

The Link between Default and Recovery Rates: Theory, Empirical Evidence and Implications

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Abstract

This paper analyzes the association between aggregate default and recovery rates on credit assets, and seeks to empirically explain this critical relationship. We examine recovery rates on corporate bond defaults, over the period 1982-2002. Our econometric univariate and multivariate models explain a significant portion of the variance in bond recovery rates aggregated across all seniority and collateral levels. The central thesis is that aggregate recovery rates are basically a function of supply and demand for the securities, with default rates playing a pivotal role. Such a link would bring about a significant increase in both expected and unexpected losses as measured by some widespread credit risk models, and would affect the procyclicality effects of the New Basel Capital Accord. Our results have also important implications for investors in corporate bonds and bank loans, and for all markets (e.g., securitizations, credit derivatives, etc.) which depend on recovery rates as a key variable.

Keywords: credit rating, capital requirements, credit risk, recovery rate, default, procyclicality

JEL Classification Numbers: G15, G21, G28

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Introduction

Credit risk affects virtually every financial contract. Therefore the measurement, pricing and management of credit risk have received much attention from financial economists, bank supervisors and regulators, and from financial market practitioners. Following the recent attempts of the Basel Committee on Banking Supervision (1999, 2001) to reform the capital adequacy framework by introducing risk-sensitive capital requirements, significant additional attention has been devoted to the subject of credit risk measurement by the international regulatory, academic and banking communities.

This paper analyzes and measures the association between aggregate default and recovery rates on corporate bonds, and seeks to empirically explain this critical relationship. After a brief review of the way credit risk models explicitly or implicitly treat the recovery rate variable, Section 2 examines the recovery rates on corporate bond defaults over the period 1982-2002. We attempt to explain recovery rates by specifying rather straightforward linear, logarithmic and logistic regression models. The central thesis is that aggregate recovery rates are basically a function of supply and demand for the securities. Our econometric univariate and multivariate time series models explain a significant portion of the variance in bond recovery rates aggregated across all seniority and collateral levels. In Sections 3 and 4 we briefly examine the effects of the relationship between defaults and recoveries on credit VaR (value at risk) models as well as on the procyclicality effects of the new capital requirements proposed by the Basel Committee, and then conclude with some remarks on the general relevance of our results.

1. The Relationship Between Default Rates and Recovery Rates in Credit Risk Modeling: a Review of the Literature

Credit risk models can be divided into three main categories: (i) “first generation” structural-form models, (ii) “second generation” structural-form models, and (iii) reduced-form models. First generation structural-form models are the ones based on the original framework developed by Merton (1974), using the principles of option pricing. In such a framework, the default process of a company is driven by the value of the company’s assets and the risk of a

firm's default is explicitly linked to the variability in the firm's asset value¹. Under these models, all the relevant credit risk elements, including default and recovery rate (RR) at default, are a function of the structural characteristics of the firm: asset volatility (business risk) and leverage (financial risk). The RR, although not treated explicitly in these models, is therefore an endogenous variable, as the creditors' payoff is a function of the residual value of the defaulted company's assets. More precisely, under Merton's theoretical framework, the probability of default (PD) and expected RR are inversely related.²

While the original Merton model assumes that default can occur only at maturity of the debt when the firm's assets are no longer sufficient to cover debt obligations, second generation structural form models assume that default may occur at any time between the issuance and maturity of the debt, when the value of the firm's assets reaches a lower threshold level³. Under these models, the RR in the event of default is exogenous and independent from the firm's asset value. It is generally defined as a fixed ratio of the outstanding debt value and is therefore independent from the PD. This approach simplifies the first class of models by both exogenously specifying the cash flows to risky debt in the event of bankruptcy and simplifying the bankruptcy process. This occurs when the value of the firm's underlying assets hits some exogenously specified boundary.

Reduced-form models do not condition default on the value of the firm, and parameters related to the firm's value need not be estimated to implement them⁴. In addition, reduced-form models introduce separate, explicit assumptions on the dynamics of both PD and RR. These variables are modeled independently from the structural features of the firm, its asset volatility and leverage. Most reduced-form models assume an exogenous RR that is independent from the

¹ In addition to Merton (1974), first generation structural-form models include Black and Cox (1976), Geske (1977), and Vasicek (1984). Each of these models tries to refine the original Merton framework by removing one or more of the unrealistic assumptions. Black and Cox (1976) introduce the possibility of more complex capital structures with subordinated debt; Geske (1977) introduces interest paying debt; Vasicek (1984) introduces the distinction between short and long term liabilities.

² See Altman Resti and Sironi (2001) for a formal discussion of this relationship.

³ One of the earliest studies based on this framework is Black and Cox (1976). However, this is not included in the second generation models in terms of the treatment of the recovery rate. Second-generation models include Kim, Ramaswamy and Sundaresan (1993), Hull and White (1995), Nielsen, Saà-Requejo and Santa Clara (1993), Longstaff and Schwartz (1995), Finger (2002) and others.

⁴ Reduced-form models include Litterman and Iben (1991), Fons (1994), Madan and Unal (1995), Jarrow and Turnbull (1995), Das and Tufano (1995), Jarrow, Lando and Turnbull (1997), Lando (1998), Duffie and Singleton (1999), and Duffie (1998).

PD. More specifically, they take as given the behavior of default-free interest rates, the RR of defaultable bonds, as well as a stochastic intensity process for default. At each instant there is some probability that a firm defaults on its obligations. Both this probability and the RR in the event of default may vary stochastically through time, although they are not formally linked to each other^{5,6}.

During the last few years, new approaches explicitly modeling and empirically investigating the relationship between PD and RR have been developed. These include Frye (2000a and 2000b), Jokivuolle and Peura (2000), and Jarrow (2001). The model proposed by Frye draws from the conditional approach suggested by Finger (1999) and Gordy (2000b). In these models, defaults are driven by a single systematic factor – the state of the economy - rather than by a multitude of correlation parameters. The same economic conditions are assumed to cause default to rise, for example, and RRs to decline. The correlation between these two variables therefore derives from their common dependence on the systematic factor. The intuition behind Frye’s theoretical model is relatively simple: if a borrower defaults on a loan, a bank’s recovery may depend on the value of the loan collateral. The value of the collateral, like the value of other assets, depends on economic conditions. If the economy experiences a recession, RRs may decrease just as default rates tend to increase. This gives rise to a negative correlation between default rates and RRs. The model originally developed by Frye (2000a) implied recovery from an equation that determines collateral. His evidence is consistent with the most recent U.S. bond market data, indicating a simultaneous increase in default rates and losses

⁵ An exception to this assumption is represented by the model developed by Duffie and Singleton (1999). While this model assumes an exogenous process for the expected loss if default were to occur, meaning that the RR variable does not depend on the value of the defaultable claim, it allows for correlation between the default hazard-rate process and RR. Indeed, in this model the behavior of both PD and RR may be allowed to depend on firm-specific or macroeconomic variables and therefore to be correlated.

⁶ A special case of reduced form models is represented by Credit VaR models. These include J.P. Morgan’s CreditMetrics® (Finger, Gupton and Bhatia [1997]), CreditRisk+® (Credit Risk Financial Products, 1997), McKinsey’s CreditPortfolioView® (Wilson, 1998), and KMV’s CreditPortfolioManager® (McQuown, 1993). These models can indeed largely be seen as reduced-form models, where the RR is typically taken as an exogenous constant parameter or a stochastic variable independent from PD. Some of these models, such as CreditMetrics®, CreditPortfolioView® and CreditPortfolioManager®, treat the RR in the event of default as a stochastic variable – generally modeled through a beta distribution - independent from the PD. Others, such as CreditRisk+®, treat it as a constant parameter that must be specified as an input for each single credit exposure. For a comprehensive analysis of these models, see Crouhy, Galai and Mark (2000), Gordy (2000a), and Saunders and Allen (2002).

given default (LGDs)⁷ in 1999-2001⁸. Frye's (2000b and 2000c) empirical analysis allows him to conclude that in a severe economic downturn, bond recoveries might decline 20-25 percentage points from their normal-year average. Loan recoveries may decline by a similar amount, but from a higher level.

Jarrow (2001) presents a new methodology for estimating RRs and PDs implicit in both debt and equity prices. As in Frye (2000a and 2000b), RRs and PDs are correlated and depend on the state of the economy. However, Jarrow's methodology explicitly incorporates equity prices in the estimation procedure, allowing the separate identification of RRs and PDs and the use of an expanded and relevant dataset. In addition, the methodology explicitly incorporates a liquidity premium in the estimation procedure, which is considered essential in the light of the high variability in the yield spreads between risky debt and U.S. Treasury securities.

A rather different approach is the one proposed by Jokivuolle and Peura (2000). The authors present a model for bank loans in which collateral value is correlated with the PD. They use the option pricing framework for modeling risky debt: the borrowing firm's total asset value determines the event of default. However, the firm's asset value does not determine the RR. Rather, the collateral value is in turn assumed to be the only stochastic element determining recovery. Because of this assumption, the model can be implemented using an exogenous PD, so that the firm's asset value parameters need not be estimated. In this respect, the model combines features of both structural-form and reduced-form models. Assuming a positive correlation between a firm's asset value and collateral value, the authors obtain a similar result as Frye (2000a), that realized default rates and recovery rates have an inverse relationship.

Using Moody's historical bond market data, Hu and Perraudin (2002) examine the dependence between recovery rates and default rates. They first standardize the quarterly recovery data in order to filter out the volatility of recovery rates given by the variation over time in the pool of borrowers rated by Moody's. They find that correlations between quarterly recovery rates and default rates for bonds issued by US-domiciled obligors are 0.22 for post 1982 data (1983-2000) and 0.19 for the 1971-2000 period. Using extreme value theory and other non-parametric techniques, they also examine the impact of this negative correlation on credit

⁷ LGD indicates the amount actually lost (by an investor or a bank) for each dollar lent to a defaulted borrower. Accordingly, LGD and RR (recovery rate) always add to one. One may also factor into the loss calculation the last coupon payment that is usually not realized when a default occurs (see Altman, 1989).

⁸Gupton, Hamilton, and Berthault (2001) provide clear empirical evidence of this phenomenon.

VaR measures and find that the increase is statistically significant when confidence levels exceed 99%.

Bakshi et al. (2001) enhance the reduced-form models briefly presented above to allow for a flexible correlation between the risk-free rate, the default probability and the recovery rate. Based on some preliminary evidence published by rating agencies, they force recovery rates to be negatively associated with default probability. They find some strong support for this hypothesis through the analysis of a sample of BBB-rated corporate bonds: more precisely, their empirical results show that, on average, a 4% worsening in the (risk-neutral) hazard rate is associated with a 1% decline in (risk-neutral) recovery rates.

Compared to the above mentioned contributions, this study extends the existing literature in three main directions. First, the determinants of defaulted bonds' recovery rates are empirically investigated. While most of the above mentioned recent studies concluded in favor of an inverse relationship between these two variables, based on the common dependence on the state of the economy, none of them empirically analysed the more specific determinants of recovery rates. While our analysis shows empirical results that appear consistent with the intuition of a negative correlation between default rates and RRs, we find that a single systematic risk factor – i.e. the performance of the economy - is less predictive than the above mentioned theoretical models would suggest.

Second, our study is the first one to examine, both theoretically and empirically, the role played by supply and demand of defaulted bonds in determining aggregate recovery rates. Our econometric univariate and multivariate models assign a key role to the supply of defaulted bonds and show that these variables together with variables that proxy the size of the high yield bond market explain a substantial proportion of the variance in bond recovery rates aggregated across all seniority and collateral levels.

Third, our simulations show the consequences that the negative correlation between default and recovery would have on VaR models and on the procyclicality effect of the capital requirements recently proposed by the Basel Committee. Indeed, while our results on the impact of this correlation on credit risk measures (such as unexpected loss and value at risk) are in line with the ones obtained by Hu and Perraudin (2002), they show that, if a positive correlation highlighted by bond data were to be confirmed by bank data, the procyclicality effects of “Basel 2” might be even more severe than expected if banks use their own estimates of LGD (as in the “advanced” IRB approach).

As concerns specifically the Hu and Perraudin paper, it should be pointed out that they correlate recovery rates (or % of par which is the same thing) with issuer based default rates. Our models assess the relationship between dollar denominated default and recovery rates and, as such, can assess directly the supply/demand aspects of the defaulted debt market. Moreover, besides assessing the relationship between default and recovery using *ex-post* default rates, we explore the effect of using *ex-ante* estimates of the future default rates (i.e., default probabilities) instead of actual, realized defaults. As will be shown, however, while the negative relationship between RR and both *ex-post* and *ex-ante* default rates is empirically confirmed, probabilities of default show a considerably lower explanatory power. Finally, it should be emphasized that while our paper and the one by Hu and Perraudin reach similar conclusions, albeit from very different approaches and tests, it is important that these results become accepted and are subsequently reflected in future credit risk models and public policy debates and regulations. For these reasons, concurrent confirming evidence from several sources are beneficial, especially if they are helpful in specifying fairly precisely the default rate/recovery rate nexus.

2. Explaining Aggregate Recovery Rates on Corporate Bond Defaults: Empirical Results

The average loss experience on credit assets is well documented in studies by the various rating agencies (Moody's, S&P, and Fitch) as well as by academics⁹. Recovery rates have been observed for bonds, stratified by seniority, as well as for bank loans. The latter asset class can be further stratified by capital structure and collateral type (Van de Castle and Keisman, 2000). While quite informative, these studies say nothing about the recovery vs. default correlation. The purpose of this section is to empirically test this relationship with actual default data from the U.S. corporate bond market over the last two decades. As pointed out in Section 1, there is strong intuition suggesting that default and recovery rates might be correlated. Accordingly, this section of our study attempts to explain the link between the two variables, by specifying rather straightforward statistical models¹⁰.

We measure aggregate annual bond recovery rates (henceforth: BRR) by the weighted average recovery of all corporate bond defaults, primarily in the United States, over the period

⁹ See e.g. Altman and Kishore (1996), Altman and Arman (2002), FITCH (1997, 2001), Standard & Poor's (2000).

¹⁰ We will concentrate on average annual recovery rates but not on the factors that contribute to understanding and explaining recovery rates on individual firm and issue defaults. Madan and Unal (2001) propose a model for estimating risk-neutral expected recovery rate distributions, not empirically observable rates. The latter can be particularly useful in determining prices on credit derivative instruments, such as credit default swaps.

1982-2001. The weights are based on the market value of defaulting debt issues of publicly traded corporate bonds¹¹. The logarithm of BRR (BLRR) is also analysed.

The sample includes annual averages from about 1300 defaulted bonds for which we were able to get reliable quotes on the price of these securities just after default. We utilize the database constructed and maintained by the NYU Salomon Center, under the direction of one of the authors. Our models are both univariate and multivariate, least squares regressions. The univariate structures can explain up to 60% of the variation of average annual recovery rates, while the multivariate models explain as much as 90%.

The rest of this Section will proceed as follows. We begin our analysis by describing the independent variables used to explain the annual variation in recovery rates. These include supply-side aggregate variables that are specific to the market for corporate bonds, as well as macroeconomic factors (some demand side factors, like the return on distressed bonds and the size of the “vulture” funds market, are discussed later). Next, we describe the results of the univariate analysis. We then present our multivariate models, discussing the main results and some robustness checks.

2.1. Explanatory Variables

We proceed by listing several variables we reasoned could be correlated with aggregate recovery rates. The expected effects of these variables on recovery rates will be indicated by a +/- sign in parentheses. The exact definitions of the variables we use are:

BDR (-) The weighted average default rate on bonds in the high yield bond market and its logarithm (BLDR, -). Weights are based on the face value of all high yield bonds outstanding each year and the size of each defaulting issue within a particular year¹².

¹¹ Prices of defaulted bonds are based on the closing “bid” levels on or as close to the default date as possible. Precise-date pricing was only possible in the last ten years, or so, since market maker quotes were not available from the NYU Salomon Center database prior to 1990 and all prior date prices were acquired from secondary sources, primarily the *S&P Bond Guides*. Those latter prices were based on end-of-month closing bid prices only. We feel that more exact pricing is a virtue since we are trying to capture supply and demand dynamics which may impact prices negatively if some bondholders decide to sell their defaulted securities as fast as possible. In reality, we do not believe this is an important factor since many investors will have sold their holdings prior to default or are more deliberate in their “dumping” of defaulting issues.

¹² We did not include a variable that measures the distressed, but not defaulted, proportion of the high yield market since we do not know of a time series measure that goes back to 1987. We define distressed issues as yielding more than 1000 basis points over the risk-free 10-year Treasury Bond Rate. We did utilize the average yield spread in the market and found it was highly correlated (0.67) to the subsequent one year’s default rate and hence did not add value (see discussion below). The high yield bond yield spread, however, can be quite helpful in forecasting the

BDRC (-) One-year change in BDR.

BOA (-) The total amount of high yield bonds outstanding for a particular year (measured at mid-year in trillions of dollars) and represents the potential supply of defaulted securities. Since the size of the high yield market has grown in most years over the sample period, the BOA variable is picking up a time-series trend as well as representing a potential supply factor.

BDA (-) We also examined the more directly related bond defaulted amount as an alternative for BOA (also measured in trillions of dollars).

GDP (+) The annual GDP growth rate.

GDPC (+) The change in the annual GDP growth rate from the previous year.

GDPI (-) Takes the value of 1 when GDP growth was less than 1.5% and 0 when GDP growth was greater than 1.5%.

SR (+) The annual return on the S&P 500 stock index.

SRC (+) The change in the annual return on the S&P 500 stock index from the previous year.

2.2. The Basic Explanatory Variable: Default Rates

It is clear that the supply of defaulted bonds is most vividly depicted by the aggregate amount of defaults and the rate of default. Since virtually all public defaults most immediately migrate to default from the non-investment grade or “junk” bond segment of the market, we use that market as our population base. The default rate is the par value of defaulting bonds divided by the total amount outstanding, measured at face values. Table 1 shows default rate data from 1982-2001, as well as the weighted average annual recovery rates (our dependent variable) and the default loss rate (last column). Note that the average annual recovery is 41.8% (weighted average 37.2%) and the weighted average annual loss rate to investors is 3.16%¹³. The correlation between the default rate and the weighted price after default amounts to 0.75.

following year’s BDR, a critical variable in our model (see our discussion of a default probability prediction model in Section 2.6).

¹³ The loss rate is impacted by the lost coupon at default as well as the more important lost principal. The 1987 default rate and recovery rate statistics do not include the massive Texaco default since it was decided by a lawsuit which was considered frivolous resulting in a recovery rate (price at default) of over 80%.

Table 1
Default Rates, Recovery Rates and Losses

<i>Year</i>	<i>Par Value Outstanding (a)</i> (\$ MMs)	<i>Par Value of Defaults (b)</i> (\$ MMs)	<i>Default rate</i>	<i>Weighted Price after Default (Recovery Rate)</i>	<i>Weighted Coupon</i>	<i>Default Loss (c)</i>
2001	\$649,000	\$63,609	9.80%	25.5	9.18%	7.76%
2000	\$597,200	\$30,295	5.07%	26.4	8.54%	3.95%
1999	\$567,400	\$23,532	4.15%	27.9	10.55%	3.21%
1998	\$465,500	\$7,464	1.60%	35.9	9.46%	1.10%
1997	\$335,400	\$4,200	1.25%	54.2	11.87%	0.65%
1996	\$271,000	\$3,336	1.23%	51.9	8.92%	0.65%
1995	\$240,000	\$4,551	1.90%	40.6	11.83%	1.24%
1994	\$235,000	\$3,418	1.45%	39.4	10.25%	0.96%
1993	\$206,907	\$2,287	1.11%	56.6	12.98%	0.56%
1992	\$163,000	\$5,545	3.40%	50.1	12.32%	1.91%
1991	\$183,600	\$18,862	10.27%	36.0	11.59%	7.16%
1990	\$181,000	\$18,354	10.14%	23.4	12.94%	8.42%
1989	\$189,258	\$8,110	4.29%	38.3	13.40%	2.93%
1988	\$148,187	\$3,944	2.66%	43.6	11.91%	1.66%
1987	\$129,557	\$1,736	1.34%	62.0	12.07%	0.59%
1986	\$90,243	\$3,156	3.50%	34.5	10.61%	2.48%
1985	\$58,088	\$992	1.71%	45.9	13.69%	1.04%
1984	\$40,939	\$344	0.84%	48.6	12.23%	0.48%
1983	\$27,492	\$301	1.09%	55.7	10.11%	0.54%
1982	\$18,109	\$577	3.19%	38.6	9.61%	2.11%
<i>Weighted Average</i>			<i>4.19%</i>	<i>37.2</i>	<i>10.60%</i>	<i>3.16%</i>

Notes: (a) measured at mid-year, excludes defaulted issues; (b) does not include Texaco's bankruptcy in 1987; (c) includes lost coupon as well as principal loss
Source: authors' compilations

2.3. The Demand and Supply of Distressed Securities

The principal purchasers of defaulted securities, primarily bonds and bank loans, are niche investors called distressed asset or alternative investment managers - also called "vultures." Prior to 1990, there was little or no analytic interest in these investors, indeed in the distressed debt market, except for the occasional anecdotal evidence of performance in such securities. Altman (1991) was the first to attempt an analysis of the size and performance of the distressed debt market and estimated, based on a fairly inclusive survey, that the amount of funds under management by these so-called vultures was at least \$7.0 billion in 1990 and if you include those investors who did not respond to the survey and non-dedicated investors, the total

was probably in the \$10-12 billion range. Cambridge Associates (2001) estimated that the amount of distressed assets under management in 1991 was \$6.3 billion. Estimates since 1990 indicate that the demand did not rise materially until 2000-2001, when the estimate of total demand for distressed securities was about \$40-45 billion as of December 31, 2001 (see Altman and Pompeii, 2002). So, while the demand for distressed securities grew slowly in the 1990's and early in the next decade, the supply, as we will show, grew enormously.

On the supply side, the last decade has seen the amounts of distressed and defaulted public and private bonds and bank loans grow dramatically in 1990-1991 to as much as \$300 billion (face value) and \$200 billion (market value), then recede to much lower levels in the 1993-1998 period and grow enormously again in 2000-2001 to the unprecedented levels of \$650 billion (face value) and almost \$400 billion market value as of December 2001. These estimates are based on calculations in Altman and Pompeii (2002) from periodic, not continuous, market calculations and estimates.¹⁴

On a relative scale, the ratio of supply to demand of distressed and defaulted securities was something like ten to one in both 1990-1991 and also in 2000-2001. Dollarwise, of course, the amount of supply side money dwarfed the demand in both periods. And, as we will show, the price levels of new defaulting securities was relatively very low in both periods - at the start of the 1990's and again at the start of the 2000 decade.

2.4. Univariate Models

We begin the discussion of our results with the univariate relationships between recovery rates and the explanatory variables described in the previous section. Table 2 displays the results of the univariate regressions carried out using these variables. These univariate regressions, and the multivariate regressions discussed in the following section, were calculated using both the recovery rate (BRR) and its natural log (BLRR) as the dependent variables. Both results are displayed in Table 2, as signified by an "x" in the corresponding row.

We examine the simple relationship between bond recovery rates and bond default rates for the period 1982-2001. Table 2 and Figure 1 show several regressions between the two

¹⁴ Defaulted bonds and bank loans are relatively easy to define and are carefully documented by the rating agencies and others. Distressed securities are defined here as bonds selling at least 1000 basis points over comparable maturity Treasury Bonds (we use the 10-year T-Bond rate as our benchmark). Privately owned securities, primarily bank loans, are estimated as 1.4-1.8 x the level of publicly owned distressed and defaulted securities based on

fundamental variables. We find that one can explain about 51% of the variation in the annual recovery rate with the level of default rates (this is the linear model, regression 1) and as much as 60%, or more, with the logarithmic and power¹⁵ relationships (regressions 3 and 4). Hence, our basic thesis that the rate of default is a massive indicator of the likely average recovery rate amongst corporate bonds appears to be substantiated¹⁶.

The other univariate results show the correct sign for each coefficient, but not all of the relationships are significant. BDRC is highly negatively correlated with recovery rates, as shown by the very significant t-ratios, although the t-ratios and R-squared values are not as significant as those for BLDR. BOA and BDA are, as expected, both negatively correlated with recovery rates with BDA being more significant on a univariate basis. Macroeconomic variables did not explain as much of the variation in recovery rates as the corporate bond market variables explained; their poor performance is also confirmed by the presence of some heteroskedasticity and/or serial correlation in the regression's residuals, hinting at one or more omitted variables. We will come back to these relationships in the next paragraphs.

studies of a large sample of bankrupt companies (Altman and Pompeii, 2002). The supply continued to grow to over \$900 billion (face value) and about \$500 billion (market value) at the end of 2002.

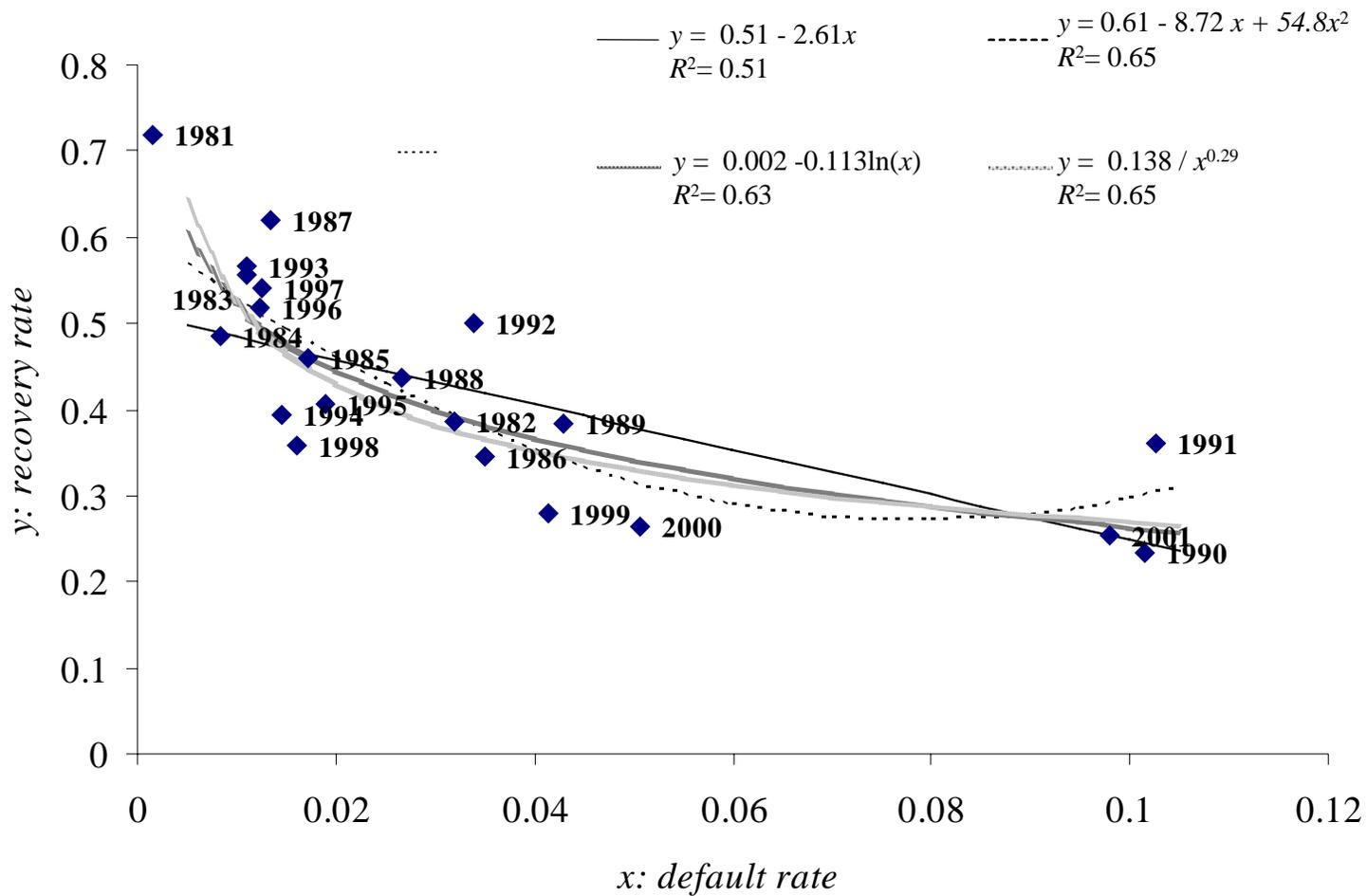
¹⁵ The power relationship ($BRR = e^{b_0} \times BDR^{b_1}$) can be estimated using the following equivalent equation: $BLRR = b_0 + b_1 \times BLDR$ ("power model").

¹⁶ Such an impression is strongly supported by a -80% rank correlation coefficient between BDR and BRR (computed over the 1982-2001 period). Note that rank correlations represent quite a robust indicator, since they do not depend upon any specific functional form (e.g., log, quadratic, power, etc.).

Table 2: Univariate Regressions, 1982-2001
Variables explaining annual recovery rates on defaulted corporate bonds

<i>(a) Market variables</i>										
<i>Regression #</i>	1	2	3	4	5	6	7	8	9	10
<i>Dependent variable</i>										
BRR	x		x		x		x		x	
BLRR		x		x		x		x		x
<i>Explanatory variables: coefficients and (t-ratios)</i>										
Constant	0.509 (18.43)	-0.668 (-10.1)	0.002 (0.03)	-1.983 (-10.6)	0.432 (24.9)	-0.872 (-20.1)	0.493 (13.8)	-0.706 (-8.05)	0.468 (19.10)	-0.772 (-13.2)
BDR	-2.610 (-4.36)	-6.919 (-4.82)								
BLDR			-0.113 (-5.53)	-0.293 (-5.84)						
BDRC					-3.104 (-4.79)	-7.958 (-4.92)				
BOA							-0.315 (-2.68)	-0.853 (-2.95)		
BDA									-4.761 (-3.51)	13.122 (-4.08)
<i>Goodness of fit measures</i>										
R-square	0.514	0.563	0.630	0.654	0.560	0.574	0.286	0.326	0.406	0.481
Adjusted R-square	0.487	0.539	0.609	0.635	0.536	0.550	0.246	0.288	0.373	0.452
F-Stat	19.03	23.19	30.61	34.06	22.92	24.22	7.21	8.69	12.31	16.67
(P-value)	0.000	0.000	0.000	0.000	0.000	0.000	0.015	0.009	0.003	0.001
<i>Residual tests</i>										
Serial correlation LM, 2 lags (Breusch-Godfrey)	1.021	1.836	1.522	2.295	1.366	2.981	1.559	1.855	3.443	2.994
(P-value)	0.600	0.399	0.467	0.317	0.505	0.225	0.459	0.396	0.179	0.224
Heteroskedasticity (White, Chi square)	0.089	1.585	0.118	1.342	8.011	5.526	2.389	1.827	0.282	1.506
(P-value)	0.956	0.453	0.943	0.511	0.018	0.063	0.303	0.401	0.868	0.471
N. obs.	20	20	20	20	20	20	20	20	20	20
<i>(b) Macro variables</i>										
<i>Regression #</i>	11	12	13	14	15	16	17	18	19	20
<i>Dependent variable</i>										
BRR	x		x		x		x		x	
BLRR		x		x		x		x		x
<i>Explanatory variables</i>										
Constant	0.364 (7.59)	-1.044 (-8.58)	0.419 (18.47)	-0.907 (-15.65)	0.458 (15.42)	-0.804 (-10.8)	0.387 (10.71)	-1.009 (-11.3)	0.418 (16.42)	-0.910 (-14.4)
GDP	1.688 (1.30)	4.218 (1.28)								
GDPC			2.167 (2.31)	5.323 (2.22)						
GDPI					-0.101 (-2.16)	-0.265 (-2.25)				
SR							0.205 (1.16)	0.666 (1.53)		
SRC									0.095 (0.73)	0.346 (1.07)
<i>Goodness of fit measures</i>										
R-square	0.086	0.083	0.228	0.215	0.206	0.220	0.070	0.115	0.029	0.060
Adjusted R-square	0.035	0.032	0.186	0.171	0.162	0.176	0.018	0.066	-0.025	0.007
F-Stat	1.69	1.64	5.33	4.93	4.66	5.07	1.36	2.35	0.53	1.14
(P-value)	0.211	0.217	0.033	0.040	0.045	0.037	0.259	0.143	0.475	0.299
<i>Residual tests</i>										
Serial correlation LM, 2 lags (Breusch-Godfrey)	2.641	4.059	0.663	1.418	0.352	1.153	3.980	5.222	3.479	4.615
(P-value)	0.267	0.131	0.718	0.492	0.839	0.562	0.137	0.073	0.176	0.100
Heteroskedasticity (White, Chi square)	2.305	2.077	2.254	2.494	0.050	0.726	2.515	3.563	3.511	4.979
(P-value)	0.316	0.354	0.324	0.287	0.823	0.394	0.284	0.168	0.173	0.083
N. obs.	20	20	20	20	20	20	20	20	20	20

Figure 1
Univariate Models
 Explaining annual recovery rates on defaulted corporate bonds



2.5. Multivariate Models

We now specify some more complex models to explain recovery rates, by adding several variables to the default rate. The basic structure of our most successful models is:

$$BRR = f(BDR, BDRC, BOA \text{ or } BDA)$$

Some macroeconomic variables will be added to this basic structure, to test their effect on recovery rates.

Before we move on to the multivariate results, Table 3 reports the cross-correlations among our regressors; values greater than 0.6 are highlighted. A strong link between GDP and BDR is shown, suggesting that default rates are, as expected, positively correlated with macro growth measures. Hence, adding GDP to the BDR/BRR relationship is expected to blur the significance of the results. We also observe a high positive correlation between BDA (absolute amount of all defaulted bonds) and the default rate.

Table 3
Correlation coefficients among the main independent variables

	<i>BDR</i>	<i>BOA</i>	<i>BDA</i>	<i>GDP</i>	<i>SR</i>
<i>BDR</i>	1.00	0.21	0.71	-0.78	-0.35
<i>BOA</i>		1.00	0.73	0.14	-0.28
<i>BDA</i>			1.00	-0.43	-0.55
<i>GDP</i>				1.00	0.10
<i>SR</i>					1.00

We estimate our regressions using 1982-2001 data in order to explain recovery rate results and to predict 2002 rates. This involves linear and log-linear structures for the two key variables – recovery rates (dependent) and default rates (explanatory) – with the log-linear relationships somewhat more significant. These results appear in Table 4.

Regressions 1 through 6 build the “basic models”: most variables are quite significant based on their t-ratios. The overall accuracy of the fit goes from 71% (65% adjusted R-square) to 87% (84% adjusted).

The actual model with the highest explanatory power and lowest “error” rates is the power model¹⁷ in regression 4 of Table 4. We see that all of the four explanatory variables have the expected negative sign and are significant at the 5% or 1% level. BLDR and BDRC are extremely significant, showing that the level and change in the default rate are highly important explanatory variables for recovery rates. Indeed the variables BDR and BDRC explain up to 80% (unadjusted) and 78% (adjusted) of the variation in BRR simply based on a linear or log-linear association. The size of the high yield market also performs very well and adds about 6/7% to the explanatory power of the model. When we substitute BDA for BOA (regressions 5 and 6), the latter does not look statistically significant, and the R-squared of the multivariate model drops slightly to 0.82 (unadjusted) and 0.78 (adjusted). Still, the sign of BDA is correct (+). Recall that BDA was more significant than BOA on a univariate basis (Table 2).

Macro variables are added in columns 7-10: we are somewhat surprised by the low contributions of these variables since there are several models that have been constructed that utilize macro-variables, apparently significantly, in explaining annual default rates¹⁸.

As concerns the growth rate in annual GDP, the univariate analyses presented in Tables 2 and 3 had shown it to be significantly negatively correlated with the bond default rate (-0.78, see Table 3); however, the univariate correlation between recovery rates (both BRR and BLRR) and GDP growth is relatively low (see Table 2), although with the appropriate sign (+). Note that, when we utilize the change in GDP growth (GDPC, Table 2, regression 5 and 6), the significance improves markedly.

When we introduce GDP to our existing multivariate structures (Table 4, regressions 7 and 8), not only is it not significant, but it has a counterintuitive sign (negative). The GDPC

¹⁷ Like its univariate cousin, the multivariate power model can be written using logs. E.g., $BLRR = b_0 + b_1 \times BLDR + b_2 \times BDRC + b_3 \times BOA$ becomes $BRR = \exp[b_0] \times BDR^{b_1} \times \exp[b_2 \times BDRC + b_3 \times BOA]$ and takes its name from BDR being raised to the power of its coefficient.

¹⁸ See e.g. Jonsson and Fridson (1996), Keenan, Sobehart and Hamilton (1999), Fridson, Garman, and Wu (1997), Helwege and Kleiman (1997), and Chacko and Mercier (2001).

variable leads to similar results (not reported). No doubt, the high negative correlation between GDP and BDR reduces the possibility of using both in the same multivariate structure.

Table 4: Multivariate Regressions, 1982-2001
Variables explaining annual recovery rates on defaulted corporate bonds

Regression # Dependent variable	Linear and logarithmic models										Logistic models				
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
BRR	x		x		x		x		x		x	x	x	x	x
BLRR		x		x		x		x		x					
<i>Explanatory variables: coefficients and (t-ratios)</i>															
Constant	0.514 (19.96)	-0.646 (-11.34)	0.207 (2.78)	-1.436 (-8.70)	0.482 (20.02)	-1.467 (-6.35)	0.529 (11.86)	-1.538 (-9.07)	0.509 (14.65)	-1.447 (-8.85)	-0.074 (-0.64)	-0.097 (-0.92)	0.042 (0.44)	0.000 (0.00)	0.000 (0.00)
BDR	-1.358 (-2.52)	-3.745 (-3.13)			-1.209 (-1.59)		-1.513 (-2.28)		-1.332 (-2.33)		12.200 (4.14)	6.713 (2.82)	5.346 (1.55)	7.421 (2.59)	6.487 (2.64)
BLDR			-0.069 (-3.78)	-0.176 (-4.36)		-0.167 (-2.94)		-0.222 (-4.64)		-0.169 (-4.17)					
BDRC	-1.930 (-3.18)	-4.702 (-3.50)	-1.748 (-3.39)	-4.389 (-3.84)	-2.039 (-3.03)	-4.522 (-3.35)	-1.937 (-3.11)	-4.415 (-4.05)	-1.935 (-3.09)	-4.378 (-3.87)	8.231 (3.339)	8.637 (3.147)	8.304 (3.282)	8.394 (3.315)	
BOA	-0.164 (-2.13)	-0.459 (-2.71)	-0.141 (-2.12)	-0.410 (-2.78)			-0.153 (-1.86)	-0.328 (-2.20)	-0.162 (-2.03)	-0.387 (-2.63)	0.742 (2.214)			0.691 (1.927)	0.736 (2.136)
BDA					-1.203 (-0.81)	-3.199 (-1.12)							8.196 (1.064)		
GDP							-0.387 (-0.43)	-2.690 (-1.62)						1.709 (0.473)	
SR									0.020 (0.192)	0.213 (1.156)					-0.242 (-0.56)
<i>Goodness of fit measures</i>															
R-square	0.764	0.819	0.826	0.867	0.708	0.817	0.767	0.886	0.764	0.878	0.534	0.783	0.732	0.786	0.787
Adjusted R-square	0.720	0.785	0.793	0.842	0.654	0.782	0.704	0.856	0.702	0.845	0.508	0.742	0.682	0.729	0.731
F-Stat	17.250	24.166	25.275	34.666	12.960	23.752	12.320	29.245	12.168	26.881	20.635	19.220	14.559	13.773	13.876
(P-value)	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
<i>Residual tests</i>															
Serial correlation LM, 2 lags (Breusch-Godfrey)	3.291	2.007	1.136	0.718	1.235	0.217	3.344	0.028	5.606	1.897	1.042	2.673	1.954	2.648	5.899
(P-value)	0.193	0.367	0.567	0.698	0.539	0.897	0.188	0.986	0.061	0.387	0.594	0.263	0.376	0.266	0.052
Heteroskedasticity (White, Chi square)	5.221	5.761	5.049	5.288	12.317	12.795	5.563	4.853	6.101	6.886	0.008	5.566	9.963	5.735	5.948
(P-value)	0.516	0.451	0.538	0.507	0.055	0.046	0.696	0.773	0.636	0.549	0.996	0.474	0.126	0.677	0.653
N. obs.	20	20	20	20	20	20	20	20	20	20	20	20	20	20	20

We also postulated that the return of the stock market could impact prices of defaulting bonds in that the stock market represented investor expectations about the future. Table 4, regressions 9-10, show the association between the annual S&P 500 Index stock return (*SR*) and recovery rates. Note the insignificant t-ratios in the multivariate model, despite the appropriate signs. Similar results (together with low R^2 s) emerge from our univariate analysis (Table 2), where the change in the S&P return (*SRC*) was also tested.

Since the dependent variable (*BRR*) in most of our regressions is bounded by 0 and 1, we have also run the same models using a logistic function (Table 4, columns 11-15). As can be seen, R-squares and t-ratios are broadly similar to those already shown above. The model in column 12, including *BDR*, *BDRC* and *BOA* explains as much as 74% (adjusted R-square) of the recovery rate's total variability. Macroeconomic variables – as before – tend to have no evident effect on *BDR*.

2.6 Robustness checks

This section hosts some robustness checks carried out to verify how our results would change when taking into account several important modifications to our approach.

Default probabilities - The models shown above are based on the actual default rate experienced in the high yield, speculative grade market (*BDR*) and reflect a coincident supply/demand dynamic in that market. One might argue that this *ex-post* analysis is conceptually different from the specification of an *ex-ante* estimate of the default rate.

We believe both specifications are important. Our previous *ex-post* models and tests are critical in understanding the actual experience of credit losses and, as such, impact credit management regulation and supervision, capital allocations, and credit policy and planning of financial institutions. On the other hand, *ex-ante* probabilities (*PDs*) are customarily used in VaR models in particular and risk management purposes in general; however, their use in a regression analysis of recovery rates might lead to empirical tests which are inevitably limited by the models used to estimate *PDs* and their own biases. The results of these tests might therefore not be indicative of the true relationship between default and recovery rates.

In order to assess the relationship between *ex ante* *PDs* and *BRRs*, we used *PDs* generated through a well-established default rate forecasting model from Moody's (Keenan,

Sobehart and Hamilton, 1999). This econometric model is used to forecast the *global* speculative grade *issuer* default rate and was fairly accurate ($R^2 = 0.8$) in its explanatory model tests¹⁹.

The results of using Moody's model to explain our recovery rates did demonstrate a significant negative relationship but the explanatory power of the multivariate models was considerably lower (adjusted $R^2 = 0.39$), although still impressive with significant t-tests for the change in PD and the amount of bonds outstanding (all variables had the expected sign). Note that, since the Moody's model is for global issuers and our earlier tests are for US dollar denominated high yield bonds, we did not expect that their PD model would be nearly as accurate in explaining US recovery rates.

Quarterly data - Our results are based on yearly values. We explored the feasibility of using higher-frequency data. We had to refrain from using monthly data simply because of missing values (several months show no defaults). Based on quarterly data, a simple, univariate estimate shows that: 1) BDR is still strongly significant, with the expected sign and a t-statistic of -5.77, 2) the R-square looks somewhat modest (23.9%) because quarterly default rates tend to be very volatile (due to some "poor" quarters with only very few defaults)²⁰.

Risk-free rates - We considered the role of risk-free rates in explaining recovery rates, since these, in turn, depend on the discounted cash flows expected from the defaulted bonds. We therefore added to our "best" models (e.g., columns 3-4 in Table 4) some "rate" variables (namely, the one-year U.S. dollar Treasury rate taken from the Federal Reserve Board of Governors, the corresponding discount rate and their logarithms). The results are disappointing, since none of these variables ever is statistically significant at the 10% level²¹.

Returns on defaulted bonds - We examined whether the return experienced by the defaulted bond market affects the demand for distressed securities, thereby influencing the "equilibrium price" of defaulted bonds. To do so, we considered the one-year return on the

¹⁹ Thus far, Moody's has tested their forecasts for the 36-month period 1999-2001 and found that the correlation between estimated (PD) and actual default rates was greater than 0.90 (Hamilton, et al., 2003). So, it appears that there can be a highly correlated link between estimated PDs and actual BDRs. By association, therefore, one can infer that accurate PD models can be used to estimate recovery rates and LGD.

²⁰ Using a moving average of 4 quarters (weighted by the number of defaulted issues) we estimated another model (using BDR, its lagged value and its square) which obtained a better R-square (72.4%) but shows highly auto-correlated residuals.

²¹ This might also be due to the fact that one of our regressors (BOA, the amount of outstanding bonds) indirectly accounts for the level of risk-free rates, since lower rates imply higher market values and vice versa. Even removing BOA, however, risk-free rates cannot be found to be significant inside our model.

Altman-NYU Salomon Center Index of Defaulted Bonds (*BIR*), a monthly indicator of the market weighted average performance of a sample of defaulted publicly traded bonds²². This is a measure of the price changes of existing defaulted issues as well as the “entry value” of new defaults and, as such, is impacted by supply and demand conditions in this “niche” market.²³ On a univariate basis, the *BIR* shows the expected sign (+) with a t-ratio of 2.67 and explains 35% of the variation in *BRR*. However, when *BIR* is included in multivariate models, its sign remains correct, but the significance is usually below 10%.

GDP dummy - We saw, in our multivariate results, that the *GDP* variable lacks statistical significance and tends to have a counterintuitive sign when added to multivariate models. The fact that *GDP* growth is highly correlated with default rates, our primary explanatory variable, looks like a sensible explanation for this phenomenon. To try and circumvent this problem, we used a technique similar to Helwege and Kleiman (1997): they postulate that, while a change in *GDP* of say 1% or 2% was not very meaningful in explaining default rates when the base year was in a strong economic growth period, the same change was meaningful when the new level was in a weak economy. Following their approach, we built a dummy variable (*GDPI*) which takes the value of 1 when *GDP* grows at less than 1.5% and 0 otherwise.

The univariate *GDPI* results show a somewhat significant relationship with the appropriate negative sign (Table 2). However, when one adds the “dummy” variable *GDPI* to the multivariate models discussed above, the results (not reported) show no statistically significant effect, although the sign remains appropriate.

3. Implications for Credit VaR Models, Capital Ratios and Procyclicality

The results of our empirical tests have important implications for a number of credit risk related conceptual and practical areas. This section reviews two key areas that can be significantly affected when one factors in that default rates are, in fact, negatively correlated with

²² More details can be found in Altman (1991) and Altman and Pompeii (2002). Note that we use a different time frame in our analysis (1987-2001), because the defaulted bond index return (*BIR*) has only been calculated since 1987.

²³ We are aware of the fact that the average recovery rate on newly defaulted bond issues could influence the level of the defaulted bond index and vice-versa. The vast majority of issues in the index, however, are usually comprised of bonds that have defaulted in prior periods. And, as we will see, while this variable is significant on an univariate basis and does improve the overall explanatory power of the model, it is not an important contributor.

recovery rates. These are (1) credit VaR models and (2) the potential impact of our findings on the procyclicality of capital requirements debated by the Basel Committee²⁴.

VaR Models - As noted in footnote 6, most credit VaR models treat recovery rates as deterministic (like in the CreditRisk+ model proposed by Credit Suisse Financial Products, 1997) or stochastic but independent from default probabilities (like in the Creditmetrics framework: Finger, Gupton and Bhatia, 1997). The impact of a negative correlation between recovery rates and default rates is generally overlooked. In order to assess this impact, we ran Montecarlo simulations on a sample portfolio of bank loans and compared the key risk measures (expected and unexpected losses) obtained by the two above-mentioned models to those generated when recovery rates are treated as stochastic and negatively correlated with PDs.

The results of our simulations are revealing, indicating that both the expected loss and the unexpected loss are vastly understated if one assumes that PDs and RRs are uncorrelated²⁵. As long as the PDs used in VaR models can be thought of as an *ex ante* estimate of actual DRs, this implies that the risk measures generated by such models are biased.

Summing up, if default rates (and PDs, which can be thought of as *ex ante* estimates of actual DRs) are found to be correlated with RRs, then not only the risk measures based on standard errors and percentiles (i.e., the unexpected losses) could be seriously underestimated, but the amount of expected losses on a given credit portfolio (on which banks' provisioning policies should be based) could also be misjudged. Therefore, credit models that do not carefully factor in the negative correlation between PDs and RRs might lead to insufficient bank reserves and cause unnecessary shocks to financial markets.

The RR/PD Link and Procyclicality Effects - Procyclicality involves the sensitivity of regulatory capital requirements to economic and financial market cycles. Since ratings and default rates respond to the cycle, the new internal ratings-based (IRB) approach proposed by the Basel Committee risks increasing capital charges, and limiting credit supply, when the economy is slowing (the reverse being true when the economy is growing at a fast rate).

²⁴ We will simply summarize here our conclusions based on several simulation analyses, discussed in greater detail in Altman, Resti and Sironi (2001).

²⁵ Both expected losses and VaR measures associated with different confidence levels tend to be underestimated by approximately 30%.

Such procyclicality effects might be thought to be exacerbated by the correlation between DRs and RRs found in our study (and in some of the contributions quoted in Section 1); in other words, low recovery rates when defaults are high would amplify cyclical effects. This would especially be true under the so-called “advanced” IRB approach, where banks are free to estimate their own recovery rates and might tend to revise them downwards when defaults increase and ratings worsen.

The impact of such a mechanism was assessed, for example, in Resti (2002), based on simulations over a 20-year period, using a standard portfolio of bank loans (the composition of which is adjusted through time according to S&P transition matrices). Two results of these simulations are worth mentioning. First, the procyclicality effect is driven more by up- and downgrades, rather than by default rates; in other words, adjustments in credit supply needed to comply with capital requirements respond mainly to changes in the structure of weighted assets, and only to a lesser extent to actual credit losses (except in extremely high default years). Second, when RRs are permitted to fluctuate with default rates, the procyclicality effect increases significantly. Moreover, bank spreads, too, become more volatile, since revisions in short-term RR estimates are factored into loan prices.

One might object that in these simulations banks basically react to short-term results, and that regulation should encourage “advanced” IRB systems to use long-term average recovery rates. However, while the use of long-term RRs would make procyclicality effects less marked, it would also force banks to maintain a less updated picture of their risks, thereby trading stability for precision.

4. Concluding Remarks

This paper analyzed the link between aggregate default rates/probabilities and the loss given default on corporate bonds, both from a theoretical and an empirical standpoint. As far as the theoretical aspects are concerned, most of the literature on credit risk management models and tools treats the recovery rate variable as a function of historic average default recovery rates (conditioned perhaps on seniority and collateral factors), but in almost all cases as independent of expected or actual default rates. This appears rather simplistic and unrealistic in the light of our empirical evidence.

We examined the recovery rates on corporate bond defaults, over the period 1982-2002, by means of rather straightforward statistical models. These models assign a key role to the

supply of defaulted paper (default rates) and explain a substantial proportion of the variance in bond recovery rates aggregated across all seniority and collateral levels.

These results have important implications for portfolio credit risk models, for markets which depend on recovery rates as a key variable (e.g., securitizations, credit derivatives. etc.), and for the current debate on the revised BIS guidelines for capital requirements on bank assets.

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