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Post-Announcement Drift

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Brown, Goetzmann and Ross (1995) document that *ex-post* conditioning can significantly bias empirical results based on observed rates of return. These results have interesting implications for cross-sectional cumulated excess return measures [CAR's] that are commonly used in the context of event studies (see Brown and Warner, 1981). Ball and Brown [1968] note an upward drift in cumulated excess returns subsequent to a positive earnings announcement surprise. Subsequent work by Foster [1977] and Foster *et al* [1984] among others has documented that this drift is related to size of the firm in question. The current state of this literature is summarized in Ball [1992].

The dynamics of the conditional price path have a direct bearing on these results. To the extent that both the magnitude of the earnings surprise and the size of the issuing firm are related to the standard deviation of excess returns, we would expect an association between the earnings announcement and drift among surviving firms. The argument runs as follows. Firms that are otherwise in financial distress are more likely to survive on a favorable earnings surprise than an unfavorable earnings surprise. In fact the *ex ante* probability of survival will be an increasing function of the magnitude of this surprise for any given level of financial distress. In a surviving sample, firms working their way out of financial distress will typically have higher earnings announcements and subsequent returns than other firms whose survival does not so nearly depend on favorable announcements. This effect works at the level of the individual security. To the extent that portfolios are formed according to the size of the earnings surprise, this effect will be magnified at the portfolio level (c.f. Lo and MacKinlay[1990]).

Event studies typically look at the impact of the earnings announcement on security prices after the announcement has been made, and then correlate this impact with the content of the earnings announcement, classifying such announcements as *good news* or *bad news*. With access to more complete information, such as the standardized unexpected earnings (SUE) upon the announcement, finer classifications are possible. These studies typically assume that before the event the expected change in security prices is zero. Knowing that a quarterly earnings announcement is to be made at some date τ in the future simply adds volatility to the *ex ante* distribution of stock prices. This increase in volatility reflects the likely magnitude of the earnings announcement, good or bad.

Thus, to capture the *ex ante* impact on security prices around earnings announcements, we can simply make the volatility, σ, rise secularly on earnings announcement dates. In such a case, the above conditional diffusion remains valid where σ is replaced by σ_t. While this is formally correct, we need more structure so that we can go back and ask different questions, such as what we expect the price p_τ at some point subsequent to an earnings announcement. To capture earnings volatility, we use a window of the kind depicted in Figure 1.

Notice that the incremental volatility in the window,

$$\textit{Incremental volatility} \equiv \frac{\kappa^2}{\Delta}\left(\tau + \frac{\Delta}{2} - \left(\tau - \frac{\Delta}{2}\right)\right) = \kappa^2$$

does not depend on Δ, and hence we can regard κ^2 as the total addition to volatility as a result of the earnings announcement. In the earnings announcement window $(\tau - \frac{\Delta}{2}, \tau + \frac{\Delta}{2}]$, the price diffusion is given as

$$dp = \frac{\sigma^2 + \frac{\kappa^2}{\Delta}}{p - \underline{p}} dt + \sqrt{\sigma^2 + \frac{\kappa^2}{\Delta}}\, dz$$

$$= \frac{\sigma^2}{p - \underline{p}} dt + \frac{\frac{\kappa^2}{\Delta}}{p - \underline{p}} dt + \sigma\, dz + \left(\sqrt{\sigma^2 + \frac{\kappa^2}{\Delta}} - \sigma\right) dz$$

As the event period Δ approaches zero, the price change conditional on the announcement will be distributed according to the distribution function

$$G(\Delta p \,|\, p) = \frac{\Phi\left(\frac{\Delta p}{\kappa}\right) - 2\Phi\left(\frac{\underline{p} - p}{\kappa}\right) + \Phi\left(\frac{2(\underline{p} - p) - \Delta p}{\kappa}\right)}{2\Phi\left(\frac{p - \underline{p}}{\kappa}\right) - 1}$$

given the immediate prior price p (Appendix), and will have an expected value

$$E(\Delta p \,|\, p) = \frac{2(p - \underline{p})\Phi\left(\frac{p - \underline{p}}{\kappa}\right)}{2\Phi\left(\frac{p - \underline{p}}{\kappa}\right) - 1}$$

that in general will be small and positive to the extent that we exclude from the analysis securities that fail due to adverse event period information. Thus we can write the security price diffusion as

$$dp = \frac{\sigma^2}{p - \underline{p}} dt + \sigma dz \qquad , \quad for \quad \tau \notin \{\alpha\ell \,|\, \ell = 1, \ldots\}$$

$$= \frac{\sigma^2}{p - \underline{p}} dt + \sigma dz + \delta_\tau(p) \ , \quad for \quad \tau \in \{\alpha\ell \,|\, \ell = 1, \ldots\} \ ,$$

where the event related change in price $\delta_\tau(p)$ is distributed according to $G(\cdot\,|\,p)$ and $\tau \in \{\alpha\ell \,|\, \ell = 1, \ldots\}$ represents the set of earnings dates separated by a time of α (three months). Figure 2 illustrates this kind of price path

Now we want to use the above analysis to determine the relevant distribution of price changes conditional on earnings announcements. In particular, conditional on observing Δp_τ on an earnings announcement date τ, what can we say about the distribution of the price change $\Delta p_{\tau'}$ on any subsequent announcement date $\tau' > \tau$? We use the following three Propositions to show that an increase in Δp_τ implies a first order stochastic increase in $\Delta p_{\tau'}$ in the surviving sample of returns. This result is consistent with empirical results found by Bernard and Thomas [1992] among others.

Given the conditional density of the change in price on the event, $g(\Delta p\,|\,p)$, define $f(p)$ as the unconditional density of the initial price p, which we assume is smooth and has no zeros on (m,∞). We shall first show that the larger the event effect Δp the more likely it was that the initial price p was lower (closer to the reservation price $\underline{p}$). In other words, for every $x>\underline{p}$, $Pr[p<x\,|\,\Delta p]$ is an increasing function of x, where, by Bayes' Theorem

$$Pr[p<x\,|\,\Delta p] = \int_m^x h(p\,|\,\Delta p)\, dp$$

$$= \int_m^x \frac{g(\Delta p\,|\,p) f(p)}{\int_m^\infty g(\Delta p\,|\,\zeta) f(\zeta)\, d\zeta}\, dp$$

A sufficient condition for this result is that changing Δp shifts the conditional density of p, $h(p\,|\,\Delta p)$ in a first order stochastically dominating fashion. To do so we first prove that the relevant density function satisfies an appropriate sufficient condition:

Proposition 1:

For any initial price density $f(.)$, the density

$$h(p \mid \Delta p) \equiv \frac{g(\Delta p \mid p) f(p)}{\int_{m} g(\Delta p \mid \zeta) f(\zeta)\, d\zeta}$$

has the single crossing property (SCP):

$$(\forall \Delta p' > \Delta p)\ (\exists p^{\circ})\ (\forall p)$$

$$p < p^{\circ} \Rightarrow h(p \mid \Delta p') > h(p \mid \Delta p)\ , \quad and$$

$$p > p^{\circ} \Rightarrow h(p \mid \Delta p') < h(p \mid \Delta p)$$

Proof: Appendix.

Raising Δp cross-sectionally on an earnings announcement date increases the probability that p on that date is closer to the reservation price $\underline{p}$. This, in turn raises the probability and the expectation that Δp will be positive on subsequent dates.

Proposition 2:

Since $h(p \mid \Delta p)$ has the SCP,

$$(\forall x) \qquad H(x \mid \Delta p) \equiv Pr(p \leq x \mid \Delta p)$$

is an increasing function of Δp.

Proof: Appendix.

This stochastic dominance result implies that excess returns on the announcement date are positively associated in the surviving sample with excess returns on subsequent announcements. An increase in Δp_{τ} on an earnings announcement date τ implies a first order stochastic increase in $\Delta p_{\tau'}$ on any subsequent announcement date $\tau' > \tau$ and an increased expected announcement effect on the subsequent date:

Proposition 3:

Let τ be an earnings announcement date and $\tau' > \tau$ represent a subsequent earnings announcement date.

$$(\forall x) \qquad Pr(\Delta p_{\tau'} > x \mid \Delta p_{\tau}) \quad and$$

$$E(\Delta p_{\tau'} \mid \Delta p_{\tau})$$

are both increasing functions of Δp_{τ}.

Proof: Appendix.

The above proposition verifies what we are seeking; we now know that an increase in Δp_τ on an earnings announcement date τ implies a first order stochastic increase in $\Delta p_{\tau'}$ on any subsequent announcement date $\tau' > \tau$.

The statistical properties of event and post event period returns are extensively analysed in Brown and Pope [1994]. Table 2 taken from that paper examines the distributional characteristics of event period and post event standardized returns conditional on a measure of the change in price over the sixty days prior to the earnings announcement. The data corresponds to that used in a study of earnings announcements by Bernard and Thomas [1992]. Returns are measured in excess of the returns on a size portfolio control, and are standardized by the standard deviation of these size excess returns, measured for each security using data 60 days prior to the announcement. The prior change in price is then measured as the 60 day holding period standardized excess return prior to the announcement.

Across the entire sample, the event period excess return, measure of positive skewness and correlation with post-announcement excess return are a generally decreasing function of the prior period change in price. Table 2 indicates that this effect is most pronounced for those firms which had the highest *a priori* probability of failure (using the Ohlson [] probability of bankruptcy measure) and yet survived into the sample. This result is broadly consistent with the survival argument presented above. However, as Brown and Pope indicate, the survival argument does not explain all of the persistence in returns subsequent to earnings announcements. Clearly other factors are also at work.

Appendix: ***The distribution of prices around earnings announcements***

*A. **The distribution of the event related change in price***

The distribution of price changes for a given earnings announcement is equivalent to the distribution of price subsequent to the earnings announcement given the price prior to the announcement, $p_{\tau+\frac{\Delta}{2}} \mid p_{\tau-\frac{\Delta}{2}}$ conditional on the security surviving the announcement $p \geq \underline{p}$.

This probability distribution is defined in terms of the probability

$$Pr[p_\Delta < X \mid p, \inf_{s\in[0,\Delta]} p_s > \underline{p}] = \frac{Pr[p_\Delta < X \ \& \ \inf_{s\in[0,\Delta]} p_s > \underline{p} \mid p]}{Pr[\inf_{s\in[0,\Delta]} p_s > \underline{p} \mid p]}$$

As before, the probability that the observed price exceeds the reservation price $\underline{p}$ is given as

$$Pr[\inf_{s\in[0,\Delta]} p_s > \underline{p} \mid p] = 2\Phi\left[\frac{p - \underline{p}}{\sigma\sqrt{\Delta}}\right] - 1$$

Now,

$$\begin{aligned} Pr[p_\Delta < X \ \& \ \inf_{s\in[0,\Delta]} p_s > \underline{p} \mid p] &= Pr[\inf_{s\in[0,\Delta]} p_s > \underline{p} \mid p] \\ &\quad - Pr[p_\Delta \geq X \ \& \ \inf_{s\in[0,\Delta]} p_s > \underline{p} \mid p] \\ &= Pr[\inf_{s\in[0,\Delta]} p_s > \underline{p} \mid p] \\ &\quad - \{Pr[p_\Delta \geq X \mid p] - Pr[p_\Delta \geq X \ \& \ \inf_{s\in[0,\Delta]} p_s \leq \underline{p} \mid p]\} \\ &= Pr[\inf_{s\in[0,\Delta]} p_s > \underline{p} \mid p] - Pr[p_\Delta \geq X \mid p] + Pr[p_\Delta < 2\underline{p} - X \mid p], \end{aligned}$$

where we have used the reflection principle to evaluate the last term. If we define the event period volatility as

$$\sigma(\Delta) = (\sigma^2 + \frac{\kappa^2}{\Delta})^{\frac{1}{2}}\sqrt{\Delta}$$

the probability distribution function is given as

$$Pr[p_{\tau+\frac{\Delta}{2}} < X \mid p_{\tau-\frac{\Delta}{2}} = p, \inf_{s\in[\tau-\frac{\Delta}{2},\tau+\frac{\Delta}{2}]} p_s > \underline{p}] = \frac{\Phi\left(\frac{X-p}{\sigma(\Delta)}\right) - 2\Phi\left(\frac{\underline{p}-p}{\sigma(\Delta)}\right) + \Phi\left(\frac{2\underline{p}-X-p}{\sigma(\Delta)}\right)}{2\Phi\left(\frac{p-\underline{p}}{\sigma(\Delta)}\right) - 1}$$

As we allow the event interval Δ to approach zero, this yields the desired probability distribution for the change in price, $\Delta p = p_{\tau+\frac{\Delta}{2}} - p_{\tau-\frac{\Delta}{2}}$:

$$G(\Delta p \mid p) = \frac{\Phi\left(\frac{\Delta p}{\kappa}\right) - 2\Phi\left(\frac{\underline{p}-p}{\kappa}\right) + \Phi\left(\frac{2(\underline{p}-p)-\Delta p}{\kappa}\right)}{2\Phi\left(\frac{p-\underline{p}}{\kappa}\right) - 1}$$

which is defined for $\Delta p > \underline{p}-p$. The associated probability density function is

$$g(\Delta p \mid p) = \frac{\varphi\left(\frac{\Delta p}{\kappa}\right) - \varphi\left(\frac{2(\underline{p}-p)-\Delta p}{\kappa}\right)}{2\kappa\Phi\left(\frac{p-\underline{p}}{\kappa}\right) - \kappa}$$

where $\varphi(z)$ is the standard Normal density function evaluated at z. This implies that the expected value of the earnings announcement price change is

$$E(\Delta p \mid p) = \frac{2(p-\underline{p})\left\{1 - \Phi\left(\frac{p-\underline{p}}{\kappa}\right)\right\}}{2\Phi\left(\frac{p-\underline{p}}{\kappa}\right) - 1}$$

which is positive, and depends on how far the pre-announcement price p is from the reservation price $\underline{p}$, in units of the event period volatility κ.

B. The relation between price change on the event and post-event performance

Proposition 1:

For any initial price density $f(.)$, the density

$$h(p \mid \Delta p) \equiv \frac{g(\Delta p \mid p) f(p)}{\int_m^\infty g(\Delta p \mid \zeta) f(\zeta)\, d\zeta}$$

has the single crossing property (SCP):

$$(\forall \Delta p' > \Delta p)\ (\exists p^\circ)\ (\forall p)$$

$$p < p^\circ \ \Rightarrow\ h(p \mid \Delta p') > h(p \mid \Delta p)\ , \quad and$$

$$p > p^\circ \ \Rightarrow\ h(p \mid \Delta p') < h(p \mid \Delta p)$$

Proof:

To prove the proposition it is sufficient to demonstrate that $\frac{\partial h}{\partial \Delta p}$ is positive for $p < p^\circ$ and negative for $p > p^\circ$. Recall that the conditional density function of Δp is

$$g(\Delta p \mid p) = \frac{\varphi\left(\frac{\Delta p}{\kappa}\right) - \varphi\left(\frac{2(\underline{p} - p) - \Delta p}{\kappa}\right)}{2\kappa\Phi\left(\frac{p - \underline{p}}{\kappa}\right) - \kappa}$$

$$= \frac{1}{\kappa}\left[2\Phi\left(\frac{p-\underline{p}}{\kappa}\right) - 1\right]^{-1} [\varphi(a+x) - \varphi(a-x)]\ , \quad where$$

$$a \equiv \frac{\underline{p} - p}{\kappa} < 0\ , \quad and$$

$$x \equiv \frac{\Delta p}{\kappa} - a$$

$$= \frac{p + \Delta p - \underline{p}}{\kappa} > 0\ .$$

Differentiating with respect to Δp we have

$$\frac{\partial g}{\partial \Delta p} = -\frac{1}{\kappa^2}\left[2\Phi\left(\frac{p-\underline{p}}{\kappa}\right)-1\right]^{-1}[(a+x)\varphi(a+x) + (a-x)\varphi(a-x)]$$

Now,

$$\frac{\partial h}{\partial \Delta p} = \frac{\partial}{\partial \Delta p}\left\{\frac{g(\Delta p \mid p)f(p)}{\int_m^\infty g(\Delta p \mid \zeta)f(\zeta)\,d\zeta}\right\}$$

$$= \left[\int_m^\infty g(\Delta p \mid \zeta)f(\zeta)\,d\zeta\right]^{-2} \times \left\{\int_m^\infty g(\Delta p \mid \zeta)f(\zeta)\,d\zeta\,\frac{\partial g}{\partial \Delta p} f(p) - g\,f(p)\frac{\partial}{\partial \Delta p}\left[\int_m^\infty g(\Delta p \mid \zeta)f(\zeta)\,d\zeta\right]\right\}$$

which has the same sign as $\frac{1}{g}\frac{\partial g}{\partial \Delta p} - C$, where

$$C = \left[\int_m^\infty g(\Delta p \mid \zeta)f(\zeta)\,d\zeta\right]^{-1} \frac{\partial}{\partial \Delta p}\left[\int_m^\infty g(\Delta p \mid \zeta)f(\zeta)\,d\zeta\right],$$

a constant independent of p. Now

$$\frac{1}{g}\frac{\partial g}{\partial \Delta p} = -\frac{1}{\kappa}\,\frac{(a+x)\varphi(a+x) + (a-x)\varphi(a-x)}{\varphi(a+x) - \varphi(a-x)}.$$

It is easily verified that the denominator has the same sign as g and is positive. Similarly, it is easily verified that the function $z\varphi(z)$ is symmetric through the origin,

$$(-z)\varphi(-z) = -z\varphi(z),$$

that it increases on $[-1,1]$, decreases outside that interval and that $z\varphi(z) \Rightarrow 0$ as $|z| \Rightarrow 0$. Figure A.1 graphs this function.

$$\frac{\partial}{\partial p}\left\{\frac{1}{g}\frac{\partial g}{\partial \Delta p}\right\} = -\frac{1}{\kappa}\frac{\partial}{\partial p}\left\{\frac{(a+x)\varphi(a+x) + (a-x)\varphi(a-x)}{\varphi(a+x) - \varphi(a-x)}\right\}$$

It is now possible to see that the numerator $(a+x)\varphi(a+x) + (a-x)\varphi(a-x)$ is negative for x in a neighborhood of zero $(x > 0)$, becomes zero at some x and then stays positive for x above that point. This verifies that g has the SCP, but to show that $\frac{1}{g}\frac{\partial g}{\partial \Delta p} - C$ has the SCP we will show that $\frac{1}{g}\frac{\partial g}{\partial \Delta p}$ declines monotonically in p:

$$\frac{\partial}{\partial p}\left\{\frac{1}{g}\frac{\partial g}{\partial \Delta p}\right\} = -\frac{1}{\kappa}\frac{\partial}{\partial p}\left\{\frac{(a+x)\varphi(a+x) + (a-x)\varphi(a-x)}{\varphi(a+x) - \varphi(a-x)}\right\}$$

$$= \frac{2e^{(a+x)^2}\gamma(a,x)}{\left\{e^{(a+x)^2/2} - e^{(a-x)^2/2}\right\}^2\kappa^2}$$

$$< 0 \ , \quad where$$

$$\gamma(a,x) \equiv 1 - e^{2ax} - 2a^2 + 2ax$$

The inequality follows since $\gamma(a,x)$ is equal to zero for a equal to zero, and is monotonically increasing in a for $a < 0,\ x > 0$. ■

Proposition 2:

Since $h(p \mid \Delta p)$ has the SCP,

$$(\forall x) \qquad H(x \mid \Delta p) \equiv Pr(p \leq x \mid \Delta p)$$

is an increasing function of Δp.

Proof:

$$H(x \mid \Delta p) = \int_{\underline{p}}^{x} h(p \mid \Delta p)\ dp \ ,$$

and, from the SCP, raising Δp raises $h(\cdot \mid \Delta p)$ for $p < p^{\circ}$ and lowers it for $p > p^{\circ}$. Since h is positive and integrates to unity, H increases in Δp as required. ■

Thus, raising Δp cross-sectionally on an earnings announcement date increases the probability that p on that date is closer to the reservation price $\underline{p}$. This, in turn raises the probability and the expectation that Δp will be positive on subsequent dates.

Proposition 3:

Let τ be an earnings announcement date and $\tau' > \tau$ represent a subsequent earnings announcement date.

$$(\forall x) \qquad Pr(\Delta p_{\tau'} \geq x \mid \Delta p_\tau) \quad and$$

$$E(\Delta p_{\tau'} \mid \Delta p_\tau)$$

are both increasing functions of Δp_τ.

Proof:

Let $f(\cdot \mid p)$ denote the density function for $\Delta p_{\tau'}$ conditional on $p_\tau = p$. From the stochastic process we know that p_τ is a sufficient statistic for the information set at date τ. Furthermore, from the SCP on $f(\cdot)$ it is clear that the upper distribution for $\Delta p_{\tau'}$

$$G(x \mid p) = \int_x^{\infty} f(\zeta \mid p)\, d\zeta$$

increases as p declines. Hence

$$Pr(\Delta p_{\tau'} \geq x \mid \Delta p_\tau) = \int_{\underline{p}} G(x \mid p) h(p \mid \Delta p)\, dp\ ,$$

and since g is a declining function of p and $h(p \mid \Delta p)$ has the SCP, it follows that increasing Δp_τ increases the value of the integral.

Similarly, letting

$$P(x \mid \Delta p_\tau) \equiv Pr(\Delta p_{\tau'} \geq x \mid \Delta p_\tau)$$

we have

$$E(\Delta p_{\tau'} \mid \Delta p_\tau) = -\int_0^{\infty} x\, dP(x \mid \Delta p_\tau)$$

$$= -xP(x \mid \Delta p_\tau)\Big|_0^{\infty} + \int_0^{\infty} P(x \mid \Delta p_\tau)\, dx$$

$$= \int_0^{\infty} P(x \mid \Delta p_\tau)\, dx$$

which increases with increases in Δp_τ. ■

The above propositions verify what we were seeking; we now know that an increase in Δp_τ on an earnings announcement date τ implies a first order stochastic increase in $\Delta p_{\tau'}$ on any subsequent announcement date

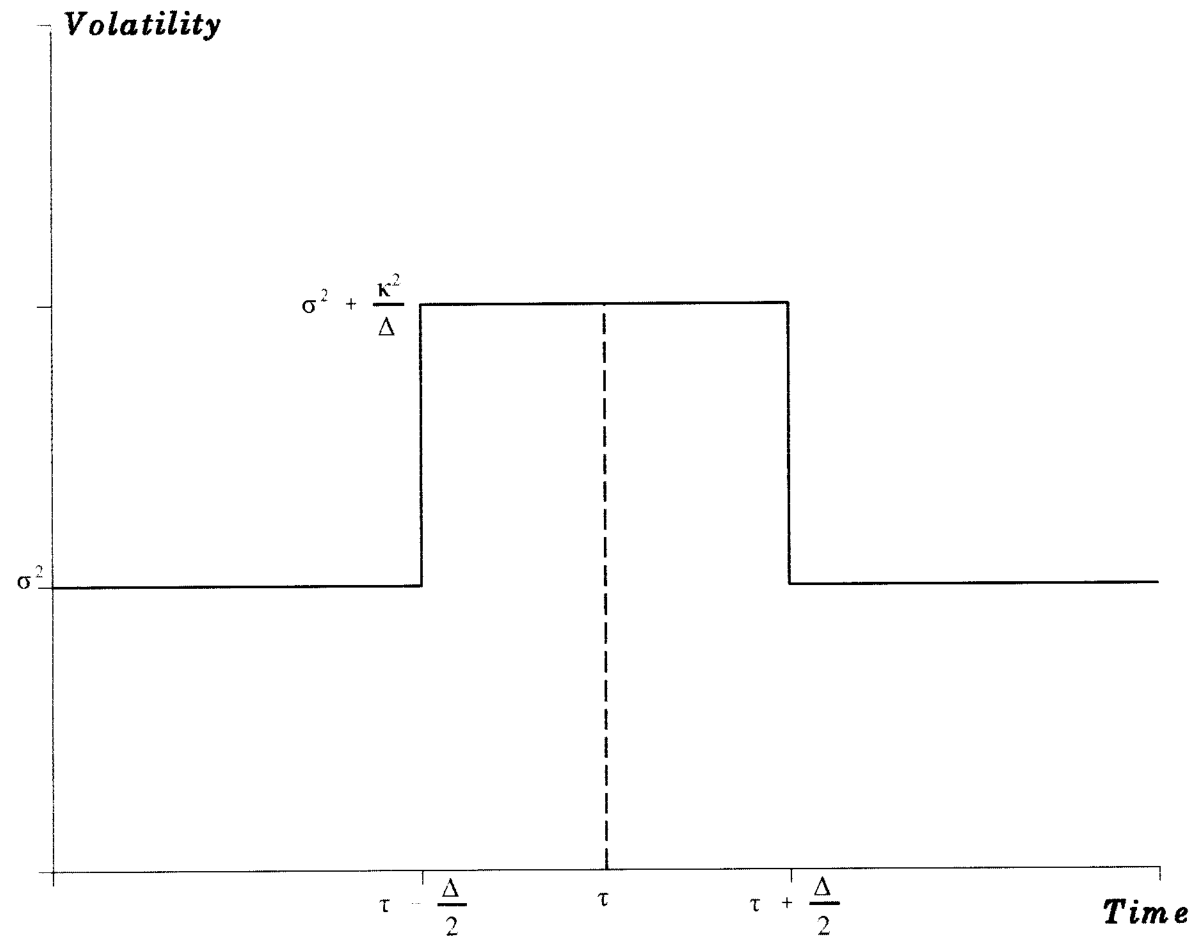

Figure 1: Volatility increase around earnings announcement

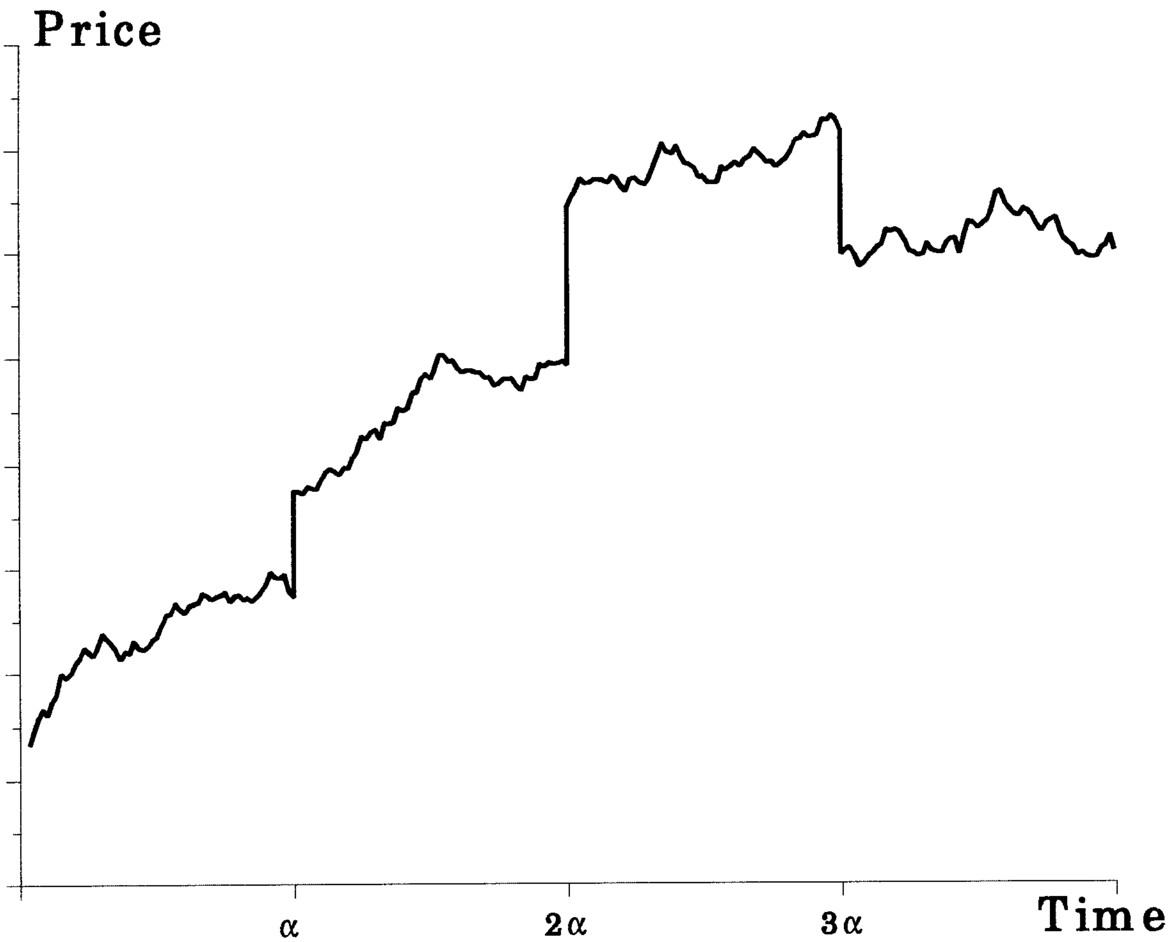

Figure 2. Sample price path with earnings announcements at dates α, 2α, and 3α.

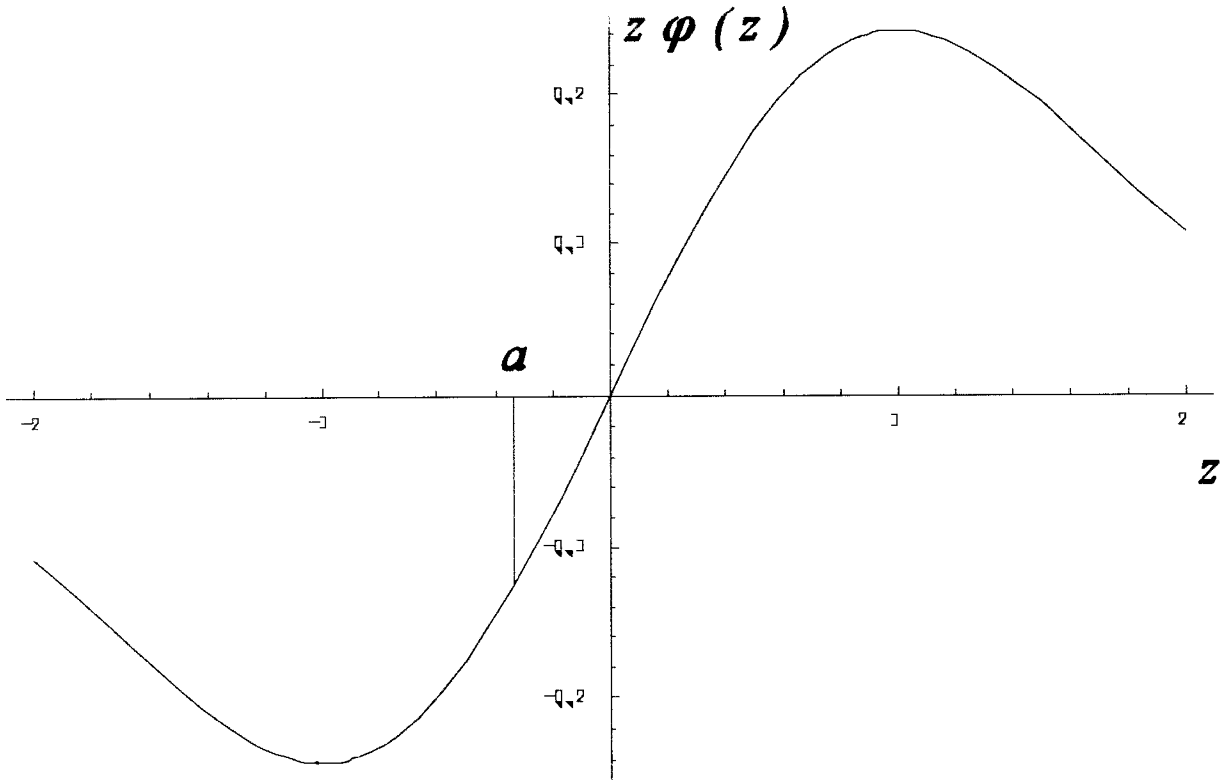

Figure A.1

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