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Understanding Fee Structures in the Asset Management Business

Anthony W. Lynch, David K Musto

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by
Anthony W. Lynch *
and
David K. Musto **

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* Department of Finance, New York University, 44 West 4th Street, NY, NY 10012. Phone: (212)998-0350. Email: alynch@stern.nyu.edu. Fax: (212)995-4233
** Wharton School, University of Pennsylvania, 2300 SH-DH, 3620 Locust Walk, Philadelphia, PA 19104-6367. Phone: (215)898-3609. Email: musto@wharton.upenn.edu. Fax : (215)898-6200.

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Understanding Fee Structures in the Asset Management Business

This paper considers the economic role of fees in aligning the incentives of money managers with those of investors. We examine a simple model in which manager effort (or investment in human and physical capital) is observed by the investor prior to her investment decision, but is not verifiable. This setup creates a positive economic role for net asset value (NAV) as a contracting variable and thus provides an explanation for the widespread use of contracts based on NAV in both the mutual and hedge fund industries. We also provide an explanation for why hedge funds use asymmetric performance fees while mutual funds typically charge a fixed fraction of NAV (even though "fulcrum" performance fees are available). Put simply, performance fees (asymmetric and "fulcrum") are better able to extract effort than a fee which is a fixed fraction of NAV. Since hedge fund managers are typically more skilled than mutual fund managers, the extra effort that can be extracted by a performance fee has a greater benefit for a hedge fund than a mutual fund. At the other extreme, a fixed fraction of NAV can do almost as well as a fraction that increases with fund return if managerial skill is low (i.e., a mutual fund). The trade-off between inducing effort and risk-sharing is also analyzed.

Understanding Fee Structures in the Asset Management Business

In recent years, the mutual fund industry has grown explosively. However, contract choices have been limited (since 1971) to those that pay the manager a fixed fraction of Net Asset Value (the fixed NAV fraction) and/or a performance-based fee that is symmetric with respect to performance relative to a benchmark and proportional to NAV (the "fulcrum" performance fee).[1] Despite the availability of the "fulcrum" performance fee, the dominant contract choice in the mutual fund industry is the fixed NAV fraction.[2] In contrast, the typical contract in the hedge fund industry (which exists along-side the mutual fund industry) has a base fixed fraction fee plus an asymmetric performance fee that is proportional to NAV (see Brown, Goetzmann and Ibbotson (1997)).[3]

Since contract design can be used to address agency problems between the fund manager and investor, understanding why mutual and hedge funds use contracts based on NAV is an important topic. At the same time, government regulation restricts the set of contracts available in the mutual fund industry. An interesting question is whether this regulation affects the attractiveness of mutual funds to investors. Finally, understanding why the typical contract choice differs so widely across

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[1]Mutual fund management fees are restricted by the Investment Company Act of 1940. The Act was originally construed to prohibit all performance fees, but a 1970 amendment exempted fees known as "fulcrum" which are symmetric in the fund's return relative to a benchmark. A typical "fulcrum" fee has the following form:
$\kappa(D_{t+1})\, D_{t+1}\, NAV_t$

where NAV_t is the Net Asset Value at the start of the period, D_{t+1} is the deviation of the fund's return from the benchmark's return over the period, and, $\kappa(.)$ is a function from $\mathbb{R}$ to $\mathbb{R}+$ with the property that $\kappa(D_{t+1}) = \kappa(-D_{t+1})$ for all $D_{t+1} \in \mathbb{R}+$. So if $\kappa(0.1)=0.05$ then $\kappa(-0.1)=0.05$. A simple "fulcrum" fee sets $\kappa(.)$ equal to a constant (see Golec(1987) and Starks(1987)).

[2]Golec (1987) identified a "fulcrum" performance fee in only 27 of 370 funds he studied; the rest charged a fixed fraction of NAV.

[3]Hedge funds are not allowed to advertize and must have less than 99 investors who all must be "accredited" (see Brown, Goetzmann, and Ibbotson (1997) for further details).

the mutual and hedge fund industries is also of interest. In particular, an interesting question is why performance fees are widely used in the hedge fund industry but "fulcrum" fees are rarely used by mutual funds.

To address these issues, this paper develops a simple model that considers the role of the fee structure in aligning fund manager incentives with those of investors.[4] We allow the investor to observe managerial effort (which is not verifiable) prior to her decision to invest, which creates an economic role for the fixed NAV fraction contract. In particular, this contract allows the manager's compensation to depend on NAV, which in turn depends on effort. We compare the fixed NAV fraction to a contract that is not currently available to the mutual fund industry, but which has a number of interesting properties. This contract (the return-based contract) charges a fee that depends solely on the fund's return, and not on the NAV of the fund.[5] We find that the fixed NAV fraction can do better, especially when return is a noisy signal of manager effort. We also investigate a contract that pays a fixed fraction of NAV at the end of the period and find that it is usually better than a contract that pays a fixed fraction of NAV at the start.

We then use the model to assess the relative attractiveness of allowing the NAV fraction to vary depend on the fund's return (return-dependent NAV fraction). This type of contract is available to both hedge and mutual funds. Hedge fund managers typically combine a fixed fraction of NAV with an asymmetric performance fee to obtain an NAV fraction that is larger when fund return is

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[4]Earlier papers addressing this issue in settings without adverse selection include Starks (1987), Stoughton (1993), Admati and Pfleiderer (1997), and Grinblatt and Titman (1989), among others.

[5] It is natural to use return-based contracts as a benchmark since return being the only signal of manager effort is an alternative information structure. With this information structure, the optimal return-based contract is the optimal contract (see, for example, Holmstrom (1979)).

high.[6] Mutual funds can use a "fulcrum" performance fee in conjunction with a fixed NAV fraction to also obtain a return-dependent NAV fraction; however, they rarely do. While the extent to which the NAV fraction can vary with return is more limited than for hedge funds, call-like payoffs are still possible using "fulcrum" fees.[7]

We will show that, when managerial ability is high, a contract with an NAV fraction that is increasing in the fund's return is much more attractive than a fixed fraction. The greater effort that the former contract induces (relative to a fixed NAV fraction) is more beneficial when that effort is more productive. Since hedge fund managers are typically regarded as the best fund managers, the ability of a return-dependent NAV fraction to better extract effort may explain its prevalence in that industry. At the other end of the spectrum, the optimal return-dependent fraction of NAV performs negligibly better than a fixed fraction when managerial ability is low: the benefit from inducing extra effort relative to the fixed fraction contract is small. Thus, any return-dependent fraction of NAV that can be obtained via a "fulcrum" performance fee must also perform only slightly better than a fixed fraction of NAV. So the reluctance of mutual funds to adopt a return-dependent fraction via "fulcrum" fees may be due to the low ability levels of mutual fund managers. Further, the unavailability of asymmetric performance fees in the mutual fund industry (which would allow even more flexibility in how the fraction of NAV could be varied with return) may be having

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[6]According to Brown, Goetzmann and Ibbotson (1997), hedge fund managers face an asymmetric performance contract with a base fixed fraction fee (around 1%) plus a percentage (around 20%) of profits above some benchmark.

[7] For example, assume a benchmark of 0% return. Then a 1% (of NAV at end) base fee, together with a 5% fee for a fund return's deviation from benchmark (multiplied by NAV at the start), implies a 0.445% fee for a -10% fund return, but a 1.455% fee for a 10% fund return. Allowing the "fulcrum" fraction to decline as the deviation increases can put a floor on the fraction charged while rewarding the manager for high returns with a bigger fraction.

only a small effect on the attractiveness of mutual funds.

Finally, we examine a more general contract, which is currently not available in the mutual fund industry and not in use in the hedge fund industry. This contract is a fixed fraction of NAV together with a fee, unrelated to NAV, that depends solely on fund return. It does better than even the return-dependent NAV fraction contract and can get quite close to the first-best, even when the opportunity set is poor. Thus, a contract that is return-based, together with a fee that is a fixed fraction of NAV, seems to be better than the existing menu of contracts available in the mutual fund industry and in use in the hedge fund industry. The implication is that less regulation by the SEC of mutual fund fee structures may make that industry even more attractive. Also, a closer investigation of the fee structures used in the hedge fund industry may be informative.

Our model has an investor with no management ability and a manager with no money. Both are risk averse (log utility), and the manager's utility decreases at an increasing rate with effort. The manager can earn either a high or low return, and the probability of a high return increases with effort. Effort is precomitted by the manger before the investor invests, and the investor can observe the effort choice before investing. However, the investor can't write the effort choice into a contract because he can't verify it in court. So he can't pay the manager per unit of effort, but has to rely instead on the contract to induce it. Section II describes the model more formally.

We consider the utility implications for the investor of a menu of contracts whose parameters are chosen to ensure that the manager gets her reservation utility. While closed form solutions for investor utility seem unattainable, we are able to numerically solve this quantity for each of the contracts.

We also consider the risk-sharing properties of the various contracts. The effort and NAV choice induced by a contract are taken as given and investor utility from the contract is compared to the utility obtained with optimal risk-sharing, rather than the contract payments. These risk-sharing properties are interesting since effort and investor utility can (and do) give different rankings across the contracts considered. The likely reason for higher effort but lower investor utility is poorer risk-sharing. For example, the optimal return-based contract often induces more effort than the fixed NAV fraction contract, but at the cost of extremely poor risk-sharing. This poorer risk-sharing can outweigh the greater effort, leaving the investor worse off than under the fixed fraction contract.

Risk-sharing is also an important consideration when comparing the fixed NAV fraction to the return-dependent NAV fraction. The latter has poorer risk sharing properties. In fact, as the fraction paid in the good state increases, the risk-sharing associated with the payments becomes poorer, even as the effort induced increases. So when managerial ability is low, the marginal cost of poorer risk sharing quickly outweighs the marginal benefit of greater effort, and the resulting improvement relative to the fixed fraction contract is small. However, when ability is high, the good state fraction must become very large relative to the bad state fraction before the marginal cost of poor risk-sharing dominates. Consequently, the improvement in investor utility relative to the fixed fraction contract is large, as is the increase in the effort level.

The rest of the paper is in four sections. Section I discusses the key features of our model and relates it to the existing literature, while the model itself is formally described in section II. Section III presents numerical solutions obtained for the different contract structures considered, and the last section concludes.

I. Features of the Model and Related Literature

Our model has two important features that together allow a positive economic role for the fixed NAV fraction contract. First, investors receive information which is not contractually verifiable about the productive behavior of fund managers. While we refer to this productive behavior by managers as effort, it is much broader than the usual notion of effort. Decisions by the manager regarding infrastructure, trading strategies, number of analysts are all dimensions of the variable we are calling effort. In particular, the manager invests in human and physical capital that he hopes will allow the fund to earn superior returns in the future. The many reports and publications about fund management contain information about these aspects of the fund. Given the high demand for these publications, investors must believe that they contain relevant information for their portfolio allocations. Client visits to a mutual fund are another source of information for investors.[8]

Second, the investor observes the effort level of the manager, and then decides how much money to place in the fund based on this observation. This seems a reasonable characterization of how investors allocate money to a fund. To the extent that the manager's productive activity is summarized in fund publications and press reports, or is observed during a client visit, investors are adjusting their portfolios in response to the information. Since the information is not verifiable, it

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[8]The ability of investors to observe managerial effort can not be underestimated. Entire periodicals (e.g. *Mutual Funds*) are devoted to the topic, and sell alongside statistical summaries of realized returns. For example, an article in the Wall Street Journal in 1996 talked about a newsletter published by Eric Kobren about particular Fidelity funds. The article discussed how money had been flowing out of Fidelity's Export Fund (despite good performance) in response to a sell recommendation in the Kobren newsletter. The information in this newsletter and investors reaction to it is an example of the mechanism that we are arguing gives the fixed fraction fee a positive economic role.

is not feasible to write a contract which ties compensation directly to this information. But since the investor observes the information prior to deciding her level of investment in the fund, the fixed fraction contract is a way around this problem. By making the manager's payment proportional to NAV, it translates the investor's positive or negative reaction to the information into more or less wealth for the manager. Thus, a fixed fraction of NAV can be an improvement over the optimal return-based contract, which allows compensation to depend only on a noisy signal of managerial effort.

While this model draws heavily on the principal-agent literature, it differs from the standard setup in one important respect: the sequencing of choices by the principal and the agent. Where the canonical setup has the agent as the last mover making an unobservable effort choice, our mutual fund model has the principal moving (choosing NAV) after the agent makes an observable effort choice. This modification delivers the strategic considerations that characterize the mutual fund industry and drive the use of the fixed fraction contract.

While we assume that the investor can observe the manager's effort level, in reality the investor is likely to observe only a signal of manager effort and not effort itself. Of course, the usefulness of the fixed fraction contract will be declining in the noisiness of this effort signal. However, since our model shows that the fixed fraction contract can dominate the optimal return-based contract when the investor's signal of effort contains no noise, our results indicate that the fixed NAV fraction contract can continue to dominate when that signal is noisy: so long as the signal is not *too* noisy.

Many reasons exist for investigating the positive economic role played by NAV as a contracting variable in the context of the fixed fraction contract. First, the reluctance of mutual funds to use (or regulators to allow) return-based or performance contracts is a reason. Second, the principal-agent literature would suggest that a return-based contract is optimal in settings in which effort is unobservable and return is the only signal of effort that the investor receives (see Holmstrom (1979)). A third reason is the presence of channels through which the fixed fraction contract has a negative impact on aligning incentives. In particular, NAV becomes a noisy signal of effort to the extent that the investor's decision depends on variables unrelated to effort. In reality, the investor's portfolio decision is likely to depend on income shocks, taste shocks, and the returns on his other investments, among other things. The induced noisiness of NAV as a signal of effort reduces its value as a contract variable.[9]

A number of papers have considered the fee's role in aligning the incentives of managers with those of investors. In particular, the extant literature argues that contract choice can help mitigate both moral hazard and adverse selection problems. For example, Bhattacharya and Pfleiderer (1985) and Heinkel and Stoughton (1994) show that the expected benefit from screening out bad managers affects the optimal contract design when manager candidates privately know their skill. Huberman and Kandel (1993) and Huddart (1995) consider how signaling considerations can lead good managers to take on excessive risk. Admati and Pfleiderer (1997) evaluate the use and choice of a benchmark when the manager has more information about stock returns than the investor. Grinblatt and Titman (1989) and Starks (1987) take skill as given, and show that asymmetric incentive fees bias the manager toward higher-variance portfolios.

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[9] Recent empirical work indicates that fund flows are related to past fund returns (see Chevalier and Ellison (1995) and Sirri and Tufano (1992)). It could be argued that contracts based on NAV are thus implicitly based on fund return. However, the unanswered question, especially given the noisy relation between fund flow and past fund return, is why contracts are not written directly on fund return. Our paper provides at least a partial answer.

Our paper is in the tradition of Stoughton (1993) and Starks (1987), focusing on the manager's incentive to allocate time and resources. But where those papers analyze managerial effort allocations that are not observable, we concentrate instead on an effort allocation which is observable, but not contractible and which occurs prior to the investor's allocation of money to the fund. Also, our paper compares contracts based on investor utility, subject to the constraint that the manager receive her reservation utility. Thus, our paper extends the earlier work by Starks (1987) and Stoughton (1993) that focussed on how the effort choice of the manager varies across different contracts.[10]

Using this basis for comparison, we also find that a return-dependent fraction of NAV can do significantly better than a fixed fraction, but only if the manager's opportunity set is high. At the other extreme, the difference between the two contracts can be negligible when the opportunity set is poor. We argued above that this result may explain the different contract choices across the hedge and mutual fund industries. However, for the argument to go through, hedge fund managers need to be more skilled. Since hedge funds are designed to bet on manager skill by being market neutral, it does not seem unreasonable. In fact, Brown, Goetzmann and Ibbotson (1997) find that the average performance of a sample of off-shore hedge funds is better than that documented for mutual funds (see for example, Gruber (1996), Elton, Gruber, Das and Hlavka (1993), Malkiel (1995) and Brown and Goetzmann (1995) among many others). While an extensive literature exists attempting to

[10]We maximize investor utility with respect to a set of contract parameters (for a given contract type), subject to incentive compatibility conditions and the constraint that the manager receive her reservation utility. As a result, the investor attains some utility U_I^*. It is worth noting that the contract parameters we obtain would solve the problem of maximizing manager utility subject to incentive compatibility conditions and the constraint that the investor receive at least U_I^* (at least for interior solutions). Thus, the ordering of contracts that we obtain is unlikely to be driven by our assumption that the manager has a reservation utility.

explain the contract structure in the mutual fund industry, our paper is the first to consider why contract structures differ across these two segments of the asset management business.[11]

II. The Model

There are two dates, 0 and 1, and two individuals, an investor and a manager. All wealth is consumed at date 1. At date 0, the investor has wealth W_0 and the manager has wealth 0. The investor's (expected) utility function is U_I=E[log(W_I)], where W_I is his date 1 wealth and E is the expectation operator, and the manager's utility function is U_M=E[log(W_M)]+log$(K-e)$, where W_M is his date 1 wealth, e is the his effort level, and K is a constant. If the manager manages money, the return is random, either $R_H=\eta+\sigma$ or $R_L=\eta-\sigma$, where η and σ are constants. Effort pays off as better management, in that the probability of earning R_H is $\frac{1}{2}+ze$, where z is a constant. If the manager doesn't manage money, he can earn expected utility U^* at some reservation activity. The riskfree rate of interest which is earned by money that is not managed is set equal to 0. This setup abstracts from risk-shifting motives for managerial choices which are important (see for example, Starks (1987), Grinblatt and Titman (1989) and Brown, Harlow and Starks (1996)) but not the focus of the current paper.[12]

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[11]Recent work by Goetzmann, Ingersoll and Ross (1997) examines the effect of "high water mark" provisions in hedge fund contracts, whereby previous years' losses have to be made up before the performance component of compensation kicks in. Also, work independent of ours by Nanda, Narayanan and Warther (1997) considers why mutual funds are structured as open- or closed-end and why some open-end funds charge loads.

[12]It is worth noting that this setup satisfies the monotone likelihood ratio property (MLRP) and the convexity of distribution function condition (CDFC) (see Hart and Holmstrom (1987) for definitions of these properties). In the canonical principal-agent problem, this setup has desirable properties. In particular, MLRP ensures that the optimal payment is monotonic in the signal (Milgrom (1981)) and MLRP and CDFC together ensure that the manager's incentive

The model's investor can be regarded as the representative investor in the fund. If there is more than one investor but they all have log utility and their starting wealths add to W_0 then the total amount those investors would place with the fund equals the amount that this representative investor would place with the fund.[13] The manager is assumed to be risk- averse since canonical principal-agent problems with risk-neutral agents obtain the first-best. The utility specification for effort has the desirable property that the marginal disutility from effort is increasing in effort. Note that the marginal disutility of effort at a given effort level is decreasing in K and that the marginal disutility of effort approaches ∞ as e approaches K. Since $e < 0.5/z$ is required for the probability of each state to be strictly positive, we always take $K<0.5/z$.

The chronology of choices goes like this. First, the investor and manager commit to the contract. Then the manager chooses effort level e, which is observable to the investor. Next the investor allocates a fraction α of his wealth to the fund, and does not have the option to renegotiate the contract. The manager invests the investor's allocation, and then date 0 finally ends. At date 1, the return is realized, the manager gets his compensation and the investor gets what is left. As discussed above, this chronology implies that the manager's effort level e is determined before the investor chooses how much to invest in the fund.

A number of contract structures are considered:

1) **RET** (return-based) contract: the manager receives H (expressed as a fraction of W_0) if the return is R_H and L (expressed as a fraction of W_0) if it is R_L.

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constraint can be replaced with a first order condition (Rogerson (1985)). The expectation, which is confirmed by our numerical solutions, is that our problem is similarly well behaved.

[13]For this result to hold for the return-based fees, the fee has to be split between the investors in proportion to their starting wealths.

2) **FOF** (fixed fraction of NAV at the start) contract: the manager receives δ of the money allocated to the fund at time 0.

3) **FOE** (fixed fraction of NAV at the end) contract: the manager receives ξ of the money in the fund at time 1.

4) **FOP** (return-dependent fraction of NAV at the end) contract: the manager receives a fraction of the money in the fund at time 1 where the fraction depends on the state; the manager receives fraction ξ_H if the return is R_H and ξ_L if it is R_L.

5) **RFE** (fixed fraction of NAV at the end plus a return-dependent fee) contract: the manager receives ξ_R of the money in the fund at time 1 plus H_R (expressed as a fraction of W_0) if the return is R_H and L_R (expressed as a fraction of W_0) if it is R_L.

Only the **FOE** and **FOP** contracts are available in the mutual fund industry and in use by hedge funds. By examining the other three contracts, we will gain a greater insight into why these are the only two contracts in use. Since the **RET** contract is the optimal contract when managerial effort is unobservable, it is a natural benchmark to use when assessing the value of NAV as a contracting variable in the current setting.[14]

The comparison between the **FOF(**δ**)** and **RET(**H,L**)** contracts illustrates the benefits of contracting on funds invested, relative to contracting on return. The **FOF(**δ**)** contract renumerates the manager solely on the funds invested at time 0 while the **RET(**H,L**)** contract allows the payment to the manager to depend on the return on the fund but not the amount invested with the manager.

[14]See Holmstrom (1979) for a discussion as to why NAV is not valuable as a contracting variable when effort is unobservable. Put simply, this information structure implies that NAV is unrelated to effort, and so NAV is of no value as a contracting variable.

Note that the **FOF(δ)** contract has undesirable risk-sharing properties relative to the **RET**(H,L) contract. The intuition is that both the manager and the investor are risk-averse in this model and yet the **FOF(δ)** contract pays the manager a fixed amount irrespective of whether R_H or R_L is realized.

This feature is avoided by letting the fee be a fraction of funds under management at the end of the period, as in the **FOE(ξ)** contract. Allowing the fee to be a fraction of the funds invested at the end of the period also accords better with industry practice (see Golec (1987), Grinblatt and Titman (1989), and Admati and Pfleiderer (1997)). However, this formulation allows compensation to depend both on the return and on α, making it difficult to assess the marginal value of α as a contracting variable. This argument provides a justification for comparing the **RET**(H,L) contract to the **FOF(δ)** contract to assess this marginal value of α. The benefits from contracting on a fixed fraction of NAV at the end rather than at the start can be assessed by comparing the **FOE(ξ)** contract to the **FOF(δ)** contract.

The incremental benefit from using a fraction of NAV that is return-dependent rather than fixed can be assessed by comparing the **FOP(ξ_H, ξ_L)** contract to the **FOE(ξ)** contract. Allowing the fraction to be higher in the good state than the bad is likely to improve managerial incentives to expend effort, but at the cost of poorer risk-sharing. The asymmetry that this **FOP** contract captures is analogous to contracts in the hedge fund industry where the manager receives a fixed fraction plus a fraction of any profit in excess of an appropriate benchmark. Here, the appropriate benchmark is the riskfree rate which is 0. So the **FOP(ξ_H, ξ_L)** can be interpreted as paying the manager a base fee of ξ_L of funds invested at time 1 and a fraction $(\xi_H - \xi_L)/\rho_H$ of any profits in excess of the benchmark, where $\rho_s = R_s/(1+R_s)$ for s=H or L. This **FOP** contract can also be obtained using a fixed fraction of NAV plus a "fulcrum" fee. In particular, an **FOP**(ξ_H, ξ_L) contract can be obtained using a base fee of (ρ_H ξ_L - ρ_L ξ_H)/ (ρ_H - ρ_L) of NAV at time 1 plus a "fulcrum" fee of (ξ_L -ξ_H)/ (ρ_H - ρ_L) of profits/losses relative to the riskless rate (so the "fulcrum" fee is negative when the fund return is negative).

Finally, an interesting question is the nature of the benefits associated with having a fraction of funds contract together with a return-based fee. One issue is the magnitude of these benefits relative to having either a fraction of funds contract alone or a return-based fee alone. This magnitude can be assessed by comparing the **RFE**(ξ_R,H_R,L_R) contract to the better of the **FOE**(ξ) and the **RET**(H,L) contracts. A second issue is the ability of a contract that is a return-dependent fraction of NAV to capture the benefits of a contract that has both a NAV-based and a return-based fee. This magnitude can be assessed by comparing the **FOP**(ξ_H, ξ_L) contract to the **RFE**(ξ_R,H_R,L_R) contract.

The z variable affects the opportunity set offered by the fund and the precision of fund return as a signal of the manager's effort e. As z increases with η held fixed, the opportunity set improves since, for a given e, the probability of the high return R_H increases. The precision of the return as a signal of e is also increasing as z increases. More formally, with a uniform prior and $e^*>e'$, the posterior likelihood ratio prob[$e^*|R_s$]/prob[$e'|R_s$] is increasing in z for s=H and is decreasing in z for s=L. Thus, the **FOF** contract is likely to be better than **RET** contract for low values of z when return is a very noisy signal of e.

When assessing a particular contract, contract parameters are chosen to maximize investor utility while insuring that the manager receives her reservation utility U^*.[15] We also characterize the first-best (**1ST**) solution for comparison purposes. Since the relative performance of the **RET** contract and the fixed fraction contracts is likely to depend on z, we examine the investor's utility from the various contracts as z is varied (holding the other parameters of the opportunity set fixed). As mentioned above, one drawback of this comparative static is that variation in z also affects the opportunity set. To ensure this effect is not driving our **RET-FOF** comparison, we also allow the parameter η to change as z is varied, so that the investor's utility from the **1ST** solution stays constant.

Anecdotal evidence together with recent work by Brown and Goetzmann (1997) suggests that hedge fund managers are more skilled than mutual fund managers. An interesting question is whether the relative attractiveness of the **FOP**(ξ_H, ξ_L) contract to the **FOE**(ξ) contract varies with the attractiveness of the opportunity set. Allowing z to vary while holding η fixed is one way to do vary the opportunity set, but the concern is the confounding effects of the associated variation in signal precision. A more direct way is to allow η to vary while holding z fixed and a third example is calculated that has this feature.

The next five subsections and the appendix describes each of the five contracts. For each contract, we derive the first-order conditions associated with the optimization problems facing the two agents. The algorithm for numerically obtaining the parameters of each contract so as to

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[15]Our paper is closest in spirit to the work of Stoughton (1993) who examines the ability of different contract structures to extract effort. In his framework, the manager must receive at least her reservation utility which is held fixed across contracts. We adopt the same assumption. In contrast, Starks (1987) compares a symmetric performance contract to a bonus contract, holding the contract parameters fixed. This difference may explain why some of her results concerning the effects of fee asymmetries on effort production differ from ours.

maximize investor utility subject to the manager receiving her reservation utility is as follows. Letting c denote the contract parameters, we solve for the optimal c for a contract in three steps: (1) determine the α which is optimal for the investor for a given set of contract parameters c and e, (2) calculate the e which is optimal for the manager given c and the impact of e on the investor's choice of α ; this gives us $U_M(c)$ and $U_I(c)$, the utilities that result from a given c, and (3) identify the c which maximizes $U_I(c)$ given that $U_M(c) \geq U^*$.[16] The last two subsections solve the first-best problem and the optimal risk-sharing problem for a given e and α.

Contract on realized returns

With the **RET** contract, the contract parameters are H and L, and the utility functions are

$$U_M(e,H,L) = (\frac{1}{2}+ze)\ \log[HW_0] + (\frac{1}{2}-ze)\ \log[LW_0] + \log[K-e] \tag{1}$$

and

$$U_I(e,\alpha,H,L) = (\frac{1}{2}+ze)\ \log[W_0(\alpha R_H+1-H)] + (\frac{1}{2}-ze)\ \log[W_0(\alpha R_L+1-L)] \tag{2}$$

With this contract, the α chosen by investor in response to the manager's choice of e does not affect the manager's utility. The manager just chooses the e which maximizes U_M given H and L, which is

$$e(H,L) = K - \frac{1}{z\log(H/L)} \tag{3}$$

Solving for the investor's optimal α given H, L and e, we get

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[16]Various safeguards are taken to ensure that the solution obtained is feasible, and that the incentive compatibility conditions of both the manager and investor are being satisfied by the solution.

$$\alpha(H,L,e) = \frac{1}{\sigma^2-\eta^2}\left[\eta + (2\sigma+R_L H - R_H L)\, z\, e - 0.5(R_L H + R_H L)\right] \quad \textbf{(4)}$$

which is linear in e. So now U_I and U_M can be expressed in terms of H and L using **(3)** and **(4)** (see the appendix). For a given set of parameters, we find the H and L which maximize $U_I\,(H,L)$ subject to $U_M(H,L)=U^*$.

Fixed Fraction of NAV at Time 0 Contract

With this **FOF** contract, the contract parameter is δ, and the utility functions are

$$U_M(e,\alpha,\delta) = \log[\delta\alpha W_0] + \log[K-e] \quad \textbf{(5)}$$

and

$$U_I(e,\alpha,\delta)=(\frac{1}{2}+ze)\log[W_0(\alpha(R_H-\delta)+1)] + (\frac{1}{2}-ze)\log[W_0(\alpha(R_L-\delta)+1)] \quad (6)$$

We solve for the optimal δ using the three steps described above: (1) determine the α which is optimal for the investor for given values of δ and e, (2) calculate the e which is optimal for the manager given δ and taking into account the α it induces; this gives us $U_M(\delta)$ and $U_I(\delta)$, the utilities that result from a given δ, and (3) identify the δ which maximizes $U_I(\delta)$ subject to $U_M(\delta)\geq U^*$.

The α which maximizes U_I given δ and e works out to be

$$\alpha(\delta,e) = \frac{2z\sigma e-\delta+\eta}{\sigma^2-(\delta-\eta)^2} \quad (7)$$

and the e which maximizes U_M given δ and $\alpha(\delta,e)$ is

$$e(\delta) = \frac{K}{2} + \frac{\delta-\eta}{4z\sigma} \quad \textbf{(8)}$$

Expressions for $U_M(\delta)$ and $U_I(\delta)$ are contained in the appendix.

Fixed Fraction of NAV at Time 1 Contract

With this **FOE** contract, the contract parameter is ξ, and the utility functions are

$$U_M(e,\alpha,\xi) = (\frac{1}{2}+ze)\ \log[W_0\alpha(1+R_H)\xi] + (\frac{1}{2}-ze)\ \log[W_0\alpha(1+R_L)\xi] + \log[K-e] \qquad \textbf{(9)}$$

and

$$U_I(e,\alpha,\xi)=(\frac{1}{2}+ze)\ \log[W_0(\alpha P_H(\xi)+1)] + (\frac{1}{2}-ze)\ \log[W_0(\alpha P_L(\xi)+1)] \qquad \textbf{(10)}$$

where $P_H(\xi)=R_H(1-\xi)-\xi$ and $P_L(\xi)=R_L(1-\xi)-\xi$. We solve for the optimal ξ using the three steps described above.

The α which maximizes U_I given ξ and e works out to be

$$\alpha(\xi,e) = \frac{-[P_H(\xi)+P_L(\xi)]}{2P_H(\xi)P_L(\xi)} - \frac{P_H(\xi)-P_L(\xi)}{2P_H(\xi)P_L(\xi)}z\ e = a(\xi) + b(\xi)e \qquad \textbf{(11)}$$

(so again α is linear in e). The e which maximizes U_M given ξ and $\alpha(\xi,e)$ in **(11)** satisfies a quadratic equation given in the appendix.

Return-dependant Fraction of NAV at Time 1 Contract

With this **FOP** contract, the contract parameters are (ξ_H, ξ_L), and the utility functions are

$$U_M(e,\alpha,\xi_H,\xi_L) = (\frac{1}{2}+ze)\ \log[W_0\alpha(1+R_H)\xi_H] + (\frac{1}{2}-ze)\ \log[W_0\alpha(1+R_L)\xi_L] + \log[K-e] \qquad \textbf{(12)}$$

and

$$U_I(e,\alpha,\xi_H,\xi_L) = (\frac{1}{2}+ze)\ \log[W_0(\alpha P_H(\xi_H)+1)] + (\frac{1}{2}-ze)\ \log[W_0(\alpha P_L(\xi_L)+1)] \qquad \textbf{(13)}$$

where $P_H(.)$ and $P_L(.)$ are defined as above. We solve for the optimal (ξ_H, ξ_L) using the three steps described above.

The α which maximizes U_I given ξ and e works out to be

$$\alpha(\xi_H,\xi_L,e) = \frac{-[P_H(\xi_H)+P_L(\xi_L)]}{2P_H(\xi_H)P_L(\xi_L)} - \frac{P_H(\xi_H)-P_L(\xi_L)}{2P_H(\xi_H)P_L(\xi_L)}z\ e = a(\xi_H,\xi_L)+b(\xi_H,\xi_L)e \tag{14}$$

which is similar to the solution for the **FOE** problem. The e which maximizes U_M given (ξ_H, ξ_L) and $\alpha\,(\xi_H, \xi_L, e)$ in **(14)** satisfies a quadratic equation given in the appendix.

Fixed Fraction of NAV at Time 1 plus Return-based Fee Contract

With this **RFE** contract, the contract parameters are the triplet (ξ_R, H_R, L_R), and the utility functions are

$$\begin{aligned} &U_M(e,\alpha,\xi_R,H_R,L_R) \\ &= (\frac{1}{2}+ze)\ \log[W_0\{\alpha(1+R_H)\xi_R+H_R\}] + (\frac{1}{2}-ze)\ \log[W_0\{\alpha(1+R_L)\xi_R+L_R\}] + \log[K-e] \end{aligned} \tag{15}$$

and

$$U_I(e,\alpha,\xi_R,H_R,L_R)=(\frac{1}{2}+ze)\ \log[W_0\{\alpha P_H(\xi_R)+1-H_R\}] + (\frac{1}{2}-ze)\ \log[W_0\{\alpha P_L(\xi_R)+1-L_R\}] \tag{16}$$

where $P_H(.)$ and $P_L(.)$ are defined as above. We solve for the optimal triplet (ξ_R, H_R, L_R) using the three steps described above.

The α which maximizes U_I given (ξ_R, H_R, L_R) and e works out to be

$$\begin{aligned} \alpha(\xi_R,H_R,L_R,e) &= \frac{-[P_H(\xi_R)(1-L_R)+P_L(\xi_R)(1-H_R)]}{2P_H(\xi_R)P_L(\xi_R)} - \frac{P_H(\xi_R)(1-L_R)-P_L(\xi_R)(1-H_R)}{2P_H(\xi_R)P_L(\xi_R)}z\ e \\ &= a(\xi_R,H_R,L_R) + b(\xi_R,H_R,L_R)e \end{aligned} \tag{17}$$

(so again α is linear in e). The e which maximizes U_M given ξ_R, H_R, L_R, and $\alpha\,(\xi_R, H_R, L_R, e)$ in **(17)** satisfies an equation given in the appendix.

First Best Problem

The first best solution pretends that the investor and manager can contract on the manager's effort, so the investor's problem is to choose the H_1, L_1, α and e which maximize

$$(\frac{1}{2}+ze)\ \log[W_0(\alpha R_H+1-H_1)]\ +\ (\frac{1}{2}-ze)\ \log[W_0(\alpha R_L+1-L_1)] \tag{18}$$

subject to

$$(\frac{1}{2}+ze)\ \log[W_0H_1]\ +\ (\frac{1}{2}-ze)\ \log[W_0L_1]+\log[K-e]\ \geq\ U^* \tag{19}$$

The numerical solution technique for this problem is outlined in the appendix.

Optimal Risk-sharing

The risk-sharing properties of a given contract can be assessed by comparing the payments to the manager under the contract to the optimal risk-sharing payments given the α and e generated by the contract. This problem, which has a closed form solution, can be stated as follows. Given α and e, choose H_{RS} and L_{RS} to maximize

$$(\frac{1}{2}+ze)\ \log[W_0(\alpha R_H+1-H_1)]\ +\ (\frac{1}{2}-ze)\ \log[W_0(\alpha R_L+1-L_1)] \tag{20}$$

subject to

$$(\frac{1}{2}+ze)\ \log[W_0H_1]\ +\ (\frac{1}{2}-ze)\ \log[W_0L_1]+\log[K-e]\ \geq\ U^* \tag{21}$$

The solution to this optimization is given by:

$$\frac{H^{RS}}{L^{RS}}\ =\ \frac{1+\alpha R_H}{1+\alpha R_L} \tag{22}$$

and

$$L^{RS} = \frac{\exp[U^*]}{W_0(K-e)}\left[\frac{1+\alpha R_H}{1+\alpha R_L}\right]^{-(0.5+ze)} \tag{23}$$

For the first best problem, H_{RS}, and L_{RS} equal H_l, and L_l. For all other problems, the actual payments to the manager in each state are likely to deviate from the optimal risk-sharing payments. Comparing the two is of interest since it allows us to assess the utility cost of suboptimal risk-sharing induced by a given contract. In fact, investor utility from a contract can be decomposed into the attainable utility given the choice of α and e induced by the contract less the cost of any suboptimal risk-sharing associated with the actual payments needed to induce those choices.

III. Results

The optimal contracts depend on the choice of parameter vector $\mathbf{p}=(W_0, K, \sigma, \eta, z, U^*)$. Although it has six parameters, the model is invariant to scale. In particular, $\{W_0, z, \sigma, \eta, K, U^*\}$ and $\{cW_0, z/c, \sigma, \eta, cK, U^*+ 2\log(c)\}$ give rise to equivalent problems for all six contracts. [17] This invariance follows from using log utility. Thus, without loss of generality, K is fixed at 0.95 throughout. Two scenarios are considered, whose parameters are chosen to capture, as much as possible, features of the U.S. mutual fund industry. In particular, (W_0, K, σ, U^*) is set equal to (10000, 0.95, 0.25, 5). The value of 0.25 for the volatility parameter was chosen to match the annual volatility of the U.S. stock market (see, for example, Fama and French (1989)). Scenario 2 increases the scale of the investor relative to that in Scenario 1 by increasing W_0 to 20000.

The parameters of Scenario 1 imply that the manager would need to receive a certain dollar

[17]The proof is available from the authors on request.

income of \$156 to obtain her reservation utility and not exert any effort. While this amount seems too low, we can scale the problem so that this certainty equivalent is a more plausible quantity. For example, if we scale by 50, this quantity becomes \$7800 which seems more reasonable. With this scaling, the wealth of the representative investor becomes \$500000 in Scenario 1 and \$1 million in Scenario 2, a range which also seems plausible.

Three numerical exercises are performed for each scenario. The first fixes η at -0.01 and allows z to vary from 0.4 to 0.5125. Since z affects the noisiness of return as a signal of effort, this experiment assess the relative performance of the different contracts as return noisiness changes. However, the opportunity improves as z increases. To assess whether this increasing attractiveness is driving results obtained in the first exercise, a second exercise is performed that allows z to vary from 0.4 to 0.5125, but also allows η to vary in such a way that the first-best investor utility stays constant (when z=0.4, $\eta = 0$). We are also interested in how an improved opportunity set impacts comparisons across the different contracts. To assess this, the third exercise fixes z at 0.4 and allows η to vary from -0.01 to 0.05.

Each figure reports results for a given scenario and numerical exercise, across one of two sets of four contracts: either **1ST**, **RET**, **FOF** and **FOE** or **1ST**, **FOE**, **FOP** and **RFE**. Results in each figure are organized into 9 panels. Panel A examines certainty equivalent dollar utilities associated with each contract, expressed as a fraction of W_0. Panel B also plots certainty equivalent dollar utilities for each contract, but assuming optimal risk-sharing given the α and e that the contract induces. Panel C reports the difference between the dollar utilities from the first two panels, which can be interpreted as the cost of suboptimal risk-sharing under each contract. Panel D and E present α and e respectively for each contract while the expected payment to the manager as a fraction of

W_0 is plotted in Panel F. Panel G shows the ratio of the manager's payment in the L state to her payment in the high state while Panel H reports the same ratio but for the optimal risk-sharing payments. Panel I plots the difference between the previous two panels and provides a criteria for assessing the direction as well as the magnitude of deviations from optimal risk shifting.

A. Establishing a Role for NAV as a Contracting Variable.

Figure 1 reports results for Scenario 1 with z allowed to vary and η fixed, and focuses on four contracts: **1ST**, **RET**, **FOF** and **FOE**. A key result of Figure 1's Panel A is that the **RET** contract is dominated by the **FOF** contract, especially for low values of z. Since return becomes a noisier signal of effort as z decreases, Figure 1 illustrates that a fee based on NAV can add value when return is a noisy signal of effort. In fact, for the parameter values of Scenario 1, the **RET** contract fails to deliver sufficient utility when z is low for the investor to prefer investing in the fund over keeping W_0.[18]

Notice that the difference between the two contracts declines as z increases. This result is consistent with the intuition that the **FOF** contract is less attractive relative to the **RET** contract when return is a precise signal of effort. However, it is not a clean test of this intuition since the opportunity set also increases with z. The results in Figure 2 address this concern by examining Scenario 1 with z varying but η also varying to keep first-best U_I constant. Panel A of Figure 2 tells the same story as Figure 1 with the improvement of the **FOF** contract over the **RET** contract declining as z increases. However, the **FOF** contract always does better than the **RET** contract in

[18] When the investor prefers to keep W_0, the manager would realize this and so would decide not to manage money. Instead, she would undertake her next best activity and receive her reservation level of utility.

Figure 2 irrespective of z.

Focusing on the case with a constant opportunity set (Figure 2), Panel E shows that the level of e induced by the **FOF** is lower than for the **RET** contract, for high values of z. So, for these values of z, the **FOF** contract induces less effort than the **RET** contract but gives the investor higher utility. Thus, it can be dangerous to evaluate contracts solely on the basis of the level of e induced. The likely reason for e giving a different ranking is differential risk sharing across the two contracts, which is confirmed by Panels B and C. Panel C shows that the utility cost of suboptimal risk-sharing is more than 5% for the **RET** contract, irrespective of the value of z, while this cost is less than 0.3% for the **FOF** contract. In fact, Panel B shows that optimal risk-sharing would result in the (α, e) pair induced by the **RET** contract giving the investor higher utility than the pair induced by the **FOF** contract, for high values of z.

So the **RET** contract needs to offer payments with poor risk-sharing properties to induce managerial effort. An interesting question is whether the investor or the manager is being forced to bear excessive risk. Panels G to I show that the ratio of actual payments in the L and H states under the **RET** contract is much lower than for the optimal payments. In particular, Panel I shows that the actual payment ratio is always less than the optimal payment ratio by more than 0.2. Thus, the manager is being forced to bear excessive risk, which is part of the reason why her expected payment is so high under the **RET** contract (see Panel F). In contrast, the actual payments ratio is too high for the **FOF** contract, and the magnitude of the difference is always more than 0.5, more than the difference for the **RET** contract (see Panel I). Despite this larger difference for the **FOF** contract, we have already noted that suboptimal risk-sharing induces a much smaller utility cost than the **RET** contract. The excessive risk-bearing by the manager under the **RET** contract has such a large utility cost effect because of the severe disutility that she gets from the low payoff in the bad state.

Finally, the **RET** contract induces a larger α than the **FOF** contract especially at high values of z (see Panel D). However, the **RET** level of α is still lower than the **1ST** level.

B. Benefits of using NAV at the End rather than at the Start.

The second question is the potential for better risk sharing by using the **FOE** contract (based on NAV at the end) rather than the **FOF** contract. Panel A of Figures 1 and 2 confirm this conjecture by showing that there is a gain from using the **FOE** contract. However, the gain is quite modest in magnitude, typically less than 1% of W_0.

In fact, Panel B of those figures show that the (α, e) pair from the **FOE** contract continues to give higher utility than the **FOF** pair ,when optimal risk-sharing payments are made under both contracts. Further, while the utility cost of suboptimal risk-sharing is higher for **FOE** than **FOF** (see Panel C), both are less than 0.3%. Thus, the **FOE** contract does better in part because it induces greater e and greater α, which is confirmed by Panels D and E of the two figures. So our results suggest that fees in the mutual fund industry are a fixed fraction of NAV at the end of the period rather than the start for risk-sharing reasons, and to induce more effort by the manager.

C. Why Performance Fees are Used in the Hedge Fund Industry but not the Mutual Fund Industry?

An interesting comparison is between the **FOE** and **FOP** contracts. Since the **FOE** contract is an **FOP** contract with the same fraction in each return state, we know that the **FOP** contract does better. The questions of interest are the magnitude of the improvement, and whether the improvement varies with the quality of the opportunity set. An assessment can be made by looking at what happens as η varies, holding z fixed. As η increases, so does the opportunity set, but without any direct impact on the precision of return as a signal of effort. Figure 3 reports results for Scenario 1 as η is varied for the four contracts, **1ST, FOE, FOP** and **RFE**. Panel A shows that the investor's utility from the two contracts (**FOE** and **FOP**) is similar when z is low. However, as η increases, the relative improvement of the **FOP** contract also increases. In fact, a roughly 1% improvement in certainty equivalent investor wealth (as a fraction of W_0) for η=-0.01 becomes an improvement of approximately 9% of W_0 when η increases to 0.05. Thus, the benefits from using a return-dependent fraction of NAV rather than a fixed fraction seem to be increasing in η.

Now differences in the opportunity set available to the manager is another way of saying differences in managerial skill. Thus, performance contracts (which allow return-dependent fractions of NAV) are likely to be more prevalent when managers are more skilled. Since hedge fund managers are typically more skilled than mutual fund managers, our model is consistent with both the widespread use of asymmetric performance contracts in the hedge fund industry, and the minimal use of "fulcrum" performance fees in the mutual fund industry.

So the next question is to understand why allowing the NAV fraction to be return-dependent is more valuable when the opportunity set is good. The key is in Panel E of Figure 3 which graphs the effort choice of the manager under the two different contracts. Put simply, the **FOP** contract is relatively better than the **FOE** contract at extracting effort from the manager. This is true irrespective of the quality of the opportunity set, since the effort choice is always higher under the **FOP** contract (Panel E of Figure 3). At the same time, the **FOP** contract has poorer risk-sharing properties than the **FOE** contract (see Panel C of Figure 3), with the manager being forced to bear excessive risk (see Panel I). As the fraction paid in the good state increases, the risk-sharing associated with the payments becomes poorer, even as the effort induced increases.

Extracting effort is more valuable when the opportunity set is of high quality. So when managerial ability is low, the marginal cost of poorer risk sharing quickly outweighs the marginal benefit of greater effort, and the resulting improvement relative to the **FOE** contract is small. However, when ability is high, the good state fraction must become very large relative to the bad state fraction before the marginal cost of poor risk-sharing dominates. Consequently, the improvement in investor utility relative to the **FOE** contract is large, as is the increase in the effort level. So the greatest improvement in investor utility going from the **FOE** to the **FOP** contract occurs when the opportunity is high, because that is when effort is most productive.

Consistent with this argument, Panel C shows that the utility cost of suboptimal risk-sharing under the **FOP** contract is increasing in the quality of the opportunity set (η). Further, Panel G shows that the extent of the asymmetry in the actual payments under the **FOP** contract is also increasing in η. While it is difficult to make statements about nonlinear payment schedules based on a two state model, the large difference in the **FOP** fraction that we observe for high η is likely to translate into the asymmetric (rather than "fulcrum") performance fees that we observe in the hedge fund industry.

D. More Complicated Contracts.

Finally, it is interesting to consider whether a more complicated contract can do substantially better than the return-dependent fraction of NAV (the **FOP** contract). The contract that we consider pays a fee that is a fixed fraction of NAV at the end of the period, together with a payment that depends solely on the fund's return (the **RFE** contract). Panel A of Figure 3 considers the certainty equivalents for the two contracts in Scenario 1 as η is varied. The **RFE** contract always does substantially better, especially when η is high. This improvement seems to be driven by the ability of the **RFE** contract to extract an effort level closer to the first-best effort level (see Panel E of Figure 3).

Panel C shows that **RFE**'s risk-sharing properties in utility cost terms are comparable to the **FOP** contract. However, as η increases, the cost of suboptimal risk-sharing is increasingly higher for the **FOP** contract than the **RFE** contract. So another reason for the **RFE** contract being more attractive for high η is better risk-sharing.

E. Robustness Checks and Comparative Statics.

Figures 4 and 5 assess whether the results described above are robust to increasing the scale of the investor. In particular, Figure 4 is analogous to Figure 2 (i.e., z increasing but η also changing to keep U_I fixed) but for Scenario 2 which has W_0 increased to 20000. Similarly, Figure 5 is the Scenario 2 analog to Figure 3 with η varying but z fixed. The results in Figures 4 and 5 are qualitatively similar to those in Figures 1 to 3 and the broadbrush conclusions of the previous subsections continue to hold. The only change of note is that the relative attractiveness of the **RET** contract increases. While still giving lower investor utility for low values of z, the **RET** contract is actually better than the **FOF** and **FOE** contracts for high z (see Panel A of Figure 4, and contrast Panel A of Figure 2). Panels C of Figures 2 and 4 reveal that the utility cost of sub-optimal risk-sharing is similar across the two Scenarios for the **RET** contract. However, Panels E of those two figures reveals that the **RET** contract is much better at extracting effort in Scenario 2 than 1.

Another question is the impact of changing the volatility of the risky asset return available to the manager. We increased σ from 0.25 to 0.5 (unreported) and found very little change in the nature of comparisons between the contracts. Of course, the amount invested in the fund by the investor (α) is much smaller. But the exact impact really depends on whether η is varying in proportion with σ or is fixed.[19]

IV. Conclusion

This paper considers the economic role of fees in aligning the incentives of money managers with those of investors. We examine a simple model in which manager effort (or investment in human and physical capital) is observed by the investor prior to her investment decision, but is not verifiable. This setup creates a positive economic role for net asset value (NAV) as a contracting variable. We consider several contract structures: in particular, contracts that are a fixed fraction of NAV at the start and end of the period, a contract that is a return-dependent fraction of NAV, the optimal return based contract, and a contract that is a fixed fraction of NAV with an adjustment that depends on fund return. Only the first two contacts, which have fees that are proportional to NAV, are available to the mutual fund industry and in use by hedge funds.

We find that a fixed fraction of NAV does better than the optimal return-based contract, especially when fund return is a noisy signal of effort. This establishes a role for contracts based on NAV and thus provides an explanation for their widespread use in both the mutual and hedge

[19] If η is varying in proportion with σ, then the **1ST**, **RET**, **FOF** and **FOP** contracts provide the same investor utility but with α varying inversely with η and σ. The same is not true of the **FOE** and **RFE** contracts since the manager receives a fixed fraction of the amount invested with the fund, whose ratio changes with η and σ irrespective of α. When η is held fixed, the effect on α and investor utility is likely to depend on whether η is positive or negative.

fund industries. Second, a fixed fraction of NAV at the end of the period does better than one based on NAV at the start, due to better risk sharing, and better effort inducement.

Third, allowing the fraction of NAV to vary with fund return induces the greatest improvement in investor utility when the opportunity set of the manager is good. In fact, when the opportunity set is poor, the improvement can be negligible. This type of contract can be implemented using a fixed fraction of NAV together with a performance fee. Since hedge fund managers are typically the better fund managers, our model provides an explanation for why "fulcrum" performance fees are not used in the mutual fund industry, but asymmetric performance fees are the dominant choice in the hedge fund industry.

Finally, we find that a contract based on return, together with a fixed fraction of NAV, can significantly improve investor utility relative to a return-dependent fraction of NAV. The implication is that less regulation by the SEC of fee structures in the mutual fund industry may make that industry even more attractive. At the same time, our other results suggest that the rapid growth in the mutual fund industry, in the face of severe restrictions on fee structures, may be due to NAV's ability as a contracting variable to address important agency problems.

Two extensions of our work would be interesting. First, allowing for multiple return states would allow a menu of non-linear functions of NAV to be analyzed. This exercise is particularly interesting for high ability managers, where non-linearities are likely to have a big impact. Second, extending the analysis to multiple periods would be of interest. Both these extensions are left to future research.

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Appendix

Contract on realized returns

Given the expressions for the manager's optimal e and the investor's optimal α, (3) and (4), we can solve for the utilities in terms of H and L:

$$U_M(H,L) = \frac{1}{2}(\log[HLW_0]) + (zK)\log[H/L] - \log[\log[H/L]] - \log[z] - 1 \qquad \textbf{(24)}$$

and

$$\begin{aligned} U_I(H,L) = \log[W_0(2\sigma + R_L H - R_H L] &+ (\frac{1}{2} + zK - \frac{1}{\log[H/L]})\log[\frac{1}{-R_L}(\frac{1}{2} + zK - \frac{1}{\log(H/L)})] + \\ &(\frac{1}{2} - zK + \frac{1}{\log[H/L]})\log[\frac{1}{R_H}(\frac{1}{2} - zK + \frac{1}{\log(H/L)})] \end{aligned} \qquad \textbf{(25)}$$

Fraction of Funds at Time 0 Contract

From expressions (7) and (8), we can calculate the α that will be chosen given δ, since we know the e that will be chosen:

$$\alpha(\delta) = \frac{2z\sigma K - (\delta - \eta)}{2(\sigma^2 - (\delta - \eta)^2)} \qquad \textbf{(26)}$$

Then we can calculate $U_M(\delta)$ and $U_I(\delta)$:

$$U_M(\delta) = \log[\frac{W_0 \delta}{8z\sigma\ (\sigma^2 - (\delta - \eta)^2)}] + 2\log[2K\sigma z + \eta - \delta] \qquad \textbf{(27)}$$

and

$$U_I(\delta) = (\frac{1}{2} + \frac{zK}{2} + \frac{\delta - \eta}{4\sigma})\log[W_0(\frac{1}{2} + \frac{\sigma(0.5 + zK)}{\sigma + \delta - \eta})] + (\frac{1}{2} - \frac{zK}{2} - \frac{\delta - \eta}{4\sigma})\log[W_0(\frac{1}{2} + \frac{\sigma(0.5 - zK)}{\sigma - \delta + \eta})] \qquad \textbf{(28)}$$

Fraction of Funds at Time 1 Contract

The e which maximizes U_M given ξ and $\alpha(\xi,e)$ in **(11)** satisfies the following quadratic equation:

$$\Lambda\ e^2\ +\ (2+(\frac{a(\xi)}{b(\xi)}-K)\Lambda)\ e+\ (\frac{a(\xi)}{b(\xi)}-K-\frac{a(\xi)}{b(\xi)}K\Lambda)\ =\ 0 \tag{29}$$

where $\Lambda\ =\ z\ \ln[\frac{1+R_H}{1+R_L}]$. We take the solution for e to this equation that maximizes U_M, and then can calculate the α that will be chosen given ξ and this e. Next we can calculate $U_M(\xi)$ and $U_I(\xi)$. Finally, we identify with a computer the set of ξ values which sets $U_M(\xi)=U^*$, and take the one that maximizes $U_I(\xi)$.

Asymmetric Fraction of Funds at Time 1 Contract

The e which maximizes U_M given (ξ_H, ξ_L) and $\alpha\ (\xi_H, \xi_L, e)$ in **(14)** satisfies the following quadratic equation:

$$\Lambda\ e^2\ +\ (2+(\frac{a(\xi_H,\xi_L)}{b(\xi_H,\xi_L)}-K)\Lambda)\ e+\ (\frac{a(\xi_H,\xi_L)}{b(\xi_H,\xi_L)}-K-\frac{a(\xi_H,\xi_L)}{b(\xi_H,\xi_L)}K\Lambda)\ =\ 0 \tag{30}$$

where $\Lambda\ =\ z\ \ln[\frac{(1+R_H)\xi_H}{(1+R_L)\xi_L}]$.

We take the solution for e to this equation that maximizes U_M, and then can calculate the α that will be chosen given (ξ_H, ξ_L) and this e. Next we can calculate $U_M\ (\xi_H, \xi_L)$ and $U_I\ (\xi_H, \xi_L)$. Finally, we identify with a computer the set of (ξ_H, ξ_L) values which sets $U_M\ (\xi_H, \xi_L)=U^*$, and take the one that maximizes $U_I\ (\xi_H, \xi_L)$.

Fraction of Funds at Time 1 plus Return-based Fee Contract

The e which maximizes U_M given ξ_R, H_R, L_R, and $\alpha(\xi_R, H_R, L_R, e)$ in (17) satisfies the following equation:

$$z \log[\frac{\alpha(1+R_H)\xi_R+H_R}{\alpha(1+R_L)\xi_R+L_R}] + b(\xi_R,H_R,L_R)\ \xi_R \left\{\frac{(0.5+ze)(1+R_H)}{\alpha(1+R_H)\xi_R+H_R} + \frac{(0.5-ze)(1+R_L)}{\alpha(1+R_L)\xi_R+L_R}\right\} - \frac{1}{K-e} \quad \textbf{(31)}$$
$$=0$$

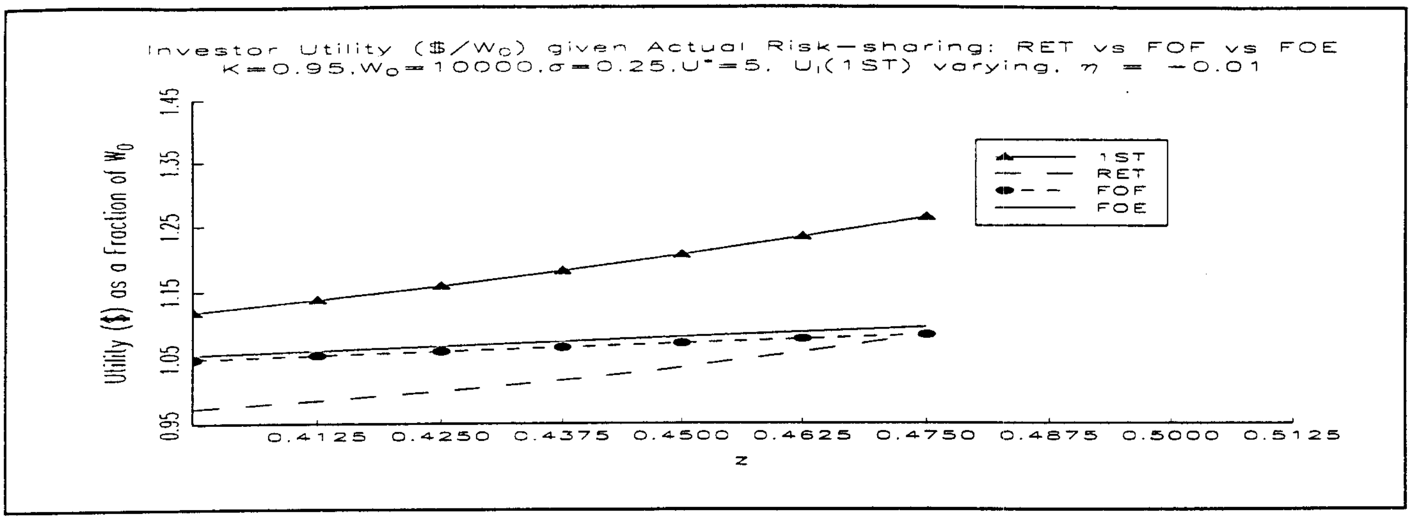
We take the solution for e to this equation that maximizes U_M, and then can calculate the α that will be chosen given (ξ_R, H_R, L_R) and this e. Next we can calculate $U_M(\xi_R, H_R, L_R)$ and $U_I(\xi_R, H_R, L_R)$. Finally, we identify numerically the set of (ξ_R, H_R, L_R) values which sets $U_M(\xi_R, H_R, L_R) = U^*$, and take the one that maximizes $U_I(\xi_R, H_R, L_R)$.

First Best Problem

The first-order conditions for the first-best problem of maximizing ? subject to ? ,rearrange into one equation with one unknown:

$$\begin{aligned} U^* = &\log[\frac{W_0}{z}] + 2\log[zK-F(\alpha)] + \log[\log[\frac{1+\alpha R_H}{1+\alpha R_L}]] \\ &+ (\frac{1}{2}-F(\alpha))\log[1+\alpha R_H] + (\frac{1}{2}+F(\alpha))\log[1+\alpha R_L] \end{aligned} \quad \textbf{(32)}$$

where $F(\alpha) = (\alpha(\eta^2-\sigma^2)+\eta)/\sigma$. We solve numerically for α, plug the solution into the other first-order conditions to find e, H_I and L_I, then calculate the U_I it delivers. The equation has multiple solutions; we choose the one with the highest U_I.



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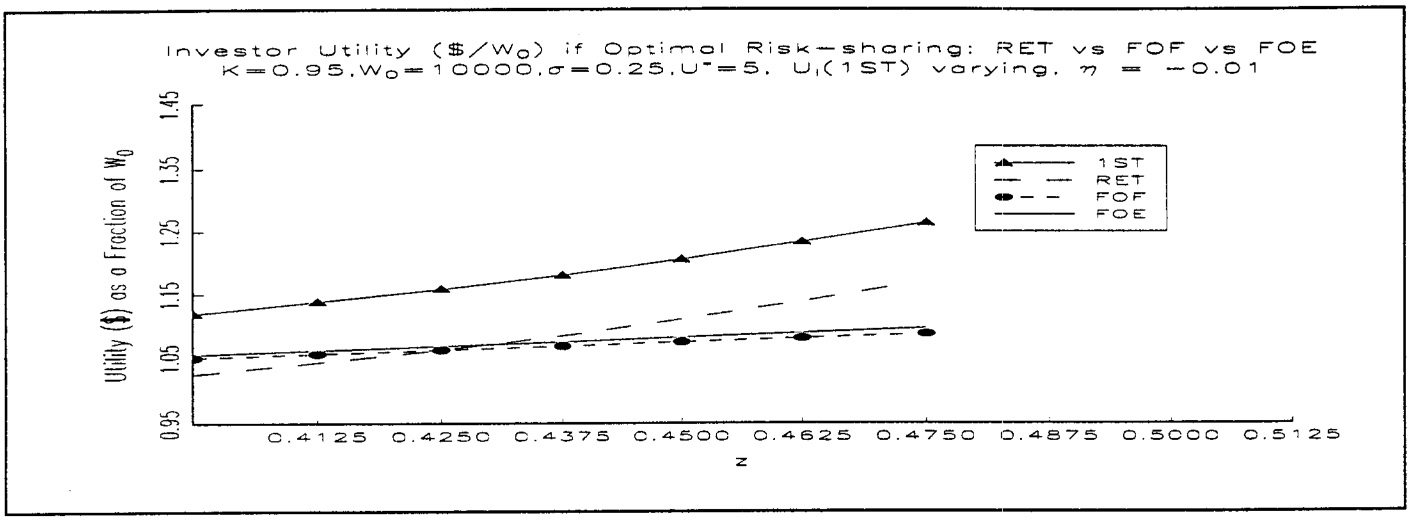

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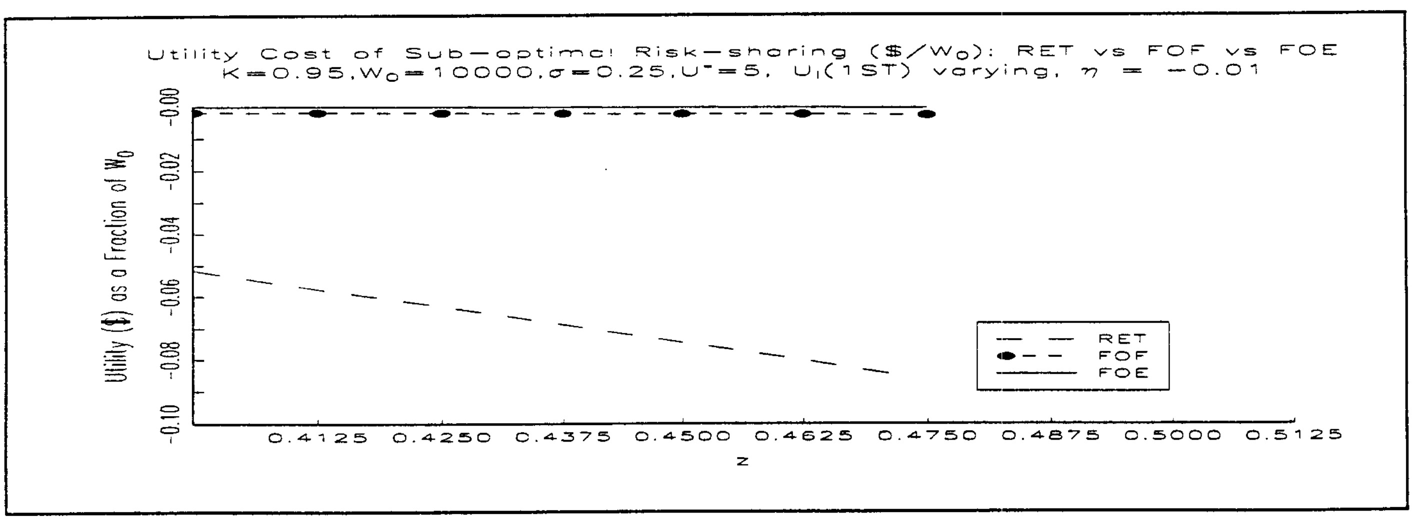
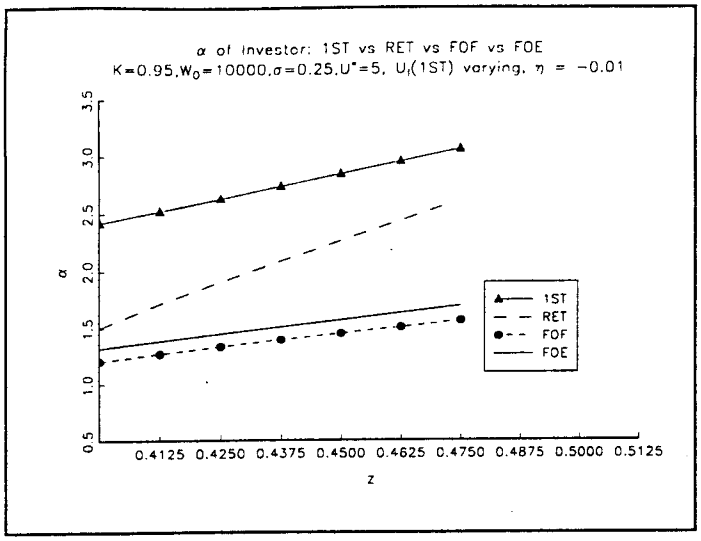


Panel C

Figure 1. Scenario 1 with (W_0, K, σ, U^*) set to (10000, 0.95, 0.25, 5). Numerical solution exercise that holds η fixed at -0.1 and varies z from 0.4 to 0.475. Comparison of 4 contracts, 1ST, RET, FOF and FOE which are described in Section II.



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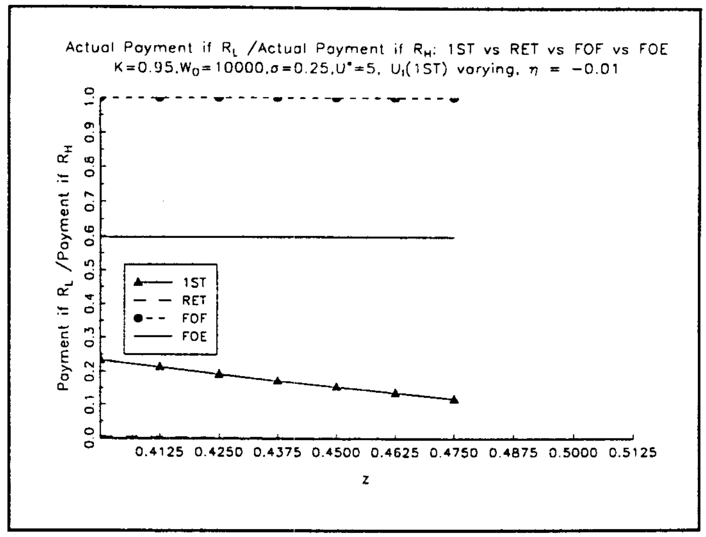

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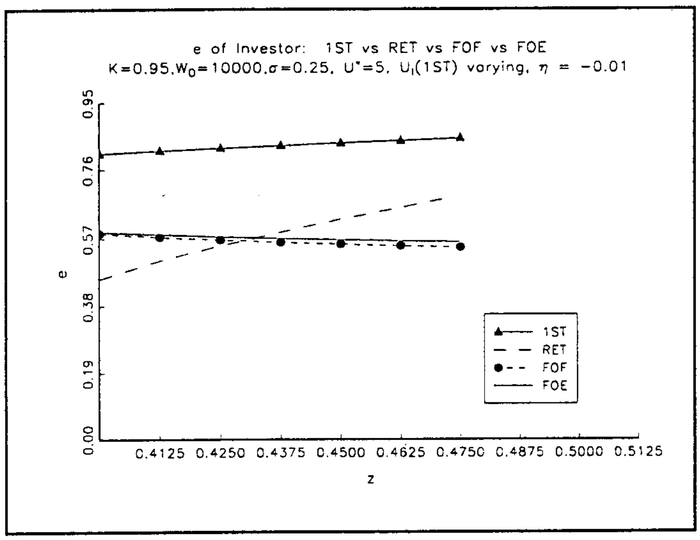

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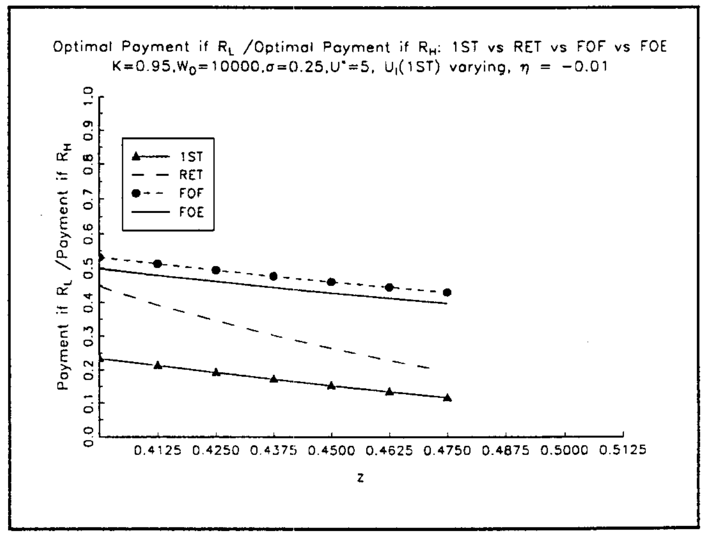

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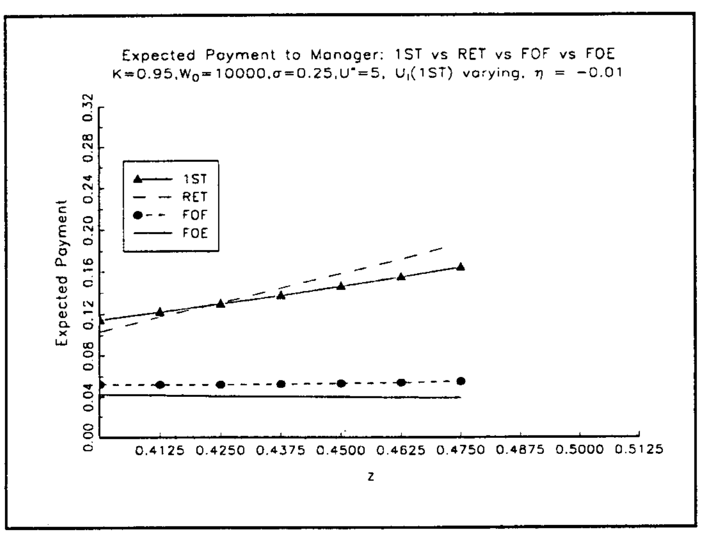

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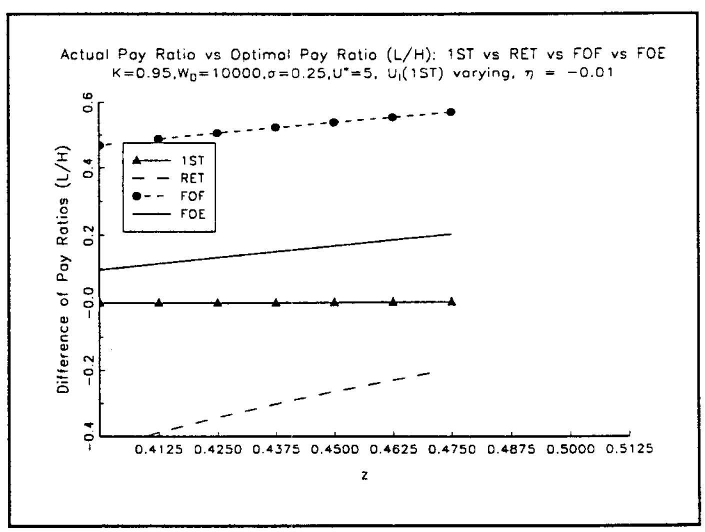
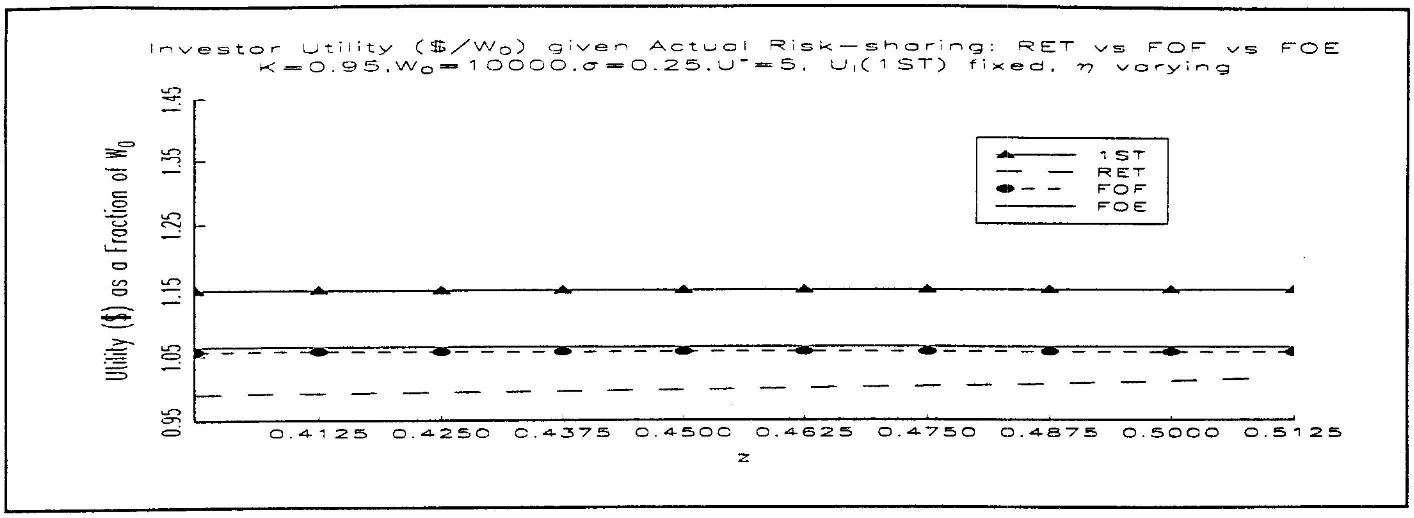


Panel I

Figure 1 (cont). Scenario 1 with (W_0, K, σ, U^*) set to (10000, 0.95, 0.25, 5). Numerical solution exercise that holds η fixed at -0.1 and varies z from 0.4 to 0.475. Comparison of 4 contracts, 1ST, RET, FOF and FOE which are described in Section II.



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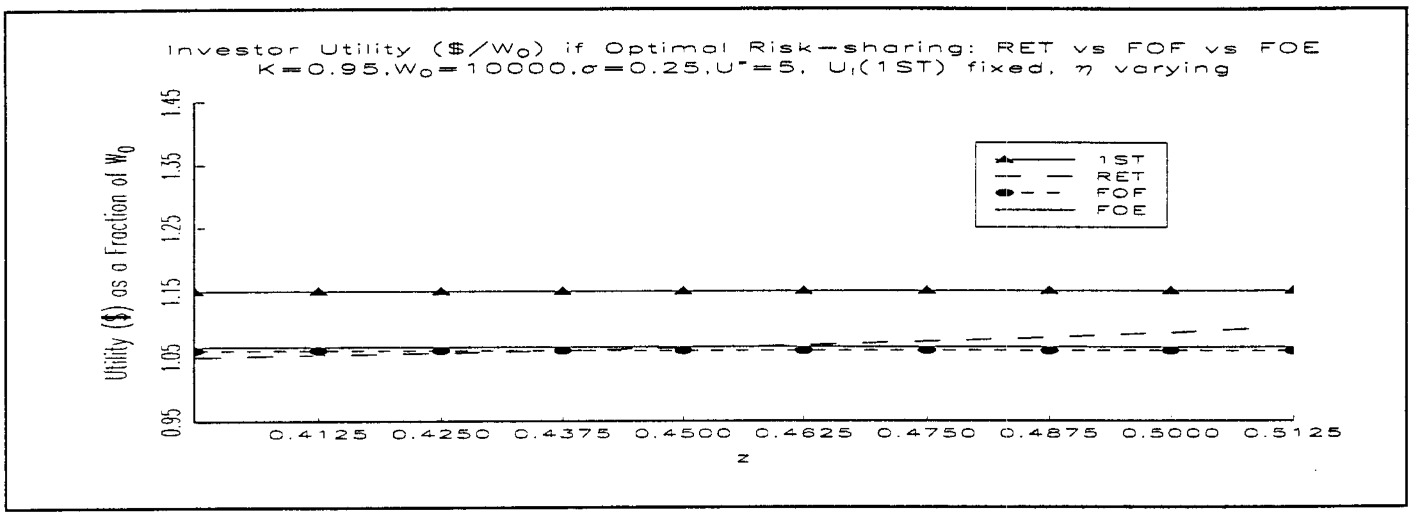

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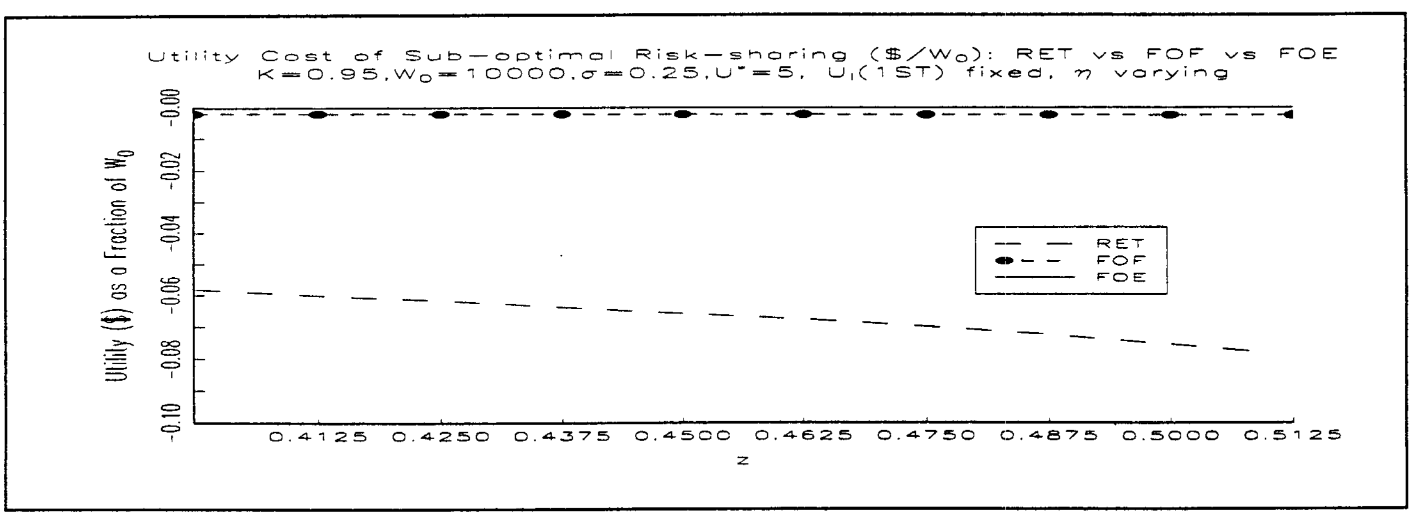
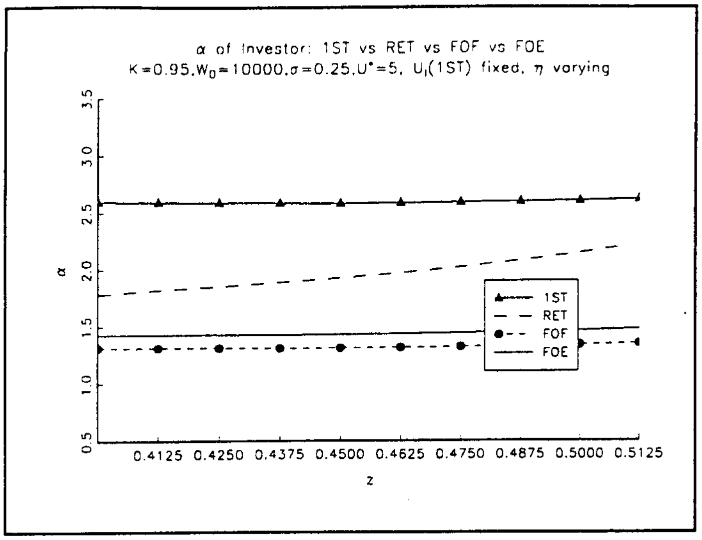


Panel C

Figure 2. Scenario1 with (W_0, K, σ, U^*) set to (10000, 0.95, 0.25, 5). Numerical solution exercise that holds U_1 fixed and allows η to vary as z varies from 0.4 to 0.5125. Comparison of 4 contracts, 1ST, RET, FOF and FOE which are described in Section II.



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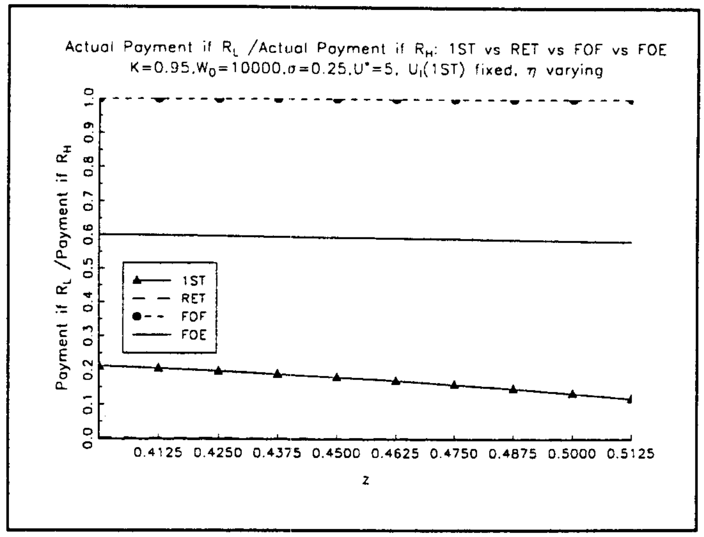

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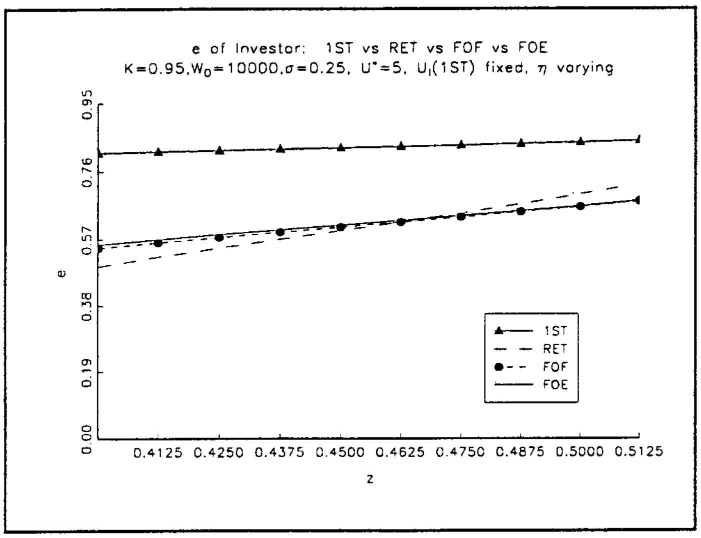

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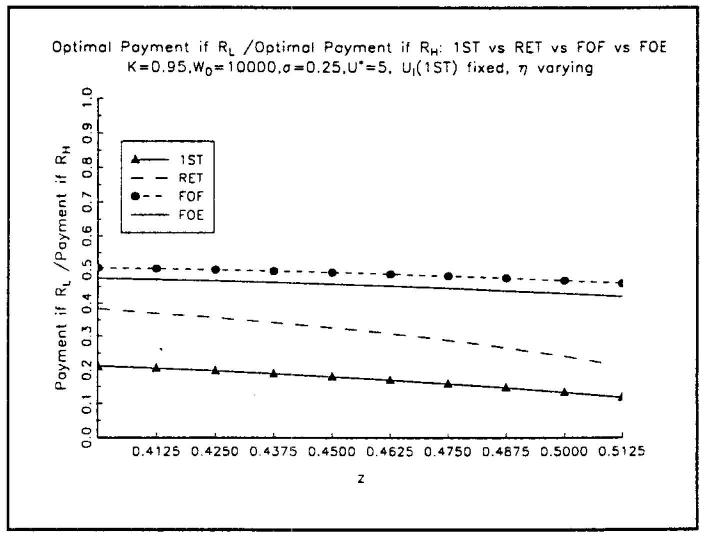

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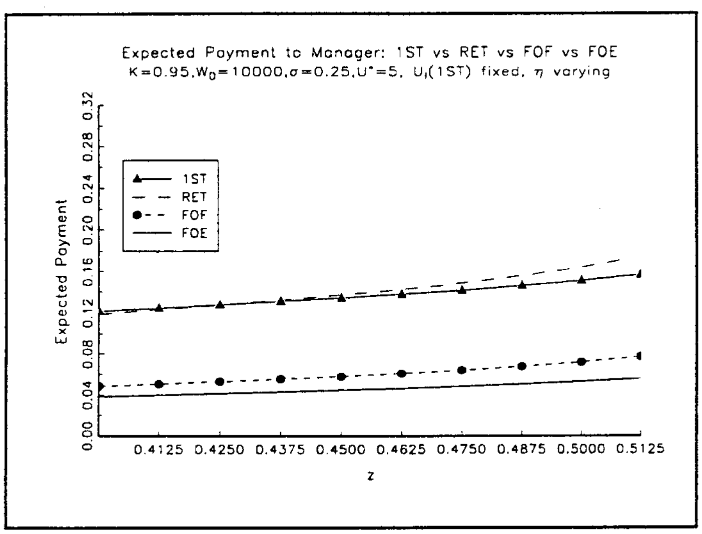

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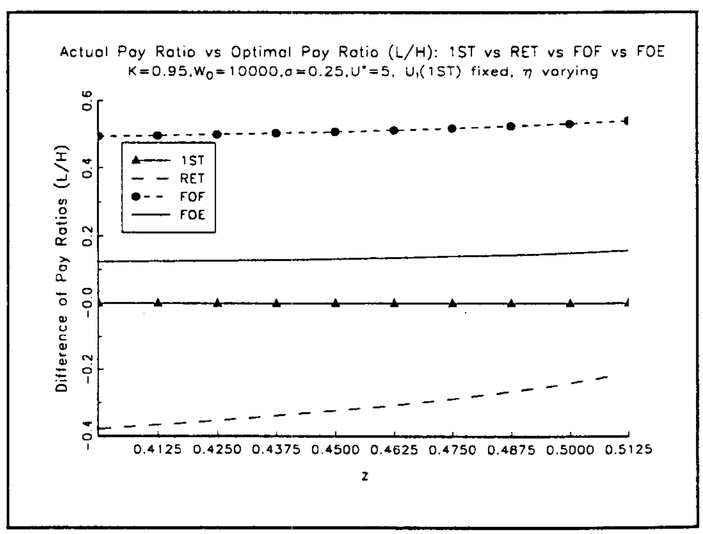
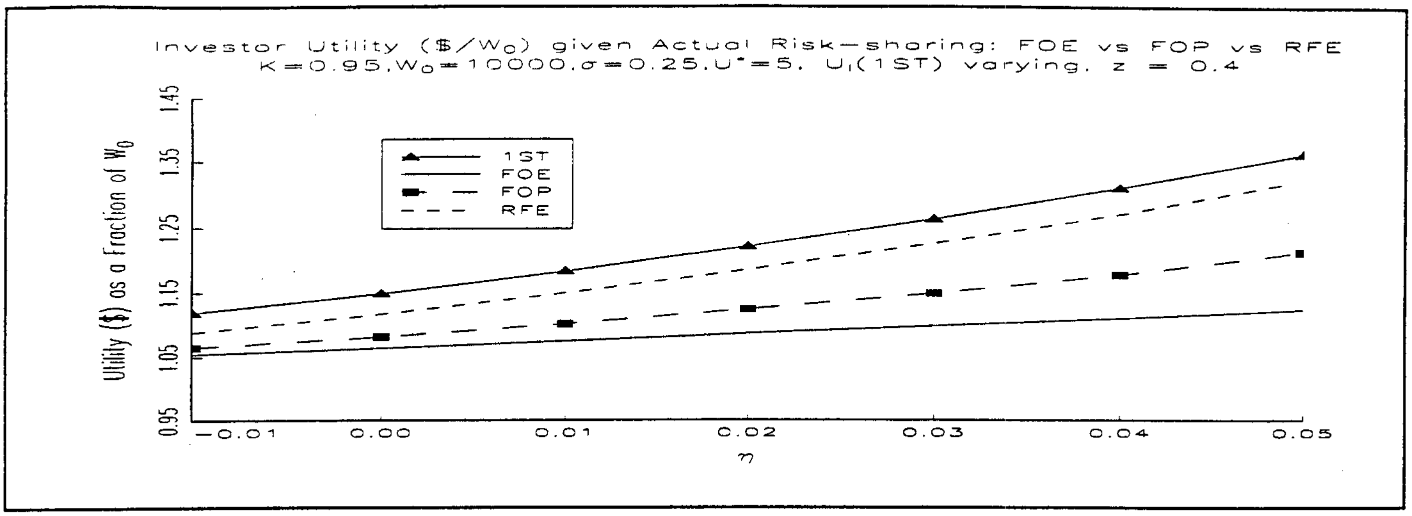


Panel I

Figure 2 (cont). Scenario1 with (W_0, K, σ, U^*) set to (10000, 0.95, 0.25, 5). Numerical solution exercise that holds U_l fixed and allows η to vary as z varies from 0.4 to 0.5125. Comparison of 4 contracts, 1ST, RET, FOF and FOE which are described in Section II.



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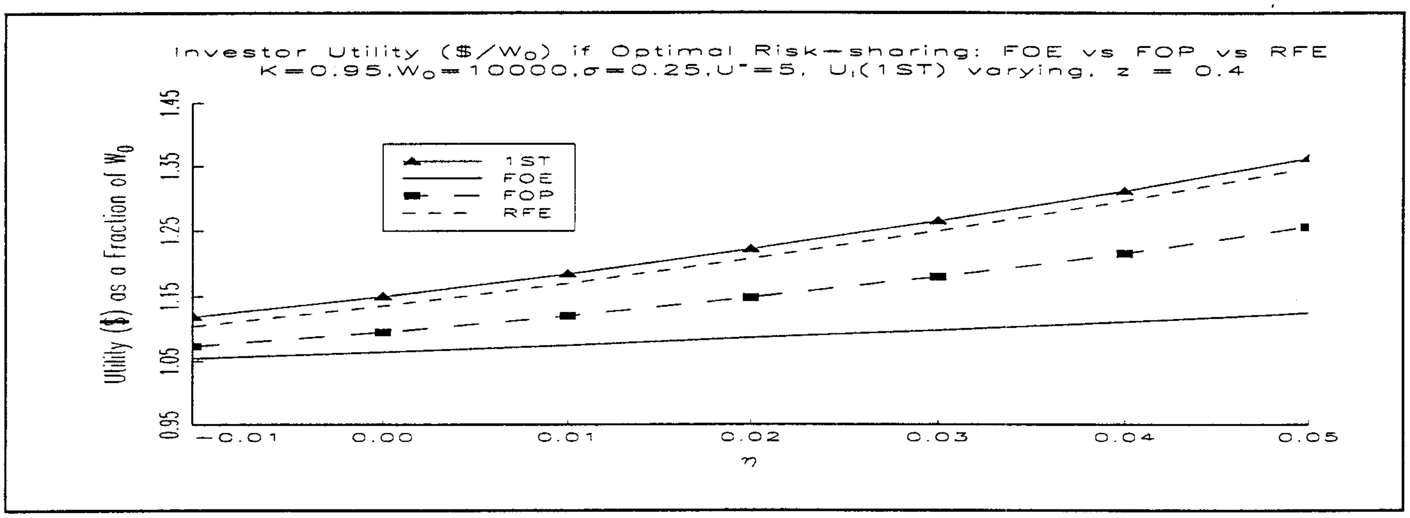

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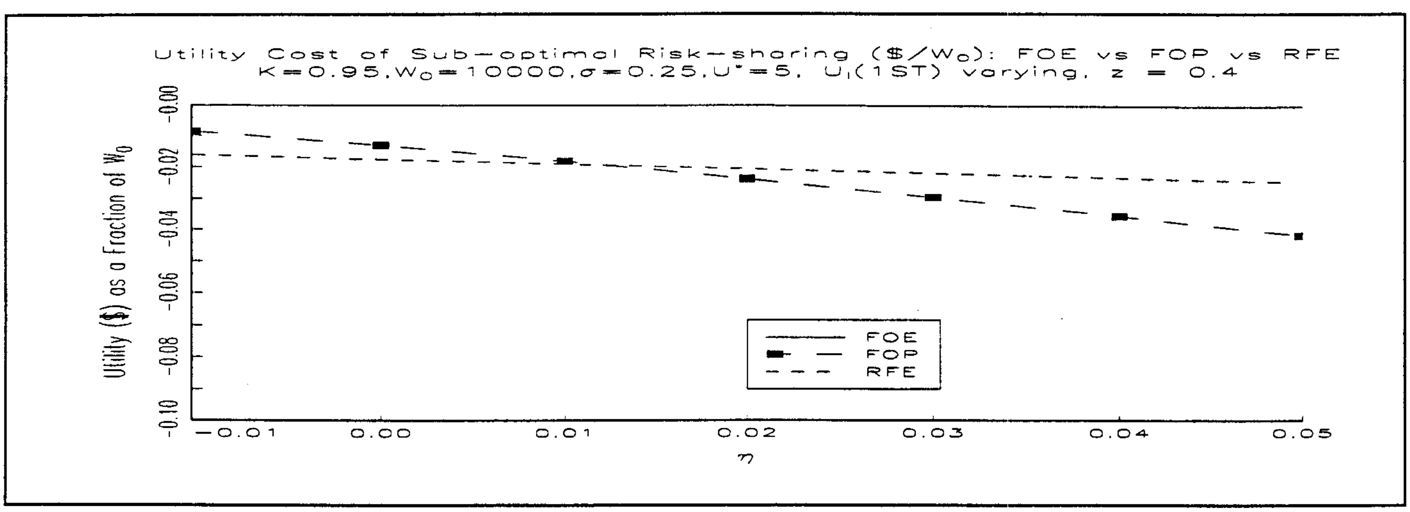
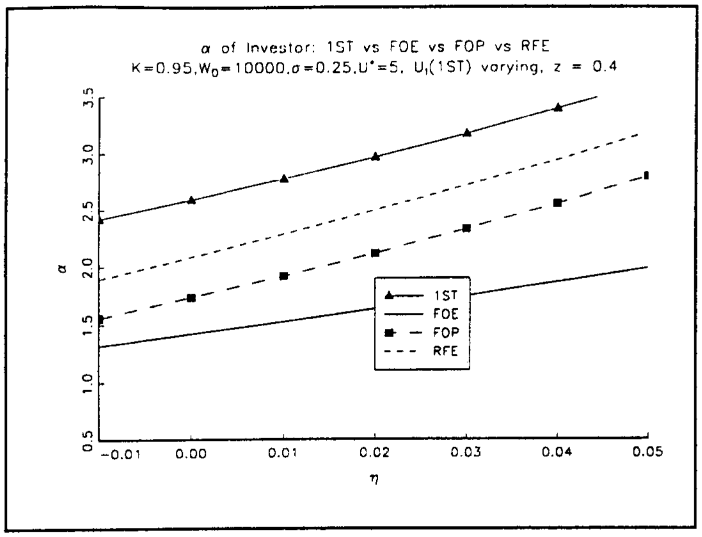


Panel C

Figure 3. Scenario1 with (W_0, K, σ, U^*) set to (10000, 0.95, 0.25, 5). Numerical solution exercise that holds z fixed at 0.4 and allows η to vary from -0.01 to 0.05. Comparison of 4 contracts, 1ST, FOE, FOP and RFE which are described in Section II.



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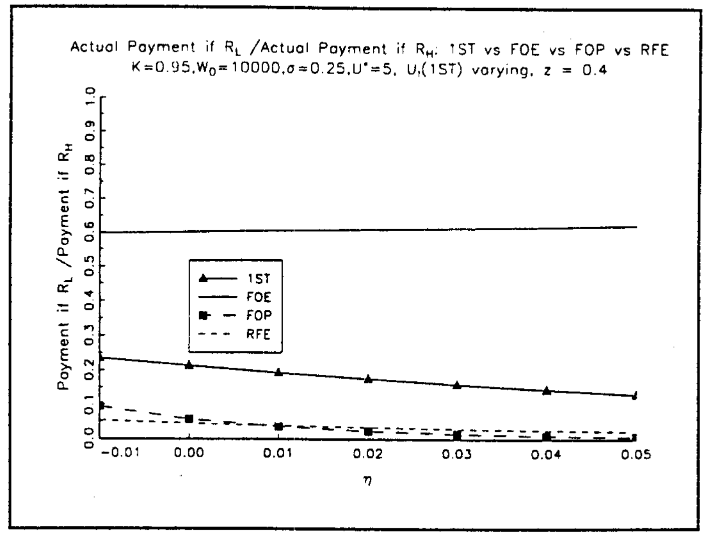

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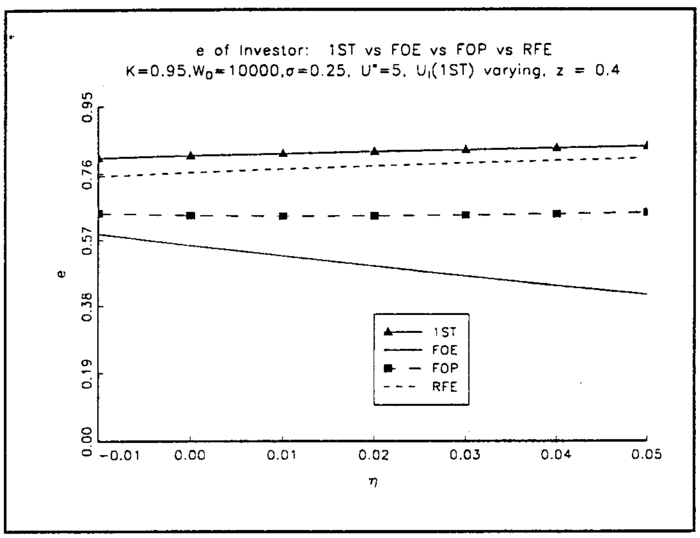

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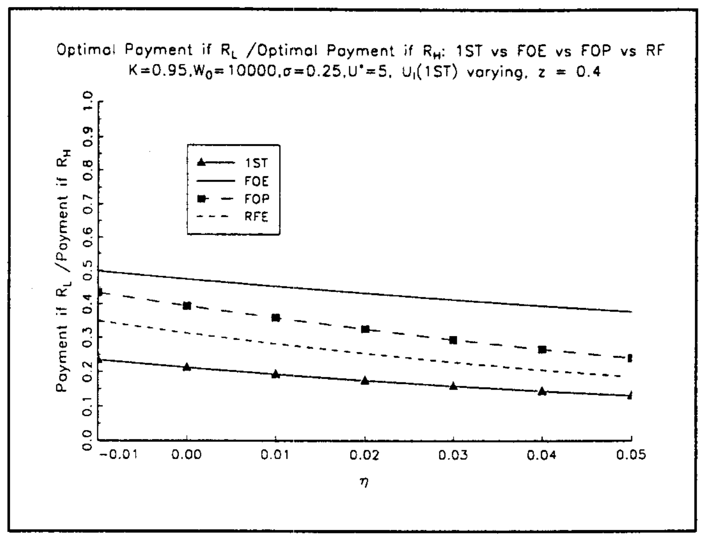

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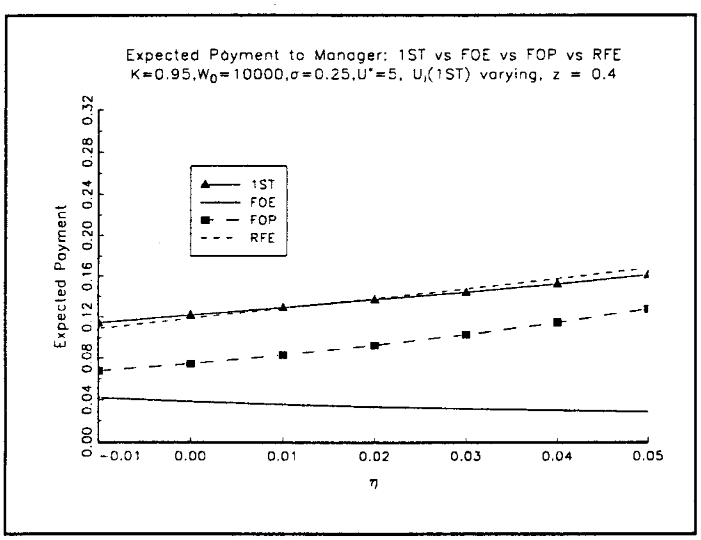

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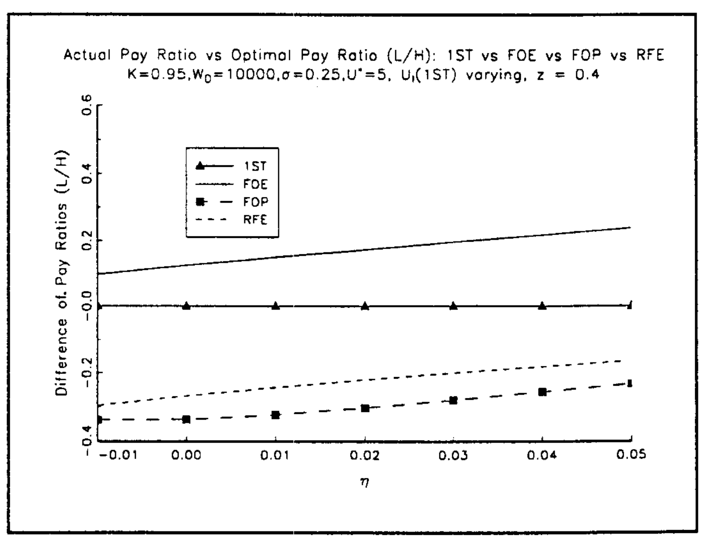
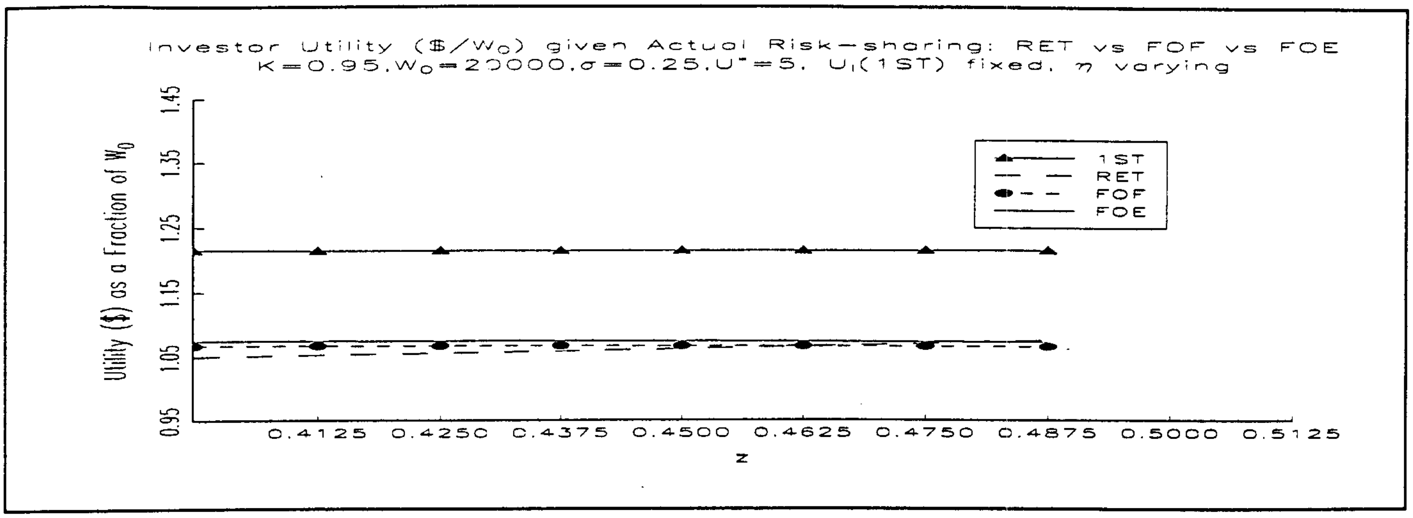


Panel I

Figure 3 (cont). Scenario1 with (W_0, K, σ, U^*) set to (10000, 0.95, 0.25, 5). Numerical solution exercise that holds z fixed at 0.4 and allows η to vary from -0.01 to 0.05. Comparison of 4 contracts, 1ST, FOE, FOP and RFE which are described in Section II.



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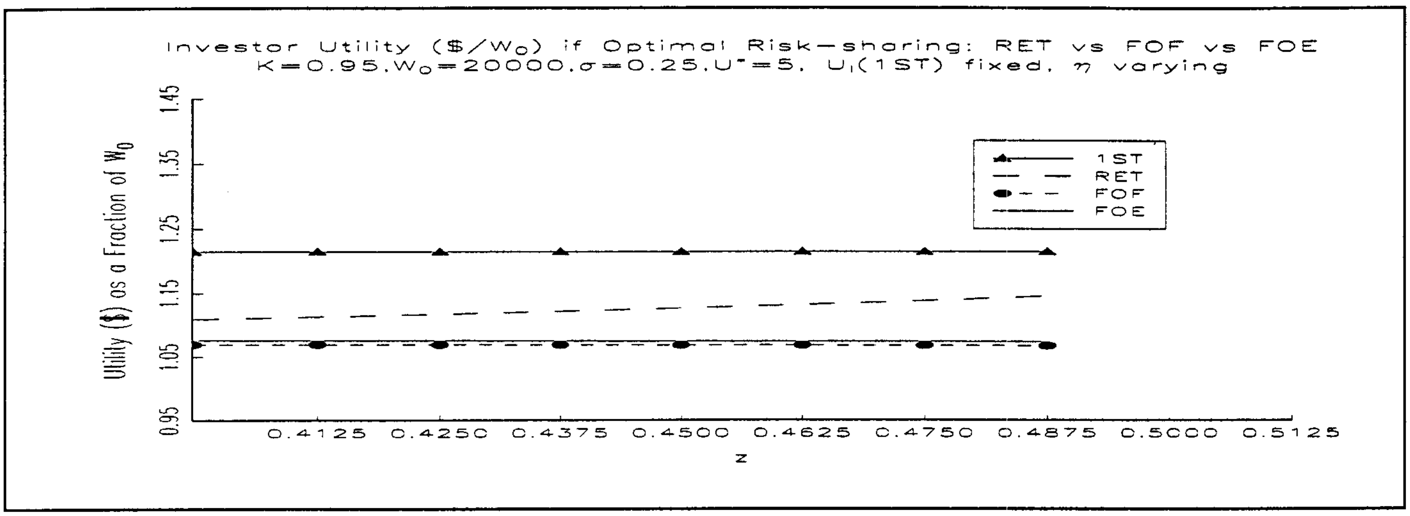

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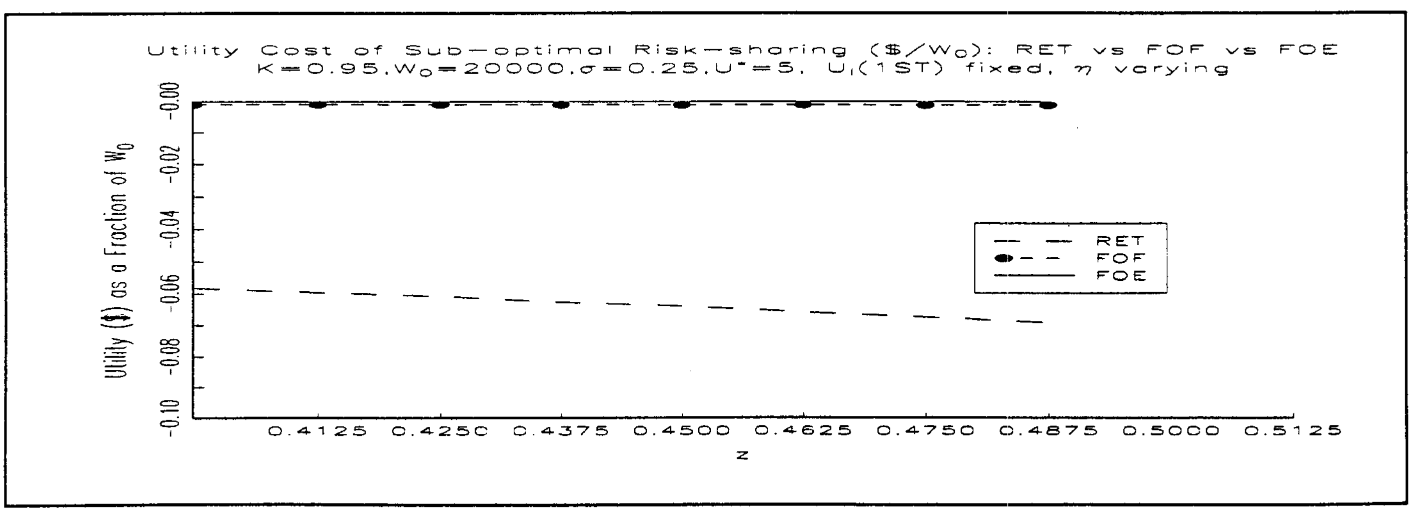
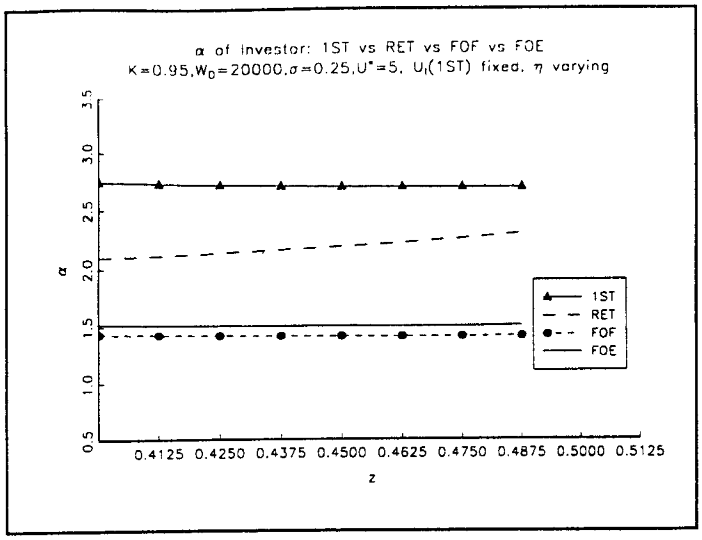


Panel C

Figure 4. Scenario 2 with (W_0, K, σ, U^*) set to (20000, 0.95, 0.25, 5). Numerical solution exercise that holds U_I fixed and allows η to vary as z varies from 0.4 to 0.5125. Comparison of 4 contracts, 1ST, RET, FOF and FOE which are described in Section II.



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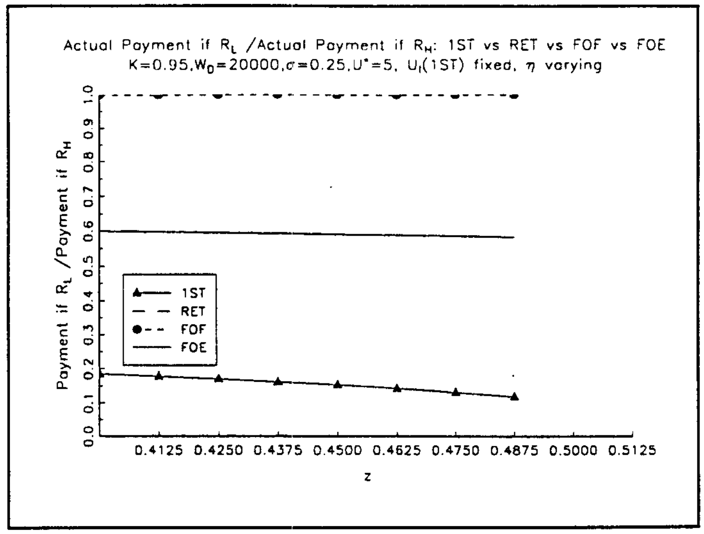

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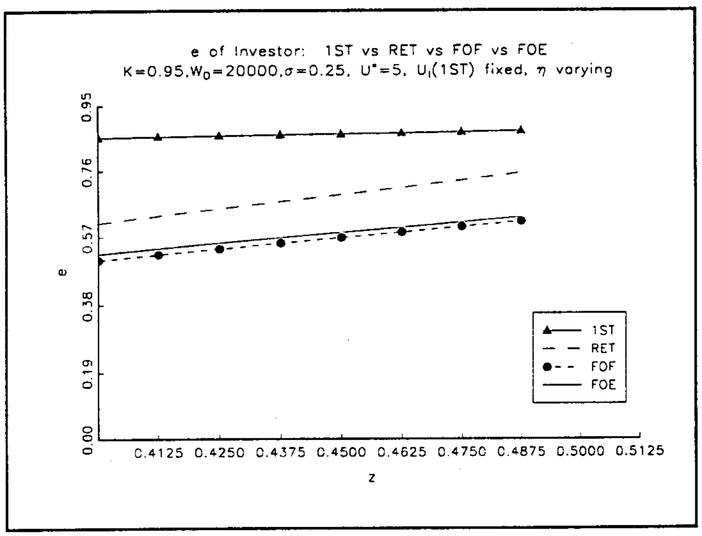

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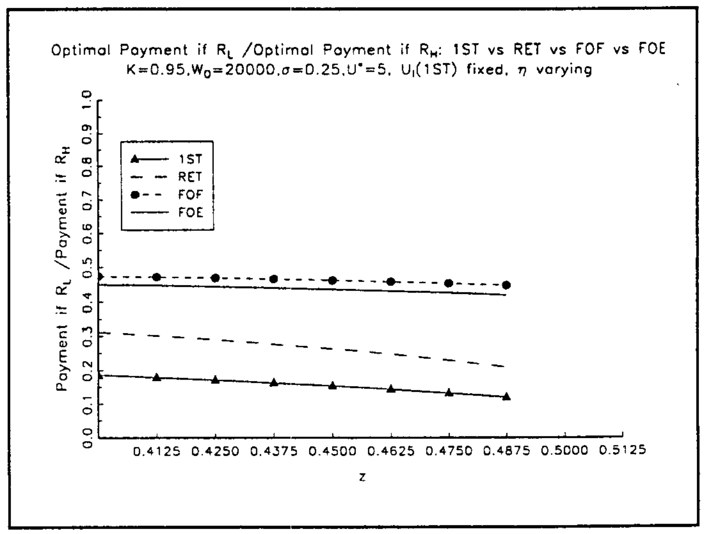

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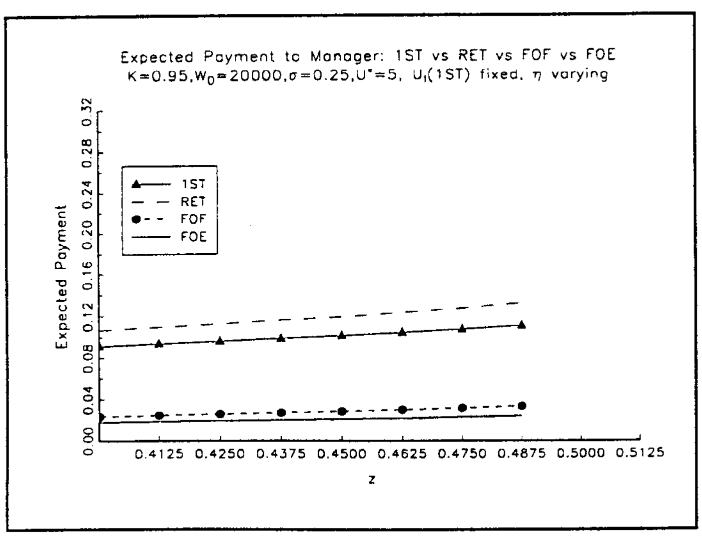

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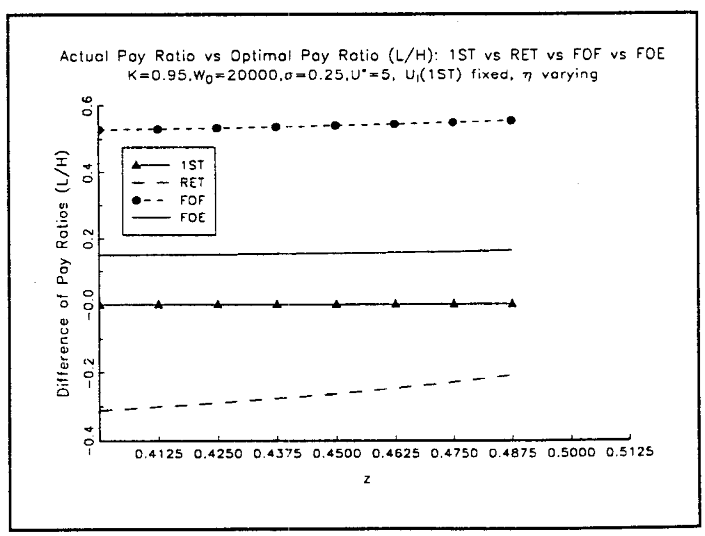
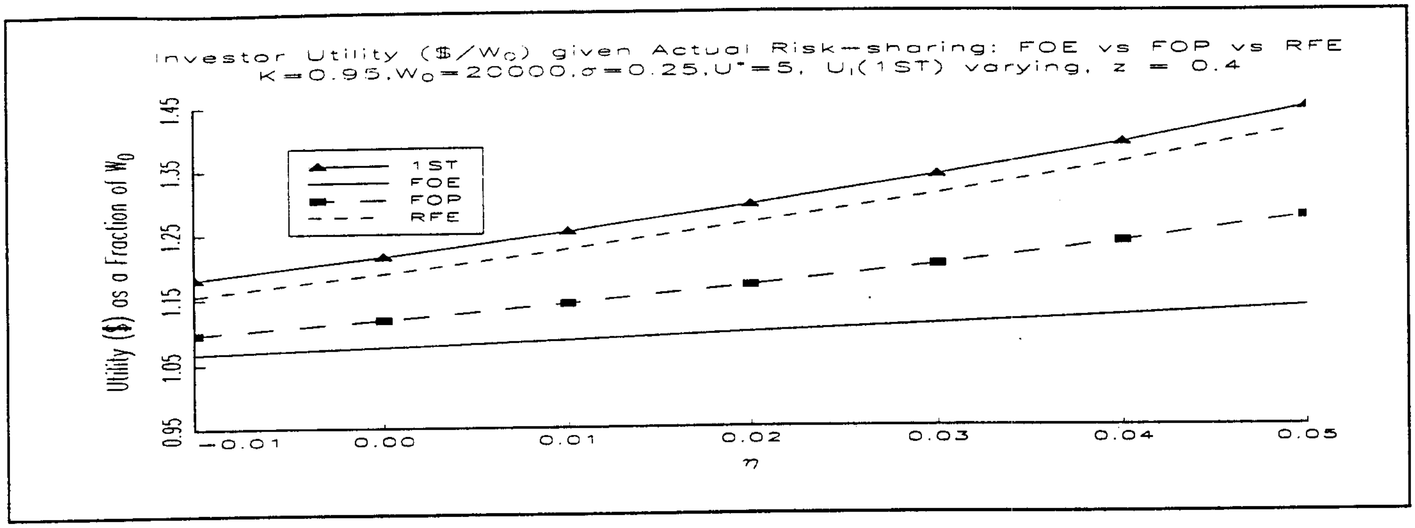


Panel I

Figure 4 (cont). Scenario 2 with (W_0, K, σ, U^*) set to (20000, 0.95, 0.25, 5). Numerical solution exercise that holds U_l fixed and allows η to vary as z varies from 0.4 to 0.5125. Comparison of 4 contracts, 1ST, RET, FOF and FOE which are described in Section II.



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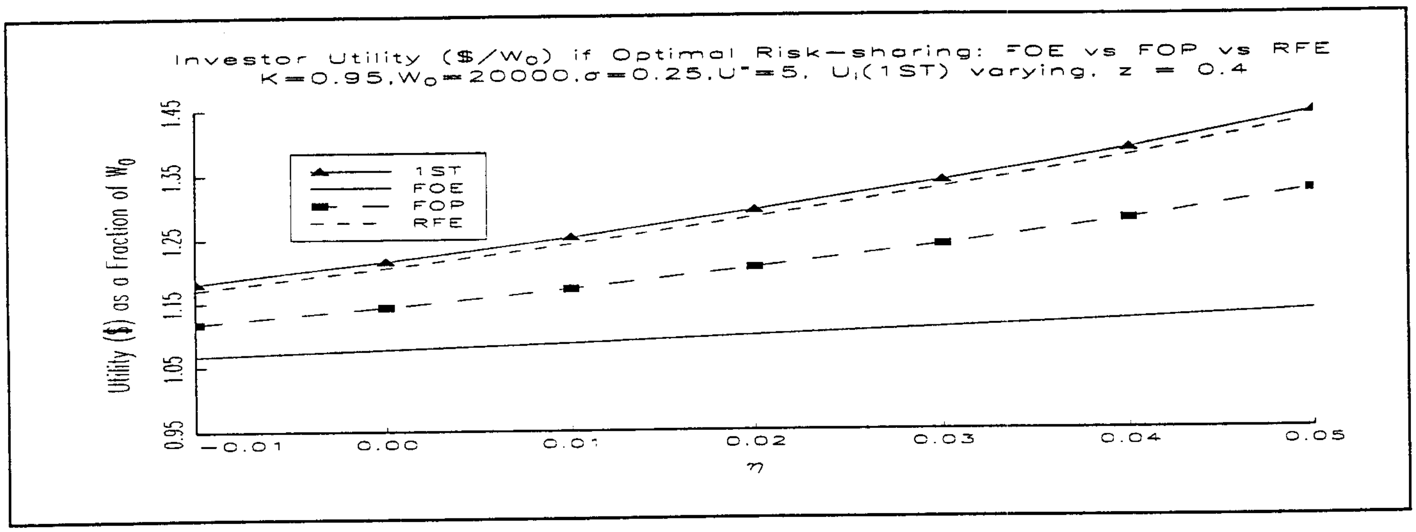

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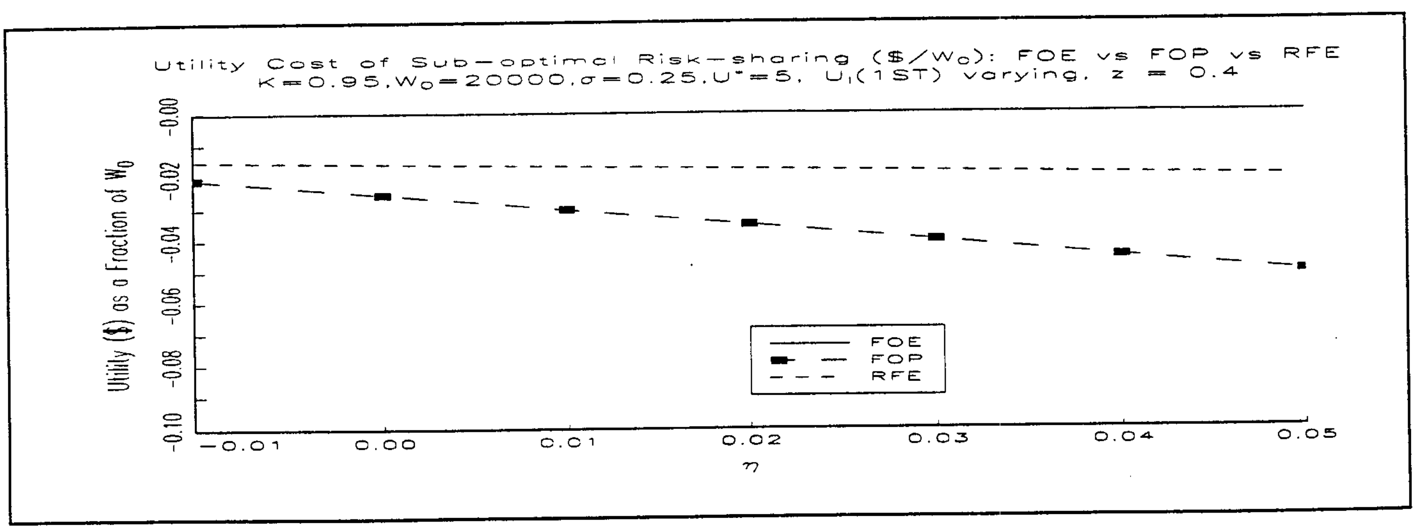
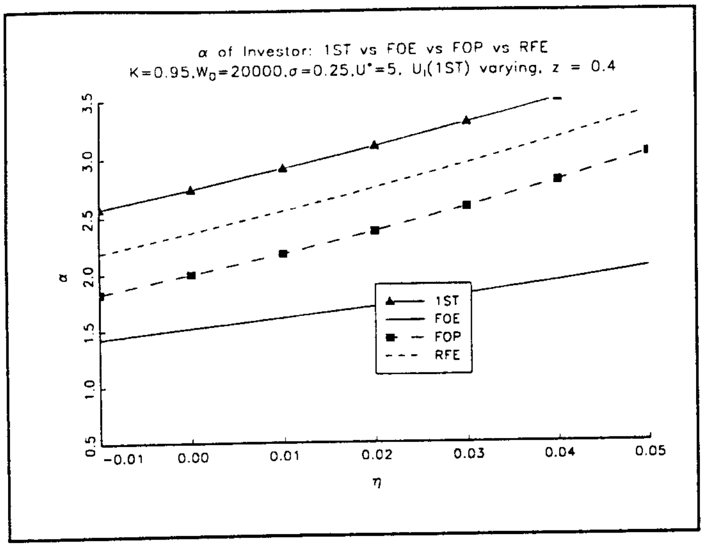


Panel C

Figure 5. Scenario2 with (W_0, K, σ, U^*) set to (20000, 0.95, 0.25, 5). Numerical solution exercise that holds z fixed at 0.4 and allows η to vary from -0.01 to 0.05. Comparison of 4 contracts, 1ST, FOE, FOP and RFE which are described in Section II.



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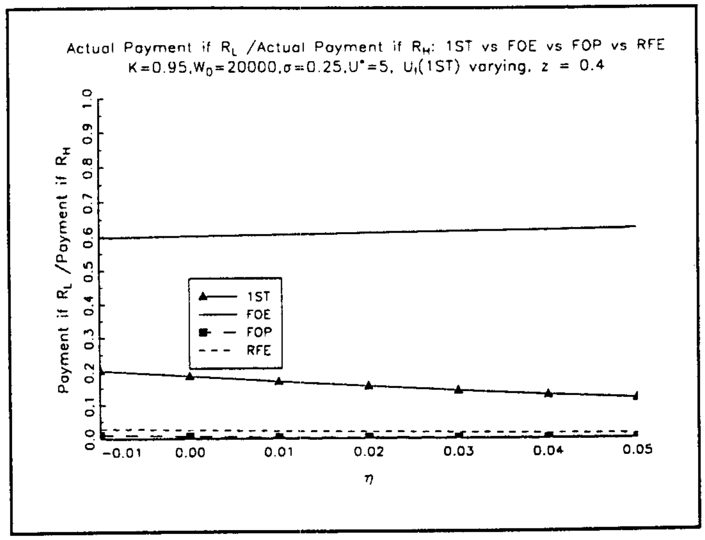

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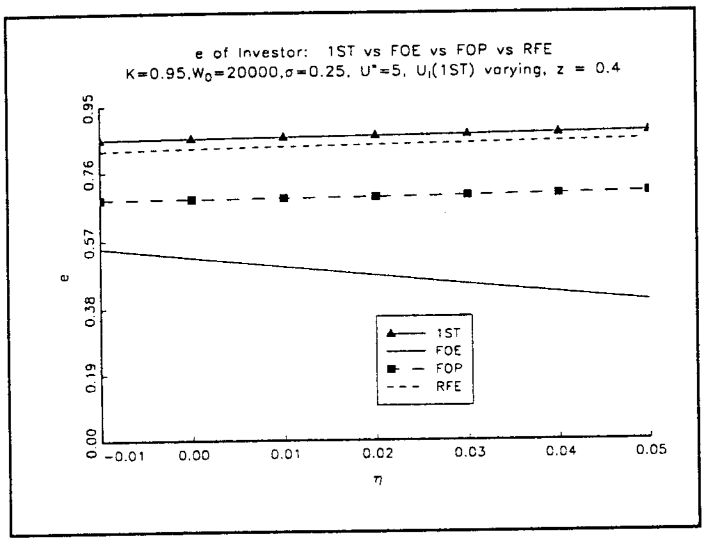

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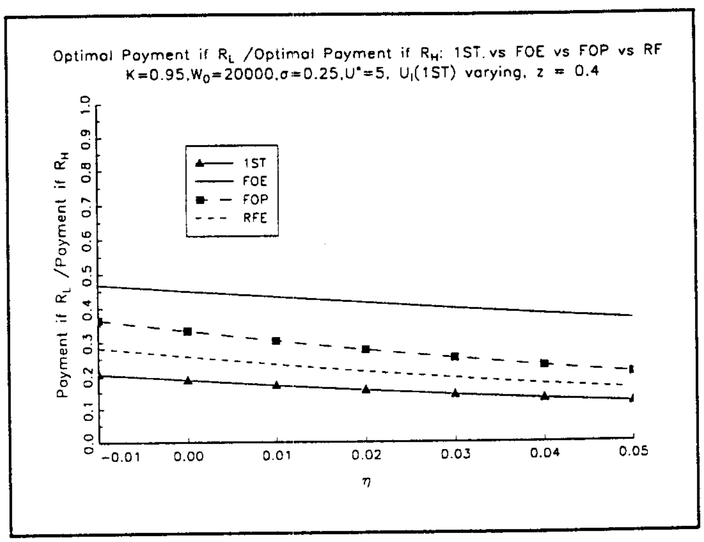

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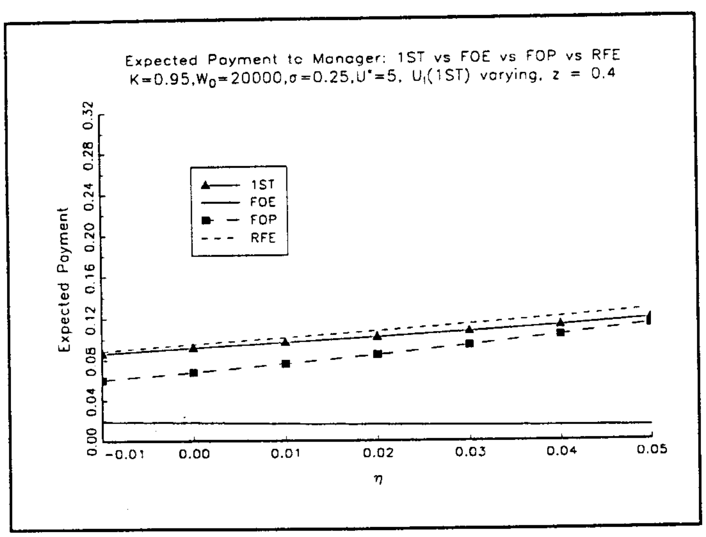

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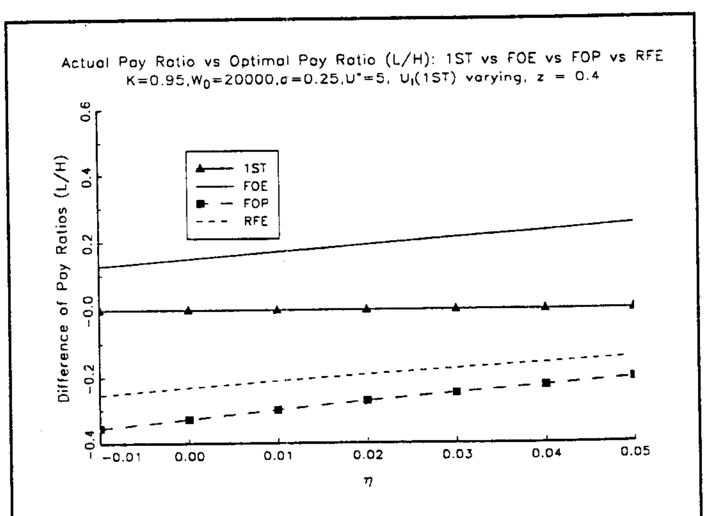

Panel I

Figure 5 (cont). Scenario2 with (W_0, K, σ, U^*) set to (20000, 0.95, 0.25, 5). Numerical solution exercise that holds z fixed at 0.4 and allows η to vary from -0.01 to 0.05. Comparison of 4 contracts, 1ST, FOE, FOP and RFE which are described in Section II.