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A SIMULATION-BASED PRICING METHOD FOR CONVERTIBLE BONDS

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Abstract

We propose a pricing model for convertible bonds based on Monte Carlo simulation that is more flexible than previous lattice-based methods because it allows to better capture the dynamics of the underlying state variables. Furthermore, the model is able to deal with embedded American-style put and call features with path-dependent trigger conditions. The simulation method uses parametric representations of the early exercise decisions and consists of two stages. In the first stage, the parameters representing the exercise strategies are optimized on a set of simulated stock prices. Subsequently, the optimized parameters are applied to a new simulation set to determine the model price. In an empirical analysis, the model is found to provide a better fit compared to previous studies.

1. Introduction

To raise capital on financial markets, companies may chose among three major asset classes: equity, bonds, and convertible bonds.[1] While issues arising from valuing equity and bonds are extensively studied by researchers in academia and industry, remarkably few articles deal with the pricing of convertible bonds. This is even more surprising as convertible bonds cannot simply be considered as a combination of equity and bonds but present their own pricing challenges.

Convertible bonds are difficult to value because, as hybrid instruments, they depend on state variables related to the underlying stock (price dynamics), the fixed-income component (interest rates and credit risk), and the interaction between them. Convertible bonds contain various types of embedded options, such as conversion, call, and put provisions that have to be accounted for in any pricing model. These options often are restricted to certain periods, may vary over time, and are subject to additional path-dependent features of the state variables, which further complicates the valuation. Certain individual convertible bonds contain innovative, pricing-relevant specifications that require a flexible valuation model capable of dealing with them. Monte Carlo simulation provides a flexible tool that is suitable for this task. The purpose of this study is to propose a pricing method for convertible bonds based on Monte Carlo simulation that accounts for these pricing difficulties, to determine the pricing impact of various specifications of the dynamics of the state variables, and to perform a pricing study of the US convertible-bond market.

Theoretical research on convertible-bond pricing can be divided into three branches. The first pricing approach aims at finding a closed-form solution to the valuation problem. It was initiated by Ingersoll (1977) who applies the contingent-claims approach to the valuation of convertible bonds. In this valuation model, the convertible-bond price usually depends on the firm value as the underlying variable. Lewis (1991) follows this line of

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[1] The Bank for International Settlements reports an amount outstanding of 299.5 billion US dollars for international convertible bonds (not including domestic issues) per June 2002 (BIS 2002). The corresponding amount for international bonds of corporate issuers excluding financial institutions is 1012.1 billion US dollars.

research and develops a formula for convertible bonds that accounts for more complex capital structures, i.e. multiple convertible-bond issues. More recently, Benninga et al. (2002) develop under very restrictive assumptions (e.g. no call option, zero dividends, and one exercise date) a closed-form solution for convertible bonds with stochastic interest rates. While very fast in computation, closed-form solutions are not feasible for valuing real-world convertible bonds because they require an analytically tractable setting and fail to account for the various convertible-bond specifications. In particular, dividends and coupon payments are modeled continuously rather than discretely, callability is restricted to few functional forms, path dependencies cannot be included, and the underlying processes are rough approximations of the true dynamics.

The second pricing approach values convertible bonds numerically with lattices. Commercially available models for pricing convertible bonds, such as Bloomberg OVCV, Monis, and SunGard TrueCalc Convertible, belong to this category. The first theoretical model was introduced by Brennan and Schwartz (1977) who apply a firm-value-based approach and a finite-difference method for the pricing task. Brennan and Schwartz (1980) extend their pricing method by including stochastic interest rates. McConnell and Schwartz (1986) develop a pricing model based on a finite-difference method with the stock value as stochastic variable. To account for credit risk, they use an interest rate that is grossed up by a constant credit spread. Since the credit risk of a convertible bond varies with respect to its moneyness, Bardhan et al. (1993) and Tsiveriotis and Fernandes (1998) propose an approach that splits the value of a convertible bond into a stock component and a straight bond component. Hung and Wang (2002) propose a method that accounts for stochastic interest rates and time-dependent default probabilities. However, they employ a non-recombining binomial tree and assume zero correlation between the state variables. Unfortunately, lattice methods have some general problems: Computing time grows exponentially with the number of state variables, path dependencies cannot be incorporated easily, and the flexibility in modeling the underlying state variables is low.

The third class of convertible-bond pricing methods uses Monte Carlo simulation and may overcome many of the drawbacks of the lattice-based approaches. The relationship between the number of state variables and computing time is linear. This aspect is especially relevant for convertible bonds since they depend, due to their complex structure, on the dynamics of various state variables, such as the stock price, interest rates, default probabilities, and, in the case of cross-currency convertibles, exchange rates. Furthermore, Monte Carlo simulation is very well suitable to account for discrete coupon and dividend payments, to realistically model the dynamics of the underlying state variables via appropriate discretization schemes, and to account for path-dependent call features. Typically, path dependencies arise from the fact that, for many convertibles, early redemption is only allowed when the stock price exceeds a certain level for a pre-specified number of days in a pre-specified period of time. Despite all the natural advantages of the Monte Carlo approach, pricing American-style options such as those present in convertible bonds is a demanding task within a Monte Carlo pricing framework. In recent years, a considerable number of important articles has addressed the problem of pricing American-style options[2] by using a combination of Monte Carlo simulation and dynamic programming. Bossaerts (1989), Li and Zhang (1996), Grant et al. (1996), Andersen (2000), and García (2002) represent the early-exercise rule with a finite number of parameters. The optimal exercise strategy and hence the price of the American-style option is obtained by maximizing the value of the option over the parameter space. Carrière (1996), Tsitsiklis and Van Roy (1999), Longstaff and Schwartz (2001), and Clément et al. (2002) apply standard backward induction and estimate the continuation value of the option by regressing future payoffs on a set of basis functions of the state variables. Tilley (1993), Barraquand and Martineau (1995), and Raymar and Zwecher (1997) present methods based on backward induction that stratify the state space and find the optimal exercise decision for each subset of state variables. Broadie and Glasserman (1997a) and Broadie et al. (1997) propose a method for calculating prices of American-style options with *simulated trees*, also called *bushy trees*, which generates two estimates, one biased high and one biased low. Broadie and Glasserman (1997b), Avramidis and Hyden (1999), Broadie et al.

[2] In general, simulation techniques only allow for a finite number of early-exercise times and hence price Bermudan options rather than continuously exercisable American options. However, for a fairly large number of early-exercise dates, the Bermudan price may serve as an approximation for the price of the American option.

(2000), and Boyle et al. (2000) develop stochastic-mesh methods with different choices for the mesh weights. Finally, Haugh and Kogan (2001) and Rogers (2001) suggest a simulation method that uses a duality approach for pricing Bermudan options. So far, simulation methods have rarely been adopted to value convertible bonds. To our knowledge, the only academically recorded attempt to price convertible bonds with Monte Carlo simulation is performed by Buchan (1997, 1998). She applies the parametric optimization approach of Bossaerts (1989) by employing the firm value as underlying variable and allowing for senior debt. However, as pointed out by García (2002), the price obtained with this procedure is biased high because both the parameters of the early-exercise decision and the convertible-bond price are jointly estimated from the same simulation set.

In this paper, we contribute a stock-based pricing method for convertible bonds building on the enhanced Monte Carlo simulation method by García (2002). This is a two-stage method designed to cope with the Monte Carlo bias that is inherent in one-stage methods. It may be called a *parametric approach* because it uses a parametric representation of the early-exercise behavior of the investor and the issuer. The first step is an optimization, in which a set of Monte Carlo simulations is used to estimate parameter values representing strategies for early exercise under the condition that both the investor and the issuer exercise their early-exercise options in an optimal way. In a next valuation stage, the optimized parameters are applied to a second set of simulated stock-price paths to determine the model price of the convertible bond. Since convertible bonds usually have long maturities, it is crucial to model the dynamics accurately. As outlined above, the inherent strength of this approach is its flexibility in incorporating the dynamics of the state variables. We therefore allow for more realistic volatility dynamics, such as models from the GARCH family. Furthermore, besides discrete coupon and dividend payments, the introduced method accounts for path-dependent call triggers as outlined in the offering circulars. Instead of using a firm-value model, the stock price is modeled directly, as proposed by McConnell and Schwartz (1986). Whereas stock-price-based models can easily be estimated with standard methods, firm-value models are notoriously hard to calibrate because the companies' asset values are not observable.[3] Since the presented method is cash-flow based, credit risk can easily be incorporated by discounting the payoffs subject to credit risk with the appropriate interest rate.

In addition to the convertible-bond pricing model with various specifications for the underlying dynamics, this study contributes an empirical analysis of the US market. Despite the large size of international convertible-bond markets, very little empirical research on the pricing of convertible bonds has been undertaken. Previous research in this area was performed by King (1986), who examines a sample of 103 American convertible bonds with a lattice-based method and the firm-value as stochastic variable. Using monthly price data and a valuation model with Cox et al. (1985) stochastic interest rates (CIR), Carayannopoulos (1996) empirically investigates 30 American convertible bonds for a one-year period beginning in the fourth quarter of 1989. Buchan (1997) uses a simulation-based approach to implement a firm-value model with a CIR term-structure model for 35 Japanese convertible bonds. Buchan (1998) performs a pricing study for 37 US convertibles issued in 1994. However, the American property of convertible bonds is not accounted for in that study. Using data with daily frequency, Ammann et al. (2003) investigate the French market by applying a binomial tree with the stock price as stochastic variable.

A drawback of many of the previous pricing studies is the small number of data points per convertible bond: Buchan (1997) tests her pricing model only for one calendar day (bonds priced per March 31, 1994)[4], King (1986) for two days (bonds priced per March 31, 1977, and December 31, 1977), and Carayannopoulos (1996) for twelve days (one year of monthly data). Our study covers a larger sample with 69 months of daily price data, ranging from May 10, 1996, to February 12, 2002 and includes 32 convertible bonds in the US market.

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[3] The practical problems associated with firm-value models are discussed in several articles on credit risk modeling, such as Jarrow et al. (1997).

[4] All but one bond were out-of-the-money on March 31, 1994.

A second drawback of the previous pricing studies is the simple modeling of the volatility of the underlying stock. This shortcoming is almost inherent to the lattice approaches adopted by King (1986), Carayannopoulos (1996), and Ammann et al. (2003). Although Buchan (1997) uses a simulation-based approach, her approach does not fully exploit the potentials provided by Monte Carlo simulation as a constant volatility is used for the stock dynamics. Because of the importance of accurately modeling the stock dynamics, we implement besides a discrete version of the standard geometric Brownian Motion a GARCH(1,1) model and a fractionally integrated GARCH model. The latter is able to capture long-run dependencies in volatility which affect the relevant risk-neutral densities of the underlying stock prices.

The paper is organized as follows: First, we introduce the convertible-bond pricing model that will be applied in the empirical investigation. Second, we describe the data set and present the specific characteristics of the convertible bonds examined. Third, we discuss the model implementation. Finally, we present results of a sensitivity analysis and the empirical study comparing theoretical model prices with observed market prices.

2. Pricing Convertible Bonds with Monte Carlo Simulation

The American Option Pricing Problem for Convertible Bonds

A standard, plain-vanilla convertible bond is a bond that additionally offers the investor the option to exchange it for a predetermined number of stocks during a certain, predefined period of time.[5] The bond usually offers semi-annual coupon payments and, in case it is kept alive, is redeemed at the time of maturity T with a pre-specified amount κN, where N is the face value of the convertible bond and κ is the final redemption ratio in percentage points of the face value. Although κ is equal to one for most convertibles, some issues are redeemed at premium with κ larger than one. Let us consider discrete times with daily frequency, i.e. time t belongs to a finite set, $t \in \{0, 1, \ldots, T\}$, where $t=0$ indicates today, and $t=T$ the day of maturity. In the case of conversion, the investor receives $n_t S_t$, where the conversion ratio n_t is the number of stocks the bond can be exchanged for, and S_t is the equity price (underlying) at time t.[6] If the underlying stock differs from that of the issuing firm, the instrument is commonly called *exchangeable*. Almost all convertible bonds additionally contain a call option, allowing the issuer to demand premature redemption in exchange for the call price K_t applicable at time t. The issuer is obliged to announce the intention to call a certain period in advance, called *call-notice period*. For many issued convertible bonds, the provisions in the offering circulars limit the call option through a *call trigger* Ξ_t. The call trigger is a pre-specified level that the conversion value $n_t S_t$ has to reach before the issuer is allowed to call the convertible. In many cases, the inequality $n_t S_t > \Xi_t$ must hold for a certain time (usually 20 out of the last 30 trading days) before the bond becomes callable. If the convertible bond is called, the investor is allowed to exercise his conversion option at any time during the call-notice period to receive the conversion value instead of the call price. Furthermore, some convertibles permit the investor to exercise a put option, giving him a fixed amount P_t in exchange for the convertible bond,

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[5] Although we specifically address convertible bonds, the proposed model is also suitable for pricing convertible preferred stocks.

[6] Some convertibles in certain markets include a conversion option that entitles the holder to exchange the bond into shares of several different companies at his discretion, adding to the complexity of the instrument.

where P_t is smaller than K_t. The embedded options in convertible bonds may be restricted to certain times: $t \in \Omega_{conv}$ for the conversion option, $t \in \Omega_{call}$ for the call option, and $t \in \Omega_{put}$ for the put option. The set of possible call times Ω_{call} consists of the *call period* as specified in the offering contract given that the call-trigger condition is satisfied.[7] Typically, maturity is the last possible date to convert the bond.

Thus, the payoff of a convertible bond depends on whether and when the investor or the issuer decide to terminate the convertible bond by exercising their option. While the investor acts to maximize the value of the convertible bond, the issuer acts in the opposite way. The outcome of this interaction may either be conversion, a call, a put, forced conversion, or regular redemption when the bond matures. Further possibilities of the issuer to influence the value of the convertible bond, apart from the call strategy, are generally limited in the offering circulars. Otherwise, the company could directly influence the conversion value by setting out the dividend policy and deciding about stock splits as well as stock-repurchase plans. Through these measures, wealth could be redistributed from convertible-bond holders to equity holders.[8]

Let τ^* be the optimal stopping time at which it is optimal for either the issuer or the investor to terminate the convertible-bond contract. The optimal stopping time of the convertible bond is defined as $\tau^*=min\{t:p(X,t)\neq 0\}$, where $p(X,t)$ is the payoff in state X at time t resulting from the convertible bond due to the exercise decisions. Formally, $p(X,t)$ is represented by:

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[7] The call-trigger condition can easily be checked in a Monte Carlo framework by a backward-loop that counts the number of days at which the conversion value was higher than the call trigger during the relevant period.

[8] This argument does not apply to exchangeables because the issuing company has no means to influence the firm value.

$$
p(X,t)=\begin{cases}
n_tS_t & \text{if} \quad n_tS_t > V_t' \quad \text{for } t\in\Omega_{conv} \quad \text{(Conversion)}\\
 & \text{and} \quad P_t \le V_t' \quad \text{for } t\in\Omega_{put}\\
P_t & \text{if} \quad P_t > V_t' \quad \text{for } t\in\Omega_{put} \quad \text{(Put)}\\
 & \text{and} \quad n_tS_t \le V_t' \quad \text{for } t\in\Omega_{conv}\\
K_t & \text{if} \quad V_t' > K_t \quad \text{for } t\in\Omega_{call} \quad \text{(Call)}\\
 & \text{and} \quad K_t \ge n_tS_t \quad \text{for } t\in\Omega_{call}\cap\Omega_{conv}\\
n_tS_t & \text{if} \quad V_t' > K_t \quad \text{for } t\in\Omega_{call} \quad \text{(Forced Conversion)}\\
 & \text{and} \quad n_tS_t > K_t \quad \text{for } t\in\Omega_{call}\cap\Omega_{conv}\\
\kappa N & \text{if} \quad t=T \quad \text{(Redemption)}\\
 & \text{and} \quad n_tS_t \le \kappa N \quad \text{for } t\in\Omega_{conv}\\
0, & \text{otherwise} \quad \text{(Continuation)}
\end{cases}
$$

where V_t' is the conditional expected value of continuation, i.e. the value of holding the convertible bond for one more period instead of exercising immediately. The presented alternatives stand for all events that will cause the convertible bond to be terminated. Besides when reaching maturity, the convertible bond will be ended by a conversion into stock, by a call, or by a put. The convertible bond is called by the issuer when the conversion value exceeds the call price and when it is permitted to call the convertible bond. Besides by the call period as stated in the offering circulars, the possibility to call the convertible bond may be restricted by the call-trigger condition. In the case of a call, the investor will convert the bond if the conversion value is above the call price (forced conversion), otherwise he will prefer to have it redeemed. As soon as the investor could receive more than the continuation value through immediate option exercise, he will terminate the contract by either converting the bond voluntarily or by putting it, whichever yields the highest payoff. Thus, the convertible bond should be kept alive only as long as $max(n_tS_t;P_t)\le V_t'\le K_t$, i.e. neither the investor nor the issuer execute their options that terminate the convertible bond.

In addition to the payoff at the time of termination, the investor receives from the convertible-bond investment all coupon payments that occurred prior to this date. Formally, the function $h(X, \tau^*)$ represents the payoff from a convertible bond in state X at time τ^*:

$$h(X,\tau^*) = p(X,\tau^*) + c(\tau^*),$$

where $p(X, \tau^*)$ is the payoff from the convertible bond at termination and $c(\tau^*)$ is the present value at time τ^* of all coupon payments[9] made during the existence of the bond, i.e. before τ^*.

The price of a convertible bond can be obtained by discounting under the risk-neutral measure all future cash flows with the risk-free interest rate r that is applicable from time zero to τ^*. Thus, valuing convertible bonds implies determining

$$V_0 = E^Q\left[e^{-r\tau^*} h(X,\tau^*)\right],$$

where V_0 is the current value of the convertible bond, τ^* is the optimal stopping time taking values in the finite set $\{0,1,\ldots,T\}$, the function $h(X,\tau^*)$ represents the payoff from a convertible bond with embedded call and put options in state X at time τ^*, and the expectation $E^Q[\cdot]$ is taken with respect to the equivalent Martingale measure Q defined using the riskless security as the numeraire.

Characterizing the Optimal Exercise Decision

Before maturity, the optimal exercise strategy implies comparing the value of immediate exercise with the value from continuing, i.e. not exercising this period. The crucial step implies determining the conditional expected value of continuation V_t'. Formally, the value at a future time t of a convertible bond which is not exercised immediately, but held for one more period, is given by

$$V_t' = E\left[e^{-r(\tau^*-t)} h(X,\tau^*) \middle| I_t\right] \qquad \text{with } \tau^* > t,$$

where I_t represents the information available at time t.

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[9] The coupon payments also include accrued interests when agreed upon in the offering contract.

While the investor will convert the bond as soon as $n_tS_t > V_t'$ for $t \in \Omega_{conv}$, the issuer will call the convertible as soon as $V_t' > K_t$ for $t \in \Omega_{call}$. Thus, at each point in time, both investor and issuer decide whether they want to exercise their option or not. In both cases, the optimal exercise boundary is the unknown continuation value of the convertible bond, V_t'. Since investor and issuer have different optimization objectives, we introduce two boundaries described by functions of the state variables and time, $G(X,t;\theta_{t,inv})$ and $G(X,t;\theta_{t,iss})$, representing the exercise strategies for the investor and issuer, respectively, where $\theta_{t,inv}$ and $\theta_{t,iss}$ are the parameter sets describing the exercise functions. If the conversion value or put price at any relevant time exceeds the corresponding exercise boundary $G(X,t;\theta_{t,inv})$, the investor will exercise her option at that time. Correspondingly, as soon as the call price falls below the call boundary $G(X,t;\theta_{t,iss})$, the convertible bond will be called. In all other cases, it will continue to exist, unless it reaches maturity.

Simulation Methodology

The pricing algorithm consists of two stages, an optimization stage and a valuation stage. In the first stage, the exercise strategies of the investor and of the issuer are estimated using a set of simulated conversion-price paths. The relevant stopping times $\tau_i(\theta_{inv}, \theta_{iss})$ for each path i and the corresponding payoffs $h(X_i, \tau_i)$ for determining the convertible-bond value in each simulation path are obtained by applying an exercise rule to the simulated paths. θ_{inv} and θ_{iss} are the parameter sets describing the exercise behavior of the investor and the issuer over time. As soon as either the conversion value or the put price exceeds the corresponding boundary, or the call price lies below the call boundary for the first time, the convertible bond will cease to exist and the occurring payoffs can be determined.

Action	Condition	Payoff
Conversion	$n_tS_t>G(X,t;\theta_{t,inv})$ and $P_t\leq G(X,t;\theta_{t,inv})$	n_tS_t
Put	$P_t>G(X,t;\theta_{t,inv})$ and $n_tS_t\leq G(X,t;\theta_{t,inv})$	P_t
Call	$G(X,t;\theta_{t,iss})>K_t$ and $K_t\geq n_tS_t$	K_t
Forced Conversion	$G(X,t;\theta_{t,iss})>K_t$ and $K_t<n_tS_t$	n_tS_t
Redemption	$t=T$ and $n_tS_t\leq\kappa N$	κN

The value of the convertible bond under its associated exercise policy can be calculated by averaging the discounted payoffs of every simulation path:

$$V(\theta_{inv},\theta_{iss})=\frac{1}{M}\sum_{i=1}^{M}e^{-r\tau_i(\theta_{inv},\theta_{iss})}h(X_i,\tau_i(\theta_{inv},\theta_{iss})).$$

X_i are realizations of the simulated conversion values and M is the number of simulation paths. The initial parameters for the exercise strategy of the investor will be altered given the parameters for the call strategy and vice versa until the algorithm finds the optimal exercise behavior for investor and issuer, represented by the parameter estimates $\hat{\theta}_{inv}$ and $\hat{\theta}_{iss}$. In the second stage, the optimized exercise strategies from the first stage are applied together with a second set of simulated conversion-price paths to determine the value of the convertible bond $V(\hat{\theta}_{inv},\hat{\theta}_{iss})$. The procedure of using a second simulation set augments the accuracy of the pricing because the optimized parameters are not applied to the same simulation set from which they were estimated.

3. Data

Daily convertible-bond prices as well as the corresponding stock prices were made available by Mace Advisers. All domestic convertible bonds on the US market outstanding as of February 12, 2002, are considered for the empirical analysis. Convertible bonds with embedded put options and cross-currency convertibles are excluded. Furthermore, to estimate the parameters of the stock dynamics, only convertible bonds with a pre-sample stock history dating back at least until January 1, 1990, are included into the sample. Moreover, for all convertible bonds in the sample, we require that a rating is provided by Standard & Poor's Bond Guide, and – to be able to account for all relevant specifications for each convertible bond in detail – that the official and legally binding *offering circulars* are available. The latter proved to be necessary because some contractual provisions are so specific that they can hardly be collected in predefined data types, and electronic databases usually lack the needed flexibility to incorporate non-standard features. Rating changes for the individual issues were followed up according to the monthly publications in Standard & Poor's Bond Guide. To account for a possible publication lag and additional potential delays of rating adjustments by the rating companies, we apply a filter that eliminates forty data points preceding rating changes that lead to a credit-spread change of at least 2 percentage points. As an additional liquidity requirement, we only consider data points with a bid-ask spread of less than 2 percentage points for both the convertible bond and the underlying stock.

After filtering the sample with these criteria, we obtain a final sample of 32 convertible bonds, with price data ranging from May 10, 1996, to February 12, 2002. A general description of these convertible bonds is presented in Table 1 and Table 2. Most analyzed convertibles include a call option, allowing the issuer to repurchase the bond for a certain price K_t, called *call price* or *early-redemption price*. When a convertible bond is called, the issuer has to notify the investor a certain period in advance about his intention to call the convertible. This provision bears some risk for the issuer in form of a failed forced conversion, in which case the issuer will have to redeem the bond in cash instead of shares. Thus, the issuer might want to avoid this eventuality by delaying the call. The call-notice period in the US market is generally 30 days. However, for the convertibles in the sample, the call-notice period varies across the individual issues with values between 15 and 30 days. Usually, the call price varies over time, but is piecewise constant. For almost all examined convertibles, early redemption through a call is restricted during certain predetermined times, referred to as *call-protection periods*. Most issues are not callable during a certain time immediately after issuance. This feature is called *hard call protection.* Subsequently, callability may be restricted by a *call condition,* according to which the issuer is allowed to call the convertible only if the conversion value $n_t S_t$ exceeds the *call trigger* Ξ_t. The protection given by the trigger is referred to as *soft call protection*. For the convertibles in the sample with a call trigger, the contractual specification of the call condition states that the conversion value must exceed the call trigger for 20 out of the last 30 trading days before the bond becomes callable. This *qualifying period* introduces a path-dependent feature that can be accounted for better by a convertible-bond-pricing method based on Monte Carlo simulation than by conventional lattice methods. Usually, the call trigger is calculated as a percentage of either the early-redemption price or the face value. More often than not, the conversion ratio n_t is constant over time. It changes in case of an alteration of the nominal value of the shares (stock subdivisions or consolidations), extraordinary dividend payments, and other financial operations that directly affect the stock price. Since stock splits are very common in the US market, the conversion ratio often changes over time and deviates quite substantially from the initial values stated in the offering circulars. To accommodate for this, we apply an equity-correction factor and use the adjusted conversion ratio at each point in time. Conversion is possible during a certain period, called *conversion period.* The conversion period starts at time t_{conv} and ends at time T_{conv}. For all issues in our sample, the end of the conversion period coincides with the maturity of the convertible bond, i.e. $T_{conv} = T$. Some convertibles in the US market are *premium-redemption* convertibles, i.e. the redemption price at maturity is above par. In this case, the final redemption price is given by κN with the final redemption ratio κ larger than one. However, in our sample, all convertible bonds have a terminal redemption price of 1000 dollars and κ is equal to one. Furthermore, while some convertible bonds in the US market are traded dirty, all bonds in our sample are traded clean.

The 32 convertible bonds in the sample correspond to 5013 data points. Figure 1 and Figure 2 present summary statistics of the convertibles for these data points and thus give a richer picture than a static breakdown performed at one date only. Since the ratings of the individual issues change substantially over time, the histogram in Figure 1 gives a more detailed picture as it presents the number of data points for each rating category. The convertibles in the sample cover all categories in the Standard & Poor's rating scheme ranging from A- to CCC-. The absence of convertible bonds with high investment grade ratings (AAA and AA) and the presence of low-rated convertibles (CCC) in our sample reflects the phenomenon that, in the US market, primarily small companies issue convertible bonds while more established companies rely on other means of financing. While a substantial degree of rating migration occurred, none of the convertible bonds in our sample actually defaulted during the examination period.

Figure 2 presents the frequency of individual convertible-bond data points for various maturity classes. While the convertible bonds in the US market have maturities of up to 30 years, the issues in our sample cover maturities ranging from half a year to slightly more than ten years and have a mean maturity of approximately five years. Thus, within the class of derivative instruments, convertible bonds have the longest maturities of all. They even largely surpass long-term options that seldom reach up to three years, with potentially important implications for accurately modeling the evolution of the stochastic variables.

4. Model Implementation

Stock Dynamics

One of the most important determinants for convertible-bond prices is the evolution of the underlying stock price over time. Although empirical evidence shows that volatility changes over time, the traditional option-pricing model as proposed by Black and Scholes (1973) assumes constant volatility. Amin and Ng (1993) and Duan (1995a,b) show that it is possible to allow for more realistic volatility patterns by making fairly weak assumptions about the joint distribution of the stock-price process and the marginal rate of intertemporal substitution of a representative investor. These analyses provide the theoretical framework for empirical option-pricing studies that employ sophisticated stochastic processes for the stock price (see e.g. Bollerslev and Mikkelsen, 1999).

While research on stock volatility is plentiful, there is no consensus on which model should be applied for forecasting. A popular approach is the implied volatility concept. With option-pricing formulas, it is possible to extract market participants' volatility estimations for various horizons from at-the-money option prices. However, for three reasons, implied volatility is not suitable as input for the forecasting task in this analysis. First, most liquid options have maturities that are much shorter than the maturities of convertible bonds. Thus, the extracted implied volatilities do not cover the time horizons needed. Second, empirical research is not conclusive with respect to whether implied volatility truly is an unbiased estimator of realized volatility and therefore might be suitable for forecasting or not. While the results reported by Day and Lewis (1992), Harvey and Whaley (1992), and Canina and Figlewski (1993) cast doubt on the hypothesis that implied volatility is an unbiased estimator of realized volatility, the study performed by Christensen and Prabhala (1998), among others, seems to support it. Finally, the accuracy of implied volatilities obtained from option data depends on the accuracy of the option pricing model used. Since we aim at examining the accuracy of the presented convertible-bond pricing model and want to avoid any biases from using a potentially wrong option-pricing model, we refrain from using implied volatility in this analysis. For these reasons, we consider other alternatives and focus on possibilities to generate forecasts with volatility models that were fitted on historical data.

Several studies indicate that stock-market volatility exhibits long-run dependencies and that these features can be captured best by fractionally integrated processes. Baillie et al. (1996) propose the FIGARCH(p,d,q) model type which is able to account for the mentioned characteristics. This model allows the conditional variance σ_t^2 to evolve in the following way:

$$\sigma_t^2 = \omega[1-\beta(1)]^{-1} + \left[1-(1-\beta(L))^{-1}\cdot\phi(L)\cdot(1-L)^d\right]\varepsilon_t^2,$$

where ω, β, ϕ as well as d are constant parameters, L is the lag operator, and ε_t are return shocks drawn from a normal distribution with a mean of zero and conditional variance σ_t^2. The stock returns are modeled as $y_t=r+\varepsilon_t$ with a drift term r.

The expression $(1\text{-L})^d$ is defined by its binomial expansion as

$$(1-L)^d = \sum_{j=0}^{\infty}\Gamma(j-d)\cdot\Gamma(j+1)^{-1}\cdot\Gamma(-d)^{-1}\cdot L^j,$$

where $\Gamma(\cdot)$ denotes the Gamma function. The FIGARCH(p,d,q) model differs from a GARCH(p,q) model as it additionally contains the fractional integrating parameter d. This allows shocks to the conditional variance to decay at a slow hyperbolic rate.

We implement a FIGARCH(1,d,1) model and, to determine the pricing impact of alternative volatility models, we also apply the discrete version of a standard geometric Brownian Motion model with constant volatility, $dS_t=S_t r dt+S_t\sigma_S dW_{S,t}$, and a GARCH(1,1) model. In the GARCH(1,1) model, the conditional variance evolves as

$$\sigma_t^2 = w + a\varepsilon_{t-1}^2 + b\sigma_{t-1}^2,$$

where ε_t are return shocks drawn from a normal distribution with a mean of zero and conditional variance σ_t^2; w, a, and b are constant parameters to be estimated.

In the empirical part, the chosen volatility models are calibrated with both simulated and historical data. The parameter set ψ is chosen to maximize the likelihood function

$$\ln L(\psi;\varepsilon_1,\varepsilon_2,...,\varepsilon_T) = -0.5\ln(2\pi) - 0.5\sum_{t=1}^{T}\left(\ln(\sigma_t^2)+\left(\frac{\varepsilon_t^2}{\sigma_t^2}\right)\right)$$

under the assumption of normally distributed innovations ε_t. Subsequently, the estimated parameters are used for the simulations. The estimated parameters of the volatility models are presented in Table 4 for each convertible bond in the sample.

Interest-Rate Dynamics

To determine the impact of a term-structure model on convertible-bond prices, we implement the pricing model with stochastic interest rates and perform a sensitivity analysis. The interest rates are assumed to follow a CIR process and are correlated with the stock market. The process in the CIR model is given by

$$dr_t = \kappa(\theta_r - r_t)dt + \sigma_r\sqrt{r_t}\,dW_{r,t},$$

where κ, θ_r, and σ_r are constant. When simulating the interest-rate paths discretely, we have to ensure that they remain positive. This is accomplished by considering the process of the natural logarithm of the interest rate, $x_t=ln(r_t)$. By Ito's Lemma we obtain the following dynamics for x_t

$$dx_t = \left(\frac{\kappa\theta_r}{\exp(x_t)} - \kappa - \frac{\sigma_r^2}{2\cdot\exp(x_t)}\right)dt + \frac{\sigma_r}{\sqrt{\exp(x_t)}}dW_{r,t}.$$

Consistently with the chosen finite time set the simulated discrete paths have a sampling interval of one day. To avoid the error due to discretization accompanied with the simple Euler scheme, we use the more accurate scheme proposed by Milstein (1978):

$$x_{t+\Delta t} = x_t + \left[\frac{\kappa\theta_r}{\exp(x_t)} - \kappa - \frac{\sigma_r^2}{2\exp(x_t)} + \frac{\sigma_r^2}{4\exp(2x_t)}\right]\Delta t + \frac{\sigma_r}{\exp(0.5x_t)}\varepsilon_{t+\Delta t} - \frac{\sigma_r}{4\exp(2x_t)}\varepsilon_{t+\Delta t}^2.$$

For the empirical analysis, a model with constant interest rates is chosen. This is motivated by the results obtained in the sensitivity analysis shown later in this study. For low correlations between the stock price and interest rates, the pricing impact of a CIR term-structure model is rather low. This applies to the analyzed convertibles with in-sample correlations ranging from –0.04 to 0.1. Hence, since the empirical analysis is computationally intensive due to the large number of data points and the applied volatility processes, the inclusion of stochastic interest rates does not seem to be appropriate. However, since computational time grows only linearly with the number of state variables, stochastic interest rates may be included in the presented approach at relatively low cost for less extensive pricing applications. This is shown in the sensitivity analysis where the model is implemented with stochastic interest rates.

We choose as input for the empirical pricing study spot interest rates obtained from the Federal Reserve. The time series of the risk-free interest rates are extracted from T-Bill and T-Note prices and cover maturities from 3 months to 30 years on a daily basis. We obtain through interpolation the complete continuous term structure of spot rates at any time. Thus, with the complete term structure available at each point in time, it is straightforward to discount the cash flows occurring at different dates with the corresponding interest rates.

Integrating Credit Risk

Since the simulation-approach presented in this paper is cash-flow based, we incorporate credit risk by discounting the cash flows subject to credit risk with the appropriate risk-adjusted interest rate. This applies to coupon payments, the final redemption payment, and the call price in the event of a call. The stock price, on the other hand, is not subject to credit risk and should therefore be discounted with the risk-free interest rate. In this approach, credit spread can be implemented as constant or as following a process correlated with other state variables.

Unfortunately, for most convertible bonds in the sample, there are no straight bonds outstanding, let alone with a maturity corresponding to that of the convertible bond, that could be used to extract the appropriate issue-specific credit spreads for the implementation. In addition, such a procedure to obtain the credit spreads has the drawback that it does not account for issue-specific characteristics of the convertible bonds, such as seniority. Thus, to obtain credit spreads, we extract from the Yield Book database monthly time series of credit spreads for several rating categories according to Standard & Poor's

Bond Guide. For all investment-grade rating categories, we further obtain monthly credit-spread time series covering four maturity classes (1-3, 3-7, 7-10, and over 10 years). While this procedure allows to account for issue-specific convertible-bond characteristics through applying the rating, it has several obstacles that potentially could influence the pricing results. First, the credit spreads represent averages of bonds outstanding within the same rating category. Second, ratings change over time and the publication we refer to only has a monthly updating frequency. Additionally, this procedure does not account for potential lags and, more importantly, differences between market valuations and the rating decisions of the company that is performing the ratings. The resulting estimation error of the credit spreads is potentially very relevant in our sample as it primarily consists of bonds with low ratings and relatively high credit spreads.

Coupons and Dividends

We accommodate for discrete coupon and dividend payments at the appropriate dates and with the appropriate frequencies: semi-annual for coupons and quarterly, semi-annually, or annually for dividends, depending on the current dividend policy of the company. For each pricing, we assume that the dividend yield at the last ex-dividend date remains constant and applies to all future dividend payments until maturity.

Numerical Implementation

We implement the algorithm in C and use as source for normally distributed random numbers the Box-Muller method. Correlated random numbers are obtained by Cholesky decomposition. Equally distributed random deviates are generated by the linear congruential generator proposed by L'Ecuyer (1988) with additional Bays-Durham shuffles as described in Press et al. (1992). Each pricing point is computed with a different starting point of the random-number sequence (seed). In order to compare the results of different pricing runs, the seed attributed to one pricing point is held constant. As optimization method for the first stage, i.e. maximizing the value of the convertible bond given a simulation set for the conversion value, we employ the simplex method originally proposed by Nelder and Mead (1965). The simplex is iterated until any additional step changing the conversion (call) boundary cannot increase (decrease) the value of the convertible bond by an amount larger than a tolerance of 0.1 dollars.

At any point in time, the exercise rules $G(X,t;\theta)$ of both the investor and the issuer are made linearly dependent on the stock price. The time to maturity is divided into 10 sub-periods of equal length with constant parameters θ_{inv} and θ_{iss}. This approach has the advantage that the American-style conversion option is applied to every time step, which in our setting is one day. Consequently, the number of parameters representing the exercise strategies may be limited while they still allow for early exercise at every day.

5. Results

The simulation-based pricing method is implemented for a sensitivity analysis and an empirical investigation of the US convertible-bond market. The sensitivity analysis investigates the pricing behavior of the model, measured using fictive convertibles under various parameter values describing the evolution of the state variables. More specifically, the results show the impact of different specifications for credit risk, interest rates, and stock volatility. The empirical study focuses on the relationship between prices generated by the model and observed market prices of a representative sample of US convertibles.

Sensitivity Analysis

As for straight bonds, credit risk has a major impact on the fair value of a convertible. In contrast to standard bonds, however, the effect of the credit spread on prices is asymmetric, because the size of the impact critically depends on whether the convertible is in- or out-of-the-money. As can be seen in Figure 3, changes in the credit spread are largest for out-of-the money convertibles, with decreasing magnitude the further the convertible is in-the-money. Naturally, for bonds with maturities of 10 years, credit-spread shocks and misspecifications of only a small magnitude potentially have a large price impact.

We now investigate the effect of introducing stochastic interest rates. Convertible-bond prices generated by a pricing model under the assumption of constant interest rates are compared with prices generated by a model that incorporates a CIR term structure. The comparison is performed in Table 3 for several initial stock prices and correlation values between the stochastic processes of the two state variables.[10] To keep the analysis simple, a zero-coupon convertible bond with no default risk and no call or put features is chosen for the valuation. The table displays the convertible-bond price under stochastic interest rates, the percentage deviation to the price obtained with constant interest rates, and the pricing accuracy of the simulation represented by the standard deviation. The pricing effect of

[10] Different initial stock prices imply different moneyness values. In analogy to standard option theory, moneyness is defined as the ratio of the conversion value, i.e. the value of shares that can be obtained by converting the bond, and the investment value, i.e. the value of the convertible bond under the hypothetical assumption that the conversion option does not exist.

stochastic interest rates is largest for at-the-money convertible bonds and fades out for higher and lower moneyness values. This result is consistent with the theory because deep in-the-money convertible bonds are almost equivalent to stocks and hence the dynamics of the interest rates should not affect their prices. Similarly, far out-of-the money convertibles are practically equivalent to straight bonds because the probability of conversion approaches zero.[11] Straight bonds must have the same prices under stochastic and constant interest rates because, in our setting, both interest-rate regimes imply the same term structure. Actually, the fact that prices of out-of-the-money bonds with and without stochastic interest rates are almost equivalent proves that the implemented term-structure model is capable of generating arbitrage-free bond prices. The presented results show further that a positive correlation between stock process and interest rates increases convertible-bond prices, whereas a negative correlation lowers them. The largest price discrepancy is obtained for a moneyness of 1.03 and a correlation of +0.5 which is the highest considered in this analysis. Overall, the inclusion of stochastic interest rates generates the highest price deviation for at-the-money convertible bonds and a high absolute correlation between stock price and interest rates.

The third sensitivity analysis investigates the effect of various stock-price dynamics on prices generated by the model. We simulate, as an example, sample paths covering 50 years (13050 trading days) for the conversion value under the assumption that the true data generating process (DGP) is a FIGARCH(1,d,1). This is motivated by the empirical fact, documented by Baillie et. al. (1996), that the conditional stock volatility seems to exhibit long-run dependencies. The parameter sets for a FIGARCH(1,d,1) model, a GARCH(1,1) model and a discrete version of the standard geometric Brownian Motion are then estimated on the sample paths to best capture the dynamics of the underlying stock. Subsequently, the estimated parameters are applied in connection with the simulation-based model to value a convertible bond with suitably chosen specifications. The results are presented in Figure 4 and Figure 5. The plots in each diagram are obtained by dividing the convertible bond

[11] The fact that, for far out-of-the-money convertibles, very few paths reach a sufficiently high level to trigger conversion at maturity is shown by the low values of the reported standard deviation.

prices obtained with the three estimated models through prices obtained with the original DGP-FIGARCH(1,d,1) and then subtracting one. The fluctuation of the lines is mainly due to the finite simulation set of 3000 paths used for each pricing point.

In Figure 4, the prices obtained with the three estimated volatility models for increasing moneyness are depicted in relation to the prices obtained with the DGP-FIGARCH(1,d,1). The moneyness is computed by dividing the conversion value through the investment value. The plots indicate that the choice of the volatility model is not crucial for both far out-of-the money convertibles and in-the-money convertibles. The small price discrepancy between the three models in the far in-the-money zone is fully attributable to the numerical pricing procedure and will consequently disappear when using more simulation paths. However, for at-the-money convertible bonds, the price deviations are quite substantial despite the relatively low unconditional volatility (13.9%) of the original path generated by the DGP. While the estimated FIGARCH(1,d,1) model provides the best fit in comparison to the DGP-prices, the prices generated by the GARCH(1,1) and Brownian Motion models perform worst, with price deviations of up to two percent.

Figure 5 depicts the price deviation for at-the-money convertible bonds and various maturities up to 30 years which corresponds to 7830 trading days under the assumption of 261 trading days per year. In order to isolate the maturity effect, we artificially keep the moneyness constant by setting interest rate, credit spread, and coupon payments equal to zero. Otherwise, an increase in maturity would alter the moneyness of the convertible bond and hence tend to reduce the pricing discrepancy. For all estimated volatility models, the price deviation from DGP-prices increases with maturity following a non-linear relationship and already for relatively short maturities reaches substantial levels. Again, the estimated FIGARCH(1,d,1) model performs best for all maturities with maximum price deviations of only up to 1.1 percent, while the GARCH(1,1) and the Brownian Motion model lead to higher price deviations with maximum values of more than 2.5 percent for the longest maturities considered. Overall, the simulation results indicate that the volatility dynamics have an important pricing impact on convertible bonds. As shown by the example, the pricing difference can be even larger than the impact of a term-structure model.

In summary, the presented simulation-based model can deal with several stochastic variables affecting the price of the convertible bond. The size of the pricing impact significantly depends on the assumed process parameters. In particular, this study shows evidence for an important influence of stock-volatility dynamics on convertible-bond prices.

Empirical Analysis of the US Convertible-Bond Market

The model will now be tested with real convertible-bond data. Theoretical prices obtained using the proposed simulation-based model are compared with convertible-bond prices observed in the US market. First, a simple model specification is implemented with standard geometric Brownian Motion (GBM) as process for the underlying stock price and with credit spreads derived from the issue-specific ratings by Standard & Poor's. Afterwards, results for alternative specifications are presented. Figure 6 exhibits the distribution of percentage deviations between model prices and empirical prices for the simple GBM specification. In general, the pricing accuracy may be defined in terms of the average deviation or in terms of the tracking error between model and market prices. The tracking error is calculated as root mean squared error (RMSE) of the difference between model and market prices. On average, generated prices are 0.34% lower than market prices, with a standard deviation of 6.04% and a RMSE of 6.05%. Previous studies based on the firm value report larger mean price deviations. King (1986) investigates a sample of 103 American convertible bonds and finds that market prices are 3.75% below model prices on average. Carayannopoulos (1996) obtains for 30 US convertible bonds and one year of monthly price data a larger price deviation, with model prices higher than market prices by 12.9% on average. Buchan (1997) investigates 35 Japanese convertible bonds and reports that model prices are below observed market prices by 1.7% on average.

To examine the obtained results more in detail, the percentage price deviation between each daily observed market price and the theoretical fair values are presented in relation to certain characteristics of the convertible bond. Figure 7 shows the obtained daily price deviations with respect to the moneyness of the convertible bond, calculated as the ratio between conversion value and investment value. The credit spread is set equal to zero to display the moneyness isolated from default risk. This proves to be useful because the credit spread is potentially subject to an estimation error, as we do not observe issue-specific credit spreads but infer them from straight-bond issues with the same rating. Since disregarding credit risk leads to moneyness values that are slightly downward-biased, at-the-money convertibles have a moneyness of less than one in Figure 7. The plot indicates that the pricing accuracy is higher for in-the-money convertible bonds than for at- and out-of-the money bonds. Table 5 presents aggregated statistics for various moneyness classes and supports these results. The standard deviation of the error decreases with higher moneyness values. For the two classes with the highest moneyness, the RMSE is lowest and there is no significant mispricing at the 5% level with average price deviations of -0.05% and -0.38%. This result can be explained theoretically because, for deep in-the-money convertibles, the probability of conversion is very high, the time value of the conversion option becomes very small, and thus the convertible presents less pricing challenges. The large error dispersion for at-the-money convertibles reflects the difficulties in pricing the option part of a convertible bond, the value of which is particularly large for at-the-money-bonds. For deep out-of-the-money convertibles, the likelihood of exercising the conversion option is near to zero and so is the value of the conversion option. Pricing a deep out-of-the-money convertible is very similar to pricing its straight-bond equivalent. It is possible that the large error dispersion of out-of-the-money convertibles is due to difficulties in determining the appropriate credit spread.

The rating categories present in the sample range from A- to CCC-. Figure 8 and Table 6 show the relative price deviations distinguishing between rating categories. While no clear pattern is discernible from Figure 8, a closer look at Table 6 reveals that the average mispricing as well as the RMSE are largest for issue with a CCC rating, possibly indicating that the applied credit spreads for these bonds are higher than assumed by the market. However, after considering all rating categories, there does not seem to exist a systematic relationship between the pricing performance and the rating categories.

In Table 7, the relative mispricings are presented for the individual issues. Of the 32 issues in the sample, 19 present higher average market prices than model prices. For all but one of them, the mispricing is statistically significant at the ten percent level. For three out of the thirteen issues with, on average, lower market prices than model prices, the deviation is not statistically significant at the ten percent level. Clear Channel Communications 2, Rite Aid, and Hexcel Corporation present the lowest absolute average pricing deviation with 0.02%, 0.03%, and 0.15%, respectively. Analog Devices, Alpharma, and Silicon Graphics are the three convertibles with the lowest RMSE: 0.0189, 0.0271, and 0.0288, respectively.

To determine the impact of choosing alternative volatility models, we implement the pricing model by assuming that the stock price follows a GARCH(1,1) process. The GARCH(1,1) model leads to an observed price difference for the entire sample of 0.36% with a standard deviation of 6.17%, which is very similar to the results obtained with standard GBM dynamics. This finding is also supported by Table 8, Table 9 and Table 10, where the sample is divided according to moneyness, credit spread, and issue. To investigate the extent of the pricing similarity between GBM and GARCH(1,1), Figure 9 and Table 11 show a direct comparison of the convertible-bond prices generated by these two models. Although, for at-the-money convertible bonds, the pricing difference is not negligible, overall, the average percentage price deviation between the models is only 0.01% which is not significantly different from zero at the ten percent level. The similarity of the results might be explained by the long maturities of the convertibles in the sample because the specific advantages of the GARCH model in volatility forecasting are larger at shorter horizons and decrease quickly over time.

To determine the pricing impact of the chosen methodology to estimate credit spreads, we create a sub-sample of convertible bonds for those companies with outstanding straight bonds. Of the 32 bonds in the sample, only 6 have straight bonds outstanding and enter the sub-sample (Clear Channel I, Clear Channel II, Corning/Oak Industries, Hilton Hotels, Rite Aid, Service Corp). For each of these convertibles, a time series of credit spreads with daily frequency was extracted and subsequently employed as input for pricing. A comparison of the pricing results for the sub-sample using the two different methods to calculate the credit spread is presented in Table 12. Using credit spreads extracted from outstanding straight bonds leads to a smaller average deviation from market prices and to a smaller RMSE. Unfortunately, both methods to obtain the credit spreads have theoretical and practical drawbacks: While the ratings are issue-specific and thus account for seniority, the spread time series corresponding to these rating categories are only averages from a sample of straight bonds and may be subject to a lag in adjustment. On the other hand, straight bonds from which credit spreads can be extracted might poorly match the maturity and seniority of the convertible bond, which may cause biases. Many companies may not even have any straight bonds outstanding.

The presented simulation-based convertible-bond pricing model offers flexibility to account for more sophisticated processes for the stock. Therefore, we perform the pricing study with a process that is able to account for the long-run volatility dynamics not captured by the GARCH(1,1). The fractionally integrated GARCH is chosen as process specification due to its proven ability to deal with these features (see e.g. Baillie et al., 1996). Since simulating this process is computationally demanding, the analysis is performed on the sub-sample with credit spreads extracted from straight bonds. In Table 12, the prices obtained by FIGARCH(1,d,1) are compared to standard geometric Brownian motion prices. As shown in the last row of the table, the introduction of FIGARCH(1,d,1) reduces the average pricing error by 0.37%. The average deviation between theoretical values and observed market prices shrinks to 0.91% with a root mean squared error of 2.85%.

These preliminary results suggest that the presented simulation-based pricing model for convertible bonds provides a good fit to data. Nevertheless, the model still leaves space for alternative processes for the state variables and offers the opportunity for additional improvements. Further studies might be performed on modeling and estimating the dynamics of the state variables and their interdependencies more accurately.

6. Conclusion

This paper presents a simulation-based pricing method for convertible bonds which incorporates credit risk, stochastic interest rates, and allows for advanced stock-price dynamics as well as a straightforward integration of discrete coupons and dividends. It extends existing approaches to be able to account for various convertible-bond characteristics such as American-style embedded options with path-dependent trigger conditions. The method uses parametric representations of the early exercise decisions and consists of two stages. First, the parameters representing the exercise strategies are optimized given a set of simulated stock prices. In the second stage, the optimized parameters are applied to a new simulation set to determine the model price.

We examine the impact of stochastic interest rates and volatility models. For realistic correlation values between stock price and interest rates, the inclusion of a term-structure model is found to have a minor effect on convertible-bond prices. The impact of choosing the correct volatility model increases with the maturity of the convertible. While GARCH and standard geometric Brownian Motion provide similar prices, a FIGARCH specification leads to a considerable price deviation. In an empirical analysis of the US convertible-bond market covering convertible-bond prices for an entire period of 69 months using daily price data, the model is found to fit market data better than previous studies, with a mean deviation of 0.34 percent.

Overall, Monte Carlo simulation appears to be the appropriate approach for valuing convertible bonds. Due to its flexibility it can much better deal with innovative features than closed-from solutions and lattice-based methods. Moreover, it offers the opportunity to further improve the pricing accuracy by capturing the subtle statistical characteristics of the state variables, which cannot be accomplished to the same extent by the other approaches.

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7. Figures and Tables

Figure 1: Number of data points according to rating categories

This diagram splits the total number of pricing points of the sample into different classes according to the S&P rating of the convertible bond as provided by Standard & Poor's Bond Guide. *Frequency* (y-axis) indicates the absolute number of pricing points for each rating category.

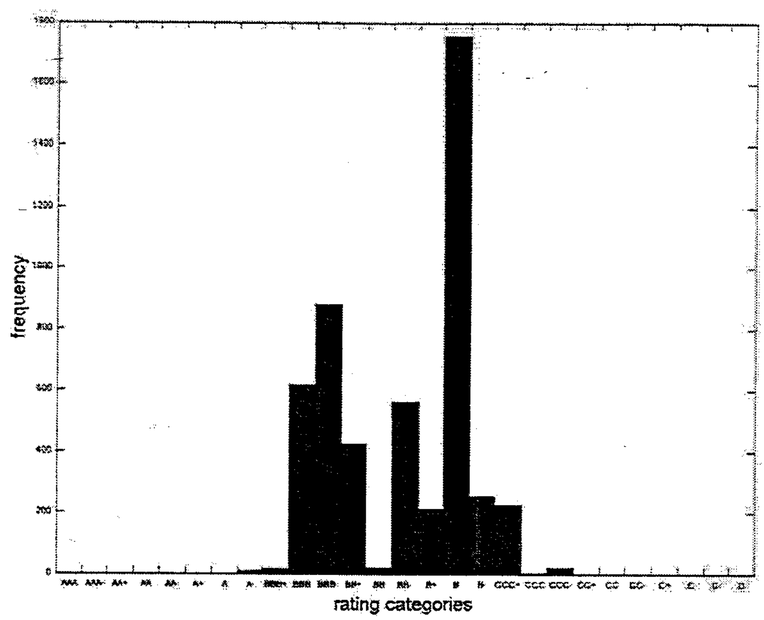

Figure 2: Number of data points according to maturity classes

This histogram splits the total number of pricing points of the sample into different classes according to the maturity of the convertible bond. The *maturity* (x-axis) is expressed in years and the *frequency* (y-axis) indicates the absolute number of pricing points for each maturity class. A maturity class of *n* covers pricing points with a time-to-maturity ranging from *n-0.5* years to *n+0.5* years.

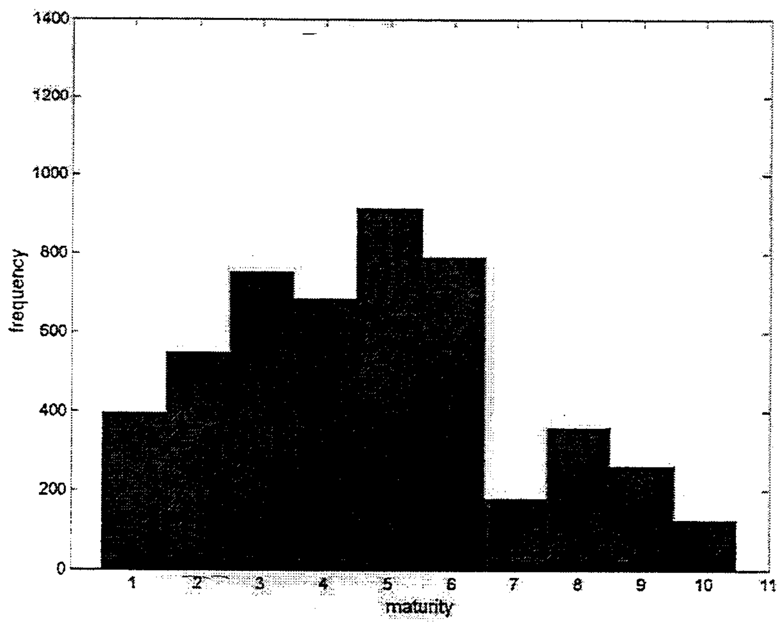

Figure 3: Impact of credit-spread shocks on convertible-bond prices

This graph shows the percentage impact of four credit-spread shocks, +500bp, +300bp, -300bp, and –500bp respectively, on the price of a non-callable convertible bond with face value F=100$, maturity T=10 years, conversion ratio γ=1.0, coupon c=0 assuming a risk free interest rate r=0.05 and a volatility of 13.8957%. The individual plots depicting relative model prices in relation to different initial conversion values are obtained by dividing model prices generated with a credit spread of 0%, 2%, 8%, and 10%, respectively, through model prices obtained by assuming a credit spread of 5%. The *conversion value* (x-axis) ranges from 0 to 150 and the *relative price deviation* (y-axis) ranges from 40% to 170%.

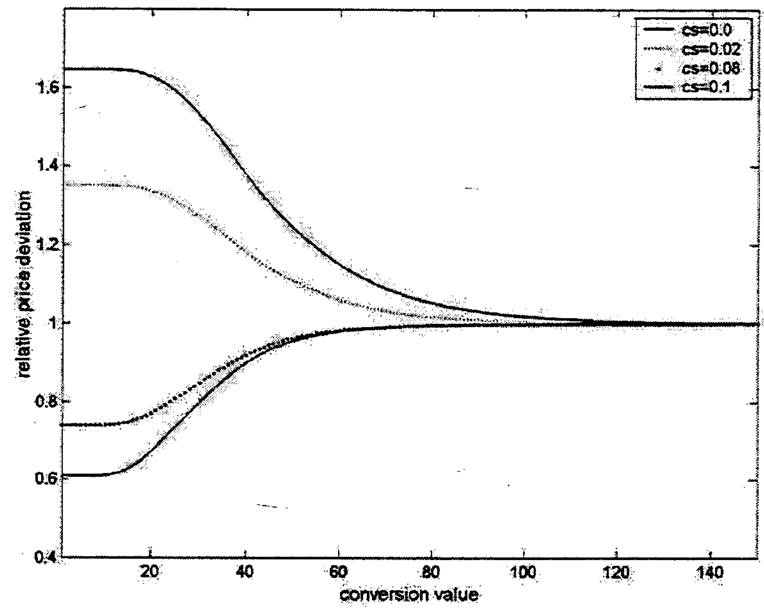

Figure 4: Relative price deviation of various volatility models at different conversion values

This figure shows the relative pricing error of various volatility models compared to convertible-bond prices obtained by assuming a FIGARCH(1,d,1) data generating process (DGP). First, a path of 50 years of daily data is generated using the DGP. Second, parameters for FIGARCH(1,d,1), GARCH(1,1), and geometric Brownian motion are estimated using this simulated time series. Third, the estimated processes are used to price at different levels of conversion value a convertible bond with the following specifications: face value F=100, maturity T=10 years, conversion ratio $\gamma=1$, coupon c=0, risk-free interest rate r=0, and credit spread cs=0. The firm pays no dividends and it is not entitled to call back the convertible bond at any time apart from maturity. The plots are obtained by dividing the convertible-bond prices obtained with the three estimated volatility models by those obtained using DGP. Prices are generated using 3000 simulation paths and a constant seed for the random numbers. The *conversion value* (x-axis) ranges from 0 to 250. The *relative price deviation* (y-axis) ranges from 0% to 2.5%.

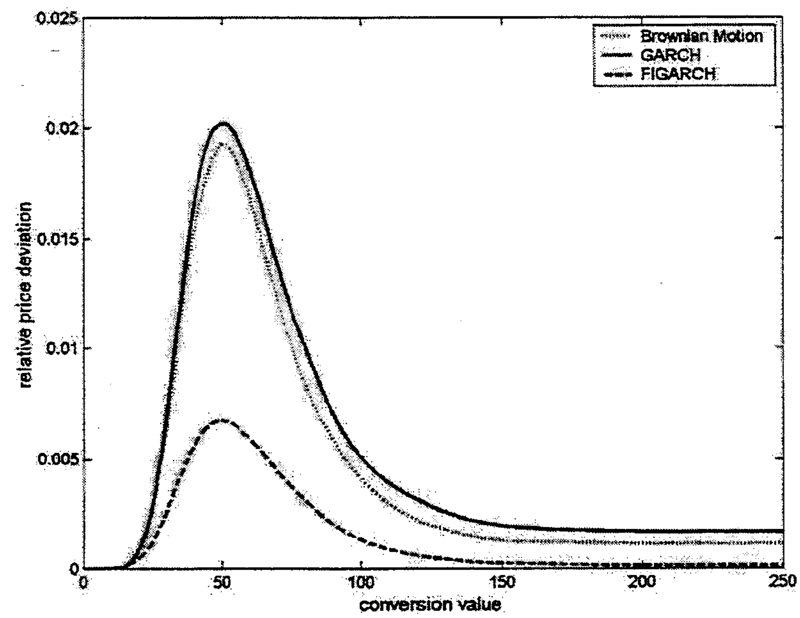

Figure 5: Relative price deviation of various volatility models at different maturities

This figure shows the relative pricing error of various volatility models compared to convertible bond prices obtained by assuming a FIGARCH(1,d,1) data generating process (DGP). First, a path of 50 years of daily data is generated using DGP. Second, parameters for FIGARCH(1,d,1), GARCH(1,1), and geometric Brownian motion are estimated using this simulated time series. Third, the estimated processes are used to price at different levels of conversion value a convertible bond with the following specifications: face value F=100, stock price S=100, conversion ratio γ=1, coupon c=0, risk free interest rate r=0, and credit spread cs=0. The firm pays no dividends and it is not entitled to call back the convertible bond at any time apart from maturity. The plots are obtained by dividing the convertible-bond prices obtained with the three estimated volatility models by those obtained using DGP. Prices are generated using 3000 simulation paths and a constant seed for the random numbers. The *maturity* (x-axis) is expressed in trading days and ranges from 0 to 8000. This corresponds to a maximum maturity of the x-axis of 30.652 years. The *relative price deviation* (y-axis) ranges from 0% to 3%.

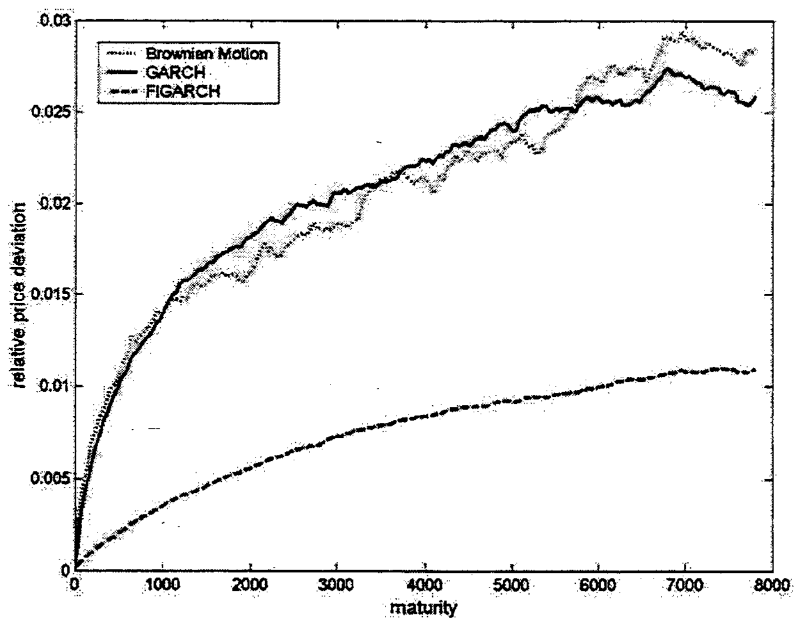

Figure 6: Distribution of percentage price deviation for the simulation-based method with geometric Brownian motion

This histogram splits the total number of pricing points of our sample into different classes according to the percentage *overpricing* as identified by the simulation-based method with standard geometric Brownian motion. *Overpricing* states the extent to which market prices are, on average, above model prices. *Overpricing* (x-axis) ranges from –40% to +40%. F*requency* (y-axis) indicates the absolute number of pricing points for each *overpricing* class.

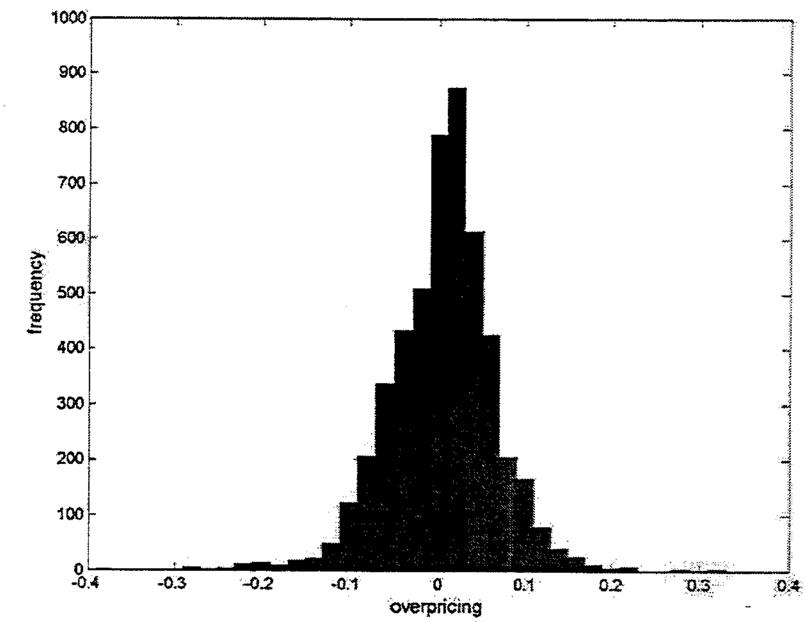

Figure 7: Deviation of prices obtained by applying Brownian motion plotted against moneyness

This graph shows the percentage price deviation between each daily observed market price in the sample and the corresponding theoretical value as generated by the simulation-based method plotted against the *moneyness* of the convertible bond. Stock returns are assumed to follow a standard geometric Brownian motion with constant volatility. The plots are obtained by dividing observed prices through model prices and subtracting one. The *moneyness* (x-axis) ranges from 0 to 7.

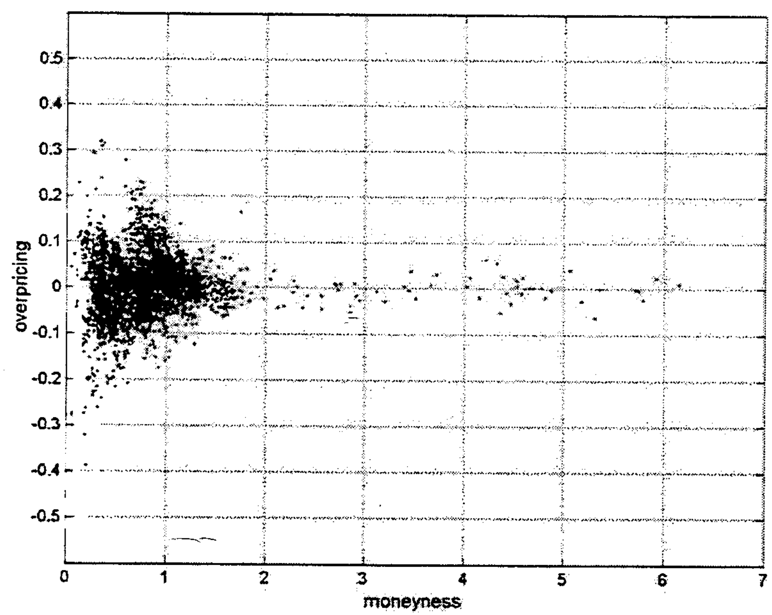

Figure 8: Percentage price deviation for various credit rating categories obtained by applying Brownian motion

This graph shows for each rating category the percentage price deviations between each daily observed market price in the sample and the corresponding theoretical fair value as generated by the simulation-based method. Stock returns are assumed to follow a standard geometric Brownian motion with constant volatility. The plots are obtained by dividing observed prices through model prices and subtracting one. The rating is attributed to each convertible bond according to Standard & Poor's Bond Guide. The data in the sample covers *rating categories* (x-axis) ranging from A- to CCC-. *Overpricing* (y-axis) ranges from –60% to +60%.

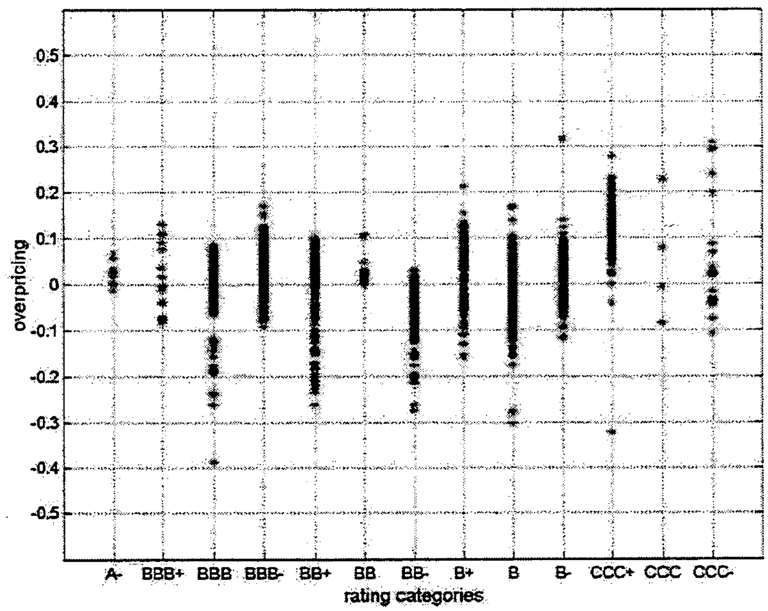

Figure 9: Deviation of prices generated by the simulation-based method obtained by Brownian Motion and GARCH plotted against moneyness

This graph shows the percentage price deviation between two theoretical fair values as generated by the GARCH(1,1)-simulation-based method and the same simulation-based method with a geometric Brownian motion. This relative price deviation for the pricing points in the sample is plotted against the moneyness of the convertible bond. The plots are obtained by dividing GARCH(1,1) prices through Brownian motion prices and subtracting one. The *moneyness* (x-axis) ranges from 0 to 7.

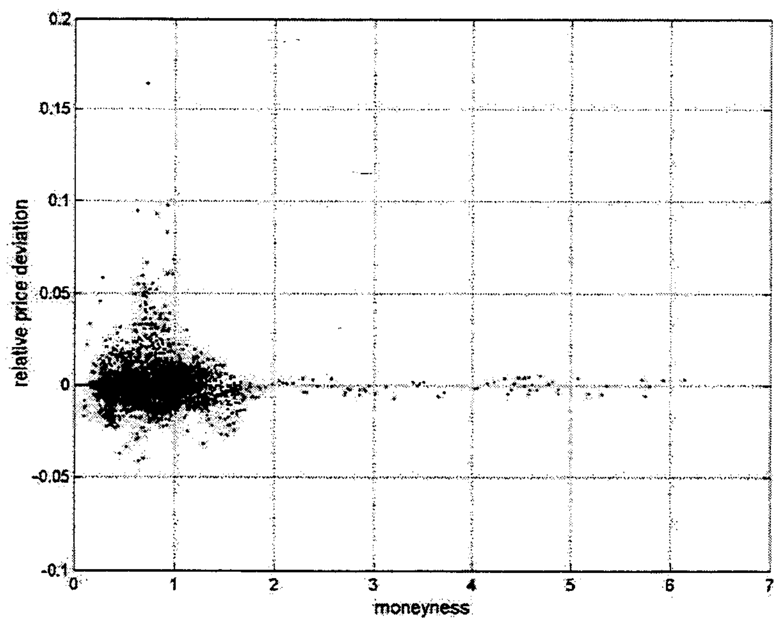

Table 1: Specifications of convertible bonds in the sample

This table gives an overview of the analyzed convertible bonds with *convertible bond* referring to the name of the issuing firm, *date of issue*, *coupon* as percentage of the face value, and *maturity*. *Size* indicates the amount outstanding in million dollars as reported by Standard & Poor's Bond Guide.

Convertible bond	*Date of issue*	*Coupon*	*Maturity*	*Size*
Adaptec Inc	28. Jan 97	4.75%	01. Feb 04	230
Alpharma Inc	25-Mar-98	5.75%	01. Apr 05	125
Analog Devices	26. Sep 00	4.75%	01-Oct-05	1200
Charming Shoppes	17. Jul 96	7.50%	15. Jul 06	138
CKE Restaurants	09-Mar-98	4.25%	15-Mar-04	159
Clear Channel 1	25-Mar-98	2.63%	01. Apr 03	575
Clear Channel 2	17. Nov 99	1.50%	01-Dec-02	900
Corning/Oak Inds	20. Feb 98	4.88%	01-Mar-08	100
Cypress Semicon	21. Jun 00	3.75%	01. Jul 05	250
Genesco Inc	06. Apr 98	5.50%	15. Apr 05	104
Healthsouth Corp	17-Mar-98	3.25%	01. Apr 03	443
Hexcel Corp	18. Jul 96	7.00%	01. Aug 03	114
Hilton Hotels	09-May-96	5.00%	15-May-06	494
Interpublic Group	26-May-99	1.87%	01. Jun 06	361
Kerr McGee Corp	21. Jan 00	5.25%	15. Feb 10	550
Kulicke & Soffa	08-Dec-99	4.75%	15-Dec-06	175
LAM Research	19. Aug 97	5.00%	01. Sep 02	310
LSI Logic	16-Mar-99	4.25%	15-Mar-04	345
NABI	02. Feb 96	6.50%	01. Feb 03	80.5
Offshore Logistics	11-Dec-96	6.00%	15-Dec-03	80
Omnicare Inc	04-Dec-97	5.00%	01-Dec-07	345
Parker Drilling	21. Jul 97	5.50%	01. Aug 04	124
Penn Treaty Amer	20. Nov 96	6.25%	01-Dec-03	74.8
Photronics Inc	22-May-97	6.00%	01. Jun 04	103
Pogo Producing	11. Jun 96	5.50%	15. Jun 06	115
Providian Financial	17. Aug 00	3.25%	15. Aug 05	402
Rite Aid	04. Sep 97	5.25%	15. Sep 02	650
Safeguard Scientific	03. Jun 99	5.00%	15. Jun 06	200
Semtech Corp	03. Feb 00	4.50%	01. Feb 07	400
Service Corp	18. Jun 01	6.75%	22. Jun 08	300
Silicon Graphics	07. Aug 97	5.25%	01. Sep 04	231
Standard Motor Prods	20. Jul 99	6.75%	15. Jul 09	90

Table 2: Further issue-specific information on convertible bonds in the sample

Rating represents the Standard & Poor's Bond Guide rating as of February 2002. *Callability* indicates whether the bond is redeemable at the option of the issuing company at any time prior to maturity during the period considered in this study. *Trigger* indicates the existence of an additional trigger condition to be satisfied in order to call the convertible. In case the issuing company wants to exercise the call option it has to notify to the issuer its intent to do so a certain number of days in advance. This period is referred to as *call notice period.* More often than not, the contractual provision specified in the legally binding offering circular states that upon call accrued interest are paid to the investor.

Convertible bond	*Rating*	*Callability*	*Trigger*	*Call notice period*	*Accrued interest paid at call*
Adaptec Inc	B-	Yes	No	15	Yes
Alpharma Inc	B	Yes	No	30	Yes
Analog Devices	BBB	Yes	No	30	Yes
Charming Shoppes	B	Yes	No	30	Yes
CKE Restaurants	CCC	Yes	No	30	Yes
Clear Channel 1	BBB-	Yes	No	15	Yes
Clear Channel 2	BBB-	No	No	30	No
Corning/Oak Inds	BBB-	Yes	No	30	Yes
Cypress Semicon	B	Yes	No	20	Yes
Genesco Inc	B	Yes	No	30	Yes
Healthsouth Corp	BB+	Yes	No	30	Yes
Hexcel Corp	CCC+	Yes	No	20	Yes
Hilton Hotels	BB+	Yes	No	30	Yes
Interpublic Group	BBB	Yes	No	30	No
Kerr McGee Corp	BBB-	Yes	No	30	Yes
Kulicke & Soffa	B-	Yes	No	30	Yes
LAM Research	B	Yes	Yes	20	No
LSI Logic	B	Yes	No	30	Yes
NABI	CCC-	Yes	No	20	Yes
Offshore Logistics	B+	Yes	No	30	Yes
Omnicare Inc	BB+	Yes	No	30	Yes
Parker Drilling	B-	Yes	No	30	Yes
Penn Treaty Amer	CC	Yes	No	15	Yes
Photronics Inc	B	Yes	No	20	Yes
Pogo Producing	BB	Yes	No	30	Yes
Providian Financial	B	Yes	No	30	Yes
Rite Aid	CCC+	Yes	No	30	Yes
Safeguard Scientific	CCC	Yes	No	20	Yes
Semtech Corp	CCC+	Yes	No	30	Yes
Service Corp	B	Yes	No	30	Yes
Silicon Graphics	CCC-	Yes	Yes	30	Yes
Standard Motor Prods	B+	Yes	No	30	Yes

Table 3: Impact of stochastic interest rates on convertible-bond prices

This table shows the percentage price impact of a term structure model on prices of convertible bonds for different initial stock prices and for different values of the correlation between the stock and the interest rates. The different initial stock prices imply different values for the moneyness of the convertible bond. Moneyness ranges from 0.03 to 5.5 with corresponding stock prices ranging from S=2 to S=350. The number of paths is 5000, with the same random-number series for each pricing. The first number indicates the absolute price of the convertible bond. The number in parentheses indicates the error of the simulated prices computed as standard deviation of the mean of the simulated discounted payoffs. For prices calculated with stochastic interest rates, the second number refers to the percentage deviation with respect to prices with constant interest rates. All convertible bonds have a face value F=100, maturity T=10 years, conversion ratio γ=1.0, coupon c=0, and credit spread cs=0. The issuing firm pays no dividends and is not entitled to call back the convertible bond at any time apart from maturity. The stock price follows a geometric Brownian motion, $dS_t=S_t r_t dt+S_t\sigma_S dW_{S,t}$, with volatility σ_S=0.25 and the instantaneous interest rate follows a one-factor CIR interest-rate process, $dr_t = \kappa(\theta_r - r_t)dt + \sigma_r\sqrt{r_t}\,dW_{r,t}$, with initial short rate r=0.04, long term interest rate θ_r=0.06, mean-reversion parameter κ=0.05, volatility σ_r=0.08, and correlations $\rho_{S,r}$ between $dW_{r,t}$ and $dW_{S,t}$ ranging from $\rho_{S,r}$=-0.5 to $\rho_{S,r}$=+0.5. The short rate is simulated using the Milstein discretization scheme. The spot rate curve resulting from the simulation with the above parameter set is upward sloping with the following continuously compounded spot rates: $r_{1/12}$=0.0400, $r_{0.5}$=0.0403, r_1=0.0407, r_2=0.0413, r_5=0.0434, r_7=0.0447, and r_{10}=0.0462.

Stock price (Moneyness)	2 (0.03)	10 (0.16)	30 (0.48)	65 (1.03)	120 (1.90)	200 (3.97)	350 (5.55)
Constant interest rates	63.03 (0.00)	63.09 (0.02)	66.43 (0.23)	84.55 (0.74)	129.21 (1.65)	204.86 (2.69)	354.10 (4.76)
Stochastic interest rates							
$\rho_{S,r}$=-0.5	63.04 0.03% (0.18)	63.07 -0.03% (0.18)	65.85 -0.87% (0.29)	83.11 -1.70% (0.77)	127.86 -1.05% (1.57)	204.09 -0.38% (2.70)	353.87 -0.07% (4.77)
$\rho_{S,r}$=-0.2	63.05 0.04% (0.18)	63.11 0.02% (0.18)	66.39 -0.05% (0.29)	84.39 -0.20% (0.76)	129.12 -0.07% (1.56)	204.84 -0.01% (2.70)	354.13 0.01% (4.76)
$\rho_{S,r}$=0.0	63.03 0.00% (0.18)	63.12 0.04% (0.18)	66.78 0.53% (0.29)	85.18 0.75% (0.75)	129.88 0.51% (1.56)	205.30 0.21% (2.69)	354.29 0.05% (4.76)
$\rho_{S,r}$=+0.2	62.99 -0.06% (0.18)	63.13 0.06% (0.18)	67.19 1.15% (0.28)	85.96 1.66% (0.75)	130.58 1.05% (1.55)	205.73 0.42% (2.69)	354.46 0.10% (4.76)
$\rho_{S,r}$=+0.5	62.93 -0.15% (0.18)	63.19 0.15% (0.18)	67.85 2.14% (0.27)	87.10 3.01% (0.74)	131.58 1.83% (1.54)	206.32 0.71% (2.68)	354.70 0.17% (4.76)

Table 4: Parameter estimates for various volatility models

σ represents the annualized unconditional volatility that was used in the implementation using Brownian Motion, estimated from as much pre-sample data as available to us (starting at least before 1990). The GARCH(1,1) equation is: $\sigma_t^2 = w + a\varepsilon_{t-1}^2 + b\sigma_{t-1}^2$. The conditional variance in the FIGARCH(1,d,1) model evolves as: $\sigma_t^2 = \omega[1-\beta(1)]^{-1} + \left[1-(1-\beta(L))^{-1}\cdot\phi(L)\cdot(1-L)^d\right]\varepsilon_t^2$. The parameters m represent the estimated drift terms in the corresponding mean equations.

Convertible bond	*Brownian Motion*	*GARCH(1,1)*				*FIGARCH(1,d,1)*				
	σ	m	ω	a	b	m	ω	β	φ	d
Adaptec Inc	0.619601	0.001584	3.07E-05	0.014910	0.965199	0.001681	0.000256	0.601290	0.508965	0.141031
Alpharma Inc	0.434762	0.000894	0.000311	0.257896	0.377281	0.000872	0.000301	0.360626	0.604310	0.012850
Analog Devices	0.524330	0.000809	5.51E-06	0.030070	0.964677	0.000707	4.76E-05	0.687196	0.519559	0.282618
Charming Shoppes	0.535953	0.000657	1.05E-05	0.029823	0.960113	0.000515	4.57E-05	0.760001	0.626982	0.269265
CKE Restaurants	0.426572	0.000575	0.00016	0.161979	0.615782	0.000697	1.77E-05	0.899319	0.872380	0.210000
Clear Channel 1	0.357913	0.001414	4.34E-05	0.097719	0.818231	0.001328	3.86E-05	0.672571	0.622976	0.221189
Clear Channel 2	0.357913	0.001414	4.34E-05	0.097719	0.818231	0.001328	3.86E-05	0.672571	0.622976	0.221189
Corning/Oak Inds	0.299630	0.000371	2.61E-05	0.100687	0.829004	0.000278	1.02E-05	0.830208	0.697323	0.334785
Cypress Semicon	0.538525	0.000692	9.85E-05	0.077597	0.831327	0.000694	8.51E-05	0.845976	0.912684	0.013669
Genesco Inc	0.600186	-0.000140	0.000143	0.095389	0.805274	-0.000170	0.000242	0.437008	0.389691	0.168209
Healthsouth Corp	0.429999	0.001952	7.25E-05	0.089714	0.807925	0.00191	5.45E-05	0.628669	0.548072	0.226892
Hexcel Corp	0.471715	0.000115	5.48E-06	0.008321	0.975708	0.000321	3.67E-06	0.966751	0.226993	0.951033
Hilton Hotels	0.352844	0.000687	1.71E-05	0.069696	0.898264	0.000672	6.82E-05	0.351397	0.256534	0.230405
Interpublic Group	0.313456	0.000873	2.14E-06	0.038887	0.955268	0.000831	7.56E-06	0.756535	0.468253	0.424235
Kerr McGee Corp	0.289180	0.000244	3.42E-06	0.046240	0.943652	0.000192	1.18E-05	0.688442	0.481415	0.323558
Kulicke & Soffa	0.641227	0.000537	1.37E-05	0.031839	0.960379	5.50E-05	9.19E-05	0.689805	0.566131	0.259942
LAM Research	0.596124	0.001008	0.000095	0.090067	0.841620	0.001021	0.000151	0.391075	0.207184	0.261515
LSI Logic	0.542819	0.000739	0.000164	0.072973	0.782621	0.000780	0.00016	0.575479	0.511618	0.164449
NABI	0.964364	0.000200	4.66E-05	0.070418	0.920182	-0.00012	7.98E-05	0.745677	0.589179	0.332907
Offshore Logistics	0.785796	0.000737	0.000338	0.098008	0.757633	3.32E-05	2.21E-05	0.855026	0.736368	0.310825
Omnicare Inc	0.411482	0.000833	4.04E-06	0.023983	0.970296	0.000985	1.27E-05	0.897241	0.763908	0.335427
Parker Drilling	0.493291	0.000218	8.76E-05	0.111824	0.798210	0.000298	9.32E-05	0.493155	0.394805	0.233519
Penn Treaty Amer	0.645585	0.001364	2.31E-05	0.112127	0.880060	0.001409	4.25E-06	0.951543	0.226614	0.986493
Photronics Inc	0.713992	0.000966	3.22E-05	0.054081	0.931388	0.000988	0.000143	0.488868	0.299369	0.285027
Pogo Producine	0.453902	6.86E-05	8.27E-06	0.043163	0.947773	-1.00E-04	2.44E-05	0.748811	0.560364	0.351629
Providian Financial	0.292776	0.000497	6.09E-06	0.058140	0.924539	0.000517	1.21E-05	0.720209	0.563881	0.310937
Rite Aid	0.385032	0.000500	2.65E-06	0.026851	0.967054	0.000527	1.93E-05	0.698083	0.523834	0.299441
Safeguard Scientific	0.675231	-0.000300	0.000115	0.007935	0.945687	1.60E-07	6.63E-05	0.891473	0.249062	0.693621
Semtech Corp	0.755131	-0.000130	9.13E-06	0.031023	0.965534	-0.000640	4.99E-05	0.802921	0.617351	0.353382
Service Corp	0.394883	0.000933	1.41E-06	0.025392	0.972872	0.000844	1.34E-05	0.781386	0.579286	0.348238
Silicon Graphics	0.508525	0.001369	0.000267	0.142702	0.599636	0.001419	0.000283	0.323649	0.381109	0.108264
Standard Motor Prods	0.410191	0.000443	1.85E-06	0.028817	0.968659	0.000384	8.84E-06	0.830239	0.627657	0.415845

Table 5: Pricing overview for various moneyness classes obtained by the simulation-based method with geometric Brownian motion

Data points indicates the number of days for which model prices are computed. *Mean percentage overpricing* states the extent to which market prices are, on average, above model prices for a given moneyness class. *Overpricing std.* is the standard deviation of the observations in the respective class. *Probability values* refer to a two-sided test for the H_0 hypothesis that model prices and observed prices are equal in the mean. *Root mean squared error* is the non-central standard deviation of the relative deviations of model prices from market prices.

Moneyness	*Data Points*	*Mean percentage overpricing*	*Overpricing std.*	*Probability values*	*Root mean squared error*
< 0.50	1242	-0.0178	0.0792	0.0000	0.0811
0.50 – 0.80	1454	0.0018	0.0574	0.2258	0.0574
0.80 – 0.95	866	0.0232	0.0555	0.0000	0.0601
0.95 – 1.05	516	0.0233	0.0442	0.0000	0.0500
1.05 – 1.20	447	0.0110	0.0336	0.0000	0.0354
1.20 – 2.00	429	-0.0005	0.0286	0.7418	0.0286
> 2.00	59	-0.0038	0.0262	0.2721	0.0263
Total sample	5013	0.0034	0.0604	0.0001	0.0605

Table 6: Pricing overview for various rating classes obtained by the simulation-based method with geometric Brownian motion

Data points indicates the number of days for which model prices are computed. *Mean percentage overpricing* states the extent to which market prices are, on average, above model prices for a given rating class. *Overpricing std.* is the standard deviation of the observations in the respective class. *Probability values* refer to a two-sided test for the H_0 hypothesis that model prices and observed prices are equal in the mean. *Root mean squared error* is the non-central standard deviation of the relative deviations of model prices from market prices.

Rating	*Data points*	*Mean percentage overpricing*	*Overpricing std.*	*Probability values*	*Root mean squared error*
A-	11	0.0196	0.0235	0.0056	0.0298
BBB+	18	-0.0017	0.0739	0.9216	0.0718
BBB	617	0.0096	0.0509	0.0000	0.0518
BBB-	881	0.0227	0.0444	0.0000	0.0498
BB+	427	0.0088	0.0717	0.0110	0.0721
BB	21	0.0264	0.0269	0.0000	0.0372
BB-	563	-0.0668	0.0437	0.0000	0.0798
B+	216	0.0271	0.0662	0.0000	0.0713
B	1751	-0.0059	0.0415	0.0000	0.0419
B-	255	0.0143	0.0577	0.0001	0.0594
CCC+	227	0.1057	0.0503	0.0000	0.1170
CCC	4	0.0473	0.1243	0.4468	0.1176
CCC-	22	0.0473	0.1150	0.0536	0.1219
Total sample	5013	0.0034	0.0604	0.0001	0.0605

Table 7: Pricing overview for different issues obtained by applying geometric Brownian motion

Data points indicates the number of days for which model prices are computed. *Mean percentage overpricing* states the extent to which market prices are, on average for a given convertible bond, above model prices as generated by the simulation-based method with standard geometric Brownian Motion. *Overpricing std.* is the standard deviation of the observations in the respective class. *Probability values* refer to a two-sided test for the H_0 hypothesis that model prices and observed prices are equal in the mean. *Root mean squared error* is the non-central standard deviation of the relative deviations of model prices from market prices.

Convertible bond	*Data points*	*Mean percentage overpricing*	*Overpricing std.*	*Probability values*	*Root mean squared error*
Adaptec Inc	545	-0.0613	0.0517	0.0000	0.0801
Alpharma Inc	296	0.0100	0.0253	0.0000	0.0271
Analog Devices	39	-0.0117	0.0151	0.0000	0.0189
Charming Shoppes	83	0.0166	0.0407	0.0002	0.0437
CKE Restaurants	248	-0.0030	0.0293	0.1109	0.0294
Clear Channel 1	240	0.0209	0.0229	0.0000	0.0309
Clear Channel 2	144	-0.0003	0.0364	0.9286	0.0363
Corning/Oak Inds	22	0.0320	0.0438	0.0006	0.0534
Cypress Semicon	124	0.0623	0.0336	0.0000	0.0707
Genesco Inc	46	0.0400	0.0295	0.0000	0.0495
Healthsouth Corp	83	-0.0501	0.0292	0.0000	0.0579
Hexcel Corp	32	0.0015	0.0439	0.8451	0.0432
Hilton Hotels	616	0.0301	0.0230	0.0000	0.0379
Interpublic Group	46	-0.0469	0.0186	0.0000	0.0504
Kerr McGee Corp	227	0.0735	0.0255	0.0000	0.0778
Kulicke & Soffa	71	-0.0493	0.0228	0.0000	0.0542
LAM Research	657	-0.0161	0.0343	0.0000	0.0379
LSI Logic	169	0.0155	0.0257	0.0000	0.0299
NABI	18	0.0580	0.125	0.0489	0.1347
Offshore Logistics	79	-0.0375	0.0468	0.0000	0.0597
Omnicare Inc	111	0.0357	0.0388	0.0000	0.0526
Parker Drilling	66	0.0591	0.0331	0.0000	0.0676
Penn Treaty Amer	65	-0.1072	0.0801	0.0000	0.1335
Photronics Inc	257	-0.0260	0.0561	0.0000	0.0618
Pogo Producing	43	0.0275	0.0397	0.0000	0.0479
Providian Financial	91	-0.0242	0.1115	0.0381	0.1135
Rite Aid	266	-0.0002	0.076	0.9698	0.0758
Safeguard Scientific	2	0.1407	0.0905	0.0279	0.1546
Semtech Corp	187	0.1103	0.0402	0.0000	0.1174
Service Corp	9	0.0311	0.0228	0.0000	0.0378
Silicon Graphics	122	0.0113	0.0266	0.0000	0.0288
Standard Motor Prods	9	0.1030	0.0433	0.0000	0.1108
Total sample	5013	0.0034	0.0604	0.0001	0.0605

Table 8: Pricing overview for various moneyness classes obtained by the simulation-based method with GARCH(1,1)

Data points indicates the number of days for which model prices are computed. *Mean percentage overpricing* states the extent to which market prices are, on average, above model prices for a given moneyness class. *Overpricing std.* is the standard deviation of the observations in the respective class. *Probability values* refer to a two-sided test for the H_0 hypothesis that model prices and observed prices are equal in the mean. The *root mean squared error* is the non-central standard deviation of the relative deviations of model prices from market prices.

Moneyness	*Data points*	*Mean percentage overpricing*	*Overpricing std.*	*Probability values*	*Root mean squared error*
< 0.50	1242	-0.0156	0.0822	0.0000	0.0836
0.50 – 0.80	1454	0.0008	0.0588	0.6144	0.0588
0.80 – 0.95	866	0.0225	0.0564	0.0000	0.0607
0.95 – 1.05	516	0.0229	0.0442	0.0000	0.0497
1.05 – 1.20	447	0.0103	0.0338	0.0000	0.0353
1.20 – 2.00	429	0.0012	0.0292	0.4025	0.0292
> 2.00	59	-0.0032	0.0258	0.3440	0.0258
Total sample	5013	0.0036	0.0617	0.0000	0.0618

Table 9: Pricing overview for various rating classes obtained by the simulation-based method with GARCH(1,1)

Data points indicates the number of days for which model prices are computed. *Mean percentage overpricing* states the extent to which market prices are, on average, above model prices for a given rating class. *Overpricing std.* is the standard deviation of the observations in the respective class. *Probability values* refer to a two-sided test for the H_0 hypothesis that model prices and observed prices are equal in the mean. The *root mean squared error* is the non-central standard deviation of the relative deviations of model prices from market prices.

Rating	*Data points*	*Mean percentage overpricing*	*Overpricing std.*	*Probability values*	*Root mean squared error*
A-	11	0.0183	0.0244	0.0129	0.0296
BBB+	18	0.0009	0.0742	0.9594	0.0721
BBB	617	0.0042	0.0525	0.0466	0.0526
BBB-	881	0.0219	0.0438	0.0000	0.0490
BB+	427	0.0089	0.0774	0.0177	0.0778
BB	21	0.0228	0.0256	0.0000	0.0339
BB-	563	-0.0670	0.0456	0.0000	0.0810
B+	216	0.0353	0.0715	0.0000	0.0796
B	1751	-0.0032	0.0432	0.0022	0.0434
B-	255	0.0155	0.0591	0.0000	0.0610
CCC+	227	0.0980	0.0584	0.0000	0.1140
CCC	4	0.0084	0.1394	0.9039	0.1210
CCC-	22	0.0440	0.1112	0.0636	0.1172
Total sample	5013	0.0036	0.0617	0.0000	0.0618

Table 10: Pricing overview for the different issues as depicted by the simulation-based method with GARCH(1,1)

Data points indicates the number of days for which model prices are computed. *Mean percentage overpricing* states the extent to which market prices are, on average for a given convertible bond, above model prices as generated by the simulation-based method with GARCH(1,1). *Overpricing std.* is the standard deviation of the observations in the respective class. *Probability values* refer to a two-sided test for the H_0 hypothesis that model prices and observed prices are equal in the mean. *Root mean squared error* is the non-central standard deviation of the relative deviations of model prices from market prices.

Convertible bond	*Data points*	*Mean percentage overpricing*	*Overpricing std.*	*Probability values*	*Root mean squared error*
Adaptec Inc	545	-0.0616	0.0528	0.0000	0.0811
Alpharma Inc	296	0.0045	0.0249	0.002	0.0253
Analog Devices	39	-0.0139	0.0150	0.0000	0.0203
Charming Shoppes	83	0.0182	0.0405	0.0000	0.0442
CKE Restaurants	248	0.0002	0.0288	0.9304	0.0288
Clear Channel 1	240	0.0215	0.0233	0.0000	0.0316
Clear Channel 2	144	0.0009	0.0363	0.7619	0.0362
Corning/Oak Inds	22	0.0336	0.0424	0.0002	0.0534
Cypress Semicon	124	0.0788	0.0340	0.0000	0.0857
Genesco Inc	46	0.0397	0.0301	0.0000	0.0496
Healthsouth Corp	83	-0.0477	0.0292	0.0000	0.0558
Hexcel Corp	32	0.0225	0.0426	0.0029	0.0476
Hilton Hotels	616	0.0260	0.0229	0.0000	0.0346
Interpublic Group	46	-0.0543	0.0152	0.0000	0.0563
Kerr McGee Corp	227	0.0722	0.0243	0.0000	0.0761
Kulicke & Soffa	71	-0.0511	0.0216	0.0000	0.0554
LAM Research	657	-0.0075	0.0349	0.0000	0.0357
LSI Logic	169	0.0230	0.0281	0.0000	0.0362
NABI	18	0.0442	0.1293	0.1470	0.1332
Offshore Logistics	79	-0.0393	0.0472	0.0000	0.0612
Omnicare Inc	111	0.0401	0.0431	0.0000	0.0587
Parker Drilling	66	0.0612	0.0347	0.0000	0.0702
Penn Treaty Amer	65	-0.1280	0.0746	0.0000	0.1479
Photronics Inc	257	-0.0334	0.0605	0.0000	0.069
Pogo Producing	43	0.0222	0.0393	0.0002	0.0448
Providian Financial	91	-0.0161	0.1171	0.1907	0.1175
Rite Aid	266	-0.0057	0.0789	0.2379	0.0789
Safeguard Scientific	2	0.1136	0.0982	0.1019	0.1332
Semtech Corp	187	0.1011	0.0521	0.0000	0.1136
Service Corp	9	0.0173	0.0440	0.2391	0.0449
Silicon Graphics	122	0.0166	0.0257	0.0000	0.0306
Standard Motor Prods	9	0.1019	0.0445	0.0000	0.1102
Total sample	5013	0.0036	0.0617	0.0000	0.0618

Table 11: Deviation of prices generated by the simulation-based method obtained by Brownian Motion and GARCH(1,1) plotted against moneyness

Data points indicates the number of data points for which model prices are computed. *Mean percentage deviation* states the extent to which GARCH(1,1) prices are, on average, above Brownian-motion prices for a given moneyness class. *Deviation std.* is the standard deviation of the observations in the respective class. *Probability values* refer to a two-sided test for the H_0 hypothesis that the model prices applying both volatility models are equal in the mean. The *root mean squared error* is the non-central standard deviation of the relative deviations of GARCH(1,1) prices from Brownian-motion prices.

Moneyness	*Data points*	*Mean percentage deviation*	*Deviation std.*	*Probability values*	*Root mean squared error*
< 0.50	1242	0.0021	0.0078	0.0000	0.0081
0.50 – 0.80	1454	-0.0010	0.0135	0.0031	0.0135
0.80 – 0.95	866	-0.0007	0.0110	0.0551	0.0110
0.95 – 1.05	516	-0.0004	0.0080	0.2259	0.0080
1.05 – 1.20	447	-0.0007	0.0066	0.0208	0.0066
1.20 – 2.00	429	0.0016	0.0075	0.0000	0.0077
> 2.00	59	0.0006	0.0032	0.1673	0.0032
Total sample	5013	0.0001	0.0103	0.3454	0.0103

Table 12: Pricing comparison between rating-based credit spreads and credit spreads directly extracted from other outstanding straight bonds of the same issuer

This table compares pricing results obtained using two different estimation methods for credit-spreads. In the *total sample* credit spreads are obtained using the individual rating in combination with average credit-spreads from all available straight-bond issues from the same credit class. As alternative to this approach, credit spreads may be extracted directly from other outstanding straight-bond issues of the same company. Since only six companies in our sample have straight bonds outstanding, the pricing comparison is based on a sub sample of the initial data. *Data points* indicates the number of days for which model prices are computed. *Mean percentage deviation* states the extent to which market prices are, on average, above model prices for a given moneyness class. *Deviation std.* is the standard deviation of the percentage difference between market prices and prices generated by the model. *Probability values* refer to a two-sided test for the H_0 hypothesis that the model prices and observed prices are equal in the mean. The *root mean squared error* is the non-central standard deviation of the relative deviations of model prices from market prices. *Simulation error* is the standard deviation of the obtained payoffs divided by the square root of the number of simulated paths.

Moneyness	*Data points*	*Mean percentage deviation*	*Deviation std.*	*Probability values*	*Root mean squared error*	*Simulation error*
Total sample with GBM	5013	0.0036	0.0617	0.0000	0.0618	
Sub sample with GBM and rating-based credit spreads	303	0.0329	0.0269	0.0000	0.0425	0.0023
Sub sample with GBM and credit spreads from outstanding straight-bonds	303	0.0128	0.0249	0.0000	0.0280	0.0020
Sub sample with FIGARCH and credit spreads from outstanding straight-bonds	303	-0.0091	0.0271	0.0000	0.0285	0.0024