Credit Risk Analysis and Security Design*

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June 2004

*We are especially indebted to an anonymous referee whose insightful comments have substantially improved the paper. Thanks also to Andres Almazan, Jacqes Cremer, Darrel Duffie, Colin Mayer, Enrico Perotti, Patrick Rey, Jean-Charles Rochet, Tony Saunders, Raghu Sundaram, Elu von Thadden (our WFA discussant), Jean Tirole, Jeff Wurgler, and seminar participants at NYU, Yale, Northwestern, Rochester, UNC Chapel Hill, INSEAD, Wisconsin-Madison, LBS, LSE, Toulouse, Amsterdam, Frankfurt, Humboldt, the WFA Meetings in Los Cabos (2003), the Oxford Finance Summer Symposium (2002), and the European Summer Symposium in Financial Markets (ESSFM) in Gerzensee (2002) for helpful comments. Inderst acknowledges financial support from the Financial Markets Group (FMG).

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We consider the security design problem of a lender who can assess the borrower's project prior to making an accept or reject decision. The lender's subjective assessment is represented by a private signal. Unless the lender extracts the full suplus from the project, her cutoff signal above which she is willing to accept the project is inefficiently high, i.e., the lender is too conservative. The unique optimal security is standard debt. Debt maximizes the lender's payoff from financing bad—i.e., low-signal—projects, thus implementing a lower cutoff signal than other securities. While the lender could, in principle, make the loan terms indirectly contingent on the signal by choosing a security from a prespecified menu, such ex-post fine-tuning is generally not optimal. Rather, it is optimal to either grant credit at standardized terms or not at all. Our model suggests a natural segmentation among lenders, whereby inside (i.e., local or relationship) lenders attract low-NPV borrowers while arm's-length lenders attract high-NPV borrowers.

1 Introduction

Technological progress notwithstanding, human judgement remains a key factor in credit decisions: "[T]he credit decision is left to the local or branch lending officer or relationship manager. Implicitly, this person's expertise, subjective judgement, and his weighting of certain key factors are the most important determinants in the decision to grant credit" (Saunders and Allen (2002)). In this paper, we consider the security design problem of a lender who evaluates the borrower's project prior to granting credit. In line we the above quote, we assume that the lender's assessment is subjective, which implies the credit decision is fully discretionary. Hence, it is the lender, and only the lender, who decides whether credit is granted.[1]

The structure of our model is simple. The lender and borrower initially agree on a security. The lender then scrutinizes the borrower's project, which generates a subjective—and therefore private—signal about the project's cash-flow distribution. High signals are good news in the sense of the Monotone Likelihood Ratio Property (MLRP). Based on the signal, the lender either accepts or rejects the borrower. The lender's incentives to accept the project depend on the value of her claims, and hence on the security in place. The optimal accept or reject decision follows a cutoff rule: accept if and only if the signal is above a certain cutoff signal. As we show, the lender is generally too conservative: unless she extracts the full surplus from the project, her privately optimal cutoff signal is strictly above the first-best cutoff signal. There thus exists a range of signals at which positive-NPV projects are rejected.

The unique optimal security in our model is standard debt. To satisfy the borrower's participation constraint, the lender must leave the borrower a positive expected payoff. Debt shifts all of the borrower's payoffs into high cash-flow states, thus maximizing the lender's payoffs in low cash-flow states. Given the positive relation between cash flows and signals due to MRLP, debt consequently maximizes the lender's expected payoffs at low signals. Accordingly, debt minimizes the lender's cutoff signal, and hence her excessive conservativism.

In principle, the lender could make the loan terms (indirectly) contingent on her signal by selecting a security from a prespecified menu. We show that the unique optimal menu consists of a single security, namely, debt.[2] If the lender offers a nontrivial menu, she will always select

[1] This is in contrast to models in which borrowers have private information. In such models, the lender typically offers a menu of contracts from which the borrower chooses. The final decision is thus made by the borrower.

[2] This is true in our base model with fixed investment size. In an extension of our model with signal-dependent

the security that is ex-post optimal for her. This "self-dealing" problem undermines the lender's commitment to leave the borrower a high expected payoff at high signals, which is necessary to satisfy the borrower's participation constraint while maximizing her own expected payoff at low signals. In other words, it is not optimal to fine-tune the loan terms after observing the signal. Instead, it is optimal to specify the loan terms ex ante and then accept or reject the borrower on the basis of these terms. This might help explain the use of quantity rationing in conjunction with standardized loan terms found in retail lending: "[L]oan decisions made for many types of retail loans are reject or accept decisions. All borrowers who are accepted are often charged the same rate of interest and by implication the same risk premium. [...] In the terminology of finance, retail customers are more likely to be sorted or rationed by loan quantity restrictions rather than by price or interest rate differences" (Saunders and Thomas (2001)).

While our argument suggests that loan terms might be insensitive with respect to interim information, it is different from intertemporal credit smoothing arguments. In our model, the loan terms do not depend on subjective or private information. They do, however, incorporate all publicly available information. By contrast, in the credit smoothing literature loan terms are insensitive with respect to both private and public information.

Our model offers a new argument for the optimality of debt based on the notion that debt minimizes lenders' excessive conservativism. This argument is evidently different from costly state verification models (Townsend (1979), Gale and Hellwig (1985)) and models with non-verifiable cash flow (Bolton and Scharfstein (1990), Hart and Moore (1998), DeMarzo and Fishman (2000)). Also, unlike in Allen and Gale (1988), risk-sharing considerations play no role in our model. Finally, in Innes' (1990) model the borrower must be incentivized to work hard. By contrast, in our model the incentive problem resides with the lender, and it is a problem of (interim) private information, not moral hazard.

In Nachman and Noe (1994), DeMarzo and Duffie (1999), and Biais and Mariotti (2003), the borrower is privately informed either before or after the security design. By contrast, in our model it is the lender who has private information. And yet, in both cases the lender receives debt. Moreover, in both cases the optimality of debt derives from the same property: it maximizes the lender's return from financing low-type projects. What is different is *why* this property implies optimality. In models of borrower private information, debt minimizes the

optimal investment sizes, the optimal menu is a (nontrivial) menu of debt contracts.

sensitivity of the lender's return with respect to the borrower's type—and thus the underpricing of high types (Myers and Majluf (1984)). In our model, by contrast, debt maximizes the set of borrower types for which the lender breaks even.

Axelson (2002), Boot and Thakor (1993), and Fulghieri and Lukin (2001) also consider incentive problems on the part of investors. Axelson examines an auction model in which a seller auctions off asset-backed securities to privately informed investors.[3] The optimal security trades off the expected underpricing against the liquidity costs of retaining cash flow. Boot and Thakor and Fulghieri and Lukin both consider costly information acquisition. To make information acquisition attractive for investors, firms issue information-sensitive claims like equity.

An important element of our model is the borrower's participation constraint. To endogenize this constraint, we embed our model in a competitive credit market where an "inside" (i.e., relationship or local) lender with superior but soft information competes with a less well-informed credit market.[4] If the project's NPV based on public information is small or negative, the insider lender can successfully compete with the market as her informational advantage allows her to weed out bad projects. If the NPV based on public information is large, however, the inside lender cannot compete: as the lender's information is soft, she inevitably captures an informational rent that prevents her from undercutting the market offer. In the end, there is a natural market segmentation among lenders, whereby inside and arm's-length lenders coexist by catering to different borrower clienteles.

The rest of this paper is organized as follows. Section 2 lays out the model. Section 3 contains all our main results: (i) the lender is too conservative, (ii) the optimal security is debt, and (iii) a menu of contracts is generally not optimal. Section 4 embeds our model in a competitive credit market. Section 5 discusses robusteness issues, such as ex-ante negotiations, interim renegotiations, the introduction of an additional interim constraint, and a more general investment technology. Section 6 concludes. All proofs are in the Appendix.

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[3] See Garmaise (2001) for a related setting.

[4] See Sharpe (1990), Rajan (1992), von Thadden (2001), and Hauswald and Marquez (2003) for related settings. In Boot and Thakor (2000), lenders can choose between relationship and transaction lending.

2 The Model

2.1 Project Technology and Credit Analysis

A penniless entrepreneur ("the borrower") has a project that requires an investment outlay k. The project's cash flow x is stochastic with support $X := [\underline{x}, \overline{x}]$, where $0 \leq \underline{x} < k$, and where $\overline{x}$ is either finite or infinite. In our base model, we assume that there is a single lender. In Section 4, we embed our model in a competitive credit market.

Prior to financing the investment, the lender performs a credit analysis. Based on the credit analysis, the lender forms subjective beliefs about the project's profitability. These beliefs can be represented by a signal $s \in [0, 1]$. As the lender's beliefs are subjective, we assume that the signal is private information.[5] The signal is drawn from the absolutely continuous distribution function $F(s)$ with $F(0) = 0$, which is common knowledge. We assume that $F(s)$ has positive density $f(s)$ everywhere in $(0, 1)$. Each signal is associated with a conditional distribution function over cash flows $G_s(x)$. We assume that $G_s(x)$ is absolutely continuous in x with $G_s(\underline{x}) = 0$ and density $g_s(x) > 0$ for all $x \in X$. Moreover, $g_s(x)$ is continuous in s for all $x \in X$. The expected project cash flow given signal s is $\mu_s := \int_X x g_s(x)dx$.

Observing a high signal is good news. Precisely, high signals put more probability mass on high cash flows in the sense of the Monotone Likelihood Ratio Property (MLRP).[6]

Assumption 1. *For any pair $(s, s') \in [0, 1]$ with $s' > s$, the ratio $g_{s'}(x)/g_s(x)$ is strictly increasing in x for all $x \in X$.*

MLRP is a common assumption in contracting models and satisfied by many standard distributions (Milgrom (1981)). To rule out trivial situations where the project's NPV is either always positive or negative, we assume that $\mu_0 < k$ and $\mu_1 > k$.

We can think of at least two inefficiencies associated with the credit analysis: (i) the lender misclassifies bad projects as good ones and vice versa, and (ii) she devotes too little effort to the analysis. The second inefficiency has been studied previously (e.g., Manove, Padilla, and

[5] A canonical example is a relationship lender with soft information about a borrower.

[6] The fact that the signal is informative suggests that the lender has expertise in evaluating projects, e.g., from having granted similar loans before (Boot and Thakor (2000)). "As a result, banks are likely to be more knowledgeable about some aspects of project quality than many of the entrepreneurs they lend to ... This is why banks are, and should be, in the project-evaluation business" (Manove, Padilla, and Pagano (2001)).

Pagano (2001), Fulghieri and Lukin (2001)). To focus on the first inefficiency, we assume that the informativeness of the signal is fixed.

2.2 Lending Process

The sequence of events is as follows. At $\tau = 0$ the lender offers a menu of contracts $T := \{t_i\}_{i \in I}$, where I is some index set.[7] A contract $t_i = t_i(x)$ in the menu specifies a repayment out of the project's cash flow. As the lender's signal is private, contracts cannot directly condition on the signal. However, the lender can make them indirectly contingent on the signal by offering a menu under which she (optimally) selects different contracts at different signals.

In practice, do lenders tell loan applicants "this is what you can expect if your loan gets approved"? At least for certain types of loans this appears to be the case. At Chase Manhattan, for instance, a major small business lender in the United States, applicants for small business loans are shown a pricing chart explaining—depending on verifiable loan characteristics such as size and maturity—what interest rate they will get if their loan is approved.[8]

While convenient, we do not need to assume that the lender makes a take-it-or-leave-it offer, however. Given that we solve for contracts (or menus) that are Pareto optimal, we would expect that the borrower and lender also choose such a contract if they bargain ex ante, in particular as there is no asymmetric information at $\tau = 0$. Also, we may allow that the borrower and lender renegotiate the initial offer after the lender has observed the signal, possibly replacing it with an entirely different contract or menu.[9] Ex-ante bargaining and interim renegotiations are considered in Sections 4.3 and 5.1, respectively.

At $\tau = 1$ the lender performs the credit analysis. Based on the resulting signal, the lender either accepts or rejects the borrower. If the lender accepts, she selects a contract from the prespecified menu and finances the investment. Cash flows are then realized at $\tau = 2$. If the

[7] This setting follows Maskin and Tirole (1992).

[8] A copy of such a pricing chart is available from the authors. The interest rate is defined in terms of spread over prime, which implies it changes on a daily basis. Like any (contract) offer, the extent to which the offer is credible depends on the offeror's reputation and the ability to enforce it. The loan officer we spoke to said that, in case the loan is rejected, the original offer will not be adjusted. His argument was that the profit margin on such loans is so small that it does not pay to haggle with rejected applicants.

[9] Renegotiation implies a mutually beneficial change of loan terms. It does not imply that the lender can unilaterally renege on her original offer.

lender rejects, the project is not financed. This latter assumption is endogenized in Section 4.

We make the standard assumption that the repayment $t_i(x)$ be nondecreasing in x. See see Innes (1990) and DeMarzo and Duffie (1999) for a motivation and further details.

Assumption 2. *The contract* $t_i(x)$ *is nondecreasing.*

The constraint that $t_i(x)$ be nondecreasing is binding at the optimum. Section 3.3 provides a brief discussion of what the optimal contract might be in the absence of this constraint.

We assume that the menu must provide the borrower with an expected payoff of $\overline{V} \geq 0$. For the most part of our analysis, we take $\overline{V}$ as given and solve the model for all (feasible) values of $\overline{V}$, hence tracing out the entire Pareto frontier of optimal contracts (or menus, respectively). In Section 4, we show how $\overline{V}$ might arise naturally from competition in the credit market.

We finally assume that only accepted borrowers can receive a payment from the lender. In particular, this rules out the possibility that the lender "buys the project" *before* performing the credit analysis. If upfront payments were possible, the first best could be attained without any implications for the security design: as the purchase price is sunk, the lender has first-best incentives to make a socially efficient credit decision. The standard argument for ruling out upfront payments is that they might attract "fly-by-night operators" (Rajan (1992), von Thadden (1995), Hellmann (2002)).[10] Rather than formally introducing such fly-by-night operators, we assume that only accepted borrowers can receive a payment from the lender.

3 Optimal Credit Decision and Security Design

3.1 The Lender's Problem

Instead of solving the lender's original problem, it is convenient to solve a restricted problem and show that its solution uniquely solves the lender's original problem.

The *lender's original problem* is to choose a menu of contracts $T := \{t_i\}_{i \in I}$ while taking into account the effect of this menu on the subsequent credit decision. The lender's optimal credit

[10] Fly-by-night operators are crooks who have no real project. While indistinguishable from true entrepreneurs ex ante, their identity is revealed during the credit analysis. This can be formalized as follows: crooks generate a signal $s = 0$ with certainty while true entrepreneurs generate a signal according to the distribution $F(s)$. Since crooks are singled out for sure at $\tau = 1$, their (potential) presence does not affect the lender's credit decision or the optimal security design. A positive upfront payment, however, would attract all crooks. If there is a potentially large pool of crooks, the lender's expected payoff would become negative.

decision is to accept the borrower if and only if the lender's payoff at the observed signal s is positive for *some* contract in the menu.[11] If the lender accepts, she selects the contract from the menu that yields her the highest expected payoff. The lender's expected gross payoff under the contract t_i and signal s is denoted by $u_s(t_i) := \int_X t_i(x) g_s(x) dx$.

The lender's expected gross payoff under the menu T and signal s is $U_s(T) := \max_{t_i \in T} u_s(t_i)$, while her net payoff is $U_s(T) - k$. To simplify the notation, we denote by $\Omega(T) \subseteq [0,1]$ the set of signals for which the lender accepts under the menu T. We refer to $\Omega(T)$ as the lender's acceptance set.

The lender's original problem is as follows. At time $\tau = 0$ the lender chooses a menu T to maximize her expected payoff

$$U(T) := \int_{\Omega(T)} [U_s(T) - k] f(s) ds,$$

where $\Omega(T) := \{s \in [0,1] | \, U_s(T) - k > 0\}$, subject to the constraint that the borrower receives at least $\overline{V}$ in expectation,

$$V(T) := \int_{\Omega(T)} [\mu_s - U_s(T)] f(s) ds \geq \overline{V}, \tag{1}$$

and the requirement from Assumption 2 that $t_i(x)$ be nondecreasing.

The *lender's restricted problem* is identical to her original problem, except that the menu T is replaced with a single contract. For convenience, we use the notation $t = t(x)$ instead of $t_i(x)$ when considering the lender's restricted problem. The optimization problem is the same as above, except that $T = \{t\}$. Hence, the lender chooses a contract $t = t(x)$ to maximize

$$U(t) = \int_{\Omega(t)} [u_s(t) - k] f(s) ds,$$

where $\Omega(t) := \{s \in [0,1] | \, u_s(t) - k > 0\}$, subject to

$$V(t) = \int_{\Omega(t)} [\mu_s - u_s(t)] f(s) ds \geq \overline{V},$$

and the constraint that $t(x)$ be nondecreasing.

The lender's restricted problem is solved in Sections 3.2-3.3. Section 3.4 shows that the solution to this problem uniquely solves the lender's original problem.

[11] If the lender is indifferent, we assume she rejects. Since this is a zero-probability even, the assumption is without loss of generality.

3.2 Optimal Credit Decision

We first consider the lender's optimal credit decision at $\tau = 1$ given some contract t in place. The result is a characterization of the lender's acceptance set $\Omega(t)$. Section 3.3 then considers the lender's optimal contract choice at $\tau = 0$, while taking into account the effect of t on the lender's credit decision.

As a benchmark, let us characterize the socially optimal—or first-best—credit decision. The first-best decision is to accept the project if and only if the NPV conditional on s, $\mu_s - k$, is positive. Given MLRP (Assumption 1) and continuity of $g_s(x)$ in s, the conditional expected cash flow μ_s is continuous and strictly increasing in s. Since $\mu_0 < k$ and $\mu_1 > k$, the first-best credit decision is characterized by a simple cutoff rule:

Lemma 1. *The first-best credit decision is to accept if $s > s_{FB}$ and to reject if $s \leq s_{FB}$. The first-best cutoff signal s_{FB} is unique, satisfies $s_{FB} \in (0,1)$, and is given by $\mu_{s_{FB}} = k$.*

At the optimal cutoff signal s_{FB} the expected project cash flow conditional on s, μ_s, equals the investment cost k. If $s < s_{FB}$ the expected project cash flow is less than k; if $s > s_{FB}$ it exceeds k. Hence, the first-best credit decision is the NPV rule, which prescribes to accept the project if and only if the project's NPV conditional on the signal is positive.[12]

We next consider the lender's privately optimal credit decision. The lender accepts the project if and only if her expected payoff at the given signal, $u_s(t) - k$, is positive. Like the first-best decision, the lender's privately optimal decision is given by a cutoff rule: accept the project if and only if $s > s^*$, where $s^* = s^*(t)$ is the lender's optimal cutoff signal.

Lemma 2. *The lender's optimal credit decision is to accept if $s > s^*$ and to reject if $s \leq s^*$. If $u_1(t) \leq k$ the lender's cutoff signal is $s^* = 1$, while if $u_1(t) > k$ the lender's cutoff signal is unique, satisfies $s^* \in (0,1)$, and is given by $u_{s^*}(t) = k$.*

Proof. See Appendix.

If $s > s^*$ the lender makes a profit, while if $s < s^*$ she makes a loss, which implies she optimally rejects. Hence, the lender's acceptance set $\Omega(t)$ is an interval: $\Omega(t) = (s^*, 1]$. By Lemma 1, the first-best acceptance set is also an interval, namely, $\Omega_{FB} = (s_{FB}, 1]$.

[12] If the NPV is exactly zero, we specify that the project be rejected. Since $s = s_{FB}$ is a zero-probability event, this assumption is without loss of generality.

We next establish whether the lender's privately optimal cutoff signal s^* lies above or below the first-best cutoff signal s_{FB}. If $u_1(t) \leq k$ the lender's cutoff signal is $s^* = 1$, which is evidently above the first-best cutoff signal. If $u_1(t) > k$, on the other hand, the lender's cutoff signal is given by $u_{s^*}(t) = k$. Since $t(x) \leq x$, we have that

$$u_{s_{FB}}(t) = \int_X t(x) g_{s_{FB}}(x) dx \leq \int_X x g_{s_{FB}}(x) dx = \mu_{s_{FB}} = k,$$

with strict inequality if and only if $t(x) < x$ on sets of positive measure. Accordingly, if $t(x) = x$ the lender (just) breaks even at $s = s_{FB}$, implying that $s^* = s_{FB}$. On the other hand, if $t(x) < x$ the lender does not break even at $s = s_{FB}$. As—by Assumptions 1 and 2 and continuity of $g_s(x)$—the lender's conditional expected payoff $u_s(t) - k$ is strictly increasing in s, her optimal cutoff signal s^* must consequently lie strictly above s_{FB}, i.e., $s^* > s_{FB}$. We thus have

Proposition 1. *Unless the lender can extract the full surplus, her credit decision is too conservative, i.e., her cutoff signal s^* lies strictly above the first-best cutoff signal s_{FB}.*

At marginal signals $s \in [s_{FB}, s^*)$, the lender's conditional expected payoff is negative. To implement the social optimum, one would have to force the lender to occasionally finance a project under which she does not break even.

The argument that the lender is too conservative is fairly general and does not hinge on Assumptions 1 and 2. In particular, it does not hinge on the assumption that $t(x)$ be nondecreasing. This follows from the fact that $u_s(t) < \mu_s$ for all $s \leq 1$ if $t(x) < x$ on sets of positive measure. Hence, if at some signal $s = \hat{s}$ it holds that $u_{\hat{s}}(t) > k$, it must also hold that $\mu_{\hat{s}} > k$, but not vice versa. In words: if the lender's optimal credit decision prescribes to accept the project, the first-best credit decision also prescribes to accept. The reverse is not true, however. Hence, even without Assumptions 1 and 2, the lender is too conservative in the sense that her acceptance set $\Omega(t)$ is strictly smaller than the first-best acceptance set Ω_{FB}. Assumptions 1 and 2 merely ensure that the acceptance sets are connected, in which case both the first-best credit decision and the lender's privately optimal credit decision are characterized by simple cutoff rules.[13] "Being too conservative" then has the intuitive interpretation that the lender uses too high a cutoff signal.

[13] While the result that the lender is too conservative holds independently of Assumptions 1 and 2, these assumptions are crucial for the optimal security design in Section 3.3, however.

3.3 Optimal Security Design

We now solve the lender's optimal security design problem at $\tau = 0$. Given Lemma 2, the lender's restricted problem reduces to maximizing

$$U(t) = \int_{s^*(t)}^{1} [u_s(t) - k] f(s) ds, \tag{2}$$

subject to

$$V(t) = \int_{s^*(t)}^{1} [\mu_s - u_s(t)] f(s) ds \geq \overline{V}, \tag{3}$$

and the constraint that $t(x)$ be nondecreasing.

By standard arguments, (3) must bind at the optimum. Substituting the binding constraint into the lender's objective function (2), the latter becomes

$$U(t) = \int_{s^*(t)}^{1} [\mu_s - k] f(s) ds - \overline{V}. \tag{4}$$

By inspection, the lender is the residual claimant to any surplus in excess of $\overline{V}$. Moreover, the only way in which $t(x)$ affects the lender's expected payoff is via its impact on the optimal cutoff signal $s^*(t)$. By implication, the lender will therefore design a contract that implements as low as possible a cutoff signal, and thus as efficient as possible a credit decision.

There are two cases. If $\overline{V} = 0$, the lender obtains the full surplus. The optimal contract then trivially has $t(x) = x$, and the credit decision is first-best efficient. If $\overline{V} > 0$, on the other hand, it holds that $t(x) < x$ on sets of positive measure. By Proposition 1, the credit decision is then inefficient. As a claimant to the residual surplus, the lender chooses $t = t(x)$ to minimize this inefficiency. Accordingly, she chooses the contract that implements the lowest feasible cutoff signal $s^*(t)$.

Clearly, if $\overline{V}$ is too large the lender cannot break even. In all other cases, a nontrivial contract under which the borrower is accepted with positive probability exists. In the following, we assume that $\overline{V}$ is sufficiently small in the above sense. (We consider the case where $\overline{V}$ is "too large" in our competition model in Section 4.) We obtain the following result.

Proposition 2. *The unique optimal security is standard debt. Precisely, there exists a unique repayment $R^* = R^*(\overline{V}) > k$ such that the unique optimal security is $t^*(x) = \min\{x, R^*\}$.*

If $\overline{V} = 0$ the optimal repayment is $R^* = \overline{x}$, which implies the lender's credit decision is first-best optimal. Conversely, if $\overline{V} > 0$ the optimal repayment satisfies $R^* < \overline{x}$, which implies the lender's cutoff signal s^* lies strictly above the first-best cutoff signal s_{FB}.

Proof. See Appendix.

If $\overline{V} = 0$, the optimal contract can be either interpreted as 100 percent equity or debt with face value $R = \overline{x}$. If $\overline{V} > 0$, on the other hand, the uniquely optimal security is debt. The intuition is simple. To satisfy the borrower's ex-ante participation constraint (3), the lender must leave him a positive expected payoff.[14] Debt shifts all of the borrower's payoff into high cash-flow states, thus maximizing the lender's payoff in low cash-flow states. Given the positive relation between cash flows and signals (by MRLP), debt maximizes the lender's expected payoff at low signals. Debt consequently implements a lower cutoff signal $s^*(t)$ than any other security, thus minimizing the lender's excessive conservativism.

The proof of Proposition 2 points to an easy-to-make mistake when thinking about possible alternative solution candidates. It is easy to find a contract that—holding the cutoff signal s^* fixed—yields both the lender and the borrower the same expected payoff as the optimal debt contract. The (incorrect) conclusion is that debt is not uniquely optimal. What is incorrect about this conclusion is that any such contract will actually implement a higher cutoff signal than debt. Hence, the thought exercise of holding s^* fixed is illegitimate.

Let us finally comment on what the optimal contract might look like in the absence of Assumption 2. In the absence of a monotonicity constraint, a "live-or-die" (LD) contract $t_{LD}(x) = x$ if $x \leq \widetilde{x}$ and $t_{LD}(x) = 0$ if $x > \widetilde{x}$ provides the lender with a higher expected payoff than debt at low signals. The flip side is that it provides the lender with a lower expected payoff at high signals. If the lender's expected payoff $u_s(t_{LD}) - k$ remains positive at high signals, standard arguments can be used to show that LD is uniquely optimal. On the other hand, if $u_s(t_{LD}) - k$ turns negative at high signals, LD—while maximizing the acceptance probability at low signals—makes the lender reject the project at high signals. The optimal nonmonotonic contract then depends on the underlying probability distributions.

3.4 Menu of Contracts

We finally show that it is suboptimal to offer a menu of contracts, which implies the solution to the lender's restricted problem constitutes the unique solution to her original problem.

Offering a menu creates a "self-dealing" problem: if the lender offered a menu, she would

[14] This highlights the importance of the ex-ante participation constraint (3), which is critical for our debt result. See Section 4 for an endogenization of this constraint and Section 5.2 for a further discussion of this issue.

always pick the contract that is ex-post optimal for her. In particular, at high signals she would not pick the optimal debt contract t^* but some other contract that offers her a greater fraction of high cash flows, e.g., a contract that is more "equity-like". To ensure that the borrower obtains $\overline{V}$ in expectation, the menu must consequently also include contracts that provide the lender with a relatively low payoff at low signals (and that are actually chosen by the lender at these signals). But this implies that the menu must implement a higher cutoff signal than the (single) optimal debt contract t^*. By restricting her choice to a single contract—namely, the optimal debt contract—the lender can thus commit to a lower cutoff signal. This maximizes the acceptance probability, which is the most efficient way to satisfy the borrower's participation constraint (3).

Proposition 3. *The unique optimal menu consists of a single contract*: *the optimal debt contract from Proposition 2. Any other menu either violates the borrower's participation constraint or implements a higher cutoff signal.*

Proof. See Appendix.

Proposition 3 shows that it is not optimal to ex-post fine tune the loan terms by selecting different contracts at different signals. Rather, it is optimal to offer a single contract and either accept or reject the borrower on the basis of this contract. While this implies that loan terms are insensitive with respect to interim information, it does not imply that they are insensitive with respect to project risk in general. Precisely, borrowers with different prior distributions $F(s)$—and hence different ex-ante expected cash flows—naturally obtain different loan terms. Similarly, the optimal contract varies with respect to investment size and other ex-ante characteristics. Moreover, such ex-ante characteristics affect the optimal contract both directly as well as indirectly via their impact on the borrower's reservation utility $\overline{V}$. In Section 4, for instance, we show that in a competitive credit market $\overline{V}$ depends on the project's ex-ante expected cash flow.

Proposition 3 suggests that, once borrowers are grouped into different categories, any further discrimination based on interim information (here: the signal s) is crude and comes in the form of an accept or reject decision. This is consistent with the notion that in retail lending accepted borrowers are commonly either granted credit at prespecified terms or not at all (see Introduction). It is also consistent with Petersen and Rajan's (1994) finding that the rate charged on small-business loans is generally insensitive with respect to measures of interim information about a borrower, while the availability of credit is sensitive with respect to such measures.

Let us conclude by pointing out that the optimality of a single contract depends on a number of (restrictive) assumptions, most notably the assumption that the investment size k is fixed. If the optimal investment size varies with the lender's signal, for instance, a single contract might not be optimal. For a formal analysis of this case, see Section 5.3.

4 Credit Market Competition

4.1 Relationship vs. Arm's-Length Lending

To endogenize the borrower's reservation utility $\overline{V}$—and thus his ex-ante participation constraint (3)—we now embed our model in a competitive credit market environment. There are two types of lenders. There is an "inside lender" ("the lender") who observes a private signal s in addition to publicly available information. Additionally, there is a competitive credit market ("the market") without access to the private signal. While stylized, we believe this setting captures some key aspects of real-world lending situations, especially small business lending.[15]

The literature offers two arguments as to what makes up an inside lender: existing lending relationships and proximity to the borrower.[16] Lending relationships might give lenders better access to soft information, e.g., through personal contacts with management and employees. Relationship lenders also interact with borrowers through other channels, e.g., borrowers often maintain checking and savings accounts with their lender, or the lender factors the borrower's accounts receivables. Such additional channels "increase the precision of the lender's information about the borrower" (Petersen and Rajan (1994)). In this spirit, Sharpe (1990), Rajan (1992), and von Thadden (2001)—like this paper—all consider a single relationship lender with a private signal about the borrower's quality who competes with an uninformed credit market.[17] As for supporting evidence, Petersen and Rajan (1994) find that 95 percent of the smallest firms in their (small business loan) sample borrow from a single bank. Across their entire sample, only 18 percent borrow from more than one bank.

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[15] Besides, having a single informed lender greatly simplifies our analysis. If there are multiple informed lenders, each with a private, imperfectly correlated signal, the analysis becomes significantly more complex. (See also Section 4.3.) We cannot, and do not want to, speculate what the optimal security might be in this case.

[16] For a survey of the relationship lending literature, see Boot (2000).

[17] For a similar setup, see also Dell'Ariccia and Marquez (2004) and Hauswald and Marquez (2003).

As for geographical proximity, a lender in close proximity to a borrower might have better knowledge of local conditions, which puts her in a better position to evaluate the borrower's project. With regard to small business lending, Petersen and Rajan (2002) (for the United States) and Degryse and Ongena (2003) (for Belgium) find that the median distance between banks and borrowers is four and 1.4 miles, respectively, suggesting that small business lending might be locally segmented. Relatedly, Guiso, Sapienza, and Zingales (2004) argue that there is "direct evidence of the informational disadvantage of distant lenders in Italy". In this spirit, and similar to our paper, Hauswald and Marquez (2002) assume that the lender that is closest to a borrower has a more informative signal than more distant lenders.

The timing is as follows. The lender and the market make offers at $\tau = 0$.[18] If the borrower goes to the lender, the lender performs a credit analysis and accepts or rejects the borrower based on her signal. At $\tau = 1$ an accepted borrower can either stay with the lender or visit the credit market. If the borrower was rejected, his only option is to visit the market. Cash flows are realized at $\tau = 2$.

For technical reasons, we assume that the market can distinguish between "fresh" borrowers and borrowers who have been previously screened. Without this assumption, there exists no pure-strategy equilibrium (e.g., von Thadden (2001), Hauswald and Marquez (2003)). Hence, the borrower may solicit offers from both the lender and the market at $\tau = 0$. If he visits the market *after* he has been screened, however, the market might make him a different offer. In practice, potential lenders would usually check a borrower's credit history before making a loan. In most countries, including the United States, credit bureaus provide this information in the form of credit reports. Such credit reports commonly also show whether other lenders have made similar inquiries in the past, including the date of the inquiry and the identity of the inquirer (Jappelli and Pagano (2002)).[19] To the extent that lenders have access to this information, they can see whether a borrower has recently sought credit, and whether he sought credit from a

[18] To ensure that the market does not fall prey to "fly-by-night operators" (see Section 2.2), we could assume that the market does not offer any upfront payments. Since fly-by-night operators generate a cash flow of $x = 0$ for sure, their expected payoff is then zero. Alternatively, we might assume that both the lender and the market can filter out crooks, but the lender has a more precise signal about "real" borrowers. Suppose all investors observe two signals. The (informed) lender observes a public signal at $\tau = 0$ and a private signal s at $\tau = 1$, where $s = 0$ if the borrower is a crook. The market observes a public signal at $\tau = 0$ and a second signal σ at $\tau = 1$. If the borrower is a crook, the second signal is $\sigma = 0$, otherwise it is identical to the first (i.e., public) signal.

[19] Jappelli and Pagano provide a copy of an actual credit report containing this (and other) information.

local or a distant lender.

4.2 Equilibrium Analysis

The project's ex-ante NPV—i.e., the NPV based on publicly available information alone—is

$$\mu := \int_0^1 (\mu_s - k) f(s)\, ds. \tag{5}$$

At $\tau = 0$ the market can offer the borrower an expected payoff of $\max\{0, \mu\}$. On the other hand, the maximum expected payoff that the informed lender can offer the borrower is

$$V_{\max} := \max_t \int_{s^*}^1 [\mu_s - u_s(t)] f(s) ds, \tag{6}$$

subject to $u_{s^*}(t) = k$ (Lemma 2), where t is a debt contract. The restriction to debt contracts follows from Proposition 2.

The competition model has a unique equilibrium: at $\tau = 0$ the borrower either visits the lender or the market, depending on the relative magnitudes of μ and $V_{\max}$. At $\tau = 1$ an accepted borrower stays with the lender, while a rejected borrower cannot obtain financing elsewhere.

Proposition 4. *The competition model has a unique equilibrium. There are two cases*:

i) Case 1: $V_{\max} > \mu$. *The inside lender offers the optimal debt contract from Proposition 2, whereby* $\overline{V} = \max\{0, \mu\}$. *The borrower goes to the inside lender, which provides him with an expected payoff of* $\overline{V}$. *The lender finances the borrower if* $s \geq s^*$ *and rejects him if* $s < s^*$. *A rejected borrower cannot obtain financing elsewhere.*

ii) Case 2: $V_{\max} < \mu$. *The borrower goes to the credit market, which provides him with an expected payoff of* μ.

Proof. See Appendix.[20]

Consider first Case 1. At $\tau = 0$ the lender's ability to make an informed decision allows her to undercut the market. By standard arguments, the unique equilibrium is to exactly match the market offer $\max\{0, \mu\}$. At $\tau = 1$ the market cannot offer more than $\max\{0, \mu\}$.[21] As the expected payoff of an accepted borrower exceeds $\max\{0, \mu\}$, he optimally stays with the lender. But if all accepted borrowers stay with the lender, rejected borrowers cannot obtain financing

[20] If $V_{\max} = \mu$ there are two equilibrium outcomes: the borrower either visits the lender or the market.

[21] A market offer of $\max\{0, \mu\}$ is sustainable only under the "most optimistic beliefs" that—in addition to all rejected borrowers—also all accepted borrowers visit the market at $\tau = 1$.

from the market as their conditional NPV $\int_0^{s^*}(\mu_s - k)f(s)/F(s^*)ds$ is negative. This endogenizes an assumption made in Section 2.2 that a rejected project will not be financed.

Proposition 4 also endogenizes the borrower's reservation utility as $\overline{V} = \max\{0, \mu\}$. If $\mu \leq 0$ the borrower cannot obtain financing from the market, which implies the lender can extract the full surplus. In contrast, if $\mu > 0$ there is true competition. As the project is already viable on the basis of public information alone, the lender must give the borrower the full ex-ante NPV μ to match the market offer. Note that, even if projects are viable on the basis of public information, the lender performs a valuable task by filtering out bad projects. The easiest way to see this is by looking at the lender's and borrower's combined profits $\mu + U(t)$: as $U(t) > 0$, the lender's screening activity has generated surplus in addition to the ex-ante NPV μ.[22]

In Case 2, the ex-ante NPV is so high that the lender cannot compete. The borrower consequently visits the credit market, where he extracts the full ex-ante NPV μ.[23] As an illustration, consider the limit case where $s_{FB} = 0$. To offer the borrower the full ex-ante NPV, the lender would have to accept the project for *all* signals $s > 0$. But this is impossible: the lender sets $s^* = s_{FB} = 0$ only if she extracts the full surplus; but then she cannot promise the borrower anything. Conversely, if the borrower gets just a tiny fraction of the surplus, the lender optimally sets $s^* > s_{FB} = 0$. Intuitively, if $s_{FB} = 0$ there are no bad projects. Absent any value added, the only way the lender can offer the borrower the full ex-ante NPV is by making zero profits herself. But this is impossible, as the lender earns an information rent of $U(t) > 0$ due to the discretionary nature of her accept or reject decision.

Clearly, it is desirable to express the choice between Case 1 and Case 2 in terms of model primitives, e.g., in terms of the investment cost k. Suppose k can vary between $\underline{k} = \mu_0$ and $\overline{k} = \mu_1$. Everything else equal, borrowers with a low k have a high ex-ante NPV and are likely to break even, while borrowers with a high k are less likely to break even. It is easy to show that if k is sufficiently large Case 1 applies, while if k is sufficiently small Case 2 applies. For intermediate $k-$values further assumptions are needed, however, since $\mu - V_{\max}$ may have multiple points of zero. It is relatively easy, however, to construct numerical examples in which all borrowers above a certain cutoff-NPV visit the credit market while all borrowers below the cutoff visit the lender. Consider the following example.

[22] Precisely, it is not the lender's screening activity per se that generates surplus, but the fact that rejected projects disappear from the market.

[23] This is the case where $\overline{V}$ is "too large" that we have ruled out in Section 3.3.

Example. Suppose the project cash flow is exponentially distributed over $x \geq 0$ with CDF $G_s(x) = 1 - e^{-(S-s)x}$ and density $g_s(x) = (S-s)e^{-(S-s)x}$, where $S > 1$. The expected cash flow conditional on s is then $\mu_s = 1/(S-s)$. To ensure that s_{FB} lies in the interior of s, we assume that $1/S < k < 1/(S-1)$, which implies that $s_{FB} = S - 1/k$. The signal distribution $F(s)$ is uniform over $S := [0, 1]$.

Appendix B shows the derivation of all the main formulas in this section for the above specifications of $G_s(x)$ and $F(s)$. The ex-ante project NPV is $\mu = \ln\left(\frac{S}{S-1}\right) - k$, which implies it is fully characterized by two parameters: the investment cost k and the distributional parameter S. Holding S constant, a lower value of k implies a higher ex-ante NPV. Conversely, holding k constant, a lower value of S implies a "better" conditional cash-flow distribution $G_s(x)$ in the sense of MLRP, and therefore a higher ex-ante NPV. Consequently, the ex-ante NPV μ is strictly decreasing in both k and S. Likewise, the maximum utility that the lender can offer the borrower, $V_{\max}$, is strictly decreasing in both k and S.

Figure 1 here

Figure 1a plots μ and $V_{\max}$ for different values of k, while Figure 1b plots μ and $V_{\max}$ for different values of S. In both cases, μ and $V_{\max}$ cross exactly once such that Case 1 in Proposition 4 applies if and only if k or S is sufficiently large. **End.**

With the usual degree of caution, we might summarize our results as follows: informed (i.e., relationship or local) lenders tend to be segmented towards borrowers who—if evaluated solely on the basis of hard, public information—are less likely to break even, and where additional, discretionary information is particularly valuable (Proposition 4, Case 1). Conversely, uninformed, arm's-length lenders tend to be segmented towards "safe" borrowers who are likely to break even in the first place, e.g., big firms with a good credit rating (Case 2).

Petersen and Rajan (2002) find that more transparent firms and firms with a good credit quality are indeed more likely to borrow at arm's length. Similarly, Denis and Mihov (2003) find that profitable firms with a high credit quality are more likely to tap public debt markets, while less profitable firms tend to borrow more from banks. Relatedly, Cole, Goldberg, and White (1999) document that big banks tend to approve loans primarily on the basis of hard information such as financial statements, while small banks are more likely to use soft information. Accordingly, big banks tend to act more as arm's-length lenders while small banks tend to act more as relationship lenders. Consistent with our stylized picture, Berger et. al (2004) find that big banks are more apt to lend at a distance and to firms that are financially more transparent, while Haynes, Ou, and Berney (1999) find that big banks tend to lend more to financially secure firms.

4.3 Discussion

Ex-ante Negotiations

Our qualitative results remain the same if—instead of the lender making a take-it-or-leave-it offer—the borrower and lender bargain over a menu of contracts. The only difference is that instead of merely matching the competitive market offer, the lender must offer the borrower the competitive market offer plus a bargaining premium.

As ex-ante negotiations take place under symmetric information, it is reasonable to assume that the borrower and lender select a point on the Pareto frontier. We have already derived the Pareto frontier by maximizing the lender's expected payoff subject to fixing the borrower's expected utility at $\overline{V} \in [0, V_{\max}]$. For each such $\overline{V}$, denote the optimal debt contract and the lender's expected payoff by $t^*(\overline{V})$ and $U(\overline{V}) := u(t^*(\overline{V}))$, respectively. Following a standard convention in bargaining theory, we assume that the Pareto frontier is smooth and concave.[24]

We use the Nash bargaining solution. Accordingly, the borrower and lender choose a pair $(\overline{V}, U(\overline{V}))$ maximizing the Nash product $(\overline{V} - \hat{V})^b (U(\overline{V}) - \hat{U})^{1-b}$, where $0 < b < 1$ reflects the borrower's bargaining power, $\hat{V} = \max\{0, \mu\}$ denotes the borrower's outside option, and $\hat{U} = 0$ denotes the lender's outside option. The solution is given by the first-order condition

$$\frac{b}{1-b} = -U'(\overline{V}) \frac{\overline{V} - \max\{0, \mu\}}{U(\overline{V})}. \tag{7}$$

Equation (7) presumes the existence of a point $(\overline{V}, U(\overline{V}))$ on the Pareto frontier with the property that $\overline{V} > \max\{0, \mu\}$ and $U(\overline{V}) > 0$. Such a point exists if and only if Case 1 in Proposition 4 applies. Moreover, if $b \to 0$ the solution to (7) converges to our previous solution $\overline{V} = \max\{0, \mu\}$. The following result is obvious.

Proposition 5. *Suppose the inside lender and the borrower bargain over a menu of contracts ex ante, where the outcome is determined by the Nash bargaining solution.*

[24] Concavity of the Pareto frontier can be ensured by introducing lotteries over contracts.

i) *If* $V_{\max} > \mu$ *Case 1 in Proposition 4 applies. The only difference is that the borrower's expected payoff* $\overline{V}$ *lies above the competitive market offer* $\max\{0, \mu\}$*, where the "bargaining premium"* $\overline{V} - \max\{0, \mu\}$ *is increasing in the borrower's bargaining power* b*.*

ii) *If* $V_{\max} < \mu$*, the lender and borrower cannot reach an agreement, and Case 2 applies.*

Informed Lending as a Monopoly?

Like Sharpe (1990), Rajan (1992), Dell'Ariccia and Marquez (2004), and others, we have assumed that there is a single informed lender competing with an uninformed credit market. In models of relationship lending, this information monopoly derives from the assumption that at the beginning of a firm's history, the firm finances its first-period project through a single lender.[25] Due to the soft information acquired in this period, the initial lender obtains (and retains) an informational advantage in all following periods.[26] The empirical findings by Petersen and Rajan (1994) that 82 percent of the firms in their sample borrow from a single bank (95 percent if they consider only the smallest firms in their sample) suggests that the notion of a single inside lender is not unreasonable.

Alternatively, the inside lender's advantage might derive from geographical proximity to the borrower. In this case, a local monopoly might arise from a fixed cost of establishing a local presence. As an illustration, consider the case of two local lenders with perfectly correlated, yet nonverifiable signals. This case is analyzed in, e.g., von Thadden (1994) and Hauswald and Marquez (2003). As each lender knows the other lender's signal, Bertrand competition at $\tau = 1$ drives profits to zero. Given the fixed cost, there exist pure-strategy equilibria where only one of the two lenders enters. If the two signals are not perfectly correlated, the analysis becomes significantly more difficult as competition at $\tau = 1$ takes place under asymmetric information.[27]

[25] Rajan (1992, Section IV.B) briefly discusses the case where the firm can initially borrow from multiple banks.

[26] "In the process of this monitoring, the lender learns more about the success of the firm's operation than do outside banks. As a result, the original lender should be in a better position to evaluate the firm's future performance" (Sharpe, 1990)).

[27] In Thakor (1996) signals are uncorrelated across lenders. As there are only two signals, all lenders who accept the borrower have the same (namely, high) signal, however. Given that all lenders know how many other lenders have accepted the borrower, competition takes place under symmetric information. Just like in the case where signals are perfectly correlated, competing banks thus make zero profits. In our model, there is a continuum of signals *and* signals are private. Hence, even if a lender knew how many other lenders have accepted, she would not know their signals—and hence their valuations—unless signals were perfectly correlated.

Even in a setting with two cash flows, two signals, and no security design, pure-strategy equilibria may not exist (Broecker (1990)). While it may still be true that a high fixed cost can create a monopoly, a formal analysis of this case is beyond the scope of this paper.

5 Robustness

5.1 Renegotiations

Under the optimal contract, the borrower is rejected at marginal signals $s \in (s_{FB}, s^*)$ even though the project has a positive NPV. This potentially provides scope for renegotiations: to make the project more attractive for the lender, the borrower might accept less favorable loan terms, e.g., a higher repayment R. This is what would happen if s were jointly observable by the borrower and lender. As s is private information, however, the borrower does not know if the true signal is $s \in (s_{FB}, s^*)$, in which case he might accept new loan terms, or if $s > s^*$, in which case changing the loan terms would merely constitute a wealth transfer to the lender. A *necessary* condition for the borrower to accept a new contract t is therefore that t implements a lower cutoff signal, i.e., $s^*(t) < s^*(t^*)$. However, such a contract would make the lender not only better off at marginal signals $s \in (s^*(t), s^*(t^*)]$ but also at higher signals $s \geq s^*(t^*)$.[28] Hence, the lender has every incentive to claim that $s \in (s_{FB}, s^*)$ even if the true signal is $s \geq s^*(t^*)$. As Proposition 6 shows, this conflict of objectives implies that the optimal contract t^* will not be renegotiated in equilibrium.

We consider the following model of renegotiation. After the signal has been realized either the borrower or the lender can offer a new contract t.[29] The offer must be accepted by the respective counterparty; otherwise the optimal contract t^* remains in effect. We have the following result.

Proposition 6. *In any (perfect Bayesian) equilibrium of the renegotiation game, the unique optimal debt contract from Proposition 2 is not renegotiated. This holds irrespective of whether the borrower or the lender can offer a new contract at the interim stage.*

Proof. See Appendix.

[28] Since t^* is standard debt, any nondecreasing contract $t \neq t^*$ that is preferred by the lender *must* award the lender a greater fraction of high cash flows. The rest of the argument follows then immediately from MLRP.

[29] The argument trivially extends to the case where the borrower and lender can offer menus of contracts.

5.2 Interim Constraint

The optimality of debt derives from the fact that the lender must satisfy the borrower's ex-ante participation constraint (3). If there was no ex-ante constraint but, say, *only* an interim constraint, our arguments would generally not apply.[30] Section 4 shows how the ex-ante participation constraint might arise naturally from competition in the credit market: to attract the borrower in the first place, the lender must offer him $\overline{V} = \max\{0, \mu\}$.

In the following, we examine to what extent our results are preserved if we *additionally* introduce an interim constraint. We consider two possible sources for an interim constraint: a wage from outside employment and an agency rent due to ex-post moral hazard.

Outside Employment Opportunity

Suppose the borrower has an outside employment opportunity, both at $\tau = 0$ and $\tau = 1$, offering him a wage $w > 0$. His ex-ante participation constraint (3) is then

$$\int_{s^*(t)}^{1} [\mu_s - u_s(t)] f(s) ds + F(s^*) w \geq \overline{V} + w, \tag{8}$$

where $\overline{V}$ is the borrower's rent on top of w.

The possibility of an outside wage creates an interim participation constraint

$$\int_{s^*(t)}^{1} [\mu_s - u_s(t)] \frac{f(s)}{1 - F(s^*)} ds \geq w, \tag{9}$$

where $f(s)/[1 - F(s^*)]$ is the posterior probability of s given that $s \geq s^*$. By inspection, (8) implies (9). In fact, if $\overline{V} > 0$ (9) is slack.

Even though (9) never binds, introducing an outside wage has a nontrivial effect as it also enters into the first best. Precisely, the first-best cutoff signal is now given by $\mu_{s_{FB}} = w + k$. The lender's optimal cutoff signal, on the other hand, is (still) given by $u_{s^*}(t) = k$, since part of the "real opportunity cost", w, is borne by the borrower.

On the one hand, the lender must leave the borrower a rent $\overline{V}$, which tends to make her too conservative. This is our previously studied effect. On the other hand, the lender does not bear the full opportunity cost, which makes her too lenient. This second effect may (partly) offset the first. In fact, if w is sufficiently large, the first best can be attained. If w increases beyond this point, the lender's cutoff signal under the optimal debt contract would drop below

[30] This is true for our base model. In our extension with moral hazard, debt is optimal regardless of whether the ex-ante or interim constraint binds. See the end of Section 5.2 for a brief discussion.

s_{FB}. To remain at the first best, the lender must consequently adjust the optimal contract. One way—but not the only way—of doing this is to offer a debt contract and pay the borrower a fixum of W if the project is financed. One interpretation of this contract is that the lender provides funds of $k + W$ but only k is invested in the project. When the project cash flow is realized the borrower can pay himself a wage W, while the lender holds a debt contract on the cash flow. The proof of the following proposition is immediate given our previous results.

Proposition 7. *Suppose the borrower has an alternative employment opportunity offering him a wage of* $w > 0$. *If* w *is sufficiently small compared to* $\overline{V}$, *our previous results continue to hold, i.e.,* $s^* > s_{FB}$ *and the unique optimal security is debt. Otherwise, the first best can be attained, e.g., with* $t(x) = -W + \max\{x, R\}$ *with* $W \leq w$, *in which case the lender holds a debt contract and the borrower receives an additional wage* W *if the project is undertaken.*

Hence, it is crucial for our debt result that the borrower extracts a rent $\overline{V}$ on top of w. Merely requiring that he be compensated for his forgone wage w is—*absent any rent extraction*—insufficient. In our basic competition model (Sections 4.1 - 4.2), for instance, we have $\overline{V} = \max\{0, \mu - w\}$, which implies we need $\mu > w$ for our debt result to hold. If the borrower has additionally some bargaining power (Section 4.3), we have $\overline{V} > \max\{0, \mu - w\}$. In this case, our debt result may hold even if $\mu < w$.

Ex-post Borrower Moral Hazard

Suppose after the project is financed but before cash flows are realized, the borrower can exert a noncontractible effort $e \in \{e_l, e_h\}$. Low effort is costless while high effort involves a private cost $c > 0$. By assumption, it is always efficient to implement the high effort. The conditional cash-flow distribution is $G(x \mid s, e)$, which is assumed to be additively separable of the form $G(x \mid s, e) = H_1(x \mid s) + H_2(x \mid e)$ with corresponding densities g, h_1, and h_2. Hence, the marginal productivity of e is independent of s.[31] The conditional expected cash flow and the lender's conditional expected payoff are $\mu(s, e)$ and $u(t \mid s, e)$, respectively.

[31] Given the assumptions in this section, we can restrict ourselves to single contracts. Absent any productive interaction between e and s, the only implication of a menu (where each contract in the menu implements high effort) would be that the lender always picks the contract that minimizes the borrower's agency rent (see Section 3.4 for a related argument). But for any such menu, the lender can implement the same expected agency rent with a single contract. This logic extends to more than two effort levels provided the lender wants to implement the same effort for all signals. Otherwise a menu of incentive contracts might be optimal.

We assume that high effort generates a "better" cash-flow distribution in the sense of MLRP. Assumption 1 is thus replaced by

Assumption 1a. *For any pair* $(s, s') \in [0, 1]$ *with* $s' > s$ *and any* $e \in \{e_l, e_h\}$, *the ratio* $g(x \mid s', e)/g(x \mid s', e)$ *is strictly increasing in* x, *while for any* $s \in [0, 1]$ *the ratio* $g(x \mid s, e_h)/g(x \mid s, e_l)$ *is strictly increasing in* x.

The first-best cutoff signal is unique and given by $\mu(s_{FB}, e_h) = k + c$. Similarly, if we rule out the trivial case where the lender always rejects, the lender's unique optimal cutoff signal is given by $u(t \mid s^*, e_h) = k$.

The lender's problem is as follows. The lender chooses a contract t to maximize

$$U(t) := \int_{s^*}^{1} [u(t \mid s, e_h) - k] f(s) ds \tag{10}$$

subject to the borrower's ex-ante participation constraint

$$\int_{s^*}^{1} [\mu(s, e_h) - u(t \mid s, e_h) - c] \, f(s) ds \geq \overline{V}, \tag{11}$$

and his (interim) incentive-compatibility constraint

$$\int_{s^*}^{1} [\mu(s, e_h) - u(t \mid s, e_h)] \, \frac{f(s)}{1 - F(s^*)} ds - c \geq \int_{s^*}^{1} [\mu(s, e_l) - u(t \mid s, e_l)] \, \frac{f(s)}{1 - F(s^*)} ds, \tag{12}$$

where $f(s)/[1 - F(s^*)]$ is the posterior probability of s given that $s \geq s^*$. Recall that the borrower does not know the true signal—and thus his actual agency rent—but only that $s \geq s^*$.

Consider the borrower's moral hazard problem in isolation. Given that $G(x \mid s, e) = H_1(x \mid s) + H_2(x \mid e)$, we can rewrite (12) as

$$\int_X [x - t(x)] h_2(x \mid e_h) dx - c \geq \int_X [x - t(x)] h_2(x \mid e_l) dx. \tag{13}$$

Accordingly, the borrower's incentive-compatibility constraint—and thus the optimal incentive contract—is independent of s or s^*. Given Assumptions 1a and 2, the unique optimal incentive contract is a call option, which implies $t(x)$ is standard debt. Intuitively, a call option maximizes the borrower's payoff in high cash-flow states (subject to Assumption 2), and thus precisely in those states where the likelihood ratio is highest, i.e., states that are most informative about high effort.[32]

[32] This line of argument follows Innes (1990).

Consider now the lender's overall problem (10)-(12). Given that $t(x)$ is debt, there exists a unique repayment R_1 defined by the binding interim constraint (13). This is the repayment the lender would require if there was no ex-ante constraint. Inserting R_1 in (12), the borrower then obtains an expected (from a $\tau = 0$ perspective) agency rent of

$$\int_{s^*(R_1)}^{1} \left[\mu(s, e_l) - u(R_1 \mid s, e_l)\right] f(s) ds. \tag{14}$$

If (14) exceeds $\overline{V}$, the solution to the lender's overall problem is to offer a debt contract with repayment R_1. In this case, the interim constraint (12) binds while the ex-ante constraint (11) is slack. Conversely, if $\overline{V}$ exceeds (14), the expected agency rent is insufficient to satisfy the borrower's ex-ante participation constraint. The lender must consequently lower R, which renders the interim constraint (12) slack. By Proposition 1, the unique optimal solution in this case is a debt contract with repayment R^* such that (11) is satisfied with equality. Overall, the borrower's expected rent is the maximum of (14) and $\overline{V}$. Hence, irrespective of whether the interim or ex-ante constraint binds, it holds that $s^* > s_{FB}$.

Proposition 8. *The unique optimal solution to the lender's problem with ex-post borrower moral hazard is to offer a standard debt contract. The optimal repayment is determined either by* (11) *or* (12)*, depending on which constraint binds. In either case, it holds that $s^* > s_{FB}$.*

Proof. See Appendix.

The fact that debt is optimal even when the ex-ante participation constraint (11) is slack depends on the specific structure of the moral hazard problem, especially Assumption 1a and the fact that $G(x \mid s, e)$ is additively separable. If s and e interact in a more complex fashion, debt—or even a single contract—might no longer be optimal. While interesting, an analysis of this case is beyond the scope of this paper.

5.3 Signal-Dependent Optimal Investment Levels

In Section 3.4 we pointed out that the optimality of a single contract depends on a number of assumptions, most notably the assumption that the investment size k is fixed. One can easily think of situations where a fixed investment is not a good assumption. For instance, the optimal investment size might depend on the productivity of the project, and hence on the lender's signal. In what follows, we analyze this case in more detail.

Our main finding is that, while a single debt contract is generally no longer optimal, the optimal menu consists exclusively of debt contracts. The intuition is similar to Section 3, except that there are now two distortions: $s^* > s_{FB}$ and/or $k(s) \neq k_{FB}(s)$ for some $s \geq s_{FB}$.[33] Consider two incentive-compatible menus that implement the *same* investment schedule $k(s)$ and cutoff signal s^*. One of them, menu D, consists exclusively of debt contracts while the other, menu ND, does not. For the same reason as in Section 3 where a single debt contract minimized the lender's information rent $U_s(t) - U_{s^*}(t)$ for all $s > s^*$, the lender's expected information rent is lower under the menu of debt contracts, D. Since s^* and $k(s)$ are the same under both menus, the borrower's expected payoff is higher under D. If the borrower's participation constraint binds under ND, it must therefore be slack under D. This slack can be used to improve efficiency—i.e., to either lower the cutoff signal s^* by increasing the repayment R in the "cutoff contract" and/or implement a more efficient investment schedule $k(s)$—until the borrower's participation constraint binds.[34] Call this adjusted menu of debt contracts D'. In the end, the borrower is equally well of under ND and D'. The latter, however, is more efficient, which implies the lender's expected payoff is higher under D'.

To verify this intuition formally, let us modify our setup as follows. Given some signal s and investment size $k > 0$, let $G_s(x \mid k)$ and $\mu_s(k)$ denote the project's conditional cash-flow distribution and conditional expected cash flow, respectively. We assume that the corresponding density $g_s(x \mid k) > 0$ is continuously differentiable in s and k over $x \in [\underline{x}, \overline{x}]$. For $k = 0$ we specify $\mu_s(k) = 0$. Moreover, we assume that $\mu_s(k)$ is strictly quasiconcave in k given s, which implies $\mu_s(k) - k$ is also strictly quasiconcave. We finally assume that $\mu_s(k) - k$ has a unique finite maximum $k_{FB}(s)$ for all s.

Given that $G_s(x \mid k)$ satisfies MLRP, $\mu_s(k)$ is strictly increasing in s for given $k > 0$, which immediately implies that $\mu_s(k_{FB}(s)) - k_{FB}(s)$ is strictly increasing in s if $k_{FB}(s) > 0$ and nondecreasing otherwise. Similar to our basic model in Section 3, we assume that $\mu_s(k_{FB}(s)) - k_{FB}(s)$ is positive for sufficiently high s and negative for sufficiently low s, implying that there

[33] This is true provided $\overline{V} > 0$. If $\overline{V} = 0$ the first best can be attained, i.e., $s^* = s_{FB}$ and $k(s) = k_{FB}(s)$ for all $s \geq s_{FB}$. See the Appendix for details.

[34] When adjusting the repayment associated with the cutoff debt contract $t(x, s^*)$ one must at the same time also adjust the repayments at higher signals $s > s^*$ to preserve incentive compatibility.

exists a unique first-best cutoff signal $0 < s_{FB} < 1$ satisfying[35]

$$\mu_{s_{FB}}(k_{FB}(s_{FB})) - k_{FB}(s_{FB}) = 0. \tag{15}$$

A menu of contracts consists of pairs $\{k(s), t(x, s)\}$ prescribing an investment level $k(s)$ and a repayment $t(x, s)$ for each (reported) signal s. Let $U_s(t(x, s), k(s)) := \int_X t(x, s) g_s(x \mid k) dx - k(s)$. As $t(x, s)$ is nondecreasing in x by Assumption 2 and $G_s(x \mid k)$ satisfies MLRP, it follows immediately from previous arguments that the lender's optimal accept or reject decision is characterized by a cutoff rule.[36] If we (again) rule out the trivial case where the lender always rejects, there then exists a unique cutoff signal $0 < s^* < 1$ satisfying

$$U_{s^*}(t(x, s^*), k(s^*)) - k(s^*) = 0. \tag{16}$$

Given that the optimal accept or reject decision is characterized by a cutoff rule, we can write the lender's problem as follows. At time $\tau = 0$ the lender chooses a menu $T := \{t(x, s), k(s)\}$ to maximize her expected payoff

$$U(T) := \int_{s^*(T)}^{1} [U_s(t(x, s), k(s)) - k(s)] f(s) ds, \tag{17}$$

subject to her incentive-compatibility constraint

$$U_s(t(x, s), k(s)) - k(s) \geq U_s(t(x, s'), k(s')) - k(s') \text{ for all } s, s' \geq s^*, \tag{18}$$

the borrower's ex-ante participation constraint

$$V(T) := \int_{s^*(T)}^{1} [\mu_s(k(s)) - U_s(t(x, s), k(s))] f(s) ds \geq \overline{V}, \tag{19}$$

the optimality condition (16), and the requirement that $t(x, s)$ be nondecreasing.[37]

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[35] Existence of s_{FB} follows from continuity of $\mu_s(k_{FB}(s)) - k_{FB}(s)$, which follows from continuity of $G_s(x \mid k)$ and the maximum theorem.

[36] If the lender's expected payoff under the contract $(k(s), t(x, s))$ is positive at some signal s, it remains positive under the *same* contract at all higher signals by Assumption 2. If the lender optimally selects a different contract at higher signals, her expected payoff may be higher (by optimality), but never lower.

[37] In Section 3.1 incentive compatibility is ensured by $U_s(T) := \max_{t_i \in T} u_s(t_i)$. As a menu now consists of two instruments, t and k, it is simpler to write the incentive-compatibility constraint as in (18). Note that the restriction to $s \geq s^*$ is without loss of generality. Alternatively, we could extend the notation to all s by specifying that $k(s) = 0$ for $s < s^*$.

To characterize the optimal repayment schedule $t^*(x,s)$, it is not necessary to fully solve (16)-(19). (In particular: it is not necessary to derive the optimal investment schedule $k^*(s)$.) As the following result shows, in *any* optimal menu T^* the associated optimal repayment schedule $t^*(x,s)$ must consist exclusively of debt contracts.[38]

Proposition 9. *The optimal menu consists exclusively of debt contracts.*

Proof. See Appendix.

6 Conclusion

With few exceptions, credit decisions are subjective and hence discretionary.[39] Discretion, in turn, implies that the lender's incentives to accept or reject depends on the value of her claims, and thus on the security in place. We show that the lender is generally too conservative in the sense that she rejects too often. The optimal contract minimizes this ex-post inefficiency. The unique optimal contract is debt. Debt minimizes the lender's losses from bad projects, thus minimizing her excessive conservativism and inclination to reject.

We also show that the fine-tuning of loan terms after interim information might not be optimal. In principle, the lender could offer a menu from which she selects a contract after obtaining information about the project. Since the lender has discretion, she would always pick the contract that is ex-post optimal for her, however. This "self-dealing" undermines the lender's commitment to leave the borrower as much payoff as possible in good states, which is necessary to maximize her own payoff in bad states. Hence, the same feature that makes debt optimal—namely, maximization of the lender's return in bad states—makes a menu *not* optimal. Instead, it is optimal to offer a single contract and either reject or accept the borrower on the basis of this contract. (As we show in Section 5.3, this might not true if the investment size is state-dependent, however.) Our results might help explain the use of standardized loan terms in conjunction with quantity rationing found in certain lending situations

[38] The problem (16)-(19) is not amenable to standard solution techniques. In a standard mechanism design problem (e.g., Fudenberg and Tirole (1992, Chapter 7)) monetary transfers are separable from allocative decisions. In our model, this would imply that t is a lump-sum transfer that depends on s but not on k. From our previous analysis, however, we know that t optimally depends on x, which in turn depends on $G_s(x \mid k)$ and thus on k.

[39] One exception is automated credit scoring as used, e.g., by credit card companies.

What makes our lender special is her ability to weed out lemons. In other investment situations, the characteristic ability of investors may be a different one. For instance, it is frequently argued that what distinguishes a venture capitalist from a bank is the venture capitalist's expertise to coach projects. The optimal contract in this case is to give the venture capitalist equity or equity-linked securities, like convertibles. In practice, ex-ante and ex-post incentives often cannot be separated, however. Banks, for instance, typically engage in ex-post monitoring while venture capitalists also engage in ex-ante screening. Exploring this tension between screening (which tends to favor debt) and ex-post incentives (which tends to favor equity) in a unified framework appears to be an interesting avenue for future research.

In the United States, banks are generally forbidden to hold significant equity stakes in nonfinancial firms. In this regard, Berlin (2000) asks: "Are U.S. banks' borrowers at a disadvantage because their lenders are too cautious when evaluating project risks and too harsh when a borrower experiences financial difficulties?" In our model, lenders are indeed too cautious. But this is not because they hold debt, but because they cannot extract the full surplus from their borrowers. On the contrary, it is debt, not equity, that makes lenders the *least* cautious. Given that our setting is based on specific, and possibly restrictive, assumptions, one must be cautious in drawing policy implications, however.

7 Appendix A: Proofs

Proof of Lemma 2. If $t(x)$ is a constant, we have that $t(x) \leq \underline{x} < k$. But this implies that $u_s(t) < k$ for all $s \leq 1$, which implies that the lender rejects for all s. Conversely, if $t(x)$ is increasing over sets of positive measure, Assumptions 1 and 2 together with continuity of $g_s(x)$ in s imply that $u_s(t)$ is continuous and strictly increasing in s. There are two cases: if $u_1(t) \leq k$ the lender rejects for all s, while if $u_1(t) > k$ she accepts if and only if $s > s^* \in (0, 1)$, where $s^* > 0$ follows from $u_0(t) \leq \mu_0 < k$. **Q.E.D.**

Proof of Proposition 2. The case where $\overline{V} = 0$ is trivial. If the lender extracts the full surplus, the unique optimal security is $t(x) = x$ for all x (or debt with repayment $R = \overline{x}$). The rest follows from Proposition 1.

Suppose $\overline{V} > 0$. The statement regarding s^* follows from Proposition 1. In the following, we show that for any (nondecreasing) non-debt contract $t = t(x)$ we can construct a debt contract

$\tilde{t} = \tilde{t}(x)$ that satisfies the borrower's participation constraint (3) and is strictly preferred by the lender. Holding the cutoff signal fixed at $s^*(t)$, choose $\tilde{t}$ such that it generates the same expected payoff for the lender as t. Accordingly, $\tilde{t}$ is uniquely defined by

$$\int_{s^*(t)}^{1} \left[\int_X z(x) g_s(x) dx \right] f(s) ds = 0, \tag{20}$$

where $z(x) := \tilde{t}(x) - t(x)$. As the cutoff signal is fixed and the lender's expected payoff is the same under t and $\tilde{t}$, the borrower's expected payoff is consequently also the same under t and $\tilde{t}$.

As $\tilde{t} \neq t$ and the lender's conditional expected payoff is continuous in s under both t and $\tilde{t}$, (20) implies that there exists at least one signal s' satisfying $s^*(t) < s' < 1$ such that $\int_X z(x) g_{s'}(x) dx = 0$. Moreover, as t is nondecreasing and $\tilde{t}$ is a debt contract, this furthermore implies that there exists a cash flow $\hat{x} \in (\underline{x}, \overline{x})$ such that $z(x) \geq 0$ for all $x < \hat{x}$ and $z(x) \leq 0$ for all $x > \hat{x}$, where both inequalities are strict over sets of positive measure. As $G_s(x)$ satisfies MLRP and $s^*(t) < s'$, it follows that $g_{s^*(t)}(x)/g_{s'}(x)$ is strictly decreasing in x. Consequently,

$$\begin{aligned} & \int_X z(x) g_{s^*(t)}(x) dx \\ = & \int_{\underline{x}}^{\hat{x}} z(x) g_{s'}(x) \frac{g_{s^*(t)}(x)}{g_{s'}(x)} dx + \int_{\hat{x}}^{\overline{x}} z(x) g_{s'}(x) \frac{g_{s^*(t)}(x)}{g_{s'}(x)} dx \\ > & \frac{g_{s^*(t)}(\hat{x})}{g_{s'}(\hat{x})} \left[\int_X z(x) g_{s'}(x) dx \right] = 0, \end{aligned} \tag{21}$$

where the last equality follows from the definition of s'.

Since $\int_X z(x) g_{s^*(t)}(x) dx > 0$, we have that

$$\int_X \tilde{t}(x) g_{s^*(t)}(x) dx > \int_X t(x) g_{s^*(t)}(x) dx = k,$$

where the equality follows from the definition of $s^*(t)$. In words: The lender's conditional expected payoff at $s = s^*(t)$ is positive under the debt contract $\tilde{t}$ but zero under the non-debt contract t (by the definition of $s^*(t)$). As $\int_X \tilde{t}(x) g_s(x) dx$ is strictly increasing in s, this implies that $\int_X \tilde{t}(x) g_{s^*(\tilde{t})}(x) dx = k$ for some $s^*(\tilde{t}) < s^*(t)$, i.e., the cutoff signal implemented by $\tilde{t}$ is strictly lower than the cutoff signal implemented by t.

To wrap up, given some arbitrary non-debt contract t, we have constructed a debt contract $\tilde{t}$ such that, if the cutoff signal was fixed at $s^*(t)$, the lender and borrower have the same expected payoff under both contracts. However, under $\tilde{t}$ the borrower is additionally accepted at signals

$s \in (s^*(\tilde{t}), s^*(t)]$, which implies both the lender and borrower benefit from switching to $\tilde{t}$. This proves that any optimal contract must be debt.

Uniqueness is straightforward. As the borrower's expected payoff is continuous in R, there exists for each $\overline{V}$ a compact set of $R-$values at which the borrower's participation constraint binds. As the lender's expected payoff is increasing in R, the largest value in this set uniquely defines the optimal debt contract t^*. **Q.E.D.**

Proof of Proposition 3. As $U_s(T)$ is the maximum over a set of continuous and nonincreasing functions, it is continuous and nonincreasing in s. Lemma 2 thus carries over to the case with a menu, i.e., the optimal credit decision is characterized by a unique cutoff signal $s^*(T)$. The case where $\overline{V} = 0$ is trivial. If $\overline{V} > 0$ the menu cannot contain $t(x) = x$, for then the lender would always choose this contract, thus violating $\overline{V} > 0$. But if $t(x) = x$ is not in the menu, Proposition 1 holds, which implies there exists a unique cutoff signal $s^*(T) > s_{FB}$.

We now have two cases. In one case the menu T is not feasible (*Case 1*). In the other case we can lower the cutoff signal by replacing T with the unique optimal debt contract t^* from Proposition 2. Since the borrower's participation constraint is satisfed under t^* (Proof of Proposition 2, last paragraph), the lender is consequently better off (*Case 2*).

Case 1. Suppose $t^* \in T$ and $s^*(T) = s^*(t^*)$. In words: The menu contains the optimal debt contract, and this contract determines the cutoff signal $s^*(T)$. We now show that if t^* is not chosen for almost all $s > s^*$, then T must violate the borrower's participation constraint (1). If t^* is not chosen for almost all $s > s^*$, there exists some $t \neq t^*$ in T and some $\hat{s}$ satisfying $s^*(T) \leq \hat{s} < 1$ such that $u_{\hat{s}}(t) \geq u_{\hat{s}}(t^*)$, i.e., t is weakly preferred to t^* after observing the signal $\hat{s}$. But this implies that t must be strictly preferred to t^* for all $s > \hat{s}$. The argument is analogous to that in the Proof of Proposition 2. Define $z(x) := t^*(x) - t(x)$. Since $t^*(x)$ is a debt contract and $t(x)$ is nondecreasing, there exists some $\hat{x} \in (\underline{x}, \overline{x})$ such that $z(x) \geq 0$ for all $x < \hat{x}$ and $z(x) \leq 0$ for all $x > \hat{x}$, where the inequalities are strict over sets of positive measure. By MLRP of $G_s(x)$, we then obtain for $s > \hat{s}$ that

$$\int_X z(x) g_s(x) dx < \frac{g_s(\hat{x})}{g_{\hat{s}}(\hat{x})} \left[\int_X z(x) g_{\hat{s}}(x) dx \right] = 0,$$

which completes the argument. Since $u_s(t) > u_s(t^*)$ for all $s > \hat{s}$, and $s^*(T) = s^*(t^*)$, the lender's expected payoff from offering the menu is strictly greater than from offering only t^*, i.e., $U(T) > U(t^*)$. But since the cutoff signal is the same, i.e., $s^*(T) = s^*(t^*)$, the borrower's expected payoff must be lower, i.e., $V(T) < V(t^*)$. However, by the construction of t^* (Proof of Proposition 2, last paragraph), we have that $V(t^*) = \overline{V}$, implying that $V(T) < \overline{V}$.

Case 2. Suppose $s^*(T) \neq s^*(t^*)$, which covers all cases not covered in Case 1. In this case, we can straightforwardly apply the logic in the Proof of Proposition 2 and show that replacing T with a single contract t^* makes the lender strictly better off. Consider the "cutoff contract" $\hat{t} \in T$ defined by $s^*(T) = s^*(\hat{t})$. (If there are several such contracts in the menu, take one of them.) Next, delete all contracts $t \neq \hat{t}$ from the menu. By construction, the cutoff signal $s^*(\hat{t})$ remains unchanged. Moreover, as the lender (weakly) prefers the deleted contracts over $\hat{t}$ for *some* $s \geq s^*(\hat{t})$, but the cutoff signal remains unchanged, the borrower is not made worse off. That is, his participation constraint (1) remains satisfied after the deletion. We can then use the argument in the Proof of Proposition 2 and replace the remaining contract $\hat{t}$ with the optimal debt contract t^*. This lowers the cutoff signal while the borrower's participation constraint binds under t^*. The lender is consequently better off by replacing T with t^*. **Q.E.D.**

Proof of Proposition 4. In the following, we use the term "positive offer" to denote an offer by the market at $\tau = 1$ under which attracted borrowers receive a positive payoff. To simplify the exposition, we make use of an observation in the Proof of Proposition 3 that for any menu T the optimal credit decision follows a simple cutoff rule. The acceptance set $\Omega(T)$ is thus some interval $[s^*, 1]$, where $s^* = s^*(T)$.

The case where $\mu \leq 0$ is straightforward. Since any positive market offer which attracts accepted borrowers also attracts all rejected borrowers, the expected NPV from financing a borrower visiting the market at $\tau = 1$ can be at most μ.[40] Given that the market must break even, it therefore cannot make a positive offer. The same holds at $\tau = 0$. Accepted borrowers thus optimally stay with the lender, which implies we are back to our basic setup with $\overline{V} = 0$. At $\tau = 0$ the lender optimally offers $t^*(x) = x$, which provides the borrower with an expected payoff of zero.

In the remainder of this proof, we consider the case where $\mu > 0$. We first consider Case 1 where $V_{\max} > \mu$. The proof proceeds in several steps. We first prove an auxiliary result stating that if the market can make a positive offer (under which it breaks even) at $\tau = 1$, then the

[40] Since $\mu \leq 0$ rejected borrowers must have a negative expected NPV. This implies that there cannot exist a market offer that is separating and generates zero profit, since rejected borrowers would inevitably be attracted by the offer.

lender's expected payoff at $\tau = 0$ cannot be positive (*Step 1*). We subsequently show that the constellation described in the proposition is an equilibrium (*Steps 2a-c*). We finally show that the equilibrium is unique (*Step 3*).

Step 1. Suppose at $\tau = 1$ an accepted borrower stays with the lender's with probability $1-\xi$, while with probability $\xi \in [0,1]$ he takes the market offer. Evidently, a rejected borrower always takes a positive market offer. Accordingly, the expected NPV of financing a borrower visiting the market at $\tau = 1$ is

$$\begin{aligned}\hat{V} := & \frac{\xi\,[1-F(s^*)]}{\xi\,[1-F(s^*)]+F(s^*)}\int_{s^*}^{1}[\mu_s-k]\,\frac{f(s)}{1-F(s^*)}ds \\ & +\frac{\xi F(s^*)}{\xi\,[1-F(s^*)]+F(s^*)}\int_{0}^{s^*}[\mu_s-k]\,\frac{f(s)}{F(s^*)}ds,\end{aligned} \tag{22}$$

where $\int_{s^*}^{1}[\mu_s-k]\frac{f(s)}{1-F(s^*)}ds$ is the expected NPV of an accepted borrower, while $\int_{0}^{s^*}[\mu_s-k]\frac{f(s)}{F(s^*)}ds$ is the expected NPV of a rejected borrower. Given that the market makes zero profit, $\hat{V}$ is also what borrowers visiting the market at $\tau = 1$ will get on average.

Since $\mu > 0$ projects are viable on an ex-ante basis. Consequently, at $\tau = 0$ the lender must offer the borrower an expected payoff of at least $\mu > 0$. Given the borrower's decision at $\tau = 1$, his ex-ante participation constraint is

$$(1-\xi)\int_{s^*}^{1}[\mu_s-U_s(T)]\,f(s)ds+[\xi\,[1-F(s^*)]+F(s^*)]\,\hat{V}\geq\mu. \tag{23}$$

Inserting (22) in (23), the latter transforms to

$$(1-\xi)\int_{s^*}^{1}[U_s(T)-k]\,f(s)ds\leq 0,$$

which implies that, regardless of ξ, the lender's expected payoff at $\tau = 0$ is nonpositive. (If (23) binds, the lender's expected payoff is zero.) Intuitively, if the market makes a positive offer at $\tau = 1$, the project gets always financed—either by the lender or by the market. But this implies that the total surplus just equals the ex-ante NPV μ. Since μ is also what the lender must leave the borrower to satisfy his participation constraint, the lender makes no profit. Put differently, if projects always get financed the lender provides no value added. But if the lender provides no value added, her profit in a competitive credit market must be zero.

Step 2a. We next show that if the market does not make a positive offer at $\tau = 1$, the lender's maximization program coincides with that in Section 3.1, which implies that the unique optimal offer is t^*. (The remaining statements in Proposition 4 are then immediate.) The argument is straightforward. If the market does not make a positive offer at $\tau = 1$, an accepted borrower always stays with the lender, while a rejected borrower obtains a payoff of zero. Moreover, since projects are viable ex ante, the lender must offer the borrower at least $\mu > 0$. The borrower's ex-ante participation constraint is then

$$\int_{s^*}^{1} [\mu_s - U_s(T)]\, f(s)ds \geq \mu,$$

which coincides with (1) for $\overline{V} = \mu$ and $\Omega(T) = [s^*, 1]$ (on the latter, see above). The rest follows from the analysis in Section 3.

To prove that the constellation described in the proposition is an equilibrium, we must also show that the converse is true, i.e., if the lender offers t^*, the market cannot make a positive offer at $\tau = 1$. Specifically, we will show that (i) even under the "most optimistic beliefs" the market cannot make an offer which attracts accepted borrowers (*Step 2b*), and (ii) rejected borrowers have a negative NPV (*Step 2c*). Together, Steps 2b-c imply that if the lender offers t^*, the market cannot make a positive offer at $\tau = 1$.

Step 2b. Since any market offer attracting accepted borrowers also attracts all rejected borrowers, the "most optimistic beliefs" are those where—besides all rejected borrowers—all accepted borrowers visit the market at $\tau = 1$. By definition, these are the prior beliefs $F(s)$. Given these beliefs, the expected NPV of a borrower visiting the market at $\tau = 1$ is μ. Let t_m denote the contract offered by the market.[41] For the market to break even, t_m must satisfy

$$\int_{s^*}^{1} [\mu_s - u_s(t_m)]\, f(s)ds + \int_{0}^{s^*} [\mu_s - u_s(t_m)]\, f(s)ds = \mu, \tag{24}$$

i.e., borrowers visiting the market at $\tau = 1$ must receive on average μ. Consider next the optimal debt contract t^*. By construction (see Section 3.1), t^* satisfies

$$\int_{s^*}^{1} [\mu_s - u_s(t^*)]\, f(s)ds = \mu. \tag{25}$$

Since $\int_0^{s^*} [\mu_s - u_s(t_m)]\, f(s)ds > 0$, (24) and (25) together (after dividing through by $1 - F(s^*)$) imply that

$$\int_{s^*}^{1} [\mu_s - u_s(t^*)]\, \frac{f(s)}{1 - F(s^*)} ds > \int_{s^*}^{1} [\mu_s - u_s(t_m)]\, \frac{f(s)}{1 - F(s^*)} ds. \tag{26}$$

[41] Below we show that if the lender offers t^*, rejected borrowers have a negative NPV. Again, this then implies that there cannot exist a market offer that is separating and generates zero profit.

The left-hand side is the expected payoff of an accepted borrower under t^*, while the right-hand side is the expected payoff of an accepted borrower under t_m.[42] Hence an accepted borrower prefers t^* over t_m even though t_m is the highest possible market offer, i.e., the offer under which the market breaks even if and only if it attracts—in addition to attracting rejected borrowers—also attracts all accepted borrowers.

Step 2c. The lender's expected payoff under t^* is

$$\int_{s^*}^{1} \left[u_s(t^*) - k\right] f(s)ds > 0,$$

where the sign follows from $u_{s^*}(t^*) = k$ and the fact that $u_s(t^*)$ is increasing in s by Assumptions 1 and 2. In conjunction with (25), this implies that

$$\int_{s^*}^{1} \left[\mu_s - k\right] f(s)ds > \mu.$$

Since $\mu := \int_{s^*}^{1} \left[\mu_s - k\right] f(s)ds + \int_{0}^{s^*} \left[\mu_s - k\right] f(s)ds$, this implies that $\int_{0}^{s^*} \left[\mu_s - k\right] f(s)ds < 0$, and therefore that

$$\int_{0}^{s^*} \left[\mu_s - k\right] \frac{f(s)}{F\left(s^*\right)} ds < 0, \tag{27}$$

i.e., the expected NPV of financing a rejected borrower is negative.[43] In a certain sense, Step 2c is the mirror image of Step 1. Starting out from the fact that the lender makes a positive profit, it shows that a rejected borrower cannot obtain financing by the market. Step 1 shows the converse, i.e., if a (rejected) borrower can obtain financing by the market, the lender cannot make a positive profit.

Step 3. It remains to establish uniqueness. Evidently, there cannot exist an equilibrium in which the lender offers $T \neq t^*$ and the market makes a positive offer at $\tau = 1$. As was shown in Step 1, the lender would then make zero profit, whereby by offering t^* she makes a positive profit (Steps 2b-c.) There also cannot exist an equilibrium in which the lender offers $T \neq t^*$ and the market does not make a positive offer at $\tau = 1$. As was shown in Step 2a, if the market does not make a positive offer at $\tau = 1$, the unique optimal offer is t^*. This completes the proof of Case 1 in Proposition 4 ($V_{\max} > \mu$).

[42] Since an accepted borrower knows he was accepted, his beliefs regarding his own type (signal) have the density $f(s)/[1 - F(s^*)]$ for $s > s^*$ and zero otherwise.

[43] If the lender offers t^*, the market knows from Step 2b that a borrower visiting the market must have been rejected. The expression in (27) is the NPV of financing a borrower conditional on knowing that he was rejected.

As for Case 2 ($V_{\max} < \mu$), if the lender approves a loan with positive probability, strict monotonicity of $U_s(T)$ implies that she realizes a strictly positive expected payoff. We know from Step 1 that in this case the market cannot make a positive offer at $\tau = 1$. Consequently, if the lender makes an offer under which the borrower is approved with positive probability, the borrower's expected payoff from approaching the lender is strictly less than $V_{\max}$. The borrower can, however, realize $\mu > V_{\max}$ by immediately turning to the market. **Q.E.D.**

Proof of Proposition 6. The proof makes use of the following auxiliary result.

Claim 1. *Take a debt contract $t(x)$ and some other contract $\widetilde{t}(x) \neq t(x)$ such that $U_{\hat{s}}(\widetilde{t}) \geq U_{\hat{s}}(t)$ holds for some $\hat{s} < 1$. Then it must hold that $U_s(\widetilde{t}) > U_s(t)$ for all $s > \hat{s}$.*

Proof. The argument follows the logic of Proposition 2. We argue to a contradiction and suppose that at some $s > \hat{s}$ we have $U_s(\widetilde{t}) \leq U_s(t)$. By continuity of $U_s(t)$ and $U_s(\widetilde{t})$, this—together with $U_{\hat{s}}(\widetilde{t}) \geq U_{\hat{s}}(t)$—implies the existence of some $\widetilde{s}$ where $\hat{s} < \widetilde{s} < s$ and $U_{\widetilde{s}}(\widetilde{t}) = U_{\widetilde{s}}(t)$. Next, $U_{\widetilde{s}}(\widetilde{T}) = U_{\widetilde{s}}(T)$, together with Assumption 2 and the fact that $t(x)$ is debt implies the existence of some value $0 < \widetilde{x} < \overline{x}$ such that $t(x) \geq \widetilde{t}(x)$ for all $x < \widetilde{x}$ while $t(x) \leq \widetilde{t}(x)$ for all $x > \widetilde{x}$, where the inequalities are strict on sets of positive measures. By Assumption 1 we thus have—in analogy to the proof of Proposition 2—that

$$U_{\hat{s}}(\widetilde{t}) - U_{\hat{s}}(t) < \frac{g_{\hat{s}}(\widetilde{x})}{g_{\widetilde{s}}(\widetilde{x})} \left[U_{\widetilde{s}}(\widetilde{t}) - U_{\widetilde{s}}(t)\right] = 0,$$

which yields a contradiction. **Q.E.D.**

Consider first the case where the borrower offers some contract $\widetilde{t}(x)$ to replace the optimal (debt) contract $t^*(x)$. If the lender prefers $\widetilde{t}(x)$ over $t^*(x)$ for some $\hat{s}$, then she strictly prefers $\widetilde{t}(x)$ for all $s > \hat{s}$ by Claim 1. Consequently, the borrower can only be better off under $\widetilde{t}(x)$ if it lowers the cutoff signal, i.e., if $s^*(\widetilde{t}) < s^*(t^*)$. In this case, the loan is approved under the new contract for all $s \geq s^*(\widetilde{t})$, implying that the offer is profitable for the borrower only if $V(\widetilde{t}) \geq V(t^*)$. Existence of some contract $\widetilde{t}(x)$ where $V(\widetilde{t}) \geq V(t^*)$ and $s^*(\widetilde{t}) < s^*(t^*)$ would contradict the optimality of t^* in the lender's original problem, however.

If the lender makes the offer we have a signaling game. For the sake of brevity, we restrict attention to equilibria where in case of indifference, the borrower accepts the lender's offer. We argue to a contradiction and suppose that in a given equilibrium there is a nonempty set of accepted new contracts denoted by T. Denote by $\widetilde{T}$ the union of T and the optimal (debt) contract $t^*(x)$. In this equilibrium, the loan is approved for all $s \geq s^*(\widetilde{T})$. Denote by $\widetilde{t}(x)$ one possible contract that is chosen at $s = s^*(\widetilde{T})$. By our previous arguments, we know that unless $s^*(\widetilde{T}) < s^*(t)$ we cannot have an equilbrium where the borrower accepts the lender's offer. By Claim 1, it then follows that *all* lenders with signals $s > s^*(\widetilde{T})$ strictly prefer to offer a new contract. Moreover, in equilibrium they will offer their most preferred contract from the set T, which implies the upper boundary for the borrower's payoff in the equilibrium under consideration is given by $T = \widetilde{t}(x)$. By our previous arguments, however, the borrower's expected payoff will then be strictly less than $V(t)$, which implies he rejects. **Q.E.D.**

Proof of Proposition 8. In the following, we prove the optimality of debt for the borrower's (interim) moral hazard problem. The rest follows from the argument in the main text.

Suppose, to the contrary, that some non-debt contract $t(x)$ was optimal. We can then find a unique debt contract $\tilde{t}(x)$ such that

$$\int_{\underline{x}}^{\overline{x}} [\tilde{t}(x) - t(x)] g(x \mid s, e_h) dx = 0. \tag{28}$$

By Assumption 2, there exists a value $\hat{x} \in (\underline{x}, \overline{x})$ such that $\tilde{t}(x) - t(x) \geq 0$ for all $x < \hat{x}$ and $\tilde{t}(x) - t(x) \leq 0$ for all $x > \hat{x}$, where both inequalities are strict over sets of positive measure. We then have that

$$\begin{aligned} \int_{\underline{x}}^{\overline{x}} [\tilde{t}(x) - t(x)] g(x \mid s, e_l) dx &= \int_{\underline{x}}^{\overline{x}} [\tilde{t}(x) - t(x)] g(x \mid s, e_h) \frac{g(x \mid s, e_l)}{g(x \mid s, e_h)} dx \\ &> \frac{g(\hat{x} \mid s, e_l)}{g(\hat{x} \mid s, e_h)} \int_{\underline{x}}^{\overline{x}} [\tilde{t}(x) - t(x)] g(x \mid s, e_h) dx \\ &= 0, \end{aligned} \tag{29}$$

where the last two lines follow from Assumption 1a and (28), respectively. From this it follows that the expected agency rent (14) is strictly lower under $\tilde{t}(x)$. It remains to show that the borrower's incentive-compatibility constraint (13) is satisfied under $\tilde{t}(x)$. Using (28) together with (29), we have that

$$\int_{\underline{x}}^{\overline{x}} [\tilde{t}(x) - t(x)] \left[g(x \mid s, e_h) - g(x \mid s, e_l) \right] dx < 0,$$

implying that (13) is satisfied. **Q.E.D.**

Proof of Proposition 9. For a given schedule of investment levels $k(s)$, define $\hat{\mu}_s := \mu(s, k(s)) - k(s)$, $\hat{U}_s := U_s(t(x,s), k(s)) - k(s)$, and $\hat{V}_s := V_s(t(x,s), k(s))$. By standard arguments, the menu is incentive compatibile if $\hat{U}_s$ is strictly increasing, continuous, and a.e. differentiable for all $s \geq s^*$. The corresponding local condition requires that $d\hat{U}_s/ds = \partial\hat{U}_s/\partial s$ at points of differentiability. We begin with two auxiliary results.

Claim 1. *If $\overline{V} > 0$ the first-best outcome cannot be attained.*

Proof. Suppose to the contrary that $\overline{V} > 0$, $s^* = s_{FB}$ and $k(s) = k_{FB}(s)$ for all $s \geq s^*$. Next, observe that if $t(x,s) = x$ for *some* $s \geq s^*$, it must hold that $d\hat{V}_s/ds = 0$. To see this, note that incentive compatibility in conjunction with $t(x,s) = x$ implies $d\hat{U}_s/ds = \partial\hat{U}_s/\partial s = \partial\hat{\mu}_s/\partial s$. Since $k(s) = k_{FB}(s)$, we have (by the envelope theorem) $d\hat{\mu}_s/ds = \partial\hat{\mu}_s/\partial s$, which implies $d\hat{U}_s/ds = d\hat{\mu}_s/ds$ and therefore $d\hat{V}_s/ds = 0$. Moreover, $s^* = s_{FB}$ implies $t(x,s^*) = x$ and therefore $\hat{V}_{s^*} = 0$. Together with the first observation, this implies that $\hat{V}_s = 0$ for all $s \geq s^*$, contradicting $\overline{V} > 0$. **Q.E.D.**

Claim 2. *For any $k > 0$, $\hat{s} < 1$, debt contract $\tilde{t}(x)$, and non-debt contract $t(x)$ satisfying $U_{\hat{s}}(t,k) \geq U_{\hat{s}}(\tilde{t},k)$, it holds that*

$$\left.\frac{dU_s(t,k)}{ds}\right|_{s=\hat{s}} > \left.\frac{dU_s(\tilde{t},k)}{ds}\right|_{s=\hat{s}}. \tag{30}$$

Proof. Differentiability of U_s follows from differentiability of $g_s(x \mid k)$. If $U_{\hat{s}}(t,k) = U_{\hat{s}}(\tilde{t},k)$ the claim follows immediately from Claim 1 in Proposition 6. As for the case $U_{\hat{s}}(t,k) > U_{\hat{s}}(\tilde{t},k)$, consider the uniquely defined debt contract $\hat{t}(x)$ given by $U_{\hat{s}}(t,k) = U_{\hat{s}}(\hat{t},k)$ (which implies that $U_{\hat{s}}(\hat{t},k) > U_{\hat{s}}(\tilde{t},k)$). Since

$$\left.\frac{dU_s(t,k)}{ds}\right|_{s=\hat{s}} > \left.\frac{dU_s(\hat{t},k)}{ds}\right|_{s=\hat{s}}$$

by Claim 1 in Proposition 6, it remains to show that

$$\left.\frac{dU_s(\hat{t},k)}{ds}\right|_{s=\hat{s}} \geq \left.\frac{dU_s(\tilde{t},k)}{ds}\right|_{s=\hat{s}}. \tag{31}$$

Partial integration yields $U_s(\tilde{t},k) = \widetilde{R} - \int_{\underline{x}}^{\widetilde{R}} G_s(x \mid k)dx$. As $\hat{R} > \widetilde{R}$, (31) holds if $dG_s(x \mid k)/ds < 0$ for all $k > 0$, $x \in (\underline{x}, \overline{x})$, and s, which holds by Assumption 1. **Q.E.D.**

We are now in the position to show that any optimal menu must contain only debt contracts. We argue to a contradiction. Suppose some menu $T = \{t(x,s), k(s)\}$ with corresponding cutoff s^* is optimal but some contracts in the menu are non-debt contracts.

Claim 3. *Consider two incentive-compatible menus with the same cutoff s^* and investment schedule $k(s)$. One of them, $\widetilde{T} = \{\tilde{t}(x,s), k(s)\}$, consists exclusively of debt contracts while* *the other, $T = \{t(x,s), k(s)\}$, does not (on a set of positive measure). It then holds that $U_s(t(x,s),k(s)) \geq U_s(\tilde{t}(x,s),k(s))$ for all $s \geq s^*$, with strict inequality for all $s > \hat{s} \geq s^*$.*

Proof. The proof follows almost immediately from Claim 2. Let $\hat{s} \in [s^*, 1)$ denote the first signal $s \geq s^*$ for which T prescribes a non-debt contract, $t(x,\hat{s})$. We can replace $t(x,\hat{s})$ with the uniquely defined debt contract $\tilde{t}(x,\hat{s})$ given by $U_{\hat{s}}(t(x,\hat{s}),k(\hat{s})) = U_{\hat{s}}(\tilde{t}(x,\hat{s}),k(\hat{s}))$. By Claim 2 and local incentive compatibility, we then have that

$$\left.\frac{\partial U_s(\tilde{t},k(\hat{s}))}{\partial s}\right|_{s=\hat{s}} < \left.\frac{\partial U_s(t,k(\hat{s}))}{\partial s}\right|_{s=\hat{s}}.$$

We continue this procedure for all other $s > \hat{s}$ for which T prescribes non-debt contracts. For signals $s > s^*$ for which T prescribes debt contracts we simply take those. In the end, we obtain an incentive-comaptible menu of debt contracts $\widetilde{T}$ with the same cutoff signal and investment schedule as T satisfying

$$\frac{\partial U_s(\tilde{t},k(s))}{\partial s} \leq \frac{\partial U_s(t,k(s))}{\partial s}$$

for all $s \geq s^*$, with strict inequality for some $s \geq \hat{s}$ on sets of positive measure. As $dU_s/ds = \partial U_s/\partial s$ by local incentive compatibility and $U_{s^*}(t(x,s^*),k(s^*)) = U_{s^*}(\tilde{t}(x,s^*),k(s^*))$ by construction, integrating yields $U_s(t(x,s),k(s)) \geq U_s(\tilde{t}(x,s),k(s))$ for all $s \geq s^*$, with strict inequality for $s > \hat{s} \geq s^*$. **Q.E.D.**

In words: starting from an arbitrary, incentive-compatible menu T we have constructed an incentive-compatible menu $\widetilde{T}$ of debt contracts such that s^* and $k(s)$ are the same under both menus but the borrower's participation constraint (19) is relaxed. In analogy to the Proof of Proposition 2, we can now fine-tune $\widetilde{T}$—either by improving s^* and/or $k(s)$—until (19) binds. Call this adjusted, final menu of debt contracts $\hat{T}$. As the borrower is no better off under $\hat{T}$ compared to T (if (19) binds under T he is equally well off, otherwise he is worse off), but $\hat{T}$ has a lower cutoff signal and/or a more efficient investment schedule than T, the lender must be strictly better off under $\hat{T}$.

The argument proceeds in two steps. First, we change the investment schedule from $k(s)$ to $\hat{k}(s) := (1-\gamma)k(s) + \gamma k_{FB}(s)$ by increasing γ. For each increase in γ, we must adjust the corresponding repayments in the menu to preserve incentive compatibility and to ensure that $U_{s^*}(\hat{t}(x,s^*),\hat{k}(s^*)) = \hat{k}(s^*)$ holds. Note that, by strict quasi-concavity of $\mu_s(k)$, $\mu_s(\hat{k}(s)) - \hat{k}(s)$ is nondecreasing in γ and strictly increasing in γ for all s with $k(s) \neq k_{FB}(s)$. Moreover, all expected payoffs change continuously in γ.

We now have two cases. In Case 1 the borrower' participation constraint (19) becomes binding at some $\gamma \leq 1$, which concludes the proof. Conversely, if (19) remains slack at $\gamma = 1$ (in which case we have the first-best investment schedule $k_{FB}(s)$), we are in Case 2. By Claim 1, it must then hold that $s^* > s_{FB}$. We can consequently increase the repayment associated with the cutoff debt contract—thereby lowering the cutoff signal—until (19) becomes binding. (To preserve incentive compatibility, we must also increase all remaining repayments in the menu, and to remain at the first-best investment level we must specify $k(s) = k_{FB}(s)$ for all "newly added signals".) By Claim 1, (19) must eventually bind at *some* signal $s^* > s_{FB}$. Irrespective of whether Case 1 or 2 obtains, we have constructed an incentive-compatible menu of debt contracts $\hat{T}$ at which the borrower's participation constraint (19) binds and that implements a more efficient investment schedule and/or a lower cutoff signal than the original menu T. **Q.E.D.**

8 Appendix B: Numerical Example in Section 4.3

The ex-ante project NPV μ is

$$\mu = \int_0^1 \mu_s f(s) ds - k = \ln\left(\frac{S}{S-1}\right) - k,$$

where $\mu_s = 1/(S-s)$ and $f(s) = 1$ from the uniform distribution. The lender's expected payoff conditional on the signal s is then (using partial integration)

$$u_s(t) = \int_0^R x g_s(x) dx = \frac{1}{S-s}[1 - e^{-(S-s)R}].$$

It follows that the lender's privately optimal cutoff signal s^* is given by

$$\frac{1 - e^{-(S-s^*)R}}{S - s^*} = k,$$

which implies that

$$R = \frac{-\ln[1 - (S - s^*)k]}{S - s^*}. \tag{32}$$

Note that s^* is strictly decreasing in R with $s^* \to s_{FB} = S - 1/k$ as $R \to \infty$. Finally, the borrower's expected payoff from going to the inside lender is

$$V(t) = \int_{s^*}^1 [\mu_s - u_s(t)] f(s) ds = \int_{s^*}^1 \frac{1}{S-s} e^{-(S-s)R} ds, \tag{33}$$

where R and s^* satisfy (32).

To obtain $V_{\max}$ we must maximize (33) with respect to R subject to (32). Since R enters in (33) both directly and indirectly via s^*, and hence via (32), we cannot express $V_{\max}$ in closed form. Instead, we must solve for $V_{\max}$ numerically; the results are shown in Figure 1.

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