Dividend Policy and Clientele Rationality

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December 14, 1999

*I would like to thank Liz Demers, Harrison Hong, Andrew Karolyi, Maureen McNichols, Terry Odean, Paul Pfleiderer, Manju Puri, Peter Reiss, Bill Sharpe, Itamar Simonson, Ingrid Werner, and especially Anat Admati, Dimitri Vayanos and Jeff Zwiebel for helpful comments and discussion. All remaining errors are my own.

Abstract

This paper examines a firm's dividend and investment policies in a model in which some investors are not aware of some of the firm's investments. These naive investors attribute all dividend changes to earnings changes, although the change in dividend is sometimes due to a new investment opportunity, beyond the firm's existing core business. For example, when a cost is incurred for such an investment and the dividend is consequently lowered, naive investors under-value the firm. The valuation of these investors influences prices since we assume all investors are risk averse. This implies that the firm will be under-priced when the investment cost is incurred. Following a similar argument, the firm is over-priced when the investment pays off. The model predicts that naive investors sell to sophisticated investors after a dividend decrease, and buy from sophisticated investors following a dividend increase. Generally, the firm's investment policy is suboptimal, typically involving under-investment. Surprisingly, having more naive investors sometimes brings the investment level closer to first best. Our model has several testable implications, in particular, the abnormal returns and trading patterns implied by naive investors' mispricing are supported by recent empirical studies.

1 Introduction

The market reaction to dividend changes is still somewhat of a puzzle. According to Miller and Modigliani [1961], in a frictionless world, dividend policy should not influence share price. In fact, in view of the higher taxes paid for dividends we may expect that decreases in dividends, if anything, will move prices up. However, many empirical papers have documented a drop in stock price after a dividend decrease is announced, and a rise in price after an increase in dividends (see e.g., Aharony and Swary [1980] and Eades, Hess, and Kim [1985]). A common explanation for these phenomena is the dividend signaling theory (e.g., Bhattacharya [1979]), which argues that firms use dividends to signal future earnings prospects. There are several problems with this explanation. First, it is plausible that there are cheaper ways to signal, e.g., using debt, which gets preferable tax treatment. Second, signaling models typically have multiple equilibria with different implications, and in order to select one, additional assumptions are needed. Finally, Benartzi, Michaely, and Thaler [1997] found that earnings tend to *increase* after dividend cuts, which is inconsistent with the signaling model's predictions.

Signaling models assume, among other things, that investors can use dividends to correctly infer information about earnings. This is not always plausible. Consider a one-time profitable project whose cost is incurred now and whose payoff is in the future (e.g., next period). If the firm has undertaken this project, and financed it using retained earnings, current dividends might be lowered and future dividends, at the time of payoff, would likely be higher. Investors must realize that the dividend drop, as well as increase, are an isolated occurrence rather than a negative or positive signal about earnings further in the future. In reality, however, it is likely that some investors are ignorant of the costs and payoffs or even the existence of some of the firm's investment opportunities. These investors may naively misinterpret the isolated dividend changes as a signal about future earnings.

This paper examines the firm's investment and dividend policies under the assumption that some (possibly a very small fraction of) investors are naive in the way suggested above. As in the signaling models, dividends in our model contain information about the earnings process.

However, unlike the signaling models, if the firm undertakes the investment, its dividends are affected too. This is because the cost of the investment is taken from, and the investment's payoff is added to, retained earnings. We assume that all investors learn about the earnings process from the dividends, but that the naive investors are ignorant with respect to the one time investment and therefore misinterpret changes in dividends.[1] The setup of the model we present is immediately consistent with existing empirical findings.[2] The model provides empirical implications that are consistent with the existing literature (e.g., Petrie [1998], Beer [1993] and others, see section 3.2 for details). In addition, the model has new empirical predictions about price and volume reactions to dividends and about investment and dividend policies.[3]

An example of a dividend cut, that was an attempt to increase future profits rather than a sign of hardship, is the cut by Florida Power and Light (FPL) on May 9, 1994. As documented in Soter, Brigham, and Evanson [1996], FPL announced a cut in its dividend in order to fund a stock repurchase plan as well as "provide the financial resources to take advantage of future growth opportunities." Upon this announcement FPL's stock price dropped by 14%. However, following this initial drop, FPL outperformed the S&P utility index in the next 18 months. In addition, as Soter, Brigham, and Evanson report, other utilities that took similar steps shortly after FPL also experienced a price drop. In contrast, utilities that took such actions later on did not experience a price drop. Soter, Brigham, and Evanson conjecture that investors realized

[1] Reckers and Stagliano [1980] surveyed individual investors and found that only 22% use information from the company as their primary source of information about the quality of management. This includes the annual or quarterly report or a company's newsletter. As a comparison, friends accounted for 11%, and stock-brokers accounted for 16%. They also found that 48% of investors casually glanced at the statement of changes in financial position in the annual report or ignored it all together. Al-Qudah, Walker, and Lonie [1991] surveyed UK analysts. They found that the most often cited main source of information about a company's capital expenditure plans is private meetings with the company (15%). Additional main sources that accounted for 17% of respondents were telephone conversations with contacts in the company and personal contacts in the company. These venues are not available to the average individual investor, for example. Chang [1982] surveyed individual investors as well as analysts. She found that the two groups differed in three of the four information sources deemed most important by each type. Thus, it seems reasonable to assume that naive investors cannot know about all of the firm's investment opportunities, as the sophisticated investors do.

[2] There are non-tax related clienteles in the model, consistent with the findings of Skinner and Gilster [1990], and Baja and Vijh [1990]. In addition, it is immediate from the model's set up that earnings can in fact be higher following a dividend decrease, as found by Benartzi, Michaely, and Thaler [1997].

[3] Some of these implications are tested, and supported, in Nelson [1999].

that good investments, stemming from deregulation, are available for these companies. The case of FPL provides anecdotal evidence that is consistent with the predictions of our model (see below). If some FPL investors are naive, in the sense above, they initially misinterpret the lower dividend and do not attribute it to investment. Thus, at this point, the firm is undervalued (as evidenced by the ensuing out-performance of the stock). As Soter, Brigham, and Evanson mention, it is possible that over time, after several utilities go through similar transitions, naive investors will learn to correctly interpret the cut in dividends in such cases. This can explain why utilities that adopted this strategy relatively late did not experience an initial price drop.

In our model, if an investment cost was incurred in the current period, the dividend will be lower. When the investment pays off the dividend will increase. These two dividend changes cause two sets of opposite effects which are, essentially, two sides of the same coin. We will term these effects the *cost effects* and *payoff effects*, respectively. The basic cost effect is that prices fall after dividend decreases, even if the decrease is due to investment. In the latter case the firm will be under-priced. The corresponding payoff effect is that prices rise after a dividend increase, and, if the investment opportunity is profitable, the firm will be over-priced. Even though sophisticated investors value the firm correctly, mispricing occurs because these investors are risk averse. In fact, the severity of mispricing is increasing in the proportion of naive investors.

Combining these two effects, the model predicts that there are positive average excess returns to a strategy that buys after dividend decreases and sells some time later, and negative excess returns to the strategy that buys after a dividend increase and sells some time later. By similar reasoning, the model implies certain trading patterns for the different types of investors. For example, after a dividend cut due to investment, sophisticated investors, such as insiders, are predicted to be net buyers, while naive investors, such as individual investors, are net sellers.[4] This latter implication, as well as the excess return predictions, are supported by

[4] In the case of the FPL dividend cut, Soter, Brigham, and Evanson note that "FPL's institutional ownership has increased since the dividend action - from 34% at the end of 1993 to 47% at the end of 1995". If we think that the remaining stock in the company is held by individual investors, which could be considered naive,

recent findings in Nelson [1999]. In particular, individual investors do indeed tend to sell after dividend decreases, where decreases are defined relative to some measure of expectations. These investors also tend to buy after dividend increases. The paper also finds positive abnormal returns, on average, following an unexpected dividends cut, and negative abnormal returns following an unexpected dividend increase. These findings support the model's predictions.

In general, the model addresses the impact of corporate dividend policy on naive investors beliefs and the total market reaction. In turn, the model also sheds light on how the potential reaction of naive investment ex ante affects the corporate investment and dividend policies. We assume that the manager, who makes the investment and dividend decisions, potentially cares about current and future prices and dividends. If the manager cares mainly about the current price, two implied cost effects are that the manager under-invests when there are naive investors present and that the more naive investors there are the less he invests. The corresponding payoff effects suggest that if the manager cares mainly about the price at the time of payoff, he may be induced to over-invest, and the magnitude of the over-investment is increasing in the fraction of naive investors.[5] Whether the cost- or payoff-effects dominate depends on the manager's sensitivity to prices and dividends. For some managerial objectives a mixture dominates (e.g., under-investment with a higher propensity to invest the more naive investors there are).

To illustrate the model's predictions we consider in more detail two specific plausible types of managerial preferences:

Myopic – This manager type puts too much weight on the current price. We'll consider an extreme such case where the manager cares only about the current price. This manager invests optimally if all investors are sophisticated, but with naive investors he will under-invest, as implied by the cost effect. For this manager cost effects always dominate and his investment level *decreases* with the fraction of naive investors.

this shift in ownership could be interpreted as naive investors selling and sophisticated investors buying after a dividend cut. In addition, as mentioned earlier, following the dividend cut and the accompanying price drop, FPL outperformed the S&P Utilities index for at least a year.

[5] These effects are hard to balance out. Even when a manager cares equally about both prices, he does not invest optimally. In fact, this manager over-invests since he gains from the payoff both in the future price, and through the sophisticated investors, in the current price.

Deferred Compensation/High Turnover (DT) – This manager obtains stock based deferred compensation, such as unvested stock options. However, he also fears being fired if his current performance (dividend) is insufficient. Therefore, the DT manager cares only about the current dividend and the future price, but myopically cares more for the former (since if he is fired he will not realize the deferred compensation at all). If this type of manager were to care equally about the current dividend and the price when the investment pays off, he would achieve first best investment levels regardless of the proportion of naive investors. Since the DT manager cares more for the current dividend, he will under-invest, but his investment level *increases* with the proportion of naive investors. The DT manager cares about the future price, therefore, the payoff-effects tend to dominate for him. He under-invests because he cares too much about the current dividend.

The literature has proposed several rationales for the payment of dividends, as well as for the effects a dividend change has on prices. One is a tax clientele theory.[6] Under this theory, a drop in dividend level will lower stock prices since investors must rebalance their portfolio to its previous dividend level, thus incurring transaction costs. Unlike our model, this explanation implies that a similar drop is predicted, for similar reasons, when dividends are increased. This, however, is not observed (e.g., Aharony and Swary [1980]).

Another theory, already mentioned above, is the signaling theory (see Bhattacharya [1979]), based on the signaling model of Ross [1977]. Beyond the problems mentioned above, signaling models have additional drawbacks. As mentioned by Bhattacharya, the signaling equilibrium may not be feasible if there is too much quality dispersion among firms.[7] Signaling models are also highly dependent on the beliefs of investors.[8] For example, if investors believed that a lower

[6]Miller and Modigliani [1961] first suggested the theory of clienteles in general. The tax clientele hypothesis has been tested with mixed success by Elton and Gruber [1970], Skinner and Gilster [1990], Hearth and Rimbey [1993] and many more. The tax clientele theory argues that the firm maintains a stable dividend policy to meet the needs of the tax clientele that is holding the firm's shares. The firm in a sense is catering to its clientele when it does not change dividend levels.

[7]Rodriguez [1992] finds explicit quality dispersion constraints leading to feasible signaling equilibria. However, he bounds dividends by *expected* cash flows in order to get his results.

[8]This is embodied in Blume and Friend [1978], based on a survey finding that managers believe that the impact of dividend changes on stock price depends on the market perception of the reason for the changes.

payout induces larger future earnings, due to superior investments, then restricting dividends would be a better signal than increasing them.[9]

A third theory to explain dividends is based on mitigating agency costs by using dividends to reduce free cash flow. A reduction in dividends leaves more cash for the manager to squander, hence reducing firm value. However, Jensen [1986] argues that regular dividend payments are likely to be weak substitutes for debt payment in controlling free cash flow problems. This is supported by Denis, Denis, and Sarin [1994].[10]

Very few behavioral models have addressed the issue of dividends.[11] Shefrin and Statman [1984] restrict individual investors' behavior based on prospect theory (see Kahneman and Tversky [1979]) and self-control problems (see Shefrin and Thaler [1981]). They claim that investors do not have the self control required to create their optimal cash stream by selling stock.[12] Unlike in our model, for some (high) dividend levels, an increase in dividends should actually lower prices in their model. This is because an increase in dividends induces weak willed investors to consume too much, and thus they must rebalance their portfolio at a cost. This is not observed in practice. Further, when looking at electric utilities, which pay very high dividends, Shelor and Officer [1994] found a higher than average positive price reaction to dividend increases.

In a related behavioral paper about capital budgeting, Stein [1996] considers a firm's investment policy when the investors in the financial market are all irrational and could be biased towards optimism or pessimism. He assumes that these investors misprice the investment the manager is considering for exogenous reasons. In contrast, in our model the direction of mis-

[9]Noe and Rebello [1996] develop a theoretical model along these lines, where restricting the dividends is in the interest of policy makers since they would at some point like to raise outside capital.

[10]Zwiebel [1996] gives a model of controlling free cash flow, when takeovers are possible, using debt together with dividends that are not driven by earnings alone. In Zwiebel's model, in equilibrium, lower debt levels and dividend levels imply better investment opportunities and thus a higher quality firm.

[11]Cyert, Kang, and Kumar [1996] suggest a behavioral model where managers are assumed to use decision rules that minimize their need to correctly anticipate events in the distant future. The behavioral characteristics of their model are very close to those suggested by Lintner [1956]. It is not, however, clear what the implications of their model are regarding stock price reaction to dividend decreases.

[12]Shefrin and Statman themselves raise some obvious drawbacks of these assumptions: a money manager can be hired to simulate dividend streams at a lower cost, or bond coupons could be used.

pricing is derived endogenously. In addition, Stein's model would predict that the average cumulative abnormal returns after an investment is made will be negative.[13] Our model, in contrast, predicts that this average will be positive, as long as the investment is reflected in a change in dividend payments. Our model's prediction is supported by findings in Nelson [1999].

The paper is organized as follows: In section 2 we present and analyze our model. This section contains the main results of the model, which include comparative statics. Subsection 3.1 summarizes the implications of the model and presents some additional predictions. Subsection 3.2 reviews empirical findings consistent with the model's implications, and discusses possible extensions of the model. Section 4 concludes. A list of notation precedes the references, and all derivations and proofs not in the text are in the appendix.

2 The Model

Our model is a three period model. For simplicity we normalize the riskless rate to zero. The firm's net earnings in period t, denoted E_t, are of the form $E_t = \mathcal{X} + e_t$ (for $t = 1, 2, 3$), where $\mathcal{X}$ is the mean of the firm's earnings process, and e_t is the current earnings deviations from the mean. We assume that $e_t \sim N(0, \frac{1}{r})$, where r is the precision of the earnings random variable, and that e_1, e_2, and e_3 are i.i.d. . We also assume that $\mathcal{X}$ is the realization of a random variable, X, that is $X \sim N(M_0, \frac{1}{\tau})$. This assumption allows investors to learn about the earnings process (see below). In period 1 the firm has an additional investment opportunity with cost I_1 (in period 1) and payoff R_2 (in period 2). We assume that I_1 and R_2 are revealed to the manager and some investors in period 1, before any dividend decision. Notice that since there is no uncertainty and the riskless rate is zero, the first best investment strategy is to invest iff $\frac{R_2}{I_1} > 1$ (i.e., the required rate of return is 1 at first best). For simplicity, we assume that the firm must pay out all net retained earnings. The manager of the firm knows the net earnings realization, E_1, and chooses the dividend D_1 (or, alternatively, whether to invest or not) subject to this

[13] In Stein's basic model, high return investments are taken regardless of the bias of irrational investors. The average abnormal return following these investments is zero. On the other hand, low return investments are only taken if investors are overly optimistic (which is when the stock is over-priced). Since Stein assumes the price converges to fair value in the long run, the prediction follows.

constraint. The second period dividend, D_2, is determined by the second period earnings, E_2, and the manager's investment decision. Third period dividends always equal third period earnings ($D_3 = E_3$). Initially, we will take the manager's investment decision as given, but later we solve for it endogenously.

All investors have CARA utility with risk tolerance parameter ρ, i.e., $u(W) = -e^{-\frac{W}{\rho}}$. They maximize $Eu(W)$ in each period, where W is total wealth over all remaining periods. All investors are unsure of the mean of the earnings process. Investors' initial belief is that $E_t \sim N(X, \frac{1}{r})$ with unknown X (the truth is that $X = \mathcal{X}$). Their prior for X is $N(M_0, \frac{1}{\tau})$, where τ is the precision of the prior and M_0 is its mean. Thus, all investors initially have the same expectation for the next dividend, namely M_0. Investors use dividends to update their beliefs about the mean of the earnings process, using conjugate priors.[14]

There are two types of investors, *naive* and *sophisticated*. While sophisticated investors are fully rational and well informed, naive investors are not as knowledgeable about the firm's investment opportunities. In fact, naive investors incorrectly believe that the firm has no further profitable investment opportunity beyond its core business. They, therefore, infer that earnings were equal to the dividend. On the other hand, the fully rational sophisticated investors know the realization of I_1 and R_2. They can also verify whether an investment was indeed made by the firm. Hence they can correctly use dividends to back out the actual earnings, which is what they use to update their beliefs on the earnings process. We assume that naive investors ignore the beliefs of sophisticated investors as well as their manifestation in prices.[15] The proportion of *naive* investors in the population is α, and the proportion of *sophisticated* investors is $1 - \alpha$.

Trading, and thus price determination, takes place *after* the dividend announcement, but *before* the actual dividend distribution.[16] Therefore, after the announcement of the third divi-

[14]The assumption that investors infer information about the firm's permanent net earnings prospects from dividends is consistent with evidence in Denis, Denis, and Sarin [1994] as well as Lee [1996] and others. As an example, Farrelly and Baker [1989] surveyed institutional investors in order to assess their views on dividends. Almost 88% of respondents thought that "the market uses dividend announcements as information for assessing security values".

[15]See Reckers and Stagliano [1980], Al-Qudah, Walker, and Lonie [1991], and Chang [1982].

[16]The timing is set this way in order to consider announcement effects.

dend, which is a liquidating dividend, there is no uncertainty left and the price in period 3 is just the dividend announced for that period, D_3. The first and second period prices, denoted by $\mathcal{P}_1$ and $\mathcal{P}_2$ respectively, are determined by investors' demands. For simplicity we also assume that there exists a riskless asset available to investors (recall that the risk-free rate is assumed to be zero). All outside financing the firm undertakes is observable by investors, and is thus normalized to zero.

For purposes of price determination in period 1, we need to pinpoint naive investors' expectations about price and demands in period 2. It is plausible that naive investors are not aware of their naivete. Therefore, it makes sense to assume that they think all investors in period 2 are the same as them, i.e., also naive. Although this assumption is consistent with our motivation for naive investors, in some cases it yields complex expressions that are hard to follow. However, qualitatively, this assumption yields the same results as another, more elegant, assumption. The latter assumption is that all investors are aware of the correct investor mix (α naive and $1-\alpha$ sophisticated), however naive investors ignore the information of sophisticated investors, as though it were wrong. Sophisticated investors simply behave rationally and hence acknowledge the presence of naive investors but ignore their incorrect information. We will use this very symmetric assumption in the rest of the paper. In the appendix the results for both assumptions are derived and compared.

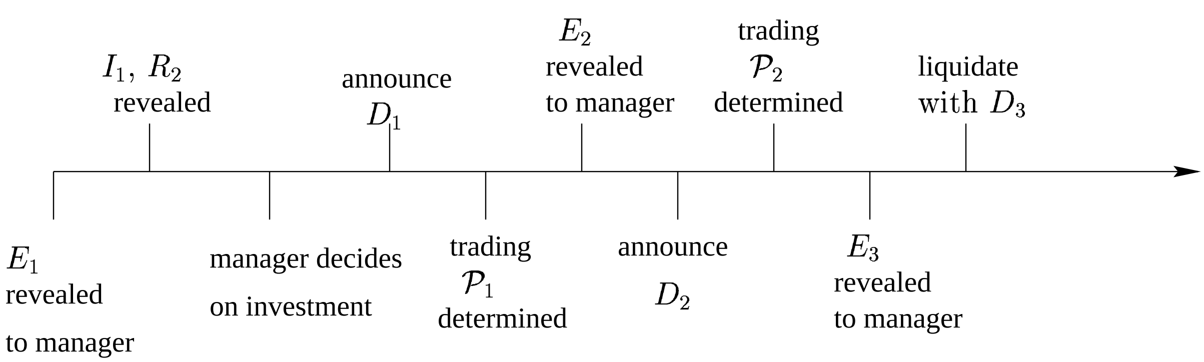

Figure 1: Time Line

The model, presented above, allows us to consider the case where both the cost and the payoff of the investment affect the beliefs of naive investors about future dividends.[17] This influences prices and hence the manager's decision. In subsection 2.1 we consider the price impact and trading patterns in this model. In subsection 2.2 we consider the effects on the manager's dividend and investment policies. In subsection 2.3 we consider the case where the manager can pre-commit to the investment before any cost is incurred.

2.1 Demands, Prices, and Trading

We start by taking the manager's investment decision as given and examining the effect of this decision on prices. Given that a dividend D_1 is announced, naive investors update their beliefs using D_1, while sophisticated investors correct for the investment, if it was taken. Naive investors now believe the mean of X is $M_1^N = \frac{\tau}{\tau+r}M_0 + \frac{r}{\tau+r}D_1$ and its precision is $\tau+r$. Using 1_{I_1} to denote an indicator that is 1 if the investment is undertaken and 0 otherwise, sophisticated investors now believe the mean of X is $M_1^S = \frac{\tau}{\tau+r}M_0 + \frac{r}{\tau+r}(D_1 + 1_{I_1}I_1)$ (also with precision $\tau + r$). Similarly, after investors saw both D_1 and D_2, naive investors will believe the mean of X is $M_2^N = \frac{\tau}{\tau+2r}M_0 + \frac{r}{\tau+2r}D_1 + \frac{r}{\tau+2r}D_2$ while sophisticated investors will believe the mean of X is $M_2^S = \frac{\tau}{\tau+2r}M_0 + \frac{r}{\tau+2r}(D_1 + 1_{I_1}I_1) + \frac{r}{\tau+2r}(D_2 - 1_{I_1}R_2)$, both with precision $\tau + 2r$.

Given these beliefs, we backtrack investors' demands starting in period 2 (period 3 price is just D_3). The appendix gives the calculations below in more detail. Let $x_j^{(t)}$ denote the period t demand of type j, where $t = 1, 2$ and $j \in \{S, N\}$ (S stands for sophisticated and N stands for naive). Naive investors solve the maximization problem

$$\max_{x_N^{(2)}} -Ee^{-x_N^{(2)}(D_2+D_3-\mathcal{P}_2)/\rho}$$

subject to $D_2 = E_2$ and $D_3 = E_3$. From this we can derive their demands:

$$x_N^{(2)} = \rho r \left(D_2 \left(1 + \frac{r}{\tau + 2r} \right) + \frac{r}{\tau + 2r} D_1 + \frac{\tau}{\tau + 2r} M_0 - \mathcal{P}_2 \right) \tag{1}$$

[17] A variation of this model would allow naive investors to learn from prices ex-post, i.e., after trading has ended and $\mathcal{P}_1$ is posted. In this case all beliefs are equal at the beginning of the second period. This is akin to the case where the investment shifts the core earnings of the firm, rather than being a one time investment. See section 4 for further discussion, and the appendix for details.

which, as expected, are linear in $\mathcal{P}_2$. Sophisticated investors solve the maximization problem

$$\max_{x_S^{(2)}} -Ee^{-x_S^{(2)}(D_2+D_3-\mathcal{P}_2)/\rho}$$

subject to $D_2 = E_2 + 1_{I_1}R_2$ and $D_3 = E_3$. From this we can derive their demands:

$$x_S^{(2)} = \rho r\left(D_2\left(1+\frac{r}{\tau+2r}\right)+\frac{r}{\tau+2r}D_1+\frac{\tau}{\tau+2r}M_0-\mathcal{P}_2+1_{I_1}\frac{r}{\tau+2r}(I_1-R_2)\right) \qquad (2)$$

Using market Clearing ($\alpha x_N^{(2)} + (1-\alpha)x_S^{(2)} = 1$) we get that the second period price is

$$\mathcal{P}_2 = D_2\left(1+\frac{r}{\tau+2r}\right)+\frac{\tau}{\tau+2r}M_0+\frac{r}{\tau+2r}D_1+(1-\alpha)1_{I_1}\frac{r}{\tau+2r}(I_1-R_2)-\frac{1}{\rho r} \qquad (3)$$

which is a weighted average of investors' valuations minus a risk premium.[18] If all investors were sophisticated the price would have been

$$D_2\left(1+\frac{r}{\tau+2r}\right)+\frac{\tau}{\tau+2r}M_0+\frac{r}{\tau+2r}D_1+1_{I_1}\frac{r}{\tau+2r}(I_1-R_2)-\frac{1}{\rho r}$$

Hence, if the investment is profitable ($I_1 - R_2 < 0$), then $\mathcal{P}_2$ is over-priced. Notice that the degree of over-pricing is proportional to α.

Plugging the price in equation (3) back into the demand functions (1) and (2) we get that demands in period 2 are

$$x_N^{(2)} = 1 + (1-\alpha)\rho\frac{r^2}{\tau+2r}1_{I_1}(R_2-I_1) \qquad (4)$$

and

$$x_S^{(2)} = 1 - \alpha\rho\frac{r^2}{\tau+2r}1_{I_1}(R_2-I_1) \qquad (5)$$

[18]More specifically, the price, $\mathcal{P}_2$, can be decomposed as follows:

- D_2 is the announced second period dividend, and $\frac{1}{\rho r}$ is the risk premium ($\frac{1}{\tau}$ is the variance of earnings).
- $\frac{r}{\tau+2r}D_2+\frac{\tau}{\tau+2r}M_0+\frac{r}{\tau+2r}D_1$ is the naive investor's assessment of the mean of future earnings (and thus the expected value of D_3 in his eyes) while $\frac{r}{\tau+2r}D_2+\frac{\tau}{\tau+2r}M_0+\frac{r}{\tau+2r}D_1+1_{I_1}\frac{r}{\tau+2r}(I_1-R_2)$ is the sophisticated investor's assessment of this mean. Thus, $(1-\alpha)1_{I_1}\frac{r}{\tau+2r}(I_1-R_2)$ is the weighted adjustment for the sophisticated investors' belief (their fraction in the population is $1-\alpha$).

Notice that demands are independent of the dividend itself, and that if the undertaken investment is profitable (i.e., $R_2 - I_1 > 0$), then a sophisticated investor holds less than a naive investor in the second period.

We now solve for the first period, taking into account that the demands are as in equations (4) and (5), and that the second period price has the form of equation (3) (i.e., using the symmetric assumption where naive investors are aware of the investor mix). To find first period demands, type j investors solve the maximization problem:

$$\max_{x_j^{(1)}} -e^{-x_j^{(1)}(D_1-\mathcal{P}_1)/\rho} E e^{-(\mathcal{P}_2(x_j^{(1)}-x_j^{(2)})+x_j^{(2)}(D_2+D_3))/\rho}$$

Naive investors ($j = N$) assume that $D_1 = E_1$ and $D_2 = E_2$, while sophisticated investors ($j = S$) assume that $D_1 = E_1 - 1_{I_1} I_1$ and $D_2 = E_2 + 1_{I_1} R_2$. All investors assume that $D_3 = E_3$. This yields the following demand functions for the two types:

$$x_N^{(1)} = \rho r \left(\frac{\tau+2r}{\tau+3r}\right)^2 \left[D_1\left(1+\frac{2r}{\tau+r}\right)+\frac{2\tau}{\tau+r}M_0 - \mathcal{P}_1 - \frac{b}{\rho r} - 1_{I_1}(1-\alpha)\frac{rb}{\tau+2r}(R_2 - I_1)\right]$$

$$x_S^{(1)} = \rho r \left(\frac{\tau+2r}{\tau+3r}\right)^2 \left[D_1\left(1+\frac{2r}{\tau+r}\right)+\frac{2\tau}{\tau+r}M_0 - \mathcal{P}_1 - \frac{b}{\rho r} + \right.$$
$$\left. +1_{I_1}\left(\alpha\frac{rb}{\tau+2r}(R_2 - I_1) + R_2 + \frac{2r}{\tau+r}I_1\right)\right]$$

where the constant b is defined as $b = 1 - \frac{r(\tau+3r)}{(\tau+2r)^2} = 1 - \frac{r}{\tau+2r} - \left(\frac{r}{\tau+2r}\right)^2$. Notice that $\frac{1}{4} < b < 1$ since both r and τ are precisions and thus positive.

Using the demands above and market clearing, we get that price in period 1 is:

$$\mathcal{P}_1 = D_1\left(1+\frac{2r}{\tau+r}\right) + M_0\frac{2\tau}{\tau+r} + (1-\alpha)1_{I_1}\left(\frac{2r}{\tau+r}I_1 + R_2\right) - \frac{2}{\rho r} - \frac{1}{\rho(\tau+2r)} \qquad (6)$$

which, again, is a weighted average of investors' valuations minus a risk premium.[19] Comparing $\mathcal{P}_1$ to the valuation of sophisticated investors, we see that $\mathcal{P}_1$ is under-priced by $\alpha 1_{I_1}\left(\frac{2r}{\tau+r}I_1 + R_2\right)$.

[19] $\mathcal{P}_1$ can be decomposed as follows:

- D_1 is the announced dividend in period 1
- $\frac{r}{\tau+r}D_1 + \frac{\tau}{\tau+r}M_0$ is naive investors' updated assessment of the mean of future earnings in each of the next two periods (and thus the expectation of $D_2 + D_3$ is $\frac{2r}{\tau+r}D_1 + \frac{2\tau}{\tau+r}M_0$). The expression $(1 - \alpha)1_{I_1}\left(\frac{2r}{\tau+r}I_1 + R_2\right)$ is the adjustment for sophisticated investors' beliefs. This expression is always

Plugging the price back into demands we get

$$x_N^{(1)} = 1 - \frac{(1-\alpha)\rho r(\tau+2r)^2}{(\tau+3r)^2} 1_{I_1} \left[\frac{rb}{\tau+2r}(R_2 - I_1) + \frac{2r}{\tau+r} I_1 + R_2 \right]$$

$$x_S^{(1)} = 1 + \frac{\alpha\rho r(\tau+2r)^2}{(\tau+3r)^2} 1_{I_1} \left[\frac{rb}{\tau+2r}(R_2 - I_1) + \frac{2r}{\tau+r} I_1 + R_2 \right]$$

where b is the constant defined above. Since $b < 1$, the square brackets are positive, and hence in equilibrium, assuming the investment is undertaken, a naive investor holds less shares than his sophisticated counterpart in period 1. In summary we get the following proposition.

Proposition 1: *If the investment is taken then*

(i) *The firm is under-priced in period 1 (compared to its true value).*

(ii) *If the investment is profitable, the firm is over-priced in period 2.*

(iii) *A sophisticated investor will hold more shares than a naive investor in period 1, and, if the investment is profitable, less shares than a naive investor in period 2.*

The naive investors will anticipate lower future dividends after seeing the low first period dividend. However, if the investment is profitable, they will expect unusually high dividends after also seeing the high dividend in period 2 (which includes payoff R_2). The sophisticated investors correctly value the firm and expect the naive investors' mispricing. If all investors start with the same holdings, then sophisticated investors buy more shares in period 1 (when the firm is under-priced) and sell those shares and more in period 2 (when the price is high).

positive and therefore sophisticated investors always have a higher valuation of the firm than naive investors, when the investment is undertaken. Thus, the firm is undervalued in period 1.

- $\frac{2}{\rho r} + \frac{1}{\rho(\tau+2r)}$ is the risk premium for the next two periods. The first part is due to the variance of the next two dividends. The second part is due to the uncertainty about the mean of the earnings process. The precision of investors' belief regarding this mean, in the next period, will be $\tau + 2r$.

Proposition 1 implies that a firm that lowers dividends due to a profitable investment is under-priced at the time the dividend is cut, and over-priced later on when the investment pays off. In the framework of this model, a firm that lowers dividends due to financial hardship should be fairly priced after the dividend decrease. Therefore, an implication of the model is that buying a firm after a dividend decrease will, on average, give positive excess returns. Ideally, the firm is held until an abnormally high dividend is paid. However, if we believe that other information may induce naive investors to update their beliefs, a shorter holding period should also yield positive, though possibly more modest, average excess returns. Conversely, the proposition implies that following a dividend increase there will be negative average excess returns.[20] In addition, equation (6) implies that the magnitude of the initial price impact is proportional to the fraction of naive investors.[21]

Another prediction that follows from proposition 1 is that after a dividend is cut due to investment, when the price drops, the sellers are mainly naive investors while the buyers are mainly sophisticated investors.[22] In order to test this hypothesis one would need to identify which investors could be considered naive and which could be considered sophisticated, in the sense of this model. Individual investors are natural candidates for naive investors.[23] Insiders seem to be natural candidates for sophisticated investors.[24] Some caution is needed, however, since insiders may have private agenda that induce them to buy when the price drops.[25]

[20]Nelson [1999] tests and finds support for these implications.

[21] In the Belgian market, Beer [1993] found no significant effect when dividends are changed. She postulates that this is because Belgian firms are mostly closely held firms. Such closely held firms have better informed investors, which can correspond to the sophisticated investors in our model. Thus, Beer's findings and explanation are consistent with this implication. Another paper that is consistent with this implication is Petrie [1998], see section 3.2.

[22]If the dividend is cut due to financial hardship we would expect no significant abnormal trading, just a price adjustment.

[23]Nelson [1999] tests this implication with respect to the trades of individual investors. Assuming that individual investors are on average more naive than other investors, the model's predictions are supported.

[24]Petrie [1998] finds that the price impact of dividend surprises is inversely related to insider activity in the stock. This is consistent with our model's prediction.

[25]E.g., insiders may try to cause price appreciation by using the effect their buying has, see Chang and Suk [1998].

2.2 Investment and Dividend Policy

In this subsection we examine the level of investment and subsequent dividend, that the manager chooses. We will assume that the manager may care about dividends as well as prices, and that his preferences are not necessarily equal to his compensation package. Part of the weights in the manager's preferences might stem from implicit considerations, such as reputation, rather than explicit considerations, such as monetary compensation given by the firm. Because of the implicit components, the firm cannot completely control the manager's preferences, thus it is reasonable to take the manager's preferences as exogenous. For simplicity, in our discussion, we will represent the manager's preferences only in terms of a compensation scheme, however, intuitively we think of his preferences as incorporating implicit considerations as well.

The general form of the manager's preferences is $w_1^p \mathcal{P}_1 + w_2^p \mathcal{P}_2 + w_1^d D_1 + w_2^d D_2 + w_3^d D_3$, where $\mathcal{P}_t$ denotes the price at time t, the dividend at time t is denoted by D_t, and w_t^p is the sensitivity to $\mathcal{P}_t$ while w_t^d is the sensitivity to D_t.[26] We will assume that all compensation parameters are non negative (i.e., $w_1^d, w_2^d, w_3^d, w_1^p, w_2^p \geq 0$).[27] Proposition 2 characterizes which preference parameters will induce over- or under-investment.

Proposition 2: *In our model, there is under-investment iff*

$$w_2^d + w_2^p < w_1^d + w_1^p \alpha \left(1 + \frac{2r}{\tau + r}\right)$$

and over-investment iff the inequality is reversed.

We can rewrite the expression in the proposition as $w_2^d - w_1^d + w_2^p < w_1^p \alpha \left(1 + \frac{2r}{\tau+r}\right)$. The left hand side is the net inducement to (over-) invest due to the dividends themselves (including their manifestation in the second period price), while the right hand side is the deterrent to

[26] This compensation structure is a generalization of the one used in Bolton and Freixas [1997], as well as Petrie [1998].

[27] Hall and Liebman [1998] document a strong relationship between firm performance and CEO compensation. In their sample, this relationship is generated almost entirely by changes in the value of CEO holdings of stock and stock options.

investment due to the naive investors' effect on the first period price. The sensitivity to second period price, w_2^p, comes in through both types of investors since undertaking the investment always increases $\mathcal{P}_2$. This is because the investment cost is financed by a dividend reduction in period 1. Since we assumed that all residual cash flows are paid out as dividends each period, the only cash flows transferred between periods are those that are invested in the project (an outlay of I_1 in period 1 is "transferred" to an income of R_2 in period 2).

Figures 2 and 3 show some partitions of the parameter space, in terms of over- and under-investment. These figures also implicitly depict the change in a preference parameter needed to keep a manager at first best investment levels, as α increases (holding all other parameters fixed). When $w_2^d + w_2^p - w_1^d > 0$, Figure 2 demonstrates that, as implied by proposition 2, in order to be at first best it must hold that

$$w_1^p = \frac{1}{\alpha}\left(\frac{w_2^d + w_2^p - w_1^d}{1 + \frac{2r}{\tau + r}}\right)$$

Figure 3 shows that w_2^p should be increased linearly in α, at a rate of $w_1^p(1 + \frac{r}{\tau+r}) + w_1^d - w_2^d$. These observations are useful, for example, when the board of directors can only influence some of the manager's preference parameters. Notice, however, that in order to keep the manager at first best, the board must know all of his preference parameters, or sensitivities. This may not be plausible. From the figures we see that when $w_2^d - w_1^d + w_2^p \leq 0$, there is always under-investment since the weight on the current dividend more than offsets the weights on elements that are affected by the investment payoff. On the other hand, in both figures we see that if w_2^p, the weight on $\mathcal{P}_2$, is relatively large then there is over-investment.

In order to illustrate the propositions in this section more clearly, we will consider two special cases of manager types. One is the **Myopic** manager, whose compensation depends solely on the current price. The Myopic manager invests optimally if all investors are sophisticated. Another type we call the **Deferred Compensation/High Turnover (DT)** manager, for whom we assume that $w_2^d = w_1^p = 0$ and $w_1^d = w_2^p + \epsilon$ with $\epsilon > 0$.[28] This is a manager that has a stock based deferred compensation package, but could be fired or penalized if current

[28]When $\epsilon < 0$ then most of the results presented for the DT manager are reversed.

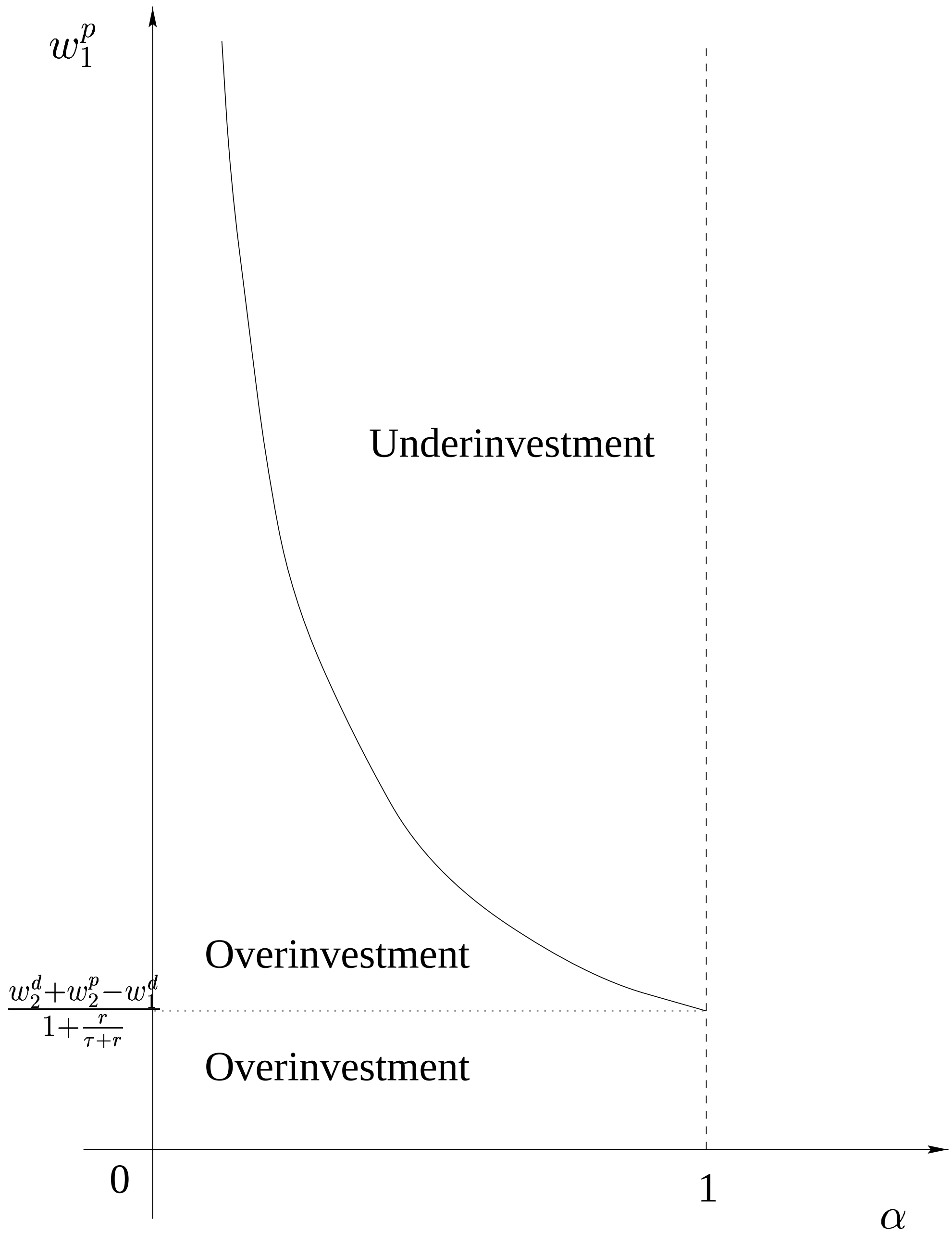

Figure 2: over- and under-investment as a function of α and w_1^p
assumes $w_2^d + w_2^p - w_1^d > 0$ (otherwise always under-invest)

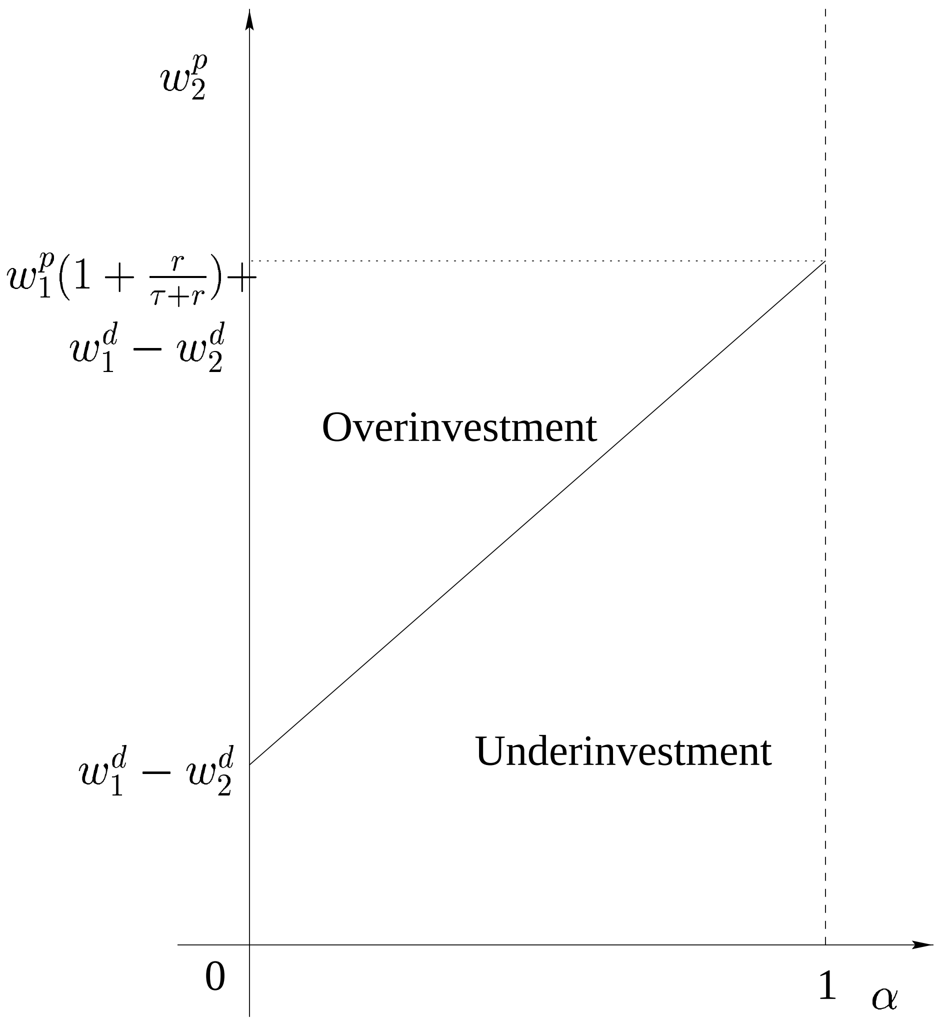

Figure 3: over- and under-investment as a function of α and w_2^p for fixed w_1^p, w_1^d, w_2^d

dividends are too low. Thus, the DT manager cares only for the current dividend and the future price, and we assume that he myopically cares more for the current dividend. If $\epsilon = 0$ then first best investment levels are achieved, regardless of the investor mix (and the expression in proposition 2 holds with equality). Therefore, if ϵ is small, the DT manager is close to the optimal compensation scheme.

Notice that when $\alpha = 0$, i.e., when all investors are sophisticated, the Myopic manager invests optimally (at first best) while the DT manager under-invests. For $\alpha > 0$, we have the following corollary.

Corollary 1: *When $\alpha > 0$, both the Myopic manager and the DT manager under-invest.*

For a derivation of the required rate of return for a generic manager, see the appendix. Proposition 2 and Corollary 1 have implications regarding investment levels. Typically, managers will under-invest, including managers like the Myopic and DT managers.[29] Recall that the latter cares only about the current dividend and the future price. However, if the manager does not care about the current dividend or price, but does care about the future price or dividend, then he may over-invest.[30]

Proposition 2 characterizes whether we get under- or over-investment. However, it does not tell us the degree of the manager's sub-optimality, nor does it tell us how the fraction of naive investors, α, affects this sub-optimality. The former point, the required rate of return for any given compensation scheme, is given in the appendix. The next proposition explicitly addresses the latter point. For simplicity, from here on we formulate the propositions in terms of the two manager types, Myopic and DT. The full characterization is in the appendix and applies to all manager types.

[29] However, some preferences can lead to both over- and under-investment, depending on the fraction of naive investors in the population. For example, the manager that cares equally about the attributes of both first and second period (i.e., $w_1^d = w_2^d$ and $w_1^p = w_2^p$) over-invests for lower proportions of naive investors ($\alpha < \frac{\tau+r}{\tau+2r}$) and under-invests otherwise ($\alpha > \frac{\tau+r}{\tau+2r}$).

[30] Such a manager could be a manager that plans on unloading some of his stock later on (after the investment pays off) and is also safe from firing since he is, for example, a majority stockholder. This indicates that a similar model, using earnings rather than dividends, may be applied to the case of IPOs: the entrepreneur can try to beef up short term earnings, at the expense of long run performance, in order to push IPO prices up.

Proposition 3: *The Myopic manager's required rate of return is increasing in α, moving away from the first best level. The DT manager's required rate of return is decreasing in α, moving closer to the first best level as α increases.*

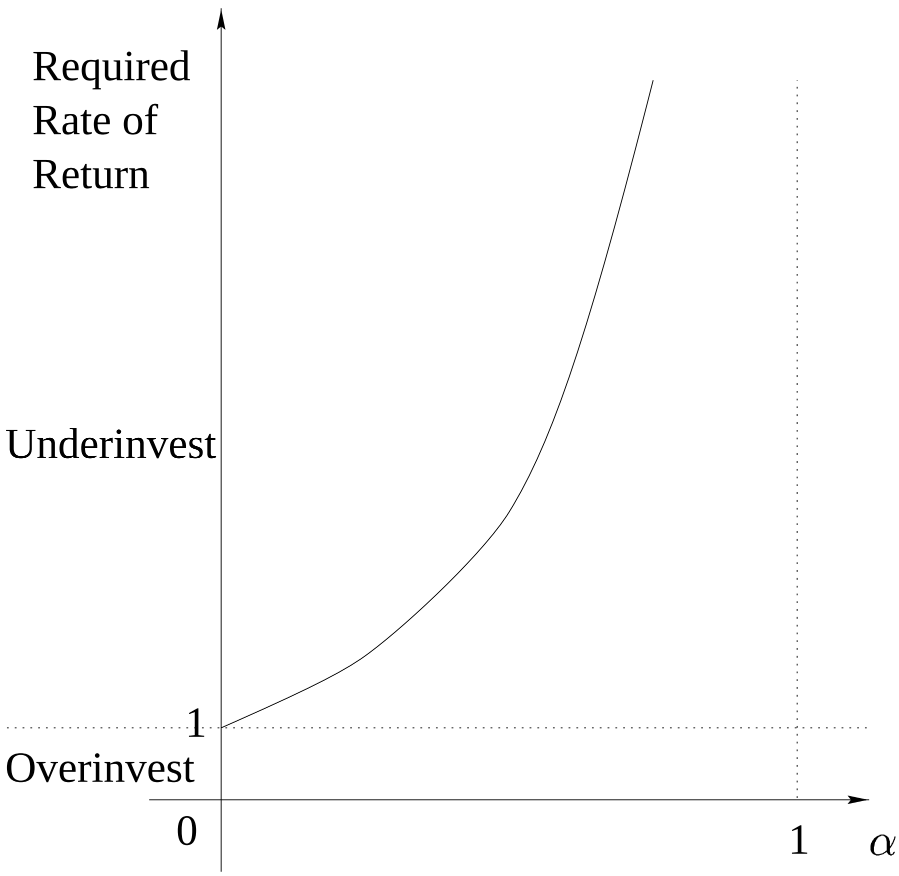

Figure 4: required rate of return for the Myopic manager as a function of α

Figures 4 and 5 qualitatively demonstrate the change in the required rate of return as α changes for the Myopic and DT managers, respectively. In both figures the dotted line denotes the optimal required rate of return, 1. In Figure 4 notice that, as α increases, the Myopic manager's required rate of return is increasing, moving further away from 1 and approaching infinity as α approaches 1. This is because the under-pricing in period 1 is proportional to the

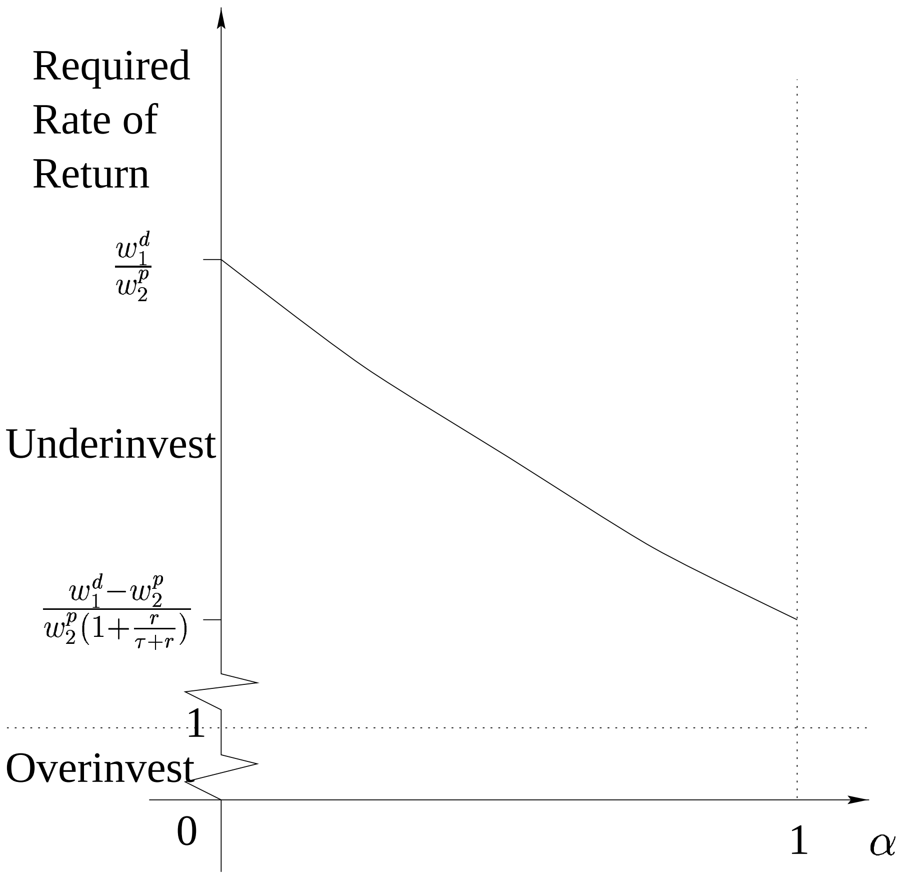

Figure 5: required rate of return for the DT manager as a function of α

fraction of naive investors. When all investors are naive the myopic manager will not take any project since he is only concerned with the current price which will not reflect the gain from the investment, only the cost. On the other hand, in Figure 5, we see that, as α increases, the DT manager's hurdle rate decreases, moving closer to the first best level. For this manager, who cares about the future price, more naive investors are a stronger inducement to invest, since their presence leads to a higher future price. However, this inducement is not enough for the DT manager to reach optimal levels since he cares more for the current dividend than for the future price. The more general formulation of Proposition 3 is given in the appendix.

Consider two companies with compensation structures similar to that of the Myopic manager, where the first company has a larger proportion of naive investors than the second. The model implies that the first company will have a higher hurdle rate than the second. Conversely, the model predicts that firms with higher insider holdings and managers that are sensitive to current price[31] will invest relatively more than firms with less insider holdings. The latter are under-investing firms that are further away from optimal hurdle levels.

Another factor that can influence the manager's decision is the quality of the prior information available about the firm. This parameter may, to some extent, be controlled by the firm's disclosure policy, or its board of directors. Superior prior information may facilitate investors' understanding of the firm's investment opportunities, and thus reduce the fraction of naive investors. In addition, better prior information can reduce the effect of the existing naive investors by decreasing the weight these investors assign to the dividends, relative to the weight on their prior beliefs. The effect of this latter consideration on the firm's investment policy is explored in the following proposition.

Proposition 4: *For any $\alpha > 0$, the Myopic manager will tend to invest more, the more precise the prior information about the firm's earnings process. However, the more precise the prior, the less likely the DT manager is to invest.*

[31] See Abrutyn and Turner [1990], Pruitt and Gitman [1991], Baker, Farrelly, and Edelman [1985], and Partington [1985].

The direction of the comparative statics in proposition 4 is compatible with that of proposition 3. Consider the marginal investment, for which the manager is indifferent between investing and not investing. Since both manager types under-invest, this investment is profitable. The less weight naive investors give to their inferences from dividends, the higher $\mathcal{P}_1$ and the lower $\mathcal{P}_2$ (recall that the investment is profitable). A more precise prior lowers the weight on the inference from dividends and raises the weight given to the prior. The Myopic manager, who cares only about $\mathcal{P}_1$, will invest more the more precise the prior. Conversely, the DT manager, who cares about $\mathcal{P}_2$ (and under-invests) will invest less, since for profitable investments $\mathcal{P}_2$ is less inflated with a more precise prior. If we assume that the board of directors has some control over the quantity and quality of information released to the public, they can use this information to induce the manager to invest at a level that is closer to optimal.

2.3 Pre-Commitment

In this subsection we consider the case where the manager can pre-commit to the investment, before the investment costs are incurred. The key is that when the manager commits, the dividends are not yet affected. Therefore, naive investors will not see any difference, compared to the case without pre-commitment. On the other hand, sophisticated investors realize that the manager has pre-committed and incorporate the new earnings into their expectations (and thus the price). In order to do this we add an additional period (period 0) to our model, before the investment cost is incurred. We denote the price in this period by $\mathcal{P}_0$, and the weight on this price in the manager's compensation by w_0^p. I.e., the manager's preferences are $w_0^p\mathcal{P}_0 + w_1^p\mathcal{P}_1 + w_2^p\mathcal{P}_2 + w_1^d D_1 + w_2^d D_2 + w_3^d D_3$. Suppose that I_1 and R_2 are revealed to the manager and sophisticated investors in period 0, rather than period 1. Further, suppose the manager can credibly commit to undertake the investment in period 0, while the cost is still incurred only in period 1. Proposition 5 characterizes the change in the manager's investment policy when this pre-commitment is possible.

Proposition 5: *Suppose that the weight on $\mathcal{P}_0$ in the manager's preferences is positive, i.e., $w_0^p > 0$, and that the manager can commit to the first period investment in period 0. Then, the manager's required rate of return will be closer to first best level, when compared to the no commitment case. However, if the manager under-invests (over-invests) without commitment he will also do so with commitment.*

In other words, pre-commitment improves the manager's investment policy but a suboptimal investment level w/o commitment remains suboptimal with pre-commitment. To understand this proposition, consider the marginal investment, i.e., the investment for which the manager is indifferent between investing and not investing. If the investment is profitable (i.e., the manager is under-investing), pre-committing to the investment will increase the valuation of sophisticated investors in period 0, since they realize the net gain from the investment. At the same time, the valuation of naive investors in period 0 does not change, since no cost is incurred yet and dividends in period 0 are not affected. Hence, if the under-investing manager cares about $\mathcal{P}_0$ and can commit to the marginal investment in period 0, he will strictly prefer to take the investment. Using similar arguments, the over-investing manager will strictly prefer *not* to take the marginal (unprofitable) investment. Thus, pre-commitment combined with sensitivity to period 0 price brings all manager types closer to first best level, which in our model is a hurdle rate of 1. Notice that the arguments above will never induce a manager to cross over the first best level. E.g., an under-investing manager will never be induced to over-invest since pre-committing to an unprofitable project will be punished in period 0 by the sophisticated investors, and cause no reaction from the naive investors.

3 Implications and Consistent Evidence

In this section we review and expand the list of implications of the model, and consider how they relate to the existing empirical literature. In subsection 3.1 we review the implications presented in section 2. Subsection 3.2 considers additional implications of the model, as well as some possible extensions. We also review empirical findings that are consistent with the model, including new empirical work that tests some of the model's unique implications.

3.1 Implications

The model's implications can be categorized into two groups: those that are dependent on the manager's preferences, and those that are independent of his preferences. We start by reviewing the independent predictions already presented in the previous section.

1. Buying a firm after a dividend decrease will on average give positive excess returns. Conversely, following a dividend increase there will be negative average excess returns.

2. After a dividend is cut, on average, the sellers are naive investors while the buyers are sophisticated. Conversely, after an increase the sellers are, on average, sophisticated investors while the buyers are mainly naive.

3. The price impact of a dividend change is proportional to the fraction of naive investors.

4. The ability to pre-commit to an investment will bring the firm closer to the optimal hurdle rate, thus improving its investment strategy.

Notice that implication 4 is independent of the manager's preferences even though it considers comparative statics on the manager's investment strategy. The manager's preferences do, of course, affect his investment decisions. Therefore, although our results are general, the paper focuses on predictions relating to two specific sets of managerial preferences. The first is based on a compensation scheme that depends only on the current price. The manager with this compensation scheme is called a *Myopic* manager. In the absence of naive investors, this manager invests at first best levels. The second compensation scheme we consider depends only on the current dividend and the future price, giving more weight to the former. A manager who has these preferences is called a *Deferred Compensation/High Turnover (DT)* manager. In order to empirically examine the implications below, we must first ascertain that the prevailing compensation structure is indeed the one assumed.[32] Notice that, since our results are general, implications can be derived for any preferences used. Below we present predictions that depend on the manager's preferences:

5. Managerial preferences drawn at random from the parameter space will most likely induce the manager to under-invest. Also, preferences that heavily rely on the current price or dividend will induce under-investment, especially when naive investors are present. Specifically, both the Myopic and the DT managers under-invest, when there are any naive investors.

6. The Myopic manager invests less the more naive investors there are. He invests more the more precise the prior information about the company in the market. Given Myopic compensation, we would expect less new investments in industries with complex or secret investments when compared to other industries.[33]

7. The DT manager invests more the more naive investors there are. He invests less the more precise the prior information. Given DT compensation, the model predicts *more* new investments in industries with complex or secret investments when compared to other industries.[34]

8. Managers whose compensation scheme depends only on the future price (when the investment pays off) over-invest.

[32]Another issue concerning testing is that in order to test the implications regarding investment, a proxy for investment level is needed. The usual method of using Tobin's q is not appropriate here since it relies on prices, which in our model are not at fair value at the time of investment. With a good proxy for investment level, the relative level of firms with different characteristics (such as fraction of insiders or quality of prior information) can be examined and compared to the model's predictions.

[33]The lower investment level can be seen from two directions. First, in complex industries it may be harder for investors to find out about a firm's investments, thus increasing the proportion of naive investors. Also, these companies have a lower precision of prior information. A similar logic, in reverse, applies for the DT manager.

[34]Implications 6 and 7 together suggest different kinds of compensation schemes in different industries. In complex industries, such as the high tech industry, the deferred stock based compensation style with high turnover, as for the DT manager, may be preferred to a scheme that concentrates on the current price. This seems to be true anecdotally.

3.2 Consistent Evidence and Possible Extensions

In this subsection we consider empirical evidence that is consistent with our model's implications, as well as additional implications and possible extensions of the model. Some of the implications presented in subsection 3.1 are consistent with several existing empirical papers, others are new. In order to test our model we must first decide which investors are likely to be naive and which are likely to be sophisticated. As we mentioned in section 2, natural candidates for naive investors are individual investors. Insiders are candidates for sophisticated investors, as are, perhaps, analysts. These observations are useful when interpreting empirical findings in the context of this model.

In a recent paper, Petrie [1998] uses positive dividend surprises data to test a signaling model where some investors are insiders and some are not. She finds that the price reaction to positive dividend surprises is inversely related to her measure of insider activity in the stock. Assuming that insiders are sophisticated investors, these findings are consistent with implication 3.[35]

Several empirical papers are consistent with the setup of our model. As mentioned in the introduction, Benartzi, Michaely and Thaler [1997] find that dividend decreases were followed by earnings increases, which is consistent with our model.[36] Skinner and Gilster [1990] find dividend clienteles that are not tax related. This is consistent with the assumptions of our model in which each firm may have a different fraction of naive investors, based, for example, on how difficult it is to obtain and interpret information about the firm. Bajaj and Vijh [1990] as well as Denis, Denis, and Sarin [1994] also find clienteles that may not be tax related.

Of the novel implications of this model, two have recently been tested. Nelson [1999] constructs samples of unexpected dividend decreases and increases, relative to an estimate of expectations. The estimate is based on the past trend in dividends, implicitly assuming that the trend will continue. Using these samples, findings include abnormal returns supporting

[35] The findings of Beer [1993] are also consistent with this implication. See footnote 21.

[36] Although Benartzi, Michaely and Thaler claim that this increase in earnings does not appear to be due to additional investments, they only consider investing the dividend change proper at twelve percent return. However, the cut in dividend could have enabled a much larger investment to take place and, as implied by our model, the manager may require a much higher rate of return in order to undertake the investment.

implication 1, as well as short term price impact paralleling the findings for nominal dividend changes (e.g., Aharony and Swary [1980]). These same dividend event samples were used to check implication 2. This was done by checking transactions of individual investors in discount brokerage accounts.[37] Nelson [1999] finds that individual investors tend to sell after dividend decreases and buy after dividend increases. Since we think of individual investors as candidates for naive investors, this finding supports implication 2.

Our model also implies that analysts can play a dual role in influencing the manager's investment policy. In general, the model suggests that the more analysts follow the firm, the more the Myopic manager invests. This follows from two effects. First, the more analysts follow a stock the less naive investors we would expect, since information about the firm's investments is more readily available. This will increase the level of investment of the Myopic manager (or equivalently decrease his required rate of return). Second, the more analysts follow the firm the more precise we would expect the prior information about the firm's earnings to be. This too decreases the Myopic manager's required rate of return.[38]

Extensions of this model may shed some light on additional issues such as cutting dividends in order to repurchase stock. Although stock repurchases can be considered an investment, our model can be extended to explicitly allow a repurchase decision with or instead of a dividend. As mentioned in Section 1, one company that cut dividends and announced a stock repurchase at the same time is Florida Power and Light (FPL). Although a stock repurchase plan is clearly an investment, the price of FPL dropped sharply when this step was taken.[39] The price increased shortly after the decline and the stock outperformed the S&P utilities index in the next 18 months. This is consistent with our model's predictions and similar to the more general findings in Nelson [1999]. Other utilities followed FPL and for the later followers there was no price drop. Soter, Brigham and Evanson [1996] postulate that over time investors may have

[37] The data for this part were generously provided by Terry Odean.

[38] For the DT manager, the effect is reversed, the more analysts follow the firm the less the DT manager invests, taking him further away from first best. This would suggest that as the firm matures and garners more analyst following, the manager's compensation should move away from the "high turnover/deferred compensation" type.

[39] A stock repurchase announcement is a declaration of intentions, not a guarantee. Stephens and Weisbach [1998] report that firms typically execute only about three quarters of the repurchase plan they announce.

learned to look for this kind of an investment when considering utilities.

The model can easily be extended to allow the possibility of raising capital to fund the investment, while leaving the dividend intact. This strategy is attractive if the pool of investors for the new issue is, on average, more sophisticated than the pool of existing shareholders. This is a plausible scenario since individual investors, which we consider naive, do not typically have direct access to secondary stock offerings. This situation is now starting to change, as individual investors achieve access to more and more offerings. Checking how popular the combination of paying a dividend and raising money is before and after such a change would be interesting.

Another straightforward extension is to allow the investment to shift core earnings. That is, assume the investment has a cost in period 1 and a (constant) payoff in each future period. If an investment of this sort is undertaken then updating beliefs based on the high dividend that includes the first payoff (in period 2) is correct. Therefore, this model can also capture the variant of our model where after seeing prices in period 1 (i.e., after trading has ended), naive investors correctly update their beliefs. This causes both investors to start period 2 with the same beliefs about the earnings process. Hence, both investors will have fully rational beliefs starting in period 2. This is modeled by a two period model that is, in essence, a subset of the model presented in section 2. In this model, the only discrepancy in beliefs is in the first period. The two period model is presented more fully in the appendix, and yields the same qualitative results as the model presented in Section 2.

4 Concluding Remarks

In this paper we examined a model where some investors are naive in the sense that they are not aware of some of the firm's investment opportunities and they do not update their beliefs based on prices. Thus, naive investors attribute all of the firm's dividend changes to earnings changes without realizing that, in some cases, the change in dividend is due to an investment opportunity. This leads them to draw inaccurate inferences. We examine the impact of undertaking such partially-ignored investment on firm price, and trading patterns.

We also consider the firm's investment and dividend policies in this context. Several of the model's implications are consistent with existing empirical literature, and the model gives, in addition, new predictions. Some of these new predictions are currently being tested, with favorable results (see Nelson [1999]), while some have not been considered yet.

The model implies that when the investment cost is incurred the firm is under-priced. In addition, if the investment is profitable, then the firm is over-priced when the investment pays off. Nelson [1999] tests the hypotheses that buying after dividend cuts yields positive average abnormal returns while buying after dividend increases yields negative average abnormal returns. These hypotheses are both supported, lending support to the model's predictions. Another implication of the model is that naive investors are the net sellers when the firm is under-priced and net buyers when the firm is over-priced. This prediction is also supported by Nelson [1999].

The price and trading results above are independent of the manager's preferences or compensation scheme. The firm's dividend and investment decisions, however, do depend on the manager's preferences. In general, except for some specific cases, the firm's investment policy will be suboptimal. Typically, the manager will under-invest. We derive the investment level and how this level changes with the proportion of naive investors. Specific compensation packages are examined. We find that the effect of the precision of prior information on the investment level is also sensitive to the manager's preferences. Finally, we consider the case where the manager can credibly pre-commit to an investment, before the investment costs are born. In this case, we find that the manager's investment policy will be closer to optimal, and the stock price at the time of commitment will be affected by the decision to invest.

There are several aspects of this model which warrant further discussion. First, we assume that naive investors do not use prices to learn about the information of sophisticated investors. This is the crux of their naivete. If instead we considered a model of asymmetric information, the problem becomes much less tractable. However, most of the intuitions from our model should carry through to a model with differing opinions, where there is some probability that the other type is right. The current model can be seen as an extreme case of such a model, where the perceived probability that the other type is correct approaches zero. This setup is consistent with the "symmetric assumption" we used when deriving the expectations of naive investors in period 1 (see the appendix for details). Second, in our model, naive investors do not update their beliefs about the investment opportunity even after observing the final first period price. As mentioned in Subsection 3.2, we can relax this assumption without altering the qualitative results.[40]

Future research directions, in the spirit of this paper, include additional rules of thumb that investors may use, and their influence on corporate policies. In addition, the model could be extended to allow for some investors that see investment opportunities that do not exist. These investors may partially attribute a dividend omission, which was due to lack of earnings, to an investment and will thus be continually disappointed in the firm as more information is released. This may cause the price to continuously drift down, as is observed in the post omission drift phenomenon.

[40]This effectively brings us to the setting of the two period model which is explored in the appendix.

5 List of Notation

E_t : firm's earnings at time t ($E_t \sim N(\mathcal{X}, \frac{1}{r})$)

r : precision of actual earnings

I_1 : cost of additional investment

R_2 : payoff of additional investment

D_t : firm's dividend at time t

1_{I_1} : indicator of investment, equals 1 if the investment was taken by firm and 0 otherwise

$\mathcal{P}_t$: price at time t

$\mathcal{P}_2^N$: naive investor's assessment (in period 1) of price in period 2

ρ : investors' risk tolerance parameter

α : proportion of naive investors

X, τ : investors' prior belief (in period 0) on firm's expected earnings, $X \sim N(M_0, \frac{1}{\tau})$

M_t^j : type j investor's mean of expected earnings in period t ($t = 0, 1, 2$; $j \in \{N, S\}$)

x_t^j : type j investor's demand in period t ($t = 0, 1, 2$; $j \in \{N, S\}$)

w_t^d : coefficient on period t dividend in manager's preferences

w_t^p : coefficient on period t price in manager's preferences

Myopic : Myopic (or price conscious) manager that cares only about current price

DT : Deferred Compensation/High Turnover manager that cares only about current dividend and future price (but more for dividend)

References

[1] Abrutyn, S. and R. Turner, “Taxes and Firms’ Dividend Policies: Survey Results”, *National Tax Journal* 43, pp. 491-496, 1990.

[2] Aharony J. and I. Swary, “Quarterly Dividends and Earnings Announcements and Stockholders Returns: An Empirical Analysis”, *Journal of Finance* 35, pp. 1-12, 1980.

[3] Al-Qudah K., M. Walker, and A. Lonie, “The Accessibility and Perceived Usefulness of Information on the Capital Expenditure Intentions of UK Quoted Companies”, *Accounting and Business Research* 22, pp. 3-12, 1991.

[4] Bajaj M. and A. Vijh, “Dividend Clienteles and the Information Content of Dividend Changes”, *Journal of Financial Economics* 26, pp. 193-219, 1990.

[5] Baker H.K., G. Farrelly, and R. Edelman, “A Survey of Management Views on Dividend Policy”, *Financial Management* 14, pp. 78-84, 1985.

[6] Beer F., “Dividend Signalling Equilibria: Quantitative Evidence from the Brussels Stock Exchange”, *Financial Review* 28, pp. 139-157, 1993.

[7] Benartzi S., R. Michaely, and R. Thaler, “Do Changes in Dividends Signal the Future or the Past?”, *Journal of Finance* 52, pp. 1007-1034, 1997.

[8] Bhattacharya S., “Imperfect Information, Dividend Policy, and the Bird in the Hand Fallacy”, *Bell Journal of Economics* 10, pp. 259-270, 1979.

[9] Blume M. and I. Friend, “The Changing Role of the Individual Investor”, *John Wiley and Sons*, New York 1978.

[10] Bolton P. and X. Freixas, “A Dilution Cost Approach to Financial Intermediation and Securities Markets”, *ECARE working paper*, December 1997.

[11] Chang L., "New Light on Investors' Information Sources", *Woman CPA*, pp. 8-11, January 1982.

[12] Chang S. and D. Suk, "Stock Prices and the Secondary Dissemination of Information: The WSJ's Insider Trading Spotlight Column", *Financial Review*, August 1998.

[13] Cyert, R., S.-H. Kang, and P. Kumar, "Managerial Objectives and Firm Dividend Policy: A Behavioral Theory and Empirical Evidence", *Journal of Economic Behavior and Organization* 31, pp. 157-174, 1996.

[14] Denis D., D. Denis, and A. Sarin, "The Information Content of Dividend Changes: Cash Flow Signaling, Overinvestment, and Dividend Clienteles", *Journal of Financial and Quantitative Analysis* 29, pp. 567-587, 1994.

[15] , Eades K., P. Hess, and E. Kim, "Market Rationality and Dividend Announcements", *Journal of Financial Economics* 14, pp. 581-604, 1985.

[16] Elton E. and M. Gruber, "Marginal Stockholder Tax Rates and the Clientele Effect", *Review of Economics and Statistics*, pp. 68-74, 1970.

[17] Farrelly G., and H. Baker, "Corporate Dividends : Views of Institutional Investors", *Akron Business & Economic Review* 20, pp. 89-100, Summer 1989.

[18] Hall B., and J. Liebman, "Are CEOs Really Paid Like Bureaucrats?", *Quarterly Journal of Economics* 113, pp. 653-691, August 1998.

[19] D. Hearth and J. Rimbey, "The Dividend-Clientele Controversy and the Tax Reform Act of 1986", *Quarterly Journal of Business and Economics* 32, pp. 68-81, 1993.

[20] Jensen M., "Agency Costs of Free Cash Flow, Corporate Finance, and Takeovers", *American Economic Review* 76, pp. 323-329, 1986.

[21] Kahneman D. and A. Tversky, "Prospect Theory" An Analysis of Decision under Risk", *Econometrica* 47, pp. 263-291, 1979.

[22] Lee B.-S., "Time-Series Implications of Aggregate Dividend Behavior", *Review of Financial Studies* 9, pp. 589-618, 1996.

[23] Lintner J., "Distribution of Incomes of Corporations Among Dividends, Retained Earnings, and Taxes", *American Economic Review*46, pp. 97-113, 1956.

[24] Miller M. and F. Modigliani, "Dividend Policy, Growth and the Valuation of Shares", *Journal of Business* 4, pp. 411-433, 1961.

[25] Nelson L., "Evidence on Abnormal Returns and Trading Around Dividend Surprises", *Stanford working paper*, 1999.

[26] Noe T. and M. Rebello, "Asymmetric Information, Managerial Opportunism, Financing, and Payout Policies", *Journal of Finance* 51, pp. 637-660, 1996.

[27] Partington G., "Dividend Policy and Its Relationship to Investment and Financing Policies : Empirical Evidence", *Journal of Business Finance and Accounting* 12, pp. 531-542, 1985.

[28] Petrie K., "The Impact of Informed Trading on Dividend Signaling", *INSEAD working paper*, 1998.

[29] Pruitt S. and L. Gitman, "The Interactions between the Investment, Financing, and Dividend Decisions of Major U.S. Firms", *Financial Review* 26, pp. 409-430, 1991.

[30] Reckers P. and A. Stagliano. "How Good are Investor's Data Sources?", *Financial Executive*, pp. 26-32, April 1980.

[31] Rodriguez R., "Quality Dispersion and the Feasibility of Dividends as Signals", *Journal of Financial Research* 15, pp. 307-315, 1992.

[32] Ross S., "The Determination of Financial Structure: The Incentive-Signalling Approach", *Bell Journal of Economics* 8, pp. 23-40, 1977.

[33] Shefrin H. and M. Statman, "Explaining Investor Preferences for Cash Dividends", *Journal of Financial Economics* 13, pp. 253-282, 1984.

[34] Shefrin H. and R. Thaler, "An Economic Theory of Self-Control", *Journal of Political Economy* 89, pp. 392-406, 1981.

[35] Shelor R. and D. Officer, "The Impact for Stockholders When Regulated Firms Revise Dividend Policy", *Review of Financial Economics* v3 n2, pp. 121-129, 1994.

[36] Skinner D. and J. Gilster, "Dividend Clienteles, the Tax-Clientele Hypothesis, and Utilities", *Financial Review* 25, pp. 287-296, 1990.

[37] Soter D., E. Brigham, and P. Evanson, "The Dividend Cut Heard Around the World: the Case of FPL", *Journal of Applied Corporate Finance* 9, pp. 4-15, 1996.

[38] Stein J., "Rational Capital Budgeting in an Irrational World", *Journal of Business* 69, pp. 429-455, 1996.

[39] Stephens C. and M. Weisbach, "Actual share reacquisitions in open-market repurchase programs", *Journal of Finance* 53, pp. 313-333, 1998.

[40] Zwiebel J. "Dynamic Capital Structure under Managerial Entrenchment", *American Economic Review* 86, pp. 1197-1215, December 1996.

A The Two Period Model - Setup and Results

We start with a simple two period model which focuses on the effects of the investment cost. For simplicity we normalize the riskless rate to zero. The firm's net earnings in period t, denoted E_t, are of the form $E_t = \mathcal{X} + e_t$ (for $t = 1, 2$), where $\mathcal{X}$ is the mean of the firm's earnings process, and e_t is the current earnings deviations from the mean. We assume that $e_t \sim N(0, \frac{1}{r})$, where r is the precision of the earnings random variable, and that e_1 and e_2 are i.i.d. . We also assume that $\mathcal{X}$ is the realization of a random variable, X, that is $X \sim N(M_0, \frac{1}{\tau})$. In period 1 the firm has an additional investment opportunity with cost I_1 (in period 1) and payoff R_2 (in period 2). We assume R_2 and I_1 are revealed to management and some investors in period 1, before any dividend decision. Notice that since there is no uncertainty and the riskless rate is zero, the first best investment strategy is to invest iff $\frac{R_2}{I_1} > 1$, i.e., the required rate of return is 1 at first best. For simplicity, we assume that the firm must pay out all net retained earnings. The manager of the firm knows the net earnings realization, E_1, and chooses the dividend D_1 (or, alternatively, whether to invest or not) subject to this constraint. For the moment we will take the manager's investment decision as given and below we will solve for it endogenously. We use 1_{I_1} to denote an indicator function that is 1 if the investment is undertaken and 0 otherwise.

All investors have CARA utility with risk tolerance parameter ρ, i.e., $u(W) = -e^{-\frac{W}{\rho}}$. They maximize $Eu(W)$ in each period, where W is total wealth over all remaining periods. All investors are unsure of the mean of the earnings process. Investors' initial belief is that $E_t \sim N(X, \frac{1}{r})$ with unknown X (the truth is that $X = \mathcal{X}$). Their prior for X is $N(M_0, \frac{1}{\tau})$, where τ is the precision of the prior and M_0 is its mean. Thus, all investors initially have the same expectation for the next dividend, namely M_0. Investors use dividends to update their beliefs about the mean of the earnings process, using conjugate priors.

There are two types of investors, *naive* and *sophisticated*. While sophisticated investors are fully rational and well informed, naive investors are not as knowledgeable about the firm's investment opportunities. In fact, naive investors incorrectly believe that the firm has no further profitable investment opportunity beyond the core business. They, therefore, infer that earnings were equal to the dividend. On the other hand, the fully rational sophisticated investors know the realization of I_1 and R_2. They can also verify whether an investment was indeed made by the firm. Hence they can correctly use dividends to back out the actual earnings, which is what they use to update their beliefs on the earnings process. We assume that naive investors ignore the beliefs of sophisticated investors as well as their manifestation in prices. The proportion of *naive* investors in the population is α, thus the proportion of *sophisticated* investors is $1-\alpha$.

Trading, and thus price determination, takes place *after* the dividend announcement, but *before* the actual dividend distribution. Thus, after the announcement of the second dividend, which is a liquidating dividend, there is no uncertainty left and the price in period 2 is just the dividend announced for that period, D_2. The first period price, denoted by $\mathcal{P}_1$, is determined by investors' demands. For simplicity we also assume that there exists a riskless asset available to investors (recall that the risk-free rate is assumed to be zero). All outside financing the firm undertakes is observable by investors, and is thus normalized to zero.

Since the payoff from the investment is included in the final liquidating dividend, investor beliefs about earnings, following the investment payoff in period 2, do not influence prices. Hence, this model only allows for mispricing due to the cost of the investment while neutralizing the effects of the investment payoff (see section 2 below). We start by taking the manager's investment decision as given and examining the effect of this decision on prices. Given that a dividend D_1 is announced, investors update their beliefs. Naive investors update their beliefs using D_1 while sophisticated investors correct for the investment, if it was taken. Naive investors now believe the mean of X is $M_1^N = \frac{\tau}{\tau+r}M_0 + \frac{r}{\tau+r}D_1$ and its precision is $\tau + r$. Using 1_{I_1} to denote an indicator that is 1 if the investment is undertaken and 0 otherwise, sophisticated investors now believe the mean of X is $M_1^S = \frac{\tau}{\tau+r}M_0 + \frac{r}{\tau+r}(D_1 + 1_{I_1}I_1)$ (also with precision $\tau + r$). Naive investors solve the maximization problem

$$\max_{x_N} -Ee^{-x_N(D_1+D_2-\mathcal{P}_1)/\rho}$$

subject to $D_1 = E_1$ and $D_2 = E_2$. They get demands

$$x_N = \rho r \left(D_1 \left(1 + \frac{r}{\tau + r}\right) + \frac{\tau}{\tau + r}M_0 - \mathcal{P}_1 \right)$$

which, as expected, are linear in $\mathcal{P}_1$. Sophisticated investors solve the maximization problem

$$\max_{x_S} -Ee^{-x_S(D_1+D_2-\mathcal{P}_1)/\rho}$$

subject to $D_1 = E_1 - 1_{I_1}I_1$ and $D_2 = E_2 + 1_{I_1}R$; and get demands

$$x_S = \rho r \left(D_1\left(1+\frac{r}{\tau+r}\right)+\frac{\tau}{\tau+r}m_0 - \mathcal{P}_1 + 1_{I_1}\left(\frac{r}{\tau+r}I_1+R\right)\right)$$

Using market clearing ($\alpha x_N + (1-\alpha)x_S = 1$) we get proposition 1.

Proposition 6: *When there is a proportion α of naive investors and $1-\alpha$ of sophisticated investors in the firm, in equilibrium, we have prices*

$$\mathcal{P}_2 = D_2 \;\; ; \;\; \mathcal{P}_1 = D_1\left(1+\frac{r}{\tau+r}\right)+\frac{\tau}{\tau+r}M_0 + (1-\alpha)1_{I_1}\left(\frac{r}{\tau+r}I_1+R\right)-\frac{1}{\rho r} \qquad (7)$$

Note that the first period price, $\mathcal{P}_1$, is a weighted average of the investors' beliefs minus a risk premium, $\frac{1}{\rho r}$. The valuation of naive investors is

$$D_1\left(1+\frac{r}{\tau+r}\right)+\frac{\tau}{\tau+r}M_0-\frac{1}{\rho r}$$

while that of the sophisticated investors (which is the true value) is

$$D_1\left(1+\frac{r}{\tau+r}\right)+\frac{\tau}{\tau+r}M_0+1_{I_1}\left(\frac{r}{\tau+r}I_1+R\right)-\frac{1}{\rho r}$$

From the valuation above we can see that the price $\mathcal{P}_1$ is lower than the "correct" price, that would hold if all investors were sophisticated, by $\alpha 1_{I_1}(\frac{r}{\tau+r}I_1+R)$. We can plug the price back into the demand functions and get that a naive investor will hold

$$x_N(\mathcal{P}_1) = 1-(1-\alpha)\rho r 1_{I_1}\left(\frac{r}{\tau+r}I_1+R\right)$$

while a sophisticated investor will hold

$$x_S(\mathcal{P}_1) = 1+\alpha\rho r 1_{I_1}\left(\frac{r}{\tau+r}I_1+R\right)$$

which is larger than the holdings of a naive investor. Together, this implies the following corollary.

Corollary 2: *If the investment opportunity is taken then, in period 1, the firm is under-priced, and a sophisticated investor will hold more shares than a naive investor. The more naive investors there are, the more severe the under-pricing and the larger the holdings of a sophisticated investor in the first period.*

The intuition for the proposition and corollary is straightforward. The naive investors infer from the lower dividend that second period earnings will also be low, thus unjustly lowering their valuation of the firm. Since the valuation of sophisticated investors is higher than both the valuation of naive investors and the price, $\mathcal{P}_1$, a sophisticated investor will tend to hold more shares of the firm than a naive investor. Notice that, since all investors are risk averse, the sophisticated investors cannot close the price gap and the firm remains under-priced in spite of their high valuations.

Given the influence of the investment decision on demands and prices, we now consider the manager's investment decision. In order to keep the discussion as general as possible, we will allow the manager's preferences to depend on all observables (i.e., prices and dividends). This generality allows for a wealth of managerial types (based on preferences), and this in turn allows us to observe which of the results presented is typical in the model, regardless of preferences, and which is based on specific preferences. We will, later on, concentrate on two specific preference types that will demonstrate the richness that this approach yields. We assume that the manager maximizes a weighted average of the current price of the firm and its two dividends (some of the weights may be zero). The manager's objective in period 1 is thus $w_1^p \mathcal{P}_1 + w_1^d D_1 + w_2^d D_2$ for some parameters w_1^p, w_1^d, w_2^d, where w_t^p is the sensitivity to $\mathcal{P}_t$, price in period t, and w_t^d is the sensitivity to D_t, the dividend in period t. Note that a value maximizing manager, which follows the optimal investment strategy, would care equally for D_1 and D_2, and not at all for the price ($w_1^d = w_2^d$ and $w_1^p = 0$). Throughout we will assume that all compensation parameters are non negative (i.e., $w_1^d, w_2^d, w_1^p \geq 0$).

Part of the weights in the manager's preferences might stem from implicit considerations, such as reputation, rather than explicit compensation, such as monetary compensation given by the firm. Because of the implicit components, the firm cannot completely control the manager's preferences, thus it is reasonable to take the manager's preferences as exogenous. For simplicity, in our discussion, we will represent the manager's preferences only in terms of a compensation scheme, however, intuitively we think of his preferences as incorporating implicit compensation as well.

Given the above general compensation scheme, the manager chooses whether or not to invest.

Proposition 7: *There is under-investment iff*

$$w_2^d \quad < \quad w_1^d + w_1^p \alpha \left(1 + \frac{r}{\tau + r}\right)$$

and over-investment if the inequality is reversed. Further, if $w_1^p > 0$ then the required rate of return is increasing in α and decreasing in the precision of the prior information, τ.

The proof of this proposition is given in appendix B. Alternatively, if $w_1^p > 0$, then there is under-investment iff $\alpha > \frac{w_2^d - w_1^d}{w_1^p\left(1+\frac{r}{\tau+2r}\right)}$ and over investment iff the inequality is reversed. The expression $w_2^d - w_1^d$ is the inducement to (over-) invest due to the dividends themselves (regardless of the mix of investors). On the other hand, $w_1^p \alpha \left(1 + \frac{r}{\tau+2r}\right)$ is the deterrent to investment due to the naive investors. We will call the effects in corollary 2 and proposition 7 *cost effects.*

As an example, following the surveys cited above,[41] consider a manager that cares only about the current price. We will call this type of manager a **Myopic** manager. Note that this manager invests at first best if all investors are sophisticated ($\alpha = 0$). Proposition 7 implies that the Myopic manager will always under-invest when there are any naive investors present. However, not all manager types under-invest. For example, a manager who cares more for the future dividend than the current dividend and does not care for the price will, naturally, always over-invest, even if all investors are sophisticated. Although the results in this paper

[41] See Baker, Farrelly, and Edelman [1985], Partington [1985], as well as the empirical findings of Hall and Liebman [1998].

can be applied to any manager type, we will follow the implications for the Myopic manager type throughout this paper. In addition, in the next section we will introduce another manager type which has a deferred compensation package. The general conditions, when not given in the text, will be given in appendix B.

Before we look at the three period model, which includes a third period after the investment payoff, let us consider the possibility of the manager pre-committing to the investment before the investment costs affect the dividend. In order to do this we add an additional period, period 0, to the two period model, before the investment cost is incurred. We denote the price in this period by $\mathcal{P}_0$, and the weight on this price in the manager's compensation by w_0^p. Suppose the manager and sophisticated investors know I_1 and R_2 in period 0, rather than period 1. Further, suppose the manager can credibly commit to undertake the investment in period 0, while the cost is still incurred in period 1. Proposition 8 characterizes the change in the manager's investment policy when this pre-commitment is possible.

Proposition 8: *If the weight on $\mathcal{P}_0$ is positive, i.e., $w_0^p > 0$, and the manager can commit to the first period investment in period 0, then his required rate of return will be closer to 1, or first best level, when compared to the no commitment case. However, if the manager under-invests (over-invests) without commitment he will also do so with commitment.*

In other words, pre-commitment improves the manager's investment policy but the investment level is still suboptimal due to the presence of naive investors. To understand this proposition, consider the marginal investment, for which the manager is indifferent between investing and not investing. If the investment is profitable (i.e., the manager under-invests), pre-committing to the investment will increase the valuation of sophisticated investors in period 0, since they realize the net gain from the investment. At the same time, the valuation of naive investors in period 0 does not change, since no cost is incurred yet so dividends in period 0 do not change. Therefore, if he can commit to it in period 0, an under-investing manager will strictly prefer to take the marginal investment. Using similar arguments. the over-investing manager will strictly prefer *not* to take the marginal (unprofitable) investment. Proposition 8 says that pre-commitment together with sensitivity to the price prior to the execution of the investment attenuates the deviation from optimality but the direction of the deviation (under- or over-investment) remains the same.

B Two Period Model - More Detailed Proofs

For proposition 7 we need to show the hurdle rate for an investment to be taken. Since the riskless rate is assumed to be 0, if the hurdle rate is above 1 the firm is under-investing and if it is below 1 it is over-investing.

Proof: Given the prices in proposition 6, the manager's period 1 maximization problem is:

$$\max_{1_{I_1} \in \{0,1\}} w_1^p \mathcal{P}_1 + w_2^d E(\mathcal{P}_2) + w_1^d D_1 \tag{8}$$

subject to prices as in (7), $D_2 = E_2 + 1_{I_1} R_2$, and $D_1 = E_1 - 1_{I_1} I_1$.

In equilibrium, the manager chooses to invest iff

$$R_2 \left((1-\alpha)w_1^p + w_2^d\right) - I_1 \left(w_1^p(1+\alpha\frac{r}{\tau+r}) + w_1^d\right) > 0$$

This is equivalent to a required rate of return of

$$\frac{R_2}{I_1} > \frac{w_1^p(1+\alpha\frac{r}{\tau+r}) + w_1^d}{(1-\alpha)w_1^p + w_2^d}. \tag{9}$$

Thus there can be under-investment when

$$w_1^p(1+\alpha\frac{r}{\tau+r}) + w_1^d > (1-\alpha)w_1^p + w_2^d$$

or, alternatively,

$$w_2^d < w_1^d + w_1^p \alpha \left(1 + \frac{r}{\tau+2r}\right)$$

as in proposition 7.

As we can see from equation (9), the required rate of return is increasing in α and decreasing in τ.

To show proposition 8 we need to add period 0 with price $\mathcal{P}_0$ and weight w_0^p in the manager's compensation. At the end of period 0 (after the dividend D_0 is announced) both types believe the mean of the earnings process is M_0. However, if the manager pre-commits to the investment, the sophisticated investors expect the total dividends to change by $R_2 - I_1$. Thus their valuation changes by this amount. If we solve in a similar way to the above we get that

$$\mathcal{P}_0 = D_0 + 2M_0 - \frac{2}{\rho r} - \frac{1}{\rho(\tau + r)} + (1-\alpha)1_{I_1}(R_2 - I_1)$$

The manager's objective is now

$$\max_{1_{I_1} \in \{0,1\}} w_0^p \mathcal{P}_0 + w_1^p E(\mathcal{P}_1) + w_2^d E(\mathcal{P}_2) + w_1^d D_1$$

subject to prices as in (7), $\mathcal{P}_0$ above, $D_2 = E_2 + 1_{I_1} R$, and $D_1 = E_1 - 1_{I_1} I_1$. Thus he will invest iff

$$\frac{R_2}{I_1} > \frac{w_1^p(1 + \alpha\frac{r}{\tau+r}) + w_1^d + w_0^p(1-\alpha)}{(1-\alpha)w_1^p + w_2^d + w_0^p(1-\alpha)}$$

Comparing this to equation (9) we see that whether there is under- or over- investment, the required rate of return will be closer to 1 if the manager commits in period 0. Notice also that if the manager under- (over-) invested without commitment he will continue to under- (over-) invest with commitment. Commitment only attenuates the deviation from first best. This gives us proposition 8.

C Three Periods - Symmetric Assumption

In this section we prove the propositions as they appear in the text, using the symmetric assumption. In the next section we find corresponding results using the naive assumption. The symmetric assumption is that all investors know that the proportion of naive investors is α. Thus, naive investors assume that prices in period 2 will be of the form of equation (3) and demands will be as in (4) and (5).

We first prove Proposition 1 which says:

If the investment is taken then

(i) *The firm is under-priced in period 1 (compared to its true value).*

(ii) *If the investment is profitable, the firm is over-priced in period 2.*

(iii) *A sophisticated investor will hold more shares than a naive investor in period 1, and, if the investment is profitable, less shares than a naive investor in period 2.*

Proof: In order to show the proposition we solve backwards starting with period 2. The derivation for period 2 is similar to the derivation for the two period model. Denote by $x_j^{(t)}$ the period t demand of type j for $j \in \{S, N\}$, where S stands for sophisticated and N stands for naive. We get that the naive investor's second period demand is:

$$x_N^{(2)} = \rho r \left(D_2 \left(1 + \frac{r}{\tau + 2r} \right) + \frac{r}{\tau + 2r} D_1 + \frac{\tau}{\tau + 2r} M_0 - \mathcal{P}_2 \right)$$

and the sophisticated investor's second period demand is:

$$x_S^{(2)} = \rho r \left(D_2 \left(1 + \frac{r}{\tau + 2r} \right) + \frac{r}{\tau + 2r} D_1 + \frac{\tau}{\tau + 2r} M_0 - \mathcal{P}_2 + 1_{I_1} \frac{r}{\tau + 2r} (I_1 - R_2) \right)$$

Clearing the market we get that the second period price is

$$\mathcal{P}_2 = D_2 \left(1 + \frac{r}{\tau + 2r} \right) + \frac{\tau}{\tau + 2r} M_0 + \frac{r}{\tau + 2r} D_1 + (1 - \alpha) 1_{I_1} \frac{r}{\tau + 2r} (I_1 - R_2) - \frac{1}{\rho r} \qquad (10)$$

which is a weighted average of investors' valuations minus a risk premium $(\frac{1}{\rho r})$.

If all investors were sophisticated the price would have been

$$D_2 \left(1 + \frac{r}{\tau + 2r} \right) + \frac{\tau}{\tau + 2r} M_0 + \frac{r}{\tau + 2r} D_1 + 1_{I_1} \frac{r}{\tau + 2r} (I_1 - R_2) - \frac{1}{\rho r}$$

Thus, if the investment is profitable (hence $I_1 - R_2 < 0$), then $\mathcal{P}_2$ is over-priced, showing part (ii). Also, notice that the degree of over-pricing is proportional to α.

Plugging the price in equation (3) back into the demand functions we get that the demand of type j in period 2 is

$$x_N^{(2)} = 1 + (1 - \alpha) \rho \frac{r^2}{\tau + 2r} 1_{I_1} (R_2 - I_1) \qquad (11)$$

$$x_S^{(2)} = 1 - \alpha\rho\frac{r^2}{\tau+2r}1_{I_1}(R_2 - I_1) \tag{12}$$

Notice that, after plugging $\mathcal{P}_2$ in, the demands are independent of the dividends. Also, if the undertaken investment is profitable (i.e., $R_2 - I_1 > 0$), then a sophisticated investor holds less than a naive investor in the second period, showing the first part of (iii).

We now solve for the first period, taking into account that the demands are as in equations (4) and (5), and that the price has the form of equation (3) (i.e., using the symmetric assumption where naive investors are aware of the investor mix). To find first period demands, type j investors solve the maximization problem:

$$\max_{x_j^{(1)}} -e^{-x_j^{(1)}(D_1-\mathcal{P}_1)/\rho}Ee^{-(\mathcal{P}_2(x_j^{(1)}-x_j^{(2)})+x_j^{(2)}(D_2+D_3))/\rho}$$

Using a similar procedure to the one in the previous section, investors update their beliefs and solve for their demands. We get that the demands are:
For a naive investor:

$$x_N^{(1)} = \rho r\left(\tfrac{\tau+2r}{\tau+3r}\right)^2\left[D_1\left(1+\tfrac{2r}{\tau+r}\right)+\tfrac{2\tau}{\tau+r}M_0-\mathcal{P}_1-\tfrac{b}{\rho r}-(1-\alpha)\tfrac{rb}{\tau+2r}1_{I_1}(R_2-I_1)\right]$$

and for a sophisticated investor:

$$\begin{aligned} x_S^{(1)} \quad = \quad & \rho r\left(\tfrac{\tau+2r}{\tau+3r}\right)^2 \quad \Big[D_1\left(1+\tfrac{2r}{\tau+r}\right)+\tfrac{2\tau}{\tau+r}M_0-\mathcal{P}_1-\tfrac{b}{\rho r} \\ & +1_{I_1}\left(\alpha\tfrac{rb}{\tau+2r}(R_2-I_1)+R_2+\tfrac{2r}{\tau+r}I_1\right)\Big] \end{aligned}$$

where the constant b is defined as $b = 1 - \frac{r(\tau+3r)}{(\tau+2r)^2} = 1 - \frac{r}{\tau+2r} - \left(\frac{r}{\tau+2r}\right)^2$. Notice that $0 < b < 1$ since both r and τ are precisions and thus positive.

Using the demands above and market clearing, we get that price in period 1 is:

$$\mathcal{P}_1 = D_1\left(1+\frac{2r}{\tau+r}\right)+M_0\frac{2\tau}{\tau+r}+(1-\alpha)1_{I_1}\left(\frac{2r}{\tau+r}I_1+R_2\right)-\frac{2}{\rho r}-\frac{1}{\rho(\tau+2r)} \tag{13}$$

which, again, is a weighted average of investors' valuations minus a risk premium (see below). If all investors were sophisticated, the price would be

$$D_1\left(1+\frac{2r}{\tau+r}\right)+M_0\frac{2\tau}{\tau+r}-\frac{2}{\rho r}-\frac{1}{\rho(\tau+2r)}+1_{I_1}\left(\frac{2r}{\tau+r}I_1+R_2\right)$$

Hence, $\mathcal{P}_1$ is under-priced by $\alpha 1_{I_1}\left(\frac{2r}{\tau+r}I_1 + R_2\right)$, showing part (i).

Finally, plugging the price back into demands we get

$$x_N^{(1)} = 1 - \frac{(1-\alpha)\rho r(\tau+2r)^2}{(\tau+3r)^2} 1_{I_1}\left[\frac{rb}{\tau+2r}(R_2 - I_1) + \frac{2r}{\tau+r}I_1 + R_2\right]$$

$$x_S^{(1)} = 1 + \frac{\alpha\rho r(\tau+2r)^2}{(\tau+3r)^2} 1_{I_1}\left[\frac{rb}{\tau+2r}(R_2 - I_1) + \frac{2r}{\tau+r}I_1 + R_2\right]$$

Since $0 < b < 1$ the square brackets are positive, thus in equilibrium, assuming the investment is undertaken, a naive investor holds less shares than his sophisticated counterpart. This shows the second part of (iii).

Next we determine the manager's required rate of return and the comparative statics on it (this is for propositions 2, 3, and 4). Here the general expressions are derived.

The manager's objective in period 1 is:

$$\max_{1_{I_1}\in\{0,1\}} E(w_1^p\mathcal{P}_1 + w_2^p\mathcal{P}_2 + w_1^d D_1 + w_2^d D_2 + w_3^d D_3)$$

the condition to invest is:

$$R_2\left(w_2^d + w_1^p(1-\alpha) + w_2^p(1 + \frac{\alpha r}{\tau+2r})\right) - I_1\left(w_1^d + w_1^p(1 + \frac{2\alpha r}{\tau+r}) + w_2^p\frac{\alpha r}{\tau+2r}\right) > 0$$

Or a required rate of return of

$$\frac{R_2}{I_1} > \frac{w_1^d + w_1^p(1+\frac{2\alpha r}{\tau+r}) + w_2^p\frac{\alpha r}{\tau+2r}}{w_2^d + w_1^p(1-\alpha) + w_2^p(1+\frac{\alpha r}{\tau+2r})}$$

To check over- and under-investment we need to compare the numerator and denominator. If the ratio above is over 1 (recall that the riskless rate is 0), then there is under-investment. If it is below 1, then there is over-investment. Thus the manager will under-invest iff

$$w_1^d + w_1^p(1 + \frac{2\alpha r}{\tau+r}) + w_2^p\frac{\alpha r}{\tau+2r} > w_2^d + w_1^p(1-\alpha) + w_2^p(1 + \frac{\alpha r}{\tau+2r})$$

or, alternatively,

$$w_1^d + w_1^p\alpha(1 + \frac{2r}{\tau+r}) > w_2^d + w_2^p$$

as in proposition 2.

To see how the required rate of return changes with α, take the derivative with respect to α. The derivative's sign is the same as the expression

$$\tau^2 w_1^p \left(w_1^d + w_1^p\right) + r\tau \left(3w_1^d w_1^p + 2w_2^d w_1^p + 5w_1^{p2} - w_1^d w_2^p + w_2^d w_2^p + 2w_1^p w_2^p + w_2^{p2}\right)$$

$$+r^2 \left(2w_1^d w_1^p + 4w_2^d w_1^p + 6w_1^{p2} - w_1^d w_2^p + w_2^d w_2^p + 4w_1^p w_2^p + w_2^{p2}\right)$$

This is independent of α and is positive for most parameter values. So for most values the required rate of return increases in α. The rate is *decreasing* if $w_1^p < \frac{w_2^p r}{\tau+2r}$ and

$$w_1^d > \frac{(\tau+3r)(\tau+2r)w_1^{p2} + r\left(w_2^d + w_2^p\right)\left(2(\tau+2r)w_1^p + (\tau+r)w_2^p\right)}{(\tau+r)\left(rw_2^p - (\tau+2r)w_1^p\right)}$$

In this case the manager's sensitivity to first period price must be quite a bit less than his sensitivity to second period price, and this is somewhat offset by bounding from below his sensitivity to the first period dividend. For the Myopic manager the required rate of return is increasing in α (since $w_1^p > \frac{w_2^p r}{\tau+2r} = 0$). For the DT manager, the required rate of return is decreasing in α since both of the above conditions hold. The first condition is $= 0 w_1^p < \frac{w_2^p r}{\tau+2r}$ and the second condition simplifies to $w_1^d > w_2^p$, which holds for the DT manager by definition. Proposition 3 follows.

To show proposition 4 we must check the derivative of the required rate of return with respect to τ. Assuming $\alpha > 0$, this has the same sign as

$$\frac{w_2^p \left(w_1^d + w_1^p + \frac{2\alpha r w_1^p}{\tau+r} + \frac{\alpha r w_2^p}{\tau+2r}\right)}{(\tau+2r)^2}$$

$$-\left(\frac{2w_1^p}{(\tau+r)^2} + \frac{w_2^p}{(\tau+2r)^2}\right)\left(w_2^d + w_1^p - \alpha w_1^p + w_2^p + \frac{\alpha r w_2^p}{\tau+2r}\right)$$

The derivative is positive iff $w_2^p > 0$ and

$$w_1^d > \frac{(\tau+2r)^2}{w_2^p}\left(\frac{2w_1^p}{(\tau+r)^2} + \frac{w_2^p}{(\tau+2r)^2}\right)\left(w_2^d + w_1^p - \alpha w_1^p + w_2^p + \frac{\alpha r w_2^p}{\tau+2r}\right)$$

$$-\left(w_1^p + \frac{2\alpha r w_1^p}{\tau+r} + \frac{\alpha r w_2^p}{\tau+2r}\right)$$

Thus, the required rate of return is decreasing is the precision of prior information (τ) for the Myopic manager. For the DT manager both conditions hold ($w_2^p > 0$ and the second condition simplifies to $w_1^d > w_2^p$). Thus the required rate of return for the DT manager is increasing in τ. This shows proposition 4.

D Three Periods - Naive Assumption

In this section we will go through the same process as in section C for the naive assumption. As we shall see the results are qualitatively, and sometimes even quantitatively, the same. The naive assumption is that, in period 1, naive investors believe that all investors in period 2 are naive.

Again, we solve backwards starting with period 2. Period 2 is the same as with the symmetric assumption, that is the price is as in equation (3) and demands are as in equations (4) and (5).

Now solve for the first period when sophisticated investors know that these will be demands and prices in period 2 but naive investors assume that their demand in period 2 will be $x_N^{(2)} = 1$ and that the price in period 2 will be

$$\mathcal{P}_2^A = D_2\left(1 + \frac{r}{\tau + 2r}\right) + \frac{\tau}{\tau + 2r}M_0 + \frac{r}{\tau + 2r}D_1 - \frac{1}{\rho r}$$

To find demands type j solves the maximization problem:

$$\max_{x_j^{(1)}} -e^{-a(x_j^{(1)}(D_1 - \mathcal{P}_1))} E e^{-a(\mathcal{P}_2^j(x_j^{(1)} - x_j^{(2)}) + x_j^{(2)}(D_2 + D_3))}$$

Recall we defined in section C the constant, $0 < b < 1$, as follows:

$$b = 1 - \frac{r(\tau + 3r)}{(\tau + 2r)^2}$$

Using the beliefs about period 2 demands, we find period 1 demands. The demand of sophisticated investors is as before (since the price, and demand in period 2 have not changed). Naive investors' demand is different now because of their expectations. It is:

$$x_N^{(1)} = \frac{r}{a}\left(\frac{\tau + 2r}{\tau + 3r}\right)^2 \left[D_1 \frac{\tau + 3r}{\tau + r} + \frac{2\tau}{\tau + r}M_0 - \mathcal{P}_1 - \frac{ab}{r}\right]$$

Using market clearing we get that price in period 1 is:

$$\begin{aligned}\mathcal{P}_1 &= D_1\left(1+\tfrac{2r}{\tau+r}\right)+M_0\tfrac{2\tau}{\tau+r}-\tfrac{2}{\rho r}-\tfrac{1}{\rho(\tau+2r)}\\ &+ (1-\alpha)1_{I_1}\left(\tfrac{\alpha br}{\tau+2r}(R_2-I_1)+R_2+\tfrac{2r}{\tau+r}I_1\right)\end{aligned}$$

In period 1, recall that the manager solves:

$$\max_{1_{I_1}\in\{0,1\}} E(w_1^p\mathcal{P}_1+w_2^p\mathcal{P}_2+w_1^dD_1+w_2^dD_2+w_3^dD_3)$$

the condition to invest is then:

$$R_2\left(w_2^d+w_1^p(1-\alpha)\left(1+\frac{\alpha rb}{\tau+2r}\right)+w_2^p(1+\frac{\alpha r}{\tau+2r})\right)$$

$$-I_1\left(w_1^d+w_1^p\left(1+\frac{2\alpha r}{\tau+r}+\frac{(1-\alpha)\alpha rb}{\tau+2r}\right)+w_2^p\frac{\alpha r}{\tau+2r}\right)>0$$

or, alternatively,

$$\frac{R_2}{I_1}>\frac{w_1^d+w_1^p\left(1+\frac{2\alpha r}{\tau+r}+\frac{(1-\alpha)\alpha rb}{\tau+2r}\right)+w_2^p\frac{\alpha r}{\tau+2r}}{w_2^d+w_1^p(1-\alpha)\left(1+\frac{\alpha rb}{\tau+2r}\right)+w_2^p(1+\frac{\alpha r}{\tau+2r})}$$

There will be under-investment iff

$$\begin{aligned}&w_1^d+w_1^p\left(1+\frac{2\alpha r}{\tau+r}+\frac{(1-\alpha)\alpha rb}{\tau+2r}\right)+w_2^p\frac{\alpha r}{\tau+2r}\\ &> \quad w_2^d+w_1^p(1-\alpha)\left(1+\frac{\alpha rb}{\tau+2r}\right)+w_2^p\left(1+\frac{\alpha r}{\tau+2r}\right)\end{aligned}$$

simplifying we get that there will be under-investment iff

$$w_1^d+w_1^p\alpha\left(1+\frac{2r}{\tau+r}\right)>w_2^d+w_2^p$$

exactly the same as for the symmetric assumption.

To see how the required rate of return changes with α, take the derivative with respect to α. This derivative has the same sign as the expression:

$$\tau^2w_1^p\left(w_1^d+w_1^p\right)+r\tau w_1^{p2}\left(5+\alpha^2b\right)+r\tau w_1^d\left((3-b+2\alpha b)\,w_1^p-w_2^p\right)$$

$$+r\tau w_1^p w_2^p (2+b-2\alpha b) + r\tau \left(w_2^{p2} + w_2^d \left((2+b-2\alpha b) w_1^p + w_2^p\right)\right)$$

$$+3r^2 w_1^{p2} \left(2+\alpha^2 b\right) + r^2 w_1^d \left((2+(-1+2\alpha) b) w_1^p - w_2^p\right)$$

$$+r^2 w_1^p w_2^p (4+b-2\alpha b) + r^2 w_2^{p2} + r^2 w_2^d \left((4+b-2\alpha b) w_1^p + w_2^p\right)$$

This expression is positive (i.e., the required rate of return is increasing) for most parameter values and α's, as was the case for the symmetric assumption. Specifically this hold for both the Myopic manager and the DT manager. However, for some parameter values the expression changes sign as α goes from 0 to 1, unlike in the case of the symmetric assumption.

If $\alpha > 0$, the derivative of the required rate of return with respect to τ has the same sign as the expression:

$$\frac{\left((1-\alpha) b w_1^p + w_2^p\right)\left(w_1^d + \left(1 + \frac{2\alpha r}{r+\tau} - \frac{(-1+\alpha)\alpha b r}{2r+\tau}\right) w_1^p + \frac{\alpha r w_2^p}{2r+\tau}\right)}{(\tau+2r)^2}$$

$$-\left(\left(\frac{2}{(\tau+r)^2} + \frac{(1-\alpha) b}{(\tau+2r)^2}\right) w_1^p + \frac{w_2^p}{(\tau+2r)^2}\right) \cdot$$

$$\left(w_2^d + (1-\alpha)\left(1 + \frac{\alpha b r}{2r+\tau}\right) w_1^p + w_2^p + \frac{\alpha r w_2^p}{2r+\tau}\right)$$

If we consider, as an example the myopic manager (for which $w_1^p > 0$ and all other preference parameters are 0), we see that the required rate of return decreases in τ both under the symmetric assumption and under the naive assumption. The above expression simplifies to

$$\frac{\alpha r \left((8+\alpha b)\ r^2 + 2\ (4-\alpha b) r\tau + (2-\alpha b)\tau^2\right)}{(-1+\alpha)\ (\tau+r)^2 \left((2+\alpha b) r + \tau\right)^2}$$

Which is negative for all values of r and τ (recall that $\alpha, b \leq 1$). Similarly, for the DT manager, under both assumptions the required rate of return increases in τ. These parallels hold for most choices of parameters.