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Abstract

Much has been made of the potential for hedging restrictions to reduce the value of executive stock options. We investigate this issue by comparing a rational utility-maximizing model that incorporates both hedging restrictions and an endogenous departure decision and a naive value-maximizing model with an exogenous departure rate. While researchers mainly use these kinds of models to compute option values, we also use the models to generate forecasts of observable variables, the size and the timing of the payoffs of exercised options, and the annual rate at which options are canceled. We show that the naive model provides just as good a description of actual exercise patterns of executives as the rational model in a sample of NYSE and AMEX firms. The more parsimonious naive model may, therefore, be better for the purpose of valuation. The naive incorporation of the exogenous departure rate in the standard American option model not only aligns predicted exercise and cancellation patterns with actual patterns, but also reduces option value by about a quarter.

1 Introduction

What is the value of the stock options that firms grant their executives from the viewpoint of the shareholders?[1] There is general agreement that it should be in shareholders' interest to link manager pay to stock price performance.[2] There is also evidence that a firm's stock price responds favorably to an announcement of the adoption of a stock or stock option manager compensation plan.[3] Whether option compensation is the best way for firms to inspire managers to perform their best, or the best way for cash-constrained firms to attract talented managers, the widespread use of option plans suggests that firms feel the benefits outweigh the costs. Nevertheless, accountants and economists are trying to quantify these costs.[4]

Much has been made of the potential for hedging restrictions to reduce the value of executive stock options.[5] We investigate this issue by comparing a rational utility-maximizing model that incorporates both hedging restrictions and an endogenous departure decision and a naive value-maximizing model with an exogenous departure rate.[6] We show that the naive model provides just as good a description of actual exercise patterns of executives as the rational model in a sample of 40 NYSE and AMEX firms. While the rational model may have more economic appeal, the more parsimonious naive model may be better for the purpose of valuing actual executive stock options.

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[1]See Lambert, Larcker, and Verrechia [1991] for valuation of options from the holder's perspective.

[2]The principal-agent literature does not, however, say that option-like sharing rules are necessarily best; the optimal shape of the sharing rule can be arbitrary. See Holmstrom and Hart [1987].

[3]See Bhagat, Brickley, and Lease [1985].

[4]FASB [1993] contains a proposal for option valuation. In December 1994, the FASB voted to require firms to disclose option value in footnotes to financial statements but not necessarily to deduct option cost from earnings.

[5]See, for example, Huddart [1994] and Kulatilaka and Marcus [1994].

[6]See Jennergren and Naslund [1993], Cuny and Jorion [1993] and Rubinstein [1994] for models of the effect of forfeiture on option value.

We start by establishing a general framework for valuing executive stock options from the viewpoint of the option writer which extends option pricing theory to the situation in which the option holder's ability to trade the option is restricted so that his exercise policy depends on non-market risks in addition to the stock price. Assuming that the non-market risks borne by the option writer are diversifiable, option valuation becomes a matter of determining the exercise policies of executives.

We consider two models of exercise policies, a rational utility-maximizing model, which improves upon the models of Huddart [1994] and Marcus and Kulatilaka [1994], and a naive value-maximizing model with an exogenous departure rate, based on the model of the Jennergren and Naslund [1993]. While researchers have mainly used these kinds of models to compute option values, we use the models to generate forecasts of observable variables, the size and the timing of the payoffs of exercised options, and the annual rate at which options are canceled. For example, the ordinary American option model predicts that at a representative firm, an option that is exercised at all will be exercised after 7.6 years when the stock price is 3.3 times the strike price, and on average, 3% of the options outstanding in any year will be canceled. Adding an 11% annual departure rate brings the average exercise time down to 5.8 years, the level of the stock price down to 2.6 times the strike price, and the cancellation rate up to 7%, which is consistent with the average values of these variables in our sample. The departure rate also brings the typical option value down from 39% to 29% of the grant date value of the underlying stock.[7]

We construct base case parametrizations of the models to best match average option exercise times, payoffs, and cancellation rates in the sample. Then we use the base case parametrizations to generate forecasts of the exercise variables at each of the forty firms that incorporate firm specific information. The two different models give remarkably

[7]Whereas Yermack [1995] reports that options represented about a third of the value of average CEO compensation in 1990 and 1991 based on their Black-Scholes value, our models suggest adjusting this fraction to one quarter–still quite a substantial component of CEO compensation.

similar forecasts under base case parametrizations, so we vary the extra parameters of the rational model in search of an improvement over the naive model. In no case does the utility-maximizing model outperform the more parsimonious naive model in explaining the cross-sectional variations in the exercise variables.

Our results suggest that once we extend standard option pricing theory with an exogenous departure rate, we gain little more by incorporating a preference-based decision process. One reason for this may be that executives have much more ability to hedge the option position than our model allows. For example, the executive can in reality short stocks that are highly correlated with his company's stock, and he can easily take short futures positions on a stock index to eliminate market risk. Another reason is that he may be more willing to hold the option, despite hedging restrictions, when he knows he has some control of the underlying asset process and the opportunity to learn information about the firm's prospects before the rest of the market.

Here is an outline of the paper. §2 provides institutional details of executive stock options. §3 discusses our approach to option valuation and presents the two alternative models. §4 compares the ability of the models to explain actual exercise behavior in a sample of NYSE and AMEX firms. The appendix offers a new interpretation of the FASB's valuation proposal.

2 Description of Executive Stock Options

The stock option gives the executive the right to buy shares of the company's stock at a predetermined strike price any time between the vesting date and the expiration date. Typically, the strike price is equal to the market price of the stock at the date the option is granted and the expiration date is ten years after the grant date. The options often vest according to a schedule. For example, 25% of the options in a given grant may vest one year from the grant date, an additional 25% may vest after two years, and so on, so that all options are vested after four years. The executive may not sell the options to another party, nor may he take the options with him if he leaves the firm. If he leaves the firm, he must either forfeit the options if they are not yet vested or if they are out of the money, or else he must exercise them before he leaves.

For tax purposes, options are either classified as qualified or nonqualified. When a nonqualified option is exercised, the difference between the market price of the stock and the strike price is taxable as ordinary income to the executive and tax-deductible for the firm. Any appreciation in the stock price from the exercise date to the date at which the executive sells the stock is taxed as a capital gain. If instead the option is tax-qualified, then the full amount of the difference between the sale price of the shares and the exercise price is treated as a capital gain. Various regulations have governed the grant and exercise of tax-qualified options over the past few decades.

Since 1981, tax-qualified options have been in the form of Incentive Stock Options (ISOs). In order to qualify for preferential tax treatment in §422 of the Internal Revenue Code, the executive may not exercise ISOs for more than $100,000 worth of stock valued at the grant date market price, and may not sell the acquired stock sooner than two years after the grant date and one year after the exercise date. In addition, the strike price must be no less than the market price at the grant date, and the options must expire within ten years of the grant date. Furthermore, the ISOs must be granted under a written option plan that is approved by shareholders and limits the total number of shares that may be optioned. Before 1986, ISOs had to be exercised in the order granted, and rather than a limit on the amount the executive could exercise in any year, there was a limit of $100,000 worth of underlying stock on the amount of these options a firm could grant in any year.

Until 1991, §16b-3 of the Securities Exchange Act prohibited insiders from selling shares they acquired through option exercise until six months after the exercise date. Since the SEC amendment in May 1991, insiders may sell shares acquired through option as early as six months after the grant date, which usually means they may sell the shares immediately after exercise.

Insiders must report the acquisition of shares through option exercises on Form 4 or Form 5. Since 1991, they must also report option grants on Form 4 or Form 5. When a firm adopts or amends a stock option plan, it must file a plan description on Form S-8. These forms sometimes give detailed descriptions of outstanding options. Firms must disclose the compensation of their top executives in their proxy statements, but descriptions of executive option holdings contained in proxy statements typically only include average exercise prices and ranges of expiration dates.

Under current accounting rules (Accounting Principles Board Opinion No. 25), a firm granting employee stock options recognizes compensation cost equal to the excess of the market price of the stock at the grant date over the exercise price. This amount is typically zero. If an employee exercises the option, his payment to the firm is credited to equity. The Financial Accounting Standards Board has been trying to develop new rules for recognizing stock option value in corporate financial statements since 1984. We discuss their latest valuation proposal in the appendix.

Another contract firms give their executives that is economically similar to a stock option is a stock appreciation right (SAR). The SAR pays the executive the difference between the market price of the stock at exercise and a predetermined strike price, which is usually equal to the stock price at the grant date. The payment may be in cash, shares, or a combination of the two. Often, SARs are issued in tandem with stock options, meaning that the exercise of one cancels the other. The payoff from a SAR is taxable as ordinary income to the executive and deductible to the firm. When a SAR is exercised, the firm must recognize compensation expense equal to the SAR payoff.

Executive stock options may contain other contractual features which distinguish them from ordinary stock options. For example, performance-based options only vest if the executive achieves a given performance goal, such as a target market share or stock price level. With a reload option, the executive may pay the exercise price with previously owned shares and then receive a new at-the-money option on the old shares just surrendered with the same expiration date as the old option.[8] Firms may also choose to reduce the strike prices of options that have become far out-of-the-money. Such a repricing may be viewed as the cancellation of the original out-of-the-money option and the grant of a new at-the-money option. Since it may be that the strike price reduction is just a mechanism for granting new options that the firm would have granted anyway, it is not obvious that the grant of such a new option takes place contingent on the outcome of the original option. On the other hand, even if the firm does not explicitly grant reload options or reduce the strikes of out-of-the-money options, grants of new options may still depend on the outcomes of old options.

3 Executive Stock Option Valuation

Executives cannot hedge their options because §16-c of the Securities Exchange Act prohibits firm insiders from selling their firm's stock short. Consequently, executives may not exercise their options according to a market-value maximizing strategy. In general, the random time τ at which the executive exercises or cancels the option may depend not only on the stock price path but also on non-market risks such as whether the executive suffers a liquidity shock or chooses to leave the firm. Therefore, it is not clear that a contingent claim approach is appropriate for valuing these options because the writer faces risks he cannot hedge with the underlying stock.

We assume that any risks that the option writer cannot hedge with the stock are idiosyncratic across different option holders. It is quite feasible to hold a diversified portfolio of short positions in executive stock options, just by holding a portfolio stocks, and in a diversified portfolio, these idiosyncratic risks become trivial. Therefore, in equilibrium, the option will be valued at its expected value with respect to idiosyncratic risks. On the other hand, a diversified portfolio of options will still be subject to stock

[8]See Arnason and Jagannathan (1994) for a valuation of reload options assuming a market-value-maximizing exercise policy.

market risks and therefore will not be worth its expected value but instead will be worth the same as a replicating portfolio of other assets. In other words, even though the option is not strictly a contingent claim on traded assets, the equilibrium option value is still $\mathrm{E}(\zeta_\tau(S_\tau - S_0)^+ 1_{\{\tau \geq t_v\}})$, where S is the stock price, t_v is the vesting date, and ζ is the usual pricing kernel.[9]

With this approach, valuing executive stock options becomes a matter of determining the exercise policies of executives. §3.1 and §3.2 present alternative models of option exercise policies.

3.1 Rational Model of the Executive's Exercise Policy

Consider an executive with non-hedgeable options as well as some outside wealth. The price of the stock underlying the option is a binomial multiplicative random walk[10] with a positive excess expected return, and the continuously compounded riskless rate, r, is constant. Every period there is some probability, q, that the executive will be in a departure state, that is, a state in which he contemplates leaving the firm. We model his reason for leaving as a monetary payoff, y, which he may decline in order to stay at the firm and leave his option alive. Stopping in the departure state may mean canceling the option or exercising when it is just barely in the money. The departure state is intended to capture the fact that executives do, indeed, sometimes cancel their options or else exercise them near the money for various idiosyncratic reasons. It is an essential element in any model of actual exercises.

The executive chooses a stopping rule to maximize his constant relative risk averse utility of terminal wealth. The state space is two binomial trees: at any time he is somewhere in the tree of stock prices, and, in addition, he is either in a departure state or not. In order to solve recursively backward from the option expiration date for the stopping rule, the executive's decision in any given state must depend only on

[9] See, for example, Harrison and Kreps [1980].

[10] See Cox, Ross, and Rubinstein[1979].

the prevailing level of the stock price and whether or not he is in a departure state, not on the past history of the stock price path. To acheive this level of simplicity, we assume that the executive exercises the options all at once, if at all. Then, because of the shape of the utility function, we may assume that the number of options is one, the initial stock price is one, and measure initial non-option wealth, x, as dollar wealth divided by the initial value of shares under option.

We also require that the executive invest outside wealth in a portfolio whose value is a path-independent function of the stock price.[11] In addition we must make an assumption about how the proceeds of an option exercise are invested. Investment in the riskless bond seems like a neutral choice,[12] but it tends to make the executive hold the option too long because the option is the only investment available to the executive that offers positive excess returns. In particular, when the stock pays dividends, there can be states in which the executive chooses to hold the option where a value-maximizing option holder would exercise it. This situation can arise even if we assume all outside wealth is invested in the stock. Indeed, if the executive is sufficiently risk-neutral, and the mean stock return is sufficiently higher than the riskless rate, he prefers to hold a levered portfolio and the option is his only means of doing so.[13]

We feel that this potential for late exercise is a limitation of investing the executives non-option wealth in the riskless asset. The option position should represent a contraint on his portfolio choice which exercising removes. A better choice is to invest the executive's outside wealth in the binomial version of the constant proportion Merton portfolio of the stock and bond that he would hold in the absence of the option, the noisy income y, and other risky assets. We are not claiming that this fund is fully optimal for the executive in the presence of the option and the noisy income source. It

[11]If we allowed the executive to choose exercise and investment strategies simultaneously, the non-negativity constraint on stock holdings might become binding along some stock price paths but not others, so the optimal portfolio value would not generally be path-independent.

[12]This is the choice made in Kulatilaka and Marcus [1994].

[13]Huddart [1994] avoids this problem by assuming the stock has zero risk premium.

is a tractable alternative to the riskless fund which never involves a short position in the stock and, in the absence of the noisy income, has the property that whenever it is optimal for the value-maximizer to exercise, it is also optimal for the utility-maximizer to exercise, because by exercising, not only does he increase the value of his assets, but he also moves to his best portfolio.

We determine the exercise rule working backwards from the expiration date to determine the highest utility value of the choices of the executive, with or without a payoff for leaving as the case may be. If he exercises or cancels and leaves, he gets the expected utility of total wealth invested to the expiration date in the binomial Merton portfolio. Note that the utility value of exercising in a non-departure state incorporates the possibility of receiving a payoff for leaving in the future. If the executive waits, then he gets the expected utility of moving into one of the four possible states the following date: up or down, and departure or non-departure.

The model abstracts from a number of aspects of the executive's situation that complicate the optimization problem. First, the option holder has some control over the underlying stock price process.[14] Next, the firm's decisions about the executive's future compensation mix may depend on the state of existing options and knowledge of this dependence may affect the executive's exercise policy. Finally, option holders may have private information about the future path of the stock price. There is no published evidence that option exercises by insiders are followed by significant abnormal returns,[15]but before 1991, the SEC's restriction on the resale of shares acquired through option may have made option exercise an ineffective way for insiders to act on private information. Now that the restriction has been lifted, it may be easier for insiders to incorporate private information in their exercise policies.

[14]For evidence that option-compensated managers increase asset variance and leverage and reduce dividends, see Agrawal and Mandelker [1987], Lambert, Lanen, and Larcker [1989], and DeFusco, Johnson, and Zorn [1990].

[15]See Seyhun [1992, footnote 20].

The rational model ignores taxes and therefore it ignores the distinction between qualified and non-qualified options. It also overlooks the slight dilution of option value that arises because the firm typically delivers newly issued shares at exercise.[16]

3.2 Naive Model of the Executive's Exercise Policy

The rational model requires unknown parameters such as a risk aversion coefficient and wealth level in addition to the all of the inputs that the Black-Scholes model requires. Consider a much simpler model in which the executive follows a value-maximizing policy given that each period there is some exogenous probability that he will leave the firm.[17] This "naive model" only requires one parameter more than the usual Black-Scholes inputs, the departure probability.

Just as in ordinary binomial American option models, we solve for the exercise policy by working backward from the expiration date, determining the exercise decision at each point. In the departure state, the executive automatically either exercises the option if it is vested and in-the-money, or else forfeits the option. The option value in that state is either its exercise value or zero. In the non-departure state, the executive only exercises the option if its exercise value exceeds the market value the option would have if he did not exercise it. The market value of the unexercised option is the discounted probability-weighted average of the four possible option values at the next date, where the probabilities of the different values are computed using the true probabilities of the departure and non-departure states, and the risk-neutral probabilities that the stock will rise or fall.

[16] See Noreen and Wolfson [1981].

[17] This is essentially a discrete-time version of the model of Jennergren and Naslund [1993].

3.3 Characteristics of the Exercise Policies

Under the rational model, with sufficiently high wealth levels, the executive tends to exercise the option in non-departure states only if the stock price is above some critical time-dependent level, an exercise pattern that is also characteristic of the value-maximizing policy.[18] Figures 1 and 2 depict exercise boundaries in the non-departure state for the naive model and various versions of the rational model.[19] To ease comparison with the naive model, we either set the departure probability q equal to zero, or else set the payoff for leaving very high, $y = 10$, so the executive always chooses to leave if faced with a departure decision.

Figure 1 demonstrates that in a utility-maximizing model, even if all the executive's outside wealth is invested in the stock, if he is sufficiently risk tolerant, there are states in which he will choose not to exercise when a value-maximizer would. Indeed, if the executive's risk aversion coefficient is A is 0.25, his exercise boundary lies above the exercise boundary of the naive model; points between the two boundaries represent states in which the executive deviates from a value-maximizing policy by waiting too long to exercise. The problem is worse when his wealth is invested in the bond. If the executive is more risk averse, $A = 2$, his exercise boundary in this model falls so low that he virtually exercises the option as soon as it gets in the money. With outside wealth invested in the Merton portfolio, the exercise boundaries for $A = 0.25$ and $A = 2$ are much closer together. The exercise policy is less sensitive to the level of risk aversion, because when we make the executive more risk averse, we simultaneously make his portfolio less risky.

Figure 2 demonstrates that as the level of the executive's outside wealth grows

[18]See Kim [1990].

[19]Since the models are discrete (here, decision dates are monthly), we cannot obtain a smooth, continuous boundary directly. To represent each boundary, we fit a quadratic through the points $\{(t, b(t)) : t = 2, 2\ 1/12, 2\ 2/12, ..., 10\}$ where $b(t)$ is the lowest stock price at time t in the binomial tree at which the executive exercises the option.

large, the exercise boundary for the utility-maximizing model tends to rise up to the exercise boundary for the value-maximizing model.

Consider the exercise decision in departure states under the utility-maximizing model when the payoff for leaving is an intermediate value. Before the vesting date, the executive tends to take the payoff only if the option is sufficiently out of the money. After the vesting date, he chooses to stay at the firm if the option is near the money, as shown in Figure 3. Note that once the option is vested, neither deep in-the-money nor deep out-of-the-money departures reduce option value much, because in those cases, the option exercise value is close to its unexercised value.

4 Empirical Study of Option Exercises

The rational model may have more theoretical appeal, but is it empirically better for valuing options than the more parsimonious naive model? We cannot answer this directly because executive stock option prices are not observable. Instead, we address the question of whether the rational model can explain actual exercise patterns better than the naive model. §4.1 describes our sample of option exercises from 40 different firms. §4.2 explains the calculation of forecasts of the observable variables implied by models. §4.3 selects base case parameters for the models. In §4.4, we judge the relative performance of two models.

4.1 Sample of Option Exercises and Cancellations

The main sample consists of average times to exercise, stock prices at the time of exercise, and vesting periods of ten-year nonqualified or incentive stock options for 40 firms on the NYSE or AMEX. We began with a collection of 70 firms for which we had option grant information that included specific grant dates and exercise prices. Nearly half of these come from a proprietary database of large firms constructed by Mark Vargus at Wharton. The database uses information from a variety of corporate filings including proxies, Forms 10-K and Forms S-8.[20] We augmented this database to include smaller firms. Starting with firms in the smallest size decile, we searched option plan prospectuses for explicit information about grant dates, strike prices, and vesting periods of ten-year options. For each firm we tried to select one option with a strike price that was distinct from all other options granted by that firm in the database and equal to the stock's market price on the grant date. If more than one option was available, we selected the last one expiring before the end of 1992. We then searched all option exercises filed with the SEC by insiders at that firm for exercises with a matching strike price, adjusted for stock splits and stock dividends.[21] If there were no exercises reported from that grant, then we selected the next earlier grant. If no grants before 1982 were available, we worked through grants from 1983 to September 1984.[22] We were able to find at-the-money option grants followed by at least one exercise for 40 firms.[23][24] These tend to be large manufacturing firms. Based on their market capitalization at the option grant date, 63% of the firms in the sample are in size deciles 8 through 10 and 25% are in deciles 5 through 7. 85% of the sample firms were in the Manufacturing Division of the Standard Industry Classification at

[20] For a description of this database, see Vargus [1994].

[21] Although it is possible that we might have selected option exercises from grants that were not in the database, we do not believe this is a serious problem. For only two firms did we find exercises with strikes that matched our option's split-adjusted strike price, but were not in the time range when our option's price was in effect. We eliminated these option grants.

[22] We preferred options that expired before the end of 1992, to avoid early exercises triggered by the anticipation of Clinton's tax increase. Tax cuts in 1981 and 1986 would seem to alter option exercise strategies only by delaying exercise from the time the cut is anticipated until it is enacted, whereas a tax hike might cause an exercise to occur several years before it would otherwise have taken place.

[23] For simplicity, we eliminated four firms that merged. In a merger, options of the acquired firm are often converted to options on the stock of the surviving firm in a way that attempts to preserve their economic value. We do not feel that excluding these cases causes any clear selection bias.

[24] One reason we were unable to find matching exercises for some of the grants may be that the options had tandem SARs which the insiders exercised instead.

their grant date, while this division contains only about half the firms in the CRSP database that were alive in 1982.

For each firm in the sample, we computed the average across exercises of the time to exercise and the ratio of the stock price at exercise to the strike price, weighted by the split-adjusted number of shares in the transaction. We also approximated a single vesting date for each grant in the following way. To each exercise we assigned a vesting date equal to the weighted average of the dates in the vesting schedule before the exercise date. For example, if an option was exercised two and a half years after the grant date and a quarter of the options in the grant vested in each year after the grant date, with full vesting after four years, then we assigned to this exercise a vesting date of one and a half years. We then took the average vesting date across exercises as the vesting date for the grant. In addition, we estimated stock return volatilities and dividend rates for each firm with monthly data from CRSP. Volatilities were estimated over the five years prior the grant date; dividend rates were estimated over the ten years from the grant date to the expiration date. We also obtained the stock price at the expiration date, normalized by the stock price at the grant date. Table 1 presents cross-sectional summary statistics for these data. For example, the options were exercised after an average of 5.83 years and the stock price at the time of exercise was 2.75 times the strike price at the time of exercise. The average volatility of firms in the sample was 31% and the average dividend rate was 3%.

We also constructed a sample of cancellation rates for 52 of the original 70 firms. We define the cancellation rate as the average fraction of outstanding options canceled per year. To measure the cancellation rate for a given firm, we took option inventories from annual reports and computed the average ratio of the number of options canceled to the sum of the number of options outstanding and half the number of options granted. In some cases, annual reports combined ten-year options and options with terms other than ten years in the same inventory, or they indicated that tandem stock appreciation rights were outstanding but did not make clear whether their exercise counted as an option exercise or cancellation. We included the firm in the sample if we could find at least three years of data that did not suffer from these problems. We generally used up to ten years of data for each firm, from 1984 to 1993. Four of the firms reduced the strike prices on their options; we treated this as a cancellation of the original option and a grant of a new option. The cancellation rates of the 52 firms range from 0.7% to 34.3% with a mean of 7.3%, a median of 4.5%, and a standard deviation of 7.1%.

4.2 Model Forecasts of Exercise and Cancellation Variables

The exercise policy prescribed by a given model implies mean values of the exercise variables–the level of the normalized stock price at the time of exercise, s_τ, and the time of exercise, τ. These forecasts are

$$\begin{aligned} \hat{s}_\tau &= \mathrm{E}(s_\tau | s_\tau > 1, t_v, r, \mu, \sigma, \delta, s_{10}, \theta \\ )\hat{\tau} &= \mathrm{E}(\tau | s_\tau > 1, t_v, r, \mu, \sigma, \delta, s_{10}, \theta) \ , \end{aligned}$$

where μ and σ are the mean and volatility of the stock return, δ is the dividend rate, and θ is the set of unknown parameters, either $\{A, x, y, q\}$ for the rational model, or q for the naive model. We emphasize that these predictions condition on the fact that the option is exercised and also account for the overall performance of the stock over the ten years from grant to expiration. Thus, these are the average of the outcomes of these variables across all stock price paths that result in an exercise at all and terminate at the level s_{10}, weighted by their conditional probability.

A given model also implies an average value for cancellation rate, cr, at the firm under the assumption that the firm grants an equal number of options to identical executives every year, each following the prescribed exercise policy:

$$\hat{cr} = \mathrm{E}(cr | t_v, r, \mu, \sigma, \delta, \theta) \ .$$

Thus, $\hat{cr}$ is the average ratio of the number of options exercised during a year to the number of options outstanding at the beginning of that year, taking into account the unconditional distribution of the ages of the options still outstanding in any year, and the likelihood of a cancellation given that age.

4.3 Base Case Parameter Selection

Our goal in this section is to choose values of the unknown parameters θ to serve as a starting point for our comparison of the rational and naive models in §4.4. These base case parametrizations also allow us to compute option values that are consistent with actual exercise patterns. We emphasize that we are not attempting to judge the relative performance of the two models at this point.

If executive stock option prices were directly observable, then we would choose values of the unknown model parameters to make model option prices best match observed prices, since valuation is our primary purpose. In the absence of observable option prices, we instead parametrize the model to match observable features of the exercise policy: the level of the stock price at exercise, time of exercise, and cancellation rate. Fortunately, these variables relate directly to the size and timing of the option payoff when it is positive, and the chance of a zero payoff, so they are fundamental to option value.

To speed computation time, we construct a representative firm whose vesting date, stock price return volatility, dividend rate, and terminal stock price are equal to the sample average values. We set the riskless rate equal to 7%, roughly the average Treasury Bill rate over the time the options were alive. We set the mean stock return equal to 15.5%, the sum of the average riskless rate over the time the options were alive and the average equity premium from 1926 to 1975. For the rational model, we fix $A = 2$, since the model is relatively insensitive to the risk aversion coefficient with outside wealth in the Merton portfolio. We then choose values for x, y, and q in the rational model, and q in the naive model to minimize

$$\frac{(\bar{s_\tau} - \hat{s}_{\tau,0})^2}{\mathrm{S}^2(s_\tau)/40} + \frac{(\bar{\tau} - \hat{\tau}_0)^2}{\mathrm{S}^2(\tau)/40} + \frac{(\bar{cr} - \hat{cr}_0)^2}{\mathrm{S}^2(cr)/52} \tag{1}$$

where $\bar{s_\tau}$, $\bar{\tau}$, and $\bar{cr}$ are, respectively, the sample averages of the normalized stock price at exercise, time of exercise, and cancellation rate, $S^2(s_\tau), S^2(\tau)$ and $S^2(cr)$ are their respective sample variances, and $\hat{s}_{\tau,0}$, $\hat{\tau}_0$, and $\hat{cr}_0$ are their respective mean values for the representative firm according to the model:

$$\begin{aligned}
\hat{s}_{\tau,0} &= \mathrm{E}(s_\tau | s_\tau > 1, t_v = \bar{t_v}, r = .07, \mu = .155, \sigma = \bar{\sigma}, \delta = \bar{\delta}, s_{10} = \bar{s_{10}}) \\
\hat{\tau}_0 &= \mathrm{E}(\tau | s_\tau > 1, t_v = \bar{t_v}, r = .07, \mu = .155, \sigma = \bar{\sigma}, \delta = \bar{\delta}, s_{10} = \bar{s_{10}}) \\
\hat{cr}_0 &= \mathrm{E}(cr | t_v = \bar{t_v}, r = .07, \mu = .155, \sigma = \bar{\sigma}, \delta = \bar{\delta})
\end{aligned}$$

All forecasts are generated assuming a monthly stock price tree and annual decision dates.

Although we have not attempted a formal estimation of the parameters, the objective function (1) would be the appropriate minimand in a generalized method of moments estimation[25] if all the sample firms were identical to the representative firm, the exercise variables were uncorrelated, and the stock return parameters were known with certainty. It is worth noting that the exercise variables, the stock price at exercise and the time of exercise, do not appear to be highly correlated. Their sample correlation coefficient is only 0.14.

Table 2 contains the parameter values resulting from this procedure as well as model forecasts of the exercise and cancellation variables and option values for the representative firm. For reference, the first row contains the sample average values of the observable variables. The second row contains model values for an ordinary transferable option that is European to the vesting date, and American thereafter. This is just the naive model with the departure rate set to zero. Note that under this standard exercise policy, options would be exercised much deeper in-the-money and much later and would have much lower cancellation rates than is typical in our sample. Under this model, the option would be worth \$0.39 when the initial stock price is \$1.

The third row in Table 2 illustrates that simply introducing a departure rate of

[25] See Hansen 1982.

11% in the value-maximizing model brings the fitted values of the observable variables much closer to the average actual values. It also substantially reduces option value. Under this calibration of the naive model, option value is only \$0.29.

For the rational model, the best fitting values of x and y are quite high to make the exercise variables and cancellation rate high enough to match the sample average values. The payoff for leaving is so high that the executive always takes it so the departure decision is independent of the stock price. This base case rational model in the fourth row of Table 2 is almost identical to the base case naive model.

Again, when the model is parametrized to match actual exercise patterns, the typical option is worth only three quarters of its fully tradeable, American call option model value. It is also worth about three quarters of the dividend-adjusted Black-Scholes value of \$0.37. Thus, while Yermack [1995] reports that options represented about a third of the value of average CEO compensation in 1990 and 1991 based on their Black-Scholes value, our models suggest adjusting this fraction to one quarter–still quite a substantial component of CEO compensation.

Firm-specific forecasts of the exercise variables in §4.4 reveal that the base case parametrizations of the naive and rational models are almost indistinguishable. Therefore, we explore a variety of other parametrizations of the rational model that we will pit against the naive model to try to detect the potential for improvement. For completeness, we present the characteristics of the representative option with these additional choices of θ in the remaining rows of Table 2. Rows 5 through 7 force x to take ever smaller values with y fixed at 10 and q chosen to minimize (1). Row 8 fixes $y = 0.15$, small enough to introduce a dependency in the departure decision, and optimizes over x and q.

Row 9 forces $y = q = 0$ and optimizes over x, so that deviations from the standard option exercise policy result solely from the hedging restrictions, with no risk of departure. Note that while $x = 3.00$ makes the forecasts of the exercise variables close to the sample averages, the cancellation rate, 0.034, is much lower than the sample average of 0.073. Comparing this calibration to the fitted calibration in Row 4 shows that two models can agree on forecasts of the exercise variables, but if their cancellation rates differ substantially, so may the option values. While the option value under the fitted model is \$0.29, it is \$0.39, a full third greater, with y and q fixed to zero. Clearly, the cancellation rate of options is an important determinant of their value. The last row of Table 2 optimizes over x and y holding q fixed at 0.2.

The models also provide the inputs necessary to implement the option valuation method proposed by the FASB. Values in the last column, *FASB value*, are equal to the probability that the option vests times the Black-Scholes option value adjusted for proportional dividends with the expiration date set equal to the option's expected life conditional on vesting. The appendix shows that this value is like the expected Black-Scholes option value across possible "expiration" dates or stopping times, assuming the distribution of the stopping time is independent of the stock price. Note that in general, this value is quite close to the correct option value under the model, *ESO value*. The FASB values are slightly less, suggesting that exercise times under both models are not independent of the stock price, but rather, depend on the stock price in a way that increases the option value.

4.4 Comparison of Rational and Naive Model Forecasts

In this section, we pit, in turn, each of the parametrizations of the rational model from the last section against the base case parametrization of the naive model. Given a model parametrization θ, we generate forecasts for each of the 40 firms that incorporate specific information about each firm: volatility, dividend rate, and terminal stock price. In particular, the forecasted level of the stock price at exercise and the time of exercise for firm i, are

$$\begin{aligned}\hat{s}_{\tau,i} &= \mathrm{E}(s_\tau | s_\tau > 1, t_v = t_{v,i}, r = .07, \mu = .155, \sigma = \sigma_i, \delta = \delta_i, s_{10} = s_{10,i}, \theta) \\ \hat{\tau}_i &= \mathrm{E}(\tau | s_\tau > 1, t_v = t_{v,i}, r = .07, \mu = .155, \sigma = \sigma_i, \delta = \delta_i, s_{10} = s_{10,i}, \theta)\ .\end{aligned}$$

Even before looking at the actual values of the exercise variables, we examine scatter plots of the rational and naive forecasts to see if the models are really different. Figure 4 plots pairs consisting of the forecasts under the naive and rational models using the base case parametrizations. The forecasts are almost identical; for both exercise variables, the pairs of forecasts lie close to the 45° line. On the other hand, Figures 5 and 6 demonstrate that under other calibrations, the rational model is capable of giving forecasts that are different than those of the naive model.

To the extent that the models differ, it is meaningful to ask whether the rational model can make superior predictions. We now consider the ability of each model, in turn, to fit and explain cross-sectional differences in the data, $s_{\tau,i}$ and τ_i, by examining the size of the forecast errors and the results of regressions of the actual variable on the forecast. Clearly, we cannot judge the absolute validity of either model this way since we do not account for the fact that our base case parametrizations are fitted in sample. Our question is whether the rational model is better.

Tables 3 and 4 contain direct measures of the bias in the model forecasts and the size of the forecast errors. Column 2 of Table 3 gives the mean error and percentage error in the model forecast of the market price at exercise:

$$\sum_{i=1}^{40}(s_{\tau,i} - \hat{s}_{\tau,i})/40$$

$$\sum_{i=1}^{40}((s_{\tau,i} - \hat{s}_{\tau,i})/\hat{s}_{\tau,i})/40$$

Column 3 gives the mean absolute error and percentage error,

$$\sum_{i=1}^{40}|s_{\tau,i} - \hat{s}_{\tau,i}|/40$$

$$\sum_{i=1}^{40}(|s_{\tau,i} - \hat{s}_{\tau,i}|/\hat{s}_{\tau,i})/40$$

and Column 4 gives the square root of the mean squared error and percentage error:

$$(\sum_{i=1}^{40}(s_{\tau,i} - \hat{s}_{\tau,i})^2/40)^{1/2}$$

$$(\sum_{i=1}^{40}((s_{\tau,i} - \hat{s}_{\tau,i})/\hat{s}_{\tau,i})^2/40)^{1/2}$$

Columns 2-4 of Table 4 give the same summary statistics of the forecast errors for the time of option exercise. Columns 5-7 in Tables 3 and 4 contain results of the following cross-sectional regressions for the different calibrations of the naive and rational models.

$$s_{\tau,i} = \alpha + \beta \hat{s}_{\tau,i} + \varepsilon_i$$

$$\tau_i = \alpha + \beta \hat{\tau}_i + \varepsilon_i$$

The standard errors of the coefficients in the tables are just the ordinary estimates from the cross-sectional regression and do not take into account uncertainty in our estimates of the option holder and stock return parameters.

Tables 3 and 4 demonstrate that the rational models show virtually no improvement over the naive model in terms of either the regression lines or the size of the forecast errors. For example, in Table 3, the naive model errors in forecasting the market price at exercise are actually among the smallest of those given by any of the models. The regression for the naive model is closest to a 45° line with an R^2 of 38%–among the highest of all the models. Table 4 examines model forecasts of the time of option exercise. In general, the regression lines are too flat and the R^2's are low under all models. In any case, none of the calibrations of the rational model give forecast errors that are markedly smaller than those of the naive model. In terms of the regression, the rational model with outside wealth equal to 0.1 looks better than the naive model–the line is steeper and the R^2 is 16%, compared with only 10% for the naive model. However, the forecast errors under this calibration of the rational model are larger than those under the naive model, and the bias is a full two years greater.

Based on these results, we can say with confidence that the rational model is no better than naive model. Despite the fact that our parameter selection methods make use of the flexibility we have in varying the extra parameters of the rational model and thus give it an advantage, a priori, the naive model fits the data at least as well, and sometimes better.

Although our results regarding the absolute quality of the naive model are only suggestive, we feel it performs surprisingly well. It appears to be a clear improvement over the ordinary American option model. This model is an promising candidate for executive stock option valuation and will be the subject of our future research.

Appendix

Accounting for employee stock options has been on the FASB's agenda since 1984 and their recent valuation proposal has come under fierce attack. Researchers and practitioners argue that the FASB's method significantly overstates true option value. This appendix provides a better understanding of the FASB's approach and demonstrates that it can also understate option value.

According to the FASB's current proposal, a firm should measure the value of employee stock options at the grant date using an option pricing model such as Black-Scholes adjusted for proportional dividends[26] with the expiration date of the option set equal to its expected life. This value should then be multiplied by the fraction of granted options expected to vest to obtain compensation cost. This cost should appear as an asset ("prepaid compensation"), and subsequently be expensed over the employee service period, typically the vesting period. This compensation cost should be adjusted later for differences between the actual and expected forfeitures and option lives, but not for changes in the market price of the stock.

Hereafter, we shall take "expected option life" in the FASB's proposal to mean the expected option life, conditional on the option vesting. Here is one way to think about this valuation procedure. Suppose the option stopping time τ is independent of the stock price path in the sense that the distribution of S_τ given that $\tau = t$ is just the conditional distribution of S_t. Let $c(t)$ be the value of an ordinary European call on the same stock with expiration date t; $c(t) = \mathrm{E}(\zeta_t(S_t - S_0)^+)$. Then the executive option

[26]See Black and Scholes [1973] and Merton [1973].

value, $\mathrm{E}(\zeta_\tau(S_\tau - S_0)^+ 1_{\{\tau \geq t_v\}})$, reduces to $\mathrm{E}(c(\tau)|\tau \geq t_v)\mathrm{P}(\tau \geq t_v)$. By contrast, the FASB value is $c(\mathrm{E}(\tau|\tau \geq t_v))\mathrm{P}(\tau \geq t_v)$, which switches the order of the expectation and the call function operators, so it only differs from the correct value because the function c is nonlinear in t.

Thus, up to nonlinearity in the call price as a function of time to expiration, the FASB's proposal is correct if the stopping time is independent of the stock price path. Now, there is no reason to believe that an executive's optimal stopping time should be independent of the stock price path, but it is also not clear that this value is biased. Kulatilaka and Marcus [1994] claim that the tendency for earlier exercises to take place at higher stock prices causes the FASB value to overstate true option value. Clearly this is potentially false for a dividend-paying stock. Consider the case that interest rates are constant and the stock price is a geometric Brownian motion with a constant continuous dividend rate, δ. The value-maximizing exercise policy for an American option on this stock prescribes exercising the option if the stock price rises above a critical stock price, and the critical level decreases as expiration approaches.[27] Thus, earlier exercises will be at higher stock prices under this policy, and, since it is the value-maximizing policy, the correct option value under this policy exceeds the FASB value.

Even if the stock pays no dividends, we can still construct examples in which the correct option value exceeds the FASB value, and the option is exercised according to a decreasing, time-dependent boundary of critical stock prices. Consider the following three-period example with a binomial stock price. The time 0 stock price is 1 and S_{t+1}/S_t is equal to u or d with equal probability. Suppose that if the stock price rises to u at time 1, the option holder exercises the option then, but otherwise, the option holder waits until time 3 to exercise. The expected option life is two periods. Let r be 1 plus the riskless rate, a constant, and suppose $d < 1 < r < u$. Also, for simplicity,

[27] See Kim [1990].

assume $ud^2 < 1 < u^2d$. Then the option is worth

$$c_1 = \tilde{p}(u-1)/r + (1-\tilde{p})\tilde{p}^2(u^2d-1)/r^3$$

and the FASB value, the value of a European option on this stock with an expiration date equal to two periods, is

$$c_2 = (\tilde{p}^2(u^2-1) + 2(1-\tilde{p})\tilde{p}(ud-1)^+)/r^2$$

where $\tilde{p} = (r-d)/(u-d)$. If we let $u = (1+\mu)e^{\sigma}/\cosh\sigma$ and $d = (1+\mu)e^{-\sigma}/\cosh\sigma$ then the continuously compounded stock return volatility is σ and the mean stock return is μ. Figure 8 plots c_1 and c_2 as functions of r, and σ with a base case of $r = 1.07$, $\sigma = 0.3$, and $\mu = 0.12$. The plots demonstrate that correct option value exceeds the FASB value in this simple example when interest rates are low, or when volatility is high.

References

Agrawal, Anup, and Gershon N. Mandelker, 1987, Managerial Incentives and Corporate Investment and Financing Decisions, *Journal of Finance* 42, 823-837.

Arnason, Sigurjon and Ravi Jagannathan, 1994, Evaluating Executive Stock Options: A Study Guide, Working Paper, University of Minnesota.

Black, Fisher, and Myron Scholes, 1973, The Pricing of Options and Corporate Liabilities, *Journal of Political Economy* 31, 637- 654.

Brickley, James A., Sanjai Bhagat, and Ronald C. Lease, 1985, The Impact of Long-Range Managerial Compensation Plans on Shareholder Wealth, *Journal of Accounting and Economics* 7, 115-129.

Brown, Stephen J., and Jerold B. Warner, 1980, Measuring Security Price Performance, *Journal of Financial Economics* 8, 205-258.

Cox, John C., Stephen A. Ross, and Mark Rubinstein, 1979, Option Pricing: A Simplified Approach, *Journal of Financial Economics* 7, 229-263.

Cuny, Charles J., and Philippe Jorion, 1993, Valuing Executive Stock Options With a Departure Decision, Working Paper, University of California at Irvine.

DeFusco, Richard A., Robert R. Johnson, and Thomas S. Zorn, 1990, The Effect of Executive Stock Option Plans on Stockholders and Bondholders, *Journal of Finance* 45, 617-627.

Financial Accounting Standards Board, 1993, Proposed Statement of Financial Accounting Standards: Accounting for Stock-based Compensation, *Financial Accounting Series* No. 127-C.

Hansen, Lars, 1982, Large Sample Properties of the Generalized Method of Moments Estimators, *Econometrica* 50, 1029-1054.

Harrison, J. Michael, and David M. Kreps, 1979, Martingales and Arbitrage in Multiperiod Securities Markets, *Journal of Economic Theory* 20, 381-408.

Holmstrom, Bengt, and Oliver Hart, 1987, Theory of Contracts. In *Advances in Economic Theory,* ed. Truman F. Bewley, 71-155. New York and Melbourne: Cambridge University Press.

Huddart, Steven, 1994, Employee Stock Options, *Journal of Accounting and Economics* 18, 207-231.

Hull, John C., 1993, *Options, Futures, and Other Derivative Securities* (Prentice-Hall, New Jersey).

Jennergren, L. Peter, and Bertil Naslund, 1993, A Comment on "Valuation of Executive Stock Options and the FASB Proposal," *The Accounting Review* 68, 179-183.

Kim, In Joon, 1990, The Analytic Valuation of American Options, *Review of Financial Studies* 3, 547-572.

Kulatilaka, Nalin, and Alan J. Marcus, 1994, Valuing Employee Stock Options, forthcoming, *Financial Analysts Journal.*

Lambert, Richard A., William N. Lanen, and David F. Larcker, 1989, Executive Stock Option Plans and Corporate Dividend Policy, *Journal of Financial and Quantitative Analysis* 24, 409-424.

Lambert, Richard A., David F. Larcker, and Robert E. Verrechia, 1991, Portfolio Considerations in Valuing Executive Compensation, *Journal of Accounting Research* 29, 129-149.

Merton, Robert C., 1969, Lifetime Portfolio Selection Under Uncertainty: The Continuous-Time Case, *The Review of Economics and Statistics* 51, 247-257.

Merton, Robert C., 1971, Optimum Consumption and Portfolio Rules in a Continuous-Time Model, *Journal of Economic Theory* 51, 373-413.

Merton, Robert C., 1973, Theory of Rational Option Pricing, *Bell Journal of Economics and Management Science* 4, 141-183.

Noreen, Eric, and Mark Wolfson, 1981, Equilibrium Warrant Pricing Models and Accounting for Executive Stock Options, *Journal of Accounting Research* 19, 384-398.

Rubinstein, Mark, 1994, On the Valuation of Employee Stock Options, Working Paper, University of California at Berkeley.

Seyhun, H. Nejat, 1992, The Effectiveness of the Insider-Trading Sanctions, *Journal of Law and Economics* 35, 149-182.

Vargus, Mark E., 1994, The Implications of Executives' Stock Option Bailout on Compensation Contracts: An Empirical Analysis, working paper, University of Pennsylvania.

Yermack, David, 1995, Do Corporations Award CEO Stock Options Effectively? *Journal of Financial Economics* 39, 237-269.

Table 1: Summary Statistics of Firms with Option Exercises

s_τ is the average normalized stock price at exercise, τ is the average time to option exercise, and t_v is the average vesting date. σ and δ are the estimated stock return volatility and dividend rate, respectively. s_{10} is the normalized stock price at expiration.

	s_τ	τ	t_v	σ	δ	s_{10}
Minimum	1.147	1.148	0.00	0.190	0.000	0.067
Maximum	8.319	9.483	4.41	0.565	0.0712	9.116
Average	2.754	5.828	1.96	0.314	0.0298	3.270
Median	2.465	6.084	2.00	0.308	0.0296	2.751
Standard Deviation	1.415	2.245	1.03	0.099	0.0173	2.252

Table 2: Sample Average Exercise Variables and Cancellation Rate and Model Forecasts for a Representative Firm

A, x, y, and q are, respectively, the coefficient of relative risk aversion, the initial level of outside wealth, the payment for leaving the firm, and the probability of a departure decision. $\hat{s}_{\tau,0}$, $\hat{\tau}_0$, and $\hat{cr}_0$ are the model forecasts of the normalized stock price at exercise, the time of exercise and the cancellation rate for the representative firm. ESO value is the market value of the option for the representative firm. FASB value is the probability that the option vests times the option value under the Black-Scholes model adjusted for proportional dividends with the expiration date set equal to the option's expected life, given that it vests.

Sample Averages							
			$\bar{s}_\tau$	$\bar{\tau}$	$\bar{cr}$		
			2.754	5.828	0.073		
Model Forecasts							
x	y	q	$\hat{s}_{\tau,0}$	$\hat{\tau}_0$	$\hat{cr}_0$	ESO value	FASB value
American Option Model							
		0	3.329	7.569	0.031	0.394	0.360
Naive Model							
		0.113	2.647	5.770	0.070	0.292	0.287
Rational Model							
342	132	0.122	2.670	5.873	0.073	0.285	0.283
5	10	0.110	2.532	5.545	0.070	0.294	0.286
1	10	0.052	2.122	4.507	0.054	0.325	0.299
0.1	10	0.062	1.681	3.094	0.063	0.270	0.260
4.67	0.15	0.114	2.652	5.933	0.041	0.377	0.337
3.00	0	0	2.543	5.753	0.034	0.389	0.347
8.18	0.30	0.2	2.486	5.386	0.056	0.345	0.316

Table 3: Model Forecasts of the Market Price of the Stock at Exercise

x is initial outside wealth, y is the payment for leaving, and q is the annual probability of a departure decision. A, the coefficent of relative risk aversion, is set to 2. The regression equation is $s_{T,i} = \alpha + \beta \hat{s}_{T,i} + \varepsilon_i$, where $s_{T,i}$ is the actual stock price at exercise for firm i and $\hat{s}_{T,i}$ is the model forecast given firm i's stock return volatility and dividend rate, its vesting date, and its terminal stock price. The mean stock return is set to 15% and the riskless rate is set to 7%. The forecast error for firm i is $s_{T,i} - \hat{s}_{T,i}$.

Parameter Setting	Forecast Errors (Percentage Errors)			Regression Coefficients (Standard Errors)		
	Mean	Mean Absolute	Root Mean Squared	α	β	R^2
			American Option Model			
$q = 0$	-0.26	1.16	1.71	2.04	0.24	0.04
	(0.00)	(0.36)	(0.47)	(0.58)	(0.18)	
			Naive Model			
$q = 0.113$	0.42	0.76	1.19	0.02	1.18	0.38
	(0.19)	(0.34)	(0.50)	(0.59)	(0.24)	
			Rational Model			
$x = 342, y = 132$	0.42	0.75	1.17	-0.04	1.20	0.40
$q = 0.122$	(0.19)	(0.33)	(0.49)	(0.58)	(0.24)	
$x = 5, y = 10$	0.40	0.76	1.20	-0.20	1.26	0.36
$q = 0.110$	(0.17)	(0.33)	(0.50)	(0.66)	(0.27)	
$x = 1, y = 10$	0.57	0.87	1.34	0.33	1.11	0.25
$q = 0.052$	(0.27)	(0.40)	(0.60)	(0.71)	(0.31)	
$x = 0.1, y = 10$	0.99	1.12	1.56	-1.18	2.23	0.37
$q = 0.062$	(0.54)	(0.61)	(0.81)	(0.85)	(0.47)	
$x = 4.67, y = 0.15$	0.20	0.84	1.27	0.27	0.97	0.22
$q = 0.113$	(0.09)	(0.32)	(0.46)	(0.80)	(0.30)	
$x = 3.00, y = 0$	0.13	0.88	1.30	0.62	0.81	0.18
$q = 0$	(0.06)	(0.32)	(0.45)	(0.78)	(0.29)	
$x = 8.18, y = 0.30$	0.33	0.83	1.26	-0.12	1.18	0.26
$q = 0.2$	(0.13)	(0.33)	(0.48)	(0.82)	(0.33)	

Table 4: Model Forecasts of the Time of Exercise

x is initial outside wealth, y is the payment for leaving, and q is the annual probability of a departure decision. A, the coefficent of relative risk aversion, is set to 2. The regression equation is $\tau_i = \alpha + \beta\hat{\tau}_i + \varepsilon_i$, where τ_i is the actual time of exercise for firm i and $\hat{\tau}_i$ is the model forecast given firm i's stock return volatility and dividend rate, its vesting date, and its terminal stock price. The mean stock return is set to 15% and the riskless rate is set to 7%. The forecast error for firm i is $\tau_i - \hat{\tau}_i$.

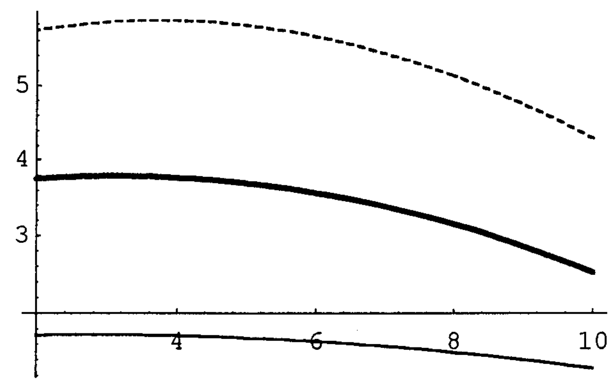
Parameter Setting	Forecast Errors (Percentage Errors)			Regression Coefficients (Standard Errors)		
	Mean	Mean Absolute	Root Mean Squared	α	β	R^2
American Option Model						
$q = 0$	-1.23	2.37	2.97	4.28	0.22	0.04
	(-0.12)	(0.34)	(0.40)	(1.29)	(0.18)	
Naive Model						
$q = 0.113$	0.23	1.95	2.29	3.33	0.45	0.10
	(0.11)	(0.40)	(0.54)	(1.26)	(0.22)	
Rational Model						
$x = 342, y = 132$	0.23	1.92	2.26	3.19	0.47	0.10
$q = 0.122$	(0.11)	(0.39)	(0.53)	(1.30)	(0.22)	
$x = 5, y = 10$	0.46	1.91	2.22	2.85	0.56	0.12
$q = 0.110$	(0.14)	(0.41)	(0.54)	(1.37)	(0.25)	
$x = 1, y = 10$	0.89	2.01	2.39	3.39	0.49	0.11
$q = 0.052$	(0.25)	(0.46)	(0.61)	(1.19)	(0.23)	
$x = 0.1, y = 10$	2.25	2.57	3.04	3.03	0.78	0.16
$q = 0.062$	(0.71)	(0.80)	(0.99)	(1.08)	(0.29)	
$x = 4.67, y = 0.15$	-0.07	2.09	2.41	3.79	0.35	0.07
$q = 0.113$	(0.04)	(0.37)	(0.43)	(1.32)	(0.21)	
$x = 3.00, y = 0$	-0.20	2.12	2.42	3.74	0.35	0.07
$q = 0$	(0.02)	(0.37)	(0.43)	(1.33)	(0.21)	
$x = 8.18, y = 0.30$	0.39	2.05	2.34	3.51	0.43	0.08
$q = 0.2$	(0.12)	(0.41)	(0.48)	(1.32)	(0.23)	

Figure 1: Exercise Boundaries for Different Levels of Risk Aversion

Each curve is a quadratic fitted through the points
$\{(t,b(t)): t = 2, 2\ 1/12, 2\ 2/12, ..., 10\}$ where b(t) is the lowest stock price at time t in the monthly binomial tree at which the option is exercised, assuming the executive is in a non-departure state. The decision trees are generated assuming a 2-year vesting date, 7% interest rate, 31% stock return volatility, 15.5% mean stock return, and a 3% dividend rate. In the rational model, the initial level of outside wealth is 2 and the payoff for leaving is 10. The annual probability of a departure state is 12.2%.

Naive model: thick line
Rational model - risk aversion coefficient 2: plain line
Rational model - risk aversion coefficient .25: dashed line

Rational Model - Outside Wealth in Stock



Rational Model - Outside Wealth in the Stock-Bond Portfolio

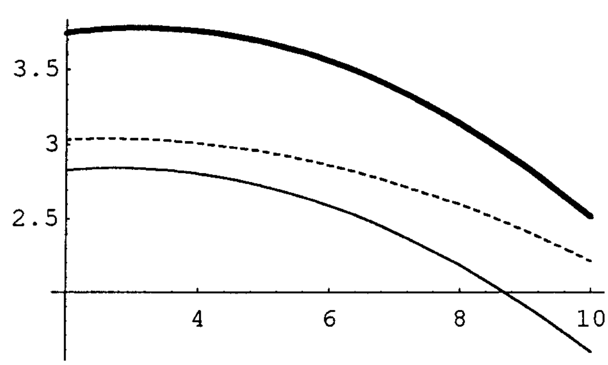


Figure 2: Exercise Boundaries for Different Levels of Initial Wealth

Each curve is a quadratic fitted through the points $\{(t,b(t)): t = 2, 2\ 1/12, 2\ 2/12, ..., 10\}$ where b(t) is the lowest stock price at time t in the monthly binomial tree at which the option is exercised, assuming the executive is in a non-departure state. The decision trees are generated assuming a 2-year vesting date, 7% interest rate, 31% stock return volatility, 15.5% mean stock return, and a 3% dividend rate. In the rational model, the coefficient of relative risk aversion is 2. The annual probability of a departure state is 0.

Naive model: thick line
Rational model - initial wealth 20: plain line
Rational model - initial wealth 1: gray line
Rational model - initial wealth 0.1: dashed line

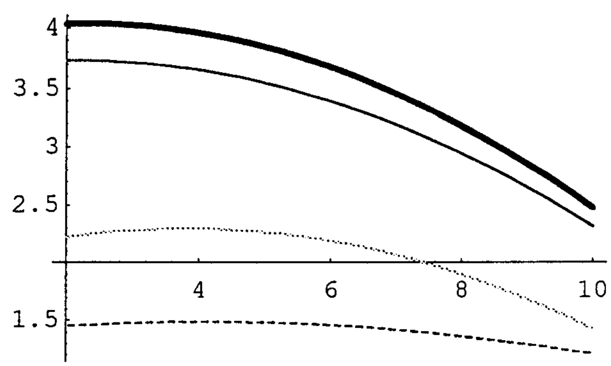


Figure 3: Rational Model Continuation Region in the Departure State

Each curve is a quadratic fitted through the points {(t,bi(t)): t = 2, 2 1/12, 2 2/12, ... 10} where b1(t), for the lower curve, is the lowest stock price at time t in the monthly binomial tree at which the option is not exercised, and b2(t), for the upper curve, is the lowest stock price above b1(t) at time t in the monthly binomial tree at which the option is exercised. The decision trees are generated assuming a 2-year vesting date, 7% interest rate, 31% stock return volatility, 15.5% mean stock return, and a 3% dividend rate. Outside wealth is invested in the stock-bond portfolio that would be optimal in a standard terminal wealth problem, the coefficient of relative risk aversion is 2, initial wealth is 4.67, the payoff for leaving is 0.15 and the annual probability of a departure state is 0.114.

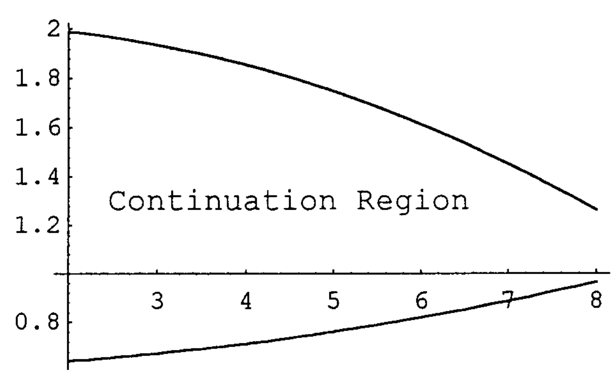
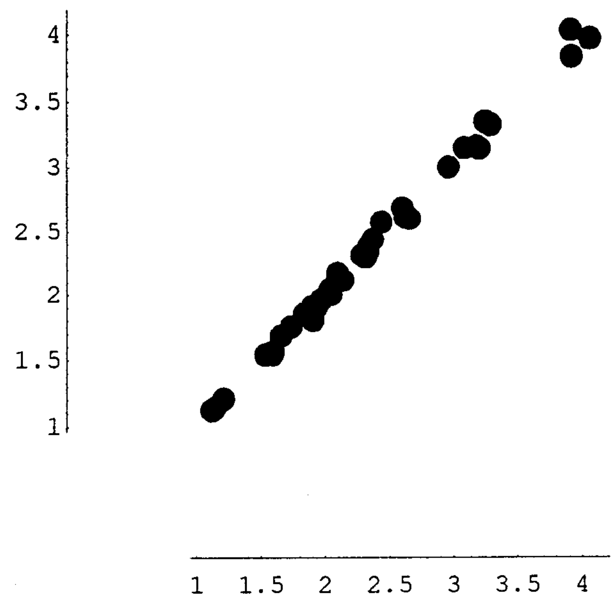


Figure 4: Rational vs. Naive Model Forecasts
(Fitted Rational Model)

Stock Price at Exercise



Time of Exercise

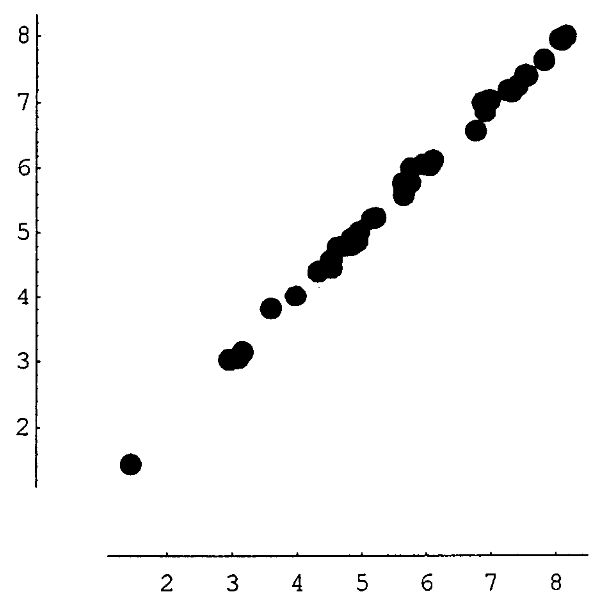
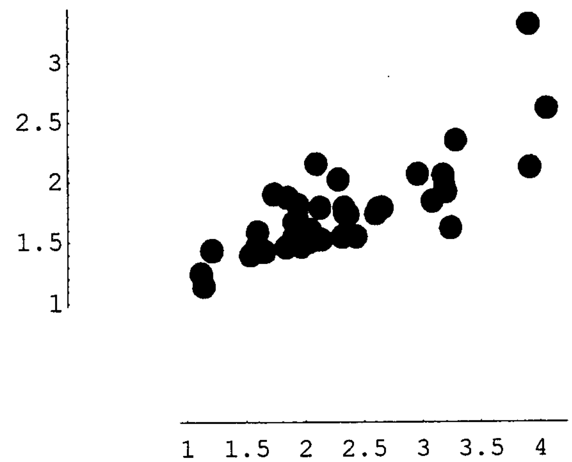


Figure 5: Rational vs. Naive Model Forecasts (Rational Model, x = 0.1)

Stock Price at Exercise



Time of Exercise

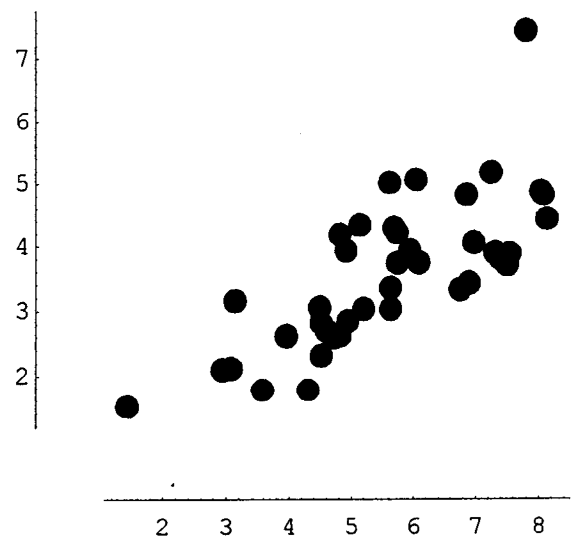
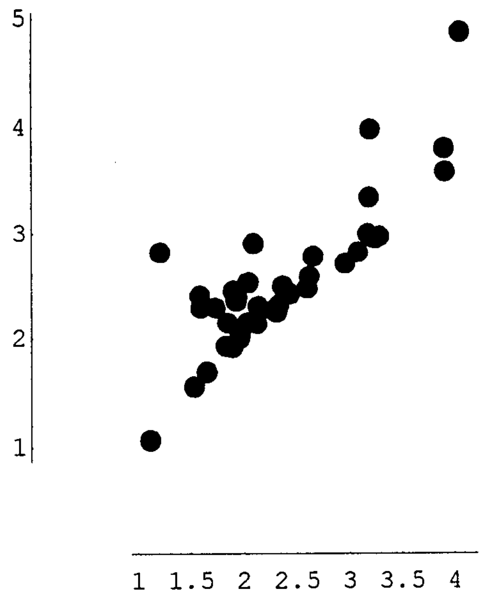


Figure 6: Rational vs. Naive Model Forecasts
(Rational Model, y = 0.15)

Stock Price at Exercise



Time of Exercise

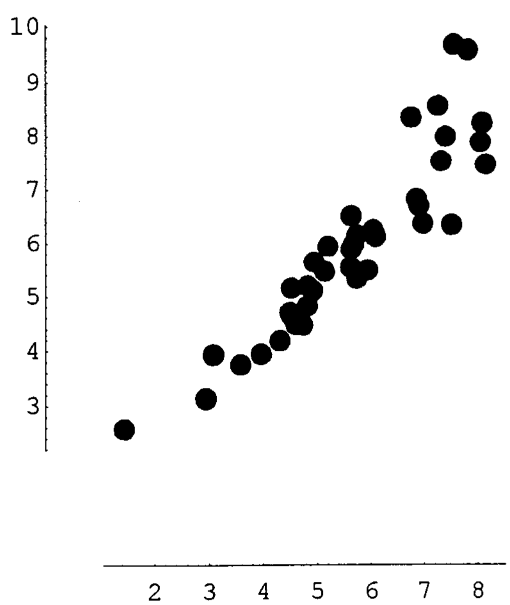
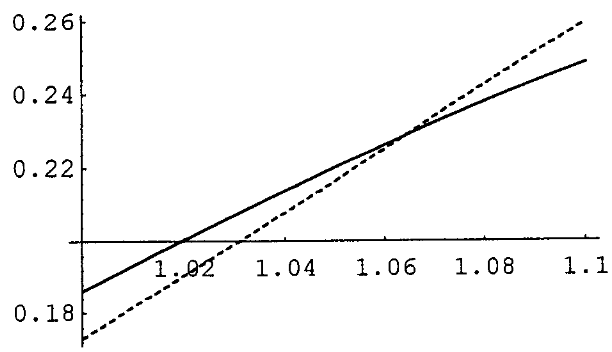


Figure 7: Option Value and the FASB Approximation

Correct value: plain line
FASB approximation: dashed line

Option Value vs. Riskless Rate



Option Value vs. Volatility