Service database of bond defaults and LGDs, Society of Actuaries database of private placement defaults and LGDs, Standard & Poor's database of bond defaults and LGDs, and Portfolio Management Data's database of LGDs) ranging from 1970 to 1999, Carey and Gordy (2003) analyze LGD measures and their correlation with default rates. Their preliminary results contrast with the findings of Frye (2000b): estimates of simple default rate-LGD correlation are close to zero. They also find that limiting the sample period to 1988-1998, estimated correlations are more in line with Frye's results (0.45 for senior debt and 0.8 for subordinated debt). The authors note that during this short period the correlation arises not so much because LGDs are low during the low-default years 1993-1996, but rather because LGDs are relatively high during the high-default years 1990 and 1991. They therefore conclude that the basic intuition behind the Frye's model may not adequately characterize the relationship between default rates and LGDs. Indeed, a weak or asymmetric relationship suggests that default rates and LGDs may be influenced by different components of the economic cycle.

Using defaulted bonds' data for the sample period 1982-2000, which includes the relatively high-default years of 1999 and 2000, Altman, Brady, Resti and Sironi (2004) find empirical results that appear consistent with Frye's intuition: a negative correlation between default rates and RRs. However, they find that the single systematic risk factor – i.e. the performance of the economy - is less predictive than Frye's model would suggest. Their econometric univariate and multivariate models assign a key role to the supply of defaulted bonds (the default rate) and show that this variable, together with variables that proxy the size of the high-yield bond market and the economic cycle, explain a substantial proportion of the variance in bond recovery rates aggregated across

[9]Hamilton, Gupton and Berthault (2001) and Altman, Brady, Resti and Sironi (2003) provide clear empirical evidence of this phenomenon.

all seniority and collateral levels. They conclude that a simple microeconomic mechanism based on supply and demand drives aggregate recovery rates more than a macroeconomic model based on the common dependence of default and recovery on the state of the cycle. In high default years, the supply of defaulted securities tends to exceed demand[10], thereby driving secondary market prices down. This in turn negatively affects RR estimates, as these are generally measured using bond prices shortly after default.

Altman et al. (2004) also highlight the implications of their results for credit risk modelling and for the issue of procyclicality[11] of capital requirements. In order to assess the impact of a negative correlation between default rates and recovery rates on credit risk models, they run Montecarlo simulations on a sample portfolio of bank loans and compare the key risk measures (expected and unexpected losses). They show that both the expected loss and the unexpected loss are vastly understated if one assumes that PDs and RRs are uncorrelated[12]. Therefore, credit models that do not carefully factor in the negative correlation between PDs and RRs might lead to insufficient bank reserves and cause unnecessary shocks to financial markets.

As far as procyclicality is concerned, they show that this effect tends to be exacerbated by the correlation between DRs and RRs: low recovery rates when defaults are high would amplify cyclical effects. This would especially be true under the so-called "advanced" IRB approach, where banks are free to estimate their own recovery rates and might tend to revise them downwards when defaults increase and and ratings worsen. The impact of such a mechanism was also assessed by Resti (2002), based on simulations over a 20-year period, using a standard portfolio of bank loans (the composition of which is adjusted through time according to S&P transition matrices). Two main results emerged from this simulation exercise: (i) the procyclicality effect is driven more by up- and downgrades, rather than by default rates; in other words, adjustments in credit supply needed to comply with capital requirements respond

[10] Demand mostly comes from niche investors called "vultures", who intentionally purchase bonds in default. These investors represent a relatively small and specialized segment of the fixed income market.

[11] Prociclicality involves the sensitivity of regulatory capital requirements to economic and financial market cycles. Since ratings and default rates respond to the cycle, the new internal ratings-based (IRB) approach proposed by the Basel Committee risks increasing capital charges, and limiting credit supply, when the economy is slowing (the reverse being true when the economy is growing at a fast rate).

mainly to changes in the structure of weighted assets, and only to a lesser extent to actual credit losses (except in extremely high default years); (ii) when RRs are permitted to fluctuate with default rates, the procyclicality effect increases significantly.

Using Moody's historical bond market data, Hu and Perraudin (2002) examine the dependence between recovery rates and default rates. They first standardize the quarterly recovery data in order to filter out the volatility of recovery rates due to changes over time in the pool of rated borrowers. They find that correlations between quarterly recovery rates and default rates for bonds issued by US-domiciled obligors are 0.22 for post 1982 data (1983-2000) and 0.19 for the 1971-2000 period. Using extreme value theory and other non-parametric techniques, they also examine the impact of this negative correlation on credit VaR measures and find that the increase is statistically significant when confidence levels exceed 99%.

Bakshi et al. (2001) enhance the reduced-form models presented in section 4 to allow for a flexible correlation between the risk-free rate, the default probability and the recovery rate. Based on some preliminary evidence published by rating agencies, they force recovery rates to be negatively associated with default probability. They find some strong support for this hypothesis through the analysis of a sample of BBB-rated corporate bonds: more precisely, their empirical results show that, on average, a 4% worsening in the (risk-neutral) hazard rate is associated with a 1% decline in (risk-neutral) recovery rates.

A rather different approach is the one proposed by Jokivuolle and Peura (2003). The authors present a model for bank loans in which collateral value is correlated with the PD. They use the option pricing framework for modeling risky debt: the borrowing firm's total asset value triggers the event of default. However, the firm's asset value does not determine the RR. Rather, the collateral value is in turn assumed to be the only stochastic element determining recovery[13]. Because of this assumption, the model can be implemented using an exogenous PD, so that the firm's asset value parameters need not be estimated. In this respect, the model combines features of both structural-form

[12] Both expected losses and VaR measures associated with different confidence levels tend to be underestimated by approximately 30%.

[13] Because of this simplifying assumption the model can be implemented using an exogenous PD, so that the firm asset value parameters need not be estimated. In this respect, the model combines features of both structural-form and reduced-form models.

and reduced-form models. Assuming a positive correlation between a firm's asset value and collateral value, the authors obtain a similar result as Frye (2000a), that realized default rates and recovery rates have an inverse relationship.

Using data on observed prices of defaulted securities in the United States over the period 1982-1999, Acharya, Bharath and Srinivasan (2003) find that seniority and security are important determinants of recovery rates. While this result is not surprising and in line with previous empirical studies on recoveries, their second main result is rather striking and concerns the effect of industry-specific and macroeconomic conditions in the default year. Indeed, industry conditions at the time of default are found to be robust and important determinants of recovery rates. This result is in contrast with to those of Altman et al. (2004) in that there is no effect of macroeconomic conditions over and above the industry conditions and is in line those results in that the effect of industry conditions is robust to inclusion of macroeconomic factors. Acharya, Bharath and Srinivasan (2003) suggest that the linkage, highlighted by Altman et al. (2004), between bond market aggregate variables and recoveries as arising due to supply-side effects in segmented bond markets may be a manifestation of Shleifer and Vishny (1992) industry equilibrium effect: macroeconomic variables and bond market conditions appear to be picking up the effect of omitted industry conditions.

7. Empirical evidence

This section of our review focuses on different measurements and the most recent empirical evidence of default recovery rates. Most credit risk models utilize historical average empirical estimates, combined with their primary analytical specification of the probability of default, to arrive at the all-important Loss-Given-Default (LGD) input. Since very few financial institutions have ample data on recovery rates by asset-type and by type of collateral, model builders and analysts responsible for Basel II inputs into their internal rate based (IRB) models begin with estimates from public bond and private bank loan markets. Of course, some banks will research their own internal databases in order to conform with the requirements of the Advanced IRB approach.

Early Empirical Evidence

Published data on default recovery rates generally, but not always, use secondary bond or bank loan prices. The first empirical study, that we are aware of, that estimated default recovery rates was in Altman, Haldeman and Narayanan's (1977) ZETA® model's adjustment of the optimal cutoff score in their second generation credit scoring model. Interestingly, these bank loan recovery estimates did not come from the secondary loan trading market -- they did not exist then -- but from a survey of bank workout-department experience (1971-1975). The general conclusion from this early experience of these departments was a recovery rate on non-performing, unsecured loans of about thirty percent of the loan amount plus accrued interest. The cash inflows for three years post-default was not discounted back to default date. We will refer to this experience as the "ultimate recovery" since it utilizes post-defaults recoveries, usually from the end of the restructuring period.

In later studies, ultimate recovery rates refer to the nominal or discounted value of bonds or loans based on either the price of the security at the end of the reorganization period (usually Chapter 11) or the value of the package of cash or securities upon emergence from restructuring. For example, Altman and Eberhart (1994) observed the price performance of defaulted bonds, stratified by seniority, upon restructuring emergence as well as the discounted value of these prices. They concluded that the most senior bonds in the capital structure (senior secured and senior unsecured) did very well in the post-default period (20-30% per annum returns) but the more junior bonds (senior subordinated and subordinated) did poorly, barely breaking even on a nominal basis and losing money on a discounted basis. Similar, but less extreme, results were found by Fridson, et. al., Merrill Lynch 2001) when they updated (1994-2000) Altman & Eberhart's earlier study which covered the period 1981-1993.

More Recent Evidence

In Table 1, we present recent empirical evidence on bank loan recoveries (Emery, Moody's, 2003) and on corporate bonds by seniority (Altman and Fanjul, 2004) based on the average prices of these securities just after the date of default. Not surprisingly, the highest median recovery rates were on senior secured bank loans (73.0%) followed by senior secured bonds (54.5%). Although the data from Moody's and Altman were from different periods and samples, it is interesting to note that the recovery on senior unsecured bonds (42.3%) was similar, but lower than senior unsecured bank loans (50.5%), with similar standard deviations (in the mid-twenty percents). The estimates of median recoveries on the senior-subordinated and subordinated bonds were virtually the same at 32.0%. Similar recoveries on defaulted bonds can be found in Varma, et. al. (Moody's, 2003). For example, Altman's mean recovery rate on almost 2000 bond default issues was 34.3% compared to Moody's 1,239 issuer-weighted mean of 35.4%.

TABLE 1 APPROXIMATELY HERE

Altman and Fanjul (2004) further breakdown bond recoveries just after the default date by analyzing recoveries based on the original rating (fallen angels vs. original rating non-investment ["junk"] bonds) of different seniorities. For example, in Table 2 we observe that senior-secured bonds, that were originally rated investment grade, recovered a median rate of 50.5% vs. just 33.5% for the same seniority bonds that were non-investment grade when issued. This is a dramatic statistically significant difference for similar seniority securities. The mean recovery rate differential was even greater. Since fallen angel defaults are much more prominent of late in the United States (e.g., close to 50% in dollar amount of defaults in 2001 and 2002 were fallen angels prior to default), these statistics are quite meaningful. The differential was almost as great (42.7% vs. 30.0%) for senior unsecured bonds. Note that for senior-subordinated and subordinated bonds, however, the rating at issuance is of no consequence, although the sample sizes for investment grade, low seniority bonds were very small. Varma, et. al., (2003) also conclude that the higher the rating prior to default, the higher the average recovery rate in default.

TABLE 2 APPROXIMATELY HERE

In Table 3, we again return to the data on ultimate recoveries, only this time the results are from Standard & Poor's assessment of bank loan and bond recoveries. These results show the nominal and discounted (by the loan's pre-default interest rate) ultimate recovery at the end of the restructuring period for well over 2,000 defaulted loans and notes. Several items are of interest. First, the recovery on senior bank debt, which are mainly secured, was quite high at 87.3% and 78.8% for nominal and discounted values respectively. Senior secured and senior unsecured notes, which include loans and bonds, had lower recoveries and the more junior notes (almost all bonds) had, not surprisingly, the lowest recoveries. Note, the differential between the nominal and discounted recovery rates diminish somewhat at the lower seniority levels.

TABLE 3 APPROXIMATELY HERE

Standard & Poor's (Keisman, 2003) also finds, not shown in any Table, that during the most recent "extreme stress" default years of 1998 to 2002, the recovery rates on all seniorities declined compared to their longer 1988-2002 sample period in Table 3. Since 1998 and 1999 were not really high default years, the results of S&P for 2000-2002 are consistent with Altman, Brady, Resti and Sironi's (2001 and 2004) predictions of an inverse relationship between default and recovery rates. Indeed, recovery rates were a relatively low 25% in the corporate bond market for both 2001 and 2002 when default rates were in the double-digits but increased to about 45% in 2003 when default rates tumbled to below average annual levels of about 4.5 percent (Altman and Fanjul, 2004).

Some recovery studies have concentrated on rates across different industries. Altman and Kishore (1996) and FITCH (2003) report a fairly high variance across industrial sectors. For Example, Verde (FITCH, 2003) reports that recovery rates in 2001 vs. 2002 varied dramatically from one year to the next (e.g., Gaming, Lodging and Restaurants recovered 16% in 2001 and 77% in 2002, Retail recovered 7% in 2001 and 48% in 2002, while transportation recovered 31% in 2001 and 19% in 2002) but returned to more normal levels in 2003.

Another issue highlighted in some studies, especially those from S&P, (Van de Castle and Keisman, 1999 and Keisman, 2003) is that an important determinant of ultimate recovery rates is the amount that a given seniority has junior liabilities below its level; the greater the proportion of junior securities, the higher the recovery rate on the senior tranches. The theory being that the greater the "equity cushion," the more likely there will be assets of value, which under absolute priority, go first in liquidation or reorganization to the more senior tranches.

8. Concluding remarks

Table 4 summarizes the way RR and its relationship with PD are dealt with in the different credit models described in the previous sections of this paper.

TABLE 4 APPROXIMATELY HERE

While in the original Merton (1974) framework an inverse relationship between PD and RR exists, the credit risk models developed during the Nineties treat these two variables as independent. The currently available and most used credit pricing and credit VaR models are indeed based on this independence assumption and treat RR either as a constant parameter or as a stochastic variable independent from PD. In the latter case, RR volatility is assumed to represent an idiosyncratic risk which can be eliminated through adequate portfolio diversification.

This assumption strongly contrasts with the growing empirical evidence - showing a negative correlation between default and recovery rates – that has been reported in the previous section of this paper and in other empirical studies (Frye [2000b and 2000c], Altman [2001], Carey and Gordy [2003], Hamilton, Gupton and Berthault [2001], Altman, Brady, Resti and Sironi [2001, 2004]). This evidence indicates that recovery risk is a systematic risk component. As such, it should attract risk premia and should adequately be considered in credit risk management applications. The potential consequences – in terms of credit risk underestimation - of the PD and RR independence assumption when these two variables are instead correlated are shown by Altman, Brady, Resti and Sironi (2004).

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Appendix A

The relationship between PD and RR in the Merton model

Merton-like default models provide us with a framework for deriving the expected recovery rate on a defaulted firm, as well as its default probability. While the latter has been given much attention by subsequent research (see e.g. Crosbie, 1999), the former has been somewhat overlooked.

We briefly review the Merton model, emphasizing its implications for recovery rates, and showing how it can be used as a theoretical guideline to investigate the empirical link between default probabilities and severity.

In Merton-like models the asset value of the firm follows a geometric Brownian motion:

$$dV_A = \mu V_A dt + \sigma_A V_A dz$$

where μ and σ_A are the firm's asset value drift and volatility rate and *dz* is a Wiener process. This implies that the log of the asset value at a given future date *t*

$$\log V_A' = \log V_A + \left(\mu - \frac{\sigma_A^2}{2}\right)t + \sigma_A \sqrt{t}\varepsilon$$

follows a normal distribution with mean $\log V_A + \left(\mu - \frac{\sigma_A^2}{2}\right)t$ and variance $\sigma_A^2 t$. In turn, the asset value at time t will follow a lognormal distribution with mean $V_A e^{\mu t}$ and variance $V_A^2 e^{2\mu t}\left(e^{\sigma_A^2 t} - 1\right)$ (see e.g. Hull, 1997, for details).

Default happens if and only if, at time *t*, the value of the firm's assets, V'_A, is lower than its debt[14] X_t. That means that the firm's *probability of default, PD*, equals:

[14] Short-term debt due at time *t* can be used instead, since the inability to repay long term debt does not, by itself, trigger insolvency.

$$PD = p[V'_A < X_t] = p[\log V'_A < \log X_t] = p\left[\log V_A + \left(\mu - \frac{\sigma_A^2}{2}\right)t + \sigma_A\sqrt{t}\varepsilon < \log X_t\right] =$$

$$= p\left[\frac{\log\frac{V_A}{X_t} + \left(\mu - \frac{\sigma_A^2}{2}\right)t}{\sigma_A\sqrt{t}} < -\varepsilon\right] = \Phi\left(-\frac{\log\frac{V_A}{X_t} + \left(\mu - \frac{\sigma_A^2}{2}\right)t}{\sigma_A\sqrt{t}}\right) = \Phi(-d_2)$$

where $\Phi(.)$ is the normal c.d.f. and d_2 is similar to the quantity used in the standard Black-Scholes option pricing formula.

When default occurs, the recovery rate *RR* is given by the ratio of the asset value to the debt[15], V'_A/X_t. The expected recovery rate therefore is $E(V'_A/X_t)$, that is $E(V'_A)/X_t$. However, this is true only if $V'_A < X_t$, otherwise no default happens and no recovery can be observed. More formally, *the expected recovery rate, RR*, can then be defined as:

$$E\left(\frac{V'_A}{X_t}\middle| V'_A < X_t\right) = \frac{1}{X_t}E(V'_A | V'_A < X_t)$$

that is, as $1/X_t$ times the mean of a truncated lognormal variable. This, in turn, is given by:

$$E(V'_A | V'_A < X_t) = e^{\mu_* + \frac{\sigma_*^2}{2}} \frac{\Phi\left(\frac{\log X_t - \mu_*}{\sigma_*} - \sigma_*\right)}{\Phi\left(\frac{\log X_t - \mu_*}{\sigma_*}\right)}$$

(see Liu et al. [1997], for a formal proof), where $\mu_* = \log V_A + \left(\mu - \frac{\sigma_A^2}{2}\right)t$ and $\sigma_*^2 = \sigma_A^2 t$ are the mean and variance of log V'_A.

Plugging these two quantities into the above equation gives the following result:

[15] Assuming that bankruptcy costs are negligible.

$$E\left(V_A' \middle| V_A' < X_t\right) = e^{\log V_A + \mu t} \frac{\Phi\left(-\frac{\log \frac{V_A}{X_t} + \left(\mu + \frac{\sigma_A^2}{2}\right)t}{\sigma_A \sqrt{t}}\right)}{\Phi\left(-\frac{\log \frac{V_A}{X_t} + \left(\mu - \frac{\sigma_A^2}{2}\right)t}{\sigma_A \sqrt{t}}\right)} = V_A e^{\mu t} \frac{\Phi(-d_1)}{\Phi(-d_2)} = E(V_A') \frac{\Phi(-d_1)}{\Phi(-d_2)}$$

where the meaning of d_1 and d_2 is similar as in the standard Black-Scholes formula.

The expected recovery rate therefore turns out to be:

$$RR = E\left(\frac{V_A'}{X_t} \middle| V_A' < X_t\right) = \frac{V_A}{X_t} e^{\mu t} \frac{\Phi(-d_1)}{\Phi(-d_2)} = E\left(\frac{V_A'}{X_t}\right) \frac{\Phi(-d_1)}{\Phi(-d_2)}$$

Figure 1 shows a graphic representation of PD and RR. The left panel shows the normal distribution for $\log V_A'$, with PD given by the grey area on the left; the right panel shows the lognomal distribution for V'_A/X_t, the expected RR being the average of the values below 1, i.e., the mean of the values in the grey tail.

FIGURE 1 APPROXIMATELY HERE

Given the expressions for PD and RR derived above, we can run sensitivity analyses on the link between those two variables. Figures 2-4 consider the case of a firm with debt (X_t) worth 80, total assets (V_A) of 100, an annual asset volatility of 20% and an expected return on assets of 5%. This base case will be indicated by dotted vertical lines in the graphs; each time, one of the three main variables (X_t, V_A and σ_A) will be shocked (both halved and doubled) to see how PD and RR change.

FIGURE 2 APPROXIMATELY HERE

First, in Figure 2, we see that an increase in debt makes default more likely, while reducing the recovery rate on the defaulted loan (this could happen when a firm has to face an unexpected liability, e.g. because of legal claims due to polluting factories, oil leaks and so on). The opposite happens in Figure 3: when the initial value of the firm's assets is revised upwards (e.g., for a pharmaceutical concern announcing a new treatment for some lethal disease), the PD shrinks and the RR grows higher.

FIGURE 3 APPROXIMATELY HERE

Finally, in Figure 4, we see what happens when asset volatility increases. This could be the case of the telecommunications industry over the last three years: as the demand for e-commerce and Internet services has slowed down, the value of the investments made in broadband lines and UMTS licences has become more uncertain. From Figure 4 we see that, in such instances, an increase in asset volatility – even leaving leverage unchanged – brings about higher default probabilities and lower recovery rates.

FIGURE 4 APPROXIMATELY HERE

Table 1

Recovery at Default* on Public Corporate Bonds (1974-2003)
and Bank Loans (1989-Q3-2003)

Loan/Bond Seniority	Number of Issues	Median %	Mean %	Standard Deviation
Senior Secured Loans	155	73.00	68.50	24.40
Senior Unsecured Loans	28	50.50	55.00	28.40
Senior Secured Bonds	220	54.49	52.84	23.05
Senior Unsecured Bonds	910	42.27	34.89	26.62
Senior Subordinated Bonds	395	32.35	30.17	24.97
Subordinated Bonds	248	31.96	29.03	22.53
Discount Bonds	136	18.25	20.93	17.64
Total Sample Bonds	1,909	40.05	34.31	24.87

***Based on prices just after default on bonds and 30 days after default on loans.**

Source: K. Emery (Moody's), 2003 (Bank Loans) and Altman & Fanjul, 2004 (Bonds).

Table 2

Investment Grade vs. Non-Investment Grade (Original Rating)
Prices at Default on Public Bonds
(1974-2003)

Bond Seniority	N. of Issues	Median Price %	Average Price %	Weighted Price %	Standard Deviation %
Senior Secured					
Investment Grade	89	50.50	54.50	56.39	24.42
Non-Invest. Grade	283	33.50	36.63	31.91	26.04
Senior Unsecured					
Investment Grade	299	42.75	46.37*	44.05*	23.57
Non-Invest. Grade	598	30.00	33.41	31.83	23.65
Senior Subordinated					
Investment Grade	11	27.31	39.54	42.04	24.23
Non-Invest. Grade	411	26.50	31.48	28.99	24.30
Subordinated					
Investment Grade	12	35.69	35.64	23.55	23.83
Non-Invest. Grade	238	28.00	30.91	28.66	21.98
Discount					
Investment Grade	--	--	--	--	---
Non-invest. Grade	113	16.00	20.69	21.24	17.23
Total Sample	**2,054**	**30.04**	**34.76**	**30.78**	**24.38**

Notes: (*) Including WorldCom, the Average and Weighted Average were 43.53% and 30.45%
Non-rated issues were considered as non-investment grade

Source: Altman and Fanjul, 2004

Table 3

Ultimate Recovery Rates on Bank Loan Defaults
Nominal and Discounted Values

(1988-2Q 2003)

	Observations	**Ultimate Nominal Recovery**	**Ultimate Discounted Recovery**	**Standard Deviation**
Senior Bank Debt	750	87.32%	78.8%	29.7%
Senior Secured Notes	222	76.03%	65.1%	32.4%
Senior Unsecured Notes	419	59.29%	46.4%	36.3%
Senior Subordinated Notes	350	38.41%	31.6%	32.6%
Subordinated Notes	343	34.81%	29.4%	34.1%

Source: Keisman, 2003, from Standard & Poor's LossStats™ Database, 2084 defaulted loans and bond issues that defaulted between 1987-2003. Recoveries are discounted at each instruments' pre-default interest rate.

Table 4 – The Treatment of LGD and Default Rates within Different Credit Risk Models

	MAIN MODELS & RELATED EMPIRICAL STUDIES	TREATMENT OF LGD	RELATIONSHIP BETWEEN RR AND PD
Credit Pricing Models			
First generation structural-form models	Merton (1974), Black and Cox (1976), Geske (1977), Vasicek (1984), Crouhy and Galai (1994), Mason and Rosenfeld (1984).	PD and RR are a function of the structural characteristics of the firm. RR is therefore an endogenous variable.	PD and RR are inversely related (see Appendix A).
Second generation structural-form models	Kim, Ramaswamy e Sundaresan (1993), Nielsen, Saà-Requejo, Santa Clara (1993), Hull and White (1995), Longstaff and Schwartz (1995).	RR is exogenous and independent from the firm's asset value.	RR is generally defined as a fixed ratio of the outstanding debt value and is therefore independent from PD.
Reduced-form models	Litterman and Iben (1991), Madan and Unal (1995), Jarrow and Turnbull (1995), Jarrow, Lando and Turnbull (1997), Lando (1998), Duffie and Singleton (1999), Duffie (1998) and Duffee (1999).	Reduced-form models assume an exogenous RR that is either a constant or a stochastic variable independent from PD.	Reduced-form models introduce separate assumptions on the dynamic of PD and RR, which are modeled independently from the structural features of the firm.
Latest contributions on the PD-RR relationship	Frye (2000a and 2000b), Jarrow (2001), Carey and Gordy (2003), Altman, Brady, Resti and Sironi (2001 and 2004).	Both PD and RR are stochastic variables which depend on a common systematic risk factor (the state of the economy).	PD and RR are negatively correlated. In the "macroeconomic approach" this derives from the common dependence on one single systematic factor. In the "microeconomic approach" it derives from the supply and demand of defaulted securities.
Credit Value at Risk Models			
CreditMetrics®	Gupton, Finger and Bhatia (1997).	Stochastic variable (beta distr.)	RR independent from PD
CreditPortfolioView®	Wilson (1998).	Stochastic variable	RR independent from PD
CreditRisk$^+$®	Credit Suisse Financial Products (1997).	Constant	RR independent from PD
KMV CreditManager®	McQuown (1997), Crosbie (1999).	Stochastic variable	RR independent from PD

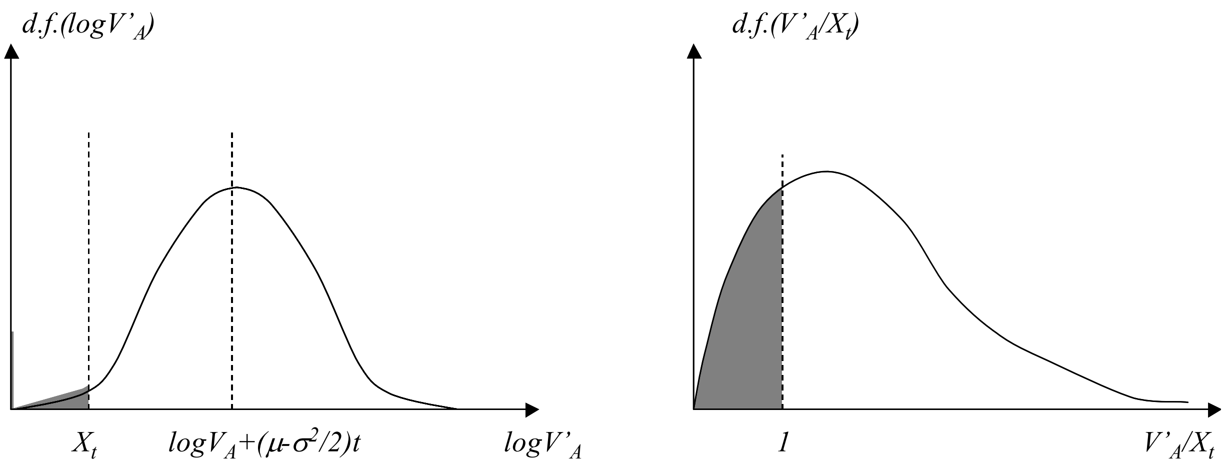

Figure 1

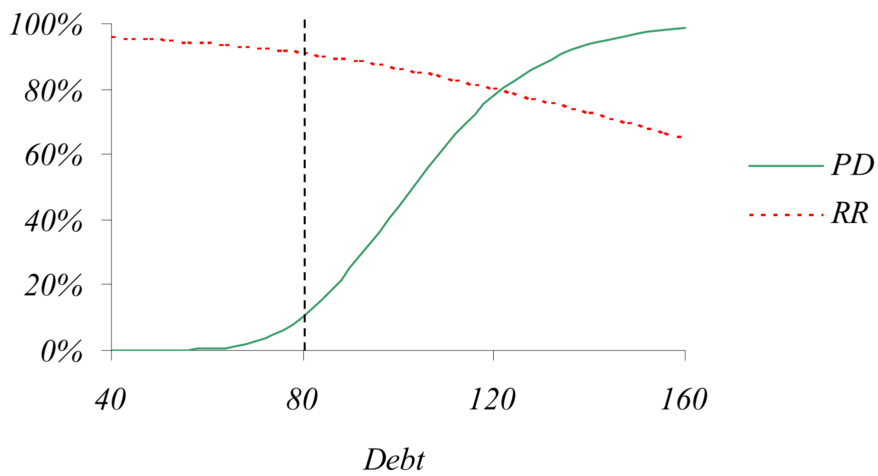

Figure 2: the effect of debt value on PD and RR

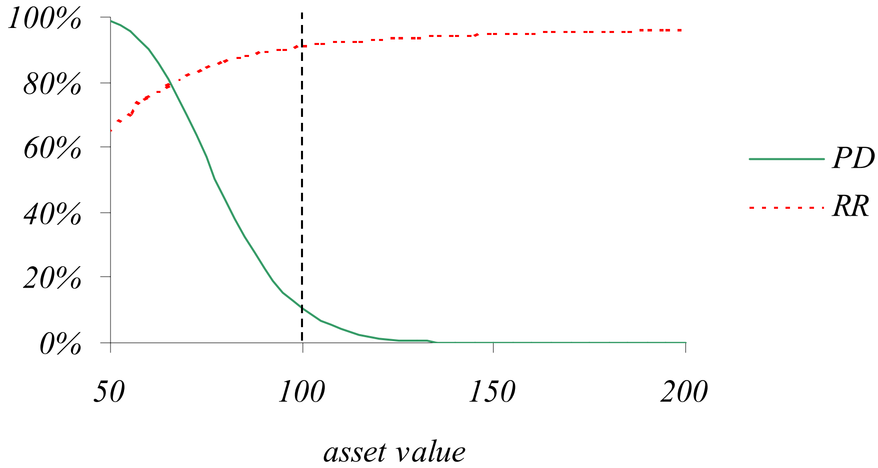

Figure 3: the effect of asset value on PD and RR

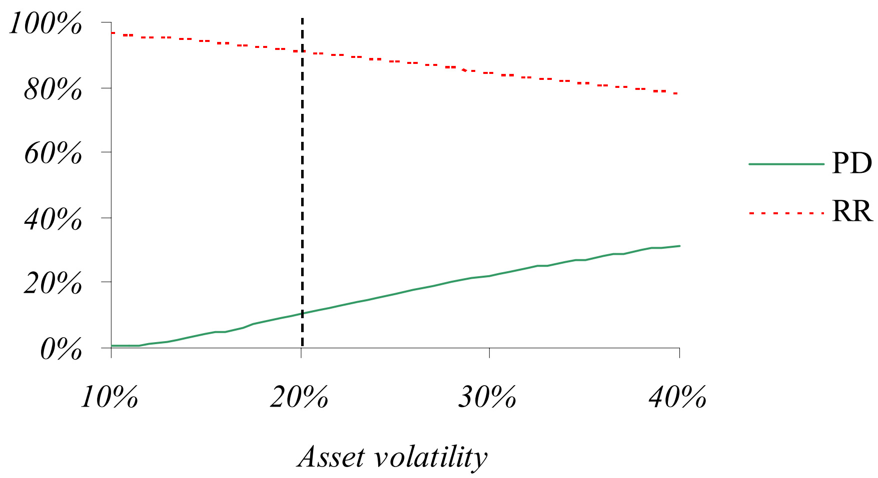

Figure 4: the effect of asset volatility on PD and RR