NEW YORK UNIVERSITY•LEONARD N. STERN SCHOOL OF BUSINESS

Department of Finance

Working Paper Series

FIN-03-044

Asset Pricing with Liquidity Risk

Viral V. Acharya and Lasse Heje Pedersen

July 17, 2003

The 2003 NYU Stern Department of Finance Working Paper Series is generously sponsored by

FitchRatings

Asset Pricing with Liquidity Risk*

Viral V. Acharya† and Lasse Heje Pedersen‡

First Version: July 10, 2000
Current Version: July 17, 2003

Abstract

This paper studies equilibrium asset pricing with liquidity risk — the risk arising from unpredictable changes in liquidity over time. It is shown that a security's required return depends on its expected illiquidity and on the covariances of its own return and illiquidity with market return and market illiquidity. This gives rise to a liquidity-adjusted capital asset pricing model. Further, if a security's liquidity is persistent, a shock to its illiquidity results in low contemporaneous returns and high predicted future returns. Empirical evidence based on cross-sectional tests is consistent with liquidity risk being priced.

---

*We are grateful for conversations with Andrew Ang, Joseph Chen, Sergei Davydenko, Francisco Gomes, Joel Hasbrouck, Andrew Jackson, Tim Johnson, Martin Lettau, Anthony Lynch, Stefan Nagel, Lubos Pastor, Dimitri Vayanos, Luis Viceira, Jeff Wurgler, and seminar participants at London Business School, London School of Economics, New York University, the National Bureau of Economic Research (NBER) Summer Institute 2002, the Five Star Conference 2002, and Western Finance Association Meetings 2003. We are especially indebted to Yakov Amihud for being generous with his time in guiding us through the empirical tests and to an anonymous referee for many valuable suggestions. All errors remain our own.

†Acharya is at London Business School and is a Research Affiliate of the Centre for Economic Policy Research (CEPR). Address: London Business School, Regent's Park, London - NW1 4SA, UK. Phone: +44 (0)20 7262 5050 x 3535. Fax: +44 (0)20 7724 3317. Email: vacharya@london.edu. Web: http://www.london.edu/faculty/vacharya

‡Pedersen is at the Stern School of Business, New York University, 44 West Fourth Street, Suite 9-190, New York, NY 10012-1126. Phone: (212) 998-0359. Fax: (212) 995-4233. Email: lpederse@stern.nyu.edu. Web: http://www.stern.nyu.edu/∼lpederse/

1 Introduction

The existing theoretical literature on frictions and asset pricing has focused on various frictions with *deterministic* severity (for instance, Amihud and Mendelson (1986), Constantinides (1986), Vayanos (1998), Vayanos and Vila (1999), Gârleanu and Pedersen (2000), Huang (2002)). Empirically, however, various measures of liquidity vary over time both for individual stocks and for the market as a whole (Chordia, Roll, and Subrahmanyam (2000), Hasbrouck and Seppi (2000), and Huberman and Halka (1999)). Hence, investors face uncertainty about liquidity, which raises the question: How does liquidity risk affect asset prices in equilibrium? We answer this question by deriving explicitly a liquidity-adjusted capital asset pricing model (CAPM) wherein there are price effects associated with *the risk of changes* in the liquidity of an individual security as well as in market liquidity.

In the liquidity-adjusted CAPM, the expected return of a security is increasing in its expected illiquidity and its "net beta," which is proportional to the covariance of its return, net of illiquidity costs, with the market portfolio's net return. The net beta can be decomposed into the sensitivity of the security's return and tradability to market downturns as well as to liquidity crises. We discuss in turn the three aspects of liquidity risk highlighted by the model and their empirical relevance.

First, the model shows that investors require a return premium for a security that is illiquid when the market as a whole is illiquid. The potential importance of this result follows from the empirically documented commonality in liquidity. In particular, Chordia, Roll, and Subrahmanyam (2000) find significant commonality in liquidity using daily data for NYSE stocks in 1992, Huberman and Halka (1999) find a systematic time-varying component of liquidity using daily NYSE data from 1996, and Hasbrouck and Seppi (2000) find weak commonality in liquidity for 30 Dow stocks over 15-minute intervals during 1994. The effect of commonality of liquidity on required returns has not yet been tested. Empirically, we find support for this prediction but its economic effect on expected returns seems small.

Second, the model shows that investors are willing to pay a premium for a security that has a high return when the market is illiquid. Pastor and Stambaugh (2001) find empirical support for this effect using monthly data over 34 years with a measure of liquidity that they construct based on the return reversals induced by order flow. Consistently, we also find empirical support for this prediction.

Third, the model implies that investors are willing to pay a premium for a security that is liquid when the market return is low. This is a new testable prediction that has not been considered in the literature. We find support for it empirically in most of our specifications and robustness tests. Further, the risk premium arising from this effect is economically significant.

We test the model cross-sectionally using the liquidity measure suggested by Amihud (2002), which is based on daily return and volume data on NYSE and AMEX stocks over the period 1963–1999. Monthly cross-sectional tests of the liquidity-adjusted CAPM demonstrate that it cannot be rejected at conventional levels of confidence. Furthermore, it fares significantly better — in terms of its R^2 for cross-sectional returns and p-values in specification tests — than the standard CAPM, even though both models employ exactly one degree of freedom.

In our tests, the three covariances described above contribute on average to a difference in risk premium between stocks with high expected illiquidity and low expected illiquidity of about 1.1% annually. 80% of this effect is attributable to the third aspect of liquidity risk, the sensitivity of a security's illiquidity to market returns. Overall, the combined effect of differences in liquidity *risk* and differences in the *level* of liquidity is 4.6% per year.[1] When we depart in the tests from the model-implied liquidity adjustment in that the risk premia on different liquidity betas are allowed to be different, the economic effect of the covariances is even higher. We conclude that the liquidity risk indeed appears to be priced.

Another result, interesting in its own right, that emerges from our empirical exercise is that illiquid securities also have high liquidity *risk*: A security which is illiquid in absolute terms, measured by its average transaction cost, also tends to have a lot of commonality in liquidity with the market liquidity, a lot of return sensitivity to market liquidity, and a lot of liquidity sensitivity to market returns. This finding points towards a fruitful direction of research aimed at understanding the sources of time-variation in liquidity at an individual stock level as well as at the market level.

The model also shows that, since liquidity is persistent,[2] liquidity predicts future returns and liquidity co-moves with contemporaneous returns. This is because a positive shock to illiquidity predicts high future illiquidity, which raises the required return and lowers contemporaneous prices. In support of

[1] We show later that sorting stocks by expected illiquidity also produces a sorting on the covariances.

[2] The persistence of liquidity is documented empirically by Amihud (2002), Chordia, Roll, and Subrahmanyam (2000, 2001), Hasbrouck and Seppi (2000), Huberman and Halka (1999), Jones (2001), and Pastor and Stambaugh (2001).

this prediction, Amihud (2002) finds a negative relation between return and unexpected illiquidity for size portfolios, Chordia, Roll, and Subrahmanyam (2001), Jones (2001), and Pastor and Stambaugh (2001) find a negative relation between market return and illiquidity, and Amihud, Mendelson, and Wood (1990) find that stocks, whose liquidity worsened more during the 1987 crash, had more negative returns.

The paper is organized as follows. Section 2 describes the economy, Section 3 derives the liquidity-adjusted capital asset pricing model and studies how liquidity predicts and co-moves with returns, Section 4 contains our empirical results, Section 5 concludes, and proofs are in the Appendix.

2 Assumptions

The model assumes a simple overlapping generations economy in which a new generation of agents is born at any time $t \in \{\ldots, -2, -1, 0, 1, 2, \ldots\}$ (Samuelson (1958)). Generation t consists of N agents, indexed by n, who live for two periods, t and $t+1$. Agent n of generation t has an endowment at time t and no other sources of income, trades in periods t and $t+1$, and derives utility from consumption at time $t+1$. He has constant absolute risk aversion A^n so that his preferences are represented by the expected utility function $-E_t \exp(-A^n x_{t+1})$, where x_{t+1} is his consumption at time $t+1$.

There are I securities indexed by $i = 1, \ldots, I$ with a total of S^i shares of security i. At time t, security i pays a dividend of D_t^i, has an ex-dividend share price of P_t^i, and has an illiquidity cost of C_t^i, where D_t^i and C_t^i are random variables.[3] The illiquidity cost, C_t^i, is modeled simply as the per-share cost of selling security i. Hence, agents can buy at P_t^i but must sell at $P_t^i - C_t^i$. Short-selling is not allowed.

Uncertainty about the illiquidity cost is what generates the liquidity risk in this model. Specifically, we assume that D_t^i and C_t^i are autoregressive processes of order one, that is:

$$
\begin{aligned}
D_t &= \bar{D} + \gamma(D_{t-1} - \bar{D}) + \varepsilon_t \\
C_t &= \bar{C} + \gamma(C_{t-1} - \bar{C}) + \eta_t\,,
\end{aligned}
$$

where $\bar{D}$, $\bar{C} \in \mathbb{R}_+^I$ are positive real vectors, $\gamma \in [0,1]$, and (ε_t, η_t) is an independent identically distributed normal process with mean $E(\varepsilon_t) = E(\eta_t) = 0$ and variance-covariance matrices $\mathrm{var}(\varepsilon_t) = \Sigma^D$, $\mathrm{var}(\eta_t) = \Sigma^C$, $E(\varepsilon_t \eta_t^\top) = \Sigma^{CD}$, and $\mathrm{var}(\varepsilon_t - \eta_t) = \Gamma$ $(= \Sigma^D + \Sigma^C - \Sigma^{CD} - (\Sigma^{CD})^\top)$.

We assume that agents can borrow and lend at a risk-free real return of $r^f > 1$, which is exogenous. This can be interpreted as an inelastic world bond market, or a generally available production technology that turns a unit of consumption at time t into r^f units of consumption at time $t+1$.

The assumptions with respect to agents, preferences, and dividends are strong. These assumptions are made for tractability, and, as we shall see, they imply natural closed-form results for prices and expected returns. The main result (Proposition 1) applies more generally, however. It holds for arbitrary utility functions as long as conditional expected net returns are normal,[4] and

[3] All random variables are defined on a probability space $(\Omega, \mathcal{F}, \mathcal{P})$, and all random variables indexed by t are measurable with respect to the filtration $\{\mathcal{F}_t\}$, representing the information commonly available to investors.

[4] The normal returns assumption is an assumption about endogenous variables that is

also for arbitrary return distribution and quadratic utility. Furthermore, it can be viewed as a result of near-rational behavior, for instance, by using a Taylor expansion of the utility function (see Huang and Litzenberger (1988), Markowitz (2000), and Cochrane (2001)). Our assumptions allow us, additionally, to study return predictability caused by illiquidity (Proposition 2) and the co-movements of returns and illiquidity (Proposition 3), producing insights that also seem robust to the specification.

Perhaps the strongest assumption is that investors need to sell all their securities after one period (when they die). In a more general setting with endogenous holding periods, deriving a general equilibrium with time-varying liquidity is an onerous task. While our model is mostly suggestive, it is helpful since it provides guidelines concerning the first-order effect of liquidity risk, showing which risks are priced. The assumption of overlapping generations can capture investors' life-cycle motives for trade (as in Vayanos (1998), and Constantinides, Donaldson, and Mehra (2002)), or can be viewed as a way of capturing short investment horizons (as in De Long, Shleifer, Summers, and Waldmann (1990)) and the large turnover observed empirically in many markets.

It should also be noted that a narrow interpretation of the illiquidity cost, C_t^i, is that it is a transaction cost such as broker fees and bid-ask spread, in line with the literature on exogenous transactions costs. More broadly, however, the illiquidity cost could represent other the real costs, for instance, arising from delay and search associated with trade execution as in Duffie,

used in standard CAPM analysis (for instance, Huang and Litzenberger (1988)). This assumption is satisfied in the equilibrium of the model of this paper, and may also be satisfied in larger classes of models.

Gârleanu, and Pedersen (2000). The novelty in our model arises from the fact that we allow this cost to be time-varying. While research on endogenous time-variation in illiquidity is sparse, in a recent paper Eisfeldt (1999) presents a model in which liquidity fluctuates with real-sector productivity and investment.

3 Liquidity-Adjusted Capital Asset Pricing Model

This section shows that, under the stylized assumption of mean-variance investors, a liquidity-adjusted version of the Capital Asset Pricing Model (CAPM) applies and its asset pricing implications are studied.

We are interested in how an asset's expected (gross) return,

$$r_t^i = \frac{D_t^i + P_t^i}{P_{t-1}^i},$$

depends on its relative illiquidity cost, defined as

$$c_t^i = \frac{C_t^i}{P_{t-1}^i},$$

on the market return,

$$r_t^M = \frac{\sum_i S^i (D_t^i + P_t^i)}{\sum_i S^i P_{t-1}^i},$$

and on the relative market illiquidity,

$$c_t^M = \frac{\sum_i S^i C_t^i}{\sum_i S^i P_{t-1}^i} \ .$$

In a competitive equilibrium of the model (henceforth referred to simply as equilibrium), agents choose consumption and portfolios so as to maximize their expected utility taking prices as given, and prices are determined such that markets clear.

To determine equilibrium prices, consider first an economy with the same agents in which asset i has a dividend of $D_t^i - C_t^i$ and no illiquidity cost. In this imagined economy, standard results imply that the CAPM holds (Markowitz (1952), Sharpe (1964), Lintner (1965), and Mossin (1966)). We claim that the equilibrium prices in the original economy with frictions are the same as those of the imagined economy. This follows from two facts: *(i)* the net return on a long position is the same in both economies; *(ii)* all investors in the imagined economy hold a long position in the market portfolio, and a (long or short) position in the risk-free asset. Hence, an investor's equilibrium return in the frictionless economy is feasible in the original economy, and is also optimal, given the more limited investment opportunities due to the short-selling constraints.[5]

These arguments show that the CAPM in the imagined frictionless economy translates into a CAPM in net returns for the original economy with illiquidity costs, that is,

$$E_{t-1}(r_t^i - c_t^i - r^f) = \lambda_{t-1} \frac{\text{cov}_{t-1}(r_t^i - c_t^i, r_t^M - c_t^M)}{\text{var}_{t-1}(r_t^M - c_t^M)} \tag{1}$$

Rewriting the one-beta CAPM in net returns in terms of gross returns, we

[5]This argument applies more generally since positive transactions costs imply that a short position has a worse payoff than minus the payoff of a long position. We impose the short-sale constraint because C can be negative in our normal setting.

get a liquidity-adjusted CAPM for gross returns. This is the main testable[6] implication of this paper:

Proposition 1 *In the unique linear equilibrium, the conditional expected return of security i is*

$$\begin{aligned} E_{t-1}(r_t^i - r^f) &= E_{t-1}(c_t^i) + \lambda_{t-1}\frac{\mathrm{cov}_{t-1}(r_t^i, r_t^M)}{\mathrm{var}_{t-1}(r_t^M - c_t^M)} + \lambda_{t-1}\frac{\mathrm{cov}_{t-1}(c_t^i, c_t^M)}{\mathrm{var}_{t-1}(r_t^M - c_t^M)} \\ &\quad -\lambda_{t-1}\frac{\mathrm{cov}_{t-1}(r_t^i, c_t^M)}{\mathrm{var}_{t-1}(r_t^M - c_t^M)} - \lambda_{t-1}\frac{\mathrm{cov}_{t-1}(c_t^i, r_t^M)}{\mathrm{var}_{t-1}(r_t^M - c_t^M)}\,, \end{aligned} \tag{2}$$

where λ_{t-1} is the risk premium,

$$\lambda_{t-1} = E_{t-1}(r_t^M - c_t^M - r^f) \tag{3}$$

Equation (2) is simple and natural. It states that the required excess return is the expected relative illiquidity cost, $E_{t-1}(c_t^i)$, as first found theoretically and empirically[7] by Amihud and Mendelson (1986)), plus four betas (or covariances) times the risk premium. These four betas depend on the asset's payoff and liquidity risks. As in the standard CAPM, the required return on

[6]Difficulties in testing this model arise from the fact that it makes predictions concerning conditional moments as is standard in asset pricing. See Hansen and Richard (1987), Cochrane (2001), and references therein. An unconditional version of (2) applies under stronger assumptions as discussed in Section 3.3.

[7]Empirically, Amihud and Mendelson (1986, 1989) find the required rate of return on NYSE stocks to increase with the relative bid-ask spread. This result is supported for amortized spreads for NYSE stocks by Chen and Kan (1996), and for Nasdaq stocks by Eleswarapu (1997), but is questioned for NYSE stocks by Eleswarapu and Reinganum (1993), and Chalmers and Kadlec (1998). Gârleanu and Pedersen (2000) find that adverse-selection costs are priced only to the extent that they render allocations inefficient. The ability of a market to allocate assets efficiently may be related to market depth, and, consistent with this view, the required rate of return has been found to decrease with measures of depth (Brennan and Subrahmanyam (1996) and Amihud (2002)). Easley, Hvidkjær, and O'Hara (2000) find returns to increase with a measure of the probability of informed trading.

an asset increases (linearly) with the covariance between the asset's return and the market return. This model yields three additional effects which could be regarded as three forms of liquidity risks.

3.1 Three Liquidity Risks

1. $\text{cov}_{t-1}(c_t^i, c_t^M)$: The first effect is that the return increases with the covariance between the asset's illiquidity and the market illiquidity ($\text{cov}_{t-1}(c_t^i, c_t^M)$). This is because investors want to be compensated for holding a security that becomes illiquid when the market in general becomes illiquid. The potential empirical significance of this pricing implication follows from the presence of a time-varying common factor in liquidity, which is documented by Chordia, Roll, and Subrahmanyam (2000), Hasbrouck and Seppi (2000), and Huberman and Halka (1999). These papers find that most stocks' illiquidities are positively related to market illiquidity, so the required return should be raised by the commonality-in-liquidity effect. The effect of commonality in liquidity on asset prices is, however, not studied by these authors; We study empirically this effect is studied in Section 4.

In this model, the risk premium associated with commonality in liquidity is caused by the wealth effects of illiquidity. Also, this risk premium would potentially apply in an economy in which investors can choose which securities to sell. In such a model, an investor who holds a security that becomes illiquid (that is, has a high cost c_t^i) can choose not to trade this security and instead trade other (similar) securities. It is more likely that an investor can trade other (similar) securities, at low cost, if the liquidity of this asset does not co-move with the market liquidity. Hence, investors would require a return premium for assets with positive covariance between individual and market illiquidity.

2. $\mathrm{cov}_{t-1}(r_t^i, c_t^M)$: The second effect on expected returns is due to covariation between a security's return and the market liquidity. We see that $\mathrm{cov}_{t-1}(r_t^i, c_t^M)$ affects required returns negatively because investors pay a premium for an asset with a high return in times of market illiquidity. Such an effect also arises in the theoretical models of Holmstrom and Tirole (2000) who examine implications of corporate demand for liquidity, and Lustig (2001) who studies the equilibrium implications of solvency constraints. Empirical support for this effect is provided by Pastor and Stambaugh (2001), who find that "the average return on stocks with high sensitivities to [market] liquidity exceeds that for stocks with low sensitivities by 7.5% annually, adjusted for exposures to the market return as well as size, value, and momentum factors." Sadka (2002) and Wang (2002) also present consistent evidence for this effect using alternative measures of liquidity.

3. $\mathrm{cov}_{t-1}(c_t^i, r_t^M)$: The third effect on required returns is due to covariation, $\mathrm{cov}_{t-1}(c_t^i, r_t^M)$, between a security's illiquidity and the market return. This effect stems from investors' willingness to accept a lower expected return on a security that is liquid in a down market. When the market declines, investors are poor, and the ability to sell easily is especially valuable. Hence, an investor is willing to accept a discounted return on stocks with low illiquidity costs in states of poor market return. We find consistent evidence of this effect in Section 4, and the effect seems economically important.

Outside our model, intuition suggests that a low market return causes wealth problems for some investors, who then need to sell. If a selling investor holds securities that are illiquid at this time, then his problems are magnified. Consistent with this intuition, Lynch and Tan (2003) find that the liquidity premium is large if the transactions costs covary negatively with wealth shocks, among other conditions. This is consistent with our effect of $\mathrm{cov}_{t-1}(c_t^i, r_t^M)$ to the extent that r^M proxies for wealth shocks. Lynch and Tan (2003) complement our paper by showing by calibration that, even if an investor chooses his holding period endogenously, the liquidity premium can be large (3.55% in one calibration). They follow Constantinides (1986) in using a partial-equilibrium framework and defining the liquidity premium as the decrease in expected return that makes an investor indifferent between having access to the asset without transaction costs rather than with them.

The three covariances thus provide a characterization of the liquidity risk of a security. While the covariance between a security's return and the market liquidity has been shown empirically to affect its expected return, the effect of the other two covariances on expected returns has not yet been examined.

Finally, note that in our model, the conditional CAPM holds for net returns, that is, returns net of illiquidity costs. The analysis is, however, focused on gross returns. The focus on gross returns is motivated by several considerations. First, computing the net return is not straightforward since it *depends on the investor's holding period*, and the holding period may be different from the econometrician's sampling period. We explain in Section 4 how we overcome this problem by separating the net return into gross return and illiquidity costs. Second, most empirical work uses some measure of gross returns and possibly some measure of illiquidity costs. Third, the model shows interesting pricing implications of co-movements in individual and market gross return and liquidity. Empirical work has documented that some of these interactions are significant (Chordia, Roll, and Subrahmanyam (2000), Hasbrouck and Seppi (2000), and Huberman and Halka (1999)) and priced (Amihud and Mendelson (1986), Amihud (2002), and Pastor and Stambaugh (2001)). Fourth, a pricing relation for gross returns and illiquidity, which is similar in spirit to (2), may hold in richer models in which net returns are not sufficient state variables. As argued above, some additional liquidity effects suggest risk premia of the same sign for the covariance terms in (2). These additional liquidity effects also suggest that the size of the risk premia need not be identical across the covariance terms. To accommodate the possibility of a richer liquidity framework, we also consider a generalized relation in our empirical work in Section 4.

3.2 Implications of Persistence of Liquidity

This section shows that persistence of liquidity implies that liquidity predicts future returns and co-moves with contemporaneous returns.

Empirically, liquidity is *time-varying and persistent* (which means that $\gamma > 0$).[8] This model shows that persistent liquidity implies that returns are *predictable*. Intuitively, high illiquidity today predicts high expected illiquidity next period, implying a high required return.

[8]See Amihud (2002), Chordia, Roll, and Subrahmanyam (2000, 2001), Hasbrouck and Seppi (2000), Huberman and Halka (1999), Jones (2001), and Pastor and Stambaugh (2001).

Proposition 2 *Suppose that $\gamma > 0$, and that $q \in \mathbb{R}^I$ is a portfolio*[9] *with $\gamma D^q_{t-1} + (1-\gamma) E(D^q_t + P^q_t \mid D^q_{t-1} = \bar{D}^q, C^q_{t-1} = \bar{C}^q) > 0$. Then, the conditional expected return increases with illiquidity,*

$$\frac{\partial}{\partial C^q_{t-1}} E_{t-1}(r^q_t - r^f) > 0. \qquad (4)$$

Proposition 2 relies on a mild technical condition, which is satisfied, for instance, for any portfolio with positive values for current dividend, mean dividend and mean price. The proposition states that the conditional expected return depends positively on the current illiquidity cost, that is, the current liquidity predicts the return.

Jones (2001) finds empirically that the expected annual stock market return increases with the previous year's bid-ask spread and decreases with the previous year's turnover. Amihud (2002) finds that illiquidity predicts excess return both for the market and for size-based portfolios.

Predictability of liquidity further implies a negative conditional covariance between contemporaneous returns and illiquidity. Naturally, when illiquidity is high, the required return is high also, which depresses the current price, leading to a low return. This intuition applies as long as liquidity is persistent ($\gamma > 0$) and innovations in dividends and illiquidity are not too correlated ($q^\top \Sigma^{CD} q$ low for a portfolio q) as is formalized in the following proposition.

Proposition 3 *Suppose $q \in \mathbb{R}^I$ is a portfolio such that $\gamma > r^f \frac{q^\top \Sigma^{CD} q}{q^\top \Sigma^C q}$. Then,* $\mathrm{cov}_{t-1}(c^q_t, r^q_t) < 0$.

[9] For any $q \in \mathbb{R}^I$, we use the obvious notation $D^q_t = q^\top D_t$, $r^q_t = \frac{\sum_i q^i (D^i_t + P^i_t)}{\sum_i q^i P^i_{t-1}}$ and so on.

Consistent with this result, Chordia, Roll, and Subrahmanyam (2001), Jones (2001), and Pastor and Stambaugh (2001) find a negative relation between the market return and measures of market illiquidity, Amihud (2002) finds a negative relation between the return on size portfolios and their corresponding unexpected illiquidity, and Amihud, Mendelson, and Wood (1990) argue that the 1987 crash was in part due to an increase in (perceived) market illiquidity.

3.3 An Unconditional Liquidity-Adjusted CAPM

To estimate the liquidity-adjusted CAPM, we derive an unconditional version. An unconditional result obtains, for instance, under the assumption of independence over time of dividends and illiquidity costs. Empirically, however, illiquidity is persistent. Therefore, we rely instead on an assumption of constant conditional covariances of innovations in illiquidity and returns.[10] This assumption yields the unconditional result that,

$$E(r_t^i - r_t^f) = E(c_t^i) + \lambda\beta^{1i} + \lambda\beta^{2i} - \lambda\beta^{3i} - \lambda\beta^{4i} \ , \tag{6}$$

[10]Alternatively, the same unconditional model can be derived by assuming a constant risk premium λ, and by using the fact that for any random variables X and Y, it holds that

$$E(\mathrm{cov}_t(X,Y)) = \mathrm{cov}(X - E_t(X), Y) = \mathrm{cov}(X - E_t(X), Y - E_t(Y)). \tag{5}$$

We note that the possible time-variation of risk premium is driven by constant absolute risk aversion in our model, but with constant relative risk aversion the risk premium is approximately constant. See Friend and Blume (1975).

where

$$
\begin{aligned}
\beta^{1i} &= \frac{\operatorname{cov}(r_t^i, r_t^M - E_{t-1}(r_t^M))}{\operatorname{var}\left(r_t^M - [c_t^M - E_{t-1}(c_t^M)]\right)} && (7) \\
\beta^{2i} &= \frac{\operatorname{cov}(c_t^i - E_{t-1}(c_t^i), c_t^M - E_{t-1}(c_t^M))}{\operatorname{var}\left(r_t^M - [c_t^M - E_{t-1}(c_t^M)]\right)} && (8) \\
\beta^{3i} &= \frac{\operatorname{cov}(r_t^i, c_t^M - E_{t-1}(c_t^M))}{\operatorname{var}\left(r_t^M - [c_t^M - E_{t-1}(c_t^M)]\right)} && (9) \\
\beta^{4i} &= \frac{\operatorname{cov}(c_t^i - E_{t-1}(c_t^i), r_t^M - E_{t-1}(r_t^M))}{\operatorname{var}\left(r_t^M - [c_t^M - E_{t-1}(c_t^M)]\right)}, && (10)
\end{aligned}
$$

and $\lambda = E(\lambda_t) = E(r_t^m - c_t^m - r_t^f)$. Next, we describe the empirical tests of this unconditional relation.

4 Empirical Results

In this section, we estimate and test the liquidity-adjusted CAPM as specified in Equation (6). We do this in five steps:

(i) We estimate, in each month t of our sample, a measure of illiquidity, c_t^i, for each individual security i. (Section 4.1.)

(ii) We form a "market portfolio" and sets of 25 test portfolios sorted on the basis of illiquidity, size, and book-to-market by size, respectively. For each portfolio and each month, we compute its return and illiquidity. (Section 4.2.)

(iii) For the market portfolio as well as the test portfolios, we estimate the innovations in illiquidity, $c_t^p - E_{t-1}(c_t^p)$. (Section 4.3.)

(iv) Using these illiquidity innovations and returns, we estimate and analyze the liquidity betas. (Section 4.4.)

(v) Finally, we consider the empirical fit of the (unconditional) liquidity-adjusted CAPM by running cross-sectional regressions based on the empirical methodology of Fama and MacBeth (1973). To check the robustness of our results, we do the analysis with a number of different specifications. (Section 4.5.)

4.1 The Illiquidity Measure

Liquidity is (unfortunately) not an observable variable. There exist, however, many proxies for liquidity. Some proxies, such as the bid-ask spread, are based on market microstructure data, which is not available for a time series as long as is usually desirable for studying the effect on expected returns. Further, the bid-ask spread measures well the cost of selling a small number of shares, but it does not necessarily measure well the cost of selling many shares. We follow Amihud (2002) in estimating illiquidity using only daily data from the Center for Research in Security Prices (CRSP). In particular, Amihud (2002) defines the illiquidity of stock i in month t as

$$ILLIQ_t^i = \frac{1}{Days_t^i} \sum_{d=1}^{Days_t^i} \frac{|R_{td}^i|}{V_{td}^i}, \tag{11}$$

where R_{td}^i and V_{td}^i are, respectively, the return and dollar volume on day d in month t, and $Days_t^i$ is the number of valid observation days in month t for stock i. Throughout our empirical analysis, $ILLIQ_t^i$ is multiplied by a scale factor of 10^6.

The intuition behind this illiquidity measure is as follows. A stock is illiquid — that is, has a high value of $ILLIQ_t^i$ — if the stock's price moves a lot in response to little volume. In our model, illiquidity is the cost of selling and, as discussed in Section 2, real markets have several different selling costs including broker fees, bid-ask spreads, market impact, and search costs. Our empirical strategy is based on an assumption that *ILLIQ* is a valid instrument for the costs of selling, broadly interpreted. Consistent with this view, Amihud (2002) shows empirically that *ILLIQ* is positively related to measures of price impact and fixed trading costs over the time period in which he has the microstructure data. Similarly, Hasbrouck (2002) computes a measure of Kyle's lambda using micro-structure data for NYSE, AMEX and NASDAQ stocks, and finds that its Spearman (Pearson) correlation with *ILLIQ* in the cross-section of stocks is 0.737 (0.473). Hasbrouck (2002) concludes that "[a]mong the proxies considered here, the illiquidity measure [ILLIQ] appears to be the best." Furthermore, *ILLIQ* is closely related to the Amivest measure of illiquidity, which has often been used in the empirical microstructure literature.[11]

There are two problems with using *ILLIQ*. First, it is measured in "percent per dollar," whereas the model is specified in terms of "dollar cost per dollar invested." This is a problem because it means that *ILLIQ* is not stationary (e.g., inflation is ignored). Second, while *ILLIQ* is an instrument for the cost of selling, it does not directly measure the cost of a trade. To solve these problems, we define a normalized measure of illiquidity, c_t^i, by

$$c_t^i = \min\left(0.25 + 0.30\, \mathit{ILLIQ}_t^i\, P_{t-1}^M\, , 30.00\right)\ , \tag{12}$$

where P_{t-1}^M is the ratio of the capitalizations of the market portfolio at the

[11]The Amivest measure of liquidity is the average ratio of volume to absolute return.

end of month $t-1$ and of the market portfolio at the end of July 1962. The P^M_{t-1} adjustment solves the first problem mentioned above, and it makes this measure of illiquidity relatively stationary. The coefficients 0.25 and 0.30 are chosen such that the cross-sectional distribution of normalized illiquidity (c^i_t) for size-decile portfolios has approximately the same level and variance as does the effective bid-ask spread reported by Chalmers and Kadlec (1998). This normalized illiquidity is capped at a maximum value of 30% in order to ensure that our results are not driven by the extreme observations of $ILLIQ^i_t$. Furthermore, a per-trade cost greater than 30% seems unreasonable and is an artifact of the effect of low volume days on $ILLIQ^i_t$.

Chalmers and Kadlec (1998) report that the mean effective spread for size-decile portfolios of NYSE and AMEX stocks over the period 1983–1992 ranges from 0.25% to 4.16% with an average of 1.11%. The normalized illiquidity, c^i_t, for identically formed portfolios has an average of 1.24%, a standard deviation of 0.37%, and matches the range as well as the cross-sectional variation reported by Chalmers and Kadlec (1998). This means that we can interpret the illiquidity measure c^i_t as directly related to (a lower bound of) the per-trade cost.

Admittedly, this is a noisy measure of illiquidity, which makes it harder for us to find an empirical connection between returns and illiquidity. This problem is alleviated in part, however, by considering portfolios rather than individual stocks.

4.2 Portfolios

We employ daily return and volume data from CRSP from July 1st, 1962 until December 31st, 1999 for all common shares (CRSP sharecodes 10 and 11) listed on NYSE and AMEX.[12] Also, we use book-to-market data based on the COMPUSTAT measure of book value.[13]

We form a market portfolio for each month t during this sample period based on stocks with beginning-of-month price between 5 and 1000, and with at least 15 days of return and volume data in that month.

We form 25 illiquidity portfolios for each year y during the period 1964 to 1999 by sorting stocks with price, at beginning of year, between 5 and 1000, and return and volume data in year $y-1$ for at least 100 days.[14] We compute the annual illiquidity for each eligible stock as the average over the entire year $y-1$ of daily illiquidities, analogously to monthly illiquidity calculation in (11). The eligible stocks are then sorted into 25 portfolios, $p \in \{1, 2, \ldots, 25\}$, based on their year $y-1$ illiquidities.

We also form 25 size portfolios for each year y during the period 1964 to 1999 by ranking the eligible stocks (as above for illiquidity portfolios)

[12] Since volume data in CRSP for Nasdaq stocks is available only from 1982 and includes inter-dealer trades, we employ only NYSE and AMEX stocks for sake of consistency in the illiquidity measure.

[13] We are grateful to Joe Chen for providing us with data on book-to-market ratios. The book-to-market ratios are computed as described in Ang and Chen (2002): [For a given month] the book-to-market ratio is calculated using the most recently available fiscal year-end balance sheet data on COMPUSTAT. Following Fama and French (1993), we define "book value" as the value of common stockholders' equity, plus deferred taxes and investment tax credit, minus the book value of preferred stock. The book value is then divided by the market value on the day of the firm's fiscal year-end.

[14] Amihud (2002) and Pastor and Stambaugh (2001) employ similar requirements for the inclusion of stocks in their samples. These requirements help reduce the measurement error in the monthly illiquidity series.

based on their market capitalization at the beginning of year y. Finally, we form portfolios sorted first in 5 book-to-market quintiles and then in 5 size quintiles within the book-to-market groups as in Fama and French (1992) and Fama and French (1993). This sample is restricted to stocks with book-to-market data in year $y-1$. When considering the portfolio properties, we use the year-y book-to-market, averaging across stocks with available book-to-market data in that year.

For each portfolio p (including the market portfolio), we compute its return in month t, as

$$r_t^p = \sum_{i \text{ in } p} w_t^{ip} r_t^i, \tag{13}$$

where the sum is taken over the stocks included in portfolio p in month t, and where w_t^{ip} are either equal weights or value-based weights, depending on the specification.[15]

Similarly, we compute the normalized illiquidity of a portfolio, p, as

$$c_t^p = \sum_{i \text{ in } p} w_t^{ip} c_t^i, \tag{14}$$

where, as above, w_t^{ip} are either equal weights or value-based weights, depending on the specification.

[15]The returns, r_t^i, are adjusted for stock delisting to avoid survivorship bias, following Shumway (1997). In particular, the last return used is either the last return available on CRSP, or the delisting return, if available. While a last return for the stock of -100% is naturally included in the study, a return of -30% is assigned if the deletion reason is coded in CRSP as 500 (reason unavailable), 520 (went to OTC), 551–573 and 580 (various reasons), 574 (bankruptcy) and 584 (does not meet exchange financial guidelines). Shumway (1997) obtains that -30% is the average delisting return, examining the OTC returns of delisted stocks. Amihud (2002) employs an identical survivorship bias correction.

The model's results are phrased in terms of value-weighted returns and value-weighted illiquidity for the market portfolio. Several studies, however, focus on equal-weighted return and illiquidity measures, for instance Amihud (2002) and Chordia, Roll, and Subrahmanyam (2000). Computing the market return and illiquidity as equal-weighted averages is a way of compensating for the over-representation in our sample of large liquid securities, as compared to the "true" market portfolio in the economy. In particular, our sample does not include illiquid assets such as corporate bonds, private equity, real estate, and many small stocks, and these assets constitute a significant fraction of aggregate wealth.[16] Therefore, we focus in our empirical work on an equal-weighted market portfolio, although we also estimate the model with a value-weighted market portfolio for robustness. Also, we use both equal- and value-weighted averages for the test portfolios.

4.3 Innovations in Illiquidity

Illiquidity is persistent. The auto-correlation of the market illiquidity, for instance, is 0.87 at monthly frequency. Therefore, we focus on the innovations, $c_t^p - E_{t-1}(c_t^p)$, in illiquidity of a portfolio when computing its liquidity betas as explained in Section 3.3.

To compute these innovations, we first define the *un-normalized* illiquid-

[16]Heaton and Lucas (2000) report that stocks constitute only 13.6% of national wealth, while non-corporate (i.e. private) equity is 13.8%, other financial wealth is 28.2%, owner-occupied real estate is 33.3%, and consumer durables is 11.1%.

ity, truncated for outliers, of a portfolio p as

$$\overline{ILLIQ}_t^p := \sum_{i \text{ in } p} w_t^{ip} \min\left(ILLIQ_t^i \, , \frac{30.00 - 0.25}{0.30\, P_{t-1}^M} \right) , \tag{15}$$

where w_t^{ip} is the portfolio weight. As explained in Section 4.1, we normalize illiquidity to make it stationary and to put it on a scale corresponding to the cost of a single trade. Hence, to predict illiquidity, we run the following regression for each portfolio:

$$\begin{aligned} \left(0.25 + 0.30\, \overline{ILLIQ}_t^p\, P_{t-1}^M\right) = a_0 \quad & + \quad a_1 \left(0.25 + 0.30\, \overline{ILLIQ}_{t-1}^p\, P_{t-1}^M\right) \\ & + \quad a_2 \left(0.25 + 0.30\, \overline{ILLIQ}_{t-2}^p\, P_{t-1}^M\right) \\ & + \quad u_t \; . \end{aligned} \tag{16}$$

Note that the three terms inside parentheses in this specification correspond closely to c_t^p, c_{t-1}^p, and c_{t-2}^p, respectively, as given by (12) and (14), with the difference that the same date is used for the market index (P_{t-1}^M) in all three terms. This is to ensure that we are measuring innovations only in illiquidity, not changes in P^M. Our results are robust to the specification of liquidity innovations and, in particular, employing other stock-market variables available at time $t-1$ did not improve significantly the explanatory power of the regression. Pastor and Stambaugh (2001) employ a specification to compute market liquidity innovations that is similar in spirit to the AR(2) specification in (16).

The residual, u, of the regression in (16) is interpreted as the standardized liquidity innovation, $c_t^p - E_{t-1}(c_t^p)$, that is,

$$c_t^p - E_{t-1}(c_t^p) := u_t \ . \tag{17}$$

For the market illiquidity series, the AR(2) specification has a R^2 of 78%. The resulting innovations in market illiquidity, $c_t^M - E_{t-1}(c_t^M)$, have a standard deviation of 0.17%. Figure 1 plots the time-series of these innovations, scaled to have unit standard deviation. The auto-correlation of these illiquidity innovations is low (-0.03) and, visually, they appear stationary. Employing AR(1) specification produces a significantly greater correlation of innovations (-0.29), whereas employing AR(3) specification produces little improvement in the explanatory power. The measured innovations in market illiquidity are high during periods that anecdotally were characterized by liquidity crisis, for instance, in 11/1973, 10/1987, the oil crisis and the stock market crash, respectively. Also, there is a string of relatively large shocks in 6–10/1998, the period in which Russia defaulted and Long-Term Capital Management suffered large losses. The correlation between this measure of innovations in market illiquidity and the measure of innovations in liquidity used by Pastor and Stambaugh (2001) is -0.33.[17] (The negative sign is due to the fact that Pastor and Stambaugh (2001) measure liquidity, whereas we follow Amihud (2002) in considering *il*liquidity.)

[17]We thank Pastor and Stambaugh for providing their data on innovations in market liquidity.

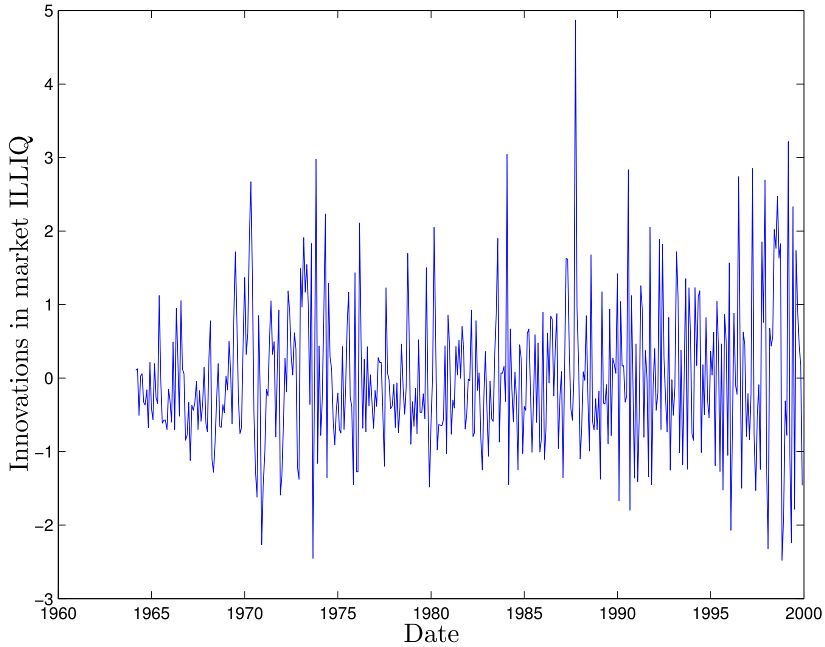

Figure 1: Standardized innovations in market illiquidity from 1964-1999.

4.4 Liquidity Risk

In this section, we present the descriptive statistics of liquidity risk, measured through the betas β^{2p}, β^{3p} and β^{4p}. We focus on the value-weighted illiquidity portfolios whose properties are reported in Table 1. Similar conclusions are drawn from examining the properties of equal-weighted illiquidity portfolios (not reported) or size portfolios (Table 10). The four betas, β^{1p}, β^{2p}, β^{3p} and β^{4p}, for each portfolio are computed as per Equation (7) using the entire time-series, that is, using all monthly return and illiquidity observations for the portfolio and the market portfolio from the beginning of year 1964 till end of year 1999. Similarly, average illiquidity $E(c^p)$ for a portfolio is computed using the entire time-series of monthly illiquidity observations for the portfolio. This approach of using the entire time-series in computing the portfolio characteristics is similar to the one adopted in Black, Jensen, and Scholes (1990) and Pastor and Stambaugh (2001).

Table 1 shows that the sort on past illiquidity successfully produces portfolios with monotonically increasing average illiquidity from portfolio 1 through portfolio 25. Not surprisingly, we see that illiquid stocks – that is, stocks with high values of $E(c^p)$ – tend to have a high volatility of stock returns, a low turnover, and a small market capitalization. Furthermore, we find that illiquid stocks also have high liquidity *risk*: they have large values of β^{2p} and large negative values of β^{3p} and β^{4p}. This is an interesting result on its own. It says that a stock, which is illiquid in absolute terms (c^p), also tends to have a lot of commonality in liquidity with the market ($\text{cov}(c^p, c^M)$), a lot of return sensitivity to market liquidity ($\text{cov}(r^p, c^M)$), and a lot of liquidity sensitivity to market returns ($\text{cov}(c^p, r^M)$). We note that all of the betas are estimated with a small error (i.e., a small asymptotic variance). Indeed, almost all of the betas are statistically significant at conventional levels.

A liquidity beta is proportional to the product of the correlation between its respective arguments and their standard deviations. As noted before, more illiquid stocks have greater volatility of returns. Furthermore, since illiquidity is bounded below by zero, it is natural that more illiquid stocks also have more volatile illiquidity innovations. This is verified in Table 1 which shows that the standard deviation of portfolio illiquidity innovations, $\sigma(\Delta c^p)$, increases monotonically in portfolio illiquidity. The higher variability of returns and illiquidity innovations are, however, not the sole drivers of the positive relationship between illiquidity and liquidity risk. The correlation coefficients between c^p and c^M (r^p and c^M) are also increasing (decreasing) in portfolio illiquidity. The correlation coefficients between c^p and r^M are decreasing in illiquidity between portfolios $1-15$ and are gradually increasing thereafter. Nevertheless, the variability of c^p ensures that the covariances between c^p and r^M are decreasing in illiquidity.[18]

This co-linearity of measures of liquidity risk is confirmed by considering the correlation among the betas, reported in Table 2. This correlation of betas is not just a property of the liquidity-sorted portfolios; it also exists at an individual stock level as is seen in Table 3. The correlation at the stock level is smaller, which could be due in part to larger estimation errors. While this correlation is theoretically intriguing, it makes it hard to empirically distinguish the separate effects of illiquidity and the individual liquidity betas.[19]

4.5 How Liquidity Risk Affects Returns

In this section, we study how liquidity risk affects expected returns. Specifically, we estimate the liquidity-adjusted CAPM (6) using the portfolios based on the sorting by illiquidity or size. Using the portfolios' betas and the illiq-

[18]These correlations are not reported in the table for sake of brevity.

[19]We have not been able to construct portfolios which allow us to better identify the separate beta effects. For instance, we have considered portfolios based on predicted liquidity betas, similar to the approach taken by Pastor and Stambaugh (2001). These results are not reported as these portfolios did not improve statistical power: The liquidity betas after portfolio formation turned out to be better sorted for illiquidity and size portfolios than for the portfolios sorted using predicted liquidity betas. We attribute this, in part, to the large estimation errors associated with predicting liquidity betas at the individual stock level.

uidity, we estimate our model (6), and subsets of its coefficients, by running cross-sectional regression using the method of Fama and MacBeth (1973). To be precise, in each month over the period 1964–1999, we run a cross-sectional regression of the excess returns on the 25 test portfolios with explanatory variables being the portfolio characteristics. The estimated coefficients are then averaged over all months.

We consider first the liquidity-adjusted CAPM (6) with the model-implied constraint that the risk premia of the different betas is the same. In doing this, we define the "net beta" as

$$\beta^{net,p} := \beta^{1p} + \beta^{2p} - \beta^{3p} - \beta^{4p}. \tag{18}$$

With this definition, the liquidity-adjusted CAPM becomes

$$E(r_t^p - r_t^f) = \alpha + kE(c_t^p) + \lambda\beta^{net,p}\,, \tag{19}$$

where we allow a non-zero intercept, α, in the estimation, although the model implies that the intercept is zero. In our model, investors incur the illiquidity cost exactly once over their holding period. The coefficient k adjusts for the difference between the monthly period used in estimation, and the typical holding period of an investor (which is the period implicitly considered in the model). More precisely, k is the ratio of the monthly estimation period to the typical holding period.[20] The average holding period is proxied by

[20] If the estimation period is equal to the holding period, then the model implies (19) with $k = 1$. If the estimation period is k times the holding period, then $E(r_t^p - r_t^f)$ is (approximately) k times the expected holding period return, and $\beta^{net,p}$ is assumed to be approximately k times the holding-period net beta. This is because a k-period return (or illiquidity innovation) is approximately a sum of k 1-period returns (or illiquidity

the period over which all shares are turned over once. Hence, we calibrate k as the average monthly turnover across all stocks in the sample.[21] In the sample of liquidity portfolios, k is calibrated to 0.034, which corresponds to a holding period of $1/0.034 \cong 29$ months. The expected illiquidity, $E(c_t^p)$, is computed as the portfolio's average illiquidity. Note that the structure of the liquidity-adjusted CAPM and its calibration using k equal to the average monthly turnover for stocks make the estimation different from the typical cross-sectional regression study in which the asset-pricing relationship is backed out from the return series and data on security characteristics such as beta, size, book-to-market, etc.

The resulting liquidity-adjusted CAPM (19) has only *one* risk premium, λ, that needs to be estimated as in the standard CAPM. Here, the risk factor is the net beta instead of the standard CAPM beta. Hence, the empirical improvement in fit is not achieved by adding factors (or otherwise adding degrees of freedom), but simply by making a liquidity adjustment.

We consider first the illiquidity portfolios. The estimated results for Equation (19) are reported in line 1 of Table 4 with value-weighted portfolios, and in Table 5 with equally-weighted portfolios. In Table 4, we report both Fama and MacBeth (1973) standard errors, and GMM standard errors that account for the pre-estimation of betas (see Shanken (1992) and Cochrane (2001)). In the other tables, we report for simplicity only the Fama and MacBeth

innovations), and because returns and illiquidity innovations have low correlation across time. The illiquidity, $E(c^p)$, however, does not scale with time period because it is an average of daily illiquidities (not a sum of such terms). Therefore, the $E(c^p)$ term is scaled by k in (19).

[21] To run the regression (19) with a fixed k, we treat the net return, $E(r_t^p - r_t^f) - kE(c_t^p)$, as the dependent variable.

(1973) standard errors.[22]

We see that the λ is positive and significant at a 1% level for value-weighted portfolios and at a 5% level for equal-weighted portfolios, and that α is always insignificant, both results lending support to our model. Further, we see that the R^2 is high, close to 82% in both tables. In line 2 of Tables 4 and 5, we see that the k and λ parameters (and their t-stats) change only modestly when k is estimated as a free parameter.

The tables also show the result of different empirical specifications with various combinations of betas and expected illiquidity cost, allowing the betas to have different risk premia and not restricting the coefficient k to be the average monthly turnover. That is, we consider components of the generalized relation

$$E(r_t^p - r_t^f) = \alpha + \kappa E(c_t^p) + \lambda^1\beta^{1p} + \lambda^2\beta^{2p} + \lambda^3\beta^{3p} + \lambda^4\beta^{4p} + \lambda\beta^{net,p} \quad (20)$$

where the risk premia, $\lambda^1, \ldots, \lambda^4, \lambda$, and the coefficient κ, are estimated as unconstrained coefficients. Our model implies the restrictions $\alpha = 0$, $\kappa = k$, and $\lambda^1 = \lambda^2 = -\lambda^3 = -\lambda^4$. The estimated market prices of liquidity risks for this generalized specification have signs that are broadly consistent with the model's prediction. In particular, a security's required return is increasing in its level of β^2 and decreasing in its level of β^3 and β^4. The

[22]Since betas are estimated accurately, their pre-estimation has a relatively modest effect on the results. Also, the serial correlation in monthly Fama-MacBeth estimates of the risk premia is low for all our tests. (It is usually lower in magnitude than 0.10 and always lower than 0.20.) The resulting bias in t-statistics, if any, is thus negligible. Furthermore, estimation of the cross-sectional regressions using the weighted least squares method to account for heteroskedasticity in the residuals produced qualitatively similar results (available from the authors upon request). Some of the specifications however suffer from the multi-collinearity problem under the weighted least squares estimation.

market prices of liquidity risks are significant at a 5% level in univariate regressions. Interestingly, the highest R^2 among the univariate regressions is the R^2 of the liquidity-adjusted CAPM (line 1).

To determine whether the liquidity risks (β^2, β^3, and β^4) matter separately from market risk (β^1), we include both β^1 and β^{net} in regressions 9 and 10. For robustness, κ is calibrated as the average turnover in the former regression, while κ is estimated as a free parameter in the latter regression. The result indicate that liquidity risk does matter. Indeed, liquidity risk may actually have a higher risk premium than market risk. For instance, Regression 9 of Table 4 shows that

$$\begin{aligned} E(r_t^p - r_t^f) &= -0.333 + 0.034E(c_t^p) - 3.181\beta^{1p} + 4.334\beta^{net,p} \\ &= -0.333 + 0.034E(c_t^p) + 1.153\beta^{1p} + 4.334\left(\beta^{2p} - \beta^{3p} - \beta^{4p}\right) \end{aligned}$$

As more coefficients are estimated simultaneously in regressions 11–15, the statistical significance is reduced but the coefficients on some betas remain significant. For instance, the coefficient related to β^4 is significant at a 5% level whenever it is included, except for value-weighted portfolios when all coefficients are estimated simultaneously (where it is marginally significant). It should also be noted that the coefficient λ^2, the risk premium on liquidity risk β^2, is either magnified substantially or reverses its sign whenever β^2 is included in the regression along with $E(c^p)$. The lack of ability to identify all the coefficients jointly may be due, at least in part, to the co-linearity of the different kinds of liquidity risk. Indeed, β^2 and $E(c^p)$ have a correlation that exceeds 0.99 for illiquidity portfolios. Of course, we must also entertain the possibility that not all these risk factors are empirically relevant.

The top panel of Figure 2 illustrates the empirical fit of the standard CAPM for the illiquidity portfolios. The middle and bottom panels show, respectively, the fit of the constrained and unconstrained liquidity-adjusted CAPM, that is, lines 1 and 15, respectively, from Table 4. We see that the liquidity adjustment improves the fit especially for the illiquid portfolios, consistent with what our intuition would suggest. We note that the number of free parameters is the same in top and middle panels, so the improvement in fit is not a consequence of more degrees of freedom.

The effect of liquidity risk on required returns seems economically meaningful. To get a perspective on the magnitude of the effects of different forms of liquidity risk, we consider the risk premium, $\lambda = 1.512$, from Row 1 in Table 4. The difference in annualized expected return between portfolio 1 and 25 that can be attributed to a difference in β^2, i.e., in the covariance, $\mathrm{cov}_{t-1}(c_t^i, c_t^M)$, between the portfolio illiquidity and the market illiquidity, is

$$\lambda(\beta^{2,p_{25}} - \beta^{2,p_1}) \cdot 12 = 0.08\%.$$

Using the Fama-MacBeth standard error of the estimate of λ and ignoring the estimation error in betas, the 95% confidence interval for the effect of β^2 is $(0.02\%, 0.13\%)$. Similarly, the annualized return difference stemming from the difference in β^3, i.e., in the covariance, $\mathrm{cov}_{t-1}(r_t^i, c_t^M)$, between the portfolio return and the market illiquidity, is

$$-\lambda(\beta^{3,p_{25}} - \beta^{3,p_1}) \cdot 12 = 0.17\%,$$

with a 95% confidence interval of $(0.05\%, 0.28\%)$. The effect of β^4, i.e., in the covariance, $\mathrm{cov}_{t-1}(r_t^i, c_t^M)$, between the security's return and the market illiquidity, is

$$-\lambda(\beta^{4,p_{25}} - \beta^{4,p_1}) \cdot 12 = 0.83\%,$$

with a 95% confidence interval of $(0.27\%, 1.39\%)$.

The total effect of liquidity risk is therefore 1.1% per year. Interestingly, of the three liquidity risks the effect of β^4, the covariation of a security's illiquidity to market returns, appears the strongest both from a statistical standpoint and in terms of the economic impact on expected returns, although this aspect of liquidity risk has not been studied before either theoretically or empirically. Our model and its tests suggest that this liquidity risk is priced: All else being equal, securities which are difficult to trade in market downturns are priced significantly lower than the ones which are relatively easy to trade in such times.

Finally, the difference in annualized expected return between portfolio 1 and 25 that can be attributed to a difference in $E(c)$, the expected illiquidity, is 3.5%, using the calibrated coefficient. The overall effect of expected illiquidity and liquidity risk is thus 4.6% per year. We note that, due to the colinearity issues, it is difficult to identify the contribution to expected returns of each liquidity risk and of liquidity itself, and the numbers given above rely on our model and on the calibrated value of k.

While this magnitude is economically significant, it is lower than the magnitude estimated by Pastor and Stambaugh (2001). This could be due to the fact that they employ a different measure of liquidity, one that is based on the principle that order flow induces greater return reversal when liquidity is lower, as theoretically motivated by Campbell, Grossman, and

Wang (1993). This difference may be in part due to the fact that they sort portfolios based on liquidity risk (in their case, β^3) whereas we sort based on the level of liquidity. Also, this could be because they do not control for the level of illiquidity which has been shown to command a significant premium in a number of studies including Amihud and Mendelson (1986), Brennan and Subrahmanyam (1996), Brennan, Chordia, and Subrahmanyam (1998), Datar, Naik, and Radcliffe (1998), Swan (2002), and Dimson and Hanke (2002). Finally, the difference could also arise because we restrict the risk premia on different liquidity betas to be the same. For instance, the magnitude of the risk premium related to β^4 is estimated to be higher in Model 14 and Model 15 of Table 4. This higher risk premium results in a per year effect of 8.21% and 9.52%, respectively, from β^4 alone.[23]

To formally test the liquidity-adjusted CAPM, we use the Wald test.[24] First, we note that we fail to reject at conventional levels the model-implied restriction that $\alpha = 0$ in the liquidity-adjusted CAPM (Row 1 of Table 4 and Table 5).

Second, we test the five model-implied restrictions $\lambda^1 = \lambda^2 = -\lambda^3 = -\lambda^4$, $\alpha = 0$, and $\kappa = k$ in context of the model with unrestricted risk premia (Row 15 of Table 4 and Table 5).

The test for $\lambda^1 = \lambda^2 = -\lambda^3 = -\lambda^4$ is asymptotically distributed as chi square with three degrees of freedom. For value-weighted illiquidity portfolios

[23]In another recent paper, Chordia, Subrahmanyam, and Anshuman (2001) find that expected returns in the cross-section are higher for stocks with low variability of liquidity, measured using variables such as trading volume and turnover. They examine the firm-specific variability of liquidity. By contrast, our model and tests suggest that it is the co-movement of firm-specific liquidity with market return and market liquidity that affects expected returns.

[24]We provide details on this standard test method in the Appendix.

in Table 4, the test statistic has a p-value of 16.2%, and for equally-weighted portfolios in Table 5, it has a p-value of 3.9%. If we add the restriction $\alpha = 0$, then the test statistic has a p-value of 22.9% for value-weighted portfolios and a p-value of 7.2% for equally-weighted portfolios.

Finally, we test jointly all the five model-implied restrictions stated above. For value-weighted illiquidity portfolios this test statistic has a p-value of 33.6%, and for equally-weighted portfolios it has a p-value of 12.4%. Overall, the model-implied restrictions are not statistically rejected. By contrast, a test of the standard CAPM restrictions that $\lambda^2 = \lambda^3 = \lambda^4 = 0$, $\alpha = 0$, and $\kappa = 0$, yields a p-value of 7.0% for value-weighted portfolios and a p-value of 0.0% for equally-weighted portfolios.

Another testable restriction implied by the model is that the risk premium λ equals $E(r_t^m - r_t^f - kc_t^m)$, the expected net return on the market in excess of the risk-free rate. This test of the liquidity-adjusted CAPM has a p-value of 6.5% in regression 1 of Table 4 and a p-value of 7.6% in regression 2. Hence, we cannot reject on a 5% level the model restriction that $\lambda = E(r_t^m - r_t^f - kc_t^m)$, but we note that the point estimate of the risk premium, λ, is somewhat larger than the sample average of the excess return of the market net of transaction costs, $E(r_t^m - r_t^f - kc_t^m)$.

Lastly, we test that all the error terms have mean equal to zero. This is perhaps the most stringent test since it requires that the model is pricing all portfolios correctly. (See Cochrane (2001) page 246 for this test.) The p-values for regressions 1, 10, and 15 are, respectively, 3.0%, 7.2%, and 4.0%. (In comparison, the CAPM has a p-value of 0.2%.) This is a borderline rejection that the model prices correctly all portfolios.

Based on our numerous specification tests, we conclude that most, but not all, model-implied restrictions are supported by the data. Importantly, the overall evidence supports that liquidity risk is priced and has explanatory power in the cross section.

To check the robustness of our results, we estimate the model with a number of different specifications. We use a value-weighted market portfolio (Table 6) and find similar results. We control for size and book-to-market (Table 7). The results are similar to the earlier results, although the standard errors increase because of the additional variables. The coefficient on β^{net} is significant in the liquidity-adjusted CAPM of Regression 1 and marginally so in Regression 2. The coefficient on book-to-market is significant in some specifications such as Regressions 1 and 2, but it is insignificant whenever β^4 is included in the regression, and when we allow β^{net} and market beta to have different risk premia. The coefficient on size is always insignificant. (Including volatility does not change the results, and volatility is not significant. These results are not reported.) Also, we consider the sub-period 1964–1981 (Table 8), and the sub-period 1982–1999 (Table 9). We see that the signs of the beta coefficients are stable across all specifications of the model. We find some, but not all, beta coefficients that are significant at conventional levels in almost all specifications. The statistical significance is reduced in the later sample.

As a further robustness check, we re-estimate our model with size-based portfolios. Table 10 shows the properties of value-weighted size-based portfolios, confirming that small-sized stocks are illiquid (in absolute terms as measured by $E(c)$) and also have high liquidity risk (as measured by the three betas β^{2p}, β^{3p} and β^{4p}). Table 11 shows that the Fama-MacBeth regressions are generally consistent with the model.

Figure 3 shows graphically the fit of the standard CAPM, and the liquidity-adjusted CAPM, with constrained and unconstrained risk premia. We see again that the liquidity adjustment improves the fit, particularly for the smaller size portfolios. Formal statistical tests cannot reject the model-implied restrictions at conventional confidence levels. The p-values of tests for value-weighted and equally-weighted size portfolios are, respectively: (i) 44.5% and 50.5% for the restrictions that $\lambda^1 = \lambda^2 = -\lambda^3 = -\lambda^4$; (ii) 61.4% and 65.5% with the additional restriction that $\alpha = 0$; and (iii) 74.9% and 73.2% with the further restriction that $\kappa = k$. This lends strong support in favor of our liquidity-adjusted CAPM. By contrast, a test of the standard CAPM restrictions that $\lambda^2 = \lambda^3 = \lambda^4 = 0$, $\alpha = 0$, and $\kappa = 0$, yields a p-value of 21.5% for value-weighted portfolios and a p-value of 5.1% for equally-weighted portfolios.

As a final robustness check, we consider portfolios sorted first in 5 book-to-market quintiles and then in 5 size quintiles within the book-to-market groups as in Fama and French (1992) and Fama and French (1993). Table 12 and Figure 4 show the models' fit of these portfolios. The results, although weaker, are similar to our previous ones. The coefficient of β^{net} is estimated to be positive and of the same magnitude as previously. The liquidity-adjusted CAPM still has a higher R^2 than the standard CAPM, while both models have lower R^2's than with the other portfolios. Figure 4 shows that CAPM does relatively poorly for book-to-market by size portfolios (adjusted $R^2 = 40.6\%$). This is consistent with the findings of Fama and French (1992) and Fama and French (1993) that beta is more or less "flat" across these portfolios. The liquidity-adjusted CAPM seems to provide a moderate improvement in the fit (adjusted $R^2 = 49.9\%$) whereas the model with unconstrained risk premia produces a significant improvement in the fit (adjusted $R^2 = 73.3\%$). It should be noted, however, that some of the risk premia estimated under the unconstrained specification are either insignificant or have incorrect signs. These results, together with Table 7, indicate that liquidity risk is important even controlling for book-to-market, but that liquidity risk may not fully explain the book-to-market effects. This is consistent with the findings of Pastor and Stambaugh (2001).

Overall, the evidence appears to be supportive of the liquidity-adjusted CAPM model. The results highlight the importance of liquidity and liquidity risk for asset prices. They also underscore the need for further empirical research aimed at better measurement of liquidity and its time-series variation, for an individual security as well as for the market.

5 Conclusion

This paper considers the effect of liquidity risk. It develops a simple pricing formula that shows that investors should worry about a security's performance and tradability both in market downturns and when liquidity "dries up." Said differently, the required return of a security i is increasing in the covariance, $\text{cov}_{t-1}(c_t^i, c_t^M)$, between its illiquidity and the market illiquidity, decreasing in the covariance, $\text{cov}_{t-1}(r_t^i, c_t^M)$, between the security's return and the market illiquidity, and decreasing in the covariance, $\text{cov}_{t-1}(c_t^i, r_t^M)$, between its illiquidity and market returns.

The model also shows why high illiquidity predicts high future returns, and why contemporaneous liquidity and returns co-move.

Hence, the model helps explain the existing empirical evidence related to liquidity risk. Further, its novel predictions are consistent with our empirical findings. In particular, we find, in a variety of specifications, that the liquidity-adjusted CAPM explains the data better than the standard CAPM. The model with net beta provides a better fit than the standard CAPM while still exploiting the same degrees of freedom. In tests of the model with unconstrained risk premia, the covariance between a security's illiquidity and the market return appears to be particularly important, an effect not previously studied in the literature. We conclude that liquidity risk indeed appears to be priced.

While the model gives clear predictions that seem to have some bearing in the data, it is decidedly simplistic. The model and the empirical results are suggestive of further theoretical and empirical work. In particular, it would be of interest to explain the time-variation in liquidity, and why stocks that are illiquid in absolute terms also are more liquidity risky in the sense of high values of all three liquidity betas. Another interesting topic is to consider liquidity premia in a general equilibrium with liquidity risk and endogenous holdings periods. We note that if investors live several periods, but their probability of living more than one period approaches zero, then our general-equilibrium economy is approached (assuming continuity). Hence, our effects would also be present in the more general economy, although endogenous holding periods may imply a smaller effect of liquidity risk (as in

Constantinides (1986)). The effect of liquidity risk is strengthened, however, if investors have important reasons to trade frequently. Such reasons include return predictability and wealth shocks (as considered in the context of liquidity by Lynch and Tan (2003)), differences of opinions (e.g. Harris and Raviv (1993)), asymmetric information (e.g. He and Wang (1995)), institutional effects (e.g. Allen (2001)), taxes (e.g. Constantinides (1983)), etc. It would be interesting to determine the equilibrium impact of liquidity risk in light of these trading motives.

A Appendix

Proof of Proposition 1:

We first solve the investment problem of any investor n at time t. We assume, and later confirm, that the price at time $t+1$ is normally distributed conditional on the time t information. Hence, the investor's problem is to choose optimally the number of shares, $y^n = (y^{n,1}, \ldots, y^{n,I})$, to purchase according to

$$\max_{y^n \in \mathbb{R}^I_+} \left(E_t(W^n_{t+1}) - \frac{1}{2} A^n \operatorname{var}_t(W^n_{t+1}) \right),$$

where

$$W^n_{t+1} = (P_{t+1} + D_{t+1} - C_{t+1})^\top y^n + r^f (e^n_t - P_t^\top y^n),$$

and e^n_t is this agent's endowment. If we disregard the no-short-sale constraint, the solution is

$$y^n = \frac{1}{A^n} \left(\operatorname{var}_t(P_{t+1} + D_{t+1} - C_{t+1})\right)^{-1} \left(E_t(P_{t+1} + D_{t+1} - C_{t+1}) - r^f P_t\right).$$

We shortly verify that, in equilibrium, this solution does not entail short selling. In equilibrium, $\sum_n y^n = S$, so equilibrium is characterized by the condition that

$$P_t = \frac{1}{r^f} \left[E_t(P_{t+1} + D_{t+1} - C_{t+1}) - A \operatorname{var}_t(P_{t+1} + D_{t+1} - C_{t+1}) S \right],$$

where $A = \left(\sum_n \frac{1}{A^n}\right)^{-1}$. The unique stationary linear equilibrium is

$$P_t = \frac{1}{r^f - 1}\left(\frac{r^f(1-\gamma)}{r^f-\gamma}(\bar{D}-\bar{C}) - A\left(\frac{r^f}{r^f-\gamma}\right)^2 \Gamma S\right) + \frac{\gamma}{r^f-\gamma}(D_t - C_t), \tag{A.1}$$

where $S = (S^1, \ldots, S^I)$ is the total supply of shares.

With this price, conditional expected net returns are normally distributed, and any investor n holds a fraction $A/A^n > 0$ of the market portfolio $S > 0$ so he is not short selling any securities. Therefore, our assumptions are satisfied in equilibrium.

Finally, since investors have mean-variance preferences, the conditional CAPM holds for net returns. See, for instance, Huang and Litzenberger (1988). Rewriting in terms of net returns yields the result stated in the proposition. □

Proof of Proposition 2:

The conditional expected return on a portfolio q is computed using (A.1):

$$\begin{aligned} E_{t-1}(r_t^q) &= E_{t-1}\left(\frac{B + r^f D_t^q - \gamma C_t^q}{B + \gamma D_{t-1}^q - \gamma C_{t-1}^q}\right) \\ &= \frac{B + r^f(1-\gamma)\bar{D}^q + r^f\gamma D_{t-1}^q - \gamma(1-\gamma)\bar{C}^q - \gamma^2 C_{t-1}^q}{B + \gamma D_{t-1}^q - \gamma C_{t-1}^q}, \end{aligned}$$

where,

$$B = \frac{r^f - \gamma}{r^f - 1}\, q^\top \left(\frac{r^f(1-\gamma)}{r^f-\gamma}(\bar{D}-\bar{C}) - A\left(\frac{r^f}{r^f-\gamma}\right)^2 \Gamma S\right).$$

The conditional expected return depends on C^q_{t-1} in the following way:

$$
\begin{aligned}
&\frac{\partial}{\partial C^q_{t-1}} E_{t-1}(r^q_t - r^f) \\
&= \frac{\gamma}{(r^f-\gamma)^2 P^2_{t-1}} \Big[-\gamma \left(B + \gamma D^q_{t-1} - \gamma C^q_{t-1}\right) \\
&\quad + \left(B + r^f(1-\gamma)\bar{D}^q + r^f \gamma D^q_{t-1} - \gamma(1-\gamma)\bar{C}^q - \gamma^2 C^q_{t-1}\right) \Big] \\
&= \frac{\gamma}{(r^f-\gamma) P^2_{t-1}} \Big[\gamma D^q_{t-1} + (1-\gamma) E(D^q_t + P^q_t \mid D^q_{t-1} = \bar{D}^q, C^q_{t-1} = \bar{C}^q) \Big].
\end{aligned}
$$

This partial derivative is greater than 0 under the conditions given in the proposition. □

Proof of Proposition 3:

The conditional covariance between illiquidity and return for a portfolio q is:

$$
\begin{aligned}
\mathrm{cov}_{t-1}(c^q_t, r^q_t) &= \frac{1}{(P^q_{t-1})^2} \mathrm{cov}_{t-1}(C^q_t\,,\, P^q_t + D^q_t) \\
&= \frac{1}{(P^q_{t-1})^2 (r^f-\gamma)} \mathrm{cov}_{t-1}(C^q_t\,,\, r^f D^q_t - \gamma C^q_t) \\
&= \frac{1}{(P^q_{t-1})^2 (r^f-\gamma)} (r^f\, q^\top \Sigma^{CD} q - \gamma\, q^\top \Sigma^C q),
\end{aligned}
$$

which yields the proposition. □

Wald Test of Parameter Restrictions: Let $\theta = (\alpha, \kappa, \lambda^1, \ldots, \lambda^4)'$ denote the 6-vector of coefficients. The Fama-MacBeth method estimates are given by

$$\hat{\theta} = 1/T \sum \theta_t,$$

where θ_t is the period-t estimates. Under independence assumptions, $\sqrt{T}(\hat{\theta}-\theta)$ is asymptotically normal with mean 0 and a variance-covariance matrix Σ, which can be estimated consistently by

$$\hat{\Sigma} = 1/T \sum(\theta_t - \hat{\theta})(\theta_t - \hat{\theta})'$$

To jointly test K parameter restrictions, consider the following null hypotheses:

$$H_0: \;\; G\theta = g$$

where G is an K-by-6 matrix and g is a K vector. In our case, G and g are chosen as follows. First, the restriction of identical risk premia are implemented by letting G's first row be $G_1 = (0,0,1,-1,0,0)$ and $g_1 = 0$. This means that $\lambda^1 = \lambda^2$. Similarly, the second and the third rows of G, are respectively, $(0,0,1,0,1,0)$ and $(0,0,1,0,0,1)$, and $g_3 = g_4 = 0$, which imply that $\lambda^1 = -\lambda^3 = -\lambda^4$.

To further impose the restriction $\alpha = 0$, we let G's 4'th row equal $(1,0,0,0,0,0)$ and $g_4 = 0$. Finally, to impose the restriction $\kappa = k$, we let the fifth row of G equal $(0,1,0,0,0,0)$ and $g_5 = k$.

Under H_0, the asymptotic distribution of $\sqrt{T}(G\hat{\theta} - g)$ is normal with mean 0 and variance-covariance matrix $G\Sigma G'$. Hence, the test statistic

$$T(G\hat{\theta} - g)'(G\hat{\Sigma}G')^{-1}(G\hat{\theta} - g)$$

is asymptotically χ^2 with K degrees of freedom.

References

Allen, F. (2001). Do Financial Institutions Matter? *Journal of Finance 56*, 1165–1175.

Amihud, Y. (2002). Illiquidity and Stock Returns: Cross-Section and Time-Series Effects. *Journal of Financial Markets 5*, 31–56.

Amihud, Y. and H. Mendelson (1986). Asset Pricing and the Bid-Ask Spread. *Journal of Financial Economics 17*, 223–249.

Amihud, Y. and H. Mendelson (1989). The Effect of Beta, Bid-Ask Spread, Residual Risk, and Size on Stock Returns. *Journal of Finance 44*, 479–486.

Amihud, Y., H. Mendelson, and R. Wood (1990). Liquidity and the 1987 Stock Market Crash. *Journal of Portfolio Management Spring*, 65–69.

Ang, A. and J. Chen (2002). Asymmetric Correlations of Equity Portfolios. *Journal of Financial Economics 63(3)*, 443–494.

Black, F., M. Jensen, and M. Scholes (1990). *The Capital Asset Pricing Model: Some Empirical Tests, in Michael Jensen (ed.),* Studies in the Theory of Capital Markets. Praeger, New York.

Brennan, M. J., T. Chordia, and A. Subrahmanyam (1998, September). Alternative factor specifications, security characteristics, and the coress-section of expected returns. *Journal of Financial Economics 49*(3), 345–373.

Brennan, M. J. and A. Subrahmanyam (1996). Market Microstructure and Asset Pricing: On the Compensation for Illiquidity in Stock Returns.

Journal of Financial Economics 41, 441–464.

Campbell, J., S. J. Grossman, and J. Wang (1993). Trading Volume and Serial Correlation in Stock Returns. *Quarterly Journal of Economics 108*, 905–939.

Chalmers, J. M. and G. B. Kadlec (1998). An Empirical Examination of the Amortized Spread. *Journal of Financial Economics 48*, 159–188.

Chen, N. F. and R. Kan (1996). Expected Return and the Bid-Ask Spread. In K. S. S. Saitou and K. Kubota (Eds.), *Modern Portfolio Theory and Applications*. Gakujutsu Shuppan Center, Osaka.

Chordia, T., R. Roll, and A. Subrahmanyam (2000). Commonality in Liquidity. *Journal of Financial Economics 56*, 3–28.

Chordia, T., R. Roll, and A. Subrahmanyam (2001). Market Liquidity and Trading Activity. *Journal of Finance*, forthcoming.

Chordia, T., A. Subrahmanyam, and V. R. Anshuman (2001). Trading Activity and Expected Stock Returns. *Journal of Financial Economics 59*, 3–32.

Cochrane, J. H. (2001). *Asset Pricing*. Princeton, New Jersey: Princeton University Press.

Constantinides, G. M. (1983). Capital Market Equilibrium with Personal Tax. *Econometrica 51*(3), 611–636.

Constantinides, G. M. (1986). Capital Market Equilibrium with Transaction Costs. *Journal of Political Economy 94*, 842–862.

Constantinides, G. M., J. B. Donaldson, and R. Mehra (2002). Junior

Can't Borrow: A New Perspective on the Equity Premium Puzzle. *Quarterly Journal of Economics 117(1)*, 269 – 296.

Datar, V. T., N. Y. Naik, and R. Radcliffe (1998). Liquidity and Stock Returns: An Alternative Test. *Journal of Financial Markets 1*, 203–219.

De Long, J. B., A. Shleifer, L. H. Summers, and R. J. Waldmann (1990). Noise Trader Risk in Financial Markets. *Journal of Political Economy 98*, 703–738.

Dimson, E. and B. Hanke (2002). The Expected Illiquidity Premium. London Business School.

Duffie, D., N. Gârleanu, and L. H. Pedersen (2000). Valuation in Dynamic Bargaining Markets. Graduate School of Business, Stanford University.

Easley, D., S. Hvidkjær, and M. O'Hara (2000). Is Information Risk a Determinant of Asset Returns? Johnson Graduate School of Management, Cornell University.

Eisfeldt, A. L. (1999). Endogenous Liquidity in Asset Markets. Kellogg Graduate School of Management, Northwestern University.

Eleswarapu, V. R. (1997). Cost of Transacting and Expected Returns in the Nasdaq Market. *Journal of Finance 52*, 2113–2127.

Eleswarapu, V. R. and M. R. Reinganum (1993). The Seasonal Behavior of Liquidity Premium in Asset Pricing. *Journal of Financial Economics 34*, 373–386.

Fama, E. F. and K. R. French (1992). The Cross-Section of Expected Stock Returns. *Journal of Finance 47(2)*, 427–465.

Fama, E. F. and K. R. French (1993). Common Risk Factors in the Returns on Stocks and Bonds. *Journal of Financial Economics 33*, 3–56.

Fama, E. F. and J. D. MacBeth (1973). Risk, Return, and Equilibrium: Empirical Tests. *Journal of Political Economy 81(3)*, 607–636.

Friend, I. and M. Blume (1975). The Demand for Risky Assets. *American Economic Review 65 (5)*, 900–922.

Gârleanu, N. and L. H. Pedersen (2000). Adverse Selection and Re-Trade. *Review of Financial Studies*, forthcoming.

Hansen, L. P. and S. F. Richard (1987). The Role of Conditioning Information in Deducing Testable Restrictions Implied by Dynamic Asset Pricing Models. *Econometrica 55*, 587–614.

Harris, M. and A. Raviv (1993). Differences of Opinion Make a Horse Race. *Review of Financial Studies 6*(3), 473–506.

Hasbrouck, J. (2002). Inferring Trading Costs from Daily Data: US Equities from 1962 to 2001. New York University.

Hasbrouck, J. and D. J. Seppi (2000). Common Factors in Prices, Order Flows and Liquidty. *Journal of Financial Economics*, forthcoming.

He, H. and J. Wang (1995). Differential Information and Dynamic Behavior of Stock Trading Volume. *The Review of Financial Studies 8 (4)*, 919–972.

Heaton, J. and D. Lucas (2000). Portfolio Choice and Asset Prices: The Importance of Entrepreneurial Risk. *Journal of Finance 55*(3), 1163–1198.

Holmstrom, B. and J. Tirole (2000). LAPM: A Liquidity-based Asset Pricing Model. *Journal of Finance 56*, 1837–1867.

Huang, C. and R. H. Litzenberger (1988). *Foundations for Financial Economics*. Englewood Cliffs, New Jersey: Prentice-Hall.

Huang, M. (2002). Liquidity Shocks and Equilibrium Liquidity Premia. *Journal of Economic Theory*, forthcoming.

Huberman, G. and D. Halka (1999). Systematic Liquidity. Columbia Business School.

Jones, C. M. (2001). A Century of Stock Market Liquidity and Trading Costs. Graduate School of Business, Columbia University.

Lintner, J. (1965). The Valuation of Risk Assets and the Selection of Risky Investments in Stock Portfolios and Capital Budgets. *Review of Economics and Statistics 47*, 13–37.

Lustig, H. (2001). The Market Price of Aggregate Risk and the Wealth Distribution. Stanford University.

Lynch, A. W. and S. Tan (2003). Explaining the Magnitude of Liquidity Premia: The Roles of Return Predictability, Wealth Shocks and State-dependent Transaction Costs. New York University.

Markowitz, H. (1952). Portfolio Selection. *Journal of Finance 7*, 77–91.

Markowitz, H. (2000). *Mean-Variance Analysis in Portfolio Choice and Capital Markets*. New Hope, Pennsylvania: Frank J. Fabozzi Associates.

Mossin, J. (1966). Equilibrium in a Capital Asset Market. *Econometrica 35*, 768–783.

Pastor, L. and R. F. Stambaugh (2001). Liquidity Risk and Expected Stock Returns. *Journal of Political Economy*, Fortcoming.

Sadka, R. (2002). Momentum, Liquidity Risk, and Limits to Arbitrage. Northwestern University.

Samuelson, P. A. (1958). An Exact Consumption-Loan Model of Interest with or without the Social Contrivance of Money. *Journal of Political Economy 66*, 467–482.

Shanken, J. (1992). On the Estimation of Beta Pricing Models. *Review of Financial Studies 5*, 1–34.

Sharpe, W. (1964). Capital Asset Prices: A Theory of Capital Market Equilibrium under Conditions of Risk. *Journal of Finance 19*, 425–442.

Shumway, T. (1997). The Delisting Bias in CRSP Data. *Journal of Finance 52*, 327–340.

Swan, P. L. (2002). Does "Illiquidity" rather than "Risk Aversion" Explain the Equity Premium Puzzle? The Value of Endogenous Market Trading. University of New South Wales.

Vayanos, D. (1998). Transaction Costs and Asset Prices: A Dynamic Equilibrium Model. *Review of Financial Studies 11*, 1–58.

Vayanos, D. and J.-L. Vila (1999). Equilibrium Interest Rate and Liquidity Premium with Transaction Costs. *Economic Theory 13*, 509–539.

Wang, A. W. (2002). Institutional Equity Flows, Liquidity Risk and Asset Pricing. University of California, Los Angeles.

Table 1: **Properties of illiquidity portfolios**.

This table reports the properties of the odd–numbered portfolios of 25 value-weighted illiquidity portfolios formed each year during 1964–1999 as described in Section 4.2. The four betas (β^{ip}) are computed for each portfolio using all monthly return and illiquidity observations for a portfolio, and an equal-weighted market portfolio. In particular, these betas based on (7), where the innovations in portfolio illiquidity and market illiquidity are computed using the AR(2) specification in (16) for the standardized illiquidity series, and the innovations in market portfolio return is computed using an AR(2) specification for the market return series that also employs available market characteristics at the beginning of the month (return, volatility, average illiquidity, log of average dollar volume, log of average turnover, all measured over past six months, and log of one-month lagged market capitalization). The t-statistics, reported in parenthesis, are estimated using GMM. The standard deviation of the portfolio illiquidity innovations is reported under the column $\sigma(\Delta c^p)$. The average illiquidity, $E(c^p)$, the average excess return, $E(r^{e,p})$, the turnover (trn), the market capitalization (size), and book-to-market (BM) are computed for each portfolio as time-series averages of the respective monthly characteristics. Finally, $\sigma(r^p)$, is the average of the standard deviation of daily returns for the portfolio's constituent stocks computed each month.

	β^{1p} $(\cdot 100)$	β^{2p} $(\cdot 100)$	β^{3p} $(\cdot 100)$	β^{4p} $(\cdot 100)$	$E(c^p)$ (%)	$\sigma(\Delta c^p)$ (%)	$E(r^{e,p})$ (%)	$\sigma(r^p)$ (%)	trn (%)	size (ml$)	BM
1	55.10	0.00	−0.80	−0.00	0.25	0.00	0.48	1.43	3.25	983.15	0.53
	(16.14)	(0.10)	(−6.18)	(−0.10)							
3	67.70	0.00	−1.05	−0.03	0.26	0.00	0.39	1.64	4.19	180.06	0.72
	(18.20)	(0.64)	(−7.60)	(−0.60)							
5	74.67	0.00	−1.24	−0.07	0.27	0.01	0.60	1.74	4.17	94.77	0.71
	(22.20)	(1.36)	(−8.13)	(−1.31)							
7	76.25	0.00	−1.27	−0.10	0.29	0.01	0.57	1.83	4.14	58.29	0.73
	(19.34)	(2.18)	(−8.04)	(−1.85)							
9	81.93	0.01	−1.37	−0.18	0.32	0.02	0.71	1.86	3.82	37.98	0.73
	(32.13)	(3.79)	(−8.38)	(−3.44)							
11	84.59	0.01	−1.41	−0.33	0.36	0.04	0.73	1.94	3.87	25.78	0.76
	(28.22)	(4.54)	(−7.80)	(−5.23)							
13	85.29	0.01	−1.47	−0.40	0.43	0.05	0.77	1.99	3.47	18.90	0.77
	(31.18)	(6.80)	(−8.07)	(−8.09)							
15	88.99	0.02	−1.61	−0.70	0.53	0.08	0.85	2.04	3.20	13.59	0.83
	(42.83)	(6.17)	(−8.06)	(−7.68)							
17	87.89	0.04	−1.59	−0.98	0.71	0.13	0.80	2.11	2.96	10.03	0.88
	(25.28)	(8.25)	(−7.66)	(−9.96)							
19	87.50	0.05	−1.58	−1.53	1.01	0.21	0.83	2.13	2.68	7.31	0.92
	(39.43)	(7.21)	(−8.19)	(−8.80)							
21	92.73	0.09	−1.69	−2.10	1.61	0.34	1.13	2.28	2.97	5.07	0.99
	(33.24)	(6.87)	(−8.03)	(−6.59)							
23	94.76	0.19	−1.71	−3.35	3.02	0.62	1.12	2.57	2.75	2.89	1.09
	(39.36)	(6.93)	(−8.39)	(−6.66)							
25	84.54	0.42	−1.69	−4.52	8.83	1.46	1.10	2.87	2.60	1.37	1.15
	(20.31)	(5.29)	(−7.27)	(−4.08)							

Table 2: **Beta correlations for illiquidity portfolios.**

This table reports the correlations of the four covariances, β^{1p}, β^{2p}, β^{3p} and β^{4p}, for the 25 value-weighted illiquidity portfolios formed for each year during 1964–1999 as described in Section 4.2. The four betas are computed for each portfolio as per (7) using all monthly return and illiquidity observations for the portfolio and the market portfolio. The monthly innovations in portfolio illiquidity and market illiquidity are computed using the AR(2) specification in (16) for the standardized illiquidity series. The monthly innovations in market portfolio return are computed using an AR(2) specification for the market return series that also employs available market characteristics at the beginning of the month.

	β^{1p}	β^{2p}	β^{3p}	β^{4p}
β^{1p}	1.000	0.562	−0.967	−0.679
β^{2p}		1.000	−0.554	−0.935
β^{3p}			1.000	0.669
β^{4p}				1.000

Table 3: **Beta correlations for individual stocks.**

This table reports the correlations of the four covariances, β^{1i}, β^{2i}, β^{3i} and β^{4i}, for the common shares listed on NYSE and AMEX during the period 1964–1999. The correlations are computed annually for all eligible stocks in a year as described in Section 4.2 and then averaged over the sample period. The four betas are computed for each stock as per (7) using all monthly return and illiquidity observations for the stock and the market portfolio. The monthly innovations in market illiquidity are computed using the AR(2) specification in (16) for the standardized market illiquidity series. The innovations in stock illiquidity are computed using a similar AR(2) specification with coefficients estimated for the market illiquidity. The monthly innovations in market portfolio return are computed using an AR(2) specification for the market return series that also employs available market characteristics at the beginning of the month.

	β^{1i}	β^{2i}	β^{3i}	β^{4i}
β^{1i}	1.000	0.020	−0.685	−0.164
β^{2i}		1.000	−0.072	−0.270
β^{3i}			1.000	0.192
β^{4i}				1.000

Table 4: **Illiquidity portfolios**

This table reports the estimated coefficients from Fama and Macbeth (1973)-type regressions of the liquidity-adjusted CAPM for 25 value-weighted illiquidity portfolios using monthly data during 1964–1999 with an equal-weighted market portfolio. We consider special cases of the relation:

$$E(r_t^p - r_t^f) = \alpha + \kappa E(c_t^p) + \lambda^1 \beta^{1p} + \lambda^2 \beta^{2p} + \lambda^3 \beta^{3p} + \lambda^4 \beta^{4p} + \lambda \beta^{net,p} \,,$$

where $\beta^{net,p} = \beta^{1p} + \beta^{2p} - \beta^{3p} - \beta^{4p}$. In some specifications, κ is set to be the average monthly turnover. The first t-statistic, reported in the parentheses, is estimated using the standard Fama–Macbeth method, and the second t-statistic, also reported in the parentheses, is computed using a GMM framework as explained in Section 4.5. The R^2 is obtained in a single cross-sectional regression, and the adjusted R^2 is reported in the parentheses.

	constant	$E(c^p)$	β^{1p}	β^{2p}	β^{3p}	β^{4p}	$\beta^{net,p}$	R^2
1	−0.556	0.034					1.512	0.822
	(−1.347)	(—)					(2.898)	(0.822)
	(−1.347)	(—)					(2.538)	
2	−0.512	0.042					1.449	0.825
	(−1.366)	(2.244)					(2.991)	(0.809)
	(−1.282)	(2.493)					(2.581)	
3	0.671	0.080						0.443
	(2.883)	(3.362)						(0.418)
	(2.931)	(3.022)						
4	−0.788		1.891					0.653
	(−1.838)		(3.333)					(0.638)
	(−1.749)		(2.838)					
5	0.669			171.134				0.537
	(2.883)			(3.476)				(0.517)
	(2.947)			(2.183)				
6	−0.371				−79.458			0.719
	(−1.117)				(−3.356)			(0.707)
	(−0.957)				(−2.309)			
7	0.606					−14.946		0.762
	(2.645)					(−3.656)		(0.751)
	(2.909)					(−2.289)		
8	−0.743						1.780	0.728
	(−1.799)						(3.411)	(0.716)
	(−1.797)						(2.985)	
9	−0.333	0.034	−3.181				4.334	0.842
	(−0.954)	(—)	(−0.993)				(1.398)	(0.836)
	(−0.811)	(—)	(−0.938)				(1.302)	
10	0.005	−0.032	−13.223				13.767	0.878
	(0.015)	(−0.971)	(−2.225)				(2.428)	(0.861)
	(0.012)	(−0.665)	(−1.801)				(1.911)	
11	−0.137			85.842	−59.725			0.810
	(−0.460)			(2.222)	(−2.687)			(0.793)
	(−0.387)			(1.921)	(−1.620)			
12	0.058				−42.504	−9.305		0.859
	(0.204)				(−1.922)	(−2.678)		(0.846)
	(0.156)				(−1.371)	(−2.221)		
13	0.060	−0.180		453.278	−46.437			0.841
	(0.205)	(−1.825)		(2.170)	(−2.117)			(0.818)
	(0.160)	(−1.274)		(1.543)	(−1.738)			
14	0.165	−0.037			−33.778	−15.142		0.874
	(0.574)	(−1.222)			(−1.529)	(−2.610)		(0.856)
	(0.580)	(−0.624)			(−0.404)	(−2.061)		
15	−0.089	0.033	0.992	−151.152	7.087	−17.542		0.881
	(−0.240)	(0.242)	(0.915)	(−0.430)	(0.147)	(−1.777)		(0.850)
	(−0.226)	(0.178)	(0.889)	(−0.300)	(0.231)	(−1.186)		

Table 5: **Illiquidity portfolios, equal weighted.**

This table reports the estimated coefficients from Fama and Macbeth (1973)-type regressions of the liquidity-adjusted CAPM for 25 equal-weighted illiquidity portfolios using monthly data during 1964–1999 with an equal-weighted market portfolio. We consider special cases of the relation:

$$E(r_t^p - r_t^f) = \alpha + \kappa E(c_t^p) + \lambda^1 \beta^{1p} + \lambda^2 \beta^{2p} + \lambda^3 \beta^{3p} + \lambda^4 \beta^{4p} + \lambda \beta^{net,p} \,,$$

where $\beta^{net,p} = \beta^{1p} + \beta^{2p} - \beta^{3p} - \beta^{4p}$. In some specifications, κ is set to be the average monthly turnover. The t-statistic, reported in the parentheses, is estimated using the standard Fama–Macbeth method. The R^2 is obtained in a single cross-sectional regression, and the adjusted R^2 is reported in the parentheses.

	constant	$E(c^p)$	β^{1p}	β^{2p}	β^{3p}	β^{4p}	$\beta^{net,p}$	R^2
1	−0.391	0.046					1.115	0.825
	(−0.835)	(—)					(2.085)	(0.825)
2	−0.299	0.062					0.996	0.846
	(−0.715)	(3.647)					(2.031)	(0.832)
3	0.614	0.082						0.634
	(2.397)	(3.808)						(0.618)
4	−0.530		1.374					0.350
	(−1.129)		(2.392)					(0.322)
5	0.602			154.046				0.759
	(2.358)			(3.731)				(0.748)
6	−0.276				−63.214			0.498
	(−0.762)				(−2.653)			(0.476)
7	0.558					−9.023		0.863
	(2.202)					(−3.615)		(0.857)
8	−0.653						1.456	0.529
	(−1.397)						(2.723)	(0.509)
9	−0.088	0.046	−2.699				3.395	0.878
	(−0.236)	(—)	(−1.364)				(1.711)	(0.873)
10	0.105	0.008	−6.392				6.800	0.901
	(0.282)	(0.384)	(−2.266)				(2.476)	(0.886)
11	0.084			122.545	−34.470			0.875
	(0.271)			(3.713)	(−1.634)			(0.863)
12	0.262				−20.377	−7.689		0.896
	(0.898)				(−0.988)	(−3.704)		(0.887)
13	0.174	−0.050		212.382	−28.493			0.882
	(0.572)	(−1.005)		(1.996)	(−1.320)			(0.865)
14	0.229	0.009			−22.808	−6.753		0.897
	(0.771)	(0.463)			(−1.067)	(−2.308)		(0.883)
15	−0.053	0.117	1.207	−346.547	33.043	−17.356		0.913
	(−0.129)	(1.208)	(0.988)	(−1.217)	(0.622)	(−2.242)		(0.890)

Table 6: **Illiquidity portfolios, value-weighted market.**

This table reports the estimated coefficients from Fama and Macbeth (1973)-type regressions of the liquidity-adjusted CAPM for 25 value-weighted illiquidity portfolios using monthly data during 1964–1999 with an value-weighted market portfolio. We consider special cases of the relation:

$$E(r_t^p - r_t^f) = \alpha + \kappa E(c_t^p) + \lambda^1 \beta^{1p} + \lambda^2 \beta^{2p} + \lambda^3 \beta^{3p} + \lambda^4 \beta^{4p} + \lambda \beta^{net,p} \,,$$

where $\beta^{net,p} = \beta^{1p} + \beta^{2p} - \beta^{3p} - \beta^{4p}$. In some specifications, κ is set to be the average monthly turnover. The t-statistic, reported in the parentheses, is estimated using the standard Fama–Macbeth method. The R^2 is obtained in a single cross-sectional regression, and the adjusted R^2 is reported in the parentheses.

	constant	$E(c^p)$	β^{1p}	β^{2p}	β^{3p}	β^{4p}	$\beta^{net,p}$	R^2
1	−1.938	0.034					2.495	0.486
	(−2.278)	(—)					(2.797)	(0.486)
2	−2.059	0.081					2.556	0.642
	(−2.322)	(3.382)					(2.811)	(0.609)
3	0.671	0.080						0.443
	(2.883)	(3.362)						(0.418)
4	0.700		0.062					0.000
	(1.224)		(0.099)					(−0.043)
5	0.647			1955.555				0.630
	(2.801)			(3.569)				(0.614)
6	−0.616				−937.655			0.627
	(−1.551)				(−3.242)			(0.610)
7	0.625					−9.829		0.704
	(2.718)					(−3.620)		(0.692)
8	−1.851						2.451	0.183
	(−2.176)						(2.748)	(0.148)
9	−1.536	0.034	−6.070				8.099	0.754
	(−2.029)	(—)	(−2.351)				(2.699)	(0.743)
10	−0.583	−0.076	−16.226				17.333	0.841
	(−0.866)	(−1.945)	(−3.211)				(3.317)	(0.819)
11	−0.216			1286.806	−613.320			0.824
	(−0.637)			(2.935)	(−2.423)			(0.808)
12	−0.129				−542.221	−6.703		0.843
	(−0.389)				(−2.171)	(−3.034)		(0.828)
13	0.000	−0.097		3380.318	−458.446			0.857
	(0.001)	(−1.772)		(2.597)	(−1.822)			(0.836)
14	0.050	−0.057			−408.975	−12.713		0.865
	(0.150)	(−1.491)			(−1.622)	(−2.711)		(0.846)
15	0.039	−0.056	0.015	−116.450	−405.451	−13.135		0.865
	(0.069)	(−0.850)	(0.019)	(−0.035)	(−1.257)	(−1.063)		(0.829)

Table 7: **Illiquidity portfolios, with control variables.**

This table reports the estimated coefficients from Fama and Macbeth (1973)-type regressions of the liquidity-adjusted CAPM for 25 value-weighted illiquidity portfolios using monthly data during 1964–1999 with an equal-weighted market portfolio. We consider special cases of the relation:

$$E(r_t^p - r_t^f) = \alpha + \kappa E(c_t^p) + \lambda^1 \beta^{1p} + \lambda^2 \beta^{2p} + \lambda^3 \beta^{3p} + \lambda^4 \beta^{4p} + \lambda \beta^{net,p} + \lambda^5 ln(size^p) + \lambda^6 BM^p\,,$$

where $\beta^{net,p} = \beta^{1p} + \beta^{2p} - \beta^{3p} - \beta^{4p}$, and the control variables $ln(size^p)$ and BM^p are, respectively, the time-series average of the natural log of the ratio of the portfolio's market capitalization at the beginning of the month to the total market capitalization, and BM is the time-series average of the average monthly book-to-market of the stocks constituting the portfolio. In some specifications, κ is set to be the average monthly turnover. The t-statistic, reported in the parentheses, is estimated using the standard Fama–Macbeth method. The R^2 is obtained in a single cross-sectional regression, and the adjusted R^2 is reported in the parentheses.

	constant	$E(c^p)$	β^{1p}	β^{2p}	β^{3p}	β^{4p}	$\beta^{net,p}$	$ln(size^p)$	B/M	R^2
1	−1.358 (−2.319)	0.034 (—)					2.158 (2.504)	0.142 (1.637)	1.076 (2.123)	0.865 (0.852)
2	−1.286 (−2.037)	0.028 (1.169)					1.970 (1.964)	0.129 (1.284)	1.120 (2.278)	0.865 (0.838)
3	−0.263 (−0.748)	−0.003 (−0.174)						−0.032 (−0.611)	1.067 (2.196)	0.841 (0.818)
4	−0.818 (−1.410)		0.798 (0.989)					0.043 (0.522)	1.350 (2.616)	0.850 (0.829)
5	−0.243 (−0.686)			−2.584 (−0.058)				−0.034 (−0.650)	1.030 (2.071)	0.841 (0.818)
6	−0.550 (−1.095)				−31.418 (−0.697)			0.032 (0.327)	1.213 (2.354)	0.844 (0.821)
7	0.166 (0.470)					−5.499 (−1.078)		−0.060 (−1.145)	0.326 (0.612)	0.850 (0.828)
8	−0.942 (−1.609)						1.061 (1.231)	0.066 (0.761)	1.329 (2.621)	0.855 (0.834)
9	−1.273 (−2.223)	0.034 (—)	−3.740 (−0.755)				6.145 (1.165)	0.155 (1.717)	0.679 (1.313)	0.869 (0.850)
10	−0.441 (−0.674)	−0.018 (−0.458)	−12.278 (−1.511)				13.565 (1.723)	0.068 (0.671)	0.159 (0.272)	0.882 (0.850)
11	−0.614 (−1.216)			21.881 (0.434)	−48.906 (−1.073)			0.060 (0.574)	1.125 (2.176)	0.845 (0.814)
12	−0.344 (−0.691)				−76.978 (−1.676)	−9.320 (−1.745)		0.087 (0.864)	0.362 (0.672)	0.864 (0.837)
13	−0.289 (−0.549)	−0.098 (−0.860)		238.283 (0.963)	−37.520 (−0.819)			0.024 (0.230)	0.732 (1.318)	0.850 (0.811)
14	0.150 (0.276)	−0.036 (−1.054)			−39.860 (−0.868)	−15.693 (−1.941)		0.009 (0.088)	−0.044 (−0.072)	0.874 (0.841)
15	−0.557 (−0.848)	0.059 (0.418)	1.300 (1.107)	−183.466 (−0.520)	−19.865 (−0.373)	−17.238 (−1.544)		0.087 (0.759)	0.253 (0.424)	0.884 (0.836)

Table 8: **Illiquidity portfolios, 1964–1981.**

This table reports the estimated coefficients from Fama and Macbeth (1973)-type regressions of the liquidity-adjusted CAPM for 25 value-weighted illiquidity portfolios using monthly data during 1964–1981 with an equal-weighted market portfolio. We consider special cases of the relation:

$$E(r_t^p - r_t^f) = \alpha + \kappa E(c_t^p) + \lambda^1 \beta^{1p} + \lambda^2 \beta^{2p} + \lambda^3 \beta^{3p} + \lambda^4 \beta^{4p} + \lambda \beta^{net,p},$$

where $\beta^{net,p} = \beta^{1p} + \beta^{2p} - \beta^{3p} - \beta^{4p}$. In some specifications, κ is set to be the average monthly turnover. The t-statistic, reported in the parentheses, is estimated using the standard Fama–Macbeth method. The R^2 is obtained in a single cross-sectional regression, and the adjusted R^2 is reported in the parentheses.

	constant	$E(c^p)$	β^{1p}	β^{2p}	β^{3p}	β^{4p}	$\beta^{net,p}$	R^2
1	−1.781	0.034					2.814	0.867
	(−3.019)	(—)					(3.403)	(0.867)
2	−1.744	0.040					2.762	0.868
	(−3.279)	(1.438)					(3.633)	(0.856)
3	0.510	0.112						0.336
	(1.460)	(3.037)						(0.307)
4	−2.098		3.337					0.777
	(−3.405)		(3.721)					(0.768)
5	0.503			248.964				0.434
	(1.446)			(3.224)				(0.410)
6	−1.332				−138.112			0.830
	(−2.906)				(−3.701)			(0.823)
7	0.402					−22.700		0.671
	(1.189)					(−3.517)		(0.657)
8	−1.967						3.082	0.833
	(−3.336)						(3.727)	(0.826)
9	−1.529	0.034	−3.591				6.000	0.877
	(−3.109)	(—)	(−0.763)				(1.294)	(0.871)
10	−1.059	−0.058	−17.523				19.086	0.903
	(−2.102)	(−1.156)	(−2.054)				(2.310)	(0.889)
11	−1.122			77.011	−120.409			0.858
	(−2.757)			(1.348)	(−3.521)			(0.845)
12	−0.905				−101.409	−9.242		0.883
	(−2.355)				(−3.119)	(−1.847)		(0.872)
13	−0.768	−0.323		735.624	−96.591			0.895
	(−1.933)	(−2.171)		(2.365)	(−2.998)			(0.881)
14	−0.710	−0.067			−85.394	−19.953		0.902
	(−1.842)	(−1.437)			(−2.731)	(−2.333)		(0.888)
15	−0.933	−0.155	0.969	267.298	−46.940	−13.440		0.906
	(−1.707)	(−0.844)	(0.617)	(0.562)	(−0.698)	(−1.000)		(0.881)

Table 9: **Illiquidity portfolios, 1982–1999.**

This table reports the estimated coefficients from Fama and Macbeth (1973)-type regressions of the liquidity-adjusted CAPM for 25 value-weighted illiquidity portfolios using monthly data during 1982–1999 with an equal-weighted market portfolio. We consider special cases of the relation:

$$E(r_t^p - r_t^f) = \alpha + \kappa E(c_t^p) + \lambda^1 \beta^{1p} + \lambda^2 \beta^{2p} + \lambda^3 \beta^{3p} + \lambda^4 \beta^{4p} + \lambda \beta^{net,p} \,,$$

where $\beta^{net,p} = \beta^{1p} + \beta^{2p} - \beta^{3p} - \beta^{4p}$. In some specifications, κ is set to be the average monthly turnover. The t-statistic, reported in the parentheses, is estimated using the standard Fama–Macbeth method. The R^2 is obtained in a single cross-sectional regression, and the adjusted R^2 is reported in the parentheses.

	constant	$E(c^p)$	β^{1p}	β^{2p}	β^{3p}	β^{4p}	$\beta^{net,p}$	R^2
1	0.668	0.034					0.211	0.394
	(1.178)	(—)					(0.336)	(0.394)
2	0.721	0.043					0.135	0.407
	(1.399)	(1.759)					(0.229)	(0.353)
3	0.831	0.047						0.398
	(2.701)	(1.590)						(0.372)
4	0.522		0.444					0.094
	(0.892)		(0.651)					(0.054)
5	0.834			93.304				0.414
	(2.716)			(1.534)				(0.388)
6	0.589				−20.804			0.128
	(1.245)				(−0.725)			(0.090)
7	0.810					−7.192		0.457
	(2.616)					(−1.445)		(0.433)
8	0.482						0.478	0.136
	(0.849)						(0.763)	(0.099)
9	0.862	0.034	−2.770				2.668	0.435
	(1.779)	(—)	(−0.636)				(0.647)	(0.410)
10	1.070	−0.007	−8.923				8.448	0.469
	(2.067)	(−0.152)	(−1.077)				(1.087)	(0.394)
11	0.847			94.674	0.959			0.414
	(1.982)			(1.814)	(0.034)			(0.361)
12	1.022				16.401	−9.369		0.494
	(2.476)				(0.556)	(−1.937)		(0.448)
13	0.888	−0.037		170.932	3.717			0.417
	(2.107)	(−0.289)		(0.615)	(0.126)			(0.334)
14	1.039	−0.006			17.838	−10.330		0.495
	(2.489)	(−0.161)			(0.578)	(−1.316)		(0.423)
15	0.755	0.221	1.015	−569.601	61.114	−21.644		0.568
	(1.535)	(1.106)	(0.678)	(−1.101)	(0.888)	(−1.494)		(0.454)

Table 10: **Properties of size portfolios**.

This table reports the properties of the odd–numbered portfolios of 25 value-weighted size portfolios formed each year during 1964–1999 as described in Section 4.2. The four betas (β^{ip}) are computed for each portfolio using all monthly return and illiquidity observations for a portfolio, and an equal-weighted market portfolio. In particular, these betas based on (7), where the innovations in portfolio illiquidity and market illiquidity are computed using the AR(2) specification in (16) for the standardized illiquidity series, and the innovations in market portfolio return is computed using an AR(2) specification for the market return series that also employs available market characteristics at the beginning of the month (return, volatility, average illiquidity, log of average dollar volume, log of average turnover, all measured over past six months, and log of one-month lagged market capitalization). The t-statistics, reported in parenthesis, are estimated using GMM. The standard deviation of the portfolio illiquidity innovations is reported under the column $\sigma(\Delta c^p)$. The average illiquidity, $E(c^p)$, the average excess return, $E(r^{e,p})$, the turnover (trn), the market capitalization (size), and book-to-market (BM) are computed for each portfolio as time-series averages of the respective monthly characteristics. Finally, $\sigma(r^p)$, is the average of the standard deviation of daily returns for the portfolio's constituent stocks computed each month.

	β^{1p} $(\cdot 100)$	β^{2p} $(\cdot 100)$	β^{3p} $(\cdot 100)$	β^{4p} $(\cdot 100)$	$E(c^p)$ (%)	$\sigma(\Delta c^p)$ (%)	$E(r^{e,p})$ (%)	$\sigma(r^p)$ (%)	trn (%)	size (ml$)	BM
1	97.87	0.38	−1.77	−5.00	5.46	1.25	1.13	2.76	4.86	0.71	1.37
	(22.50)	(5.38)	(−9.24)	(−3.45)							
3	100.00	0.20	−1.80	−4.21	2.66	0.67	0.93	2.72	4.74	1.98	1.15
	(35.16)	(8.54)	(−8.64)	(−5.02)							
5	105.04	0.12	−1.93	−2.56	1.66	0.44	0.83	2.62	4.96	3.47	1.10
	(32.70)	(7.06)	(−9.15)	(−5.59)							
7	101.27	0.08	−1.81	−1.77	1.15	0.25	0.98	2.45	4.73	5.45	1.01
	(31.58)	(7.80)	(−7.51)	(−5.95)							
9	101.11	0.05	−1.80	−1.18	0.78	0.16	0.84	2.38	5.29	8.09	0.93
	(38.24)	(8.74)	(−8.30)	(−7.72)							
11	97.95	0.03	−1.75	−0.78	0.62	0.11	0.81	2.28	5.20	11.61	0.91
	(34.84)	(7.40)	(−7.25)	(−6.81)							
13	96.30	0.02	−1.64	−0.57	0.49	0.08	0.79	2.18	5.09	16.73	0.84
	(45.84)	(6.36)	(−6.66)	(−6.53)							
15	91.81	0.01	−1.57	−0.41	0.41	0.06	0.69	2.08	4.92	24.45	0.85
	(28.71)	(4.61)	(−6.61)	(−4.72)							
17	85.88	0.01	−1.44	−0.25	0.36	0.03	0.73	1.96	4.86	36.59	0.79
	(42.70)	(4.28)	(−6.07)	(−4.62)							
19	83.56	0.00	−1.40	−0.16	0.31	0.02	0.65	1.85	4.73	57.12	0.77
	(35.98)	(2.18)	(−5.94)	(−2.46)							
21	77.79	0.00	−1.25	−0.06	0.28	0.01	0.62	1.75	4.54	96.24	0.74
	(34.91)	(0.96)	(−5.45)	(−1.11)							
23	69.60	0.00	−1.12	−0.03	0.27	0.01	0.50	1.63	4.20	186.57	0.71
	(20.88)	(0.59)	(−4.66)	(−0.55)							
25	53.34	0.00	−0.78	−0.00	0.25	0.00	0.49	1.42	2.80	988.69	0.52
	(16.53)	(0.05)	(−4.06)	(−0.06)							

Table 11: **Size portfolios.**

This table reports the estimated coefficients from Fama and Macbeth (1973)-type regressions of the liquidity-adjusted CAPM for 25 value-weighted size portfolios using monthly data during 1964–1999 with an equal-weighted market portfolio. We consider special cases of the relation:

$$E(r_t^p - r_t^f) = \alpha + \kappa E(c_t^p) + \lambda^1 \beta^{1p} + \lambda^2 \beta^{2p} + \lambda^3 \beta^{3p} + \lambda^4 \beta^{4p} + \lambda \beta^{net,p}\,,$$

where $\beta^{net,p} = \beta^{1p} + \beta^{2p} - \beta^{3p} - \beta^{4p}$. In some specifications, κ is set to be the average monthly turnover. The t-statistic, reported in the parentheses, is estimated using the standard Fama–Macbeth method. The R^2 is obtained in a single cross-sectional regression, and the adjusted R^2 is reported in the parentheses.

	constant	$E(c^p)$	β^{1p}	β^{2p}	β^{3p}	β^{4p}	$\beta^{net,p}$	R^2
1	−0.087	0.047					0.865	0.910
	(−0.255)	(—)					(1.993)	(0.910)
2	−0.059	0.056					0.823	0.912
	(−0.191)	(1.885)					(2.010)	(0.904)
3	0.647	0.111						0.583
	(2.639)	(2.721)						(0.565)
4	−0.265		1.144					0.757
	(−0.749)		(2.443)					(0.747)
5	0.668			151.962				0.619
	(2.714)			(2.725)				(0.603)
6	−0.045				−51.693			0.793
	(−0.156)				(−2.470)			(0.784)
7	0.640					−9.871		0.698
	(2.641)					(−2.657)		(0.685)
8	−0.236						1.077	0.810
	(−0.686)						(2.483)	(0.801)
9	−0.043	0.047	−0.770				1.562	0.912
	(−0.146)	(—)	(−0.289)				(0.603)	(0.908)
10	−0.055	0.054	−0.168				0.984	0.912
	(−0.180)	(1.054)	(−0.034)				(0.209)	(0.900)
11	0.118			80.542	−38.030			0.912
	(0.464)			(1.981)	(−1.951)			(0.904)
12	0.155				−34.828	−5.108		0.896
	(0.626)				(−1.782)	(−1.817)		(0.886)
13	0.199	−0.188		334.539	−35.211			0.918
	(0.790)	(−0.816)		(1.035)	(−1.835)			(0.906)
14	0.093	0.056			−38.954	−0.213		0.907
	(0.368)	(1.136)			(−1.964)	(−0.043)		(0.894)
15	0.224	−0.408	−0.079	742.841	−42.800	7.933		0.929
	(0.619)	(−1.403)	(−0.070)	(1.622)	(−0.846)	(1.152)		(0.911)

Table 12: **Book-to-market by size portfolios.**

This table reports the estimated coefficients from Fama and Macbeth (1973)-type regressions of the liquidity-adjusted CAPM for 25 value-weighted book-to-market by size portfolios using monthly data during 1964–1999 with an equal-weighted market portfolio. We consider special cases of the relation:

$$E(r_t^p - r_t^f) = \alpha + \kappa E(c_t^p) + \lambda^1 \beta^{1p} + \lambda^2 \beta^{2p} + \lambda^3 \beta^{3p} + \lambda^4 \beta^{4p} + \lambda \beta^{net,p}\,,$$

where $\beta^{net,p} = \beta^{1p} + \beta^{2p} - \beta^{3p} - \beta^{4p}$. In some specifications, κ is set to be the average monthly turnover. The t-statistic, reported in the parentheses, is estimated using the standard Fama–Macbeth method. The R^2 is obtained in a single cross-sectional regression, and the adjusted R^2 is reported in the parentheses.

	constant	$E(c^p)$	β^{1p}	β^{2p}	β^{3p}	β^{4p}	$\beta^{net,p}$	R^2
1	0.200	0.045					0.582	0.406
	(0.699)	(—)					(1.466)	(0.406)
2	0.453	0.167					0.182	0.541
	(1.716)	(3.546)					(0.456)	(0.499)
3	0.598	0.189						0.530
	(2.559)	(3.162)						(0.510)
4	0.109		0.748					0.262
	(0.370)		(1.764)					(0.229)
5	0.637			303.217				0.540
	(2.702)			(3.086)				(0.520)
6	0.343				−27.769			0.201
	(1.493)				(−1.552)			(0.166)
7	0.629					−14.231		0.511
	(2.698)					(−2.903)		(0.490)
8	0.107						0.729	0.288
	(0.373)						(1.838)	(0.257)
9	0.529	0.045	−8.289				8.275	0.503
	(2.009)	(—)	(−2.240)				(2.341)	(0.481)
10	0.187	0.387	18.229				−17.458	0.571
	(0.679)	(3.330)	(2.072)				(−2.039)	(0.510)
11	0.663			311.642	1.994			0.541
	(3.113)			(3.698)	(0.110)			(0.499)
12	0.696				5.158	−15.370		0.514
	(3.270)				(0.278)	(−3.573)		(0.470)
13	0.704	−0.112		494.141	3.193			0.543
	(3.078)	(−0.475)		(1.158)	(0.172)			(0.477)
14	0.597	0.175			−0.222	−1.007		0.530
	(2.714)	(1.615)			(−0.012)	(−0.099)		(0.463)
15	−0.395	−0.031	4.545	397.770	195.128	0.380		0.789
	(−1.128)	(−0.112)	(3.487)	(0.533)	(3.240)	(0.019)		(0.733)

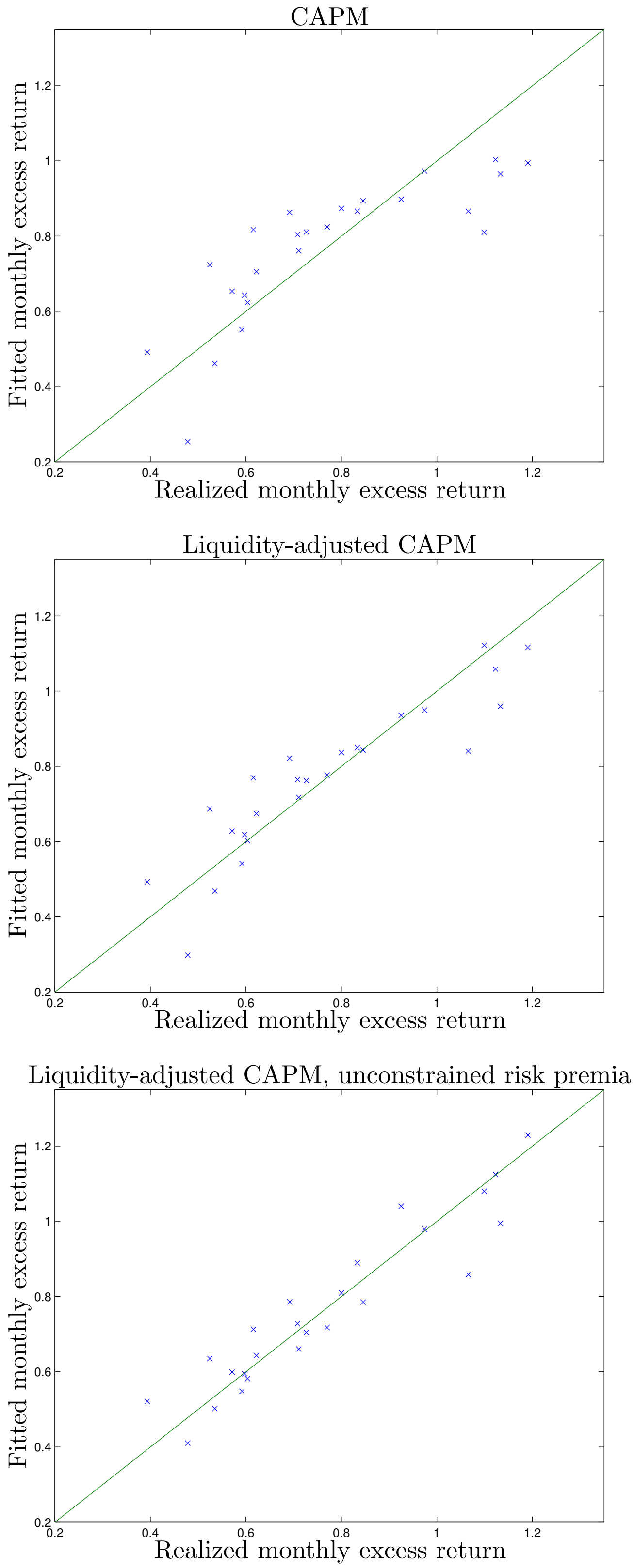

Figure 2: **Illiquidity portfolios:** The top panel shows the fitted CAPM returns vs. realized returns using monthly data 1964–1999 for value-weighted illiquidity portfolios. The middle panel shows the same for the liquidity-adjusted CAPM, and the lower panel shows the relation for the liquidity adjusted CAPM with unconstrained risk premia.

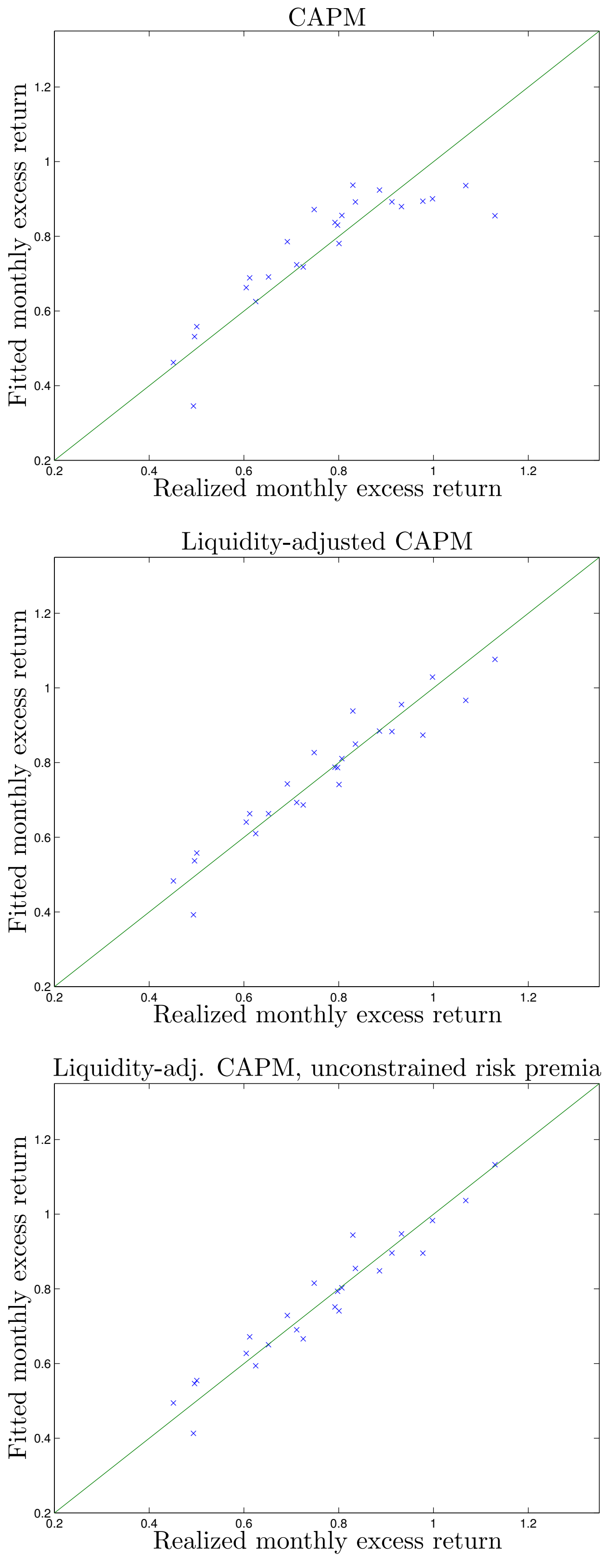

Figure 3: **Size portfolios:** The top panel shows the fitted CAPM returns vs. realized returns using monthly data 1964–1999 for value-weighted size portfolios. The middle panel shows the same for the liquidity-adjusted CAPM, and the lower panel shows the relation for the liquidity adjusted CAPM with unconstrained risk premia.

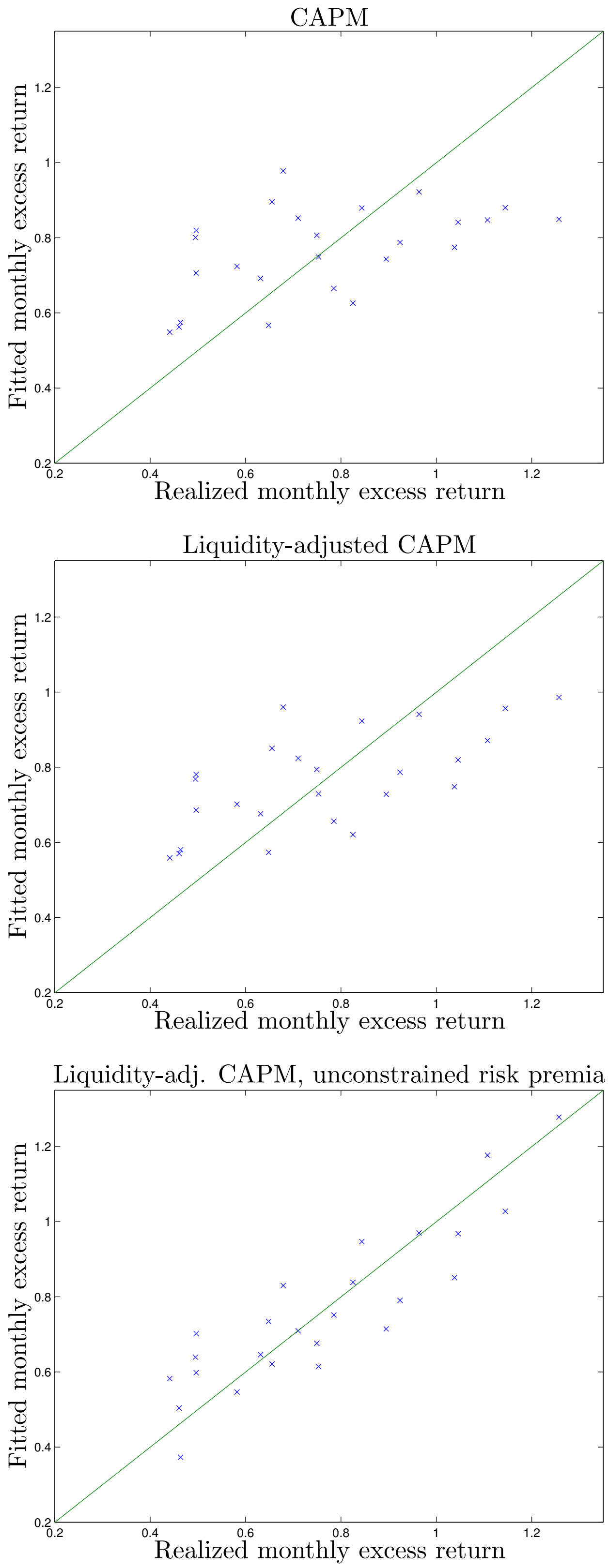

Figure 4: **Book-to-market by size portfolios:** The top panel shows the fitted CAPM returns vs. realized returns using monthly data 1964–1999 for value-weighted BM-size portfolios. The middle panel shows the same for the liquidity-adjusted CAPM, and the lower panel shows the relation for the liquidity adjusted CAPM with unconstrained risk premia.