Option-based tests of interest rate diffusion functions

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Abstract

The consistent finding in papers that estimate the interest rate diffusion function is that interest rate volatility is an increasing function of the spot rate. This paper introduces and implements regression tests of monotonic diffusion functions using an implied volatility proxy for objective volatility.

Using the case of three-month LIBOR rates, this paper documents that the relationship between interest rate levels and interest rate volatility is insignificant over the period 1985 through 1998. While rate volatility is clearly stochastic, it is not characterized by an increasing function (either linear or nonlinear) of rates.

I. Introduction

Recently, a number of papers have estimated generalized diffusion processes for short term interest rates using a variety of techniques. Ait-Sahalia (1996a, 1996b), Stanton (1997), Conley, Hansen, Luttmer, and Scheinkman (1997), and Bandi (1998). Other than Ait-Sahalia (1996b), these papers focus on the drift function, providing inference about the stationarity of interest rates.

However, each of these papers also estimates the diffusion function, which relates instantaneous interest rate volatility to a function of the level of the short term interest rate. This specification is intended to incorporate the time-varying volatility evidenced in interest rate time-series. While the estimation techniques and model specifications vary, the consistent finding in these papers is that volatility is an increasing function of the spot rate.

This paper evaluates the performance of diffusion functions, in which volatility is an increasing function of the interest rate, using volatilities inferred from interest rate options prices. Option-based volatility estimates are useful, because they do not require a complete specification of the volatility process, which may or may not be accurate. In addition, the ability to "observe" volatility rather than use a proxy based on a squared return, lends additional power to testing methods. Amin and Ng (1997) have found that Eurodollar futures option implied volatilities provide accurate forecasts of Eurodollar interest rate volatility.

Several recent papers have considered the adequacy of generalized diffusions as models for the equity index return process using option market data. Dumas, Fleming, Whaley (1998) evaluate deterministic volatility function models (generalized diffusion models) by measuring out-of-sample option pricing performance of these models. Their paper documents time-instability in the diffusion function, indicating that these models provide a poor description of the equity returns process.

Rosenberg (1999) also finds evidence indicating that equity diffusion functions, in which the only state volatility function variable is the index level, are misspecified. Rosenberg suggests a dynamic volatility function which incorporates at-the-money implied volatility as a state variable. Jackwerth and Rubinstein (1996) find that deterministic volatility models are outperformed by an ad-hoc alternative.

Generalized diffusion models are relatively new to the equity index returns area, with the exception of the constant-elasticity of variance model (Cox, 1996). One reason may be that representing equity index volatility (which is generally found to be stationary) as a stationary function of a non-stationary variable (the index level) is difficult.

In the interest rate modeling area, generalized diffusion models, in which the interest rate volatility is a function of the level, are generally accepted as providing an adequate characterization of the data. A detailed comparison of a number of parametric specifications is given in Chan, Karolyi, Longstaff, and Sanders (CKLS, 1992). They state: “We find that the most successful continuous-time models in capturing the dynamics of the short term interest rate are those that allow the volatility of interest rate changes to be highly sensitive to the level of the riskless rate (p. 1209).”

This work has been extended using non-parametric estimates of the diffusion function. Ait-Sahalia (1996a, p. 544, Figure 4), Stanton (1997, p. 1996, Figure 5), and Bandi (1998, pp. 30-31, Figures 6a and 7a) document a monotonically increasing relationship between interest rates and interest rate volatility. An exception is Ait-Sahalia (1996b, Figures 4c and 5c) although no confidence intervals are given, so it is difficult to interpret the significance of the decline in volatility with interest rates which occurs in the range from 0% to 11% (p. 409, Figure 4c) or 0% to 8% (p. 411, Figure 5c). In the same paper, the FGLS results (p. 417, Figure 7b) show an increasing relationship between returns and volatility over the observed data range.

There is a developing literature that combines GARCH-type stochastic volatility and level dependent volatility. These models generally find that the addition of a stochastic volatility factor improves model performance significantly and results in substantially smaller (but positive) estimates of interest rate elasticity. See, for example, Brenner, Harjes, and Kroner (1996, Figure 4, p.102) as well as Koedijk, Nissen, Schotman, and Wolff (1997, Table 2, p.112). In related work, Longstaff and Schwartz (1992) reject a level dependent volatility model in favor of a two factor model, while Heston and Nandi (1999) document mixed performance for two-factor model without a level effect.

Using the case of three month LIBOR rates, this paper documents that the relationship between interest rate levels and interest rate volatility is insignificant over the period 1985 through 1998. While rate volatility is clearly stochastic, it is not characterized by an increasing function (either linear or nonlinear) of rates. The presence of significant estimates of this relationship in previous papers may be due to spurious inference.

The remainder of this paper is organized as follows. Section II formulates and implements tests of monotonicity in diffusion functions. Section III concludes the paper.

II. Option-based tests of diffusion functions

II.a. The testing regime

Generalized diffusion models are defined by a stochastic differential equation in which the drift function, $\mu(r_t)$, and the diffusion function $\sigma(r_t)$ depend of state variable (r_t). In the interest rate modeling context, r_t is an instantaneous interest rate, usually proxied by a short term interest rate.

(1) $$dr_t = \mu(r_t)dt + \sigma(r_t)dW_t$$

Existing parametric specifications for $\sigma(r_t)$ include $\sigma(r_t) = \sigma$, (Vasicek, 1977), $\sigma(r_t) = \sigma\sqrt{r_t}$ (Cox, Ingersoll, and Ross, CIR, 1985), $\sigma(r_t) = \sigma r_t^{\gamma}$ (Chan, Karolyi, Longstaff, and Sanders, 1992), and $\sigma(r_t) = \beta_0 + \beta_1 r_t + \beta_2 r_t^{\beta 3}$ (Ait-Sahalia, 1996b). Recent papers such as Ait-Sahalia (1996a), Stanton (1997), and Bandi (1998) use non-parametric techniques to estimate the diffusion function.

The tests used in this paper are based on estimating the sign of the diffusion function. If the diffusion function is monotonically increasing (decreasing) in instantaneous volatility, then its first derivative with respect to volatility is positive (negative). Hence, the expected value with respect to volatility is positive (negative). This suggests the following regression tests:

(2) Test 1: $\sigma_t = \alpha_1 + \beta_1 r_t + \varepsilon_t$ $\quad$ $\mathrm{E}\left[\frac{\partial \sigma_t(r_t)}{\partial r_t}\right] = \beta_1$

$\sigma_t = \hat{\alpha}_1 + \hat{\beta}_1 r_t + \varepsilon_t$ $\quad$ $t_{\hat{\beta}_1} \leq 2$ reject monotonically increasing function

(3) Test 2: $\sigma_t - \sigma_{t-1} = \alpha_2 + \beta_2(r_t - r_{t-1}) + \varepsilon_t$ $\quad$ $\mathrm{E}\left[\frac{\partial \sigma_t(r_t)}{\partial r_t}\right] = \beta_2$

$\sigma_t - \sigma_{t-1} = \hat{\alpha}_2 + \hat{\beta}_2(r_t - r_{t-1}) + \varepsilon_t$ $\quad$ $t_{\hat{\beta}_2} \leq 2$ reject monotonically increasing function

This paper proposes a linear regression of option implied volatilities (and changes) against interest rate levels (and changes) to estimate the parameters β_1 and β_2, which measure the expected volatility response to interest rate changes. Newey-West (1987) heteroskedasticity and autocorrelation consistent standard errors are used to evaluate the statistical significance of the estimated parameters. Positive and statistically significant estimates of β_1 and β_2, based on robust t-statistics, would be in accord with the predictions of existing models.

The proposed regression tests measure the global relationship between interest rates and rate volatility. It is not necessary to estimate the entire empirical volatility function — or to specify the nature of its non-linearity — to test the differences in the global predictions of existing models versus the data. Since the existing models predict a locally positive derivative at all points, this corresponds to a positive expected derivative.

A difference between option implied volatility and the instantaneous volatility defined by the diffusion function is that option implied volatility reflects volatility over the remaining life of the option, which in this paper ranges from 10 to 100 days. Even so, volatilities over different maturities are typically highly correlated. This suggests that implied volatilities are a good instrument for instantaneous volatilities; hence, it is likely that this difference will have little effect on the results of Test 1. In addition, Test 2 should not be subject to this concern, since it utilizes daily changes which are a reasonable proxy for instantaneous changes.

II.b. Implied volatility as an estimator of objective volatility

A great deal of research has focused on whether equity option implied volatilities are unbiased and sufficient predictors of future volatility. Examples of this literature include Day and Lewis (1992), Canina and Figlewski (1993), Christensen and Prabala (1998), Fleming (1998), and many others.

The volatilities extracted from option prices are risk-neutral rather than objective; but, as long as there is approximately a constant or proportional risk-adjustment this should have no effect on measurement of correlation of rate levels and rate volatility. To the extent that the risk adjustment is stochastic and uncorrelated with the actual volatility and interest rate levels, there will be noise added to the results, reducing the power of the test, but there should be no bias. In this sense, implied volatility is acting as an instrument for true, but unobserved, volatility.

To provide a more specific justification for the use of at-the-money Eurodollar option implied volatilities as an estimator of LIBOR volatility, several tests are performed. (See Section II.c for a description of the data and technique used to obtain the implied volatilities). First, the average implied volatility is compared to the annualized sample standard deviation of daily LIBOR returns (log-differences). The sample standard deviation is 18.17% over the sample period, while the average option implied volatility over the period is 14.38%. There is a statistically significant difference between these estimates. However, as mentioned previously, if there is a constant difference between risk-neutral and objective volatility, this will not affect the tests.

A GARCH(1,1) model of objective volatilities over this period is estimated and the conditional volatilities are compared with implied volatilities. A regression of implied volatilities on conditional volatilities provides an r-squared of 61%; the correlation coefficient is .78.

As a final comparison, implied volatilities on each date are correlated with and regressed on realized volatilities measured as (annualized) LIBOR return standard deviations over the subsequent twenty days. The regression r-squared is 34.08% and the correlation is .5838.

These results from comparisons of implied and objective volatilities indicates that there is a great deal of common information in these variables. Hence, it is natural to utilize implied volatility as an instrument for objective volatility.

II.c. Implementing the tests

The volatility time-series is estimated using implied volatilities calculated using the full history of Eurodollar futures options prices. This database was obtained from the Futures Industry Institute, along with a database of Eurodollar futures prices. Eurodollar futures options commenced trading in March of 1985, so the model tests cover the period from 1985 through 1998.

While this is a shorter sample period than the estimation period used in the previously mentioned papers (often beginning in the 1973 with a separate estimation after the “Fed experiment” from 1980-1982), the tests are still quite useful. If existing models are unable to fit the interest rate volatility behavior over the most recent thirteen year period, it seems reasonable to conclude that they are inadequate.

Implied volatilities are calculated each day by selecting the nearest maturity, closest to-the-money call and put options with at least ten, but no more than 100 days until expiration, and at least five contracts traded. Daily option settlement prices are used in the implied volatility estimation. These prices have the advantage of being synchronized with the corresponding futures price at the end of the day, using several exchange determined rules. Under most circumstances, the settlement price is the average of the highest and lowest transaction prices in the last 30 seconds of trading. For additional details, see Rule 813 of the Chicago Mercantile Exchange Rulebook.

The Black (1976) formula is numerically inverted, with the futures price and strike price expressed in rates and the option price measured in basis points. The futures rate is the difference between 100 and the futures settlement price, the strike rate is the difference between 100 and the option strike price, the discount rate is the contemporaneous three-month British Banker’s

Association LIBOR rate, and the time until expiration the number of calendar days until expiration. While the Eurodollar contract has American exercise-style, implied volatility comparisons using the Black (1976) model and the Barone-Adesi and Whaley (1987) model (which incorporates the value of early exercise) indicate the differences are not significant in this context.

The interest rate series used is three-month LIBOR, as reported daily by the British Banker's Association. The series is chosen because it is the underlying asset for the Eurodollar futures contract; as such, it is the rate used in calculation of the final settlement price. The rate is determined based on a survey of the lending rates to prime customers by 16 banks which are major participants in the London Eurodollar Market.

The first panel of Table 1 describes characteristics of the three-month LIBOR interest rate series used in the tests. Over this period, LIBOR (in annualized terms) ranges from a low of 3.13% to a high of 10.62%. The first order autocorrelation is nearly equal to one, suggesting non-stationarity. This is confirmed by an augmented Dickey-Fuller test with five lags. There is also strong evidence for stochastic volatility, documented by an ARCH test p-value less than .0001.

The second panel of Table 1 describes characteristics of Eurodollar futures option data used to estimate at-the-money implied volatilities. Using the screening rules mentioned previously, there are 3170 of 3388 trading days on which implied volatilities are calculated. Implied volatilities (in annualized terms) range from a low of 3.38% to a high of 43.97%. The average implied volatility is 14.38% with a standard deviation of 5.50%.

Figure 1 plots the time-series of interest rates and interest rate volatilities. The peak in interest rates over this period is in 1989 with a trough in 1992. The peaks in interest rate volatility are around the 1987 stock market crash and in 1992. Interestingly, the 1992 volatility peak corresponds to a period of low interest rates, and the most recent period of stable rates is associated with declining rate volatility.

Figures 2 plots LIBOR volatility against LIBOR. If the diffusion function is increasing in the interest rate, there should be a scatter around a positively sloped curve. Figure 2 suggests that the relationship (either positive or negative) between rates and volatility is tenuous, at best. Especially noticeable is the high volatility at low rates shown in the left portion of the figure. High rates, shown in the right portion of the figure, are associated with average, rather than the highest levels of volatility. Figure 3, which plots volatility changes against interest rate changes, suggests that there is no discernable relationship between these variables.

Table 2 presents the results using formal tests described in Section II.a. The p-values for the parameter estimates are calculated using Newey-West (1987) heteroskedasticity and autocorrelation consistent standard errors.

The first regression test, of volatility on interest rate levels, results in a beta parameter (β_1) which is statistically insignificant. The point estimate of β_1 is -.20, so in addition to insignificance, the sign is incorrect. The second test, of volatility changes on interest rate changes, has similar results. The point estimate of β_1 is -.16, which is also statistically insignificant. From these tests, it is concluded that there is no statistically significant relationship between Eurodollar interest rates and interest rate volatility over the period 1985 through 1998.

Further evidence is provided by comparing the fitted volatilities from several parametric specifications with the "true" volatilities inferred from option prices. In particular, the CKLS (1992) specification ($\sigma(r_t) = \sigma r_t^{\gamma}$) is adopted to estimate fitted volatilities at γ levels of .5, 1.5, and 1.75. The σ parameter, for each choice of γ, is selected to scale the estimates to match the average implied volatility (14.38%) over the sample period.

The first choice of γ provides estimates consistent with the CIR (1985) model, the second choice is close to the empirical estimate in CKLS (1992), and the third estimate is at the center of the acceptable range found by Conley, Hansen, Luttmer, and Scheinkman (1997). Daily LIBOR fitted volatilities are obtained by evaluating each specification using the three-month BBA LIBOR rate over the period from 1985 through 1998.

Figure 4 plots these LIBOR fitted volatilities against LIBOR implied volatilities. The fitted volatilities are correlated with each other due to their dependence on the same factor, and the magnitude of the elasticity parameter determines the dispersion of the fitted volatilities. Pearson correlation coefficients for the fitted volatilities and implied volatility are -0.09, -0.04, and -0.02. Only the first is statistically significant at the 1% level, and it has the incorrect sign.

A final comparison with parametric level dependent volatility models is provided by estimating the CKLS volatility function using maximum likelihood estimation for daily LIBOR returns over the sample period. The estimated σ is statistically significant and equal to .0089 which corresponds to an annualized volatility of 17%. The estimated γ is -.02 and is not statistically significant.

II.d. Interpreting the results

These results are at odds with the prediction of existing models that there is a monotonically increasing relationship between interest rates and rate volatility. Hence, previously estimated generalized diffusion based interest rate models appear to be misspecified.

The presence of significant estimates of a positive level and volatility relationship in previous papers may be caused by spurious inference due to the particular choice of sample period in previous studies or misspecification of the drift and diffusion functions and the impact of outlying observations.

To test the sample dependency of the levels effect, a CKLS model is estimated using daily LIBOR returns from 1973 - 1998, 1973 - 1982, and 1983 - 1998. For the full sample, the γ parameter is statistically significant and equal to .48; for the first period, the γ parameter is statistically significant and equal to -.02; and, for the second period, the γ parameter is statistically significant and equal to -.06.

In models with nonlinear mean reversion, misspecification of the drift function may generate spurious inference for the diffusion function. Ait-Sahalia (1996b, Figure 7b, p. 417) uses FGLS to estimate the drift function and plots the squared residuals (a proxy for instantaneous volatility) against interest rate levels. This figure suggests a strong relationship between these variables. However, notice that the estimated nonlinear drift function predicts large rate increases when rates are low and large decreases when rates are high. If, in fact, the drift function is a scalar, then the misspecified nonlinear drift will generate large residuals at very high and low rates, which will be reflected in a parabolic estimate of the diffusion function.

As a comparison, Figure 5 graphs squared one-day LIBOR returns against LIBOR levels over the period 1985 through 1998. This plot shows that there is no strong relationship between volatility and interest rates over the sample period. A regression of squared returns on LIBOR and LIBOR squared indicates that there is a statistically significant nonlinear relationship, but that the predictive ability of LIBOR for squared returns is very small with a regression adjusted R-squared of .06%. The lack of variability in the fitted squared returns, also plotted in Figure 5, confirms that the explanatory power of LIBOR is low.

These results suggest that the appearance of a strong volatility and interest rate relationship in models with nonlinear mean-reversion may be an artifact of misspecification of the drift function. Evidence against a nonlinear drift function is provided in several recent studies including Jones (1998), Pritsker (1998), and Chapman and Pearson (1999).

III. Conclusions

This paper has proposed and implemented regression tests of monotonicity in interest rate diffusion functions. These tests are based on the measured relationship between observed interest rates (and changes) and interest rate volatility (and changes) inferred from option prices.

Evidence from the Eurodollar market suggests that interest rate volatility is stochastic but unrelated to rate levels. If there is a level effect, it is very weak compared to other factors that drive interest rate volatility. This suggests that generalized diffusion models provide a poor characterization of the data, and that interest rate models with stochastic volatility (e.g. Longstaff and Schwartz (1992); Fong and Vasicek (1992); Chen and Scott (1992); Brenner, Harjes, and Kroner (1996); Koedijk, Nissen, Schotman, and Wolff (1997); Andersen and Lund (1997); and Heston and Nandi (1999)) are warranted.

Bibliography

Ait-Sahalia, Y., 1996a, "Nonparametric Pricing of Interest Rate Derivative Securities," *Econometrica,* 64, 527-560.

Ait-Sahalia, Y., 1996b, "Testing Continuous Time Models of the Spot Interest Rate," *Review of Financial Studies*, 9, 385-426.

Amin, K. and V. Ng, 1997, "Inferring Future Volatility from the Information in Implied Volatility in Eurodollar Options: A New Approach," *Review of Financial Studies*, 10, 333-367.

Andersen, T. G. and J. Lund, 1997, "Estimating Continuous-time Stochastic Volatility Models of the Short term Interest Rate," *Journal of Econometrics*, 77, 343-377.

Bandi, F., 1998, "Local Time in the Econometrics of Continuous-time Finance: An Application to the Short-term Interest Rate Dynamics," Manuscript.

Barone-Adesi, G. and R. E. Whaley, 1987, "Efficient Analytic Approximation of American Option Values," *Journal of Finance*, 42, 301-320.

Black, F., 1976, "The Pricing of Commodity Contracts," *Journal of Financial Economics*, 3, 167-179.

Brenner, R. J., R. H. Harjes and K. F. Kroner, 1996, "Another Look at Models of the Short Term Interest Rate," *Journal of Financial and Quantitative Analysis*, 31, 85-107.

Canina, L. and S. Figlewski, 1993, "The Informational Content of Implied Volatility," *Review of Financial Studies*, 6, 659-681.

Chan, K. C., G. A. Karolyi, F. A. Longstaff and A. B. Sanders, 1992, "An Empirical Comparison of Alternative Models of the Short-Term Interest Rate," *Journal of Finance*, 47, 1209-1227.

Chapman, D. and N. Pearson, 1999, "Is the Short Rate Drift Actually Nonlinear?," Manuscript.

Chen, R. and L. Scott, 1992, "Pricing Interest Rate Options in a Two-Factor Cox-Ingersoll-Ross Model of the Term Structure," *Review of Financial Studies*, 5, 613-636.

Christensen, B. J. and N. R. Prabhala, 1998, "The relation between implied and realized volatility," *Journal of Financial Economics*, 50, 125-150.

Conley, T. G., L. P. Hansen, E. G. J. Luttmer and J. A. Scheinkman, 1997, "Short-term Interest Rates as Subordinated Diffusions," *Review of Financial Studies*, 10, 525-577.

Cox, J., 1996, "The Constant Elasticity of Variance Pricing Model and Its Implications for Option Pricing," *Journal of Portfolio Management*, 23, 15-17.

Cox, J. C., J. E. Ingersoll, Jr. and S. Ross, 1985, "A Theory of the Term Structure of Interest Rates," *Econometrica*, 53, 385-407.

Day, T. E. and C. M. Lewis, 1992, “Stock Market Volatility and the Information Content of Stock Index Options,” *Journal of Econometrics*, 52, 267-287.

Dumas, B., J. Fleming and R. E. Whaley, 1998, “Implied Volatility Functions: Empirical Tests,” *Journal of Finance*.

Fleming, J., 1998, “The Quality of Market Volatility Forecasts Implied by S&P100 Index Option Prices,” *Journal of Empirical Finance*, 5, 317-345.

Fong, H. G. and O. A. Vasicek, 1992, “Interest Rate Volatility as a Stochastic Factor,” Manuscript.

Heston, S. and S. Nandi, 1999, “A Two-Factor Model for Pricing Bonds and Interest Rate Derivatives with GARCH Volatility: Analytical Solutions and their Applications,” Manuscript.

Jackwerth, J. and M. Rubinstein, 1996, “Recovering Stochastic Processes from Option Prices,” Manuscript.

Jones, C. S., 1998, “Nonlinear Mean Reversion in the Short Term Interest Rate,” Manuscript.

Koedijk, K. G., F. Nissen, G. J. A., P. C. Schotman and C. C. P. Wolff, 1997, “The Dynamics of Short Rate Volatility Reconsidered,” *European Finance Review*, 1, 105-130.

Longstaff, F. A. and E. S. Schwartz, 1992, “Interest Rate Volatility and the Term Structure - a Two-Factor General Equilibrium Model,” *Journal of Finance*, 47, 1259-1282.

Newey, W. and K. West, 1987, “A Simple Positive Semi-Definite, Heteroskedasticity and Autocorrelation Consistent Covariance Matrix,” *Econometrica*, 55, 703–708.

Pritsker, M., 1998, “Nonparametric Density Estimation and Tests of Continuous Time Interest Rate Models,” *Review of Financial Studies*, 11, 449-487.

Rosenberg, J., 1999, “Implied Volatility Functions: A Reprise,” Manuscript.

Stanton, R., 1997, “A Nonparametric Model of Term Structure Dynamics and the Market Price of Interest Rate Risk,” *Journal of Finance*, 52, 1973-2002.

Vasicek, O. A., 1977, “An Equilibrium Characterization of the Term Structure,” *Journal of Financial Economics*, 5, 177-188.

Table 1
Data description

Three-month BBA LIBOR
Daily, 1985-1998

	Levels	Changes
N	3388	3388
Mean	6.33%	0.00%
Median	6.06%	0.00%
Maximum	10.62%	0.38%
Minimum	3.13%	-0.69%
Std. Dev.	1.79%	0.06%
Skewness	-0.0002	-1.0967
Kurtosis	2.2237	17.5801
Autocorrelation	0.9990	0.0697
Aug. Dickey-Fuller stat.	-1.71	-23.23
ARCH p-value	<.0001	<.0001

Eurodollar futures option data (at-the-money nearest maturity call and put)
Daily, 1985-1998

	N	Mean	Std. dev.	Minimum	Maximum
Moneyness (K/Ft - 1)	3170	0.0001	0.0131	-0.0369	0.0369
Time until expiration (days)	3170	54.01	26.01	10.00	100.00
Trading volume (contracts)	3170	5255.68	5783.63	20.00	64173.00
Implied volatility (annual)	3170	14.38%	5.50%	3.88%	43.97%

This table reports characteristics of the interest rate data and option data used in the diffusion function tests. The interest rate data is three-month LIBOR reported daily by the British Banker's Association (BBA). The augmented Dickey-Fuller (1979) test statistic measures the presence of non-stationarity (a unit root) using five lags. A test statistic smaller than -2.56 indicates rejection of the null of a unit root in the data. The ARCH test p-value is the p-value from Engle's (1982) ARCH test which measures the presence of stochastic volatility as evidenced by correlation of squared residuals using five lagged squared residuals. Kurtosis is reported in excess terms.

The futures options data is daily data distributed by the Futures Industry Institute (FII) for all contracts traded from the inception of trading in March 1985 through 1998. Only nearest maturity (between 10 and 100 days), nearest to the money, put and call contracts with daily volume of at least 5 contracts are used in this study. For each option contract, the corresponding futures contract settlement price is obtained using a database from the FII. Futures prices and strike prices are converted to rates (F_t = 100 - futures price; K = 100 - strike price). The option proportional moneyness is $K/F_t - 1$. Time until expiration is measured in calendar days. Implied volatilities are obtained by numerically inverting the Black (1976) formula, using this data, along with the BBA three-month LIBOR rate as the discount rate.

Table 2
Option-based tests of interest rate diffusion functions

Regression of volatility on interest rates

Variable	Coefficient	Std. Error	t-Statistic	Prob.
$\alpha 1$	0.16	0.0117	13.37	0.0000
$\beta 1$	-0.20	0.1646	-1.20	0.2290
Adjusted R-squared	0.0038			
Durbin-Watson stat.	0.0575			
F-probability	0.0003			
Number observations	3170			

Regression of volatility changes on interest rate changes

Variable	Coefficient	Std. Error	t-Statistic	Prob.
$\alpha 2$	0.00	0.0002	-1.31	0.1905
$\beta 2$	-0.16	0.7248	-0.21	0.8301
Adjusted R-squared	-0.0003			
Durbin-Watson stat.	2.3294			
F-probability	0.6825			
Number observations	3170			

This table reports tests of interest rate diffusion functions based on regressions of volatility levels on interest rates and volatility changes on interest rate changes. Interest rate (LIBOR) volatilities are estimated using Eurodollar futures option at-the-money implied volatility. LIBOR rates are daily three-month LIBOR rates reported by the British Banker’s Association. Standard errors and t-statistics are calculated using the Newey and West (1987) heteroskedasticity and autocorrelation consistent procedure with eight lags.

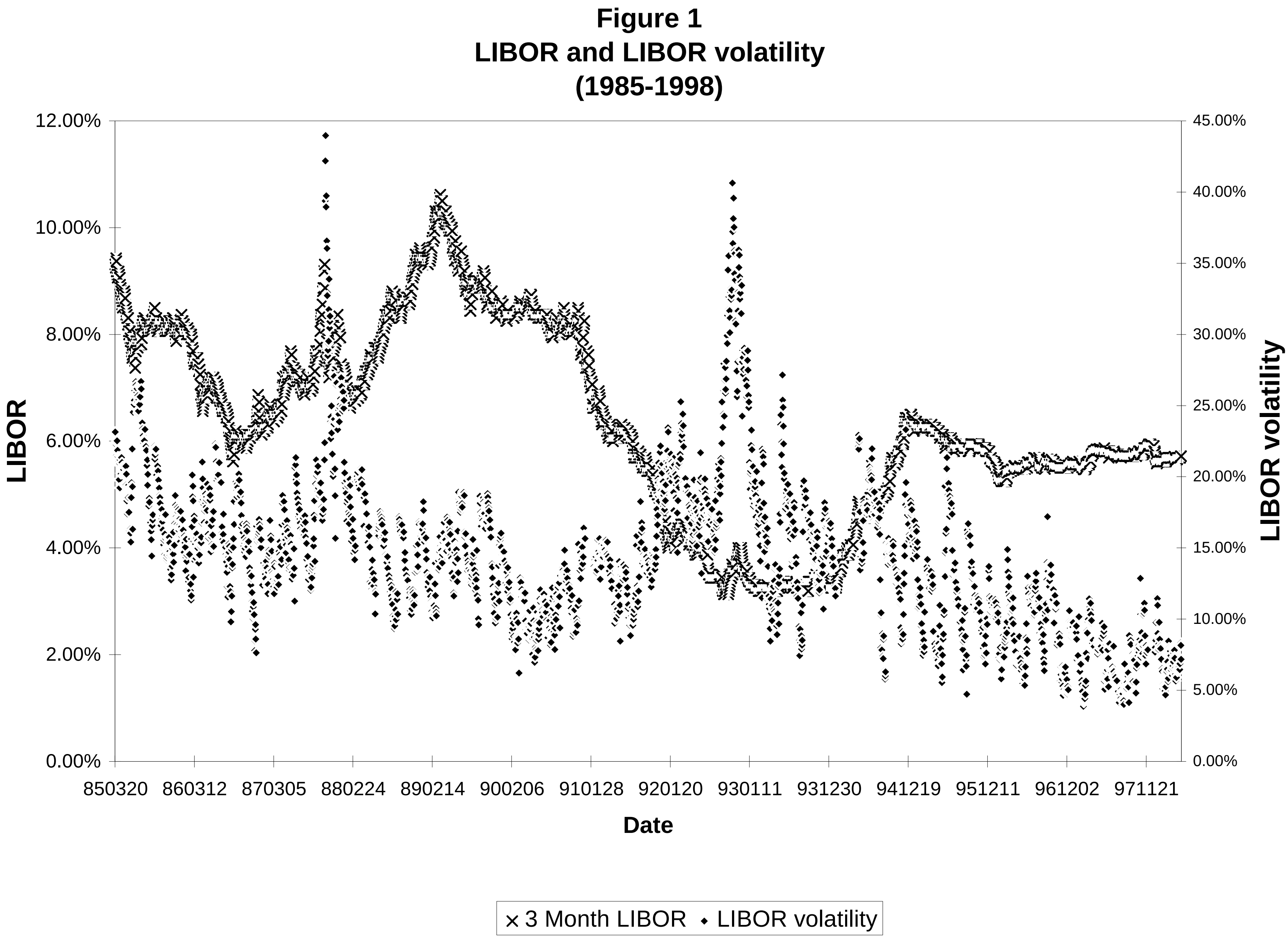
Figure 1
LIBOR and LIBOR volatility
(1985-1998)
12.00%
10.00%
8.00%
6.00%
4.00%
2.00%
0.00%
LIBOR
45.00%
40.00%
35.00%
30.00%
25.00%
20.00%
15.00%
10.00%
5.00%
0.00%
LIBOR volatility
850320
860312
870305
880224
890214
900206
910128
920120
930111
931230
941219
951211
961202
971121
Date
3 Month LIBOR
LIBOR volatility

Figure 2
Volatility and interest rates
(LIBOR, 1985-1998)

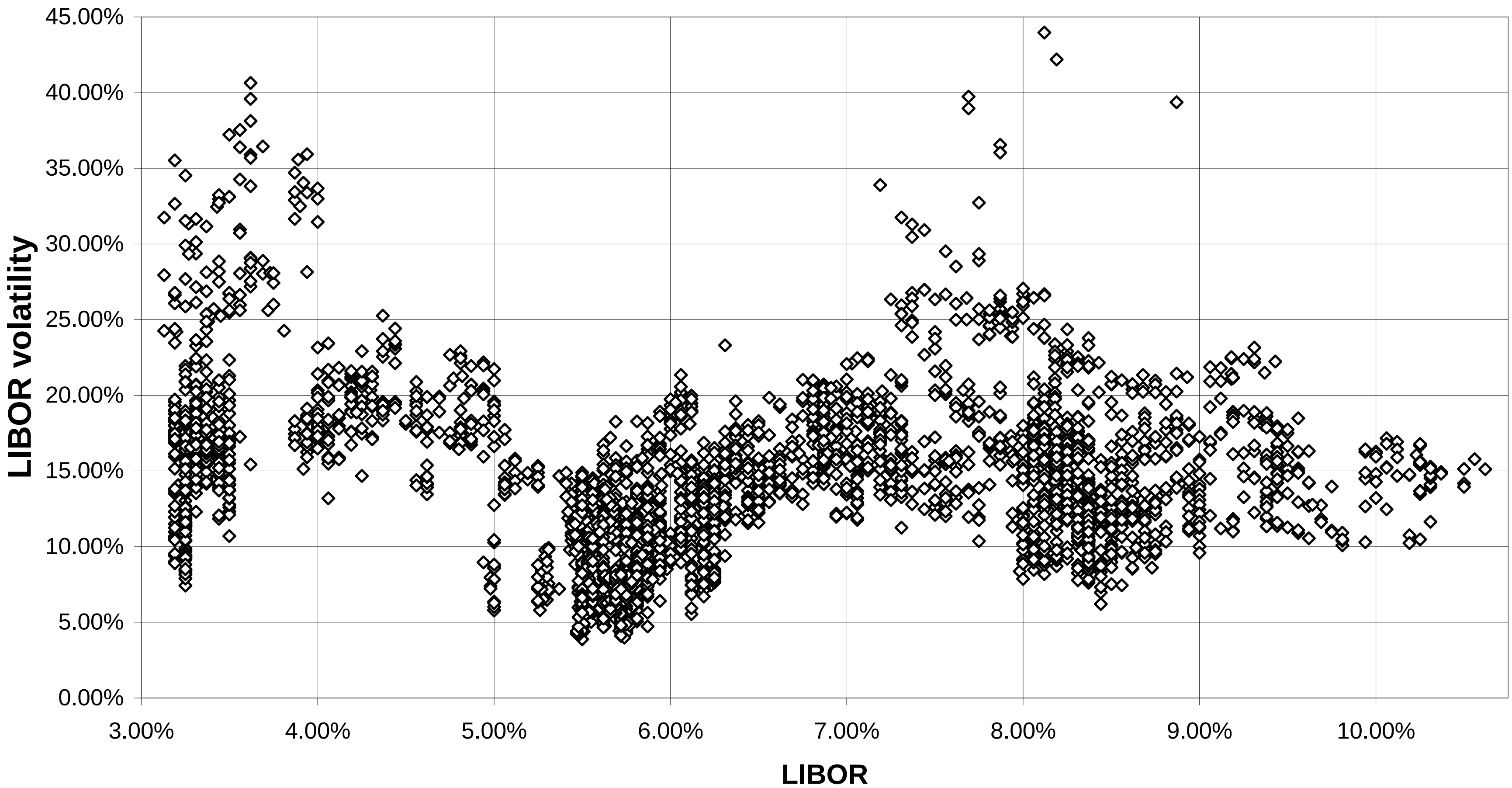

Figure 3
Volatility changes and interest rate changes
(LIBOR, 1985-1998)

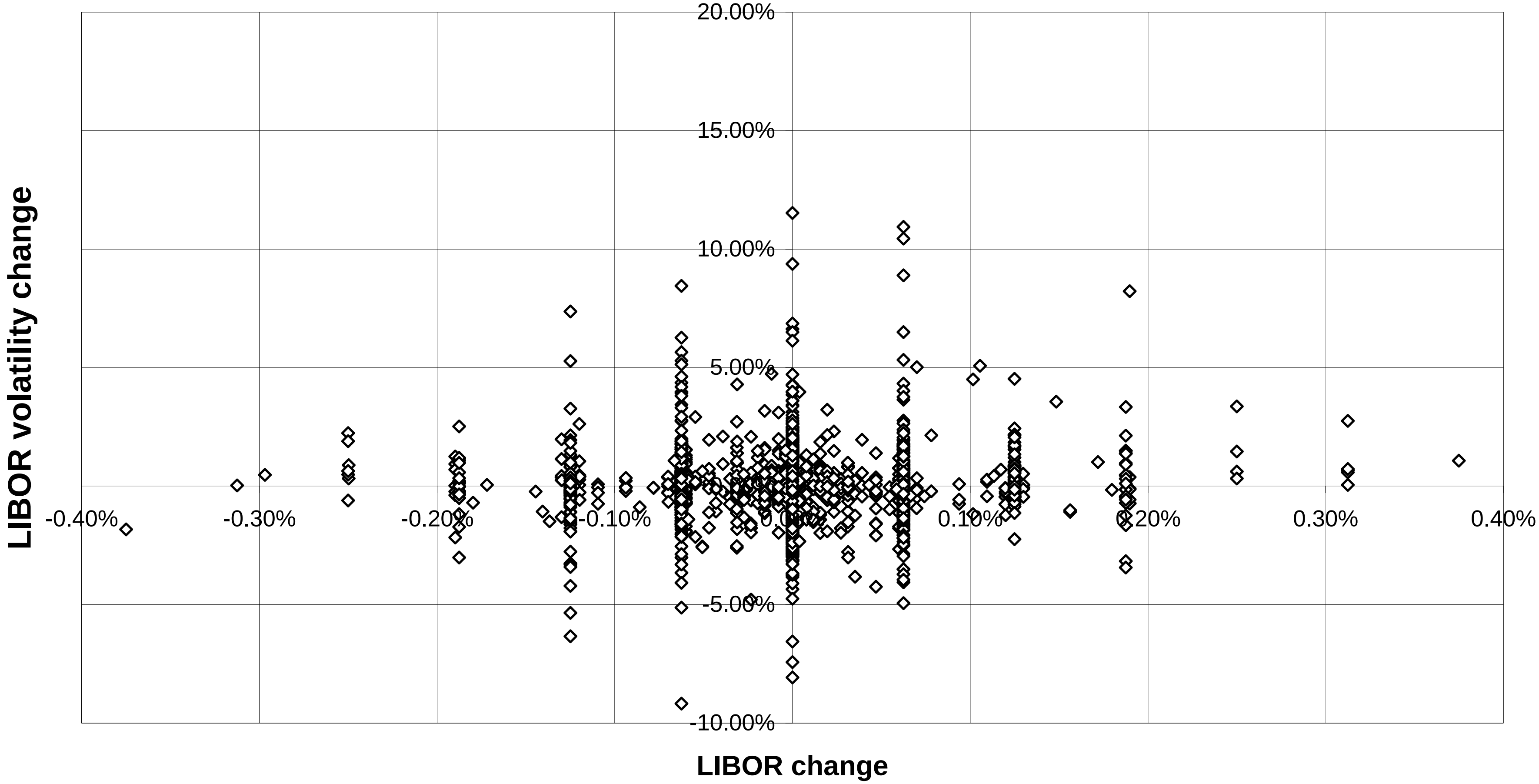

Figure 4
LIBOR implied and fitted volatilities
Daily, 1985-1998

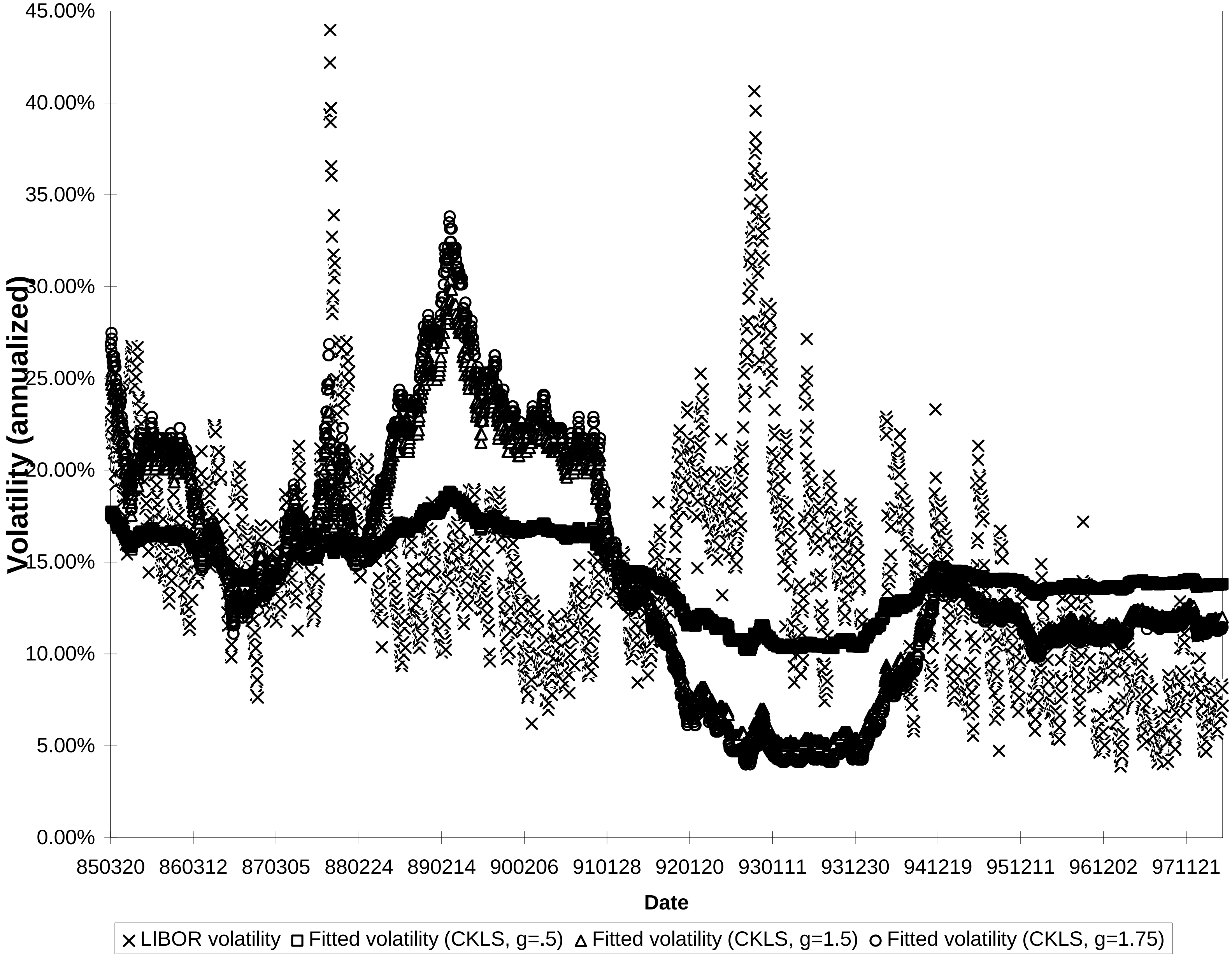

Figure 5
LIBOR and LIBOR squared return
Daily, 1985-1998