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Behavior Based Manipulation

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Abstract

If investors are not fully rational, what can smart money do? This paper provides an example in which smart money can strategically take advantage of investors' behavioral biases and manipulate the price process to make profit. The paper considers three types of traders, behavior-driven investors who have two behavioral biases (momentum trading and dispositional effect), arbitrageurs, and a manipulator who can influence asset prices. We show that, due to the investors' behavioral biases and the limit of arbitrage, the manipulator can profit from a "pump and dump" trading strategy by accumulating the speculative asset while pushing the asset price up, and then selling the asset at high prices. Since nobody has private information, manipulation investigated here is completely trade-based. The paper also endogenously derives several asset pricing anomalies, including the high volatility of asset prices, momentum and reversal.

JEL: G12, G18

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Behavioral studies in economics and finance, such as Kahneman and Tversky (1974, 1979, 2000), Tversky and Kahneman (1986), Barberis, Shleifer, and Vishny (1998), Thaler (1999), suggest that economic agents are less than fully rational[1]. They are often psychologically biased. Their psychological biases, together with "limits of arbitrage", lead to asset price' deviations from fundamental values and may generate a large number of anomalies that cannot be easily explained in the rational expectations paradigm.

While it is important to identify plausible causes for asset pricing anomalies, most investors would be more interested in knowing how to take advantage of other people's behavioral biases to make money. In this paper, we build an equilibrium model to demonstrate how "smart money" can profit from other investors' irrational behaviors. The model has three classes of investors: a manipulator, behavior-driven investors, and arbitrageurs. Behavior-driven investors are not fully rational, whose behavioral biases used in the model are momentum trading and unwillingness to sell losers. These two psychological biases are supported by many theoretical and empirical studies, including Hong and Stein (1999), Odean (1998), Shefrin and Statman (1985), among others.

Arbitrageurs play a critical role in preventing large price jumps and market crash, but because of the limits of arbitrage, they cannot fully eliminate asset price's deviation from fundamental value.

The manipulator is a large investor who is a price setter rather than a price taker. As a deep-pocket investor, he lures momentum investors into the market by pumping up the stock price and then dumps the stock to make a profit by taking advantage of the disposition effect and the limits of arbitrage.

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[1] Barberis and Thaler (2003) and Hirshleifer (2001) provide detailed surveys of the behavior literature.

Numerous empirical studies suggest that there exist trading strategies that can yield positive abnormal returns presumably because of asset pricing errors. For example, Jegadeesh and Titman (1993) report that investors can make substantial abnormal profits by buying past winners and selling past losers[2]. These studies have several common characteristics. First, they are based all on observed or realized prices. Naturally, the realized prices are the result of interactions among a large number of investors. Therefore, it is difficult to rely only on the empirical studies to identify the roles played by different investors in price determination. Second, the trading strategies such as the momentum trading documented in the empirical literature usually takes the price process as exogenous. This methodology is valid only if the investors who follow these strategies, in total, are price-takers. Investors cannot actively affect price processes for profit-making purpose.

A distinctive feature of our model is its explicit investigation of how smart money (the manipulator) interacts with irrational traders and what profit the manipulator makes from exploiting other investors' behavioral biases. In other words, the manipulator in our model manipulates the price process to create more chances for the irrational investors to make mistakes. This is an important feature, but largely assumed away in the existing behavioral finance literature. For instance, Barberis, Shleifer, and Vishny (1998, BSV henceforth) have a representative agent model in which trading does not occur. Daniel, Hirshleifer, and Subrahmanyam (1998, DHS henceforth) consider two classes of traders, the informed (I) and the uninformed (U). However, since prices in their model are set by the risk-neutral informed traders, the formal role of the uninformed is minimal there. Hong and Stein also model two classes of traders--news-watchers and momentum traders. News-watchers only care about what news they observe, while momentum traders make

[2] Lesmond, Schill, and Zhou (2003) argue that the profit of the momentum strategy documented by Jegadeesh and Titman is illusory because of transactions costs. Lesmond, Schill and Zhou's result therefore provides positive evidence for the argument of "limits of arbitrage."

decisions based only on price changes. No trader purposefully chooses a trading strategy to take advantage of other people's behavioral biases.

Moreover, the price movement in our model is completely trade based. It neither resorts to information asymmetry nor depends on the fundamental risk of the asset. Almost all other behavior-based asset pricing theories, however, depend on fundamental-related information or news in some ways. As we will discuss subsequently, this feature allows us to investigate purely trade based market manipulation.

Finally, our model produces somewhat similar correlations among prices, turnover, and volatility to the model of investor overconfidence by Scheinkman and Xiong (2003). In our model, the manipulator's strategic action, together with other investors' behavioral biases, not only brings the manipulator himself profit, but also brings about excess volatility, excess trading, short-term price continuation, and long-term price reversal. This feature helps us to further understand why investors trade and why asset prices sometimes fluctuate continually without any significant news on earnings and other fundamental variables. It also provides a purely trade-based explanation on some well known empirical anomalies, such as price momentum and reversal.

The rest of the paper is structured as follows. The next section reviews the literature of manipulation. Section 2 sets up the theoretical model. Section 3 solves the model for the "pump and dump" strategy and then extends the model to include the "dump and cover" strategy.. Section 4 investigates the implications of the model on several well-known asset pricing anomalies. Section 5 provides some empirical evidence from recent studies of market manipulation that is consistent with our model. Section 6 concludes.

1. A Review of the Manipulation Literature

Market manipulation is an issue that is almost as old as the earliest speculative market. The prevalence of "pump and dump" or "dump and cover" strategies was widely reported in financial press at the beginning of the 17th century when the Amsterdam Stock Exchange was founded. Even though market manipulation might be much more severe in the early years of financial markets, it is too early to say that manipulation is no longer of importance. In modern financial markets, manipulations are often taken in hidden ways that cannot be easily detected and outlawed. In many emerging markets where market regulations are weak, manipulation is still quite rampant.[3] Even in the relatively well-regulated US market, Aggarwal and Wu (2003) have documented hundreds of cases of price manipulation in the 1990s. .

Following Allen and Gale (1992), we classify manipulation into three categories: *information-based* manipulation, *action-based* manipulation, and *trade-based* manipulation.

Information-based manipulation is taken by releasing false information or spreading misleading rumors. The operation of "trading pools" in the United States during the 1920s gives examples of information-based manipulation. A group of investors would combine to form a pool: first to buy a stock, then to spread favorable rumors about the firm, and finally to sell out at a profit.[4] The striking cases of Enron and the Worldcom in

[3] For example, China's worst stock-market crime in 2002 was a scheme by seven people, including two former China Venture Capital executives, accused of using $700 million and 1,500 brokerage accounts nationwide to manipulate the company share price.

[4] An example of information-based manipulation is the case of Texas Gulf Sulphur Company in the 1960s (Jaffe 1974). In late 1963 drillings by its engineers struck huge mineral deposits. Between November 1963 and mid-April 1964, company officials tried hard to convince the public that the opposite was true, by falsifying evidence, while accumulating company shares and options. On April 12, 1964, the company even issued a press release stating that the technical

2001 might also be related to information-based manipulation. Van Bommel (2003) shows the role of rumors in facilitating price manipulation.

Benabou and Laroque (1992) show that if an opportunistic individual has privileged information and his statements are to certain extent viewed as credible by investors, he can profitably manipulate asset markets through strategically distorted announcements. As privileged information is noisy and learning remains incomplete, opportunistic individuals (corporate officers, financial journalists, or "gurus") can manipulate the market repeatedly, even though their manipulation power is limited in the long run by public's constant reassessment of their credibility. In a related article, John and Narayanan (1997) discuss market manipulation through inside information and the role of insider trading regulations. They show that the existing disclosure rule of the Securities and Exchange Commission (SEC) creates incentives for an informed insider to manipulate the stock market by sometimes trading in wrong direction (i.e., buying with bad news and selling with good news about the firm). By doing so, the insider can effectively reduce the informativeness of his subsequent trade disclosure because the market is not sure whether an insider's buying (selling) indicates good (bad) news. Consequently, the insider maintains his information superiority for a longer period of time and uses it to reap large profits in later periods by trading in the "right" direction. These profits more than make up for the losses suffered by trading in the wrong direction initially.[5]

Action-based manipulation is based on actions (other than trading) that change the actual

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evidence was inconclusive; four days—and a large number of shares—later, the company admitted that deposits had in fact been found. Mahoney (1999), however, question the empirical validity of the existence of manipulation in the 1920s.

[5] In addition, Vila (1989) presents an example of information-based manipulation where the manipulator shorts the stock, releases false information and then buys back the stock at a lower price.

or perceived value of the assets. Bagnoli and Lipman (1996) investigate action-based manipulation using take-over bids. In their model, a manipulator acquires stock in a firm and then announces a take-over bid. This leads to a price run up of the firm's stock. The manipulator therefore is able to sell his stock at the higher price. Of course, the bid is dropped eventually.

The Securities Exchange Act of 1934 established extensive provisions aimed at eliminating manipulation. By regulating information disclosure and restricting and monitoring the trading activities of the directors, managers, and insiders, the Act has successfully made market manipulation more difficult. The types of manipulation that the Act effectively outlawed are mainly information-based and action-based. As a matter of fact, regulating information disclosure of public companies has now become one of the most important tasks of virtually all securities regulation bodies across the world.

Trade-based manipulation, however, is much more difficult to eradicate. It occurs when a large trader or a group of traders attempt to manipulate the price of an asset simply by buying and then selling, without taking any publicly observable action to alter the asset value or releasing false information to change the price. This type of manipulation could be of great importance empirically. Hedge funds often buy and then sell substantial blocks of stock, even though they are apparently not interested in taking over the firm. In our opinion, these large buying/selling activities could be taken sometimes for the purpose of trade-based manipulation.

Allen and Gale (1992) build a model showing that trade-based manipulation is possible in a rational expectations framework. The Allen and Gale model has three trading dates (indexed by $t = 1, 2, 3$) and three types of traders, a continuum of identical rational investors, a large informed trader who enters the market at date 1 if and only if he has private information, and a large manipulator who observes whether the informed trader has the private information. The manipulator has a small but positive probability to enter the market and to mimic the informed trader's action when the informed trader actually has no private information. The manipulator is able to achieve a positive profit under certain conditions because there can exist a pooling equilibrium in which the investors are uncertain whether a large trader who buys shares is a manipulator or an informed trader.[6]

Aggarwal and Wu (2003) present a theory and some empirical evidence on stock price manipulation in the United States. Extending the framework of Allen and Gale (1992), they consider what happens when a manipulator can trade in the presence of other *rational* traders who seek out information about the stock's true value. In a market with manipulators, they show more information seekers imply a greater competition for shares, making it easier for a manipulator to enter the market and potentially worsening market efficiency.

There are several other articles investigating manipulation. Camerer (1998) tests whether naturally occurring markets can be strategically manipulated using a field experiment with racetrack betting. Kumar and Seppi (1992) develop a model of manipulation in futures markets. Hart (1977) investigates the conditions of equilibrium price process under which manipulation is possible. He considers conditions under which profitable

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[6] Allen and Gale made several assumptions to make the trade-based manipulation possible in their model. First, the small investors must be much more risk averse than the large traders. The manipulation may not be possible if the informed trader is as risk averse as or even more risk averse than the small rational investors. Second, the probability of manipulation shall be sufficiently small. Third, private information still plays a crucial role in the model. Fourth, the informed trader's trading decision depends on whether he receives the private information, but not on the content of his private information. Namely, when the informed trader receives his private information, he will purchase the same quantity of the stock no matter what he receives is good news or bad news; when he does not receive the private information, he will not enter the market even though he is risk neutral and the expected asset return is positive. To some extent, the informed trader himself seems to be less than fully rational.

speculation is possible in an infinite horizon deterministic economy. He finds that manipulation is possible if the economy is dynamically unstable or if demand functions are non-linear and satisfy some technical conditions. Jarrow (1992) extends Hart's analysis to a stochastic setting with time dependent price process. He shows that profitable manipulation is possible if the manipulator can corner the market. He also demonstrates the manipulator can achieve a positive profit if he is able to establish a price trend and trade against it. To conserve space, we are sorry to skip many other important articles in this literature.

Our investigation of manipulation is based on a different setup and generates several new insights. First, because our model does not rest on information asymmetry or fundamental risk, manipulation investigated here is therefore purely trade-based. Second, our model does not depend on various market frictions discussed in the literature (e.g., Jarrow 1992), such as corners, short squeezes, etc. Third, and most importantly, we derive the equilibrium price process endogenously by constructing manipulator's trading strategies based on certain well-documented behavioral biases of investors. Theoretically, the large trader can manipulate the price process repeatedly and frequently as long as there are investors who have those behavioral biases specified in the model.

The contributions of our work are multi-fold. First, the paper provides an application of behavioral theories documented in the literature to endogenously derive several well-known asset pricing anomalies. Second, we provide an additional example of trade-based manipulation, distinct from the model of Allen and Gale (1992)--that does not impose assumptions on information asymmetry or the probability of manipulation. Third, we illustrate a possibility of trade-based manipulation based on realistic assumptions about behavior that have been well documented empirically. One may view our paper as a companion paper of Allen and Gale (1992). They study the possibility of price manipulation under rational expectations with information asymmetry while we provide a case of market manipulation under behavioral bias and limits to arbitrage but with no fundamental risk or information asymmetry.

2. The Model Economy

We consider a discrete-time market in which there exist a speculative asset and a riskfree bond. The riskfree bond yields a zero net return each period of time. There are three classes of investors, a manipulator, arbitrageurs, and behavior-driven traders, who buy and sell the speculative asset following their own rules. The characteristics of these investors are described in detail in the following assumptions.

Assumption 1. We consider a discrete-time economy that begins at time $t = 0$, and ends at time $t = T$ (namely, $t = 0,1,2,\cdots,T$). *A continuum number of new behavior-driven investors, with measure 1, enter the market at the beginning of each period t. They are price-takers and each of them has a probability of* q_1 *to buy a share of the speculative asset if the price of the asset at time t>0,* P_t*, is greater than the asset price at time t-1,* P_{t-1}*. If* $P_t \le P_{t-1}$*, each new behavior-driven investor has a probability of* q_2 *to buy a share of the speculative asset, where* $q_2 < q_1$.

At the beginning of the economy, t=0, the price of the speculative asset (P_0*) is equal to the fundamental value of the asset, and the behavior-driven investors are endowed with* q_1 *shares of the speculative asset in total. Those investors who own the speculative asset at the beginning of the economy take* P_0 *as the initial acquiring cost per share of the* *speculative asset.*

The new behavior-driven investors at time t>0 who do not buy the speculative asset choose to leave the market right away. The old generations of behavior-driven investors who entered the market before t>0 do not buy any more shares at time t. Behavior-driven investors like to take quick profits. They sell their shares as soon as they have made a profit and then leave the market. Consider a behavior-driven investor who buys a share of the speculative asset at time t and has not sold his share by the beginning of time $t+k \quad (k>0)$*. If* $P_t < P_{t+k}$*, he shall liquidate his share in the period of* $t+k$ *for sure; if* $P_t \geq P_{t+k}$*, he will have a probability of* $q_3 < 1$ *to liquidate his share in the period of* $t+k$*. Behavior-driven investors leave the market right after they have liquidated their shares.*

This assumption is made on the basis of two important empirical observations: trend chasing (momentum trading) and dispositional effect.

In an original article, Jegadeesh and Titman (1993) find that the winners of the stock market over past several months tend to outperform in the next several months as well. This phenomenon is now termed as momentum and has been well documented in the behavioral finance literature. In the BSV model, momentum can occur because of the investors' conservatism. Hong and Stein (1999), as introduced earlier, explicitly add momentum traders—traders buying stocks after a price increase—to their model. Many other researchers, including DeLong, Shleifer, Summers, and Waldermann (1990) and Cutler, Summers, and Poterba (1990), have also investigated momentum trading or positive feedback trading. The simplest way of motivating positive feedback trading is extrapolative expectations. Namely, as investors form expectations by extrapolating trends, they buy into price trends. This can be due to some important psychological biases of investors, including representativeness and the law of small numbers (Barberis and Thaler, 2003).

Robert J. Shiller (2002, p14) has the following vivid description on momentum trading or feedback trading:

When speculative prices go up, creating successes of some investors, this may attract public attention, promote word-of-mouth enthusiasm, and heighten expectations for further price increases. ... This process in turn increases investor demand, and thus generates another round of price increases. ... The high prices are ultimately not sustainable, since they are high only because of expectations of further price increases. ...

The story about tulip mania in Holland in the 1630s provides us a real example on momentum trading with little fundamental news (Charles MacKay (1841, pp 118-119) or Shiller (2002, p15)):

Many individuals grew suddenly rich. A golden belt hung temptingly out before the people, and one after another, they rushed to the tulip marts, like flies around a honey pot. ... At last, however, the more prudent began to see that this folly could not last forever. Rich people no longer bought the flowers to keep them in their gardens, but to sell them again at cent per cent profit. It was seen that somebody must lose fearfully in the end. As this conviction spread, prices fell, and never rose again.

Dispositional effect is another well-documented empirical phenomenon. According to Shefrin and Statman (1985), Odean (1998), Grinblatt and Han (2001), etc., investors, especially the individual ones, are more likely to sell stocks that have gone up in value relative to their purchase price, rather than stocks that have gone down. Two behavioral explanations for the dispositional effect have been suggested in the literature. The first explanation suggests that investors may have a biased belief in mean-reversion. The second explanation relies on prospect theory and narrow framing.

Assumption 2. *There is a manipulator in the market who is a large market player and is able to influence the asset price. In other words, the manipulator is a price-setter rather than a price-taker. He enters the market at time 1 without any initial endowment of the speculative asset. At each period of time* $t \geq 1$*, the manipulator sets a price target for that period and then submits his order to clear the market at the target price.*

The assumption that the manipulator is a large trader is conventional in the literature on trade-based manipulation. In order to move the market with strategic trading, the manipulator must have the power to influence the price (see Jarrow (1992) and Allen and Gale (1992)).

Assumption 3. *There is also a continuum number of arbitrageurs, with measure 1, enters the market at time t=1. They are price-takers and trade shares of the speculative asset based on recent price movements. If the price moves up in the current period, they sell some shares to take profits. If the price goes down, they buy. Formally, they submit the following orders at time t:*

$$D_{a,t} = -\alpha\left(P_t - P_{t-1}\right) = -\alpha\left(\Delta P_t\right) \qquad (1)$$

where α>0.

Although the new trades of the arbitrageurs in each period only depend on short term price movements, the total position of the speculative asset held by the arbitrageurs, Q_t, is negatively proportional to price deviation from fundamentals. This is because the arbitrageurs have already held a portfolio of $Q_{t-1} = \sum_{j=1}^{t-1} -\alpha\left(P_j - P_{j-1}\right) = -\alpha\left(P_{t-1} - P_0\right)$ shares of the speculative asset at time $t-1$, if they buy additional $-\alpha\left(P_t - P_{t-1}\right)$ shares at time t, the total position of the speculative asset held by them will be $Q_t = -\alpha\left(P_t - P_0\right)$ shares. The arbitrageurs play two roles in our model. First, they provide necessary liquidity to the market so that trading can take place at equilibrium for each period. For instance, if the manipulator wants to move the asset price up by submitting a purchasing order, there must be some investors selling sufficient number of shares of the speculative asset. Because the behavior-driven investors in a sense are momentum followers, a new class of investors is therefore needed in the model. Second, our model rules out fundamental risk. The arbitrageurs' trading strategy ensures that the price of speculative asset will not move away from fundamentals explosively. We call α arbitrage parameter and will discuss its meaning and implication further in the next section.

Assumption 4. *Although the manipulator enters the market at time 1, the market already existed at time 0. The price of the speculative asset at time 0 was* P_0*, which was equal to the fundamental value of the asset. There were* q_1 *behavior-driven investors who held one share of the speculative asset per person at the market close of day 0.*

The manipulator tends to move the asset price up by a fixed amount of $\delta > 0$ *for* t_u *(*$t_u > 1$*) consecutive periods from day 1 to day* t_u*. That is*

$$P_t - P_{t-1} = \delta > 0, \qquad t = 1,2,\cdots,t_u. \qquad (2)$$

By the close of day t_u*, the manipulator has accumulated certain number of shares of the* *speculative asset. He starts liquidating his shares from day* t_u+1 *and keeps doing so until he has sold all of his shares by time T-1 for some* $T > t_u + 1$. *We define* $t_d = T - 1 - t_u$ *as the length of time the manipulator takes to liquidate his shares accumulated by time* t_u.

In order to ensure market equilibrium for each day, δ shall satisfy certain condition as discussed subsequently. Assumption 4 is not the only possible assumption that can make manipulation profitable, but is a simple one.

Assumption 5. T*he manipulator leaves the market right after he has sold all his shares at T-1. The market ends at time T and by then investors receive a liquidating dividend of* P_0 *for each share of the speculative asset.*

Assumption 5 is not really needed for discussing the manipulation issue in the model. We make this assumption here following the convention in the literature and the widespread belief that in the long run, fundamental rules. The assumption is useful in discussing certain asset price anomalies such as long-term reversal. It is easy to see from assumptions 3 and 5 that the net purchases of arbitrageurs are zero over the whole time periods.

Here, we assume the speculative asset has no fundamental risk. We also assume that there is no heterogeneous information. This does not mean that fundamental risks and information asymmetry are not important in the real market or in market manipulation. Rather, we use this simplified setup to highlight the manipulator's trading strategies when the market is not fully rational. With this simple framework, we demonstrate that manipulation is possible even if there is no information asymmetry on asset fundamentals.

More importantly, using this setup, we are able to explain the speculative dynamics of the prices of assets (commodities) with stable fundamental value, such as the tulip mania mentioned above.

3. Results and Interpretations

To solve the model, we first find out the accumulated holding of the speculative asset by the manipulator at the market close by time t_u. We have the following proposition.

Proposition 1: *By the market close at day* t_u, *the manipulator has accumulated* $N = \alpha \cdot t_u \cdot \delta$ *shares of the speculative asset with an average cost of* $P_0 + \left(\frac{1+t_u}{2}\right)\delta$ *per share.*

Proof: By Assumptions 1 to 3, it follows immediately that for each period t, such as $1 \le t \le t_u$, the manipulator shall buy $\alpha \cdot \delta$ shares at a price of $P_0 + t\delta$. A simple calculation yields the statement in Proposition 1. ■

Proposition 1 highlights the important impact of arbitrage on the manipulator's trading strategy. To move the price of the speculative asset by an amount of δ, the manipulator must purchase $\alpha \cdot \delta$ shares of the asset. If α is sufficiently large, the manipulator must have a very deep pocket to move the market. Put another way, when there is no limit of arbitrage, namely, $\alpha \to \infty$, it is almost impossible for the manipulator to "pump and dump" the speculative asset. Therefore, the assumption of the "limits of arbitrage" is essential for the manipulator's trading strategy to work. Proposition 1 also suggests that, the higher the original price of the asset, the more money the manipulator needs to put up for purchasing the shares. This implies, ceteris paribus, small cap stocks are more likely to be subject to price manipulation.

We first consider a simple but interesting case in which behavior-driven investors are extremely unwilling to take losses, namely, $q_3 = 0$. This is a strong implication of the dispositional effect that has been supported by several empirical studies, such as Odean (1998) and Grinblatt and Han (2001)

Proposition 2: *If $q_3 = 0$, then the manipulator can sell his shares at a high price $P_3 = P_0 + t_u\delta$ from time $t = t_u + 1$ through time $t = T - 1$ by appropriately choosing a positive δ. By doing so, the manipulator's total profit is $N \cdot \left(\frac{t_u - 1}{2}\delta\right) = \alpha \cdot \left(\frac{t_u - 1}{2}t_u\right) \cdot \delta^2$. The trading volume stays at $\alpha\delta + q_1$ shares per period from time $t = 1$ to time $t = t_u$. From time $t = t_u + 1$ to time $t = T - 1$, trading volume per period is q_2 shares--the manipulator sells q_2 shares to new behavior-driven investors each period.*

Proof: Set $\delta = \frac{t_d \cdot q_2}{t_u \cdot \alpha}$. Because $q_3 = 0$, behavior-driven investors will not sell their shares without a profit. The manipulator is able to sell q_2 shares to the new behavior-driven investors each period from day $t = t_u + 1$ through time $t = T - 1$ by maintaining the equilibrium price at $P_{t_u} = P_0 + t_u\delta$. The average selling price is P_{t_u} per share. As a result, the manipulator's total profit is

$$\pi = N \cdot \left[P_{t_u} - \left(P_0 + \frac{1 + t_u}{2}\delta\right)\right] = \alpha \cdot \left(\frac{t_u - 1}{2}t_u\right) \cdot \delta^2 \qquad (3)$$

Now, we consider trading volume. From time $t=1$ to time $t=t_u$, the price of speculative asset rises by δ per time period. By assumptions, the old behavior-driven investors sell q_1 shares of the speculative asset while the new behavior-driven investors buy q_1 shares in any period t, such as $1 \le t \le t_u$. In the mean time, the arbitrageurs sell $\alpha\delta$ shares each time period. In order to clear the market, the manipulator has to buy $\alpha\delta$ shares. The total trading volume from time $t=1$ to time $t=t_u$ is therefore $\alpha\delta + q_1$ shares per period.

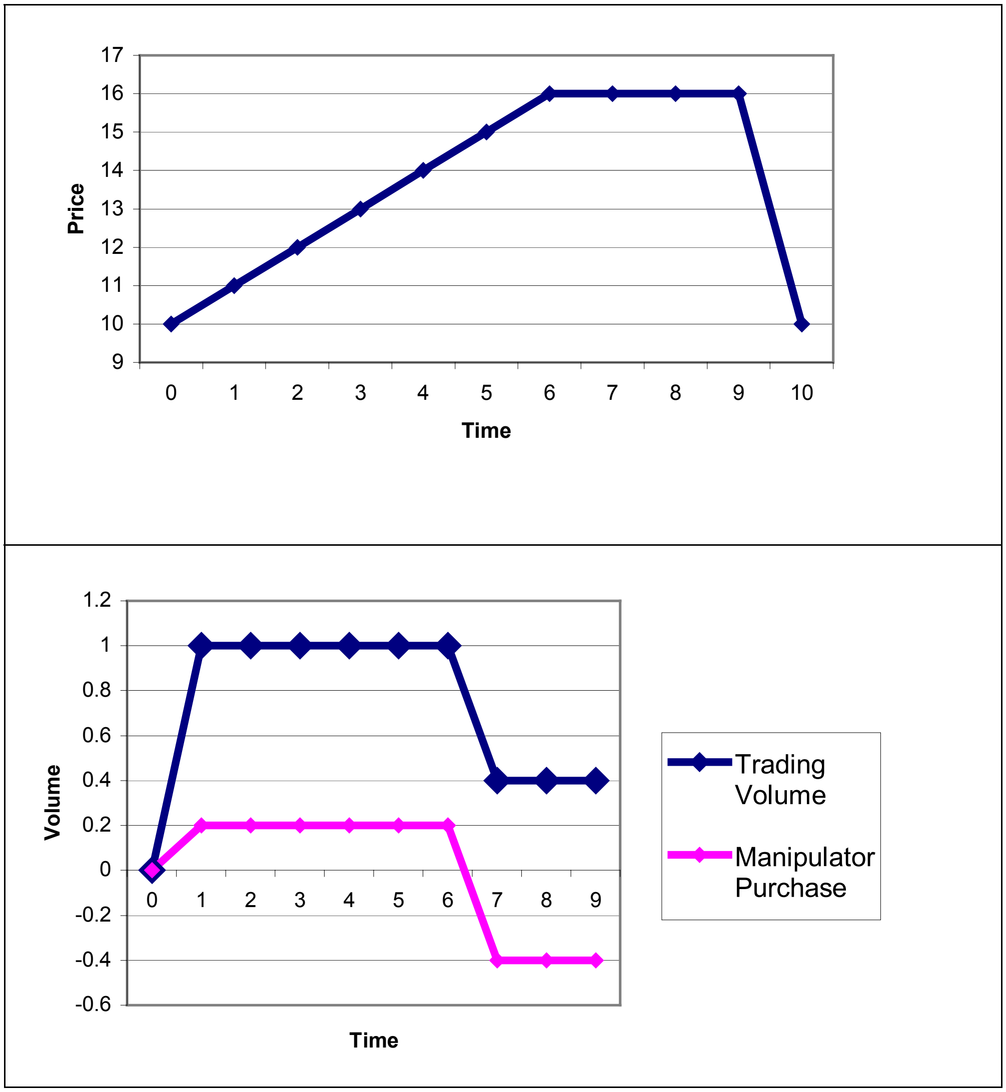
Because the asset price remains constant from time $t=t_u+1$ to time $t=T-1$, the arbitrager will not trade in this case. The old behavior-driven investors who still own the shares at time $t>t_u$ should have bought at the peak price $P_{t_u}=P_0+t_u\delta$ and must not sell because they have not made any profits. On the other hand, the new behavior-driven investors choose to buy q_2 shares at time $t>t_u$. In order to clear the market, the manipulator has to sell q_2 shares. The total trading volume in this case is q_2 shares.

■

Figure 1 presents the price dynamics, total trading volume and the buying/selling pattern of the manipulator based on Proposition 2. We use a positive number for the manipulator's buying volume and a negative number for his selling volume. The steep rise in asset price and the purchase by the manipulator clearly demonstrates his "pumping" strategy, while the negative trading volume and a flat price shows the constant sell of his position to the behavioral investors. In the final period T=10, behavioral investors and arbitrageurs settle their shares at the price equal to fundamental value. The manipulator is out of the market, thus there is no trading volume.

Figure 1: Price Dynamics and Trading Volume when $q_3 = 0$

(Assuming $t_u = 6, t_d = 3, \alpha = 0.1, q_1 = 0.8, q_2 = 0.4, q_3 = 0$**)**



This proposition illustrates a special case in which the manipulator profits from the biases of behavior-driven investors who are more likely to chase a trend and are not willing to sell losers. In contrast, behavior investors lose money on average. The behavior-driven investors who enter the market at early stage can make profits but those who enter the market lately surfer severe losses. The arbitrageurs in this case can make a profit because they shorted shares at prices higher than P_0 from time $t=1$ to time $t=t_u$ and are able to cover their short positions at the fundamental value P_0 at the end. However, if the arbitrageurs had to cover their short positions before T, they might suffer a loss. Because $q_2 < q_1$, the proposition indicates that the trading activities are more active in an up market than in a down market. This finding is consistent with the typical empirical observations.

The proposition also indicates that both short-term momentum and long-term reversal phenomena can be generated in our behavior model even without fundamental shocks: The price of the speculative asset rises for several consecutive periods but moves down eventually.

In general, when $q_3 > 0$, the situation will become more complicated. The following propositions illustrate several possible solutions to the model.

Proposition 3: *Suppose that* $h \equiv q_2 - q_1 \cdot q_3 > 0$. *Then the manipulator can sell his shares at a high price* $P_{t_u} = P_0 + t_u \delta$ *from time* $t = t_u + 1$ *through time* $t = T - 1$ *by appropriately choosing a positive* $\delta < \dfrac{h}{t_u \cdot \alpha \cdot q_3} = (q_2 - q_1 \cdot q_3) \big/ (t_u \alpha \cdot q_3)$. *By doing so, the manipulator's total profit is still* $\pi = N \cdot \left(\dfrac{t_u - 1}{2} \delta \right) = \alpha \cdot \left(\dfrac{t_u - 1}{2} t_u \right) \cdot \delta^2$. *The trading volume remains at* $\alpha\delta + q_1$ *shares per period from time* $t=1$ *to time* $t = t_u$. *From time* $t = t_u + 1$ *to time* $t = T - 1$, *trading volume per period is* q_2 *shares and the* *manipulator is able to sell* $h_j \equiv (1-q_3)^{j-1} h$ *shares at time* $t = t_u + j$ *(* $j \geq 1$ *).*

Proof: If the manipulator maintains the equilibrium price at $P_{t_u} = P_0 + t_u \delta$ from time $t = t_u + 1$ through time $t = T - 1$, the arbitrageurs will neither sell nor buy during these $t_d = T - 1 - t_u$ periods according to Assumption 3. Assumption 1 indicates that, at time $t = t_u + 1$, the new behavior-driven investors will buy q_2 shares in total, while the old behavior-driven investors will sell totally $q_1 \cdot q_3$ shares. Therefore at time $t = t_u + 1$, all behavior-driven investors will have a net purchase of $h_1 = h \equiv q_2 - q_1 \cdot q_3$ shares. At time $t_u + 2$, the net purchase of the speculative asset by old and new behavior-driven investors will be $h_2 \equiv q_2 - (q_1 + h) \cdot q_3 = (1 - q_3) h$. By the method of induction, we can prove that for any $t = t_u + j$, such as $0 < j \leq t_d$, the net purchase of the speculative asset by all old and new behavior-driven investors will be

$$h_j = (1 - q_3)^{j-1} h,$$

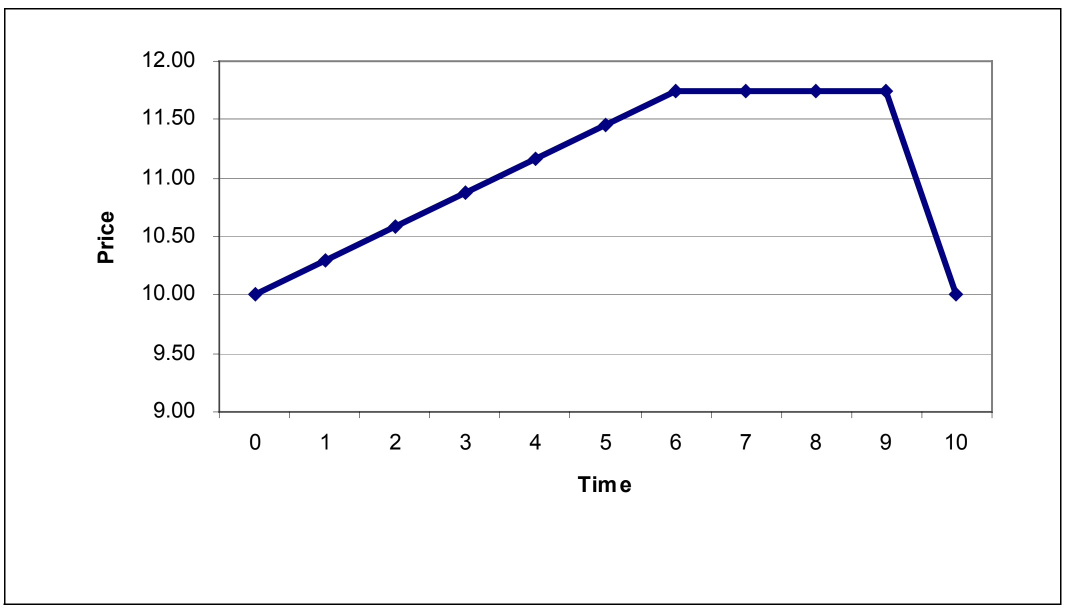
provided that the asset price is maintained at $P_{t_u} = P_0 + t_u \delta$ from time $t = t_u + 1$ through time $t = t_u + j$.

Since the arbitrageurs will not trade when the asset price is stable, the manipulator must sell $h_j = (1 - q_3)^{j-1} h$ shares at time $t = t_u + j$ to clear the market at a price of P_{t_u}. As a result, the total number of shares he can sell at price P_{t_u} from time $t = t_u + 1$ through time $t = T - 1$ equals $\sum_{j=1}^{t_d} h_j = \frac{\left\{1 - (1 - q_3)^{t_d}\right\}}{q_3} \cdot h$. If the manipulator chooses $\delta = \frac{\left[1 - (1 - q_3)^{t_d}\right] / q_3}{t_u \alpha} \cdot h$, then he can sell all of his shares by time $T - 1$. It follows immediately that if T goes to infinity, then $\delta \to \dfrac{h}{t_u \alpha \cdot q_3}$.

The trading volumes *from* time $t = 1$ *to time* $t = t_u$ *can be obtained directly from the proof of Proposition 2. From time* $t = t_u + 1$ *to time* $t = T - 1$*, because the price remains constant, the arbitrager will not trade while the new behavior-driven investors will buy* q_2 *shares each period of time. On the other hand, our proof above shows that the old behavior-driven investors will totally sell* $q_2 - h_j$ *shares at time* $t = t_u + j$ *(*$j \geq 1$*). As a result, the manipulator has to sell* h_j *shares to clear the market at time* $t = t_u + j$ *and the total trading volume at time* $t = t_u + j$ *is* q_2. ■

Figure 2: Price Dynamics and Trading Volume when $q_3 = 0.3$

(Assuming $t_u = 6, t_d = 3, \alpha = 0.1, q_1 = 0.8, q_2 = 0.4, q_3 = 0.3$**)**



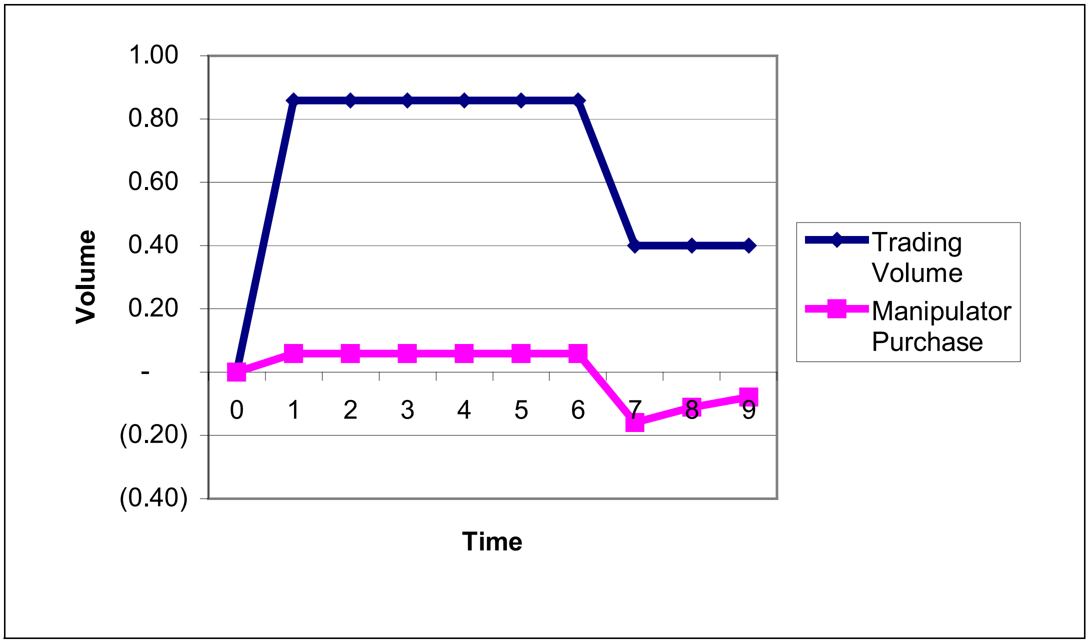


Figure 2 presents the price dynamics, total trading volume and the buying/selling pattern of the manipulator based on Proposition 3. Comparing Figure 1 and 2, we can see that the price rise is less steep when $q_3 > 0$. The trading volume is also smaller by the manipulator, since in this case he needs to take into consideration the selling by loss-making investors. Because $h_{j+1} < h_j$, Proposition 3 indicates that the manipulator's speed to liquidate his shares slows down gradually. This is because as time goes by, more and more behavior-driven investors have accumulated some shares of the speculative asset and will exert higher selling pressure on the market.

The condition $\delta < \dfrac{h}{t_u \cdot \alpha \cdot q_3} = {(q_2 - q_1 \cdot q_3)}\big/{(t_u \alpha \cdot q_3)}$ imposed in the Proposition implies that the total number of shares N accumulated by the manipulator up to time t_u must be limited, namely $N = t_u \alpha \delta < \dfrac{h}{q_3} = {(q_2 - q_1 \cdot q_3)}\big/{q_3}$, if the manipulator hopes to liquidate all his shares at the high price P_{t_u}. This result is quite intuitive. For liquidity reason, the manipulator is not able to sell too many shares without moving the price down. The restriction imposed on δ or N will also impose an upper bound for the profit made by the manipulator following the strategy described in the Proposition

$$\pi = \alpha \cdot \left(\frac{t_u - 1}{2} t_u \right) \cdot \delta^2 = \alpha \cdot \left(\frac{t_u - 1}{2} t_u \right) \cdot \left[\frac{1 - (1 - q_3)^{t_d}}{t_u \alpha} \cdot \frac{h}{q_3} \right]^2$$

$$= \frac{1}{2\alpha} \cdot \left[\frac{(t_u - 1)}{t_u} \right] \cdot \left[1 - (1 - q_3)^{t_d} \right]^2 \left[\frac{q_2}{q_3} - q_1 \right]^2 < \frac{1}{2\alpha} \cdot \left[\frac{q_2}{q_3} - q_1 \right]^2 . \qquad (4)$$

Corollary 4: *The manipulator's profit π given in Proposition 3 is a decreasing function of the arbitrage parameter α. In particular, when* $\alpha \to +\infty$, $\pi \to 0$.

Corollary 4 reemphasize the role of the "limits of arbitrage" in our manipulation model. The intuition is straightforward. If the arbitrageurs trade very aggressively against the manipulator, it will be very difficult for the manipulator to move the price up. To move the price up by a given amount, $u = t_u \delta$, by time t_u, the manipulator has to accumulate a large portfolio of $N = \alpha \cdot u$ shares of the speculative asset. This is not only a matter of the depth of the manipulator's pocket as mentioned earlier. More importantly, as the behavior-driven investors only provide a limited net purchase of the speculative asset, it is impossible for the manipulator to liquidate all his shares of the speculative asset to the behavior-driven investors. Therefore, in a market where arbitrage is unlimited, the manipulator cannot be successful even if there are investors whose behaviors are biased.

In terms of Proposition 3, the arbitrageurs are able to make a profit by shorting the asset at prices higher than the fundamental value P_0 and then covering their short positions at the fundamental price P_0 eventually. This means that it is not necessarily good for the arbitrageurs to take too aggressive actions to preventing the market price of the speculative asset from deviating from its fundamental value. If the arbitrage strength parameter α is too large to make manipulation possible, the arbitrageurs will lose their opportunities to make profit as well.

Corollary 5: *The manipulator's profit π given in Proposition 3 is a decreasing function of* q_3. *Provided* $h \equiv q_2 - q_1 \cdot q_3 > 0$, *the profit π is also an increasing function of* q_2 *but a decreasing function of* q_1.

As previously mentioned, in our model, the manipulator can make a profit, to a large extent, due to the dispositional effect--the unwillingness of certain investors to sell losers. The smaller the q_3, the stronger is the dispositional effect. Moreover, it would be easier for the manipulator to make a profit if there are more behavioral investors who can provide liquidity (higher q_2). A higher q_1 appears to be negative for manipulator profits, since the manipulator needs to worry more about the selling by behavioral investors who entered the market and bought q_1 shares at time t_u if $q_3 > 0$. In our model, behavioral investors cash out immediately during the price run up so momentum investing (q_1) plays little role in price determination when $t < t_u$.

Proposition 6: *Suppose that* t_d *is fixed, that* $N = t_u \alpha\delta > \left[1-(1-q_3)^{t_d}\right]\left(\frac{q_2}{q_3} - q_1\right)$, *and that* $h \equiv q_2 - q_1 \cdot q_3 > 0$. *If the manipulator prefers to maintain the price unchanged at* P_{t_u} *(if possible) for k (*$0 \le k < t_d$*) periods and then let the price drop by an equal amount* η *(*$\eta > 0$*), that is* $P_t - P_{t-1} = -\eta$ *for* $t = t_u + k + 1, \cdots, T-1$, *one obtains:*

$$(a)\quad \eta = \frac{t_u \alpha\delta - \left[1-(1-q_3)^{t_d}\right]\left(\frac{q_2}{q_3} - q_1\right)}{(t_d - k)\alpha} \tag{5}$$

(b) The manipulator's capital gain is

$$\pi = \alpha \cdot \left(\frac{t_u - 1}{2} t_u\right) \cdot \delta^2 - \frac{(1-q_3)^k - (1-q_3)^{t_d} - (t_d - k) q_3 (1-q_3)^{t_d}}{q_3^2} \cdot h\eta$$
$$- \alpha \frac{(t_d - k)(t_d - k + 1)}{2} \eta^2 \tag{6}$$

The trading volume remains at $\alpha\delta + q_1$ shares per period from time $t = 1$ to time $t = t_u$. From time $t = t_u + 1$ to time $t = t_u + k$, trading volume per period is q_2 shares and the manipulator is able to sell $h_j \equiv (1-q_3)^{j-1} h$ shares at time $t = t_u + j$ $(1 \le j \le k)$. From time $t = t_u + k + 1$ to time $t = T - 1$, trading volume per period is $q_2 + \alpha\eta$ and the manipulator is able to sell $h_j + \alpha\eta = (1-q_3)^{j-1} h + \alpha\eta$ shares at time $t = t_u + j$ ($j > k$).

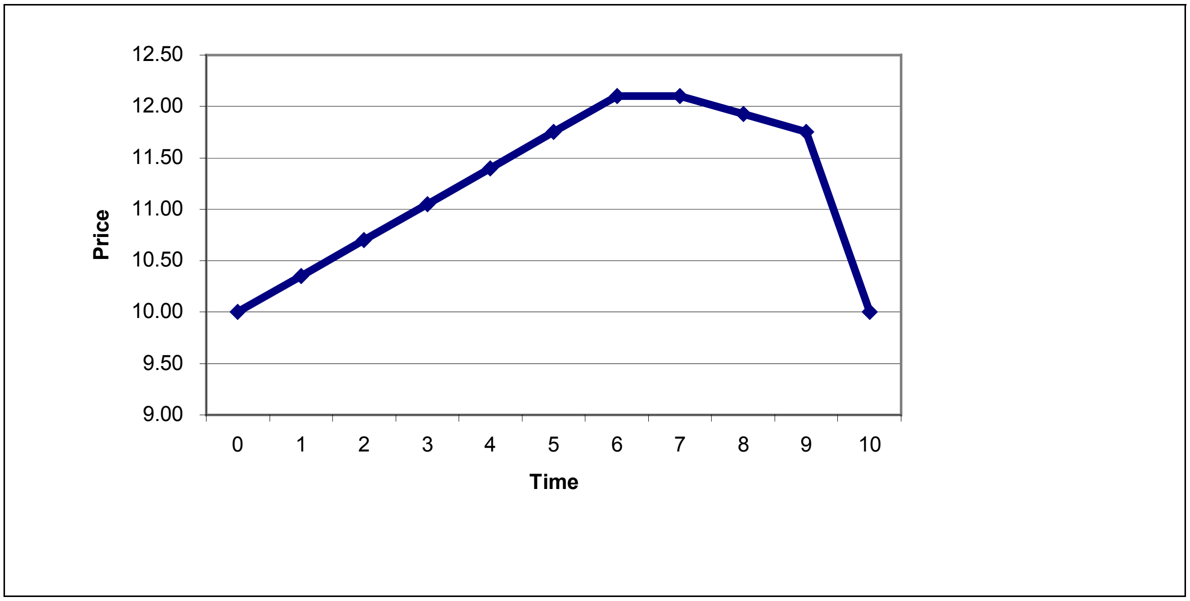
Proof: It follows immediately that if $h \equiv q_2 - q_1 \cdot q_3 > 0$ and $N = t_u \alpha\delta >$ $\left[1-(1-q_3)^{t_d}\right]\left(\frac{q_2}{q_3} - q_1\right)$, the manipulator is able to sell shares to the behavior-driven investors and to maintain the price unchanged at P_{t_u} for k periods after $t = t_u$ for some non-negative k as long as $k < t_d \equiv T - 1 - t_u$. Following the proof of Proposition 3, we obtain that if the price does not rise, the total number of shares bought by the behavior-driven investors minus shares sold by them from $t = t_u + 1$ through $t = T - 1$ is equal to $\frac{\left[1-(1-q_3)^{t_d}\right]}{q_3} \cdot h$. Therefore, the manipulator must sell totally $t_u \alpha\delta - \left\{\left[1-(1-q_3)^{t_d}\right] h \big/ q_3\right\}$ shares to the arbitrageurs. By Assumption 3, the total number of shares bought from $t = t_u + k + 1$ through $t = T - 1$ shall be $\alpha(t_d - k)\eta$. The market clearing condition gives part (a) of the proposition. Part (b) of the Proposition can be proved with tedious calculations.

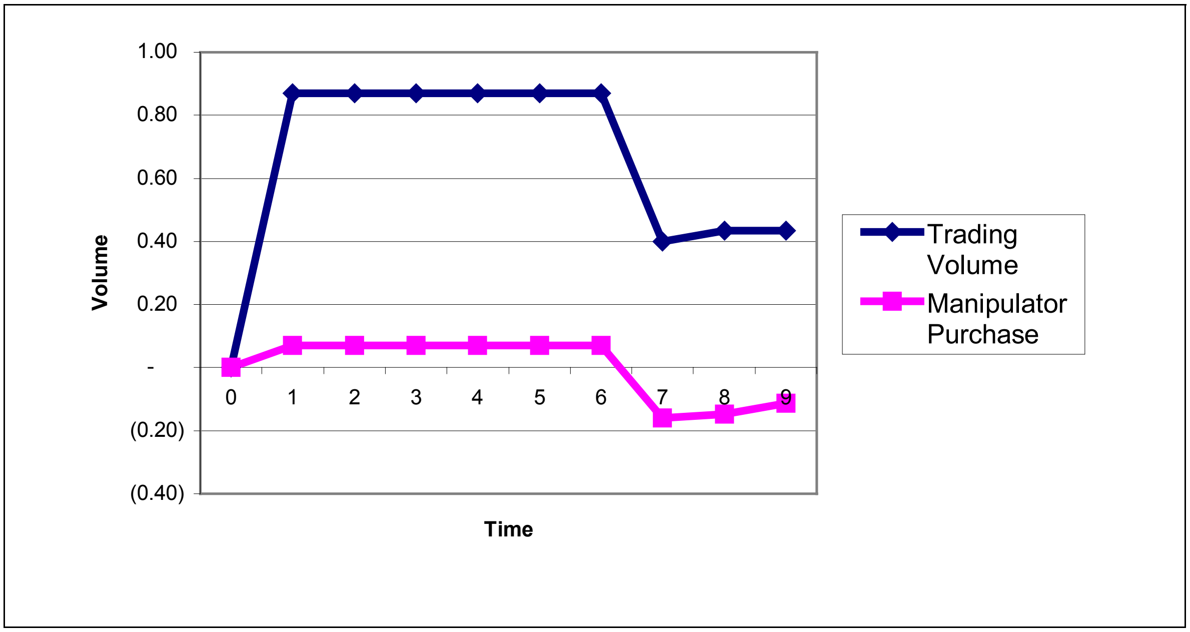
The trading volumes *from* time $t = 1$ *to time* $t = t_u$ *and from time* $t = t_u + 1$ *to time* $t = t_u + k$ *can be obtained directly from the proof of Proposition 3. From time* $t = t_u + k + 1$ *to time* $t = T - 1$, *the new behavior-driven investors will buy* q_2 *shares each period of time, while the old behavior-driven investors will totally sell* $q_2 - h_j$ *shares at time* $t = t_u + j$ *(* $j > k$ *). On the other hand, because the price drops by η each period of time, the arbitrager will buy* $\alpha\eta$ *shares at time* $t = t_u + j$ *(* $j > k$ *). As a result, the manipulator needs to sell* $h_j + \alpha\eta$ *shares to clear the market at time* $t = t_u + j$ *and the total trading volume at time* $t = t_u + j$ *is* $q_2 + \alpha\eta$ *(* $j > k$ *).*■

Figure 3 presents the price dynamics, total trading volume and the buying/selling pattern of the manipulator based on Proposition 6. The proposition demonstrates a very clear pattern of short-term momentum and long-term reversal. Asset price rises for several consecutive periods and then drops down gradually and continually after reaching the peak. As a matter of fact, if the size of the speculative asset accumulated by the manipulator by time $t = t_u$ is sufficiently large, the price is bounded to reverse its up-trend some day as it is impossible for the manipulator to sell his shares by maintaining the price at the peak level or letting the price keep rising. The differences in trading patterns before and after *time* $t = t_u + k$ indicate that if the manipulator wants to liquidate his shares in a quick manner, he must accept lower selling prices. This is consistent with our intuition on asset liquidity.

Figure 3: Price Dynamics and Trading Volume when $k < t_d$

(Assuming $t_u = 6, t_d = 3, \alpha = 0.1, q_1 = 0.8, q_2 = 0.4, q_3 = 0.3, k = 1, \delta = 0.35$)





Comparing Figure 2 and 3, we can see that the initial price rise could be more steep when the manipulator would let the sell price to fall after time $t = t_u + k$. The trading volume is also larger by the manipulator before and after the price peak, since in this case he needs to buy more shares to push the price higher while also sell more shares to liquidate his position. A simple computation shows that the manipulator makes more profit by letting the sell price to fall after time $t = t_u + k$.

Corollary 7: *Consider a special case of Proposition in which* $t_d = 1$*, that is, the manipulator needs to liquidate his shares accumulated from time* $t = 1$ *to time* $t = t_u$ *quickly within one period time. One obtains:*

(a) $$\eta = t_u \delta - \frac{h}{\alpha} \tag{7}$$

(b) The manipulator's capital gain is

$$\pi = \left[h - \left(\frac{\alpha}{2} \right) t_u \delta - \left(\frac{\alpha}{2} \right) \delta \right] t_u \delta \tag{8}$$

Corollary 8: *In Corollary 7, the manipulator can make a profit if and only if*

$$\delta < \frac{2h}{\alpha (t_u + 1)}. \tag{9}$$

Corollary 9: *Suppose that* t_u *is fixed and that* $t_d = 1$*, then the manipulator's maximum profit is obtained by setting* $\delta = \dfrac{h}{\alpha \cdot (t_u + 1)}$ *(for* $h > 0$*). By doing so, the manipulator's profit is* $\dfrac{t_u}{t_u + 1} \cdot \dfrac{h^2}{2\alpha}$.

Corollaries 8 and 9 provide us a quite intuitive result. If the manipulator has to complete a cycle of manipulation in a quick manner, he shall not move the price too slowly as by doing so, he will not be able to move the price up by a significant amount. On the other hand, he shall not be too greedy by moving the price too rapidly either, because by doing so, he will have to accumulate too large a position in the speculative asset and will not be able to liquidate the position at favorable prices.

To provide more intuition about Proposition 6, we now consider another special case of the Proposition in which $k = 1$ and $t_d = 2$. The following result can be obtained.

Proposition 10: *Suppose that* $h \equiv q_2 - q_1 \cdot q_3 > 0$ *. Consider a special case of Proposition 6 in which* $k = 1$ *and* $t_d = 2$*. One obtains:*

(a) $$\eta = \frac{t_u \alpha \delta - (2 - q_3)\, h}{\alpha} \tag{10}$$

(b) The manipulator can obtain a maximum trading profit by setting

$$\delta = \frac{(3 - q_3)h}{\alpha(t_u + 1)} > \frac{\left(1 - (1 - q_3)^2\right) / q_3}{\alpha \cdot t_u} h = \frac{(2 - q_3)}{\alpha \cdot t_u} h . \tag{11}$$

Proof: Part (a) follows immediately from Proposition 6. By the assumption, the manipulator shall sell h shares at time $t_u + 1$ and remaining $\alpha \cdot t_u \cdot \delta - h$ shares at time $t_u + 2$. Because the average cost of the manipulator's position in the speculative asset is $P_{t_u} - \frac{t_u - 1}{2} \delta$ per share, the manipulator's capital gain is given by:

$$\pi = \frac{t_u - 1}{2} \delta \cdot h + (\alpha \cdot t_u \cdot \delta - h)\left(\frac{t_u - 1}{2} \delta - \eta\right) \tag{12}$$

With tedious calculations, one can find that

$$\frac{\partial \pi}{\partial \delta} = \left[-\alpha(t_u + 1)\delta + (3 - q_3)h\right] t_u \tag{13}$$

and that

$$\frac{\partial^2 \pi}{\partial \delta^2} = [-\alpha(t_u + 1)]t_u < 0. \tag{14}$$

The first order condition with respect to δ yields $\delta = \frac{(3 - q_3)h}{\alpha(t_u + 1)}$. It is straightforward to verify that as $t_u \geq 2$, the inequality in (11) also holds. This completes the proof of part (b). ■

Recall from Proposition 3 that $\frac{1-(1-q_3)^2}{\alpha \cdot t_u}\frac{h}{q_3} = \frac{(2-q_3)}{\alpha \cdot t_u}h$ is the manipulator's optimal choice of δ if he wants to liquidate all his shares at the high price P_{t_u} in two periods after t_u. Inequality (11) in proposition 10 indicates that taking such a conservative position in the speculative asset in order to liquidate it at a very high price is not necessary the best choice for the manipulator if he does not have to cash in within one period. Comparing Proposition 10 with Corollary 9, we find that what profit the manipulator makes depends on how soon he needs to liquidate his position. The manipulator's patience in the process of liquidation pays off.

Proposition 11: *If* $h \equiv q_2 - q_1 \cdot q_3 < 0$, *manipulation considered in our model will not be profitable.*

Proof: If $h < 0$, the number of shares bought by the new behavior-driven investors will be smaller than that sold by the old behavior-driven investors whenever the price of the speculative asset stops rising. Therefore, when the manipulator liquidates his position, the behavior-driven investors as a whole will also sell. This prevents the manipulator from taking advantage of the irrationality of behavior-driven investors by liquidating his position to them at high prices. ■

Proposition 11 highlights again the importance of q_3, a measure of dispositional effect. Manipulation can be successful if and only if q_3 is sufficiently small, that is, the behavior-driven investors' unwillingness to take a loss is sufficiently strong. The proposition also indicates that even if there exist irrational investors, there is still no guarantee that a manipulation can be successful. This result has important implications for financial practices. For example, in the real world, the investors' behavior can be affected by many unpredictable factors and can show dramatic fluctuations from time to time. In other words, q_3 can be a random variable with a large variance. As a result, what consequence a real-world manipulation can bring is quite uncertain. If the manipulator miscalculates q_3 and over-estimates investors' unwillingness to take losses, he may well end up with a loss.

We have many ways to extend our model to allow for the randomness of price changes. For example, the manipulator can choose a different δ_t for each time period $t\left(t=1,2,\cdots,t_u\right)$ instead of a fixed δ. As long as δ_t remain positive for all time periods $t\left(t=1,2,\cdots,t_u\right)$, the results discussed in this section will be unchanged. We can also demonstrate that with appropriate choice of parameter values, the manipulator can make a profit even if he lets δ_t be negative occasionally for some $t\left(t=1,2,\cdots,t_u\right)$.and produces a price process that appears random but with an upward trend. As these extensions are straightforward, to conserve space, we will not discuss them in detail. The readers who are interested in these extensions can contact us directly.

Extension to Bear Raid (Dump and Cover)

It is worth noting that the model parameters may change over time according to different market conditions. While Assumption 1 may describe the trading of behavioral investors during a bull market for the asset, it is conceivable that each new behvavior-driven investor could have a probability of q_1 or q_2 to short a share during a bear market. In this case, it is straightforward to show that we can construct an example of a "dump and cover" strategy by modifying Assumption 1. This is done by assuming that behavior-driven investors are bearish and only take short positions[7] and by stating how the short-sellers may cover their positions as follows:

A continuum number of new behavior-driven investors, with measure 1, enter the market at the beginning of each period t. They are price-takers and each of them has a probability of q_1 *to short a share of the speculative asset if the price of the asset at time t>0,* P_t *, is less than the asset price at time t-1,* P_{t-1}*. If* $P_t > P_{t-1}$*, each new behavior-driven investor has a probability of* q_2 *to short a share of the speculative asset, where* $q_2 < q_1$.

The new behavior-driven investors at time t>0 who do not short the speculative asset choose to leave the market right away. The old generations of behavior-driven investors who entered the market before t>0 do not sell any more shares at time t. Behavior-driven investors like to take quick profits. They cover their short positions as soon as they have made a profit and then leave the market. Consider a behavior-driven investor who shorts a share of the speculative asset at time t and has not covered his share by the beginning

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[7] For example, during a bear market, when the market goes down, the investor will short 0.8 shares (q_1). And when the market goes up, the investor will short 0.3 shares (q_2). Thus, $q_2 < q_1$. The intuition here is that behavioral investors are bearish and follow a negative momentum. They will take short positions no matter what and short more shares when the market is down. They will only buy to cover their position. This is similar to our original set up, where behavioral investors are bullish. They will take long positions no matter what. They will only sell to liquidate their position. They buy more when market is up.

of time $t+k$ $(k>0)$. *If* $P_t > P_{t+k}$, *he shall cover his short in the period of* $t+k$ *for sure; if* $P_t \le P_{t+k}$, *he will have a probability of* $q_3 < 1$ *to cover his short in the period of* $t+k$. *Behavior-driven investors leave the market right after they have covered their shorts.*

To conserve space, we will not provide parallel proofs of Propositions 1-10.[8] The intuitions are quite similar. What make this bear raid possible are the investors' behavioral biases and the limit of arbitrage. Just as before, the manipulator can profit from his strategic trading by establish a large short position of the speculative asset while pushing asset price down, and then cover his position at low prices to take profits. The dispositional effect plays a critical role in making profitable manipulation possible. Because of this effect, the speed of price rise when the manipulator buys will be slower than that of price decline when the manipulator shorts. For simplicity, we will only discuss the case of "pump and dump" for the rest of the paper.

The introduction of this paper has briefly discussed the behavior finance literature. The behavioral or psychological biases discussed here are shown to generate both the incentives and the ability of the smart money to manipulate asset prices through strategic buying and selling. Our result suggests that as long as there are a significant number of irrational traders, market manipulation may occur.

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[8] For example, one can easily show Proposition 2 holds by setting: $\delta = \frac{t_d \cdot q_2}{t_u \cdot \alpha} < 0$. Because $q_3 = 0$, behavior-driven investors will not cover their shorts without a profit. The manipulator is able to buy q_2 shares from the new behavior-driven investors each period from day $t = t_u + 1$ through time $t = T-1$ by maintaining the equilibrium price at $P_{t_u} = P_0 + t_u\delta$. The average selling price is P_{t_u} per share. The manipulator's total profit is $\pi = \alpha \cdot \left(\frac{t_u - 1}{2} t_u \right) \cdot \delta^2$ Note here that t_u actually stands for the time period the asset price is being pushed downwards.

4. Other Implications of the Model

Our model not only provides a new and distinctive example of manipulation, but also sheds light on the cross-section of asset returns as well as some well-known asset pricing anomalies such as excess volatility, short-term momentum, and long-term reversal.

Shiller (1981, 1989) and Le Roy and Porter (1981) suggest that the historical volatility of stock prices in the United States are simply too high to be justified by the fundamental variations. Campbell and Cochrane (1999) argue that the high volatility of the stock market can possibly be caused by changing risk aversion of the investors. They propose a habit formation framework in which changes in consumption relative to habit lead to changes in risk aversion and hence the volatility of asset returns. In out model, the fundamental value of the speculative asset does not change at all. However, as the large trader move the price with his strategic trading, the price goes up and down from time to time for no fundamental reason.

Short-term momentum and long-term reversal are the other two popular empirical phenomena, as introduced in the previous sections.

BSV build a model that incorporates two updating biases, conservatism (the tendency to underweight new information relative to priors) and representativeness (the law of small numbers) to explain these phenomena. When a company announces surprisingly good earnings, conservatism means that investors react insufficiently and therefore prices will drift up subsequently. After a series of good news, though, representativeness causes people to overreact and pushes the price up too high.

DHS stress biases (overconfidence) in the interpretation of private, rather than public information. If the private information is positive, overconfidence means that investors will push prices up too high relative to fundamentals. Future public information will gradually pull prices back to their true value, leading to long-term reversals. To get momentum, DHS assume that public information alters the investors' confidence in an asymmetric fashion, a phenomenon known as self-attribution bias. Public news that confirms the investors' private information strongly increases their confidence in the private information. Disconfirming public news, though, is largely ignored, and the investors' confidence in the private information remains unchanged. This asymmetric reaction means that initial overconfidence is on average followed by even greater overconfidence, generating momentum.

Hong and Stein (1999) assume that private information diffuses slowly through the population of news watchers, since news watchers are unable to extract each others' information from prices, the slow diffusion means that the private information is not fully priced in an immediate way, generating momentum. On the other hand, momentum traders buy into price trend, which preserves momentum, but also generate price reversals. Since momentum traders do not know the extent of news diffusion, they keep buying into price trend even after the price has reached fundamental value, generating an overreaction that is reversed in the long run.

In our model, there exist both price momentum and reversal because the manipulator keeps buying the speculative asset initially, pushing the price up period by period; he then keeps selling to make profits, pushing the price down. The presence of momentum traders and the limits of arbitrage allows the manipulator to establish a price momentum while the existence of loss aversion and short-term arbitrageurs gives the manipulator a chance to sell at a profit even when the price is coming down.

Hong, Lim, and Stein (2000) document that there is a significant cross-sectional difference in momentum across different stocks. Small cap stocks usually show strong momentum, but large cap stocks do not[9]. The result of this paper is consistent with their finding. In this paper, as in Jarrow (1992), a large trader can be a manipulator because he has the power to affect (manipulate) the price. Obviously, it is much easier for someone to manipulate a small cap stock than to manipulate a large cap stock. Therefore, the price momentum and reversal generated by manipulation shall be more prominent for small cap stocks. One may also argue that higher transaction cost for small stocks would limit arbitrage, leading to easier price manipulation. However, higher transaction cost would also deter momentum trading. Thus, the effect of transaction cost on price manipulation is somewhat ambiguous.

Scheinkman and Xiong (2003) use a model of investor overconfidence that produces correlations among prices, turnover, and volatility. Their basic insight is that when investors have heterogeneous beliefs about the value of a stock and short sales are costly, the ownership of a share of the stock provides an opportunity (option) to profit from other investors' over-valuation. They show that the resale option leads to high speculative trading volume and contributes a speculative component to stock prices. In addition, fluctuations on the option value add to stock price volatility.

In our model, the manipulative trading by the large investor lures momentum traders into the market, pushes the stock price up and generates price volatility. Thus, our model implies that volume is higher in an up market (when asset price rises) than in a down market. Our model also suggests that volume is positively correlated with volatility (a

[9] Lesmond, Schill, and Zhou (2003) argue that this phenomenon is actually a price effect related to trading costs.

high δ means a high volatility).[10] The difference between our model and that of Scheinkman and Xiong (2003) is that their model needs some news about asset fundametals while ours is purely based on market manipulation. This feature helps us understand why asset prices sometimes fluctuate continually without seemingly to have any news on earnings or other fundamentals. It is important to note that, without observing the manipulator's trades, it will be quite hard to distinguish manipulative trading from speculative trading by using data only on price and trading volume. The only clue that might help investors detect the presence of manipulation is excessive trading volume and price movement without news on fundamentals.

5. Empirical Support of Our Model

While market manipulation has been extensively covered by the popular press, few academic studies have empirically examined the issue. The difficulty lies in the fact that the activity is often unobservable because of its secrecy. Several recent studies, however, have used data on government prosecution or firm-level trading data to document the existence of market manipulation. Their studies have lent some empirical support to our model.

Aggarwal and Wu (2003) provide evidence from SEC actions in cases of stock manipulation. They find that more illiquid stocks are more likely to be manipulated and manipulation increases stock volatility. They show that stock prices rise throughout the manipulation period and then fall in the post-manipulation period. More importantly, they demonstrate that stock display higher trading volume, higher price appreciation, and

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[10] In some cases, our model may also produce comparable correlations among prices, turnover, and volatility to Scheinkman and Xiong (2003). A simple computation would show that the correlations between price and trading volume are positive in all three figures in section 3. The difference between our model and that of Scheinkman and Xiong (2003) is that their model explicitly derives a positive correlation between price LEVEL and volume.

higher volatility during the manipulation period. These results are consistent with some assumptions as well as the main results of this paper. They suggest that stock market manipulation may have important impacts on asset pricing.

Using a unique daily trade level data set from the main stock market in Pakistan, Khwaja and Mian (2003) distinguish between trades done by brokers on their own behalf and those done as intermediaries for outside investors. They find that brokers earn at least 8% higher returns on their own trades. While neither market timing nor liquidity provision offer sufficient explanations for this result, they find compelling evidence for a specific trade-based "pump and dump" price manipulation scheme. The price patterns generated by such "pump and dump" is quite consistent with those described in section 3, where a manipulator with deep pocket use trade-based schemes to fool behavioral investors.

All the above results suggest that manipulation could cause large price distortions in the market and thus it is a legitimate target for government regulation. They find that potentially informed parties such as corporate insiders, brokers, underwriters, large shareholders and market makers are likely to be manipulators. Aggarwal and Wu suggest government regulation should discourage manipulation while encouraging greater competition for information.

Khwaja and Mian (2003), however, argue that the implementation of these market regulations will not be easy, since these rules aimed reducing manipulation will be actively resisted by brokers because of the sizable manipulation rents extracted by brokers from such schemes.

6. Conclusions

It is now widely believed that investors are not fully rational. If so, what can the smart money do? This paper provides an example in which smart money can strategically take advantage of investors' behavioral biases and manipulate price process to make profit. It builds a model in which there are three types of traders, behavior-driven investors who have two major behavioral biases, momentum trading and the tendency to sell winners rather than losers, arbitrageurs, and a manipulator who can influence asset prices. It shows that due to the investors' behavioral biases and the limit of arbitrage, the manipulator can profit from his strategic trading by accumulating the speculative asset while pushing asset price up, and then selling the asset to take profits. The dispositional effect plays a critical role in making profitable manipulation possible. Because of this effect, the speed of price decline when the manipulator sells will be slower than that of price rise when the manipulator buys.

Conventional wisdom suggests that smart money's speculation tends to make the market efficient by offsetting the foolishness of some investors. The efficient market theory, as it is commonly expressed, asserts that when irrational optimists buy an asset, smart money sells; when irrational pessimists sell an asset, smart money buys, thereby eliminating the effect of the irrational traders on asset price and preventing asset price from deviating from its fundamental value. This paper provides a shocking counterexample. Smart money may create "market inefficiency", by driving asset prices away from their fundamental value, rather than forcing asset prices to converge to their fundamental values. This possibility poses a new challenge. As the manipulator relies on neither inside information nor visible actions (other than trading), his manipulation is difficult to be detected and ruled out.

Our investigation is preliminary in nature and many directions for future research remains open. For example, one can consider a more complicated and more realistic case in which the large trader can have both privileged information and market moving power. With this setup, manipulation is possible and more realistic because irrational investors cannot rationally figure out whether the large trader's trading is based on his private information or simply based on his manipulation scheme. One can also consider an extension of the current model in which the manipulator's trading strategy is endogenously determined based on profit optimization. This extension is interesting because we can learn more about the price dynamics.

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