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PRICE DISCOVERY IN TICK TIME

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Abstract

In this paper we propose a tick time model for the quote setting process on Nasdaq using a time series of all quote updates by the most active dealers and ECN's (Electronic Communication Networks). The model includes duration effects in the volatility of the efficient price and in the covariance of quote updates with the efficient price. As a measure of price discovery we define information shares in tick time. When aggregated to calendar time they provide an alternative for the Hasbrouck (1995) information shares.

In the empirical analysis we compare quotes from two ECN"s (Island and Instinet), and three wholesale market makers for 20 actively traded stocks with varying liquidity. We find that volatility does not increase with the duration between quote updates, and that longer quote durations lead to lower price discovery. In terms of price discovery we find that ECNs tend to dominate the liquid stocks, whereas market makers are important for less liquid stocks.

Keywords: Price Discovery, Tick Time models, Nasdaq, Ultra-high frequency data, Microstructure.

JEL Classifications: C32, G15.

1 Introduction

Price discovery is the process of how different information sources contribute to the evolution of the underlying value of an asset. In a fragmented market with multiple dealers, like the Nasdaq, each dealer contributes to the price discovery process. Interesting questions are which dealer contributes most, how quick the discovery process works, and how it depends on market circumstances like liquidity, volatility and trading intensity.

An important measure for the price discovery contribution is the information share defined by Hasbrouck (1995). Hasbrouck (1995) defines information shares as the part of the variance of the random walk component of returns that can be attributed to a particular market or dealer. Unfortunately this variance decomposition is not unique when innovations to returns are contemporaneously correlated. Hasbrouck (1995) suggests alternative Choleski decompositions to establish upper and lower bounds. This works well in many cases when the number of different markets or market participants is small and the differences among them are large enough.

The intervals between upper and lower bounds can be very wide, however, if we wish to discriminate between more than two (groups of) participants. A typical application, and the one we will revisit in this paper, is the question on how informative different ECNs (Electronic Communication Networks) are for the price discovery process on Nasdaq. Huang (2002) applies the Hasbrouck (1995) methodology to a variety of stocks to study the relationship between ECN's and traditional dealers. He estimates a vector error correction model for quotes issued by five different groups of market participants sampled at one minute intervals. For some stocks the information share of an important ECN like Island can range between 25 and 85 percent depending on the order of the variance decomposition. In this case the information share measure does not seem to be an informative statistical measure.

Baillie, Booth, Tse, and Zabotina (2002) show that upper and lower bounds can differ substantially, when the contemporaneous correlation between innovations is high, even if returns are observed at one minute intervals. Since the contemporaneous correlation becomes smaller the higher the sampling frequency, tick time data would be the preferred observation frequency. The use of tick time data for estimating price discovery among Nasdaq dealers is the main topic of this paper.

In developing information shares based on a tick time model we deviate from Hasbrouck (1995) in three ways. First, we use a different model for the dynamics of quote changes. In the traditional vector autoregression of Hasbrouck (1995) the vector of quotes changes will contain many zeros in tick time, since for most observations only a single dealer will refresh his quote. For a multivariate time series analysis some time aggregation will be required to construct a vector of non-zero quote updates for all dealers for every time interval. An unobserved components specification is more suitable for ultra-high frequency data, since it does not require a complete vector of dealer quotes in each time interval. The model we propose is an extension of the structural time series model of Hasbrouck (1993) to a setting with multiple dealers where quotes arrive in tick time.

Second, we allow for duration effects on the quote dynamics. Time is an important factor in microstructure (Engle and Patton, 2004). It affects the volatility of the efficient price (Engle, 2000) and also has an impact on the information content of dealer quote updates (Dufour and Engle, 2000). Both effects will be included in the model.

Third, we slightly change the definition of information shares. Our definition follows the calendar time measures proposed by De Jong and Schotman (2004) for an unobserved components model. It exploits the structural interpretation of the unobserved components model, in contrast to the reduced form expressions in Hasbrouck (1995). We first define price discovery measures in tick time, where they are a function of the time between quote innovations. Additionally, we integrate these measures over time to obtain their calendar time equivalents. The aggregated calendar time information shares are closely related to the Hasbrouck (1995) information shares.

In our empirical part we examine the quotes of two ECN's (Island and Instinet) and three wholesale market makers at Nasdaq for 20 actively traded stocks with varying liquidity. These market makers are selected as the most active in terms of quoting frequency. For a tick time model it is more natural to consider the quote setting behavior of individual dealers instead of classes of dealers as in Huang (2002). Considering individual dealers is also in line with Schultz (2003) who finds a lot of heterogeneity among individual market makers.

As a preview of the results, we find that volatility does not evolve in calendar time, but in tick time as predicted by Clark (1973). The information flow to the efficient price is in general less at longer durations. We confirm the hypothesis of Easley and O'Hara (1992), which states that long durations convey no information. Similar results were found by Dufour and Engle (2000), and Engle and Patton (2004).

Price discovery measures in tick time appear strongly dependent on durations.

Some dealers reveal information when durations are short whereas others reveal information when durations are long. Aggregating to calendar time we can often clearly identify the dominant dealer. In terms of price discovery we find that Island, the most important of the ECNs, tends to dominate the liquid stocks, whereas market makers dominate the less liquid stocks.

The remainder of this paper is structured as follows. Section 2 discusses specification and estimation of the model. Section 3 defines the measures for price discovery in tick time and derives the calendar time aggregation of these measures. In section 4 we discuss the data. Section 5 presents the results of the model and the price discovery measures. Finally section 6 concludes and provides suggestions for further research.

2 A Model for Quotes in Tick Time

In this section we introduce a structural time series model for quote data in tick time. The model is an extension of the unobserved components model of Hasbrouck (1993) and theoretically motivated by the asymmetric information model of Glosten and Harris (1988).

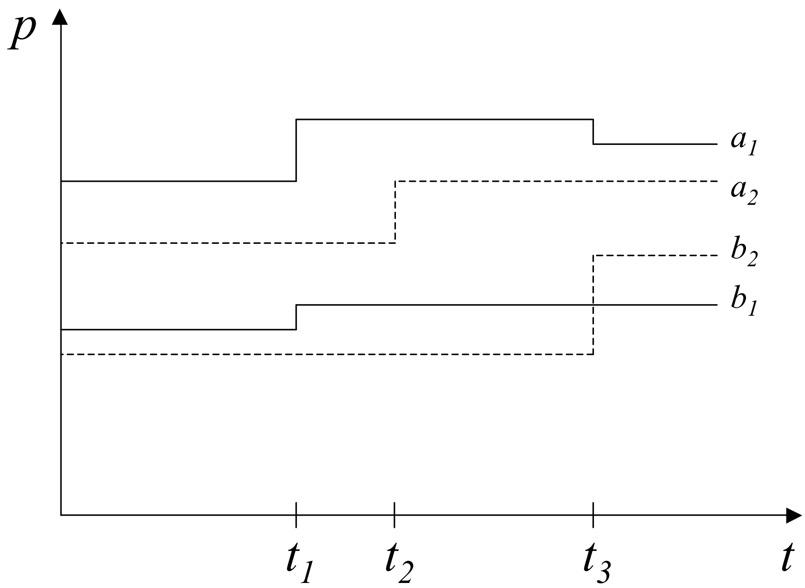

Figure 1: Quote Arrivals
The figure shows bid and ask quotes for two dealers. Bid quotes are denoted b_1 and b_2; ask quotes a_1 and a_2, respectively.

We consider a dealer market where M dealers issue bid and ask quotes. Quotes arrive at times t_ℓ ($\ell = 1, \ldots, L$). Figure 1 illustrates the quote arrival process. At t_1

dealer 1 increases her bid and ask quote simultaneously. Next, at t_2, dealer 2 increases only her ask. At time t_3 both dealers change one of their quotes. When sampling in tick time we are interested in all quotes that are updated. These can be updates in bid and/or ask quotes of one or more dealers. The time between two consecutive quote arrivals is the duration $\tau_\ell = t_\ell - t_{\ell-1}$. We will specify the dynamics of quote updates conditional on durations.

With M dealers in the market we have $2M$ different time series of quotes. Let q_ℓ be the $(2M \times 1)$ vector of all standing quotes at time t_ℓ. The bid (ask) of dealer i corresponds to element $2i-1$ $(2i)$ of q_ℓ.

Our interest is in modeling the dynamics of quote updates. We define the $(k_\ell \times 2M)$ selection matrix J_ℓ, containing the rows of the identity matrix that correspond the elements of q_ℓ that are updated at time t_ℓ. The selection matrices at times t_1, t_2 and t_3 in figure 1 would be

$$J_1 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}, \qquad J_2 = \begin{pmatrix} 0 & 0 & 0 & 1 \end{pmatrix}, \qquad J_3 = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

The vector of updated quotes is $J_\ell q_\ell$. We assume the time series of updated quotes can be described by an unobserved components model,

$$J_\ell q_\ell = J_\ell \left(c + \iota m_\ell + u_\ell \right), \tag{1}$$

where m_ℓ is the permanent component of the price, which is common to all dealers and will be referred to as the "true" or efficient price of the asset; ι is the $2M$-vector of ones; c is an $2M$-vector of constants; and u_ℓ is a $2M$-vector of temporary deviations from m_ℓ.

The permanent component is assumed to follow a random walk,

$$m_\ell = m_{\ell-1} + \sigma_\ell r_\ell, \tag{2}$$

with duration dependent volatility (σ_ℓ) and with an innovation term (r_ℓ) with unit variance. By allowing for volatility to depend on duration we can test for the time scale in which the price process evolves. Clark (1973) argues that the relevant time scale for the price process is the rate at which information arrives to the market. He measures this arrival rate by the amount of volume that is transacted. Harris (1987) and Ané and Geman (2000) show that it is not so much volume that drives the price process but the arrival of orders. We specify volatility as a function of duration,

$$\sigma_\ell = \sigma \tau_\ell^{\delta_1}, \tag{3}$$

The parameter δ_1 measures the impact of quote durations on the volatility of the random walk. If $\delta_1 = \frac{1}{2}$ the random walk is said to evolve in calendar time. In this case the variance of $(m_p - m_s)$ is equal to $(\sum_{\ell=s+1}^{p} \tau_\ell)\sigma^2$, and thus proportional to the length of the calendar time interval.[1] When $\delta_1 = 0$, the variance of the random walk is not affected by the time between quote updates. In this case the random walk evolves in tick time. In this case the calendar time variance is proportional to the number of quote updates.

The constants c measure the average spread between a bid- or ask-quote and the efficient price. This term captures e.g. the order processing costs of the market maker. To reduce the number of parameters we assume that bid and ask deviations from the efficient price are symmetric, meaning that the elements in the constant c are of the same magnitude (negative for bid, positive for ask).

The last term in (1) is the transitory component u_ℓ, which is related to informational asymmetries among dealers about the true price, dealer specific inventory costs and other sources of microstructure noise such as price discreteness. Glosten and Harris (1988) argue that the prices of informed market participants are correlated with the innovation in the efficient price. The model of Glosten and Harris (1988) explains the dynamics of transaction prices, whereas our model is formulated for quote updates. Since we only look at those quotes that are updated at time t_ℓ, the quote series $J_\ell q_\ell$ are information events like transactions prices. We therefore split u_ℓ in two components,

$$u_\ell = \alpha_\ell r_\ell + e_\ell, \tag{4}$$

where the $2M$-vector α_ℓ measures the asymmetric information, and e_ℓ is idiosyncratic noise. As for the evolution of the efficient price, we allow this information component to depend on the time between quote innovations,

$$\alpha_\ell = \alpha \tau_\ell^{\delta_2} \sigma, \tag{5}$$

where α is a $2M$-vector of parameters. The parameter δ_2 controls the impact of duration on the asymmetric information. If $\delta_2 < 0$, quotes become less informative at longer durations. This would be consistent with Easley and O'Hara (1992) who hypothesize that long durations convey no information. Dufour and Engle (2000)

[1] In the empirical model duration is normalized by dividing it by the average quote duration over the whole sample. This normalization only affects the σ parameter, which now refers to the volatility per tick, instead of the volatility per second. It facilitates interpretation of σ across different stocks.

and Engle and Patton (2004) find that the price impact of trades is largest at short durations.

Although durations between quote innovations of a particular dealer will be interesting as well, our primary interest here is on the duration between any quotes. This provides a measure for the speed of the market and we can test whether asymmetric information is larger during active periods or less active periods. In this sense we also capture part of the intradaily differences between open, mid-day and close, as durations tend to be smaller near the open and close and larger in the mid-day.

The idiosyncratic quote noise e_ℓ is uncorrelated with r_ℓ and τ_ℓ and has covariance matrix Ω. We assume that Ω has a block diagonal structure,

$$\Omega = \begin{pmatrix} \Omega_1 & & \\ & \ddots & \\ & & \Omega_M \end{pmatrix}, \tag{6}$$

where each Ω_i is a (2×2) matrix. The idiosyncratic bid and ask quotes of the same dealer are correlated, but not correlated with innovations in other dealer quote updates. Dealer specific noise includes inventory effects and a remaining microstructure noise. Theoretical models for inventory costs adhere to the notion that when a dealer receives inventory she will lower her ask to induce a trade at the opposite side, but also lowers her bid to avoid receiving additional inventory. Therefore, a specific dealer may wish to alter both quotes simultaneously due to the inventory position she has. Simultaneous changes in the quotes of different dealers can be due to asymmetric information both dealers have, which is captured by α.

A second reason for assuming the block diagonal structure is related to identification. To identify covariances between idiosyncratic noise of two different dealers, we would need to observe simultaneous quote updates of these two dealers. Since most quote updates J_ℓ contain changes of the quotes of a single dealer, only a small fraction of the data would be available for estimation of the off-diagonal blocks of Ω.

With the block diagonal structure for Ω no other restrictions are necessary for the full identification of all parameters. The only problematical case would be when both duration parameters δ_1 and δ_2 would be equal to zero, and at the same time all quote updates would be by a single dealer. Appendix 6 discusses the identification in more detail.

To estimate the model we put it in state space form for a time series process with missing observations. Following Harvey (1989) we write

$$\begin{aligned} J_\ell q_\ell &= J_\ell \left(c + \iota m_\ell + u_\ell\right), \\ m_\ell &= m_{\ell-1} + \sigma_\ell r_\ell, \\ u_\ell &= \alpha \tau_\ell^{\delta_2} \sigma r_\ell + e_\ell, \end{aligned} \tag{7}$$

Estimation of (7) is done by Maximum Likelihood using the Kalman Filter. As the underlying process is a random walk, the system cannot be initialized using the long run mean of the underlying process. We use a diffuse prior to initialize the random walk process and exclude the first 50 observations in the calculation of the likelihood function. To incorporate the potentially large price change overnight the model is re-initialized everyday with a diffuse prior. However, the parameters are estimated over the whole sample period.

As the model considers quote updates, the dimension of the quote vector will frequently be one. This makes computations in the Kalman Filter recursions straightforward and numerically fast for most observations. However since this dimension changes over time the model will not converge to a steady state and the full recursion will have to be computed for each observation. For this reason the optimization of the likelihood is computationally intensive.

3 Price Discovery

In this section we define price discovery measures in tick time using the unobserved components model of the previous section. These tick time measures are subsequently aggregated to calendar time equivalents.

3.1 Price Discovery in Tick time

We consider three measures to summarize the quote setting behavior of dealers. The first measure considers how dealers incorporate information in the efficient price into their quote innovations. The second measure considers the contribution of quote innovations of each dealer to the evolution of the efficient price. The last measure combines the two to obtain the information share of each dealer analogously to Hasbrouck (1995).

To explore the implications of the model we consider the following scenario. Suppose at t_ℓ each dealer issues bid and ask quotes and that the previous efficient price

$m_{\ell-1}$ is known to all dealers. Quote updates reflect both the change in efficient price, $m_\ell - m_{\ell-1}$ and the dealer noise e_ℓ. Substituting (2) and (4) in (1) the innovation of the dealer quotes is equal to

$$\begin{aligned} v_\ell &= q_\ell - \mathsf{E}[q_\ell | m_{\ell-1}, q_{\ell-1}] \\ &= (\iota\sigma_\ell + \alpha_\ell)\, r_\ell + e_\ell \\ &= \left(\iota + \frac{\alpha_\ell}{\sigma_\ell}\right)(m_\ell - m_{\ell-1}) + e_\ell \end{aligned} \tag{8}$$

From the decomposition (8) we obtain the first measure of the price discovery process. The duration dependent regression coefficient,

$$\beta_\ell = \iota + \frac{\alpha_\ell}{\sigma_\ell}, \tag{9}$$

shows how much of the change in the efficient price is immediately reflected in the quote update. As this measure refers to dealer efficiency and reveals how transparent the efficient price process is to a particular dealer, we refer to this measure as *dealer liquidity*. The more of the innovation in the random walk that is incorporated in dealer quotes, the more liquid they are. Using (5) and (3) for the dependence of α_ℓ and σ_ℓ on duration τ_ℓ we find

$$\beta(\tau) = \iota + \alpha\tau^{(\delta_2 - \delta_1)}, \tag{10}$$

as the $2M$-vector measuring dealer liquidity. It is a function of the duration between quote innovations with crucial parameters α, δ_1 and δ_2. At long durations this measure will converge to ι when $\delta_1 > \delta_2$. This is theoretically the most likely case, since we expect $0 < \delta_1 < \frac{1}{2}$ and $\delta_2 < 0$. The sign of α determines whether 1 is an upper or a lower bound for the respective element in $\beta(\tau)$. When α is larger than zero, $\beta(\tau)$ will be larger than one. This indicates that this quote incorporates more information of the random walk and vice versa. Hence, whether a dealer is more or less efficient depends solely on the sign of α.

For the second measure we consider the reverse regression, relating the change in the efficient price to innovations in dealer quotes similar to Hasbrouck's (1995) analysis of price discovery,

$$\Delta m_\ell = \gamma_\ell' v_\ell + \eta_\ell, \tag{11}$$

where γ_ℓ is an $2M$-vector which expresses the change of efficient price conditional on innovations in dealer quotes, and η_ℓ is the part of the change in the efficient price that is orthogonal to v_ℓ. Contrary to the reduced form variance decomposition in Hasbrouck (1995) the change in the efficient price is not a deterministic function of the quote innovations. The structural model (1) leads to a remainder term η_ℓ in (11).[2] Being regression parameters, γ_ℓ is defined as

$$\gamma_\ell = \text{Var}(v_\ell)^{-1}\text{Cov}(v_\ell,\, \Delta m_\ell). \tag{12}$$

For the variance in dealer quote innovations we find

$$\text{Var}(v_\ell) = \Sigma_\ell = \sigma_\ell^2 \beta_\ell \beta_\ell' + \Omega \tag{13}$$

The covariance between the change in the efficient price and dealer quote innovations follows directly from (8) as

$$\text{Cov}(v_\ell,\, \Delta m_\ell) = \sigma_\ell^2 \beta_\ell \tag{14}$$

To solve for γ_ℓ, note that the structure of Σ_ℓ is that of a full matrix Ω plus a symmetric rank one correction $\sigma_\ell^2 \beta_\ell \beta_\ell'$. We can guess a solution $\gamma_\ell = a\Omega^{-1}\beta_\ell$ and solve for a using the equality $\Sigma_\ell \gamma_\ell = \sigma_\ell^2 \beta_\ell$, leading to

$$\gamma_\ell = \frac{\sigma_\ell^2}{1 + \sigma_\ell^2 \beta_\ell' \Omega^{-1} \beta_\ell}\, \Omega^{-1} \beta_\ell. \tag{15}$$

As with $\beta(\tau)$ we can make the functional dependence on τ explicit through

$$\gamma(\tau) = \frac{\sigma^2 \tau^{2\delta_1} \Omega^{-1} \beta(\tau)}{1 + \sigma^2 \tau^{2\delta_1} \beta(\tau)' \Omega^{-1} \beta(\tau)}. \tag{16}$$

Again the regression coefficients are a function of the duration τ_ℓ. Contrary to $\beta(\tau)$ this second measure depends strongly on the idiosyncratic noise Ω^{-1}. Assuming all other factors constant, this measure increases for a particular dealer when the corresponding elements in Ω^{-1} increase (i.e. the particular dealer has low idiosyncratic noise) and vice versa.

The last price discovery measure is defined as the proportional contribution of dealer quote innovations to the innovation in the efficient price and resembles the Hasbrouck (1995) *information shares*. This measure is determined as the fraction of the variance of the change in the efficient price that can be attributed to innovations in the quote updates of a dealer. Taking the variance on both sides of (11) gives

$$\sigma_\ell^2 = \gamma_\ell' \Sigma_\ell \gamma_\ell + \kappa_\ell^2, \tag{17}$$

[2] De Jong and Schotman (2004) further explore the relations between the structural unobserved components model and the reduced form vector error correction model.

where κ_ℓ^2 is the variance of η_ℓ and is implicitly defined by (17). Since in general $\kappa_\ell^2 > 0$ in (17), the R^2 of regression (11) will not be equal to one.

From elementary linear regression analysis we can write the R^2 of (11) as

$$R_\ell^2 = \frac{\gamma_\ell' \Sigma_\ell \gamma_\ell}{\sigma_\ell^2} = \gamma_\ell' \beta_\ell \tag{18}$$

The last equality in (18) immediately suggests how to define the contributions to price discovery of the individual dealers. Let $\beta_{i,\ell} = (\beta_{i,\ell}^b \; \beta_{i,\ell}^a)'$ and $\gamma_{i,\ell} = (\gamma_{i,\ell}^b \; \gamma_{i,\ell}^a)'$ be the (2×1) subvectors of β_ℓ and γ_ℓ that correspond to the bid and ask quotes of dealer i. Then we define the information shares analogously to DeJong and Schotman (2004) as

$$\text{IS}_{i,\ell} = \beta_{i,\ell}' \gamma_{i,\ell} = \left(\frac{\sigma_\ell^2}{1 + \sigma_\ell^2 \beta_\ell' \Omega^{-1} \beta_\ell} \right) \beta_{i,\ell}' \Omega_i^{-1} \beta_{i,\ell}. \tag{19}$$

Due to the block-diagonal structure of Ω we obtain a unique variance decomposition, unlike in the reduced form vector error correction models of price discovery. The information shares sum to R_ℓ^2. For the interpretation of the information shares we can expand the last part of (19) as

$$\text{IS}_{i,\ell} = \frac{\phi_{i,\ell}}{1 + \sum_{j=1}^{M} \phi_{j,\ell}}, \tag{20}$$

where

$$\phi_{i,\ell} = \frac{\sigma_\ell^2}{1 - \rho_i^2} \left(\left(\beta_{i,\ell}^b / \omega_i^b\right)^2 + \left(\beta_{i,\ell}^a / \omega_i^a\right)^2 - 2\rho_i \left(\beta_{i,\ell}^b / \omega_i^b\right) \left(\beta_{i,\ell}^a / \omega_i^a\right) \right) \tag{21}$$

and

$$\Omega_i = \begin{pmatrix} (\omega_i^b)^2 & \rho_i \omega_i^b \omega_i^a \\ \rho_i \omega_i^b \omega_i^a & (\omega_i^a)^2 \end{pmatrix}$$

The important element in the information shares are the ratios $\beta_{i,\ell}^p / \omega_i^p$ $(p = a, b)$. It divides the impact of the dealer quote with respect to the change in the efficient price by the idiosyncratic noise of the dealer. Since all elements of β_ℓ depend on τ_ℓ, the ordering of the information shares can differ depending on the duration towards the most recent quote update. The expression for $\phi_{i,\ell}$ simplifies to the simple ratio $(\sigma_\ell \beta_{i,\ell} / \omega_i)^2$ when we would just include either the bid, or ask or mid-quote in the vector q_ℓ.

Looking back at (8), it should be noted that we consider the entire quote vector q_ℓ, whereas only the sub-vector $J_\ell q_\ell$ has been updated. Strictly speaking we should therefore consider the relation between the innovation in $J_\ell q_\ell$ and Δm_ℓ in defining β_ℓ and γ_ℓ. Since we do not have a model of who will be the next dealer to issue a quote, we do not have anything to say about predicting the selection matrix J_ℓ conditional on $J_{\ell-1}$ and $q_{\ell-1}$. Proceeding conditional on J_ℓ, the innovation in $J_\ell q_\ell$ will be equal to $J_\ell v_\ell$. The only effect of this on the information measure β_ℓ is that we obtain the sub-vector $\tilde{\beta}_\ell = J_\ell \beta_\ell$ instead of the full β_ℓ. In the reverse regression we likewise find

$$\tilde{\gamma}_\ell = \frac{\sigma_\ell^2}{1 + \sigma_\ell^2 \tilde{\beta}_\ell' \tilde{\Omega}^{-1} \tilde{\beta}_\ell} \tilde{\Omega}^{-1} \tilde{\beta}_\ell, \tag{22}$$

with $\tilde{\Omega}_\ell = J_\ell \Omega J_\ell'$. Due to the block diagonal structure of Ω, and because most quote updates involve both the bid and ask of the same dealer, this will in most cases be a rescaled sub-vector of γ_ℓ.

So far all measures have a subscript ℓ to indicate that they are conditional on the duration τ_ℓ. In this way the price discovery measures depend on the latest quote arrival, and do not provide an overall picture of the price discovery process. As a more relevant measure the next subsection will aggregate the information contents of a series of quote arrivals.

3.2 Calendar Time Aggregation

In the previous section we discussed measures for price discovery of single quote updates. In this section we define similar measures of price discovery over time. To obtain these measures we aggregate the tick time measures to fixed time intervals.

Let us start by fixing some notation. In calendar time we refer to the present time with the suffix (t), while we use the subscript ℓ to denote variables in tick time. To establish the link between calendar time and tick time, let $\ell(t)$ represent the closest observation preceding to time t, so that $q(t) = q_{\ell(t)}$ and similarly $q_\ell = q(t_\ell)$. Between times t and $t-1$ there are $\ell(t) - \ell(t-1)$ quote updates.

In calendar time innovations in dealer quotes are decomposed as

$$v(t) = q(t) - \mathsf{E}[q(t)|m(t-1), q(t-1)] = \iota \Delta m(t) + u_{\ell(t)} \tag{23}$$

with $\Delta m(t) = m(t) - m(t-1)$. The innovation in quotes is equal to the change in the efficient price plus the noise around this efficient price observed at the last interval. The change in the efficient price from $\ell(t-1)$ to $\ell(t)$ is the sum of all the interjacent changes. Only the most recent change is part of both $\Delta m(t)$ and $u_{\ell(t)}$. We can write (23) as

$$v(t) = \iota \sum_{j=\ell(t-1)+1}^{\ell(t)-1} \sigma_j r_j + \sigma_{\ell(t)} \beta_{\ell(t)} r_{\ell(t)} + e_{\ell(t)}. \tag{24}$$

From (24) we can compute the calendar price discovery measures $\beta(t)$, $\gamma(t)$ and $\text{IS}(t)$. Start by defining the moments

$$\begin{aligned}
\mathsf{E}[\Delta m(t)^2] &= \sum_{j=\ell(t-1)+1}^{\ell(t)} \sigma_j^2 \\
\mathsf{E}[v(t)\Delta m(t)] &= \iota(\sigma(t)^2 - \sigma_{\ell(t)}^2) + \sigma_{\ell(t)}^2 \beta_{\ell(t)} \\
\mathsf{E}[v(t)v(t)'] &= (\sigma(t)^2 - \sigma_{\ell(t)}^2)\iota\iota' + \sigma_{\ell(t)}^2 \beta_{\ell(t)} \beta_{\ell(t)}' + \Omega
\end{aligned} \tag{25}$$

We will frequently use the shorthand notation

$$\begin{aligned}
\sigma(t)^2 &= \mathsf{E}[\Delta m(t)^2], \\
\tilde{\sigma}(t)^2 &= \sigma(t)^2 - \sigma_{\ell(t)}^2, \\
\Sigma(t) &= \mathsf{E}[v(t)v(t)'].
\end{aligned} \tag{26}$$

The volatility $\sigma(t)$ increases with the length of the interval and is a function of all durations in this time interval. This immediate implies that the covariance matrix $\Sigma(t)$ converges to a matrix of rank one proportional to $\iota\iota'$. This is simply a consequence of cointegration with a single common trend. Over long enough intervals all quote updates will be perfectly correlated.

Dealer liquidity $\beta(t)$ is the response of innovations in the dealer quotes to a change in the efficient price over the time interval. This is the regression coefficient of quote innovations $v(t)$ on the change in the efficient price $\Delta m(t)$. From (25) we find

$$\beta(t) = \frac{\mathsf{E}[v(t)\Delta m(t)]}{\mathsf{E}[\Delta m(t)^2]} = \frac{\tilde{\sigma}(t)^2}{\sigma(t)^2}\iota + \frac{\sigma_{\ell(t)}^2}{\sigma(t)^2}\beta_{\ell(t)}, \tag{27}$$

which is now a function of all the durations between $t-1$ and t. Recalling the definition of β_ℓ in (9), it is seen that $\beta(t)$ is again linear in α. Elements of $\beta(t)$ have a lower bound of one when the corresponding element of α is positive and an upper bound of one when it is negative.

Since $\sigma(t)^2$ (and $\tilde{\sigma}(t)^2$) increase with the length of the interval, the parameter $\beta(t)$ will converge to the unit vector ι when the calendar time interval increases. The rate of convergence towards ι depends on the parameters δ_1 and δ_2. Thus when intervals are larger, dealers incorporate information about the true price more efficiently into their quotes and possible asymmetries in information that dealers have about the innovation in the efficient price disappears. Over long enough time intervals, all dealers will follow the movements of the underlying market price.

For the price discovery measure $\gamma(t)$ we again consider the reverse regression,

$$\Delta m(t) = \gamma(t)' v(t) + \eta(t) \tag{28}$$

with has regression coefficients

$$\gamma(t) = \Sigma(t)^{-1} \beta(t) \sigma(t)^2 \tag{29}$$

The derivation of $\gamma(t)$ is slightly more involved than it was for the tick time equivalent γ_ℓ, since $\Sigma(t)$ is now a symmetric rank two correction on Ω. We guess a solution

$$\gamma(t) = c_\iota \Omega^{-1} \iota + c_\beta \Omega^{-1} \beta_{\ell(t)}, \tag{30}$$

with scalars c_ι and c_β for which we provide the explicit solution in appendix A. The constants depend on all parameters of the model and all durations in period t.

Finally for the information shares in calendar time we consider the R^2 of (28),

$$R^2(t) = \gamma(t)' \beta(t) \tag{31}$$

and decompose it in the information shares

$$\begin{aligned} \mathrm{IS}_i(t) &= \beta_i(t)' \gamma_i(t) \\ &= d_{\iota\iota} \left(\iota_2' \Omega_i^{-1} \iota_2 \right) + d_{\iota\beta} \left(\iota_2' \Omega_i^{-1} \beta_{i,\ell(t)} \right) + d_{\beta\beta} \left(\beta_{i,\ell(t)}' \Omega_i^{-1} \beta_{i,\ell(t)} \right), \end{aligned} \tag{32}$$

with ι_2 a vector of 2 ones, and scalars $d_{\iota\iota}$, $d_{\iota\beta}$ and $d_{\beta\beta}$ that follow directly from (27) and (30) and are given explicitly in appendix A. When the sampling interval increases the change in dealer quotes will to a large extent represent the change in the efficient price. As a consequence the R^2 of the regression of the change in the efficient price on dealer quote innovations converges to one. The convergence of R^2 to one together with the convergence of $\beta(t)$ to ι, implies that both the elements of $\gamma(t)$ as well as the information shares $\mathrm{IS}_i(t)$ will sum to one for large intervals. Details are in appendix A. We therefore conclude that information shares are the only relevant measure in calendar at sufficiently large intervals.

4 Data

We use data provided by Nastraq. The Nastraq data set contains all trades and quotes that occur within normal trading hours at Nasdaq. The dealer quote data contains all quotes issued within normal trading hours, time stamped to the nearest second. Most important, it contains the identity of the dealer that issues the quote. We select 20 actively traded companies with different liquidity listed at Nasdaq during February 1999. The selected stocks and their symbols are reported in table 1.

Our data are similar to the data used by Huang (2002). His data are from different months in 1999. Contrasting to our approach, he creates categories of different types of dealers. We prefer to consider quotes of individual dealers as quoting behavior is heterogenous even within categories (see Schultz, 2003). Additionally, the model developed in section 2 is more suited to describe individual quoting behavior. We consider dealer quotes of the five largest dealers in terms of quoting frequency. This leads to the selection of two ECNs, Island and Instinet, and three wholesale market makers, which differ depending on the stock considered.

Since we are interested in modelling the innovations in dealer quotes, we remove all quotes that do not change. For example, when dealer i innovates only her ask, the corresponding bid quote is removed. When a dealer issues multiple quotes at the same second, the last quote in this sequence is selected.[3] Due to these selection criteria the number of updated quotes is just the bid and/or ask of a single dealer in more 80% of all cases.

INSERT TABLE 1 HERE

We also remove outliers before estimation. Outliers often occur when a dealer is unwilling to trade. In this case she will issue a quote far away from the inside (best quote in the market). This can happen on one side of the market when a dealer is unwilling to take on more inventory. Also, ECNs are not obliged to make the market on both sides of the market, and in some cases do not issue a quote on one particular side of the market. In this case a zero-quote is reported by Nastraq. Although these quotes send a strong signal to the other marker participants, this is something not considered here as we focus on the issue of price discovery. We define a quote as an outlier when it is more than \$ 2 away from the average of the past 50 quotes.

In table 1 we also report some summary statistics of the data after filtering. Our sample contains very actively traded stocks like Dell as well as less actively traded stocks like Starbucks. Since so many quote updates are updates by a single dealer, it will be difficult to estimate covariances between the idiosyncratic component of the

[3] This occurs frequently in the case of Island ECN, where many small trades are matched within Island itself. Every time such a transaction occurs, Island will send a new best standing quote to Nasdaq, which in many cases merely reflects a change in the quote depth. A change in quote depth is not considered and all quotes that reflect a change in depth are removed.

noise of different dealers. Average quote durations are just the inverse of the total number of observations. They range from three seconds for the most active stocks to half a minute for the less active stocks. The variance in durations can also lead to identification problems is the variance in durations. The statistics table 1 are reassuring in this respect, since durations for all stocks are highly volatile and will not to cause any identification problems for δ_1 or δ_2.

5 Results

In this section we provide the results of the model proposed in section 2. We start by providing results for the duration parameter estimates of (7). Next we address the measures of price discovery, first in tick time, then in calendar time.

5.1 Duration Effects

In table 2 we present the estimates of the duration parameters δ_1 and δ_2, including their standard errors in parentheses. For δ_1, which measures the effect of quote durations on the innovation in the efficient price, the values of interest are $\delta_1 = \frac{1}{2}$, in which case the price process is said to evolve in calendar time, and $\delta_1 = 0$, in which case the process evolves in tick time. Results for δ_1 show that we can reject the hypothesis that the price process evolves in calendar time. In most cases δ_1 is either insignificantly different from zero, or significantly negative. These findings provide evidence for a price process that evolves in tick time and confirms findings of Ané and Geman (2000). The calendar time variance of the random walk process therefore increases if many quote innovations occurred over a predefined period of time. Negative values for δ_1 can be explained by the fact the we consider the quote process, which is not necessarily the process that drives the flow of information. If the true information arrival process (e.g the transactions process) evolves faster than the quoting process, then negative values for δ_1 are expected.

INSERT TABLE 2 HERE

The parameter δ_2 measures the duration dependence of the asymmetric information component α. We find that in 17 cases δ_2 is significantly negative, which entails that long quote durations lead to less asymmetric information and short durations to more. These results confirm findings of Easley and O'Hara (1992), who hypothesize that long durations convey no information. In our case we find that asymmetries have decreased.

The combined results for δ_1 and δ_2 implicate that periods with short durations are periods with higher (calendar time) volatility and more asymmetric information. Consequently, volatile periods are periods where asymmetries are large and vice versa. For financial markets, durations tend to be shorter near the open and close of the market and longer in the middle of the trading day. Hence these are the periods when volatility is higher and asymmetries are larger.

5.2 Price Discovery per Quote Innovation

In this section we present the results for the three price discovery measures in tick time as proposed in section 3.1. As shown, these measures are functions of quote durations and are therefore affected by the parameter estimates of δ_1 and δ_2. We present detailed results for three representative stocks in the sample (Intel, CMGI and Apple) and show results for all stocks at average durations. We start by presenting the results for these three stocks to provide more insight into price discovery measures.

INSERT FIGURE 2 HERE

Apart from being affected by quote durations, price discovery measures dependent on the parameter estimates of α and Ω. To provide some insight into how these parameters affect price discovery measures, we show these estimates for the three selected stocks in figure 2. The top part of each graph displays the estimates for α. An α significantly higher/lower than zero indicates that a dealer has more/less exposure to the innovation in the efficient price than the average of all dealers and affects the price discovery measures for this particular dealer. For α, estimates differ substantially among dealers, with highest values for market maker 1, market maker 2 and market maker 1 for Intel, CMGI and Apple respectively. The bottom part in each graph displays estimates for the elements in Ω. These elements measure the amount of variance of a dealer quote innovation that is not attributable to the efficient price process. A low idiosyncratic noise variance therefore indicates that this particular dealer tracks the efficient price closely. Size of the idiosyncratic variance differs substantially among dealers. For Intel and CMGI, the Island ECN has the lowest idiosyncratic dealer noise. For Apple, market maker 2 has the lowest. There are further quite substantial differences in size of Ω among the different stocks. In general, Intel has lower idiosyncratic noise than CMGI and Apple, indicating that dealers track the efficient price more effectively for Intel.

The first measure for price discovery ($\beta(\tau)$) measures the innovations in dealer quotes that are due to a change in the efficient price. As detailed in section 3.1 this measure depends on the parameters δ_1, δ_2 for convergence. Differences in dealer liquidity are fully attributable to differences in α.

INSERT FIGURE 3 HERE

In figure 3 we show $\beta(\tau)$ for the three selected stocks as a function of quote duration. For all stocks $\beta(\tau)$ converges towards unity when quote durations are longer, as $\delta_2 < \delta_1$. This is expected, because when quote durations are long, dealers have time to learn about the information present in the efficient price. However, for all stocks, convergence is still not achieved even when durations are as long as one minute. This indicates that asymmetries are persistent among dealers. The measure for dealer liquidity is most dispersed at short durations indicating the largest asymmetry in information. Further, the dispersion is larger, the lower the activity of the stock. Hence, asymmetric information is more pronounced in less active stocks.

For Intel we find that market maker 1 has the highest measure of dealer liquidity. For CMGI and Apple, these are market maker 2 and market maker 1, respectively. As noted, this "dominance" is completely attributable to the α of the particular dealers as can be seen from figure 2.

INSERT FIGURE 4 HERE

The second measure for price discovery ($\gamma(\tau)$) considers the reverse regression of the efficient price change on dealer quote innovations. This measure examines the effect of a single quote innovation on the change in the efficient price. Again this measure depends on quote durations. As detailed in section 3.1, this measure is highly dependent on the idiosyncratic dealer noise and is higher for dealers with low entries in the idiosyncratic covariance matrix Ω and higher for dealer with higher values for $\beta(\tau)$, or equivalently α.

In figure 4 we show $\gamma(\tau)$ for the three selected stocks. Generally, the dispersion in $\gamma(\tau)$ is largest at short duration, where the dominating dealers also obtain their highest value for price discovery. For Intel we observe a clear and persistent dominance of the Island ECN at all durations, for Apple we observe this persistent dominance for market maker 2. These results are mainly driven by the low idiosyncratic noise components for both dealers as shown in figure 2. In the case of CMGI, results are mixed. At short very durations market maker 2 dominates at longer durations. The dominance of market maker 2 at short durations is attributable to the high value of $\beta(\tau)$ at short duration. At longer duration $\beta(\tau)$ decreases for market maker 2 and the low idiosyncratic noise of Island dominates over this asymmetry effect.

INSERT FIGURE 5 HERE

The third measure for price discovery ($IS(\tau)$), referred to as the information share, is determined by considering the R^2 of the regression of the efficient price change on the innovation in dealer quotes, which is the inner product of $\beta(\tau)$ and $\gamma(\tau)$. This total R^2 is decomposed to individual information shares using the imposed structure on Ω. As it combines the measures $\beta(\tau)$ and $\gamma(\tau)$, the information share will be higher when the idiosyncratic dealer noise is lower and $\beta(\tau)$ is higher. As it considers the product of both these measures, the impact of $\beta(\tau)$ will be larger in the information share than in the measure for price discovery.

In figure 5 we display the graphs for the three stocks. Similar to the other measures, information shares are also highest at short durations. For the individual stocks we observe that Island is the dominating dealer for Intel and is followed by market maker 1, giving the same results as for the $\gamma(\tau)$ measure. The dominance of Island is again due to the low idiosyncratic variance. For CMGI we find the highest information share for market maker 2, which can be attributed to the higher value of $\beta(\tau)$. The Island ECN increases in importance at longer durations, which can be attributed to the strong increase in $\gamma(\tau)$ at longer durations. For Apple market maker 1 dominates, based on its relatively high measure for both $\beta(\tau)$ and $\gamma(\tau)$. However, the information share decreases quickly at longer durations.

INSERT TABLE 3 HERE

The results presented above lead to several conclusions for price discovery in general. First, we have seen that price discovery measures among dealers differ most at short durations. Hence, most asymmetry in price discovery measures is observed at short durations. Second, the dominating dealer achieves its highest information share at short durations. As durations are shorter near the open and close of the market, higher values for information shares are mainly observed during these periods. This suggests that the structural model proposed in section 2 captures at least some of the intraday dynamics of information asymmetry. Third, parameter estimates for δ_1 led to the conclusion that calendar time volatility is higher over periods where durations are short. Combining this result with the results for price discovery leads to the conclusion that price discovery tends to be larger when market volatility is higher, which confirms findings of Martens (1998).

To discuss the results for the other stocks in the sample we first report estimates for α and Ω. In table 3 we present the estimates for the α parameters in (7). Although these α's are not that informative on their own, they do indicate large heterogeneity among the different dealers. Apart from affecting price discovery measures, this heterogeneity indicates that dealer should not be grouped into categories (as in Huang, 2002) as their characteristics are very different. Moreover, the results also show that heterogeneity is present in the bid and ask quotes for many dealers. Interesting are the large values for α for particular dealers (e.g. market maker 1 for Dell, and market maker1 and 2 for Peoplesoft), which will affect the results for the price discovery measures.

INSERT TABLE 4 HERE

Table 4 presents average estimates of bid and ask quotes of the idiosyncratic dealer noise variance as estimated from (7). Averages are reported, as elements in Ω do not differ substantially between bid and ask quote of a particular dealer. Average variances do differ between dealers, indicating that quoting behavior differs considerably between dealers. For most of the active stocks in the sample (Cisco, Dell, Microsoft and Intel) we find that the Island ECN has the lowest idiosyncratic dealer noise. For the less active stocks in the sample other dealer have lower variances (e.g. Apple, Compuware and Starbucks). These results clearly have their implications for $\gamma(\tau)$ and $IS(\tau)$, which we discuss later.

For all stocks, measures for price discovery in tick time are reported in table 5. This table reports these measures at average durations. Panel A reports the measure for dealer liquidity.[4] Similar to the large differences in α we also find large differences in measures for dealer liquidity. Especially for Peoplesoft and Dell extremely large values for dealer liquidity are found.

[4]Recall that durations are scaled by dividing them by the average duration. At average durations $\tau = 1$ and therefore $\beta(\tau) = \iota + \alpha$.

INSERT TABLE 5 HERE

Panel B reports the measure for price discovery at average durations. All values are multiplied by 100 and can be interpreted as a the percentage of change in the efficient price due to a change in a particular dealer quote (bid or ask). Generally we find that the Island ECN has the highest price discovery measure for the most active stocks in the sample (Cisco, Dell, Intel and Microsoft). Other interesting findings are the zeros reported for the stocks of Amazon and Peoplesoft, where only one dealer seems to matter for price discovery. Also interesting is the high price discovery measure of market maker 1 for Starbucks. All these results are driven by large negative covariances between idiosyncratic noise terms. For most stocks and dealers we find small positive covariances, which is expected according to theories of inventory models. We therefore do not add too much value to these results.

Panel C reports results for information shares, again reported at average durations and multiplied by 100. These measures are reported per dealer and not per individual quote, as for the other two measures. There are some interesting results found in this panel. Generally, we find that information shares per quote innovation are quite low, mostly around 5%. Specifically, we find that the Island ECN dominates for most of the active stocks in the sample (Cisco, Intel and Microsoft). For less active stocks, other dealers dominate.

The last column reports the total regression R^2, or the sum of the information. This number indicates how much of the variance of the efficient price is attributable to quote changes of all dealers. Apart from the three stocks with huge information shares (Amazon, Peoplesoft and Starbucks), on average 14.30% of the variance of the efficient price can be explained by the quotes of the top five dealers.

5.3 Calendar Time Aggregation

In the previous part we discussed results for price discovery measures per quote update. When these measures are aggregated they provide useful information on how information is incorporated into prices over time and allow us to determine the total price contribution of dealers. Another benefit of time aggregation is that the measure we find is comparable to traditional measures, like Hasbrouck (1995) information shares. In this part we aggregate to one minute and five minute frequencies, as these frequencies are most commonly used in other studies.

In section 3.2 we showed how calendar time measures depend on $\sigma(t)^2$, which is a function of all durations that occur within the aggregation interval. Hence, we compute these measures by considering the entire sample of observed durations. Since each period t is the sum of a different set of durations τ_ℓ ($\ell = \ell(t-1) + 1, \ldots, \ell(t)$), we end up with distributions for each price discovery measure. We start again by presenting price discovery measures for the three selected stocks in detail and subsequently present summary results for all stocks.

INSERT FIGURE 6 HERE

In figure 6 we show the distributions of $\beta(t)$ at 60 and 300 seconds (only the distributions of the bid quotes are shown, ask quotes are similar). Depending on the sign of α these measures are higher or lower than one. Recall that when the time period over which we aggregate increases, this measure converges to one. This convergence is clearly seen for all three stocks. For Intel, dealers incorporate most of the information in the efficient price into their quotes at a 60 second aggregation level. As it is the most active stock of the three selected ones, it has most quote innovations occurring within an aggregation interval. Hence, convergence for Intel occurs quickly. Next, for CMGI we observe that there is still quite some dispersion in the liquidity measure at the 60 second aggregation level. At five minutes these measures have converged. For Apple, measures for liquidity have not converged, even at the five minute level. So even at five minute frequencies, dealers are not able to incorporate all the information of the efficient price into their quotes.

INSERT FIGURE 7 HERE

Results for $\gamma(t)$ are shown in the distribution plots in figure 7 (we show these measures for ask quotes, bids are similar). For all stocks, we observe that $\gamma(t)$ increases when the aggregation interval increases. We also find that the distribution of $\gamma(t)$ narrows at the longer aggregation interval. In terms of dominance, we find that Island unambiguously has the highest value for $\gamma(t)$ for both Intel and CMGI. For Apple results are less clear as distributions are very wide. Market maker 2, however, tends to have the highest value for $\gamma(t)$. These findings are in line with the measures in tick time, where Island dominated price discovery for both Intel and CMGI and market maker 2 dominated for Apple.

INSERT FIGURE 8 HERE

Finally, in figure 8 we show the distribution of $IS(t)$ per dealer for the three selected stocks. These graphs show similar distributions as for $\gamma(t)$, since . The reason for this is that $IS_i(t)$ is defined as the inner product of $\beta_i(t)$ and $\gamma_i(t)$. As $\beta_i(t)$ converges to unity at longer aggregation intervals, $IS_i(t)$ approximately equals the sum of $\gamma_i(t)$ for bid and ask quote. For CMGI and Apple the calendar time results for the information shares differ substantially from the tick time equivalents. For CMGI we found that $IS(\tau)$ is highest for market maker 2 and for Apple $IS(t)$ is highest for market maker 1, which was mainly due to the large values for $\beta(\tau)$. For $IS(t)$, Island has the highest value for CMGI and market maker 2 for Apple. The difference can again be attributed to the convergence of $\beta_i(t)$. Hence the information shares in calendar time only depend on the idiosyncratic dealer noise.

INSERT FIGURE 9 HERE

In figure 9 we show the sum of the information shares (R^2 of (28)) for one, two and five minute aggregation intervals. When we move to longer sampling intervals more information is incorporated in the efficient price of the stock. In the case of Intel and CMGI, all information is almost fully incorporated after five minutes. In the case of Apple we see that after five minutes there is still a substantial amount of dealer noise.

INSERT TABLE 6 HERE

For all stocks in the sample we report results for price discovery measures in calendar time in tables 6, 7 and 8. Reported results are at 60 and 300 second aggregation levels and averages of the measures of price discovery are reported.

In table 6 we report results for $\beta(t)$. These results again show the convergence to unity. Most stocks have achieved this convergence after five minutes, like Intel, but for the less liquid stocks this measure still has not converged (see e.g. QWST). Hence for the less active stocks $\beta(t)$ still affect $\gamma(t)$ and $IS(t)$.

INSERT TABLE 7 HERE

Table 7 reports average values for $\gamma(t)$ at a one and five minute aggregation level. Generally, price discovery measures increase when the sampling interval increases. At both sampling intervals we see that Island has very high measures for price discovery compared to the other dealers, especially for the more active stocks in the sample

(Cisco, Dell, Intel, Microsoft). For the less active stocks, however, it is mostly a market maker that dominates in price discovery (see Novell, Nextel, Peoplesoft, Qwest and Starbucks). Instinet never dominates the information shares.

INSERT TABLE 8 HERE

Results for information shares per dealer are shown in table 8. Similar to $\gamma(t)$ information shares increase when the sampling interval increases, as more of the variance of efficient price changes can be explained by dealer quote innovations. Again we note that for most stocks, and for all the liquid stocks Island is dominant. For the most liquid stocks, Cisco and Dell, 77% and 87% of the variance of the efficient price change is explained by Island. For the less active stocks (e.g. Apple, Novell, Nextel, Starbucks) information shares are dominated by other dealers.

The last column in table 8 reports the total R^2 for each stock. There is a clear increase in R^2 when the aggregation interval increases. However, the total amount of variance attributable to all dealers per stock differs. In general, R^2 is higher for the more active stocks (Cisco and Dell) and lower for the less active stocks (Novell). These results have implications for e.g. realized volatility (see Andersen et al., 2001), where microstructure noise is an important feature.

6 Concluding Remarks

This paper has introduced a model for dealer quoting behavior in tick time. Quote innovations are modelled as they arrive using an unobserved components model. As the model is defined in tick time, we additionally measure the effect of quote durations on the volatility of the efficient price process and the effect of quote durations on asymmetric information.

Given the model we define three measures for price discovery, one relating to dealer liquidity, one to price discovery and one to information shares. Consequently, these measures are aggregated to calendar time equivalents.

In our empirical results we find that more volatility is generated when durations between quote innovations are short. We also find that there is more asymmetric information in dealer quotes at short durations. Combining these two results leads to the conclusion that there is more asymmetric information when volatility is high.

The measures for price discovery in tick time show that all three measures address different features of the price discovery process. We find that in most cases the Island

ECN dominates the price discovery measure (as it is mainly based in the idiosyncratic dealer noise component), but other market maker dominate in terms of information share (due to the asymmetric information component). In calendar time the most relevant measure is the information share. For most active stocks we find that the Island ECN dominates, for less active stocks a wholesale market maker dominates.

Finally, the sum of the information shares can be used as an indication of how much microstructure noise remains in aggregated dealer quotes. It is also informative about the speed of price discovery. This information can be used for studies on e.g. realized volatility.

In developing our model we have remained close to the analysis of the quote setting process of Huang (2002). Extension of the model can go in several directions.

First, a time of the day effect can be accommodated for in the model. For example α parameters, which measure the asymmetric information can be redefined to depend on the time of the day. Similarly, the innovation in the efficient price can depend on the time of the day. Part of this time of the day effect is already captured by the inclusion of the quote durations. As durations have an intradaily pattern, measures for price discovery will also have an intradaily pattern.

Second, additional variables can be added to the model. For example, the volume quoted by a particular dealer might be an indicator of the amount of asymmetric information that a particular dealer may posses. Additionally, a quote innovation that brings a particular dealer to the inside (the best quote in the market) may contain more information than any other quote innovation (see Frijns and Schotman, 2004).

Third, in this paper we consider durations between quote innovations for the asymmetric information component, as we are interested in seeing how asymmetric information differs between active and tranquil periods. The model may be extended by including dealer specific quote durations, where we can measure how the speed at which a particular dealer quotes affects its asymmetric information, but also affects the asymmetric information component of other dealers.

Fourth, adding transaction prices and a buy/sell indicator, as in Barclay, Hendershott and McCormick (2003), enables a more precise estimation of the asymmetric information component. Within the unobserved components model, transaction prices are easily included even if they evolve at a different pace compared to quote innovations.

A Calendar Time Price Discovery Measures

This appendix provides the derivation of $\gamma(t)$ and $IS_i(t)$ in calendar time. We start by repeating the definition of $\gamma(t)$ in (29)

$$\gamma(t) = \Sigma(t)^{-1}\beta(t)\sigma(t)^2 \tag{A1}$$

and the earlier derived moments

$$\begin{aligned} \sigma(t)^2 &= \sum_{j=\ell(t-1)+1}^{\ell(t)} \sigma_j^2 \\ \Sigma(t) &= \tilde{\sigma}(t)^2 \iota\iota' + \sigma_{\ell(t)}^2 \beta_{\ell(t)}\beta_{\ell(t)}' + \Omega \\ \tilde{\sigma}(t)^2 &= \sigma(t)^2 - \sigma_{\ell(t)}^2, \\ \beta(t) &= \frac{\tilde{\sigma}(t)^2}{\sigma(t)^2}\iota + \frac{\sigma_{\ell(t)}^2}{\sigma(t)^2}\beta_{\ell(t)}, \end{aligned} \tag{A2}$$

To simplify notation we will use the shorthand ℓ for $\ell(t)$, when there can be no confusion that we mean the last quote update of period t, from time $t-1$ to t. As another useful shorthand notation we define the (2×2) matrix

$$V = \begin{pmatrix} V_{\iota\iota} & V_{\iota\beta} \\ V_{\iota\beta} & V_{\beta\beta} \end{pmatrix} = \begin{pmatrix} \iota'\Omega^{-1}\iota & \iota'\Omega^{-1}\beta_\ell \\ \iota'\Omega^{-1}\beta_\ell & \beta_\ell'\Omega^{-1}\beta_\ell \end{pmatrix} \tag{A3}$$

Except $V_{\iota\iota}$ the elements of V depend on the duration $\tau_{\ell(t)}$.

To solve for $\gamma(t)$ we guess the solution

$$\gamma(t) = c_\iota\Omega^{-1}\iota + c_\beta\Omega^{-1}\beta_\ell \tag{A4}$$

and substitute in

$$\Sigma(t)\gamma(t) = \sigma(t)^2\beta(t) \tag{A5}$$

to obtain

$$\begin{gathered} \left(\tilde{\sigma}(t)^2\iota\iota' + \sigma_\ell^2\beta_\ell\beta_\ell' + \Omega\right)\left(c_\iota\Omega^{-1}\iota + c_\beta\Omega^{-1}\beta_\ell\right) = \sigma(t)^2\left(\frac{\tilde{\sigma}(t)^2}{\sigma(t)^2}\iota + \frac{\sigma_{\ell(t)}^2}{\sigma(t)^2}\beta_\ell\right) \\ \Updownarrow \\ c_\iota\left(\tilde{\sigma}(t)^2 V_{\iota\iota} + 1\right)\iota + c_\iota\sigma_\ell^2 V_{\iota\beta}\beta_\ell + c_\beta\tilde{\sigma}(t)^2 V_{\iota\beta}\iota + c_\beta\left(\sigma_\ell^2 V_{\beta\beta} + 1\right)\beta_\ell = \sigma(t)^2\iota + \sigma_\ell^2\beta_\ell \end{gathered} \tag{A6}$$

Equating coefficients on ι and β_ℓ to determine the constants c_ι and c_β gives a system of 2 linear equations in two unknowns

$$\begin{pmatrix} \tilde{\sigma}(t)^2 V_{\iota\iota} + 1 & \tilde{\sigma}(t)^2 V_{\iota\beta} \\ \sigma_\ell^2 V_{\iota\beta} & \sigma_\ell^2 V_{\beta\beta} + 1 \end{pmatrix}\begin{pmatrix} c_\iota \\ c_\beta \end{pmatrix} = \begin{pmatrix} \tilde{\sigma}(t)^2 \\ \sigma_\ell^2 \end{pmatrix} \tag{A7}$$

with solution

$$\begin{pmatrix} c_\iota \\ c_\beta \end{pmatrix} = \frac{1}{D} \begin{pmatrix} V_{\beta\beta} - V_{\iota\beta} + 1/\sigma_\ell^2 \\ V_{\iota\iota} - V_{\iota\beta} + 1/\tilde{\sigma}(t)^2 \end{pmatrix} \tag{A8}$$

in which

$$D = \left(V_{\iota\iota} + 1/\tilde{\sigma}(t)^2\right)\left(V_{\beta\beta} + 1/\sigma_\ell^2\right) - V_{\iota\beta}^2 \tag{A9}$$

Of particular interest are the expressions for $\gamma(t)$ for small and large time intervals. First, if the interval is reduced to a single quote update, (A8) reduces to the tick time solution (15) with $c_\iota = 0$ and $c_\beta = 1/(V_{\beta\beta} + 1/\sigma_\ell^2)$,

$$\lim_{\tilde{\sigma}(t)^2 \to 0} \gamma(t) = \gamma_\ell = \frac{\sigma_\ell^2}{1 + \sigma_\ell^2 V_{\beta\beta}} \beta_\ell \tag{A10}$$

Second, for a very large interval, we take the limit as $\tilde{\sigma}(t)^2 \to \infty$,

$$\bar{\gamma} = \lim_{\tilde{\sigma}(t)^2 \to \infty} \gamma(t) = \frac{\left(V_{\beta\beta} - V_{\iota\beta} + 1/\sigma_\ell^2\right)\Omega^{-1}\iota + \left(V_{\iota\iota} - V_{\iota\beta}\right)\Omega^{-1}\beta_\ell}{V_{\iota\iota}\left(V_{\beta\beta} + 1/\sigma_\ell^2\right) - V_{\iota\beta}^2} \tag{A11}$$

Premultiplying $\bar{\gamma}$ by ι' and using the definitions of $V_{\iota\beta}$ and $V_{\beta\beta}$ establishes that the elements of $\bar{\gamma}$ sum to one. To gain more insight in $\bar{\gamma}$ we go back to the original parameterization with α_ℓ. Recalling (9),

$$\beta_\ell = \iota + \frac{\alpha_\ell}{\sigma_\ell}, \tag{A12}$$

we have

$$\bar{\gamma} = \frac{\left(V_{\beta\beta} - 2V_{\iota\beta} + V_{\iota\iota} + 1/\sigma_\ell^2\right)\Omega^{-1}\iota + \left(V_{\iota\iota} - V_{\iota\beta}\right)\Omega^{-1}\alpha_\ell/\sigma_\ell}{V_{\iota\iota}\left(V_{\beta\beta} + 1/\sigma_\ell^2\right) - V_{\iota\beta}^2} \tag{A13}$$

Also expressing $V_{\iota\beta}$ and $V_{\beta\beta}$ in terms of α_ℓ gives

$$\begin{aligned}
V_{\iota\iota} - V_{\iota\beta} &= \iota'\Omega^{-1}\iota - \iota'\Omega^{-1}\beta_\ell \\
&= -\iota'\Omega^{-1}\alpha_\ell/\sigma_\ell = -V_{\iota\alpha}/\sigma_\ell \\
V_{\beta\beta} - 2V_{\iota\beta} + V_{\iota\iota} &= \beta_\ell'\Omega^{-1}\beta_\ell - 2\iota'\Omega^{-1}\beta_\ell + \iota'\Omega^{-1}\iota \\
&= \alpha_\ell'\Omega^{-1}\alpha_\ell/\sigma_\ell = V_{\alpha\alpha}/\sigma_\ell^2 \\
V_{\iota\iota}V_{\beta\beta} - V_{\iota\beta}^2 &= V_{\iota\iota}\left(\iota + \alpha_\ell/\sigma_\ell\right)'\Omega^{-1}\left(\iota + \alpha_\ell/\sigma_\ell\right) - \left(V_{\iota\iota} + V_{\iota\alpha}/\sigma_\ell\right)^2 \\
&= \frac{1}{\sigma_\ell^2}\left(V_{\iota\iota}V_{\alpha\alpha} - V_{\iota\alpha}^2\right)
\end{aligned} \tag{A14}$$

Substituting all coefficients of (A14) in (A13) we finally obtain

$$\bar{\gamma} = \frac{(1 + V_{\alpha\alpha})\Omega^{-1}\iota - V_{\iota\alpha}\Omega^{-1}\alpha_\ell}{V_{\iota\iota} + V_{\iota\iota}V_{\alpha\alpha} - V_{\iota\alpha}^2} \tag{A15}$$

The limiting value $\bar{\gamma}$ does not depend on σ or δ_1, meaning that the price discovery parameters are independent of the process of the efficient price. Since α_ℓ depends on τ_ℓ and δ_2, the limiting information shares still depend on the most recent duration.

For the information shares we need the (2×1) sub-vectors $\beta_i(t)$ and $\gamma_i(t)$ corresponding to the bid and ask quotes of dealer i. Combining (A2), (A4), (A8) and (A9) we get

$$\begin{aligned} \mathrm{IS}_i(t) &= \gamma_i(t)'\beta_i(t) \\ &= \left(c_\iota \Omega^{-1}\iota_2 + c_\beta \Omega^{-1}\beta_{i,\ell}\right)' \left(\frac{\tilde{\sigma}(t)^2}{\sigma(t)^2}\iota_2 + \frac{\sigma_\ell^2}{\sigma(t)^2}\beta_{i,\ell}\right) \\ &= d_{\iota\iota}\left(\iota_2'\Omega_i^{-1}\iota_2\right) + d_{\iota\beta}\left(\iota_2'\Omega_i^{-1}\beta_{i,\ell}\right) + d_{\beta\beta}\left(\beta_{i,\ell}'\Omega_i^{-1}\beta_{i,\ell}\right), \end{aligned} \tag{A16}$$

with

$$\begin{aligned} d_{\iota\iota} &= c_\iota \frac{\tilde{\sigma}(t)^2}{\sigma(t)^2} \\ d_{\iota\beta} &= c_\iota \frac{\sigma_\ell^2}{\sigma(t)^2} + c_\beta \frac{\tilde{\sigma}(t)^2}{\sigma(t)^2} \\ d_{\beta\beta} &= c_\beta \frac{\sigma_\ell^2}{\sigma(t)^2} \end{aligned} \tag{A17}$$

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Table 1: Summary Statistics

This table lists the twenty Nasdaq stocks included in the sample. The number of quote updates is the total number of times a dealer either changes a bid or an ask. The number of single quotes is the frequency of observations for which only one dealer issues a new quote. The last two columns provide the average and standard deviation of the durations between quote updates.

Symbol	Company name	# quote updates	% single quotes	Average duration	Std. dev. duration
AAPL	Apple Computer Inc.	29,787	89.63	15.71	28.73
AMAT	Applied Materials Inc.	105,090	83.58	4.77	5.95
AMGN	Amgen Inc.	40,279	83.97	12.32	22.24
AMZN	Amazon.com, Inc.	150,710	80.52	3.44	4.39
ATHM	At Home Corporation	76,435	86.50	6.34	9.40
CMGI	CMGI, Inc.	90,401	87.06	5.34	7.75
COMS	3Com Corporation	61,049	89.56	7.68	11.34
CPWR	Compuware Corporation	33,301	90.55	13.92	31.03
CSCO	Cisco Systems Inc.	164,480	80.85	3.13	3.52
DELL	Dell Computer Corporation	177,850	77.39	3.02	3.44
INTC	Intel Corporation	171,260	76.92	3.15	3.56
MSFT	Microsoft Corporation	151,110	80.82	3.42	3.94
NOVL	Novell Inc.	18,909	87.88	25.08	43.93
NXTL	Nextel Communications CL-A	19,556	91.63	23.42	42.23
ORCL	Oracle Corporation	87,774	85.81	5.56	7.46
PSFT	Peoplesoft Inc.	24,601	91.12	18.74	32.02
QWST	Qwest Communications Intl Inc.	44,459	88.33	10.68	18.12
SBUX	Starbucks Corporation	14,320	90.06	32.43	64.92
SUNW	Sun Microsystems Inc.	128,370	82.43	4.20	5.15
WCOM	MCI WorldCom Inc.	88,550	83.33	5.66	7.41

Table 2: Duration Parameters

This table presents the estimates for the duration parameters δ_1 and δ_2 in (7). Standard errors are in parentheses. Stocks are referred to by the ticker symbols explained in table 1. δ_1 measures the impact of time on the innovation in the efficient price process and has values of interest of $\delta_1 = \frac{1}{2}$, which represents a process that evolves in calendar time and $\delta_1 = 0$, which represents a process that evolves in tick time.

Stock	δ_1	δ_2	Symbol	δ_1	δ_2
AAPL	-0.06 (0.03)	-0.44 (0.05)	INTC	-0.17 (0.02)	-0.23 (0.04)
AMAT	0.02 (0.02)	-1.20 (0.17)	MSFT	-0.20 (0.02)	-0.13 (0.03)
AMGN	0.04 (0.02)	-0.21 (0.17)	NOVL	-0.23 (0.04)	-0.59 (0.05)
AMZN	0.03 (0.01)	0.50 (0.02)	NXTL	-0.05 (0.03)	-0.44 (0.08)
ATHM	0.01 (0.02)	-1.26 (0.18)	ORCL	0.02 (0.02)	-0.82 (0.05)
CMGI	0.07 (0.01)	-0.11 (0.04)	PSFT	0.00 (0.02)	-0.23 (0.03)
COMS	-0.11 (0.02)	-0.51 (0.06)	QWST	-0.14 (0.02)	0.23 (0.03)
CPWR	0.00 (0.02)	-0.72 (0.13)	SBUX	-0.01 (0.03)	-0.32 (0.06)
CSCO	-0.07 (0.01)	-0.24 (0.04)	SUNW	-0.06 (0.02)	-0.96 (0.11)
DELL	-0.13 (0.01)	-0.30 (0.03)	WCOM	-0.11 (0.02)	-1.17 (0.14)

Table 3: Quote Specific Relations to the Efficient Price

This table presents the estimates for the vector α in (7). Standard errors are in parentheses. Stocks are referred to by the ticker symbols explained in table 1. Dealers include the ECN's Island (ISLD) and Instinet (INCA) plus the three most active individual wholesale dealers. Parameters differ for bid and ask quotes.

	ISLD		INCA		MM1		MM2		MM3	
Stock	bid	ask	bid	ask	bid	ask	bid	ask	bid	ask
AAPL	-1.02	-0.56	1.24	0.58	3.17	2.01	0.98	-1.49	-0.82	-0.21
	(0.20)	(0.21)	(0.37)	(0.26)	(0.40)	(0.35)	(0.19)	(0.25)	(0.36)	(0.28)
AMAT	-0.03	-0.06	0.07	0.38	0.49	0.33	-0.08	-0.12	0.02	-0.63
	(0.02)	(0.02)	(0.05)	(0.10)	(0.13)	(0.11)	(0.05)	(0.06)	(0.14)	(0.19)
AMGN	-1.59	-0.69	0.13	0.14	-1.44	-0.97	-1.70	-1.70	0.17	-0.13
	(0.43)	(0.42)	(0.20)	(0.28)	(0.69)	(0.60)	(0.93)	(1.09)	(0.41)	(0.43)
AMZN	0.00	0.12	0.93	1.38	-0.33	-0.66	-0.97	-5.21	-0.12	-0.05
	(0.05)	(0.05)	(0.08)	(0.08)	(0.12)	(0.13)	(0.16)	(0.13)	(0.20)	(0.18)
ATHM	-0.08	-0.11	0.08	0.04	0.38	0.28	-0.15	-0.16	-0.13	-0.17
	(0.03)	(0.04)	(0.04)	(0.03)	(0.13)	(0.10)	(0.06)	(0.07)	(0.08)	(0.09)
CMGI	-0.35	-0.61	0.35	0.44	-0.39	-0.11	1.68	1.39	-1.03	-1.46
	(0.08)	(0.09)	(0.15)	(0.18)	(0.17)	(0.19)	(0.15)	(0.15)	(0.21)	(0.23)
COMS	-0.07	-0.02	1.27	0.79	-1.21	-1.02	0.00	-0.24	0.00	1.49
	(0.08)	(0.07)	(0.17)	(0.18)	(0.28)	(0.30)	(0.14)	(0.22)	(0.17)	(0.28)
CPWR	-0.22	-0.31	-0.12	0.04	0.59	0.37	0.79	0.89	0.12	-0.26
	(0.09)	(0.11)	(0.08)	(0.08)	(0.21)	(0.19)	(0.32)	(0.36)	(0.18)	(0.18)
CSCO	-0.41	-0.42	0.44	0.48	0.55	0.03	0.76	0.80	-3.00	-2.92
	(0.07)	(0.08)	(0.10)	(0.09)	(0.18)	(0.25)	(0.20)	(0.25)	(0.22)	(0.23)
DELL	-0.47	-0.47	0.12	0.26	4.54	5.34	0.01	0.60	-2.39	-2.84
	(0.04)	(0.04)	(0.08)	(0.07)	(0.34)	(0.37)	(0.12)	(0.16)	(0.11)	(0.18)
INTC	0.00	-0.35	-0.28	-0.03	0.67	1.04	-1.39	-1.01	-1.39	-1.55
	(0.09)	(0.08)	(0.09)	(0.08)	(0.15)	(0.16)	(0.10)	(0.11)	(0.20)	(0.19)
MSFT	-0.07	-0.07	-0.10	0.05	-0.81	-0.18	-1.31	-1.22	-2.80	-2.73
	(0.09)	(0.09)	(0.10)	(0.09)	(0.19)	(0.23)	(0.12)	(0.18)	(0.17)	(0.18)
NOVL	1.16	-0.07	1.69	0.16	1.73	-2.58	0.30	1.00	0.15	-0.07
	(0.32)	(0.12)	(0.28)	(0.11)	(0.32)	(0.44)	(0.15)	(0.20)	(0.16)	(0.16)
NXTL	-1.44	-1.25	-0.49	-0.01	1.89	1.23	1.51	-1.32	0.16	-1.50
	(0.39)	(0.42)	(0.23)	(0.34)	(0.45)	(0.42)	(0.48)	(0.62)	(0.20)	(0.38)
ORCL	-0.34	-0.46	0.20	-0.04	2.62	-2.15	-0.27	-0.05	-0.42	-0.26
	(0.06)	(0.06)	(0.07)	(0.09)	(0.22)	(0.18)	(0.07)	(0.08)	(0.12)	(0.15)
PSFT	0.93	-0.43	1.90	0.18	11.98	9.51	8.09	9.20	-0.24	2.33
	(0.36)	(0.21)	(0.30)	(0.26)	(1.44)	(1.28)	(1.03)	(1.14)	(0.54)	(0.56)
QWST	0.99	0.22	4.57	5.46	-0.45	0.23	2.79	3.44	0.70	0.37
	(0.69)	(0.43)	(0.47)	(0.42)	(0.42)	(0.63)	(0.71)	(1.11)	(0.19)	(0.21)
SBUX	0.14	-0.08	1.21	0.22	1.87	1.28	-0.52	-1.10	-0.27	-0.16
	(0.19)	(0.32)	(0.34)	(0.23)	(0.41)	(0.31)	(0.34)	(0.43)	(0.24)	(0.19)
SUNW	-0.08	-0.31	-0.04	0.00	-0.55	-0.50	-1.07	-1.61	0.10	0.10
	(0.06)	(0.08)	(0.07)	(0.06)	(0.11)	(0.14)	(0.19)	(0.27)	(0.11)	(0.13)
WCOM	-0.02	-0.12	0.24	-0.13	0.65	0.49	-0.26	-0.36	0.17	-0.05
	(0.04)	(0.05)	(0.06)	(0.05)	(0.17)	(0.15)	(0.08)	(0.09)	(0.08)	(0.06)

Table 4: Average Variance of the Idiosyncratic Dealer Noise Components

This table presents average variances (average over bid and ask quote) of the idiosyncratic dealer noise components as estimated in (7). Stocks are referred to as detailed in table 1. Dealers include the ECN's Island (ISLD) and Instinet (INCA) plus the three most active individual wholesale dealers. Parameters differ among dealer, but not for bid and ask quotes (not reported).

Stock	ISLD	INCA	MM1	MM2	MM3
AAPL	0.47	0.75	0.47	0.33	0.51
AMAT	0.09	0.34	0.29	0.30	0.58
AMGN	0.14	0.41	0.22	0.29	0.22
AMZN	0.03	0.32	0.34	0.10	0.52
ATHM	0.17	0.45	0.38	0.51	0.66
CMGI	0.10	0.50	0.43	0.39	0.62
COMS	0.14	0.42	0.50	0.49	0.38
CPWR	0.67	0.61	0.43	0.61	0.78
CSCO	0.01	0.15	0.25	0.39	0.18
DELL	0.01	0.15	0.47	0.43	0.18
INTC	0.03	0.12	0.19	0.10	0.22
MSFT	0.02	0.09	0.17	0.11	0.10
NOVL	1.03	0.95	0.83	0.63	0.56
NXTL	1.80	0.57	0.69	0.65	0.39
ORCL	0.12	0.37	0.43	0.26	0.28
PSFT	0.41	0.62	1.24	1.38	0.77
QWST	0.61	0.56	0.45	0.60	0.18
SBUX	1.09	0.54	0.25	0.66	0.33
SUNW	0.08	0.21	0.30	0.28	0.33
WCOM	0.10	0.16	0.27	0.15	0.29

Table 5: Dealer Liquidity in Tick Time Measured at Average Duration

This table presents measures for price discovery in tick time for average durations. Stocks are referred to as detailed in table 1. Included dealers are the ECN's Island (ISLD)and Instinet (INCA) and the three most active individual wholesale dealers. Panel A presents the results for the measure of dealer liquidity ($\beta(\tau)$), Panel B for price discovery ($\gamma(\tau)$) and Panel C for information shares ($IS(\tau)$). The results in Panel B and C are multiplied by 100.

	ISLD		INCA		MM1		MM2		MM3	
Stock	bid	ask	bid	ask	bid	ask	bid	ask	bid	ask
Panel A: Dealer Liquidity $\beta(\tau)$ at average duration										
AAPL	-0.02	0.44	2.24	1.59	4.17	3.01	1.98	-0.49	0.18	0.79
AMAT	0.97	0.94	1.07	1.38	1.49	1.33	0.92	0.88	1.02	0.37
AMGN	-0.59	0.31	1.13	1.14	-0.44	0.03	-0.70	-0.70	1.17	0.87
AMZN	1.00	1.12	1.93	2.38	0.67	0.34	0.03	-4.21	0.88	0.95
ATHM	0.92	0.89	1.08	1.04	1.38	1.28	0.85	0.84	0.87	0.83
CMGI	0.65	0.39	1.35	1.44	0.61	0.89	2.68	2.39	-0.03	-0.46
COMS	0.93	0.98	2.27	1.79	-0.21	-0.02	1.00	0.76	1.00	2.49
CPWR	0.78	0.69	0.88	1.04	1.59	1.37	1.79	1.89	1.12	0.74
CSCO	0.59	0.58	1.44	1.48	1.55	1.04	1.76	1.80	-2.00	-1.92
DELL	0.53	0.53	1.12	1.26	5.54	6.34	1.01	1.60	-1.39	-1.84
INTC	1.00	0.65	0.72	0.97	1.67	2.04	-0.39	-0.01	-0.39	-0.55
MSFT	0.93	1.07	0.90	1.05	0.19	0.82	-0.31	-0.22	-1.81	-1.73
NOVL	2.16	0.93	2.69	1.16	2.73	-1.58	1.30	2.00	1.15	0.93
NXTL	-0.44	-0.25	0.51	0.99	2.89	2.23	2.51	-0.32	1.16	-0.50
ORCL	0.66	0.54	1.20	0.96	3.62	-1.15	0.73	0.95	0.58	0.74
PSFT	1.93	0.57	2.90	1.18	12.98	10.51	9.10	10.20	0.76	3.33
QWST	1.99	1.22	5.57	6.46	0.55	1.23	3.79	4.45	1.70	1.37
SBUX	1.14	0.92	2.21	1.22	2.87	2.28	0.48	-0.10	0.73	0.84
SUNW	0.92	0.69	0.96	1.00	0.45	0.50	-0.07	-0.61	1.10	1.10
WCOM	0.98	0.88	1.24	0.87	1.66	1.49	0.74	0.64	1.17	0.95
Panel B: Price Discovery $\gamma(\tau)$ at average duration ($\times$100)										
AAPL	-0.03	0.24	0.81	0.59	2.22	0.64	1.96	-0.43	-0.07	0.50
AMAT	2.31	3.59	0.70	0.86	1.27	1.11	0.50	0.51	0.34	0.14
AMGN	-1.16	0.56	0.65	0.54	-0.51	-0.09	-0.60	-0.46	1.21	0.59
AMZN	0.00	0.00	0.00	0.00	0.00	0.00	-17.85	-23.86	0.00	0.00
ATHM	2.45	2.28	0.76	0.80	1.57	1.35	0.68	0.66	0.41	0.44
CMGI	3.87	2.32	1.25	1.27	0.68	1.02	4.24	3.84	0.11	-0.40
COMS	1.89	1.75	1.57	0.90	-0.11	-0.01	0.60	0.37	0.66	1.46
CPWR	0.44	0.45	0.49	0.73	2.43	1.92	1.33	1.45	0.59	0.25
CSCO	3.65	3.79	0.62	0.65	0.46	0.28	0.32	0.31	-1.14	-0.74
DELL	5.23	6.42	0.39	0.55	0.68	0.90	0.13	0.20	-0.73	-0.57
INTC	1.58	1.55	0.32	0.45	0.53	0.65	-0.28	-0.01	-0.06	-0.14
MSFT	1.96	2.02	0.33	0.40	0.04	0.17	-0.13	-0.07	-0.57	-0.52
NOVL	0.73	0.31	0.97	0.47	1.79	-1.24	1.32	1.63	0.64	0.75
NXTL	-0.12	-0.03	0.45	0.64	1.98	1.49	1.75	-0.62	1.59	-0.68
ORCL	1.25	1.04	0.67	0.50	2.01	-0.83	0.71	0.81	0.27	0.62
PSFT	0.00	0.00	0.00	0.00	4.05	4.50	0.00	0.00	0.00	0.00
QWST	0.46	0.43	1.45	2.12	0.23	0.45	1.06	1.07	1.48	1.16
SBUX	0.06	0.03	0.20	0.07	18.23	16.92	0.03	-0.01	0.08	0.10
SUNW	1.35	1.29	0.57	0.59	0.22	0.17	-0.07	-0.28	0.41	0.33
WCOM	0.82	0.97	0.63	0.47	0.50	0.46	0.41	0.32	0.27	0.20

Table 5 (Continued)

Panel C: Information Shares $IS(\tau)$ at average duration ($\times 100$)

Stock	ISLD	INCA	MM1	MM2	MM3	R^2
AAPL	0.10	2.75	11.21	4.11	0.38	18.55
AMAT	5.61	1.93	3.38	0.90	0.39	12.21
AMGN	0.86	1.36	0.22	0.74	1.94	5.12
AMZN	0.01	0.00	0.00	99.98	0.00	99.99
ATHM	4.27	1.65	3.89	1.13	0.73	11.67
CMGI	3.42	3.51	1.32	20.55	0.18	28.98
COMS	3.47	5.17	0.02	0.89	4.29	13.84
CPWR	0.65	1.20	6.50	5.11	0.84	14.30
CSCO	4.34	1.86	1.00	1.12	3.70	12.02
DELL	6.22	1.13	9.50	0.44	2.07	19.36
INTC	2.58	0.66	2.21	0.11	0.10	5.66
MSFT	3.99	0.71	0.15	0.06	1.92	6.38
NOVL	1.87	3.16	6.85	4.97	1.44	18.29
NXTL	0.06	0.86	9.06	4.61	2.19	16.78
ORCL	1.39	1.29	8.24	1.29	0.62	12.83
PSFT	0.00	0.00	99.92	0.03	0.00	99.95
QWST	1.43	21.72	0.68	8.80	4.11	36.74
SBUX	0.10	0.53	90.85	0.02	0.14	91.64
SUNW	2.13	1.15	0.18	0.18	0.82	4.46
WCOM	1.66	1.20	1.51	0.51	0.51	5.39

Table 6: Average Values for Dealer Liquidity ($\beta(t)$)

This table presents averages for dealer liquidity ($\beta(t)$) per dealer quote. This measure is defined in (27). Stocks are referred to by the ticker symbols explained in table 1. Panel A shows this measure aggregated to 60 seconds, panel B to 300 seconds. When the aggregation interval increases, all measures converge to one.

	ISLD		INCA		MM1		MM2		MM3	
Stock	bid	ask	bid	ask	bid	ask	bid	ask	bid	ask
Panel A: 60 Second Intervals										
AAPL	0.89	0.94	1.13	1.06	1.34	1.22	1.11	0.84	0.91	0.98
AMAT	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.99
AMGN	0.81	0.92	1.02	1.02	0.83	0.89	0.80	0.80	1.02	0.98
AMZN	1.00	1.02	1.14	1.21	0.95	0.90	0.85	0.22	0.98	0.99
ATHM	1.00	1.00	1.00	1.00	1.01	1.01	1.00	1.00	1.00	1.00
CMGI	0.98	0.96	1.02	1.03	0.97	0.99	1.12	1.10	0.93	0.90
COMS	1.00	1.00	1.08	1.05	0.92	0.94	1.00	0.99	1.00	1.10
CPWR	0.99	0.99	0.99	1.00	1.03	1.02	1.04	1.04	1.01	0.99
CSCO	0.98	0.98	1.02	1.02	1.02	1.00	1.03	1.03	0.87	0.87
DELL	0.98	0.98	1.01	1.01	1.19	1.23	1.00	1.03	0.90	0.88
INTC	1.00	0.98	0.99	1.00	1.03	1.05	0.93	0.95	0.93	0.93
MSFT	1.00	1.00	0.99	1.00	0.95	0.99	0.92	0.93	0.83	0.84
NOVL	1.17	0.99	1.25	1.02	1.26	0.62	1.04	1.15	1.02	0.99
NXTL	0.82	0.84	0.94	1.00	1.24	1.16	1.19	0.83	1.02	0.81
ORCL	0.99	0.99	1.01	1.00	1.07	0.94	0.99	1.00	0.99	0.99
PSFT	1.16	0.93	1.33	1.03	3.05	2.63	2.39	2.58	0.96	1.40
QWST	1.52	1.11	3.39	3.85	0.76	1.12	2.46	2.80	1.37	1.19
SBUX	1.02	0.99	1.21	1.04	1.33	1.23	0.91	0.80	0.95	0.97
SUNW	1.00	0.99	1.00	1.00	0.99	0.99	0.97	0.96	1.00	1.00
WCOM	1.00	1.00	1.01	1.00	1.02	1.01	0.99	0.99	1.00	1.00
Panel B: 300 Second Intervals										
AAPL	0.97	0.99	1.03	1.02	1.08	1.05	1.03	0.96	0.98	0.99
AMAT	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
AMGN	0.96	0.98	1.00	1.00	0.96	0.97	0.95	0.95	1.00	1.00
AMZN	1.00	1.00	1.03	1.05	0.99	0.98	0.97	0.81	1.00	1.00
ATHM	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
CMGI	0.99	0.99	1.01	1.01	0.99	1.00	1.03	1.02	0.98	0.98
COMS	1.00	1.00	1.02	1.01	0.98	0.99	1.00	1.00	1.00	1.02
CPWR	1.00	1.00	1.00	1.00	1.01	1.00	1.01	1.01	1.00	1.00
CSCO	1.00	1.00	1.00	1.00	1.00	1.00	1.01	1.01	0.97	0.97
DELL	1.00	1.00	1.00	1.00	1.04	1.05	1.00	1.01	0.98	0.98
INTC	1.00	1.00	1.00	1.00	1.01	1.01	0.99	0.99	0.99	0.99
MSFT	1.00	1.00	1.00	1.00	0.99	1.00	0.98	0.99	0.97	0.97
NOVL	1.04	1.00	1.06	1.01	1.07	0.90	1.01	1.04	1.01	1.00
NXTL	0.96	0.96	0.98	1.00	1.06	1.04	1.05	0.96	1.01	0.95
ORCL	1.00	1.00	1.00	1.00	1.02	0.99	1.00	1.00	1.00	1.00
PSFT	1.04	0.98	1.08	1.01	1.47	1.38	1.32	1.36	0.99	1.09
QWST	1.12	1.03	1.54	1.65	0.95	1.03	1.33	1.41	1.08	1.04
SBUX	1.01	1.00	1.06	1.01	1.10	1.07	0.97	0.94	0.99	0.99
SUNW	1.00	1.00	1.00	1.00	1.00	1.00	0.99	0.99	1.00	1.00
WCOM	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

Table 7: Average values for price discovery ($\gamma(t)$)

This table presents averages for price discovery ($\gamma(t)$) per dealer quote. This measure is defined in (30). Stocks are referred to by the ticker symbols explained in table 1. Panel A shows this measure aggregated to 60 seconds, panel B to 300 seconds. When the aggregation interval increases, all measures add up to one.

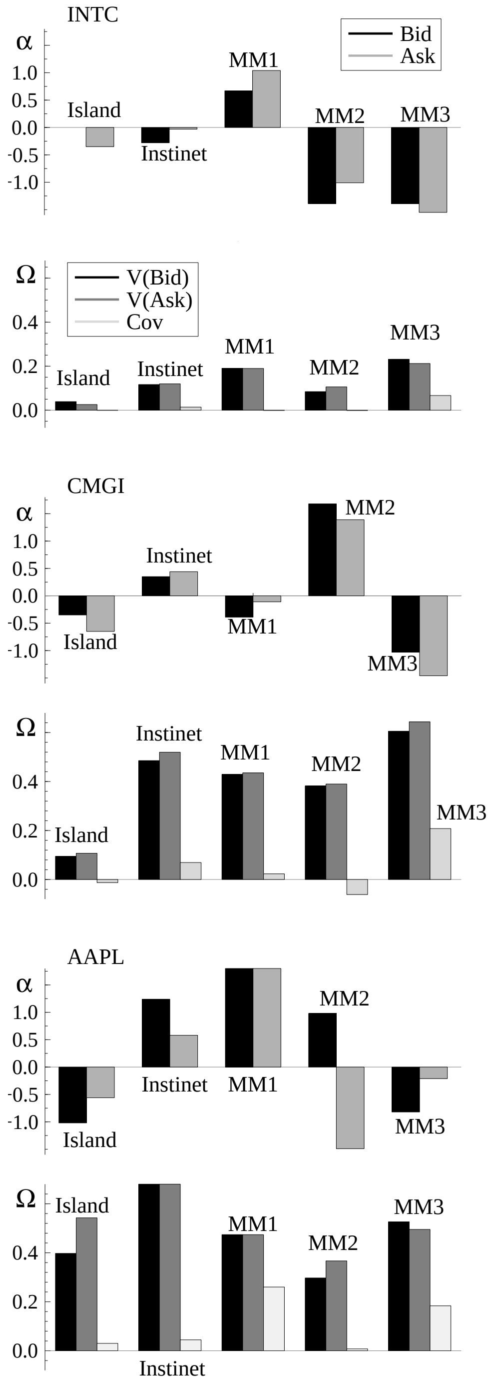
	ISLD		INCA		MM1		MM2		MM3	
Stock	bid	ask	bid	ask	bid	ask	bid	ask	bid	ask
Panel A: 60 Second Intervals										
AAPL	0.03	0.02	0.02	0.02	0.02	0.02	0.04	0.03	0.01	0.02
AMAT	0.14	0.22	0.04	0.04	0.05	0.05	0.03	0.03	0.02	0.02
AMGN	0.06	0.06	0.03	0.02	0.06	0.06	0.03	0.02	0.05	0.04
AMZN	0.30	0.38	0.02	0.03	0.01	0.01	0.07	0.04	0.01	0.01
ATHM	0.13	0.13	0.04	0.04	0.06	0.05	0.04	0.04	0.02	0.03
CMGI	0.24	0.21	0.04	0.04	0.04	0.04	0.07	0.07	0.02	0.02
COMS	0.10	0.08	0.03	0.03	0.02	0.02	0.03	0.02	0.03	0.03
CPWR	0.03	0.03	0.03	0.03	0.07	0.06	0.04	0.04	0.02	0.02
CSCO	0.30	0.32	0.02	0.02	0.02	0.01	0.01	0.01	0.02	0.01
DELL	0.34	0.41	0.01	0.02	0.01	0.01	0.01	0.00	0.01	0.01
INTC	0.12	0.18	0.03	0.03	0.02	0.03	0.05	0.04	0.01	0.02
MSFT	0.16	0.14	0.03	0.03	0.02	0.02	0.03	0.02	0.02	0.02
NOVL	0.01	0.01	0.01	0.01	0.01	0.00	0.02	0.02	0.01	0.01
NXTL	0.01	0.00	0.03	0.02	0.03	0.02	0.02	0.01	0.04	0.02
ORCL	0.12	0.12	0.03	0.03	0.03	0.03	0.06	0.05	0.03	0.05
PSFT	0.02	0.03	0.02	0.02	0.08	0.09	0.00	0.00	0.02	0.01
QWST	0.01	0.02	0.02	0.02	0.02	0.02	0.02	0.01	0.05	0.05
SBUX	0.00	0.00	0.00	0.00	0.32	0.30	0.00	0.00	0.01	0.01
SUNW	0.11	0.13	0.04	0.04	0.03	0.02	0.04	0.04	0.03	0.02
WCOM	0.05	0.07	0.03	0.04	0.02	0.02	0.04	0.03	0.01	0.02
Panel B: 300 Second Intervals										
AAPL	0.06	0.05	0.03	0.04	0.04	0.04	0.09	0.07	0.04	0.04
AMAT	0.19	0.30	0.05	0.05	0.07	0.07	0.04	0.05	0.02	0.03
AMGN	0.12	0.10	0.04	0.04	0.10	0.10	0.06	0.04	0.07	0.06
AMZN	0.32	0.41	0.03	0.03	0.02	0.02	0.08	0.05	0.01	0.01
ATHM	0.20	0.19	0.05	0.06	0.08	0.08	0.06	0.06	0.03	0.04
CMGI	0.29	0.25	0.04	0.04	0.05	0.05	0.08	0.08	0.03	0.03
COMS	0.17	0.15	0.06	0.04	0.04	0.04	0.05	0.04	0.06	0.05
CPWR	0.05	0.05	0.05	0.06	0.13	0.11	0.06	0.06	0.04	0.04
CSCO	0.37	0.40	0.03	0.03	0.02	0.02	0.01	0.01	0.03	0.02
DELL	0.39	0.48	0.01	0.02	0.01	0.01	0.01	0.00	0.02	0.01
INTC	0.19	0.28	0.05	0.05	0.04	0.04	0.08	0.07	0.02	0.02
MSFT	0.26	0.24	0.05	0.05	0.03	0.03	0.05	0.04	0.04	0.03
NOVL	0.02	0.02	0.02	0.02	0.02	0.01	0.04	0.04	0.03	0.04
NXTL	0.02	0.01	0.06	0.05	0.05	0.05	0.04	0.04	0.08	0.06
ORCL	0.18	0.18	0.05	0.05	0.05	0.04	0.09	0.08	0.05	0.07
PSFT	0.06	0.08	0.04	0.04	0.12	0.14	0.01	0.00	0.04	0.03
QWST	0.03	0.04	0.02	0.02	0.06	0.05	0.03	0.02	0.10	0.11
SBUX	0.00	0.00	0.01	0.01	0.41	0.38	0.01	0.01	0.01	0.01
SUNW	0.17	0.21	0.07	0.07	0.06	0.04	0.06	0.06	0.04	0.04
WCOM	0.11	0.14	0.06	0.07	0.04	0.04	0.07	0.07	0.03	0.03

Table 8: Averages for Information shares per dealer

This table presents Average results for information shares. This measure is obtained as the inner product of $\beta(t)$ and $\gamma(t)$.The first five columns present the information shares per dealer $IS_i(t)$, the last column reports the sum of the information shares (R^2). Stocks are referred to by the ticker symbols explained in table 1. Panel A presents the measure aggregated to 60 seconds and panel B to 300 seconds. When the aggregation interval increases, The sum of all information shares goes to one.

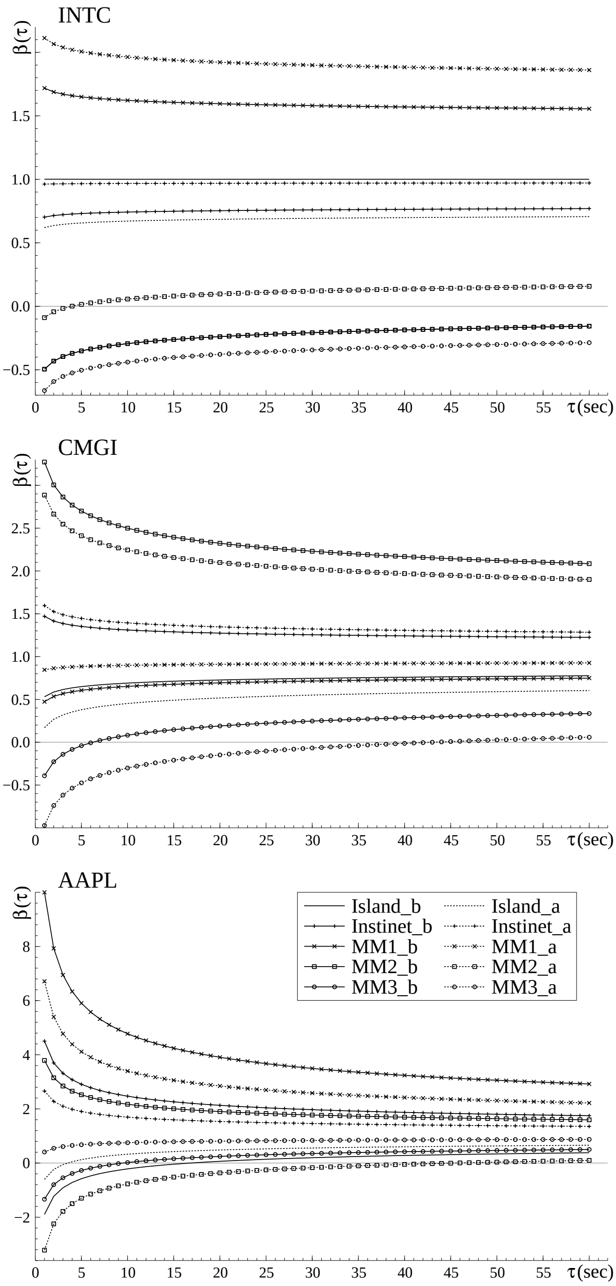
Stock	Island	Instinet	MM1	MM2	MM3	R^2
Panel A: 60 Second Intervals						
AAPL	0.04	0.03	0.03	0.07	0.03	0.20
AMAT	0.35	0.07	0.10	0.06	0.04	0.62
AMGN	0.13	0.04	0.11	0.06	0.07	0.41
AMZN	0.00	0.00	0.00	1.00	0.00	1.00
ATHM	0.26	0.07	0.11	0.08	0.05	0.57
CMGI	0.46	0.07	0.09	0.13	0.05	0.80
COMS	0.17	0.05	0.05	0.05	0.06	0.38
CPWR	0.05	0.06	0.13	0.07	0.04	0.35
CSCO	0.61	0.04	0.03	0.02	0.05	0.75
DELL	0.74	0.03	0.01	0.01	0.03	0.82
INTC	0.29	0.07	0.05	0.09	0.03	0.53
MSFT	0.28	0.05	0.03	0.05	0.04	0.45
NOVL	0.01	0.02	0.01	0.04	0.03	0.12
NXTL	0.01	0.04	0.04	0.03	0.06	0.18
ORCL	0.23	0.07	0.05	0.11	0.08	0.54
PSFT	0.00	0.00	0.98	0.00	0.00	0.98
QWST	0.03	0.03	0.04	0.03	0.09	0.22
SBUX	0.00	0.01	0.74	0.01	0.01	0.77
SUNW	0.23	0.08	0.06	0.07	0.05	0.49
WCOM	0.12	0.07	0.04	0.07	0.03	0.33
Panel B: 300 Second intervals						
AAPL	0.11	0.07	0.07	0.16	0.08	0.49
AMAT	0.50	0.10	0.14	0.09	0.06	0.89
AMGN	0.22	0.08	0.20	0.11	0.12	0.73
AMZN	0.00	0.00	0.00	1.00	0.00	1.00
ATHM	0.39	0.11	0.16	0.12	0.07	0.85
CMGI	0.55	0.09	0.11	0.15	0.06	0.96
COMS	0.32	0.10	0.09	0.09	0.11	0.71
CPWR	0.10	0.10	0.24	0.12	0.07	0.63
CSCO	0.77	0.05	0.03	0.02	0.06	0.93
DELL	0.87	0.03	0.01	0.01	0.03	0.95
INTC	0.46	0.11	0.07	0.15	0.05	0.84
MSFT	0.49	0.09	0.05	0.09	0.08	0.80
NOVL	0.03	0.04	0.03	0.09	0.06	0.25
NXTL	0.03	0.11	0.09	0.08	0.14	0.45
ORCL	0.36	0.10	0.08	0.17	0.12	0.83
PSFT	0.00	0.00	1.00	0.00	0.00	1.00
QWST	0.07	0.07	0.09	0.06	0.21	0.50
SBUX	0.01	0.01	0.88	0.01	0.01	0.92
SUNW	0.39	0.14	0.09	0.12	0.08	0.82
WCOM	0.25	0.14	0.08	0.14	0.06	0.67

Figure 2: Parameter Estimates for Three Selected Stock



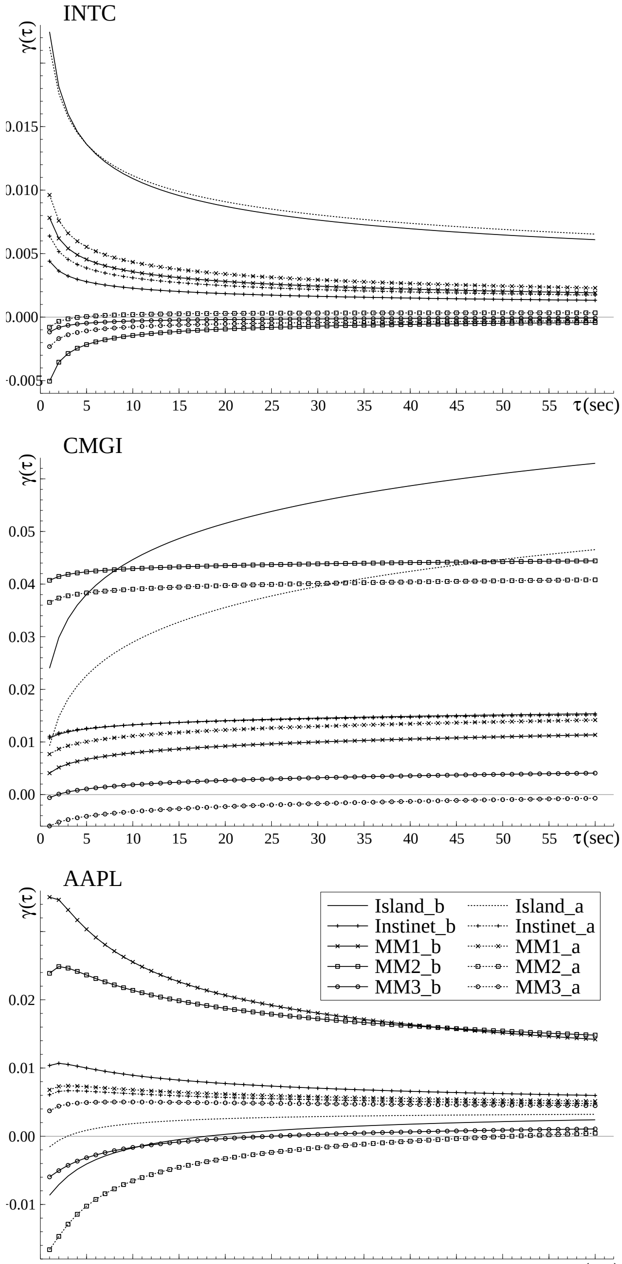
Note: These graphs show parameter estimates for asymmetric information (α) and idiosyncratic dealer noise (Ω) for three stocks, Intel, CMGI and Apple. The top part of each graphs shows α, the bottom part Ω.

Figure 3: Tick time measure for dealer liquidity ($\beta(\tau)$) as a function of duration



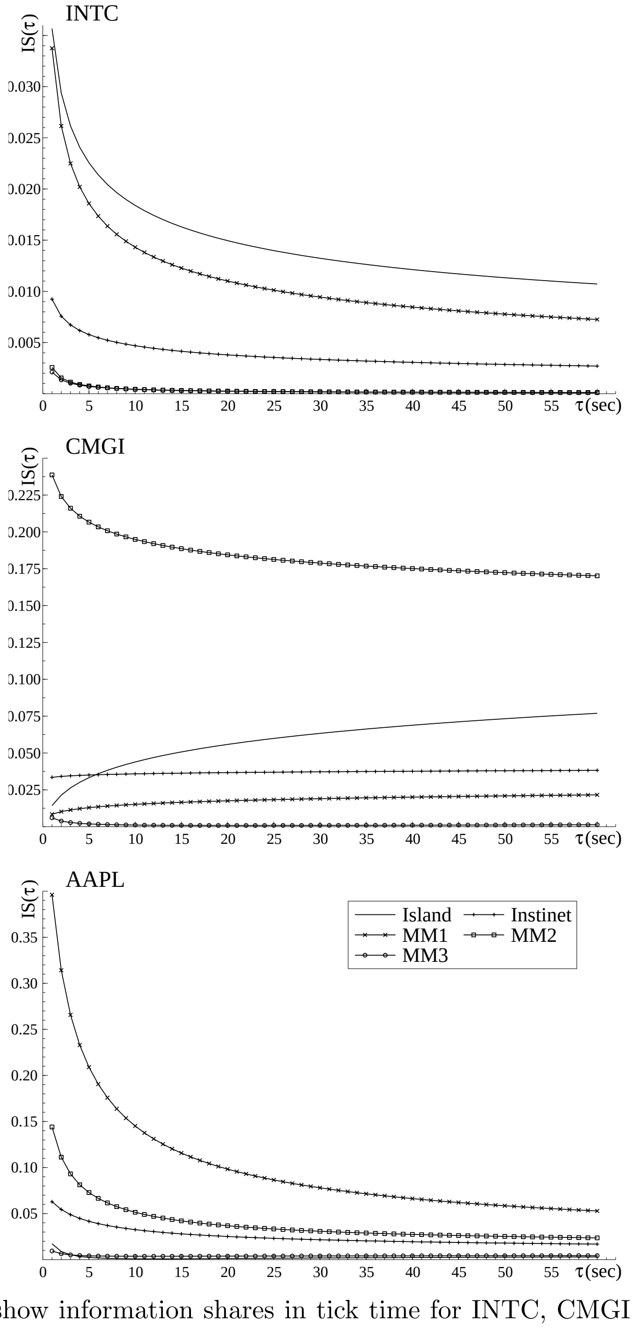
Note: These graphs show the dealer liquidity measures in tick time for INTC, CMGI and AAPL. These measures are plotted as a function of duration.

Figure 4: Tick time measure for price discovery ($\gamma(\tau)$) as a function of duration



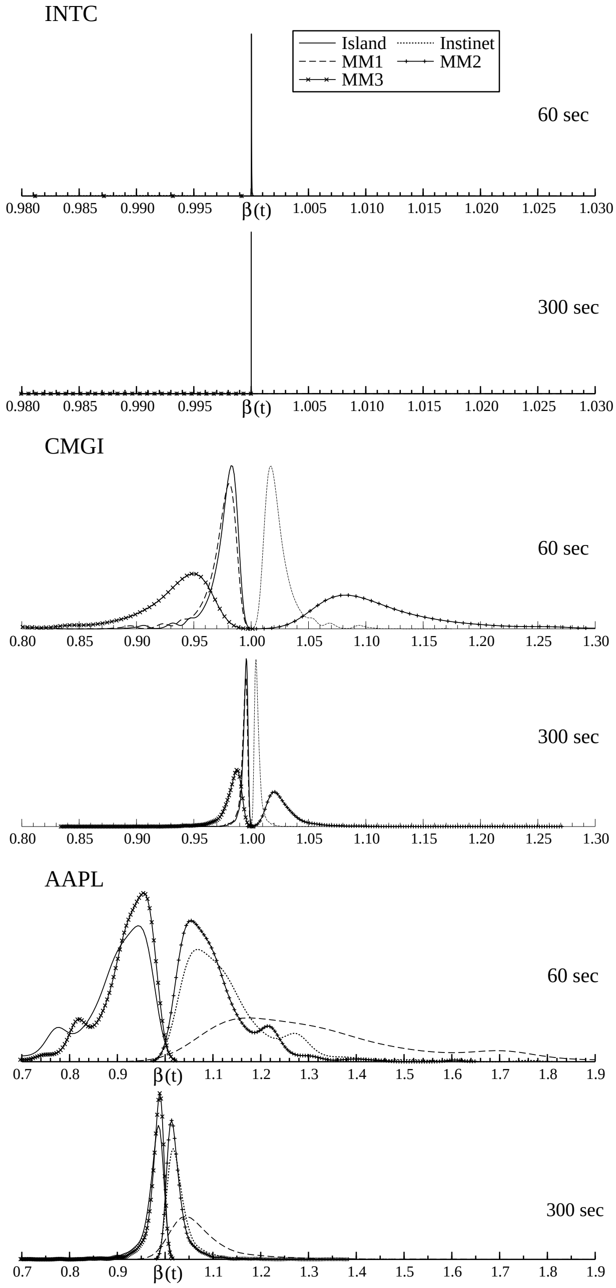
Note: These graphs show the price discovery measures in tick time for INTC, CMGI and AAPL. These measures are plotted as a function of duration.

Figure 5: Tick time measure for Information Shares as a function of duration



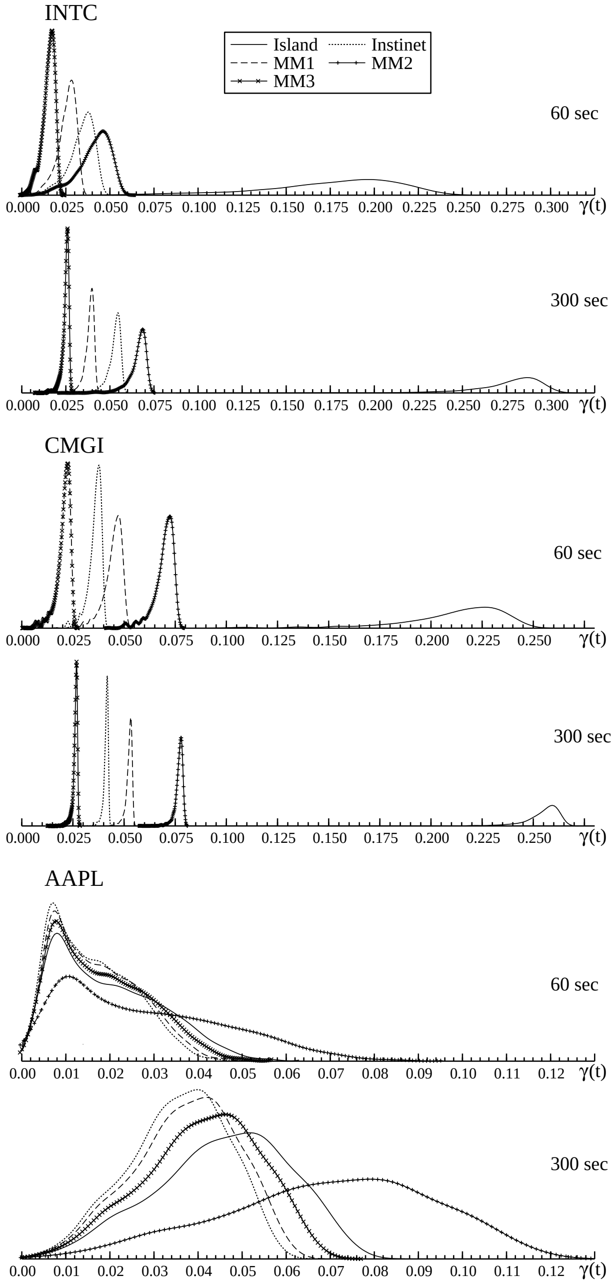
Note: These graphs show information shares in tick time for INTC, CMGI and AAPL per dealer. These measures are plotted as a function of duration.

Figure 6: Dealer liquidity in Calendar time



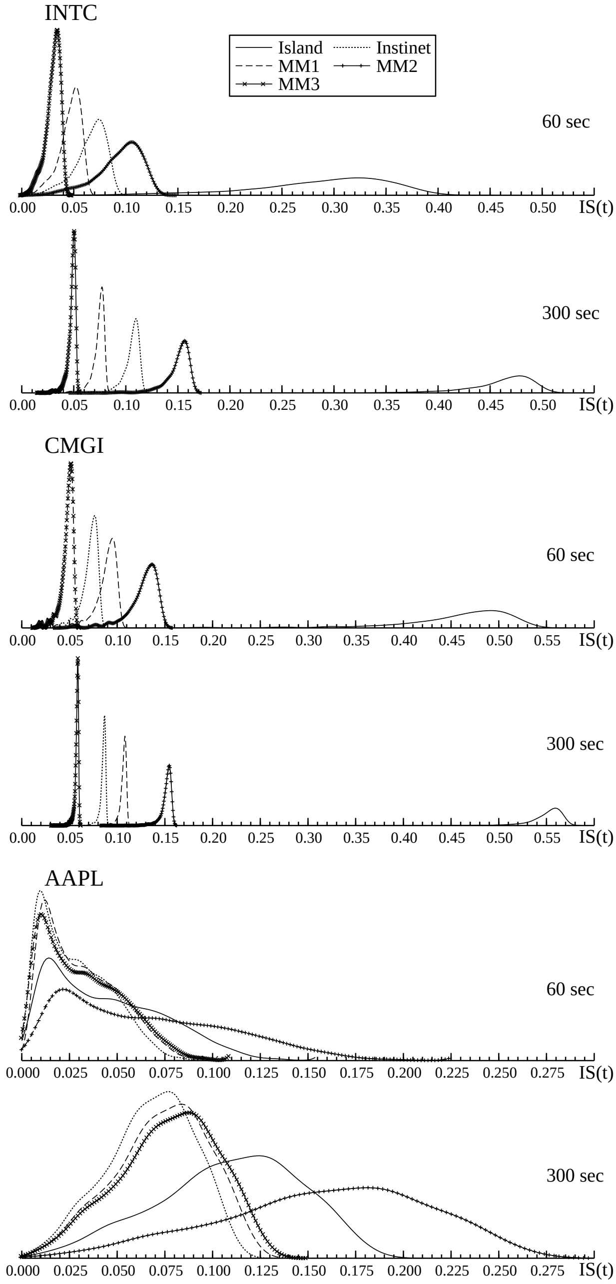
Note: These graphs show the distributions of the calendar time measures for dealer liquidity. These distributions are shown for the bids of INTC, CMGI and AAPL. Island is not shown as its value is always 1. The aggregates are shown at 60 and 300 second intervals.

Figure 7: Price Discovery in Calendar time



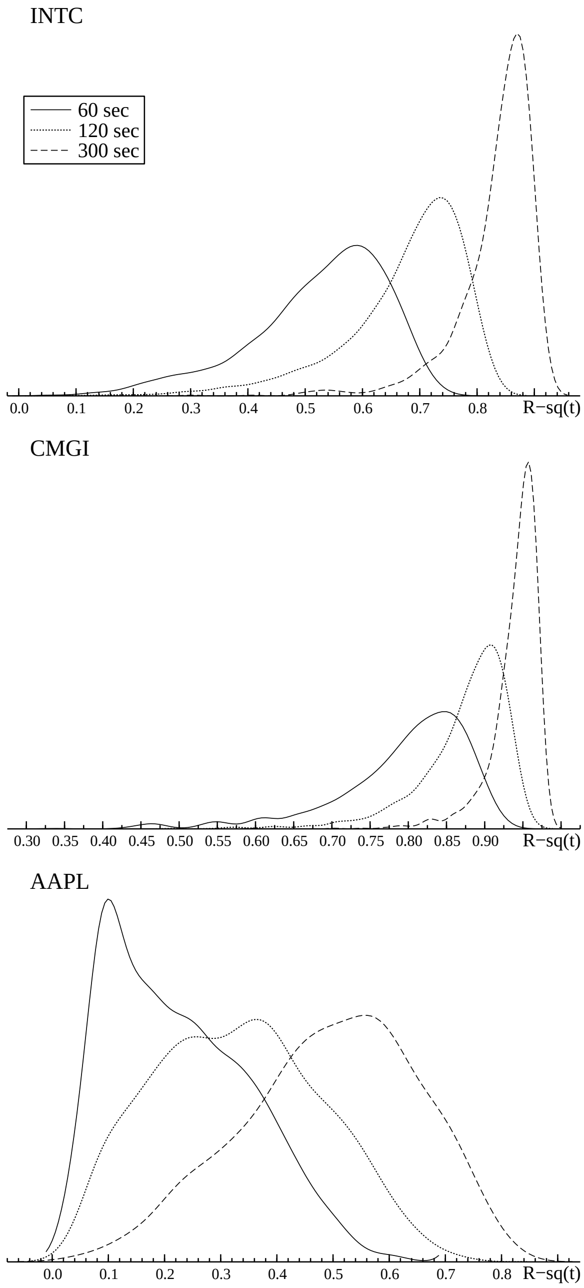
Note: These graphs show the distributions of the calendar time measures for price discovery. These distributions are shown for the asks of INTC, CMGI and AAPL. The aggregates are shown at 60 and 300 second intervals.

Figure 8: Information Shares in Calendar time per dealer



Note: These graphs show the distributions of the calendar time measures for Information Shares. These distributions are shown for INTC, CMGI and AAPL. The aggregates are shown at 60 and 300 second intervals. This measure is obtained by taking the inner product of the measure for dealer efficiency and price discovery. Using the specific structure of the model and by applying the matrix inversion lemma, the inner product can be decomposed to dealers.

Figure 9: Calendar time aggregated Information



Note: These graphs show the distributions of the total information incorporated in dealer quotes over a specific time interval. These distributions are shown for INTC, CMGI and AAPL. The aggregates are shown at 60, 120 and 300 second intervals.