Portfolio Performance and Agency

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Abstract

The evaluation and compensation of portfolio managers is an important problem for practitioners. Optimal compensation will induce managers to expend effort to generate information and to use it appropriately in an informed portfolio choice. Our general model points the way towards analysis of optimal performance evaluation and contracting in a rich model. Optimal contracting in the model includes an important role for portfolio restrictions that are more complex than the sharing rule. The agent's compensation gives the agent approximately to a benchmark return plus an incentive fee equal to a portfolio measure that is approximately the excess of return above the benchmark. This measure is often used by practitioners but is simpler than the Jensen measure and other measures commonly recommended in the academic literature. In addition to the excess return above the fixed benchmark, the manager is given some additional incentive to take a position that deviates from the benchmark to remove an incentive to tend towards being a "closet indexer." Efficient contracting involves restrictions on what portfolio strategies can be pursued, and prior communication of the information gathered.

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1 Introduction

The delegation of portfolio management from an investor to a portfolio manager is one of the most important agency relationships[1] in finance, and the appropriate compensation of portfolio managers is an ongoing topic of debate among practitioners and regulators. An optimal contract between investor and portfolio manager should give the manager incentives to expend effort in gathering information and to make good use of the information in the portfolio selection process. In a traditional agency problem, the optimal trade-off between incentives and risk-sharing is obtained by a sharing rule that defines the contractually specified share of profits to go to the manager. In a portfolio problem, we show that specifying a sharing rule alone does not implement the optimal contract. We use the revelation principal to show that the optimal contract can be implemented by restricting the manager to a menu of portfolio strategies, one strategy for each possible signal. The restriction to the menu is similar in spirit to restrictions in practice on what portfolio strategies managers are permitted to follow, although the specific restrictions are different from what we see in practice. In some cases, we can obtain a full theoretical derivation of the optimal contract, while in other cases we derive numerically a first-order solution which is not guaranteed to be optimal.

Closely related to the issue of contracting with a portfolio manager is the evaluation of a portfolio manager's performance, since an evaluation of performance will appear implicitly or explicitly in any nonconstant compensation. In our analysis, the portfolio manager expends costly effort on research that generates information, in the form of a signal that is correlated with returns, that can be used to generate superior performance.[2] We analyze optimal contracts in a rich model of security returns and a rich set of portfolio strategies and contracts, in an ideal case which abstracts from such impediments to optimal contracting as career concerns.[3] We find that opti-

[1]Stole (1993) has an excellent survey of the agency literature.

[2]Admittedly, it may not be realistic to assume that portfolio managers have the ability to outperform the market, since attempts to document consistent superior performance have failed. However, we need some assumption along these lines if we are going to rationalize significant fees for active management, and at least some managers and their clients believe superior performance is possible.

[3]This paper also abstracts from the usual hierarchical structure of separate management of different asset classes and any difficulties in preventing the manager from undoing incentives through private portfolio trades.

mal contracting provides the manager with a payoff that is an uninformed benchmark plus a multiple of the difference between the client's return and an uninformed benchmark, and that the manager's choice of portfolio is limited to a menu.

In the traditional agency problem, both the agent's costly effort and stochastic factors beyond the agent's control influence the output of a productive activity owned by the principal. The principal chooses a compensation contract for the agent that trades off optimal incentives to expend effort with the damage to risk-sharing caused by exposing the agent to too much risk from the factors beyond the agent's control. Even under the assumption of risk-neutrality of the principal, this traditional problem is a rich topic for study and has generated a substantial literature. Our agency problem in portfolio management differs in several important ways from the traditional agency problem. First, because the activity that requires effort is gathering information, the form of the agency problem is different from the outset. In particular, the signal about returns is important intermediate information that the manager can report (explicitly or implicitly through a choice from a menu of portfolio strategies) to improve contracting. A second important difference is the possibility that the manager might not use the information as the principal would like. We distinguish the first-best contract (in which effort choice can be dictated by a social planner), the second-best contract (in which the agent must be given the incentive to choose the planned effort level but the information is publicly verifiable), and the third-best contract (in which the agent must be given proper incentives to choose the effort and act as intended on the information). Another minor difference between our analysis and most of the agency literature is that we assume the principal is risk averse: it is unreasonable to assume risk-neutrality of the principal in the portfolio context, since a risk-neutral price-taker would derive infinite benefit from any superior information in a portfolio problem.[4]

Our formal specification of the model is a bit different from the traditional

[4]By risk-neutrality we mean the original definition, namely that the agent is indifferent among all payoffs with the same mean. This is different from, for example, linear von Neumann-Morgenstern preferences over positive wealth with a non-negative wealth constraint. These preferences are risk averse since getting a positive amount for sure is preferred to a risky gamble with the same mean and a positive probability of a negative payoff. Indeed, we can think of these preferences as coming from the concave extended-real utility function (as concavity is defined by Rockafeller) with $u(w) = w$ for $w \geq 0$ and $u(w) = -\infty$ for $w < 0$.

approach and for good reason. Traditionally, there is a pool of money that is invested by the manager who is then paid out of the proceeds in an amount described by a contractually specified sharing rule[5]. This is appealing, because it seems at first blush to conform to actual practice. However, implicit in this traditional approach there are significant restrictions on the possible types of contracts, and the differences between our general model and more traditional models of agency in investment do correspond to institutions we see in practice. Specifically, unlike the usual sharing rule, we find that (1) the incentive contract depends on a market or benchmark return as well as the portfolio's performance, (2) the manager is restricted in what portfolio strategies can be chosen, and (3) the manager should reveal information about the planned strategy at the start of the period, after performing the research but before any investment returns are realized. We should emphasize that these features are not assumptions of our model; rather, our model is allowed to choose a traditional contract but tells us that a contract with these features will do better.

Given the rich set of possible institutions (including as examples the three circumstances we have just listed), how are we to make sure that our model specification has captured all possible useful institutions? For example, how do we know *a priori* we cannot do even better by dividing the money into six pools, having the manager choose a different portfolio on each, and paying the manager a nonlinear function of the realizations on all the pools? The answer is a concept from mechanism design, the *revelation principle*, which says that we cannot do better than the best *direct mechanism* in which you report all your information subject to contractual guarantees on how that information will be used. The economic idea is simple: whatever is the ultimate outcome in equilibrium in a proposed institution, make choices and give ultimate payoffs that are the same function of the signals and random realizations as in the institution, substituting reported signals for the actual ones. Because the original outcome was an equilibrium outcome, the agent will not have any incentive to misreport the signal. This shows that a direct mechanism can achieve any outcome that is available in any general institution.

In the present context, here is how the direct mechanism works. First, the owner of the assets to be managed proposes a contract covering compensation of the manager, a rule for choosing the investment strategy as a function of

[5]See for example Ross (1974), Dybvig and Spatt (1986), Zender (1988), and Stoughton (1993).

reported signal, and planned managerial effort. (It is really not essential for the investor to make this choice, and it could be the manager who chooses or the result of a bargaining process. Any of these devices would select the same efficient frontier of contracts in our model.) Then, the manager decides whether to participate, and if so, expends effort. Nature provides the manager with a signal, the quality of which is a function of the effort expended. The manager reports the signal and based on the contractual schedule an accountant sets the portfolio policy. Finally, security returns are realized, the manager is payed according to the contract, and the remainder of the portfolio payoff goes to the investor. Formally, this game is solved by having the principal choose the equilibrium subject to a budget constraint for investment, a participation constraint to ensure the manager is offered at least the reservation utility, and incentive compatibility constraints to ensure that the agent exerts effort and reports the signal as the principal plans.

The statement of this formal model may seem strange, and in particular announcing the signal and handing the portfolio over to an accountant for execution is not a good literal description of practice. However, the statement of the model is merely a device for finding the optimal institution, which can then be interpreted in more familiar terms. Once we have solved the formal model, we then need to turn to the task of interpreting the solution, which is where we obtain our short list of deviations from the traditional sharing rule. This interpretation is to some extent a matter of taste or dictated by what we see in practice, since any formal solution of the model is consistent with a variety of essentially equivalent institutions. This is the point at which the issue of performance measurement arises in our analysis; and we can ask the question of how the optimal contract is related to traditional or potential performance measures.

Due to the complexity of the manager's choice problem, we are not able to prove that there exists a solution to the third-best problem. However, we provide a numerical solution to the first-order version of the problem. (Since we cannot prove that the first-order solution is a solution to the original problem, there is a very real possibility that we do not have the correct solution, even approximately. Unfortunately, this is the state of our knowledge at this point in time.) Compared to the known solution of the second-best problem, the numerical solution of the third-best penalizes the agent for reporting a signal that is not very informative (i.e., is close to the unconditional mean). In the second-best, the manager is exposed to more risk than the manager would prefer, and would choose to underreport the magnitude of signals to reduce that risk exposure. Penalizing the manager for reporting a relatively neutral signal (or equivalently rewarding the manager for taking a significant position) reverses this incentive. This is different in form but similar in intent to practitioners' desire to avoid "closet indexers" who collect fees for active management but actually choose a portfolio close to the index.

It is useful to mention what our model does not try to do. For example, Bhattacharya and Pfleiderer use a screening model to study self-selection of portfolio managers, while we use an agency model to study incentives for managers to exert effort.. Our assumption is that managers do not know themselves how good they are, and that trying to screen managers by their own confidence levels is not a good idea. Indeed, we do not want to hire a manager who is very confident about outperforming the market by 40%/year with essentially no risk, since we are likely to think such a manager is overconfident and naive about the functioning of the market. Another example of what our model does not do is to analyze career concerns, i.e., the effect of the manager's portfolio performance on future salaries or funds under management and the impact this has on incentives. One interpretation of this is that we are analyzing a manager in the last year before retirement and that there is no way to sell the manager's track record. Alternatively, our problem covers the case in which the career concerns are somehow neutralized. The model in this paper can be extended to include career concerns, and using the tools developed here to analyze that problem is a promising extension.

Stoughton (1993) has very similar goals to the current paper. In a model with costly effort to gather information and a portfolio choice between a stock and a bond, that analysis considers the optimal linear contract and a particular quadratic contract. In the linear case, the manager can undo any incentives in the compensation schedule by choosing a less risky portfolio than the client might intend, and therefore the performance-based contract is ineffective. In the quadratic case, there is a sort of limiting result that says that the contract approaches the first-best as the client becomes less and less risk averse.

Unfortunately, Stoughton (1993) has some conceptual problems. For the case of the linear contract in that paper, the model assumes the manager makes an unconstrained portfolio choice or (supposedly equivalently) what signal to report. In fact, choosing what signal to report (as in the current paper) would be more general than the analysis in that paper, since choosing a portfolio as a function of the signal that does not take on all real values is a way of enforcing a menu restriction. This can be useful in inducing effort to gather information by preventing the manager from choosing too safe a portfolio that would be attractive in the absence of information gathering. We can still interpret Stoughton's linear model as showing the ineffectiveness of linear contracts in the presence of unlimited portfolio choice, and the same point is made in a slightly different model of Admati and Pfleiderer (1997), with the same conceptual problem. Another conceptual problem with Stoughton (1993) is that for the case of the quadratic contract, the sense of convergence (small difference in utility) to the first-best as the client becomes less risk averse is not robust to different utility representations, since the limit is along a sequence of different utility functions and we are free to pick a different affine transform for each point along the sequence. In fact, the difference between the first-best certainty-equivalent and the certainty-equivalent for the quadratic model converges to a positive constant as risk aversion increases.[6] In this economically relevant sense (which is invariant to affine transforms), there is no convergence. This is because even when the client is nearly risk neutral and does not care about noise in the contract, there still has to be non-vanishing compensation for the risk borne by the manager (which is greater than first-best in order to induce the required effort). For the particular quadratic contract under study, there is also an efficiency loss to not exploiting the rents the manager could obtain by collecting the market risk premium.

Another related paper is Zender (1988), which provides a number of results. The Jensen measure is shown to be the optimal linear contract in a reduced-form model for a mean-variance world in which the expenditure of costly effort influences the conditionally efficient portfolio in a particular way. In the same model, it is shown that a Mirrlees (1974) forcing contract (with a small probability of extreme punishments when the realization is grossly incompatible with the signal) can get arbitrarily close to the first-best if we allow for a general nonlinear contract.[7] Finally, an interpretation of the problem in terms of continuous-time agency model of Holmström and Milgrom (1987) is given. The main weaknesses of Zender (1988) are that the

[6]In Stoughton (1993), the agent's utility is $U_B(w) = -\exp(-bw)$ and therefore the certainty equivalent is the inverse of this function $CE(u) = -\log(-u)/b$. Then we can compute from (29) and (30) in Stoughton (1993) (using also (4)), that the difference in certainty equivalents for small b is approximately $-\log(1 - 2a\gamma_2/H)/(2a)$, which is a positive constant.

[7]There would presumably be a Mirrlees (1974) forcing contract in the model of Stoughton (1993) as well if it were extended to admit general nonlinear contracts.

mapping from effort to efficient portfolio is a black box and that it is unclear what underlying model it is a reduced form for, or indeed whether the the optimal contract in the reduced form is also optimal in the underlying model. It should be mentioned that Sung (1995) has a nontrivial extension of Holmstrom and Milgrom (1987) that includes control of variance as well as mean, and one of the applications of the general result is to a portfolio agency problem similar to the one described in Zender (1988).

2 The General Problem

The general formulation of the problem is as follows. We assume that the set of states which can be distinguished by security payoffs is complete. Let ω denote such a state. We will refer to ω as a market state and will denote by $p(\omega)$ the price of a claim which pays a dollar in state ω.

Although market prices incorporate all publicly available information there is additional information which is costly to obtain but which would aid in investing because it allows one to make better forecasts of which market state will be observed in the future. This information is conveyed by a signal s which is observed by the agent.[8] The informativeness of the signal depends on $a \in [0, 1]$, the effort expended by the agent in information gathering. We model this as follows. Let $f^U(s, \omega)$ and $f^I(s, \omega)$ be two joint densities[9] of ω and s which we refer to as the uninformative and the informative density respectively. We assume that both joint densities have the same support and further that the marginal distributions $f^s(s)$ of s and $f^\omega(\omega)$ of ω are the same in both cases. We let $f^U(s, \omega) = f^s(s) f^\omega(\omega)$ so that uninformative signals are independent of market states (hence the term "uninformative"). If the agent expends effort $a \in [0, 1]$ then the joint distribution will be the mixture distribution

$$a f^I(s, \omega) + (1 - a) f^U(s, \omega).$$

[8]It is important that the market is NOT complete in signal states. In models in which investors are allowed to trade across signal states the resulting equilibrium is generally fully revealing, which would destroy the basic structure which we are trying to capture in this model.

[9]The term "density" may be confusing to some. It is not really necessary that ω and s have densities with respect to Lebesgue measure. In particular, most of the results we demonstrate in this paper will still hold if ω and s are discrete random variables. In that case just reinterpret the integrals as being with respect to counting measure on some finite set of points.

The effort level a can be interpreted as the probability of getting a signal drawn from the informative joint density $f^I(s, \omega)$ instead of the uninformative density $f^U(s, \omega)$. We assume without loss of generality that under the informative density s and ω are positively correlated.

Let w_0 be the initial value of the portfolio and let U^P and U^A denote the principal's and agent's von Neumann-Morgenstern utility function for end-of-period wealth respectively. Let c denote the agent's cost function for costly effort, measured in utils, and defined on $[0, \overline{a})$ for some $\overline{a} \in (0, 1)$. The cost function c is strictly increasing and strictly convex and satisfies $c'(0) = 0$ and $c'(\overline{a}) = \infty$. The cost function and utility functions are assumed to be twice continuously differentiable, and the densities are assumed to be positive everywhere.

With the above setup the agency problem is to design a contract which gives the agent the incentive to expend effort (the amount of which is chosen by the principal) in information gathering. The contract should also give the agent incentive to use the information obtained in the best way (from the principal's standpoint). When people talk about such incentive contracts they usually have in mind something like the following:

- Agent chooses a portfolio strategy based on private information
- Principal assigns a performance score as a function of the portfolio return and perhaps a benchmark return as well
- Agent receives the fee associated with the performance measure as stipulated in the contract

But this is only one of many possible mechanisms one could imagine. There is no guarantee that a restriction to mechanisms of this type would not be binding and that the contract obtained would not be optimal in the space of all contracts.

Rather than assuming a particular mechanism we appeal to the revelation principle[10] and look at contracts with the property that the manager truthfully reports the signal. We shall use $\phi(s, \omega)$ to denote the fee paid to the agent given reported signal s and market state ω and $V(s, \omega)$ to denote the portfolio payoff to the principal. We make no restrictions on the form of $\phi(s, \omega)$ or $V(s, \omega)$. The revelation principle tells us that any mechanism

[10]See Green and Laffont (1977), Holmström (1978), and Myerson (1979) for discussions of the revelation principle.

which implements the optimal contract can be expressed in this form. After solving the general problem we shall return to the question of whether the optimal contract can be implemented by a mechanism of the type described above.

The general problem then is

Problem 1 *Choose $\phi(s,\omega)$, a, and $V(s,\omega)$, to maximize*

$$(1) \qquad \int\int U^P(V(s,\omega))(f^U(s,\omega)+a(f^I(s,\omega)-f^U(s,\omega)))dsd\omega$$

subject to the agent's participation constraint[11]

$$(2) \qquad \int\int U^A(\phi(s,\omega))(f^U(s,\omega)+a(f^I(s,\omega)-f^U(s,\omega)))dsd\omega - c(a) = u_0$$

the budget constraint

$$(3) \qquad (\forall s)\int_\Omega (V(s,\omega)+\phi(s,\omega))p(\omega)d\omega = w_0$$

and incentive compatibility, namely that $a' = a$ and $s^R(\cdot) = I(\cdot)$ maximize

$$(4) \qquad \int\int U^A(\phi(s^R(s),\omega))(f^U(s,\omega)+a'(f^I(s,\omega)-f^U(s,\omega)))dsd\omega - c(a')$$

where $s^R(s)$ is the signal which the manager reports when the true signal is s and $I(\cdot)$ denotes the identity function. Equation (4) states that of all reporting strategies the agent optimally chooses truthful signal reporting and also optimally chooses effort level a.

Note that (4) requires the contract to be incentive compatible in both effort and signal reporting. The constraint is needed if effort is not directly observable and if the agent could misreport the signal after having expended effort. We call a contract *First Best* if it assumes that neither of these is a problem. In a first-best world it can be verified by the principal whether the agent really expended effort a and truly reported the signal obtained. The first-best contract maximizes (1) subject to (3) and (2).

We will call a contract *Second Best* if truthful reporting is assumed not to be a problem. So a second-best contract need only be incentive compatible for effort.

We refer to the solution of the general problem as the *Third Best* contract.

[11] In principle, this could be an inequality constraint. This is without loss of generality when the utility function is unbounded below (as in our log utility examples). This could matter if there is a limit to how much an agent can be punished (as when there is no indentured servitude or debtor's prison). Obviously, these are interesting cases, but they are just not the focus of this paper.

A Transformed and Specialized Problem

It turns out that it will be convenient in what follows to change choice variables as in Grossman and Hart (1983). If we define

$$u^A(s,\omega) \equiv U^A(\phi(s,\omega))$$

$$u^P(s,\omega) \equiv U^P(V(s,\omega))$$

we can rewrite the above problem in an equivalent form in which the choice variables are $u^A(s,\omega)$ and $u^P(s,\omega)$. To write the budget constraint we must also define inverse utility functions I^A and I^P and then

$$\phi(s,\omega) = I^A(u^A(s,\omega))$$

and

$$V(s,\omega) = I^P(u^P(s,\omega)).$$

In order to obtain explicit solutions to these problems we assume that both principal and agent have log utility so that $I^A(x) = I^P(x) = \exp(x)$. One way in which this simplifies the problem above is that it allows us to reduce both the number of choice variables and the number of constraints. To do this we take the agent's utilities as given and maximize the objective subject to the budget constraint to obtain the principal's indirect utility as

$$(5) \qquad u^P(s,\omega) = \log\left(B^P(s)\frac{f^\omega(\omega) + a(f^I(\omega|s) - f^\omega(\omega))}{p(\omega)}\right)$$

where

$$B^P(s) = w_0 - \int \exp(u^A(s,\omega))p(\omega)d\omega$$

is the principal's budget share. Equation (5) can be taken to be an application of the usual formula for optimal consumption given log utility and complete markets (in this case conditional on s).

The principal's expected utility can now be computed as

$$(6) \qquad \int \log\left(w_0 - \int \exp(u^A(s,\omega))p(\omega)d\omega\right) f^s(s)ds$$

$$+ \int\int \log\left(\frac{af^I(\omega|s) + (1-a)f^\omega(\omega)}{p(\omega)}\right)(af^I(s,\omega) + (1-a)f^U(s,\omega))dsd\omega$$

Note that the second term depends only on effort, a, and not on the agent's utilities. Note also that the first term is concave in the agent's utilities.[12]

The difficulty with solving for the second-best and third-best contracts is the incentive compatibility constraint. We can simplify this somewhat by examining the agent's problem

Problem 2 *Choose a and $s^R(\cdot)$ to maximize*

$$\int\int u^A(s^R(s),\omega)(f^U(s,\omega) + a'(f^I(s,\omega) - f^U(s,\omega)))dsd\omega - c(a'). \tag{7}$$

The first order conditions of this problem are

$$\int\int u^A(s^R(s),\omega)(f^I(s,\omega) - f^U(s,\omega))dsd\omega = c'(a) \tag{8}$$

and

$$(\forall s)\int \frac{\partial u^A(s^R(s),\omega)}{\partial s^R}(f^U(s,\omega) + a(f^I(s,\omega) - f^U(s,\omega)))d\omega = 0 \tag{9}$$

Since these must hold for any solution to this problem we may be able to use them in place of the incentive compatibility constraint. This approach of using the first order conditions for the agent's problem in place of the incentive compatibility constraints in the main problem is called the *first order approach*. The difficulty with this approach is that in general it is not equivalent to solving the original problem. However in the second-best case it is justified. Note that $f^U(s,\omega)+a(f^I(s,\omega)-f^U(s,\omega))$ is linear in a and recall that we have assumed that $c(a)$ is convex. This means that (7) is concave in a. Hence (8) is both necessary and sufficient for the agent's choice-of-effort problem. Hence we are justified in using (8) as the IC constraint in the second-best problem[13]. Unfortunately we have no such result for the truth telling constraint. Nevertheless we shall use (9) as the second IC constraint in the third best problem.

These changes lead to the following problem

[12]This means we can ignore the second term when solving the problem of what contract will implement a particular effort level. Afterward in optimizing over effort levels this term will of course be necessary.

[13]Rogerson (1985) attributes Holmström (1984) with pointing out the appeal of the mixture model over the more complex convexity condition of Mirrlees (1976) studied by Rogerson (1985). See also Grossman and Hart (1983) and Hart and Holmström (1987).

Problem 3 *Choose $u^A(s,\omega)$ and a to maximize*

$$(10)\quad \int \log\left(w_0 - \int \exp(u^A(s,\omega))p(\omega)d\omega\right) f^s(s)ds$$

$$+\int\int \log\left(\frac{af^I(\omega|s) + (1-a)f^\omega(\omega)}{p(\omega)}\right)(af^I(s,\omega) + (1-a)f^U(s,\omega))dsd\omega$$

subject to the participation constraint

$$(11)\quad \int\int u^A(s,\omega)(af^I(\omega|s) + (1-a)f^\omega(\omega))f^s(s)d\omega ds - c(a) = u_0,$$

the incentive compatibility of effort

$$(12)\quad \int\int u^A(\omega,s)(f^I(\omega|s) - f^\omega(\omega))f^s(s)d\omega ds = c'(a),$$

and the incentive compatibility of signal reporting

$$(13)\quad (\forall s)\int \frac{\partial u^A(\omega,s)}{\partial s}(af^I(\omega|s) + (1-a)f^\omega(\omega))d\omega = 0.$$

Solution of this problem can proceed in two stages. In the first stage we solve the above problem for a fixed a to find the contract which will induce the agent to take that effort level. Then we can search over a to find the optimal effort level. Below we shall show the solution to the first stage for the first-best, second-best, and third-best cases.

3 The First-best Contract

The first-best contract solves Problem 3 neglecting the incentive compatibility constraints of equations (12) and (13). In a first-best contract we expect to find that there is optimal risk sharing between the agent and the principal. This means that the marginal utility of wealth for the agent should be proportional to the principal's marginal utility in all states.

The first order condition for u^A is

$$(14)\quad \frac{\exp(u^A(s,\omega))p(\omega)}{B^P(s)} = \lambda_R(f^\omega(\omega) + a(f^I(\omega|s) - f^\omega(\omega)))$$

where λ_R is the Lagrange multiplier on the participation constraint. Multiplying both sides by $B^P(s)$ and integrating both sides with respect to ω we obtain

$$B^A(s) = \lambda_R B^P(s).$$

Substituting this into the budget constraint we have that

$$B^P(s) = \frac{w_0}{1+\lambda_R}$$

from which we obtain

$$u^A(s,\omega) = \log\left(\frac{w_0\lambda_R}{(1+\lambda_R)}\frac{f^\omega(\omega) + a(f^I(\omega|s) - f^\omega(\omega))}{p(\omega)}\right) \tag{15}$$

and

$$u^P(s,\omega) = \log\left(\frac{w_0}{(1+\lambda_R)}\frac{f^\omega(\omega) + a(f^I(\omega|s) - f^\omega(\omega))}{p(\omega)}\right). \tag{16}$$

So in the first-best contract with log utility the optimal contract is a sharing rule which gives the agent a fixed proportion of the payoff of the portfolio independent of the signal. So, as expected, optimal risk sharing obtains.

Equation (16) has an interesting interpretation. Suppose that rather than hiring the agent the principal were to manage the portfolio but without the benefit of the information gathering efforts of the manager. The payoff to the principal would be

$$w_0\frac{f^\omega(\omega)}{p(\omega)}.$$

Since this portfolio involves no superior information and because it would be the principal's optimal portfolio in the absence of the agent we call it the "benchmark" portfolio. Similarly the quantity $\frac{f^I(\omega|s)}{p(\omega)}$ can also be interpreted as the gross return on a portfolio which depends on the agent's signal. Hence, in the first-best world both the principal and the agent receive a payoff which is equal the payoff of an investment of a certain amount in the benchmark portfolio plus a constant times the excess return of some "informed portfolio" over the benchmark.

4 The Second-best Contract

To obtain the second-best contract we solve the same problem as in the first-best case with the additional constraint (12) which states that the contract is incentive compatible for effort. The first-order condition for $u^A(s,\omega)$ is

$$(17) \quad \frac{\exp(u^A(s,\omega))p(\omega)}{B^P(s)} = \lambda_R(f^\omega(\omega) + a(f^I(\omega|s) - f^\omega(\omega))) + \lambda_a(f^I(\omega|s) - f^\omega(\omega))$$

where λ_a is the Lagrange multiplier on the IC constraint. Proceeding as before we obtain

$$(18) \quad u^A(s,\omega) = \log\left(\frac{w_0\lambda_R}{(1+\lambda_R)}\frac{f^\omega(\omega) + (a+\frac{\lambda_a}{\lambda_R})(f^I(\omega|s) - f^\omega(\omega))}{p(\omega)}\right)$$

but the expression for the principal's utility is the same as in the first-best case. Hence we see that the second-best contract does not exhibit optimal risk sharing.

Equation (18) is very similar to (15) except that we have $a+\lambda_a/\lambda_R$ rather than a multiplying the second term. We know λ_R will be positive because of the positive marginal utility of wealth. At the optimal a we will have λ_a positive as well. This means that the second-best contract gives the agent more exposure to the excess return portion of the contract. This is what gives the manager the incentive to expend effort a. It seems crucial that the manager cannot misreport the signal; if the manager could do so, reporting a less extreme signal is likely going to undo (to some extent) this artificially high exposure and it may not be possible for the principal to impose useful incentives in the contract beyond what is optimal effort in the agent's own portfolio.

There is another difference between this contract and the first-best contract which is not apparent at first glance. Notice that the first best contract is defined for all $a \in [0,1]$. In general, the optimal a can lie anywhere in that interval[14] but the contract will be of the same form. For the second-best contract, we must have $a + \lambda_a/\lambda_R \in [0,1]$. To see why, let us rewrite the agent's payoff from (18) as

$$\frac{w_0\lambda_R}{(1+\lambda_R)}\frac{(1-k)f^\omega(\omega) + kf^I(\omega|s)}{p(\omega)}$$

[14]Which level of effort is optimal for the problem will depend on c,f^I,f^U, and p.

where $k = a + \lambda_a/\lambda_R$. If $k > 1$ then the agent's consumption will be negative in states for which $f^\omega(\omega) > f^I(\omega|s)$. These states represent times in which a low market state occurs when the agent's signal would have predicted a high market state or the converse. But log utility goes to minus infinity at zero and therefore negative consumption with a positive probability is more than a large punishment and is in fact infeasible.

As $a + \lambda_a/\lambda_R$ increases (varying λ_R to maintain the participation constraint), the corresponding value of $c'(a)$ that will satisfy (12) increases. Denote by $\overline{c'}$ the value of $c'(a)$ which satisfies (12) for $a + \lambda_a/\lambda_R = 1$, i.e.

$$\overline{c'} \equiv \int\int \log\left(\frac{w_0\lambda_R}{(1+\lambda_R)}\frac{f^I(\omega|s)}{p(\omega)}\right)(f^I(\omega|s) - f^\omega(\omega))f^s(s)d\omega ds$$

For effort levels which do not satisfy $c'(a) \leq \overline{c'}$ the second-best can be approached by a sequence of contracts that look very much like the contract with $a + \lambda_a/\lambda_R = 1$ but with additional punishment in remotely possible states to obtain incentive compatible effort. Specifically, we have the following theorem.

Theorem 1 *When $c'(a) \leq \overline{c'}$ the solution of the second-best problem is of the form (18). For $c'(a) > \overline{c'}$ the second-best can be achieved in the limit as $n \uparrow \infty$ of a sequence of contracts of the form*[15]

$$(19) \quad u^A_n(\omega, s) \equiv \nu_{0n} + \log\left(\frac{f^I(\omega|s)}{p(\omega)}\right) - \nu_{1n}(\omega < -n)(s > n)$$

for n sufficiently large, where ν_{1n} is chosen to satisfy incentive compatibility of effort (12) and ν_{0n} is chosen to satisfy the participation constraint (11).

Proof See appendix ∎

We call this a forcing contract but it differs from what is usually meant by this term. Mirrlees (1974) gave conditions under which the first-best solution to an agency problem can be approached by a sequence of contracts with larger and larger punishments in a smaller and smaller set of extreme

[15]By $(s > n)$ we mean the indicator of that condition, i.e.,

$$(s > n) \equiv \begin{cases} 1 & s > n \\ 0 & \text{otherwise} \end{cases}.$$

states. Mirlees' forcing contract works when likelihood ratios become unbounded in extreme states. Note that the mixing form of the densities we have assumed precludes this happening: for two interior values of a, the ratio of their likelihoods is always bounded uniformly across ω. For this case the punishments are chosen to maintain incentives so that each element in the sequence is feasible. The limiting contract gives the principal at least as much value as any feasible contract but is not itself feasible because it provides too little incentive for effort and gives too much utility to the agent. This can be interpreted as an ϵ-equilibrium because you can get arbitrarily close to the second-best with some element along the sequence.

5 The Third-best Contract

The third-best solves Problem 3 with all the constraints. The first order condition for u^A is

$$\begin{aligned}(20)\quad \frac{\exp(u^A(s,\omega))p(\omega)}{B^P(s)} &= \lambda_R(f^\omega(\omega) + a(f^I(\omega|s) - f^\omega(\omega))) \\ &\quad + \lambda_a(f^I(\omega|s) - f^\omega(\omega)) - a\lambda_s(s)\frac{\partial f^I(\omega|s)}{\partial s} \\ &\quad - \lambda_s'(s)(af^I(\omega|s) + (1-a)f^\omega(\omega))\end{aligned}$$

where $\lambda_s(s)$ is the Lagrange multiplier on the truthful reporting constraint. In this case we have

$$B^P(s) = \frac{w_0}{1 + \lambda_R - \frac{\lambda_s'(s)}{f^s(s)}}$$

and

$$B^A(s) = \frac{w_0(\lambda_R - \frac{\lambda_s'(s)}{f^s(s)})}{1 + \lambda_R - \frac{\lambda_s'(s)}{f^s(s)}}.$$

It does not seem possible to solve for $\lambda_s(s)$ analytically. We can, however, gain insight into this problem by solving the problem numerically for given choices of c, f^I, f^U, and p. For the moment we shall take the level of effort a to be given exogenously. The reason for this is that we wish to compare the functional forms of the first-best, second-best, and third-best contracts.

Choosing a optimally in each case would make this comparison more difficult since in general a different level of effort will be optimal in a first-best world versus a second-best world versus a third-best world. By specifying a exogenously we can interpret the differences between the contracts as being due only to the addition of the IC constraints.

For the "informed" and "uninformed" joint density of ω and s, we assume joint normality with the same marginals and with and without correlation $\rho > 0$. We think of this as a model of market timing, with ω representing the demeaned log market return in the usual lognormal model over one year. Then,

$$(21)\quad f^s(s) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{s^2}{2\sigma^2}\right),$$

$$(22)\quad f^\omega(\omega) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{\omega^2}{2\sigma^2}\right),$$

$$(23)\quad f^I(\omega, s) = \frac{1}{2\sigma^2\pi\sqrt{1-\rho^2}} \exp\left(-\frac{(\omega^2 - 2\rho\omega s + s^2)}{2\sigma^2(1-\rho^2)}\right),$$

and

$$(24)\quad f^I(\omega|s) = \frac{1}{\sigma\sqrt{2\pi(1-\rho^2)}} \exp\left(-\frac{(\omega - \rho s)^2}{2\sigma^2(1-\rho^2)}\right).$$

And, state prices are consistent with Black-Scholes and can be computed as the discount factor times the risk-neutral probabilities.

$$(25)\quad p(\omega) = e^{-r}\frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(\omega + \mu - r)^2}{2\sigma^2}\right)$$

In these expressions, r is the riskfree rate, μ is the mean return on the market, σ is the standard deviation of the market return. Without loss of generality, the signal s has mean 0 and the same variance as the log of the market return.

We work with discretized versions of f^I, f^U, and p with N market states and M signal states. One approach to numerical solution of this problem would be to solve problem 3 directly as a constrained optimization problem in $N \times M$ variables (the agent's utility in each market and signal state). However this approach becomes quite difficult as N and M get large. Instead we adopt the following approach. Begin by choosing positive constants for the $M + 2$

Lagrange multipliers, λ_R, λ_a, $\lambda_s(s_1)$, $\ldots$, $\lambda_s(s_M)$. The first order condition (20) gives us the agent's utilities as a function of these Lagrange multipliers. This means that we can view the constraints (11),(12), and (13) as a system of $M+2$ equations in $M+2$ unknowns. This can be solved by standard equation solving routines except that we must impose non-negativity on the Lagrange multipliers. We accomplish this by letting the routine choose the exponential of the Lagrange multipliers. This converts the problem into a system of $M+2$ equations and $M+2$ unknowns with no constraints. The only additional complication is that in the first order condition (20) includes the derivative of $\lambda_s(s)$. We calculate this by a central difference approximation. Because of this we choose the initial values of $\lambda_s(s)$ to be some "smooth" function of the signal. For the first-best and second-best problems we can use the same basic technique with fewer choice variables and fewer equations to solve.

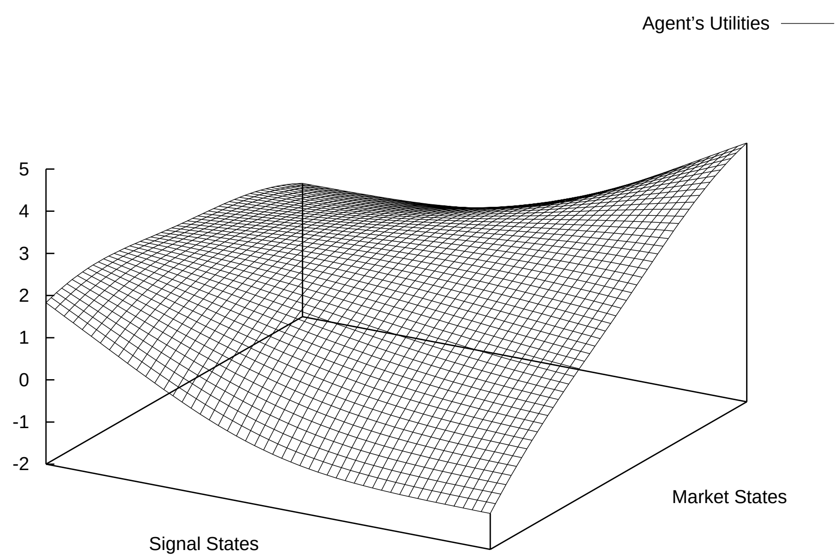

Figure 1: First-Best Contract

The agent's utilities from the first best problem are plotted in Figure 1. This is the same solution we obtained in (15) above. The parameters used are $\mu = 0.15$, $\sigma = 0.2$, $\rho = 0.5$, $r = 0.05$, and $w_0 = 100$. We chose a cost of effort function such that $u_0 - c(a) = 2$ and $c'(a) = 0.04$ at $a = 0.5$. This effort level was chosen because it corresponded to a smooth second-best contract.

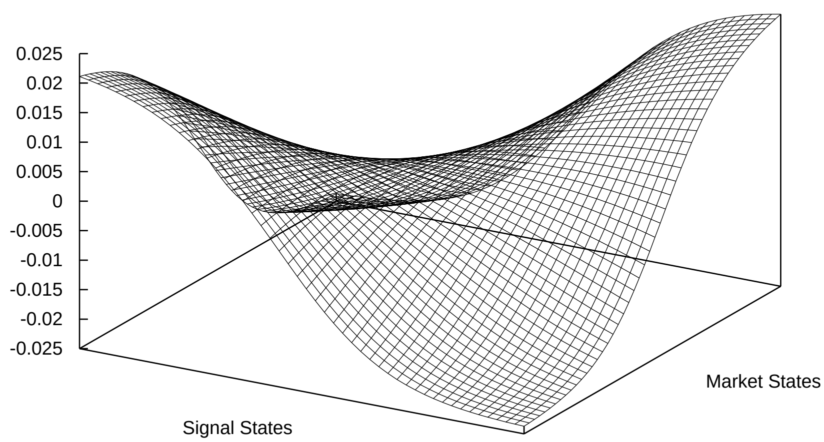

Figure 2: Second Best minus First Best Utility Levels

A visual inspection of the solution to the second and third-best contracts at these parameter values is not very instructive. However we can gain insight by examining how the contract changes when we move from first-best to second-best to third-best. Figure 2 plots the agent's utilities in the second-best minus the agent's utilities in the first-best. Notice that this gives exactly what we would expect given our analytical solution in equation (18). When signal and market are both high, $f^I(\omega|s) > f^\omega(\omega)$, so the agent is rewarded in those states. In the other corners of the distribution, the agent has less utility than in the first-best case. This provides the incentive to exert effort.

Figure 3 plots the agent's utilities in the third-best minus the agent's utilities in the second-best. The difference between these two contracts is that the third-best provides incentives to truthfully report. As noted earlier, an agent facing a second-best contract will tend to be overly conservative in re-

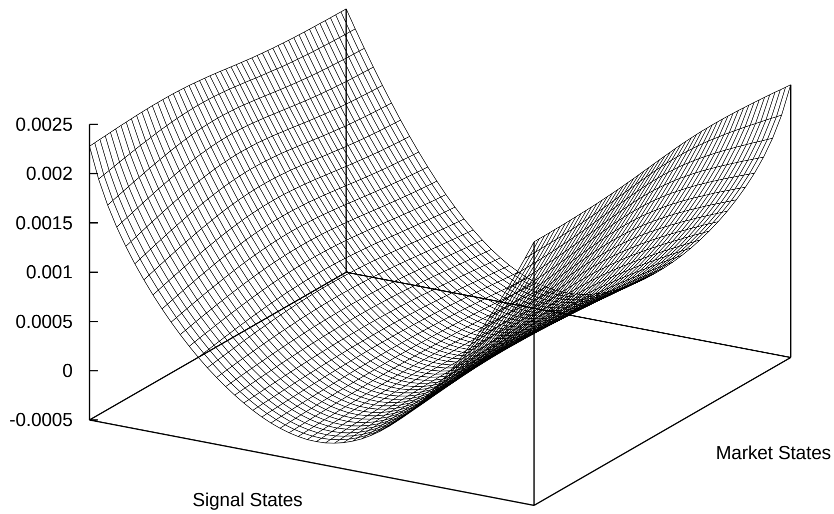

Figure 3: Third Best minus Second Best Utility Levels

porting to undo the extra exposure the second-best contract offers compared to the first best. Intuitively then we expect that the third-best contract will offer a reward for reporting more extreme signal states. This is very clearly the case. It appears in Figure 3 that this incentive is only a function of the signal state but an examination of the first order conditions for the problem show that this is not necessarily the case.

If we choose a higher value of $c'(a)$ we obtain a forcing contract as in Figure 4. Note the punishments in the corner states. Just as in the smooth case the second-best contract is not third-best. Because the punishments are concentrated in one state the agent would just misreport that state as being the one next to it. To make this contract third-best the punishment must be spread out across signal states in the extreme market states to maintain truthful reporting. This is the situation you see in figure 5. This figure corresponds to numerically solving the same problem as in Figure 4 but imposing the truthful reporting constraints.

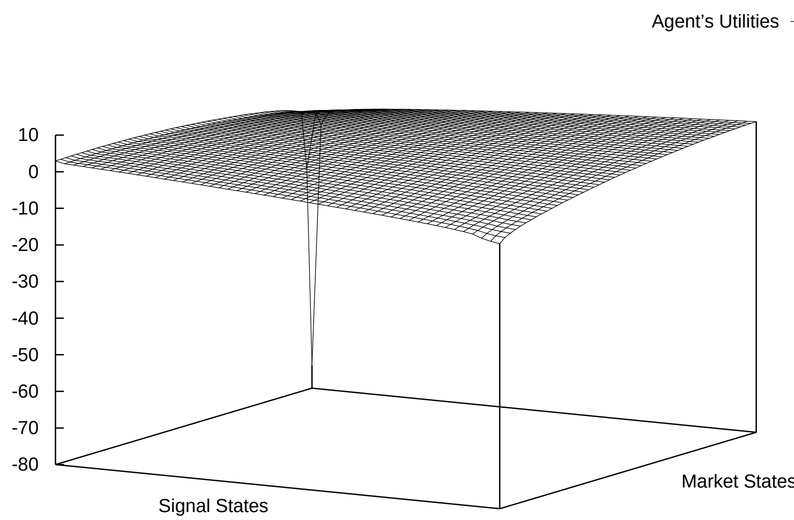

Figure 4: Second Best: Forcing Case

6 Institutional Structures

Performance Measurement

Now that we have addressed the solution of the optimal contracting problem we return to the issue of what mechanisms can achieve this optimum. In particular we are interested in whether the manager's compensation can be expressed as a function of a performance measure which depends only on the managed portfolio return and the return of some portfolio which does not depend on the signal. Given the form of the equations (15) and (18) it should not be surprising that this is indeed the case in the first-best and second-best cases.

Denote the gross return on the managed portfolio (including fees) by

$$R^P = \frac{\phi(s,\omega) + V(s,\omega)}{w_0}.$$

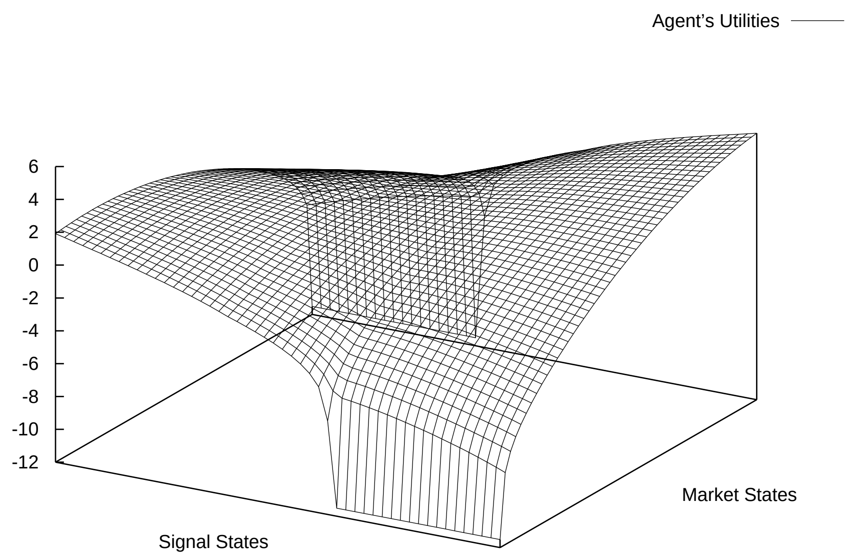

Figure 5: Third Best: Forcing Case

In the second-best case this is given by

$$R^P = \frac{f^\omega(\omega)}{p(\omega)} + \left(a + \frac{B^P \lambda_a}{w_0}\right)\left(\frac{f^I(\omega|s)}{p(\omega)} - \frac{f^\omega(\omega)}{p(\omega)}\right).$$

The first-best case is the same but with $\lambda_a = 0$. Let $R^B = \frac{f^\omega(\omega)}{p(\omega)}$ denote the gross return on the benchmark portfolio mentioned earlier. Now we can write $\frac{f^I(\omega|s)}{p(\omega)}$ as a function of R^P and R^B such that the agent's fee can be written

$$\phi(s, \omega) = B^A \left(R^B + k(R^P - R^B)\right)$$

where

$$k = \frac{a + \frac{\lambda_a}{\lambda_R}}{a + \frac{B^P \lambda_a}{w_0}} = \frac{a + \frac{\lambda_a}{\lambda_R}}{a + \frac{\lambda_a}{1+\lambda_R}} \geq 1$$

This suggests the following intuitive decomposition into a performance measure

$$m = R^P - R^B$$

and a fee schedule which is increasing in the performance measure, i.e. $B^A(R^B + km)$. This decomposition is, of course, not unique. The fact that the performance measure is the excess return of the portfolio over a benchmark has intuitive appeal. Measuring performance relative to a benchmark is common practice in the portfolio management industry. Note that in the first-best case $k = 1$ so the fee is just a fraction, B^A, of the assets under management.

Portfolio Restrictions

From the discussion above one might assume that the performance measure and fee schedule suggested are sufficient to implement the optimal contract in the first-best and second-best case. In fact this is not true. It depends on the implementation of the portfolio strategy.

Note that in setting up the model we have not specified who implements the portfolio strategy. By appealing to the revelation principle and requiring truthful reporting we have side-stepped this issue. If the agent truthfully reports the signal then for modeling purposes it doesn't matter who actually chooses the portfolio. Both the agent and the principal know the signal and know what should be done. Therefore either the agent, the principal, or an impartial third party could be assigned this task (provided implementation is verifiable). Traditionally, agency models have had the principal choosing the contract subject to the agent's approval; we can think of this assumption as a device that will map out all efficient contracts as we vary the reservation utility level. Thus, our results should be applicable to any efficient bargaining mechanism.[16]

In previous sections we have derived $\phi(s,\omega)$ which, through the budget constraint, determines $V(s,\omega)$. For each signal state $\phi(s,\omega) + V(s,\omega)$ gives the terminal value of the portfolio (before fees) as a function of the market state. The set of all such functions is not the entire collection of payoff functions which are available to the agent; this set is much richer. Hence the optimal contract can be seen as a menu of payoff functions, one for each signal state. For each payoff function there is a portfolio strategy which will return that payoff. If the agent picks only from this menu then the performance measure and fee schedule above will give the appropriate payment to the manager in each state. If the manager is not constrained to choose from the menu then there is no guarantee that the performance measure or fee schedule will provide the correct incentives.

Therefore, to implement an optimal contract there must be a way to restrict the set of strategies available to the manager. Previous examinations of the performance evaluation question in the literature do not restrict how the manager achieves superior performance. Actual investment guidelines, on the other hand, are full of such restrictions. Common restrictions for asset allocation include restrictions on the universe of assets and ranges for proportions in the various assets; while common restrictions for management within an asset class are limitations on market capitalization or style (growth versus income) of stocks, credit ratings or durations of bonds, restrictions on use of derivatives, maximum allocations to a stock or industry, and increasingly portfolio risk measures such as duration, beta, or tracking error.

Although our model is consistent with the existence of portfolio restrictions it is not clear how the restrictions we see in practice are related to the restrictions arising from our models. Recall that in the second-best case the agent bears more risk than in the first-best. Hence, the agent's incentive is

[16]Mapping out the efficient contracts in this way might not be possible in a more general model for which agents have important information at the outset, for example about their own preferences.

to take on less risky positions. The third-best contract offers explicit rewards for taking risky positions. So in our model, the restrictions force the manager to choose some curvature (in each signal state, the payoff is a concave function of the market state); i.e., the manager must choose a contract that takes a stand about what the market will do and cannot shirk and undertake low effort—this is related to the point made in Stoughton (1993) and Admati and Pfleiderer (1997) point about the ineffectiveness of linear contracts given flexible portfolio choice. This nonlinearity to prevent a neutral prediction is related to a concern that practitioners have about "closet indexers" who collect a fee for active management but actually track the market very closely. In practice most portfolio restrictions seem designed to limit the amount of risk the manager can take on rather than to encourage the manager to take on risk. Plan sponsors usually try to limit closet indexing by looking at the track record and portfolio style rather than imposing *a priori* limits on the dynamic strategy.[17]

It seems like an interesting avenue for future research to see how to rationalize portfolio restrictions to limit risk as seen in practice. One natural way to do this would be to generalize our model to include career concerns: a manager may want to take lots of risk this period in order to have a chance of generating outstanding returns that will increase the manager's wages or money under management in the future. Another reason to limit risk is to guard against the possibility that the manager has an unreasonably high opinion of the quality of the information. This would make the most sense in a context in which the expenditure of effort can be measured another way, for example through the evaluation of security analysis performed in the manager's investment process.

7 Conclusion

We have proposed a new model of optimal contracting in the agency problem in delegated portfolio management. We have shown that in a first-best world with log utility the optimal contract is a linear sharing rule over a portfolio which is equal to a benchmark portfolio plus an excess return of a portfolio which depends on the agent's signal. In a second-best world the contract is

[17]Some practitioners seem to think that an active portfolio cannot have too many assets (by the usual textbook analysis of residual variance as a portfolio becomes more diversified), but this argument assumes no special information on the part of the manager.

of the same form except that the agent's payout is more tilted toward the excess return to give incentives to the agent to work hard. We have also shown numerical results for the third-best case, which is close to the second-best in the examples we have examined. These results have been demonstrated in the context of a realistic return model and the derived performance measurement criterion looks more like the simple benchmark comparisons used by practitioners than the more elaborate measures (such as the Jensen measure, Sharpe measure, or various marginal-utility weighted measures). In addition, the optimal contract includes restrictions on the set of permitted strategies and also includes prior communication of information. These institutional features are more similar to practice than other existing agency models in finance.

We have only just started to tap the potential of this framework to tell us about agency problems in portfolio management. Although some of the general results obviously extend to stock selection models as well as the market timing examples given in this paper, it will be interesting to see the exact form of the contracts for stock-pickers. Analyzing career concerns will be an interesting variant: in this case, the current client has to take as given the manager's incentives to demonstrate superior performance this period in order to attract new clients or achieve a larger wage next period. In this case, there is probably a limit to the extent to which the client can neutralize the impact of career concerns. On a related note, it seems reasonable to solve problems for which the manager's utility function (as well as consumption) is bounded below, given that the actual economy has restrictions on indentured servitude.

In the model, we have obtained a lot of mileage from assumptions that allow us to look at an equivalent formulation in which the manager simply reports information and does not actually manage the money. However, there are aspects of performance (such as quality of execution) that are not handled adequately in this way. It would be useful to have a fuller exploration of when the reporting formulation is equivalent and of what happens otherwise. Another ambitious extension would include explicitly the two levels of portfolio management we see in practice, with the separation of responsibilities for asset allocation across asset classes and management of subportfolios in each asset class. The ultimate beneficiaries have to create incentives for the overall manager to hire and compensate the asset class managers, and this could be modeled as a hierarchy of agency contracts.

Proof of Theorem 1

PROOF The text already explained why the second-best is achieved by a first-order solution of the form (18) when $c'(a) \leq \overline{c'}$. For $c'(a) > \overline{c'}$ let

$$(26)\quad M^* = c'(a) - \int\int \log\left(\frac{f^I(\omega|s)}{p(\omega)}\right)(f^I(\omega|s) - f^\omega(\omega))f^s(s)d\omega ds,$$

which is positive by definition of $\overline{c'}$, and let

$$(27)\quad u^* \equiv \hat{u} - \int\int \log\left(\frac{f^I(\omega|s)}{p(\omega)}\right)(af^I(\omega|s) + (1-a)f^\omega(\omega))f^s(s)d\omega ds$$

where $\hat{u} = u_0 + c(a)$. Substituting (19) into (12) and (11) and invoking these definitions, we find that

$$(28)\quad \nu_{1n} = \frac{M^*}{-\int_{\omega<-n}\int_{s>n}(f^I(\omega|s) - f^\omega(\omega))f^s(s)d\omega ds}$$

and

$$(29)\quad \nu_{0n} = u^* + M^* \frac{\int_{\omega<-n}\int_{s>n}(af^I(\omega|s) + (1-a)f^\omega(\omega))f^s(s)d\omega ds}{-\int_{\omega<-n}\int_{s>n}(f^I(\omega|s) - f^\omega(\omega))f^s(s)d\omega ds}$$
$$\stackrel{n\uparrow\infty}{\longrightarrow} u^* + M^*(1-a),$$

since $f^I(\omega|s)/f^\omega(\omega) \to_{n\uparrow\infty} 0$ uniformly for $\omega < -n$ and $s > n$. Now consider the pointwise limit of $u_n^A(\omega, s)$, which is

$$(30)\quad u_*^A(\omega, s) \equiv u^* + M^*(1-a) + \log\left(\frac{f^I(\omega|s)}{p(\omega)}\right).$$

It is not hard to show that there is uniform convergence of the corresponding consumption $\exp(u_n^A(\omega, s)) \to \exp(u_*^A(\omega, s))$ (since the ν_1 term reduces consumption towards 0 only in states for which consumption was already very small), and consequently principal's utility also converges to that corresponding to $u_*^A(\omega, s)$ as $n \uparrow \infty$. Note that u_*^A itself is not feasible: it gives too little incentive for effort and too much utility to the agent.

We have left to show that u_*^A, which has the same value for the principal as the limit of the sequence u_n^A, gives the principal at least as much value as any feasible u^A. Let $Eu^P(u^A)$ represent the principal's expected utility as computed by (5) given an agent's utility pattern u^A. Then, we can use concavity of Eu^P to bound the value for any feasible u^A by taking the derivative at u^A_*:

$$
\begin{aligned}
(31)\quad & Eu^P(u^A) \\
&= \int \log\left(w_0 - \int p(\omega)\exp(u^A(\omega,s))d\omega\right) f^s(s)ds \\
&\leq \int \log\left(w_0 - \int p(\omega)\exp(u^A_*(\omega,s))d\omega\right) f^s(s)ds \\
&- \int\int \frac{p(\omega)\exp(u^A_*(\omega,s))}{w_0 - \int p(\omega)\exp(u^A_*(\omega,s))d\omega}(u^A(\omega,s) - u^A_*(\omega,s))f^s(s)d\omega ds \\
&= Eu^p(u^A_*) - \frac{\exp(u^* + M^*(1-a))}{w_0 - \exp(u^* + M^*(1-a))} \\
&\quad \times \int\int (u^A(\omega,s) - u^A_*(\omega,s))f^I(\omega|s)f^s(s)d\omega ds
\end{aligned}
$$

We are done if we can show the integral in the last right-hand expression is zero. Now (11), (27), (30), and the fact that densities integrate to 1 imply that

$$(32)\quad \int\int (u^A(\omega,s) - u^A_*(\omega,s))(af^I(\omega|s) + (1-a)f^\omega(\omega))f^s(s)d\omega ds = -M^*(1-a)$$

and (12), (26), (30), and the fact that densities integrate to 1 imply that

$$(33)\quad \int\int (u^A(\omega,s) - u^A_*(\omega,s))(f^I(\omega|s) - f^\omega(\omega))f^s(s)d\omega ds = M^*.$$

Adding (32) and (1-a) times (33), we get

$$(34)\quad \int\int (u^A(\omega,s) - u^A_*(\omega,s))f^I(\omega|s)f^s(s)d\omega ds = 0,$$

as was required to complete the proof. ∎

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