Debt, Investment, and Product Market Competition*

by

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January 1999

*I gratefully acknowledge the comments of Laurie Simon Hodrick, Peter DeMarzo, Michael Fishman, Bjorn Jorgensen, Kenneth Kavajecz, Elizabeth Odders-White, Yakov Amihud, and seminar participants at Boston College, Duke University, Harvard University, New York University, Northwestern University, Ohio State University, Stanford University, University of Colorado at Boulder, University of Michigan, University of Minnesota, and University of Texas at Austin. All remaining errors are of course my own. Comments are welcome.

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Abstract

Recent empirical literature on the interaction between capital structure, investment, and product market decisions suggests that debt leads to lower investment expenditures and weaker product market competition. Theoretical literature in this area has been unable to fully explain this finding (perhaps because all theoretical papers look only at two of the above decisions). This paper develops a model which examines all three decisions and shows that debt and investment can be substitutes in a model where firms rationally take on debt. Furthermore, it is demonstrated that when firms compete with prices in the product market, an increase in debt leads to lower investment and higher prices.

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I Introduction

This paper explicitly models the interaction of capital structure, investment, and product market decisions. When firms compete in imperfect product markets, both debt and investment give the firm a strategic advantage in the product market. Increasing investment expenditures leads to a lower marginal cost of production and commits the firm to produce more in the product market. Increases in debt lead the firm to care only about high realizations of demand in which the firm benefits from a larger production run.[1,2] This commits the firm to higher production in the product market. Debt and investment may be substitutes or complements in this model. An increase in debt leads to more production in the product market; thus, the firm benefits from a lower marginal cost and wants to invest more. However, an increase in debt also increases the probability of bankruptcy, and the shareholders receive no benefit from the investment when the firm is insolvent. Since the shareholders pay for the investment, a higher probability of bankruptcy leads the firm to want less investment. Debt and investment are substitutes when the latter effect dominates. This paper shows that positive marginal revenue is a sufficient condition for debt and investment to be substitutes. If the firms compete in prices in the product market, then increases in debt will induce the firm to charge a higher price. With a higher price and debt overhang the firm will unambiguously want to lower investment spending. In this case debt and investment are substitutes and thus, an increase in debt leads to lower investment and weaker product market competition (i.e., the firm charges a higher price).

[1] Throughout the paper I refer to the firm as representing the interests of the shareholders.

[2] When a firm is heavily levered it will remain solvent only when demand is high. It is in these high demand states where the shareholders receive a positive payoff and thus care about the profits of the firm. In these states the firm makes larger profits if it produces more relative to low demand states.

Empirical evidence

Recent empirical research has demonstrated a link between capital structure, investment expenditures, and product market behavior. Chevalier (1995) examines competition in the supermarket industry after one firm undergoes a leveraged buyout (LBO). Chevalier finds that when supermarkets increase leverage, investment decreases and prices rise. Phillips (1995) finds that in three out of four industries, controlling for demand and marginal cost changes, as leverage increases, investment decreases and industry prices increase.[3] Kovenock and Phillips (1994) find that when industry concentration is high, firms which recapitalize are less likely to invest and more likely to close plants. Kaplan (1989) also finds that firms which undergo LBOs decrease capital expenditures. These empirical results are consistent with the theoretical model presented in this paper.

Past theoretical literature, the strategic bankruptcy effect

The above empirical work suggests that following a significant increase in leverage, a firm invests less and increases price. Furthermore, the competition invests more and also increases price. Chevalier concludes that the empirical evidence supports the predatory theories put forth by Fudenberg and Tirole (1986), Poitevin (1989), and Bolton and Scharfstein (1990). This literature also includes a paper by Brander and Lewis (1988). In these papers a firm which proceeds with a leverage recapitalization becomes a "weaker" competitor in the product market. Rival firms engage in predatory product market behavior in an attempt to drive the levered firm out of the market (through bankruptcy) and thus capture larger rents in the future. These papers show that a rival may increase output or lower price in an attempt to drive the highly levered competition into bankruptcy. These papers are not fully consistent with the empirical evidence on

[3]The one industry in which Phillips does not find this result is characterized with a small minimum scale size and no barriers to entry. This is evidence that the industry is a contestable market which would lead firms to behave in the product market as if they were competing in a perfectly competitive market. In a perfectly competitive market there is no reason to believe that capital structure would effect product market decisions.

three levels. First, if this behavior was occurring we would expect to see price decreases in the industry following an LBO or a leverage restructuring during the predatory period. In all the industries examined a price increase follows a major leverage recapitalization.[4] Second, these theoretical papers do not address the investment decision, and hence do not make predictions on the affect of leverage changes on the firm's investment and its rivals' investment. Third, there is no rational reason for firms to take on debt in these models. A leverage restructuring will only decrease firm value by exposing the firm to predatory behavior. This paper presents a model which explicitly examines the investment and product market decisions after a change in leverage where a non-zero debt level may be optimal.

The limited liability effect

The limited liability literature contends that, by altering the incentives of shareholders and managers, debt commits a firm to behave more aggressively in the product market. This literature includes papers by Brander and Lewis (1986), Maksimovic (1988), and Rotemberg and Scharfstein (1990). In these papers the limited liability of equity induces the firm, which makes decisions to maximize shareholder value, to produce more when demand is random. This increases *ex ante* expected firm value. Showalter (1995) shows that if firms are competing with prices in the product market then the limited liability effect induces the firm and its rivals to increase prices after an increase in leverage. With the addition of Showalter's paper, this literature demonstrates that leverage can have different effects on the product market depending on the type of competition. These papers make no predictions on how leverage effects the investment decision because the investment decision is omitted from these models.

[4]All the theoretical models relate to firms competing in oligopolies. The gypsum industry has characteristics the imply the product market is closer to perfectly competitive and thus I discount these findings.

Debt overhang effect

Myers (1977) demonstrates that debt can cause a firm to underinvest, because some of the benefits of investment accrue to the debtholders, while the entire cost is borne by the shareholders. Myers assumes that all uncertainty is resolved prior to the investment decision. The resulting under investment consists of not taking positive NPV projects. In Myers model, firms which suffer from this underinvestment problem decide to undertake no investment spending (not just lower their investment spending). This model predicts that firms which take on leverage would either choose zero investment or their investment choice would be unaffected by the debt. There are no benefits to debt in this model, and given the choice of capital structure firms would choose all equity. This paper generalizes Myers' result by showing that underinvestment persists when uncertainty is not resolved prior to the investment decision and when there is a continuum of possible investment choices.

The relationship between investment and product market competition

Brander and Spencer (1983) discuss the relation between investment and product market competition. They show that an increase in investment, which lowers the marginal cost of production, commits a firm to a more aggressive position in the product market. When firms compete in a Cournot oligopoly and have different marginal costs, the firms with lower marginal costs produce more and obtain larger profits. Under this theory, larger investment leads to lower prices and higher output. Brander and Spencer assume that firms have no debt and, consequently, cannot comment on any capital structure effects on investment or on the product market decisions.

The remainder of the paper is organized as follows. Section II describes the model. Section III characterizes the equilibrium output market decision taking firms' investment and debt levels as fixed. The equilibrium investment choice taking firms' debt levels as given is characterized in section IV. Section V characterizes the equilibrium debt choice. Section VI determines how an increase in demand affects the results. Several numerical examples are presented in Section VII. These examples demonstrate many of the effects established in the theoretical model. Section VIII discusses possible extensions to the model, and section IX concludes the paper.

II The Model

The model is a three-stage game with an uncertain product market environment. There are two firms which compete in the product market. They produce goods that are substitutes, but not necessarily identical. Each firm has access to an investment project that lowers its marginal cost of production. For instance, this investment could be though of as a technical innovation in the production process for the industry.[5] Upon observing the realization of this innovation, a firm has the option to alter its capital structure prior to making its investment decision. Only long term, non-renegotiable debt which matures after the product market decision, will affect the investment decision.[6] Debt is sold in a competitive market, with the proceeds distributed to the firm's shareholders. After deciding how much to invest, each firm then selects a quantity to produce, and the price is determined by the market. The shareholders and debtholders are both risk neutral, and the managers make decisions to maximize the equity value of the firm. In each stage the firms observe the outcome from the previous stage(s) and then move simultaneously.

[5]For example, the development of scanners which register the price automatically at the checkout lane in supermarkets. Investing in this technology increases the number of customers a cashier can serve in a given time; thus, lowering the total and marginal cost of serving customers.

[6]In reality firms also have the option of issuing long-term debt after investment is sunk and prior to the product market decision. At this point, however, the debt decision would be identical to that in the Brander and Lewis paper. It may be interesting to add this feature to future models to see if firms want to add more debt (to increase their commitment to an aggressive product market position) after investment. This feature may also allow firms to decrease their investment or debt levels from stage 2 and stage 1 since they would be able to commit just prior to the quantity stage.

Stage 1	Stage 2	Stage 3
Firms choose D^i D^i is public knowledge Debtholders pay $W^i(D^i,D^j)$ Firms receive W^i cash	Firms choose I^i I^i is public knowledge Shareholders receive $(W^i - I^i)$	Firms choose q^i q^i is public knowledge z is realized $R^i(q^i,q^j,z)$ is received debt is paid shareholders are paid

In the first stage each firm chooses its face value of debt D^i: i=1,2. Debt, which is due at the end of stage 3, is sold at fair market value, W^i, and the proceeds are used for the investment with any residual cash available for distribution to the shareholders. If stage-3 profits are sufficient to pay the debtholders in full, then the remainder of the profits are distributed to the shareholders. There is no discounting, thus W^i equals the expected stage-3 payoff to debtholders given the face value of each firm's debt.[7] Note that the debt is sold at the fair market value given the investment incentives that the firm will have after the debt issue. The debt holders know that cash can be distributed to shareholders, and can determine what the firm's investment will be given it is maximizing shareholder wealth. This will not lead the firm to issues large amounts of debt and try to distribute all the proceeds to the shareholders. If the face value of debt is large enough to insure bankruptcy in the final stage the shareholders will chooses zero investment and distribute all the cash in stage 2. The debt holders know this will happen and will pay the expected value of the firm given no investment will occur after the debt issue.[8] The stage-1 debt must be long-term debt that exists when the investment decision arises. This cannot be short-term debt that would expire before the quantity decision is made, or debt that can be renegotiated prior to the quantity decision. The firms choose these debt

[7]The results of the paper are robust to adding a discount factor to account for the timing of the cashflows.

[8]For example, suppose expected profits of the firm are $100 (with a maximum profits of $500) if it chooses zero investment. If this firm issues debt with face value of $1000, D = $1000, and thus is guarantee to be bankrupt then all the proceeds of the debt issue will be paid to shareholders in stage 2. The debt holders will pay $100 for the debt issue, because this is their expected payoff given no investment takes place. Thus, the firm cannot keep increasing the face value of debt and collect more and more cash to distribute immediately to the shareholders. The same logic holds for an additional equity issue, if further funds are need to complete the desired investment.

levels optimally, anticipating the effect on the investment decisions in stage 2, and the quantity decisions in stage 3.

In stage 2 the firms choose investment levels I^i: i=1,2. Investment is paid for immediately with the cash on hand from the debt issue. If cash on hand in insufficient to cover the desired investment then shareholders pay the shortfall. If cash on hand is sufficient to cover desired investment then the excess cash is distributed to the shareholders. Several circumstances are strategically equivalent to requiring that equity holders pay the shortfall. For example, the firm can issue equity to raise the necessary capital to cover the shortfall. In this situations the firm's incentives in stage 3 remain unchanged since the debt outstanding remains fixed. The current equity holders must give up one dollar of expected stage 3 payoff for each dollar of capital that the firm raises for additional investment.[9] Since the players are risk neutral this is equivalent to equity holders paying for the shortfall up front.[10]

In stage 3 the firms play a Cournot game choosing quantities q^i: i=1,2. At the end of stage 3 demand is realized, the price is set to clear the market, and firms receive their profits. It is not necessary that all the uncertainty be resolved after quantities are chosen, only that there is some uncertainty remaining when quantities are chosen. Shareholders receive the residual revenue after variable costs and debtholders are paid.

The strategy of each firm consists of a debt level for stage 1, an investment level for stage 2 which is a function of the debt levels chosen in stage 1, and a quantity for stage 3 which is a function of the debt

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[9]There is nothing to restrict the firm from issuing additional equity and paying a dividend (or issuing more equity then necessary to cover the short fall and paying a dividend with the excess cash form the issue). Assuming that new equity is sold at a fair market price given the firm's incentives to maximize the existing shareholders' wealth (which must be the case in a subgame perfect equilibrium with a competitive secondary equity market) for every dollar raised through a new equity issue the existing shareholders must give up one dollar of equity. Thus, there is no way for the existing shareholders to profit at the expense of new shareholders buy selling them claims which will be worth less than they pay.

[10]The results would be similar if the firm were allowed to issue junior debt. Equityholders would still bear the cost of the additional investment, as the new debtholders would demand an expected payoff equal to the capital they invest. Allowing junior debt, however, needlessly complicates the model.

levels and investment levels that both firms have chosen. Sequential subgame perfect sequential equilibrium strategies are determined.[11] This equilibrium concept implies that, at each stage, each firm makes decisions correctly anticipating the rival's choice and the corresponding subgame equilibrium outcome of the remaining stages.

The revenue received by firm i in stage 3 is denoted $R^i(q^i,q^j,z)$, where q^i is the quantity produced by firm i, q^j is the quantity produced by firm j, and z is a random variable that captures the uncertainty in the output market. The firm's are producing goods which are substitutes, and z represents the level of demand for this type of product, thus the realization of z is the same for both firms. The random variable z is distributed continuously on ($\underline{Z}$, $\overline{Z}$), with density f(z). As firm j increases its quantity, firm i's total and marginal revenues decrease. It is assumed that marginal revenue is decreasing in quantity. I assume that a higher z corresponds to a better state of the world. This can be thought of as a higher realization of uncertain demand. It is also assumed that marginal revenue is higher in better states of the world.[12] These assumptions can be summarized with the following inequalities (subscripts i and j denote derivatives with respect to q^i and q^j respectively):

(1a) $R^i_j<0$

(1b) $R^i_{ij}<0$

(1c) $R^i_{ii}<0$

(1d) $R^i_z>0$

(1e) $R^i_{iz}>0$

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[11]See Selten (1995,1975), or Myerson (1978) for a definition of subgame perfect equilibrium. Kreps and Wilson (1982) develop sequential equilibrium.

[12]This condition will hold if an increase in z shifts the marginal revenue curve of the firm upward. Brander and Lewis (1986) call this assumption the normal case. For example, if the demand schedule facing the firm is downward slopping, and an increase in z shifts the demand curve up, then this condition is satisfied.

The investment level of firm i is denoted by I^i. The total cost of production for firm i is $C^i(q^i,I^i)+I^i$. All costs in excess of the initial investment are included in C^i (this can be thought of as total variable cost), and this cost, C^i, is assumed to be decreasing in I^i. Marginal cost is always positive and additional investment is assumed to lower the marginal cost. The following inequalities summarize the assumptions on the cost function. Note that marginal cost is denoted $c^i=C^i_i$, the subscript I denotes the derivative with respect to I^i, and the subscript II denotes the second derivative with respect to I^i.

(2a) $C^i_I<0$

(2b) $C^i_{II}>0$

(2c) $c^i>0$

(2d) $c^i_I<0$

The cashflows received by the shareholders are as follows. In stage 2, shareholders receive the difference between the cash received from the debt issue and the cost of investment, $(W^i - I^i)$. Note that this is equivalent to a specification where the shareholders receive the cash for the debt issue in stage 1 and pay the entire investment in stage 2 and thus later in the paper I refer to the shareholders as responsible for the entire investment expenditure.[13] In stage 3, shareholders receive the residual revenue after variable costs are incurred and debtholders are paid. If revenues net of variable costs are not sufficient to pay the debtholders then shareholders' stage-3 payoff is zero and debtholders receive all residual income after variable costs are paid. For simplicity, I assume that revenue always exceeds variable costs.[14] The total payoff to the shareholders of firm i is

(3) $\max[R^i(q^i,q^j,z)-C^i(q^i,I^i)-D^i,0]+W^i- I^i$.

[13]This is true because for every additional dollar spent on investment the shareholders forgo a dollar of cash, or a dollar of expected value.

[14]This eliminates the possibility that the limited liability of both debtholders and shareholders with respect to the payment of variable costs affects the outcome of the game. If for some states z, $C^i>R^i$, than it may be possible for the manager to increase the value of debt and equity at the cost of the suppliers and the workers, thereby making the suppliers an additional class of junior debtholders in the firm.

Let V^i be the expected value of total cashflows to the shareholders of firm i. The following assumptions on V^i are necessary to guarantee the existence of a Nash equilibrium (recall that subscripts i, j, I, and J, denote derivatives with respect to q^i, q^j, I^i and I^j respectively):

(4) $V^i_{ij}<0$

(5) $V^i_{IJ}<0$

(6) $V^i_{ii}V^j_{jj}-V^i_{ij}V^j_{ji}>0$

(7) $V^i_{II}V^j_{JJ}-V^i_{IJ}V^j_{JI}>0$.

Equation (4) holds if the marginal profit to firm i from increasing quantity is decreasing in firm j's quantity. Equation (5) holds if the marginal profit to firm i from increasing investment is decreasing in firm j's investment. Equations (6) and (7) are analogous to assuming that own effects of output and investment (respectively) on marginal profit are greater than cross effects, or equivalently, that output and investment reaction functions slope downward. These are standard assumptions for Cournot type models, and they assure reaction function stability and yield a unique Nash equilibrium.[15]

III Output Market Equilibrium

The subgame perfect sequential Nash equilibrium is determined using backward induction. Thus, the output market equilibrium of stage 3 is solved first, taking the results of stage 1 and stage 2 as given. The optimal quantity choice in stage 3 is determined by assuming that the debt levels are fixed at D^i and D^j, and that the investment is sunk at I^i and I^j. Investments I^i and I^j determine the cost function of each firm, $C^i(.|I^i)$ and $C^j(.|I^j)$. I now define the term *debt demand state*.

<u>Definition:</u> *Debt demand state* is the minimum state of demand which allows the firm to stay solvent in stage 3, i.e., min $\{z\in(\underline{Z}, \overline{Z})|\ R^i(q^i,q^j,z)-C^i(q^i,I^i) \geq D^i\}$.

[15]Brander and Lewis (1986) point out that, although these are standard assumptions, there exist simple Cournot models where these assumptions are violated. One model, however, in which these assumptions hold is when z is uniformly distributed, demand is linear, and marginal cost conditional on investment is constant.

Debt demand state refers to the minimum state z such that the debtholders are paid in full. Define $\hat{z}^i$ implicitly by

(8) $R^i(q^i,q^j,\hat{z}^i)-C^i(q^i,I^i)-D^i=0.$

Then $\hat{z}^i$ is the debt demand state for firm i. Assume $\underline{Z}<\hat{z}^i<\overline{Z}$. When $z=\hat{z}^i$, the firm's profits are exactly enough to completely pay off the debtholders. In states $z>\hat{z}^i$, the shareholders receive positive cashflow in stage 3, and their total payoff is $R^i(q^i,q^j,z)-C^i(q^i,I^i)-D^i-I^i+W^i$. In states $z<\hat{z}^i$, the firm's revenues in stage 3 are insufficient to pay the face value of the debt, and shareholders receive zero stage-3 payoff. Thus, total cashflow to shareholders in these states is W^i-I^i. It is useful to note the dependence of $\hat{z}^i$ on D^i,D^j,I^i,I^j,q^i, and q^j. Taking derivatives of equation (8) yields:

(9a) $$\frac{d\hat{z}^i}{dD^i}=\frac{1}{R_z^i(q^i,q^j,\hat{z}^i)}>0$$

(9b) $$\frac{d\hat{z}^i}{dD^j}=0$$

(9c) $$\frac{d\hat{z}^i}{dI^i}=\frac{C_I^i}{R_z^i(q^i,q^j,\hat{z}^i)}<0$$

(9d) $$\frac{d\hat{z}^i}{dI^j}=0$$

(9e) $$\frac{d\hat{z}^i}{dq^i}=\frac{C_i^i-R_i^i(q^i,q^j,\hat{z}^i)}{R_z^i(q^i,q^j,\hat{z}^i)}$$

(9f) $$\frac{d\hat{z}^i}{dq^j}=\frac{-R_j^i(q^i,q^j,\hat{z}^i)}{R_z^i(q^i,q^j,\hat{z}^i)}>0.$$

The expression in (9a) is positive, so as firm i's debt level increases, its debt demand state increases. This is not surprising given that revenue increases with the state variable z. Inequality (9c) is negative because investment lowers the cost of producing any quantity, q^i, while leaving the revenue function unchanged.

The expected value to the shareholders of firm i is

$$(10) \qquad V^i = \int_{\hat{z}^i}^{\bar{z}} [R^i(q^i, q^j, z) - C^i(q^i, I^i) - D^i] f(z)dz - I^i + W^i.$$

The cashflow, W^i, has already been received by shareholders at the time of the quantity decision. Therefore, the quantity choice at stage 3 will not affect this cashflow and, $dW^i/dq^i=0$. Assuming that each firm chooses quantity to maximize the value of equity, each firm is maximizing equation (10). The first order condition is:[16]

$$(11) \qquad V^i_i = \int_{\hat{z}^i}^{\bar{z}} [R^i_i(q^i, q^j, z) - c^i(q^i, I^i)] f(z)dz = 0.$$

The first term of (11) is the expected marginal revenue received by firm i's shareholders when q^i is increased, and the second term is the expected marginal cost paid by firm i's shareholders when q^i is increased. The integral is over the states z in which there are residual claims for the shareholders. The second order condition to guarantee a maximum is:

SOC: $V^i_{ii}<0$.

Theorem 1a. An increase in debt, D^i, by firm i causes an increase in their quantity, q^i, and a decrease in the competitor's quantity, q^j, i.e., $\frac{dq^i}{dD^i} > 0$ and $\frac{dq^j}{dD^i} < 0$.

[16]The complete first order condition is:

$$V^i_i = \int_{\hat{z}^i}^{\bar{z}} [R^i_i(q^i, q^j, z) - c^i(q^i, I^i)] f(z)dz - [R^i(q^i, q^j, \hat{z}^i) - C^i(q^i, I^i) - D^i] f(\hat{z}^i) \frac{d\hat{z}^i}{dq^i} = 0,$$

but the second half of the first order condition is zero by equation (8). Thus, equation (11) characterizes the quantity that firm i will choose.

Theorem 1b. If $V^i_{iI}>0$ then an increase in investment by firm i causes an increase in their quantity, q^i, and a decrease in the competitor's quantity, q^j. If $V^i_{iI}<0$ then an increase of investment by firm i has the opposite effects, i.e., causes a decrease in their own quantity, q^i, and an increase in the competitor's quantity, q^j.

Proof. See Appendix.

The first part of theorem 1 confirms that debt in this model has the same strategic effects that Brander and Lewis discovered. The only one reason for a firm to have debt in this model is to commit to a greater output. This strategic use of debt causes the competing firm to lower production in stage 3. When firm i increases its debt, this increases the debt demand state, $\hat{z}^i$, of firm i. Shareholders are only concerned about states, z, where stage-3 profits are sufficient to repay the debtholders, i.e., $z > \hat{z}^i$. Marginal revenue is increasing in z, so when $\hat{z}^i$ increases, the marginal revenue relevant to shareholders also increases. With higher marginal revenue, conditional on $z>\hat{z}^i$, the firm will increase production. Firm i is committed to a more aggressive product market position, and this causes the competitor to reduce output.

The second part of theorem 1 characterizes how investment affects the product market decision. When a firm increases its investment, this has two effects on the product market decision. First, the firm now has lower costs. With lower costs the firm has an incentive to increase production. This is the strategic effect of investment analyzed by Brander and Spencer (1983). Second, due to lower costs of production the firm is less likely to be bankrupt. When making the product market decision, the equity holders now care about more states of demand, z, under which the firm has residual cash flow. Shareholders are now optimizing over a larger set of possible states, and states where the marginal benefit of production is the smallest are being added. This induces a desire for the firm to decrease the amount that it produces. The sign of V^i_{iI} captures these two opposing effects and determines which one dominates.

IV Investment Choice Equilibrium

After the debt levels are determined and observed, the firms decide how much to invest in cost reduction. Recall that investment lowers the marginal cost of production, which in turn can lower total production costs. Investment is observed prior to the quantity choice of stage 3; thus, a firm can gain the strategic advantage of having lower marginal cost through additional investment. When a firm is observed to have a lower marginal cost, the rival will decrease their output, allowing the firm to increase both output and expected profits.

The investment levels are chosen, taking the debt levels of each firm as given and correctly anticipating the output market equilibrium that will occur in stage 3, which is the simultaneous solution of each firm's first order condition in the quantity game, equation (11). The quantities both firms will choose in stage 3 can be written as functions of the investment levels they chose in stage 2, i.e., $q^i=q^i(I^i,I^j)$ and $q^j=q^j(I^i,I^j)$. Substituting these functions into the shareholder value equation yields expected profits to shareholders as a function of investment. The firms choose investment levels to maximize the value of their equity. The expected value to firm i shareholders when the firm invests I^i is

$$V^i = \int_{\hat{z}^i}^{\bar{z}} [R^i(q^i(I^i,I^j),q^j(I^i,I^j),z) - C^i(q^i(I^i,I^j),I^i) - D^i]f(z)dz - I^i + W^i.$$

The term under the integral is the expected revenue firm i will earn net of debt payments. The minus I^i reflects the fact that investment costs must be paid up front by the shareholders. The first order condition for optimal investment is:[17]

(12) $$V_I^i = \int_{\hat{z}^i}^{\bar{z}} [R_I^i(q^i,q^j,z) - C_I^i(q^i,I^i)]f(z)dz - 1 = 0$$

[17]For notational convenience the arguments to the quantity function are dropped. For the remainder of this section $q^i=q^i(I^i,I^j)$. Also note: $R_I^i = \frac{dR^i}{dq^i}\frac{dq^i}{dI^i} + \frac{dR^i}{dq^j}\frac{dq^j}{dI^i}$, and $C_I^i = \frac{dC^i}{dI^i} + \frac{dC^i}{dq^i}\frac{dq^i}{dI^i}$. There is an additional term in the first order condition, $-[R^i(\hat{z}^i) - C^i - D^i]\frac{d\hat{z}^i}{dI^i}$, which is zero by (4).

$$\int_{\hat{z}^i}^{\bar{Z}} [R_I^i(q^i, q^j, z) - C_I^i(q^i, I^i)] f(z)dz = 1$$

and the corresponding second order condition is:

SOC: $V^i_{II}<0$.

Theorem 2. If $R_I^i(q^i, q^j, \hat{z}^i) - C_I^i(q^i, I^i) > 0$, then debt and investment are substitutes and an increase in debt causes an increase in the rival's investment. If $R_I^i(q^i, q^j, \hat{z}^i) - C_I^i(q^i, I^i) < 0$, then debt and investment are complements and an increase in debt causes a decrease in the rival's investment.

Proof. See Appendix.

Theorem 2 states that $R_I^i(q^i, q^j, \hat{z}^i) - C_I^i(q^i, I^i) > 0$ is the necessary and sufficient condition for debt and investment to be substitutes. When a firm increases its debt level this has two effects on the investment decision. First, the firm has the incentive to invest more to lower costs because it will be increasing the quantity it produces in the product market (see theorem 1). Second, an increase in debt magnifies the debt overhang problem. Since the shareholders pay for the entire cost of the investment and only receive benefits from the investment if the firm is solvent, an increase in debt through an increase in the likelihood of bankruptcy gives the firm an incentive to lower investment expenditures. When $R_I^i(q^i, q^j, \hat{z}^i) - C_I^i(q^i, I^i) > 0$, the debt overhang effect dominates, and debt and investment are substitutes. Under this condition, an increase in debt allows a decrease in investment while leaving the strategic commitment in place. This helps to mitigate the over-investment problem (in the sense that with zero debt the firm will invest more than that which minimizes total cost of production[18]) and allows the firm to move towards efficient investment levels.

An increase in debt by a firm increases the commitment of that firm to produce aggressively in the product market at the time when investment decisions are made. This has two effects on the rival's

[18] This is shown by Brander and Spencer (1983).

investment decision. The competing firm knows it will face more aggressive competition in the product market, which results in lower revenues in all states of the world; this causes the rival to reduce investment. The competing firm also knows that it will be facing a firm which invests less, which causes the competitor to increase investment. The latter effect dominates when $R^i_I(q^i, q^j, \hat{z}^i) - C^i_I(q^i, I^i) > 0$. If $R^i_I(q^i, q^j, \hat{z}^i) - C^i_I(q^i, I^i) < 0$, then the former effect dominates and debt and investment are complements. In this case, an increase in the debt of firm i causes an increase in investment for firm i, and a decrease in investment for firm j.

Theorem 3. If marginal revenue is positive then debt and investment are substitutes.[19]

Proof. See Appendix.

Theorem 3 states that a sufficient, but not necessary, condition for debt and investment to be substitutes is that marginal revenue is positive. Although we do not expect this condition to always hold, there are situations when we expect $R^i_i > 0$, and the condition, $R^i_I(q^i, q^j, \hat{z}^i) - C^i_I(q^i, I^i) > 0$, may still hold when marginal revenue is not positive.

V Debt Choice Equilibrium

In stage 1 the firms choose debt levels simultaneously. These debt levels are chosen with the knowledge of how they will affect both the quantity equilibrium and the investment equilibrium, which are the simultaneous solutions to each firm's first order conditions, equations (11) and (12) respectively. The buyers of debt have perfect information, and there is a competitive market for the sale of debt. W^i is the fair

[19] Moreover, if marginal revenue is positive at the relevant debt demand state then debt and investment are substitutes (see proof).

market value of the debt issued. W^i will be the expected payoff to debt holders given the face value of each firm's debt, D^i and D^j, and how those debt levels will affect the investment and quantity choice of each firm. Let $D = (D^i,D^j)$. Then each firm's investment level can be written as a function of D, i.e., $I^i(D)$. Let $I = (I^i(D),I^j(D))$. Then each firm's quantity choice can be written as a function of D. Substituting these relations into equation (10), yields the expected value to shareholders expressed as a function of debt levels:

$$V^i = \int_{\hat{z}^i}^{\bar{Z}} [R^i(q^i(D),q^j(D),z) - C^i(q^i(D),I^i(D)) - D^i]f(z)dz - I^i(D) + W^i.$$

Similarly, W^i can be expressed as a function of debt levels as follows:

$$W^i = \int_{\underline{Z}}^{\hat{z}^i} [R^i(q^i(D),q^j(D),z) - C^i(q^i(D),I^i(D))]f(z)dz + D^i(1 - F(\hat{z}^i))$$

The term under the integral of W^i is the profits of firm i. This is integrated over the states of nature when the firm is bankrupt. In these states the debtholders receive all of the firm's profits. The second term is the cashflow received by debtholders when the firm can completely pay its debt obligations (the face value of the debt) multiplied by the probability that the firm will be solvent. Note that $\int_{\hat{z}^i}^{\bar{Z}} [D^i]f(z)dz = D^i(1 - F(\hat{z}^i))$, which implies that

$$V^i = \int_{\hat{z}^i}^{\bar{Z}} [R^i(q^i(D),q^j(D),z) - C^i(q^i(D),I^i(D))]f(z)dz - I^i(D) + \int_{\underline{Z}}^{\hat{z}^i} [R^i(q^i(D),q^j(D),z) - C^i(q^i(D),I^i(D))]f(z)dz.$$

Firm i maximizes V^i with respect to the choice of debt level. The first order condition is[20]

(13) FOC: $\dfrac{dV^i}{dD^i}$

(13.1) $= [\int_{\hat{z}^i}^{\bar{Z}} [R^i_i(q^i,q^j,z) - c^i(q^i,I^i)]f(z)dz]\dfrac{dq^i}{dD^i}$

(13.2) $+ [\int_{\underline{Z}}^{\hat{z}^i} [R^i_i(q^i,q^j,z) - c^i(q^i,I^i)]f(z)dz]\dfrac{dq^i}{dD^i}$

[20] Note that I can be written as a function of D, and q can be written as a function of D and I, which can further be represented as just a function of D. Once q is expressed solely as a function of D (incorporating the implicit change in I) the following first order condition is correct.

(13.3) $$+[\int_{\hat{z}^i}^{\overline{Z}} R^i_j(q^i,q^j,z)f(z)dz + \int_{\underline{Z}}^{\hat{z}^i} R^i_j(q^i,q^j,z)f(z)dz]\frac{dq^j}{dD^i}$$

(13.4) $$+[\int_{\hat{z}^i}^{\overline{Z}} [R^i_I(q^i,q^j,z) - C^i_I(q^i,I^i)]f(z)dz - 1]\frac{dI^i}{dD^i}$$

(13.5) $$+[\int_{\underline{Z}}^{\hat{z}^i} [R^i_I(q^i,q^j,z) - C^i_I(q^i,I^i)]f(z)dz]\frac{dI^i}{dD^i}$$

(13.6) $$+[\int_{\hat{z}^i}^{\overline{Z}} R^i_J(q^i,q^j,z)f(z)dz + \int_{\underline{Z}}^{\hat{z}^i} R^i_J(q^i,q^j,z)f(z)dz]\frac{dI^j}{dD^i} = 0.$$

The first and fourth terms of (13) are zero by equations (11) and (12) respectively. The second term,

(14) $$[\int_{\underline{Z}}^{\hat{z}^i} [R^i_i(q^i,q^j,z) - c^i(q^i,I^i)]f(z)dz]\frac{dq^i}{dD^i},$$

is the change in the value of the debt caused by the induced change in q^i. This is the additional agency cost of adding debt due to the change in quantity that firm i chooses. $R^i_i(.)$-$c^i(.)$ is the change in profits caused by a change in q^i. This is integrated over the states, z, in which the firm is in default to determine the expected change in debt value when q^i changes. In the demand states in which the firm is bankrupt the change in the value of the debt is the change in profits (revenue - costs) since here the debtholders receive all of the firm's profits.[21] This is multiplied by the change in q^i caused by a change in D^i, $\frac{dq^i}{dD^i}$. This term is less than zero by theorem 1 and equation (11), along with the assumption that $R^i_{iz}>0$. Theorem 1 states that $dq^i/dD^i > 0$. Equation (11), along with the assumption that $R^i_{iz}>0$, implies that

$$R^i_i(q^i,q^j,\hat{z}^i) - c^i(q^i,I^i) < 0.$$

Equation (11) states that the integral from $\hat{z}^i$ to $\overline{Z}$ of $R^i_i(.)$ - $c^i(.)$ is equal to zero. Since marginal revenue is increasing in z and marginal cost is constant in z, it must be the case that $R^i_i(.)$ - $c^i(.)<0$ at $\hat{z}^i$. As z

[21]In states where the firm is not bankrupt a change in the firm's quantity does not affect the payoff to debtholders since they receive the same payoff, the face value of debt, regardless of the profits of the firm.

decreases below $\hat{z}^i$, the value of the above equation decreases further; therefore, $R^i_i(.) - c^i(.) < 0$ for all $z \leq \hat{z}^i$. This confirms that equation (14) is less then zero, which means that an increase in debt magnifies the conflict between debtholders and equityholders with respect to desired quantity choice. When a firm increases its debt this induces an increase in the quantity produced. The firm chooses quantity to maximize equity value, and thus this is the quantity that is best for the shareholders. As shown above this increase in quantity decreases the debt value of the firm. The difference between the quantity that would maximize the debt value and the quantity that would maximize the equity value is increasing as the debt level increases.

The third term is the change in value to equityholders and debtholders caused by the induced change in q^j. This is the strategic value of additional debt on the quantity game. The first integral is the change in the equity value of the firm and the second integral is the change in the debt value of the firm.

(15) $$\int_{\hat{z}^i}^{\bar{z}} R^i_j(q^i, q^j, z) f(z) dz \frac{dq^j}{dD^i} + \int_{\underline{z}}^{\hat{z}^i} R^i_j(q^i, q^j, z) f(z) dz \frac{dq^j}{dD^i}$$

$$= \left[\int_{\underline{z}}^{\bar{z}} R^i_j(q^i, q^j, z) f(z) dz \right] \frac{dq^j}{dD^i}.$$

Equation (15) is the strategic effect of debt on the quantity game. Since an increase in debt by firm i causes q^j to decrease (from theorem 1), and $R^i_j < 0$ by assumption (1a) both terms of equation (15) are greater than zero. The strategic effect of debt causes the rival firm to decrease their output which increases both the debt value and the equity value of the firm.

The fifth term is

(16) $$\int_{\underline{z}}^{\hat{z}^i} [R^i_I(q^i, q^j, z) - C^i_I(q^i, I^i)] f(z) dz \frac{dI^i}{dD^i},$$

which is the change in agency costs of debt on the investment game. This term can be positive or negative since the induced change in investment can make the debtholders either better or worse off. Thus, increasing debt can either increase of decrease the conflict between debt and equity over the desired choice of investment. R^i_I - C^i_I is the change in profits caused by a change in investment. This is integrated over the states where the firm is bankrupt to get the expected change in profits, from a change of investment, in the states where the debt holders receive all the profits. This is multiplied by the change in investment induced by a change in debt. This yields the change in the debt value of the firm from the induced change in I^i. If marginal revenue in stage 3 is positive then (16) is negative, and under this condition an increase in debt also exacerbates the conflict between shareholders and debtholders with respect to the optimal investment choice. If equation (16) is positive then an increase in debt reduces the agency costs from the conflict between debt and equity over the choice of investment level and this reduction in agency costs increases firm value.

The last term is

$$(17) \qquad \int_{\hat{z}^i}^{\bar{z}} R^i_J(q^i, q^j, z) f(z) dz \frac{dI^j}{dD^i} + \int_{\underline{z}}^{\hat{z}^i} R^i_J(q^i, q^j, z) f(z) dz \frac{dI^j}{dD^i}$$

$$= \int_{\underline{z}}^{\bar{z}} R^i_J(q^i, q^j, z) f(z) dz \frac{dI^j}{dD^i},$$

which is the strategic value of additional debt on the investment game. R^i_J is the change in revenues to firm i from a change in the investment of the rival. The first term of (17) is the change in revenues when the firm is solvent, and the second term is the change in revenues when the firm is bankrupt. This is the effect of the induced change in I^j on the equity and debt value of firm i respectively. If marginal revenue is positive then an increase in debt by firm i causes firm j to increases investment which decreases the equity value and the debt value of firm i. This can been seen by observing that the final term of equation (13) is negative when $R^i_i>0$.[22]

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[22] $R^i_J = R^i_i \frac{dq^i}{dI^j} + R^i_j \frac{dq^j}{dI^j} + R^i_z \frac{dz}{dI^j} < 0$ when $R^i_i>0$, because $R^i_j<0$ by (1a), and $\frac{dz}{dI^j} = 0$ (see footnote 12). The result follows from theorem 1. The sign of equation (17) follows from theorem 2.

When marginal revenue is positive then increasing the face value of debt has four effects on the value of the firm, three which lower the value of the firm and one which raises the value of the firm. More debt lowers the value of the firm by increasing the conflict between debt and equityholders with regard to the desired choice of investment and quantity. Debt also lowers the value of the firm through the effect on the competition's choice of investment. When the firm increases its debt level the rival increases its investment which lowers the value of the firm. An increase in debt raises the value of the firm through the effects on the competition's choice of quantity. In particular, an increase in debt causes the competition to decrease quantity. When the firm's debt level is small, the conflict between debtholders and equityholders is insignificant, and the negative effect of an increase in debt due to these conflicts is negligible. Specifically, when D=0 there are no debtholders, and therefore no conflict between the claimants. This becomes evident from examining equations (14) and (16) and noting that D=0 implies that $\hat{z}^i = \underline{Z}$; therefore, equations (14) and (16) are both equal to zero. Increasing debt from zero has two effects on firm value. Firm value is increased due to the decrease in rival output. Firm value is decreased due to the increase in rival investment.[23]

VI Increasing Demand

This section examines how an arbitrary shift in the distribution of demand changes each of the individual effects characterized in section V. The effects are examined by holding all other variables constant and determining how each term changes as the underlying distribution of demand is increased.

Let g(.) be the probability density function of a random variable distributed continuously on $[\underline{Z}', \overline{Z}']$. Assume that the distribution g(.) dominates f(.) in the sense of First Degree Stochastic Dominance.

[23]Recall that in this case it is assumed that marginal cost is positive, thus debt and investment are substitutes.

This means for any z, $G(z) \leq F(z)$, which implies that $\underline{Z}' \geq \underline{Z}$ and $\overline{Z}' \geq \overline{Z}$. First consider the agency costs of debt on the quantity game, which is equation (14)

$$(14) \qquad [\int_{\underline{Z}}^{\hat{z}^i} [R_i^i(q^i, q^j, z) - c^i(q^i, I^i)] f(z) dz] \frac{dq^i}{dD^i}.$$

For each state $z < \hat{z}^i$ the term under the integral in equation (14) is negative and increasing in z. If the distribution of demand is shifted to g(.) then equation (14) becomes

$$(14') \qquad [\int_{\underline{Z}'}^{\hat{z}^i} [R_i^i(q^i, q^j, z) - c^i(q^i, I^i)] g(z) dz] \frac{dq^i}{dD^i}.$$

Equation (14') is larger than equation (14) because the term under the integral is negative, increasing in z, and g(.) dominates f(.). This means that if the support of demand increases *ex ante* the agency cost of debt on the quantity decision is reduced. Note that in the extreme case as demand increase to the point of the debt becoming riskless the agency cost of debt goes to zero, as expected.

Second examine the strategic value of debt on the quantity game, equation (15),

$$(15) \qquad = [\int_{\underline{Z}}^{\overline{Z}} R_j^i(q^i, q^j, z) f(z) dz] \frac{dq^j}{dD^i}.$$

When the distribution of z is shifted to g(.), then equation (15) becomes

$$(15') \qquad = [\int_{\underline{Z}'}^{\overline{Z}'} R_j^i(q^i, q^j, z) g(z) dz] \frac{dq^j}{dD^i}.$$

Whether equation (15') is larger or smaller than (15) depends on the sign of R^i_{jz}, i.e., whether the integrand is increasing or decreasing in z. Recall that R^i_j is negative, that is, as firm j increase its output the revenue to firm i falls. The sign of R^i_{jz} tells us whether the revenue of firm i falls more or less in reaction to an increase of rival output when demand is higher. If $R^i_{jz} > 0$, which means that when demand is higher an increase in rival output decrease own firm revenue by less than when demand is lower, then the strategic value of debt is smaller when demand increases. The strategic value of debt is the effect of committing to a large output and therefore inducing the rival firm to decrease its output. If a reduction in rival output increase own firm revenue by less (which it does when demand is higher and $R^i_{jz} > 0$) then the strategic value of debt decreases. If, on the other hand, $R^i_{jz} < 0$ then the strategic value of debt is larger when demand is higher.

Higher demand may increase or decrease the agency costs associated with the investment choice, equation (16), and the result depends on several parameters. For instance if $V_{iI} > 0$ and $R^i_I > 0$ then it must be the case that $R^i_I(.)-C^i_I>0$ at $\hat{z}^i$ (since $R^i_I(.)-C^i_I>0$ when $R^i_I>0$ and $C^i_I<0$ (from 2a)). From theorem 2 $R^i_I(.)-C^i_I>0$ implies that $dI^i/dD^i<0$. In this situation equation (16) is negative so an increase in debt magnifies the conflict between debt and equity holders over the optimal investment choice. Under these conditions the agency costs of debt associated with the investment choice decreases when demand is higher. Under different initial conditions, however, it is possible that these agency costs increase when demand is higher.

The change in the strategic value of debt on the investment game, equation (17), depends on the sign of R^i_{Jz}, and dI^j/dD^i. If $R^i_{Jz}<0$ ($R^i_{Jz}>0$), which means that as the realization of demand, z, increases the revenue received by firm i decreases (increases) as the competitor increases their investment spending, and $dI^j/dD^i<0$ ($dI^j/dD^i>0$), which means that as firm i increases its debt level this induces the competitor to decreases (increases) their investment, then as demand increases the strategic value of debt on the investment decision increases (decreases).

VII Numerical Examples

To better understand how the choice variables change in equilibrium with variation in the distribution of z, I have computed equilibria in numerical examples with discrete random demand. I assume that the firms produce identical goods, and that the inverse demand function is

$$P = A[z] - q^1 - q^2.$$

There are seven equally likely states of demand, z = 1, 2, ..., 7, with A[z] increasing with z. The cost of production for firm i is $C^i(q^i,I^i) = (40 - \sqrt{I^i})q^i$.

First I examine how equilibrium decisions change as demand increases. These examples show that as demand increases the equilibrium investment and quantities increase. The equilibrium debt level, however, does not move monotonically as demand increases. Second I examine how the equilibrium decisions change as the variance of demand states increases. It is assumed that the variance increases through a mean preserving spread. When the variance of demand increases the equilibrium quantities increase. This is the only variable that changes monotonically when the uncertainty about demand increases.

Increasing Demand

I first examine the case of increasing demand. These results are reported in table I. In case 1, the seven possible states of demand are 90, 95, 100, 105, 110, 115, and 120. Case 1 has a unique symmetric sequential Nash equilibrium in debt demand state, investment, and quantity. Demand is discrete, and consequently there is a range of possible debt levels that will support each debt demand state. In equilibrium, firms issue debt such that the debt demand state is demand state $z=3$ or $A(z)=100$. For this to occur, D must belong to the interval [430.25, 491.83]. The equilibrium investment level is $I=174.44$, and the equilibrium quantity is $q=27.74$. Given these choices, the expected price is 49.53, and each firm's expected profit is $456.16.

In case 2, demand increases by one over the case 1 values in each state. Case 2 demands are 91, 96, 101, 106, 111, 116, and 121. As demand increases from case 1 to case 2, the equilibrium debt demand state remains constant. The expected demand relevant to shareholders ($E[\, A(z) \mid z \geq 3 \,]$) increases from 110 to 111. This generates an increase in investment to 179.46 and quantity to 28.13. The debt levels that support this equilibrium increase to the interval [450.07, 510.28]. Expected price and expected profits are also up to 49.74 and $471.29 respectively. In this example an increase in demand increases all endogenous choice variables, expected price, and expected profits.

Case 3 has demand again increasing by one in each state. Case 3 demands are 92, 97, 102, 107, 112, 117, and 122. Due to this increase in demand, the debt demand state decreases from 3 to 2. Any debt levels within the interval [405, 461.07] support the equilibrium. This interval is below the case 2 equilibrium, which shows that the minimum and maximum debt levels that support the equilibrium are not monotonic in level of demand. Investment and quantities are up to 267.42 and 28.62 respectively. The large jump in investment spending is due to the fact that debt and investment are substitutes for commitment. By lowering the commitment of debt in stage 1 the firms now try to commit more with investment in stage 2. The fact that most of the additional investment spending is for commitment can be seen by examining the optimal investment levels in case 2 and case 3. In case 2, actual investment is just below optimal investment, while in case 3, actual investment in significantly above optimal investment. The only explanation for investing higher than optimal levels is to commit to an aggressive product market position in stage 3. Expected price is up slightly to 49.76 and expected profits are up to $480.

In Case 4, the demand states are 91.5, 96.5, 101.5, 106.5, 111.5, 116.5, and 121.5. These demand levels are between case 2 and case 3 demand levels. This case has two symmetric sequential Nash equilibria, which allows us to examine how the endogenous variables react when only the debt demand state changes. Obviously, the support of equilibrium debt levels decreases when the debt demand state falls. The firm now uses debt to eliminate a lower number of demand states, and, therefore, less debt is needed. The firm increases investment in an attempt to make up for the commitment lost by the lower debt. Firms over-invest in the second equilibrium. Although shareholders are now concerned about a lower demand state, quantity increases. This is due to the large increase in investment, which simultaneously lowers marginal cost and increases commitment. Both of these effects induce larger outputs by the firm.

Table 2 shows cases of increasing demand around the case with two equilibria. Moving from case 1 to case 3 yields the unexpected result that an increase in demand can create a decrease in both price and expected profits. This is caused by the firms' increase in investment spending. In case 3 the firms’ debt demand state drops to z=2 from z=3 in case 1. To compensate for the loss of commitment from stage 1, the firms now use investment strategically. Investment increases from 180.47 to 264.35, which is a move from under-investment to over-investment. This over-investment eliminates any gains in profits that would otherwise have been realized from an increase in demand.

These examples show that an increase in demand has monotonic effects on both investment and quantity choices. The interval of debt levels that support the equilibria, however, does not move monotonically as demand increases. The investment result will generalize due to the fact that all the effects on investment as demand increases are positive. The quantity result may not be robust. As demand increases, the debt demand state decreases. This implies that the shareholders must now consider lower states of demand when choosing quantity. This has the direct consequence of lowering the desired quantity. The increase in investment has two indirect effects,[24] both of which increase the desired quantity. There is no reason to believe that the direct effect cannot dominate for some parameter values.

For comparison Table 5 examines the case of no investment. This is the Brander and Lewis model. The equilibrium debt demand state is state 6. As demand increases, the maximum and minimum elements of the support of equilibrium debt levels increase. Quantity also increases with demand. Expected price and expected profits also rise. Table 6 considers the case with no debt. This is the Brander and Spencer model. Here the debt demand state is 1 (since there is no debt). As demand increases, investment and quantity increase. Expected price and expected profits also increase.

Changes in Demand Variance

Table 3 illustrates how the decision variables change as the variance of demand increases. The variance is increased through a mean preserving spread. The average demand in each case is 100. The

[24] Investment lowers the marginal cost of production, which causes an increase in quantity. Investment also causes the competing firm to lower its production, which also causes an increase in own quantity.

probabilities of each state remain the same, while the support of possible demand states increases. As the variance of the demand states increases, the value of committing with debt increases. When the variance is high then the expected demand realization conditional upon $z > \hat{z}^i$ is larger than when the variance is small. The firm chooses investment and quantity based on expected demand conditional on the firm having enough profits to pay back the debt. Thus, when the variance of demand is high the same level of debt commits the firm to more aggressive output market decisions than when demand variance is low. This increases the relative propensity of firms to use debt instead of investment as a commitment device. This is why as the variance of demand increases we see the debt demand state increasing and investment decreasing.

In this example as variance increases, investment, expected price, and expected profits decrease, while quantity increases. Debt levels are not monotonic in the variance of demand. As the variance increases, the equilibrium debt demand state rises. This precipitates a lower investment level due to the fact that debt and investment are substitutes for strategic commitment. The quantity produced increases due to the increase in expected demand relevant to shareholders. This has a direct positive effect on investment levels, as an increase in quantity makes more investment favorable. In these cases, the commitment effect dominates.

These examples, however, all have sufficiently large increases in the variance to produce an increase in the equilibrium debt demand state. Table 4 examines increases in the variance where the debt demand state remains constant. Here we see that the monotonicity of quantity, expected price, and expected profit remain intact; however, investment is increasing rather than decreasing over these cases. This increase in investment is due to the fact that quantity increases, but the debt demand state remains the same, so there is no negative effect on investment.

From these limited examples, quantity is the only decision variable that is monotonically increasing in variance. These examples allow the support of the demand distribution to move. If the variance of demand increases with the support remaining constant, we may see more decision variables changing monotonically.

VIII Extensions

There are two natural modifications of this model which are worthwhile to investigate. First, it is interesting to examine how debt and investment interact when the order of the decision variables are changed. For instance, a similar model where the firms first choose investment levels and then choose capital structure can be investigated. Second, it is natural to be curious about whether the results of this model are robust to different types of product market competition.

Consider a model where first, firms choose investment levels; second, firms choose capital structure; and finally, firms compete under Cournot competition in the product market. First note that from the point in the game where firms choose capital structure to the end game, the game is exactly the same as that developed by Brander and Lewis.[25] The firms will always choose positive debt levels due to the strategic effect of debt on the product market.

Investment affects both the product market decision and the capital structure decision. Investment has a strategic effect on the product market decision. Brander and Spencer demonstrated that by lowering the cost of production, the firm will be committed to producing more in the product market. Investment has two effects on the capital structure stage (both of these effects have consequences on the product market stage indirectly). First, investment lowers the cost of production. This decreases the agency costs of debt in two ways. With lower costs of production there is an increase in stage-3 cashflow available to distribute to claimants, regardless of the state of demand; thus, ceteris paribus the value of the debt increases.[26] With

[25] At the stage when the firms are making their capital structure decision, investment is fixed and thus this subgame reduces to the game developed by Brander and Lewis.

[26] Recall that investment is paid for up front and excess cash is distributed to shareholders, thus an increase in investment only increases cash flow in the final stage through lower costs of production.

lower agency costs, the firm would want to increase the amount of debt issued (as long as the benefits of the debt are not changed). Second, investment can change the strategic commitment value of debt. Whether investment increases or decreases the commitment value of debt depends on second derivatives. If the strategic value of debt is increased through lower costs, then debt and investment will always be compliments. If lower costs decrease the strategic value of debt, then debt and investment can be substitutes.

The second interesting extension is to investigate the model with Bertrand competition in the product market. It is well known that many results of theoretical models change when the type of product market competition is changed from Cournot to Bertrand. This is due to the fact that in Cournot competition the firms choices are substitutes, where in Bertrand competition the firms choice variables are complements.

How would the interaction of debt and investment behave if the original model were changed so the firms first choose capital structure, second choose investment, and third compete with prices in the product market? Debt has only a strategic effect on the product market in this model. Showalter (1995) shows that if the Brander and Lewis model is changed to Bertrand competition in the product market, then an increase in debt causes both firms to increase price (and thus decrease quantity produced). Debt is still used strategically, but instead of committing the firms to aggressive output market positions, debt allows the firms to coordinate on high prices. This is the opposite of the result with Cournot competition.

Debt has two effects on the investment choice. An increase in debt leads to lower production which makes the firm want to invest less. More debt also magnifies the debt overhang problem which also leads the firm to decrease its investment. Hence, with Bertrand competition, debt and investment are substitutes. This result may also be obtained from the original model; thus, changing from Cournot to Bertrand competition may not affect the interaction between the capital structure and the investment choice.

IX Conclusion

This paper illustrates how capital structure, investment, and product market decisions are interrelated. The results obtained when modeling the interaction of only two of these choices may be reversed when the third choice variable is introduced. These decisions are analyzed under a specific structure where the decisions are assumed to occur sequentially. First, financial structure is chosen, second, investment is chosen, and finally, firms choose quantities to be distributed to the product market. Under this configuration, the financial structure decision has an effect on both the investment and the product market stages of the game. These effects are due to the limited liability of equity. In the product market, debt commits the firm to an aggressive output decision, causing an increase in own output and a decrease in rival's output. Debt has two effects on the investment decision. First, the firm increases output, which raises the marginal benefit of lower marginal cost, and thus the firm wants to increase investment. Second, investment is used strategically as a commitment to high outputs in the product market. The complete effect of debt on investment depends on whether investment and debt are substitutes or compliments for commitment in the product market.

This paper provides necessary and sufficient conditions for debt and investment to be substitutes. For example, when investment is paid for by shareholders, debt and investment are substitutes if marginal revenue is positive. This mitigates the over-investment result obtained by Brander and Spencer (1983).

In addition, this paper demonstrates how recent empirical evidence on the relation between debt, investment, and product market behavior can be consistent with a model emphasizing the limited liability of equity. This framework is perhaps more palatable than the predatory theories because firms rationally take on debt. Whereas in the predatory models the optimal debt level is zero. Further research is necessary to distinguish between these two theories.

APPENDIX

Proof of theorem 1.

Totally differentiate equation (11) and the corresponding first order condition for firm j with respect to q^i, q^j, and D^i. This gives the following equations:

$V^i_{ii}dq^i+V^i_{ij}dq^j+V^i_{iD}dD^i=0$

$V^j_{ji}dq^i+V^j_{jj}dq^j+V^j_{jD}dD^i=0.$

Note: $V^j_{jD} = -[R^j_j(q^i,q^j,\hat{z}^i)-C^j_j(q^j,I^j)]d\hat{z}^j/dD^i = 0$, because $d\hat{z}^j/dD^i = 0$ (see equation (9b)). Thus, the equations simplify to

$V^i_{ii}dq^i+V^i_{ij}dq^j=-V^i_{iD}dD^i$

$V^j_{ji}dq^i+V^j_{jj}dq^j=0.$

Solving these equations simultaneously gives:

(A1) $dq^i/dD^i = -V^j_{jj}V^i_{iD}/A$ where $A=V^i_{ii}V^j_{jj}-V^i_{ij}V^j_{ji}$

(A2) $dq^j/dD^i = V^j_{ji}V^i_{iD}/A$

$A>0$ by assumption, $V^j_{jj}<0$ by SOC, $V^j_{ji}<0$ by (4), so the signs of the above equations are determined by the sign of V^i_{iD}.

$V^i_{iD} = -[R^i_i(q^i,q^j,\hat{z}^i)-c_i(q^i,I^i)]f(\hat{z}^i)d\hat{z}^i/dD^i$

Equation (9a) shows that $d\hat{z}^i/dD^i > 0$. $R^i_{iz}>0$ by assumption, this means that as z increases marginal revenue increase. This along with the fact that the first order condition (equation (11)) is satisfied guarantees that $[R^i_i(q^i,q^j,\hat{z}^i)-c_i(q^i,I^i)] < 0$ (i.e., at $\hat{z}^i$ marginal revenue is less than marginal cost). This means that $V^i_{iD} > 0$, which implies that equation (A1) > 0 and (A2) < 0.

To derive dq^i/dI^i and dq^j/dI^i totally differentiate (11) with respect to q^i, q^j, and I^i. Also totally differentiate the corresponding FOC for firm j with respect to the same variables (i.e., q^i, q^j, and I^i). This yields the following equations:

$V^i_{ii}dq^i+V^i_{ij}dq^j+V^i_{iI}dI=0$

$V^j_{ji}dq^i+V^j_{jj}dq^j+V^j_{jI}dI=0$

Note using Leibnitz's rule: $V^j_{jI} = \int_{\hat{z}^j}^{\bar{Z}} [R^j_{jI}(q^1,q^2,z)-c^j_I(q^i,I^i)]f(z^j)dz - [R^i_i(q^1,q^2,z)-c^i(q^i,I^i)]f(\hat{z}^j)d\hat{z}^j/dI = 0$ because firm j's revenue and cost function are not effected by firm i's investment (so $R^j_{jI} = 0$ and $C^j_{jI} = 0$) and $d\hat{z}^j/dI = 0$ by (9d). $V^i_{iI} = \int_{\hat{z}^i}^{\bar{Z}} [R^i_{iI}(q^1,q^2,z)-c^i_I(q^i,I^i)]f(z)dz - [R^i_i(q^1,q^2,z)-c^i(q^i,I^i)]f(\hat{z}^i)d\hat{z}^i/dI$. Firm i's revenue does not depend on its investment (i.e., $R^i_{iI} = 0$) and firm i's cost are not dependent on the state of the world (c^i_I constant across states z), thus $V^i_{iI} = -(f(\bar{Z})-f(\hat{z}^i))c^i_I(q^i,I^i) - [R^i_i(q^1,q^2,z)-c^i(q^i,I^i)]f(\hat{z}^i)C^i_I/R^i_z(q^1,q^j,\hat{z}^i)$. This leaves the following equations:

$V^i_{ii}dq^i+V^i_{ij}dq^j=[(f(\bar{Z})-f(\hat{z}^i))c^i_I + [R^i_i(q^1,q^2,z)-c^i(q^i,I^i)]f(\hat{z}^i)C^i_I/R^i_z(q^1,q^j,\hat{z}^i)]dI^i$

$V^j_{ji}dq^i+V^j_{jj}dq^j=0$

Solving these equations simultaneously yields

(18) $dq^i/dI^i=V^j_{jj}[(f(\bar{Z})-f(\hat{z}^i))c^i_I+[R^i_i-c^i]f(\hat{z}^i)C^i_I/R^i_z(\hat{z}^i)]dI^i/A$ where $A=V^i_{ii}V^j_{jj}-V^i_{ij}V^j_{ji}$

(19) $dq^j/dI^i=-V^j_{ji}[(f(\bar{Z})-f(\hat{z}^i))c^i_I+[R^i_i-c^i]f(\hat{z}^i)C^i_I/R^i_z(\hat{z}^i)]dI^i/A$

Equation (18) is greater than zero if $V^i_{iI}>0$ (or equivalently if $-(\bar{Z}-\hat{z}^i)c^i_I > [R^i_i-c^i]f(\hat{z}^i)C^i_I/R^i_z]$) because $A>0$ by assumption, $\bar{Z}>\hat{z}^i$, $c^i_I<0$ by (2d), and $V^j_{jj}<0$ by SOC.

Equation (19) is the opposite sign of equation (18) because $V^j_{ji}<0$ see equation(4). QED

Proof of theorem 2.

To solve for dI/dD^i and dJ/dD^i first totally differentiate (12) with respect to I^i, I^j, D^i. We also totally differentiate the corresponding FOC V^j_J with respect to the same variables. This yields the following equations:

$V^i_{II}dI+V^i_{IJ}dJ+V^i_{ID}dD^i=0$

$V^j_{JI}dI+V^j_{JJ}dJ+V^j_{JD}dD^i=0.$

$$V^j_J = \int_{\hat{z}^j}^{\bar{Z}}[R^j_J(q^i,q^j,z)-C^j_J(q^j,I^j)]f(z)dz$$

$dV^j_J/dD^i=-[R^j_J(q^i,q^j,\hat{z}^i)-C^j_J(q^j,I^j)]d\hat{z}^j/dD^i=0$ because $d\hat{z}^j/dD^i=0$.

Thus the resulting equations are

$V^i_{II}dI+V^i_{IJ}dJ=-V^i_{ID}dD^i$

$V^j_{JI}dI+V^j_{JJ}dJ=0$

Solving these equations using simultaneously yields

(20) $dI/dD^i=-V^i_{ID}V^j_{JJ}/B$ where $B=V^i_{II}V^j_{JJ}-V^i_{IJ}V^j_{JI}$

(21) $dJ/dD^i=V^i_{ID}V^j_{JI}/B.$

We know $V^j_{JJ}<0$ by (SOC) and $B>0$ by (7), so the sign of dI^i/dD^i is the same as the sign of V^i_{ID}. Solving for V^i_{ID} gives

$V^i_{ID}=-[R^i_I(q^i,q^j,\hat{z}^i)-C^i_I(q^i,I^i)]f(\hat{z}^i)d\hat{z}^i/dD^i.$

$d\hat{z}^i/dD^i>0$ from (9a). Whenever $[R^i_I(q^i,q^j,\hat{z}^i)-C^i_I(q^i,I^i)]>0$ then $V^i_{ID}<0$ and $dI^i/dD<0$, alternatively whenever $[R^i_I(q^i,q^j,\hat{z}^i)-C^i_I(q^i,I^i)]<0$ then $V^i_{ID}>0$ and $dI/dD>0$. From (5) $V^j_{JI}<0$ thus dI^j/dD is the opposite sign of dI/dD from (20) and (21).

Notice that if $[R^i_I(q^i,q^j,\hat{z}^i)-C^i_I(q^i,I^i)]=0$ $V^i_{ID}=0$ and thus (20) and (21) are both equal to zero. Which means that a change in debt of firm i has no effect on the investment of either firm.

Proof of theorem 3.

Note $C^i_I<0$ by (2a), thus $[R^i_I(q^i,q^j,\hat{z}^i)-C^i_I(q^i,I^i)]>0$ whenever $R^i_I(q^i,q^j,\hat{z}^i)>0$. $R^i_I(q^i,q^j,\hat{z}^i)$ is the change in revenue in the worst state of the world that is relevant to shareholders when investment increases.

$R^i_I(q^i,q^j,\hat{z}^i)=R^i_i(q^i,q^j,\hat{z}^i)dq^i/dI^i+R^i_j(q^i,q^j,\hat{z}^i)dq^j/dI^i$

Observe that $dq^i/dI^i>0$ $dq^j/dI^i<0$ by equation(18) and equation(19) respectively and $V^i_{iI}>0$ by assumption. Thus whenever $R^i_i>0$ this implies that $[R^i_I(q^i,q^j,\hat{z}^i)-C^i_I(q^i,I^i)]>0$. In fact even if $R^i_i<0$, if $R^i_i(q^i,q^j,\hat{z}^i)dq^i/dI^i \geq -R^i_j(q^i,q^j,\hat{z}^i)dq^j/dI^i$ then $[R^i_I(q^i,q^j,\hat{z}^i)-C^i_I(q^i,I^i)]>0$ still holds. Actually even if $R^i_i(q^i,q^j,\hat{z}^i)dq^i/dI^i < -R^i_j(q^i,q^j,\hat{z}^i)dq^j/dI^i$, if $R^i_I(q^i,q^j,\hat{z}^i)> C^i_I(.)$ then $[R^i_I(q^i,q^j,\hat{z}^i)-C^i_I(q^i,I^i)]>0$ still holds. Although we don't expect this condition to always hold there are many situations when we expect $R^i_i>0$ and the condition still holds in many circumstances when marginal revenue is not positive, thus we do expect this condition to be satisfied in many instances.

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Table 1
Increasing Demand

Probability	Demand States			
	Case 1	Case 2	Case 4[27]	Case 3
1/7	90	91	91.5	92
1/7	95	96	96.5	97
1/7	100	101	101.5	102
1/7	105	106	106.5	107
1/7	110	111	111.5	112
1/7	115	116	116.5	117
1/7	120	121	121.5	122

Symmetric Nash Equilibrium results:

debt demand state	3	3	3|2	2
Min Debt	430.25	450.07	460.01|394.89	405
Max Debt	491.83	510.28	519.29|452.29	461.07
Investment	174.44	179.46	182.00|263.58	267.42
Quantity	27.74	28.13	28.33|28.41	28.62
Expected price	49.53	49.74	49.84|49.68	49.76
Expected profits	456.16	471.29	478.95|472.61	480.00
Optimal Investment [q^2/4]	192.38	197.82	200.65|201.78	204.78
W[min debt]	388.38	406.99	416.24|382.80	392.57
W[max debt]	432.36	450	458.58|432	440.63

[27]In case 4 there are two symmetric Nash equilibrium. One at debt demand state 2 and one at debt demand state 3. Allowing cheap talk would lead to the equilibrium at debt demand state 3.

Table 2
Increasing Demand

Probability		Demand States	
	Case 1	Case 2[28]	Case 3
1/7	91.2	91.5	91.6
1/7	96.2	96.5	96.6
1/7	101.2	101.5	101.6
1/7	106.2	106.5	106.6
1/7	111.2	111.5	111.6
1/7	116.2	116.5	116.6
1/7	121.2	121.5	121.6

Symmetric Nash Equilibrium results:

debt demand state	3	3|2	2
Min Debt	454.06	460.01|394.89	396.88
Max Debt	513.70	519.29|452.29	454.06
Investment	180.47	182.00|263.58	264.35
Quantity	28.21	28.33|28.41	28.45
Expected price	49.78	49.84|49.68	49.69
Expected profits	474.35	478.95|472.61	474.09
Optimal Investment [q^2/4]	198.95	200.65|201.78	202.35
W[min debt]	410.65	416.24|382.80	384.73
W[max debt]	453.25	458.58|432	433.74

[28]In case 2 there are two symmetric Nash equilibrium. One at debt demand state 2 and one at debt demand state 3. Allowing cheap talk would lead to the equilibrium at debt demand state 3.

Table 3
Increasing Variance of Demand

Probability	Demand States		
	Case 1	Case 2	Case 3
1/7	88	85	70
1/7	92	90	80
1/7	96	95	90
1/7	100	100	100
1/7	104	105	110
1/7	108	110	120
1/7	112	115	130

Symmetric Nash Equilibrium results:

Debt state	2	3	4
Min Debt	372.43	339.11	160.09
Max Debt	396.42	405.82	390.49
Investment	212.82	150.41	119.00
Quantity	25.53	25.75	28.64
Expected price	48.94	48.49	42.73
Expected profits	387.88	384.12	271.49
Optimal Investment [q^2/4]	162.95	165.77	205.06
W[min debt]	361.28	302.99	106.33
W[max debt]	381.84	350.64	237.97

Table 4
Increasing Variance of Demand

Probability	Demand States			
	Case 1	Case 2	Case 3	Case 4
1/7	85	83.5	82	76
1/7	90	89	88	84
1/7	95	94.5	94	92
1/7	100	100	100	100
1/7	105	105.5	106	108
1/7	110	111	112	116
1/7	115	116.5	118	124

Symmetric Nash Equilibrium results:

Debt state	3	3	3	4
Min Debt	339.11	302.99	267.65	190.49
Max Debt	405.82	388.21	370.02	425.86
Investment	150.41	152.73	155.07	109.68
Quantity	25.75	25.95	26.15	27.49
Expected price	48.49	48.09	47.70	45.02
Expected profits	384.12	378.08	371.89	316.18
Optimal Investment [q^2/4]	165.77	168.35	170.96	188.93
W[min debt]	361.28	266.17	229.66	136.06
W[max debt]	381.84	327.04	302.78	272.76

Table 5
Brander and Lewis Example
[No Investment]
Increasing Demand

Probability	Demand States			
	Case 1	Case 2	Case 3	Case 4
1/7	90	91	91.5	92
1/7	95	96	96.5	97
1/7	100	101	101.5	102
1/7	105	106	106.5	107
1/7	110	111	111.5	112
1/7	115	116	116.5	117
1/7	120	121	121.5	122

Symmetric Nash Equilibrium results:

debt demand state	6	6	6	6
Min Debt	462.91	477.75	485.01	492.63
Max Debt	602.87	619.18	627.71	636
Investment	0	0	0	0
Quantity	25.83	26.17	26.33	26.5
Expected price	53.33	53.67	53.83	54
Expected profits	344.44	357.61	364.28	371
W[min debt]	229.26	301.88	307.62	313.34
W[max debt]	332.25	343.80	349.97	355.86

Table 6
Brander and Spencer Example
[No Debt]
Increasing Demand

Probability	Demand States			
	Case 1	Case 2	Case 3	Case 4
1/7	90	91	91.5	92
1/7	95	96	96.5	97
1/7	100	101	101.5	102
1/7	105	106	106.5	107
1/7	110	111	111.5	112
1/7	115	116	116.5	117
1/7	120	121	121.5	122

Symmetric Nash Equilibrium results:

debt demand state	1	1	1	1
Min Debt				
Max Debt				
Investment	344.90	355.59	361	366.45
Quantity	27.86	28.29	28.5	28.71
Expected price	49.29	49.43	49.5	49.57
Expected profits	431.12	444.49	451.25	458.06
Optimal Investment [q^2/4]	194.04	200.08	203.06	206.07