A Theory of Housing Collateral, Consumption Insurance and Risk Premia

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ABSTRACT

In a model with housing collateral, the ratio of housing wealth to total wealth shifts the conditional distribution of asset prices and consumption growth. A decrease in house prices reduces the collateral value of housing, increases household exposure to idiosyncratic risk, and increases the conditional market price of risk. The model quantitatively accounts for conditional asset pricing moments, cross-sectional variation in value portfolio returns and key unconditional asset pricing moments. The increase of the equity premium and Sharpe ratio when collateral is scarce matches the increase observed in US data. The model also generates a return spread of value firms over growth firms of the magnitude observed in the data. Assets with payoffs that lay farther in the future, are less risky. Growth stocks are such long duration assets.

Introduction

Two aspects of financial markets are often studied separately. One is their role in allocating resources. The evidence suggests that countries with good institutions tend to reduce risk by individuals (Gertler and Gruber (2002)). It also suggests that those same countries have better aggregate economic performance (Levine (1997)). The other aspect of financial markets is asset returns. Arbitrage-free theory implies a *pricing kernel* that values all traded assets. In principle, this common ingredient should affect the behavior of returns over time and generate connections between returns on different kinds of assets. Research along these lines has documented a number of striking differences in expected returns: between equity and bonds (Mehra and Prescott (1985)), between equity at different points in time (Lettau and Ludvigson (2003)), between size- and value-weighted portfolios (Fama and French (1992)), and between bonds of different maturities (Backus and Zin (1994)).

We argue that modelling these two aspects together leads to deeper insights about each. We build a dynamic general equilibrium model that we think approximates the modest frictions inhibiting perfect risk-sharing in an advanced economy like the US. The model is based on two ideas: that debts can be enforced only to the extent that they are collateralized, and that the primary source of collateral is housing. Our emphasis on housing, rather than financial assets, reflects two features of the US economy: the participation rate is much higher than for equity (two-thirds of households own their own house), and the aggregate value of housing in the US is roughly double the value of equity owned by households (Flow of Funds data). In addition, our earlier work (Lustig and VanNieuwerburgh (2004b)) shows that asset returns are correlated with the amount of available housing collateral, both in the time-series and in the cross-section, giving this approach some empirical plausibility.

The first insight is that amount of housing collateral significantly shifts the risk-sharing possibilities of households. Risk-sharing patterns, explored empirically for metropolitan regions in Lustig and VanNieuwerburgh (2004a), confirm the collateral mechanism. The second insight is that the same variable that indexes the risk-sharing possibilities among households, also indexes the investment opportunity set. The main contribution of this paper is to show that time variation in housing collateral can quantitatively account for three of the aforementioned differences in expected returns: between equity and the risk-free asset, between equity at different points in time, and between size and value portfolios.

More specifically, our model generates the following features. First, it predicts high and volatile equity premia, as well as high Sharpe ratios, when collateral is scarce. The Sharpe ratio is volatile. We document similar dynamics for the excess returns and the Sharpe ratio for US data. A direct implication of the time-series variation in equity premia is that the housing collateral ratio ought to predict future excess returns. The model quantitatively replicates the predictability pattern found in the data.

Second, the collateral model generates a large value premium and the higher Sharpe ratios for value stocks than growth stocks. Following Lustig and VanNieuwerburgh (2004b), we excess regress returns on value decile portfolios on the empirical counterparts of the model's risk factors. We simulate artificial book-to-market decile excess returns by using these empirical factor loadings and the model's risk factors. The model simulation generates a return difference between the extreme value and growth portfolios of 6 percent, as high as in the data. Moreover, the Sharpe ratio on value portfolios is twice as high as the Sharpe ratio on growth portfolios. The model generates a value premium because short duration assets, such as value stocks, are more risky than long duration assets, such as growth stocks. Lettau and Wachter (2004) and Hansen, Heaton, and Li (2004) take a similar approach to explain the value premium. The model generates a decreasing term structure of consumption strips. The return spread between a basket of consumption strips with a duration of 5 years ('value') and a basket with a duration of 40 years ('growth') is 6 percent.

Third, the model generates empirically plausible unconditional aggregate asset pricing moments.

The benchmark general equilibrium model that explains both risk-sharing dynamics and asset prices is the Lucas (1978) model. It predicts perfect risk-sharing and implies that the only risk factor relevant for asset prices is aggregate consumption growth Breeden (1979). There is a wealth of empirical evidence against full consumption insurance at different levels of aggregation: at the household level (e.g. Attanasio and Davis (1996) and Cochrane (1991)), the regional level (e.g. Hess and Shin (1998) and Lustig and VanNieuwerburgh (2004a)) and the international level (e.g. Backus, Kehoe, and Kydland (1992)). The asset pricing predictions have been tested and rejected (e.g. Hansen and Singleton (1983)). Its failure occurs both along the time-series and the cross-sectional dimension. First, because aggregate consumption growth is approximately i.i.d., the CCAPM implies a market price of risk that is approximately constant and there is very little variation in the Sharpe ratio. Second, the covariance of asset returns with consumption growth explains only a small fraction of the variation in the cross-section of stock returns of firms sorted in portfolios according to size and value characteristics. It has been understood for more than a decade that frictions are needed to bring the consumption-based model closer to the data. Incomplete markets with exogenous borrowing constraints, short sales constraints or transaction costs (e.g. Telmer (1993) and Heaton and Lucas (1996)), failed to deliver sufficiently large deviations from the benchmark model. The goal of our paper is to introduce a different kind of friction, associated with the housing market, and to ask to what extent it can generate realistic asset price predictions. We show that for a plausibly calibrated housing collateral process, we significantly improve on the predictions of the canonical CCAPM, both in terms of the time-series variation in conditional asset pricing moments and the cross-sectional variation in returns.

Changes in the value of the housing stock perturb the risk sharing technology. The households in this economy trade contingent claims to insure against labor income risk. These claims have to be fully backed by the value of their housing wealth. The model relaxes the assumption that contracts are perfectly enforceable (Alvarez and Jermann (2000)). As in Lustig (2003), we allow households to forget their debts. The new feature of our model is that each household owns part of the housing stock. Housing provides utility services and collateral services. When a household chooses to forget its debts, it loses all its housing wealth but its labor income is protected from creditors. The household is not excluded from trading.[1] The lack of commitment gives rise to collateral constraints. Their tightness depends on the abundance of housing collateral. We measure this by the *housing collateral ratio*: the ratio of collateralizable housing wealth to total wealth. An increase in the housing collateral ratio increases the scope for risk sharing. It decreases the conditional dispersion of consumption growth across households.

Two channels in the model deliver time variation in the conditional market price of risk, at different frequencies. First, a drop in the housing collateral ratio adversely affects the risk sharing technology that enables households to insulate their consumption from labor income shocks. This makes households demand a higher price to bear risk in times with low housing collateral. This is the source of low frequency variation in the market price of risk. Second, households are more exposed to binding collateral constraints when the cross-sectional dispersion of labor income shocks increases, typically when aggregate consumption growth is low (Constantinides and Duffie (1996)).

[1] we model the outside option as bankruptcy with loss of all collateral assets. In Kehoe and Levine (1993), Krueger (2000), Krueger and Perri (2003), and Kehoe and Perri (2002), limited commitment is also the source of incomplete risk-sharing across US households and across countries respectively. In contrast, the outside option upon default is exclusion from future participation in financial markets. In our model, all promises are backed by all collateral assets. Geanakoplos and Zame (2000) and Kubler and Schmedders (2003) think of individual assets collateralizing individual promises in an incomplete markets economy.

Because the aggregate weight shock increases after a series of large aggregate consumption growth shocks, the conditional standard deviation of the stochastic discount factor rises. This is the source of high frequency variation in the market price of risk. The combination of both ingredients can explain the time-series variation in asset prices and the cross-section of value-portfolio returns.

The model includes a second channel that transmits housing shocks to asset prices: non-separable preferences. If utility is non-separable in non-durable consumption and housing services, households want to hedge against shocks to the share of housing consumption in total consumption. This introduces composition risk, which is the focus of recent work by Piazzesi, Schneider, and Tuzel (2004) and Yogo (2003). If housing services and consumption are complements and rental price growth is positively correlated with returns, then households command a larger risk premium.[2] Our collateral effect does not hinge on the non-separability of preferences. Instead, it relies on imperfect consumption insurance among heterogenous households induced by occasionally binding collateral constraints. We use non-separable preferences to quantify the relative effects of the composition risk mechanism and the collateral mechanism. We find that only the latter can quantitatively account for the observed volatility in conditional asset pricing moments and for the value premium.

We organize the paper as follows. Section 1 describes the environment, characterizes equilibrium allocations and defines the pricing kernel. Section 2 discusses the model computation and calibration. Asset pricing results from a simulation of the model are discussed in section 3. Section 5 concludes. Section 6 contains all figures and tables. The appendix contains some details of the model (A.1, A.3), proofs of the propositions (A.2) and a description of the data (A.5). It also discusses the results for an economy where preferences are recursive, rather than additive (A.4).

1. Model

This section starts with a complete description of the environment in section 1.1. The next section (1.2), sets up the household problem in time zero trading environment and characterizes equilibrium allocations using stochastic consumption weight processes. The growth rate of an *aggregate* consumption weight process drives the consumption growth of the unconstrained households; these households price random payoffs. The stochastic discount factor captures the risk of binding sol-

[2] Dunn and Singleton (1986) and Eichenbaum and Hansen (1990) report substantial evidence against the null of separability in a representative agent model with durable and non-durable consumption, but they conclude that introducing durables does not help in reducing the pricing errors for stocks.

vency constraints. Section 1.3 explains how the dynamics of the collateral ratio affect equilibrium allocations. To gain intuition, section 1.4 describes the dynamics in a simple two-agent economy. Section 1.5 introduces sequential trading and discusses conditions under which these equilibria coincide with time zero trading equilibria. In section 1.6 we define value and growth stocks in the model and show that the model can only generate a value premium if the pricing kernel contains a permanent component.

We consider the simplest model of housing markets that delivers variation in the amount of housing collateral relative to total wealth. The housing market has efficient rental markets or spot markets for housing services. Ownership and consumption of housing are completely separated. We calibrate the persistence of the consumption/housing expenditure ratio to the data. Variation in the expenditure ratio changes the value of the housing tree relative to the value of the other, non-durable consumption tree.

1.1. Environment

This economy is populated by a continuum of infinitely lived households. The structure of uncertainty is twofold. $s = (y, z)$ is an event that consists of a household-specific component $y \in Y$ and an aggregate component $z \in Z$. These events take on values on a discrete grid $S = Y \times Z$. We use $s^t = (y^t, z^t)$ to denote the history of events. S^t denotes the set of possible histories up until time t. s follows a Markov process with transition probabilities π that obey:

$$\pi(z'|z) = \sum_{y' \in Y} \pi(y', z'|y, z) \; \forall z \in Z, y \in Y.$$

Because of the law of large numbers, $\pi_z(y)$ denotes both the fraction of households drawing y when the aggregate event is z and the probability that a given household is in state y when the aggregate state is z.

We use $\{x\}$ to denote an infinite stream $\left\{x_t(s^t)\right\}_{t=0}^{\infty}$. There are two types of commodities in this economy: a consumption good and housing services. These consumption goods cannot be stored. We let $\{c(\theta_0, s_0)\}$ denote the stream of consumption and we let $\{h(\theta_0, s_0)\}$ denote the stream of housing services of a household of type (θ_0, s_0). The households rank consumption streams according to the criterion:

$$U(\{c\},\{h\}) = \sum_{s^t|s_0}\sum_{t=0}^{\infty}\delta^t\pi(s^t|s_0)u\left(c_t(\theta_0,s^t),h_t(\theta_0,s^t)\right), \tag{1}$$

where δ is the time discount factor. The households have power utility over a CES-composite consumption good:

$$u(c_t,h_t) = \frac{\left[c_t^{\frac{\varepsilon-1}{\varepsilon}} + \psi h_t^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{(1-\gamma)\varepsilon}{\varepsilon-1}}}{1-\gamma}.$$

$\psi > 0$ converts the housing stock into a service flow. ε is the intratemporal elasticity of substitution between non-durable consumption and housing services [3]

The aggregate endowment of the non-durable consumption good is denoted $\{e\}$. The growth rate of the aggregate endowment depends only on the current aggregate state: $e_{t+1}(z^{t+1}) = \lambda(z_{t+1})e_t(z^t)$. Aggregate consumption c^a equals the aggregate endowment e. Each of the households is endowed with a claim to a labor income stream $\{\eta\}$. The labor income share $\hat{\eta}(y_t,z_t)$ only depends on the current state of nature. The level of labor income is given by $\eta(y_t,z^t) = \hat{\eta}(y_t,z_t)e(z^t)$. The aggregate endowment is the sum of the individual endowments:

$$\sum_{y'\in Y}\pi_z(y')\hat{\eta}_t(y',z) = 1,\ \forall z,t\geq 0,$$

The aggregate endowment of housing services is denoted $\{h^a\}$. Rather than specifying a process for $\{h^a\}$, we specify a process for the expenditure ratio of non-durable to housing services consumption $\{r\}$, where $r(z^t) = \frac{c^a(z^t)}{\rho(z^t)h^a(z^t)}$ and $\rho(z^t)$ denote the relative price of a unit of housing services. The natural logarithm $\log(r)$ is specified as an autoregressive process and additionally depends on the endowment growth rate $\lambda(z^t)$. Let R be the domain of r.

We use $p_t(s^t|s_0)$ to denote the state price deflator: it is the price of a unit non-durable consumption to be delivered in state s^t, in units of time zero consumption. Finally, $\Pi_{s^t}[\{d\}]$ denotes the price of claim to $\{d\}$ in units of s^t consumption, $\Pi_{s^t}[\{d\}] = \sum_{s^\tau|s^t}\sum_{\tau=0}^{\infty}\left[p_{t+\tau}\left(s^\tau|s^t\right)d_{t+\tau}\left(s^\tau|s^t\right)\right]$.

Markets open only at time zero. Households purchase a complete, state-contingent consumption

[3] The preferences belong to the class of homothetic power utility functions of Eichenbaum and Hansen (1990). Special cases are separability ($\varepsilon = \gamma^{-1}$) and Cobb-Douglas preferences ($\varepsilon = 1$).

plan $\{c(\theta_0, s_0), h(\theta_0, s_0)\}$ subject to a single, time zero budget constraint:

$$\Pi_{s_0}\left[\{c(\theta_0, s_0) + \rho h(\theta_0, s_0)\}\right] \leqslant \theta_0 + \Pi_{s_0}\left[\{\eta\}\right], \tag{2}$$

where θ_0 is the initial non-labor wealth. We use Θ_0 to denote the initial distribution of non-labor wealth holdings.

Solvency Constraints Households can default on their debts. When the household defaults, it keeps its labor income in all future periods. The household is not excluded from trading, even in the same period. However, all collateral wealth is taken away. As a result, the markets impose a solvency constraints that keep the households from defaulting. There is one such constraint for each node s^t on the household's trades:

$$\Pi_{s^t}\left[\{c(\theta_0, s_0) + \rho h(\theta_0, s_0)\}\right] \geq \Pi_{s^t}\left[\{\eta\}\right]. \tag{3}$$

The constraints depend on the state prices as well as the rental price. Changes in equilibrium prices determine the tightness of the constraints, as we describe in the next section.

1.2. Equilibrium Prices and Allocations

We define an equilibrium with time zero markets and characterize the equilibrium allocations. These Kehoe and Levine (1993) equilibria are essentially Arrow-Debreu equilibria. Appendix A.1 fills in the details.

Definition. *For given initial state z_0 and for given distribution Θ_0, an equilibrium consists of prices $\left\{p_t(s^t|s_0), \rho(z^t|z_0)\right\}$ and allocations $\left\{c_t(\theta_0, s^t), h_t(\theta_0, s^t)\right\}$ such that*

- *For given prices $\left\{p_t(s^t|s_0)\right\}$, the allocations solve the household's problem of maximizing (1) subject to (2) and (3) (except possibly on a set of measure zero).*
- *Markets clear for all t, z^t:*

$$\begin{aligned} \sum_{y^t} \int c_t(\theta_0, y^t, z^t) d\Theta_0 \frac{\pi(y^t, z^t|y_0, z_0)}{\pi(z^t|z_0)} &= c_t^a(z_t) = e_t(z^t). & (4) \\ \sum_{y^t} \int h_t(\theta_0, y^t, z^t) d\Theta_0 \frac{\pi(y^t, z^t|y_0, z_0)}{\pi(z^t|z_0)} &= h_t^a(z_t) & (5) \end{aligned}$$

To determine the equilibrium consumption of households, it is helpful to examine the dual of the household maximization problem. Let $U_0(\{c\},\{h\})$ denote the total utility from consuming $\{c\}$ and $\{h\}$. For given prices $\{p,\rho\}$ a household with label (w_0, s_0) minimizes the cost $C(\cdot)$ of delivering initial utility w_0 to itself:

$$\begin{aligned} C(w_0, s_0) &= \min_{\{c,h\}} \left(c_0(w_0, s_0) + h_0(w_0, s_0)\rho_0(s_0)\right) \\ &\quad + \sum_{s^t} p(s^t|s_0)\left(c_t(w_0, s^t|s_0) + h_t(w_0, s^t|s_0)\rho_t(s^t|s_0)\right) \end{aligned}$$

subject to the initial promised utility constraint: $U_0(\{c\},\{h\}) \geq w_0$, and the solvency constraints (3), one for each node s^t. The initial promised value w_0 is determined such that the household spends its entire initial wealth: $C(w_0, s_0) = \theta_0 + \Pi_{s_0}[\{\eta\}]$. There is a monotone relationship between θ_0 and w_0.

Stochastic Consumption Weights Let $\{\gamma(\theta_0, s_0)\}$ denote the sequence of multipliers on the solvency constraints (3) imposed on household (θ_0, s_0). We define $\xi_t(\theta_0, s^t)$ to be household (θ_0, s_0)'s cumulative Lagrange multiplier:

$$\xi_t(\theta_0, s^t) = \ell(\theta_0, s_0) + \sum_{\tau=0}^{t} \sum_{s^\tau \preceq s^t} \gamma_\tau(\theta_0, s^\tau).$$

We refer to $\xi_t(\theta_0, s^t)$ as the *consumption weight* in state s^t for household (θ_0, s_0) in the dual problem. The initial weight $\ell(\theta_0, s_0)$ is the inverse of the Lagrange multiplier on the initial promised utility constraint, $\xi_0(\theta_0, s_0) = \ell$. $\{\xi(\theta_0, s_0)\}$ is a non-decreasing stochastic process.

The process $\xi_t^a(z^t)$ is the aggregate weight $\int \xi_t(y^t, z^t)^{\frac{1}{\gamma}} d\Phi_t(z^t)$. Φ_0 is the initial cross-sectional distribution over $\ell(\theta_0, s_0)$, implied by the initial wealth distribution Θ_0. $\Phi_t(z^t)$ is the distribution over weights after aggregate history z^t. $\xi_t^a(z^t)$ summarizes to what extent constraints bind *on average*.

If the solvency constraint does not bind, the household's weight remains unchanged. When the constraint binds, its weight increases to a cutoff level $\ell^c(y_t, z^t)$ that depends only on the current event y_t.

$$\begin{aligned} \xi_t &= \xi_{t-1} \text{ if } \xi_{t-1} > \ell^c(y_t, z^t), \\ \xi_t &= \ell^c(y_t, z^t) \text{ otherwise.} \end{aligned}$$

This imputes limited memory to the allocations: a household's individual history y^{t-1} is erased whenever it switches to a state with binding constraints. This is the amnesia property, present in many endogenously incomplete markets models (Ljungqvist and Sargent (2004)).

Risk-Sharing Rule There is a mapping from the multipliers at s^t to the equilibrium allocations of both commodities. We refer to this mapping as the risk-sharing rule. This rule flows from the optimality conditions of the dual household problem and the market clearing conditions. Henceforth, we express individual-specific variables as functions of (ℓ, s^t) rather than (θ_0, s^t). We conjecture a linear risk sharing rule: the consumption share and the housing services share is a function of the household's own consumption weight and an aggregate sum of these weights:

$$c_t(\ell, s^t) = \frac{\xi_t(\ell, s^t)^{\frac{1}{\gamma}}}{\xi_t^a(z^t)} c_t^a(z^t) \text{ and } h_t(\ell, s^t) = \frac{\xi_t(\ell, s^t)^{\frac{1}{\gamma}}}{\xi_t^a(z^t)} h_t^a(z^t). \tag{6}$$

Indeed, this rule satisfies the first order condition for non-durable and housing services consumption and the market clearing conditions.

When a household switches to a state with a binding constraint, its consumption share increases. Everywhere else, its consumption share is drifting downwards at the rate $\Delta \log \xi_{t+1}^a$. Shocks to $\xi_t^a(z^t)$ reflect aggregate shocks to the wealth distribution. Because they reflect an inability to insure against income shocks they can be interpreted as liquidity shocks.

The perfect commitment environment is a benchmark for understanding this risk sharing rule. Because households are never constrained, the individual weight stays constant and is equal to the initial consumption weight: $\xi_t(s^t) = \xi_0(s_0) = \ell$. The aggregate weight process reflects the initial wealth distribution and is constant: $\xi^a(z_0) = \int \ell(y_0, z_0)^{\frac{1}{\gamma}} d\Phi_0(z_0)$. Consumption shares are constant; consumption levels only move with aggregate consumption. There is full insurance.

Rental Prices In equilibrium, all households equate the ratio of marginal utilities for these commodities:

$$\rho_t(z^t) = \psi \left(\frac{h_t(\theta_0, s^t)}{c_t(\theta_0, s^t)} \right)^{\frac{-1}{\varepsilon}} = \psi \left(\frac{h_t^a(z^t)}{c_t^a(z^t)} \right)^{\frac{-1}{\varepsilon}}.$$

The price of rental services is a function of the aggregate history z^t only. As a result, all households have the same equilibrium expenditure ratio $r_t(\ell, s^t) = r_t(z^t) = \frac{c_t^a}{\rho_t h_t^a}$. The price of a claim to the aggregate housing dividend is $\Pi_{z^t}[\{\rho h^a\}]$.

Stochastic Discount Factor For pricing purposes there is a stand-in consumer whose preferences are defined over "twisted" aggregate non-durable and housing services consumption processes:

$$U^w(\{c^a\}, \{h^a\}) = U\left(\left\{\frac{c^a}{\xi_t^a}\right\}, \left\{\frac{h^a}{\xi_t^a}\right\}\right)$$

The risk sharing rules (16) determines a household's intertemporal marginal rate of substitution (IMRS). In each state, the payoffs are priced by the household with the highest IMRS. This maximum is attained for the unconstrained households. If not, there would be an arbitrage opportunity. The implied stochastic discount factor (SDF) is

$$m_{t+1} = m_{t+1}^a \left(\frac{\xi_{t+1}^a}{\xi_t^a}\right)^{\gamma}, \tag{7}$$

The SDF consists of two parts. First, without the collateral constraints, ours is a representative agent economy. If utility is non-separable, the housing market introduces a novel risk factor: shocks to the non-housing expenditure share.

$$m_{t+1}^a = \delta \left(\frac{c_{t+1}^a}{c_t^a}\right)^{-\gamma} \left(\frac{\alpha_{t+1}^a}{\alpha_t^a}\right)^{\frac{\varepsilon\gamma-1}{\varepsilon-1}}.$$

where α^a is the aggregate non-durable expenditure share: $\alpha_t^a = \frac{c_t^a}{c_t^a + \rho_t h_t^a} = \frac{r_t}{1+r_t}$. This is the IMRS of the representative agent economy with non-separable preferences who consumes the aggregate non-durable and housing services endowment. When preferences are separable ($\varepsilon = \gamma^{-1}$), m^a is the SDF of the Lucas (1978) economy.

The second part of the SDF is the growth rate of the aggregate consumption weight process ξ_{t+1}^a. It reflects the risk of binding solvency constraints. When many households are severely constrained in state z^{t+1}, that state's price increases, because the unconstrained households experience high marginal utility growth: $\xi^a(z^{t+1}) > \xi^a(z^t)$. When nobody is constrained, the aggregate consumption weight process stays constant ($\xi^a(z^{t+1}) = \xi^a(z^t)$,) and the Breeden-Lucas stochastic discount factor re-emerges. The risk of binding solvency constraints endogenously creates *heteroscedasticity* in the SDF.

The heteroscedasticity of the SDF, and with it the time-variation in the Sharpe ratio, comes from two mechanisms. The first is the wealth distribution dynamics induced by the solvency constraints. A larger fraction of agents draws higher labor income shares $\widehat{\eta}(y, z)$ when aggregate consumption growth is low. As a result of the persistence of labor income shocks, cutoff levels $\widetilde{\omega}_t(\omega, s^t)$ are higher. This increases the size of the aggregate weight shock $\Delta \log \xi^a_{t+1}$ and the SDF in low aggregate consumption growth states.[4] The second source of heteroscedasticity relies on endogenous movements in the collateral ratio; it is this paper's contribution. Movements in the expenditure ratio and endogenous movements in the stochastic discount factor drive movements in the housing collateral ratio. When collateral is scarce, households' solvency constraints bind more frequently (see section 1.3).

No arbitrage implies that the return on a security j, R^j_{t+1}, satisfies $E_t[m_{t+1}R^j_{t+1}] = 1$. We think of the return on the market as a levered claim to the aggregate non-durable endowment.

1.3. Collateral Supply

The tightness of the constraints depends on the ratio of aggregate housing wealth to total aggregate wealth. To see this, we aggregate the solvency constraints across households and define the housing collateral ratio $my\left(z^t\right)$:

$$my(z^t) = \frac{\Pi_{z^t}\left[\{h^a \rho\}\right]}{\Pi_{z^t}\left[\{c^a + h^a \rho\}\right]} = \frac{\Pi_{z^t}\left[\left\{\frac{c^a}{r}\right\}\right]}{\Pi_{z^t}\left[\left\{c^a\left(1+\frac{1}{r}\right)\right\}\right]}. \tag{8}$$

Persistent variation in the non-durable expenditure ratio r gives rise to persistent variation in my and in the amount of risk sharing that can be sustained. So, the housing collateral ratio indexes the risk-sharing capacity in the economy. We make this more formal in the next propositions.

If the total consumption claim is priced high enough, then perfect risk sharing can be sustained. Denote the price of a claim under perfect risk-sharing by $\Pi^*[\{\cdot\}]$.

Proposition 1. *Perfect risk sharing can be sustained if and only if*

$$\Pi^*_z\left[\left\{c^a\left(1+\frac{1}{r}\right)\right\}\right] \geq \Pi^*_{z,y}\left[\{\eta(y,z)\}\right] \text{ for all } (y,z,r)$$

Each household can consume the average endowment without violating its solvency constraint.

The following proposition states that an economy with more housing collateral (lower r) has lower cutoff weights, allowing for more consumption smoothing. In the limit perfect risk-sharing obtains. Conversely, a decrease in the supply of collateral brings the cutoff rules closer to their

[4]Constantinides and Duffie (1996) build a negative correlation between the dispersion of consumption growth across households and aggregate stock returns in their model to generate large risk premia, drawing on earlier work by Mankiw (1986). The model of Lustig (2003) is a different version of this.

upper bound: the labor income shares. In the limit, as the collateral disappears altogether, the households revert to autarky. The following proposition makes this point more formally.

Proposition 2. *Assume utility is separable. Consider 2 economies with* $r^1_\tau(z^\tau) < r^2_\tau(z^\tau)$ *for all* $z^\tau \geq z^t$. *Then the cutoff rules satisfy* $\ell^{1,c}(y_t, z^t) \leq \ell^{2,c}(y_t, z^t)$. *As* $r_\tau(z^\tau) \to \infty$ *for all* $z^\tau \geq z^t$, $\ell^c(y_t, z^t) \to \hat{\eta}(y_t, z^t)$. *Conversely, as* $r_\tau(z^\tau) \to 0$ *for all* $z^\tau \geq z^t$, $\ell^c(y_t, z^t) \to 0$

Perturbations of the r process also change the equilibrium aggregate weight process. An economy with a uniformly higher r process and less collateral as a result has higher liquidity shocks and lower interest rates on average.

Corollary 1. *Consider 2 economies with* $r^1_t(z^t) < r^2_t(z^t)$ *for all* z^t. *Fix the distribution of initial multipliers across economies:* $\Phi^1_0(z_0) = \Phi^2_0(z_0)$. *Then* $\left\{\xi^{a,1}_t(z^t)\right\} \leq \left\{\xi^{a,2}_t(z^t)\right\}$ *and* $r^{f,1} \geq r^{f,2}$.

The proposition and corollary illustrate the mechanism that underlies the time-variation in the equilibrium market price of aggregate risk by comparing two economies with different collateral processes $\{r\}$. In sections 2 and 3, we calibrate the evolution of the expenditure ratio r, simulate the model and investigate the *equilibrium* changes in the conditional moments of the aggregate weight process.

1.4. Two-agent Example

To provide more intuition for the collateral mechanism, we briefly consider the case of two agents, two aggregate states $z = (re, ex)$ and two idiosyncratic income states $y = (hi, lo)$. The vector (ω, r), where ω is the consumption share of agent 1, offers a complete description of the state space.

Suppose r is fixed. The policy rule for next period's consumption share of agent 1 is characterized by a lower and an upper bound:

$$\begin{aligned} \omega'(\omega, y, r) &= \omega \text{ if } \overline{\omega}(y, r) > \omega > \underline{\omega}(y, r) \\ &= \underline{\omega}(y, r) \text{ if } \omega < \underline{\omega}(y, r) \\ &= \overline{\omega}(y, r) \text{ if } \omega > \overline{\omega}(y, r) \end{aligned}$$

This agent's consumption share is constant as long as he stays in the same state. If she switches to the high labor income state, then his consumption share jumps up, while the other agent's consumption share jumps down, as in Alvarez and Jermann (2000).[5]

The Collateral Effect The size of the jump depends on the level of the non-housing expenditure ratio r. The cutoff value $\underline{\omega}(s,r)$ satisfies the solvency constraint for agent 1:

$$\Pi_{(s,r)}\left[\left\{\widehat{\eta}^1 c^a\right\}\right] = \Pi_{(s,r)}\left[\left\{c_t^a\left(1+\frac{1}{r}\right)\underline{\omega}\right\}\right] = \left(1+\frac{1}{r}\right)\Pi_{(s,r)}\left[\left\{c_t^a\underline{\omega}\right\}\right]$$

From the solvency constraint for agent 2, we find $\overline{\omega}(s,r)$:

$$\Pi_{(s,r)}\left[\left\{\left(1-\widehat{\eta}^1\right)c_t^a\right\}\right] = \left(1+\frac{1}{r}\right)\Pi_{(s,r)}\left[\left\{c_t^a\left(1-\omega_t\right)\right\}\right]$$

From the definition of the cutoff weights and keeping the pricing functional Π constant, the above implies that

$$\frac{\partial\underline{\omega}(s,r)}{\partial r} > 0 \; and \; \frac{\partial\overline{\omega}(s,r)}{\partial r} < 0.$$

The cutoff weights become tighter as the non-durable expenditure ratio r increases. Consequently, there is less risk-sharing as the housing collateral ratio decreases.

Perfect risk sharing is feasible if solvency constraints are satisfied when each agent consumes half of the aggregate endowment in all states:

$$\Pi^*_{(s,r)}\left[\left\{\widehat{\eta}^1 c^a\right\}\right] \leq \frac{1}{2}\left(1+\frac{1}{r}\right)\Pi^*_{(s,r)}\left[\left\{c^a\right\}\right] \text{ for all } s$$

In the limit, as r decreases, the cutoff consumption shares no longer bind and perfect risk sharing is feasible.

On the other hand as r increases, then housing contributes no collateral in the limit. Only autarchy is feasible: $\underline{\omega}(hi,z,r) = \widehat{\eta}^1_{hi,z}$, $\overline{\omega}(lo,z,r) = \widehat{\eta}^1_{lo,z}$. To see why, note that $\Pi^*_{(s,r)}\left[\left(\widehat{\eta}^1\right)c^a\right] + \Pi^*_{(s,r)}\left[\left(\widehat{\eta}^2\right)c^a\right] = \Pi^*_{(s,r)}\left[c^a\right]$.

All previously mentioned insights still hold when r is stochastic, provided it is persistent enough. The stochastic discount factor in this 2-agent economy is

$$m_{t+1} = m^a_{t+1}\left(\min\frac{\omega(s_{t+1},r_{t+1})}{\omega(s_t,r_t)}\right)^{-\gamma}.$$

[5]The two-agent risk-sharing rule maps into the continuous-agent rule with $\omega = \frac{\xi_1^{1/\gamma}}{\xi_1^{1/\gamma}+\xi_2^{1/\gamma}}$.

The size of the aggregate weight shock in the second part of the SDF depends on r. I.e. r governs the size of the jump in consumption shares when an agent switched from the low to the high income state.

When r is high enough, there is no collateral and

$$\min \frac{\omega(s_{t+1}, r_{t+1})}{\omega(s_t, r_t)} = \frac{\hat{\eta}^1_{lo,z'}}{\hat{\eta}^1_{hi,z}} \; or \; \frac{1-\hat{\eta}^1_{hi,z'}}{1-\hat{\eta}^1_{lo,z}},$$

while, if r is low enough, consumption shares are constant:

$$\min \frac{\omega(s_{t+1}, r_{t+1})}{\omega(s_t, r_t)} = 1$$

Comparing one economy with low r to another with high r, low r means abundant collateral and small liquidity shocks, while a high current expenditure ratio r means low collateral and large liquidity shocks. The volatility of the SDF will be higher in the high r economy, because larger jumps occur when the agents switch between *hi* and *lo*. The conditional volatility of the SDF is essentially constant when r is low.

The average risk premium is large because we set $\widehat{\eta}^1_{hi,re} > \widehat{\eta}^1_{hi,ex}$ and $\widehat{\eta}^1_{lo,re} < \widehat{\eta}^1_{lo,ex}$. This increase in the dispersion of idiosyncratic shocks when aggregate consumption growth is low delivers larger liquidity shocks when aggregate consumption growth is low.

We exploit the first mechanism to impute the right low frequency variation to the market price of risk. The high frequency variation comes from the second mechanism.

1.5. Sequential Trading

This section describes a sequential trading arrangement. It illustrates the nature of the collateral constraints in a more intuitive way. We then argue that the equilibrium with sequential trading can be mapped into the time zero trading equilibrium, defined earlier.

The financial markets are complete. Households trade a complete set of contingent claims a in forward markets. $a_t(\ell, s^t, s')$ is a promise made by agent (ℓ, s_0) to deliver one of unit the consumption good if event s' is realized in the next period. These claims trade at a price $q_t(s^t, s')$. All prices are quoted in units of the non-durable consumption good. ρ_t denotes the rental price; $p^h_t(z^t)$ denotes the (asset) price of the housing stock.

Household Problem The household problem is to maximize utility over non-durable consumption and rental services (1) subject to the following collateral constraints and wealth constraints. At the start of the period, the household purchases goods in the spot market $c_t(\ell, s^t)$, rental services in the rental market $h_t^r(\ell, s^t)$, contingent claims in the financial market and shares in the housing stock $h_{t+1}^o(\ell, s^t)$ subject to a wealth constraint:

$$c_t(\ell, s^t) + \rho_t(z^t) h_t^r(\ell, s^t) + \sum_{s'} q_t(s^t, s') a_t(\ell, s^t, s') + p_t^h(s^t) h_{t+1}^o(\ell, s^t) \leq W_t(\ell, s^t).$$

Next period wealth is:

$$W_{t+1}(\ell, s^t, s') = \eta_{t+1}(s^t, s') + a_t(\ell, s^t, s') + h_{t+1}^o(\ell, s^t) \left[p_{t+1}^h(s^t, s') + \rho_{t+1}(s^t, s') \right].$$

All of a household's state-contingent promises are backed by the cum-dividend value of its housing h_{t+1}^o, owned at the end of period t. In each node s^t, households face a separate *collateral constraint* for each event s':

$$-a_t(\ell, s^t, s') \leq h_{t+1}^o(\ell, s^t) \left[p_{t+1}^h(s^{t+1}) + \rho_{t+1}(s^{t+1}) \right], \text{ for all } s^t, s'. \tag{9}$$

The collateral constraints prevent bankruptcy because households are not allowed to borrow more in a given state than the cum-dividend value of the housing stock in that state.[6]

Competitive Equilibrium

Definition. *Given a distribution over initial wealth and endowments Θ_0, a competitive equilibrium is a feasible allocation $\left\{c(\ell, s^t), h^r(\ell, s^t), a_{t-1}(\ell, s^t), h^o(\ell, s^t)\right\}$ and a price vector $\left\{q_{t-1}, p^h, \rho\right\}$ such that (1) for given prices and initial wealth, the allocation solves each household's maximization problem and (2) the markets for the consumption good, the housing services, the contingent claims and housing shares clear.*

The equilibria in the economy with sequential trading are equivalent to the time zero Kehoe and Levine (1993) equilibria, if the equilibrium interest rates are high enough.

Proposition 3. *If the interest rates are high enough, the sequential equilibrium allocations can be supported as a Kehoe-Levine equilibrium.*

[6] Off the equilibrium path, default results is the loss of housing collateral. This mimics foreclosure under Chapter 7 of US bankruptcy legislation.

The proof is in appendix A.3 and follows Alvarez and Jermann (2000).

To show the equivalence, we define the market state price $p_t(z^t)$ as the product of the Arrow prices for the events along a path z^t:

$$p_t(z^t) = q_{t-1}(z^{t-1}, z')q_{t-2}(z^{t-1}) \ldots q_0(z^1),$$

where $p_t(z^t)$ is the price at time 0 of a unit of consumption to be delivered at node z^t.

By iterating forward on the collateral constraints in (9), substituting for the time 0 budget constraint, and imposing a no-arbitrage condition on $\{p^h\}$, the sequence of collateral constraints can be restated as a non-negativity constraint on net wealth in every history:

$$\Pi_{s^t}\left[\{c(\ell, s_0) + \rho h(\ell, s_0)\}\right] \geq \Pi_{s^t}\left[\{\eta\}\right], \ \forall s^t, t \geq 0. \tag{10}$$

1.6. Value Premium

Growth stocks (value stocks) can be thought of as a basket of consumption strips that is heavily weighted towards longer (shorter) maturities (Dechow, Sloan, and Soliman (2002) and Lettau and Wachter (2004)). Consumption strips are claims to period $t+k$ aggregate consumption (c_{t+k}), where k is the horizon in years.

Formally, the multiplicative (one year) equity premium on a non-levered claim to the stream of aggregate consumption $\{c_k\}_{k=1}^{\infty}$, $E_0 R^e_{0,1}[\{c_k\}]$, can be written as a weighted sum of expected excess returns on consumption strips:

$$\begin{aligned} 1+\nu_0 = 1 + E_0[R^e_{0,1}[\{c_k\}]] &= E_0 M_1 E_0\left(\frac{\sum_{k=1}^{\infty} E_1 M_k c_k}{\sum_{k=1}^{\infty} E_0 M_k c_k}\right) = \sum_{k=1}^{\infty} \frac{E_0 M_k c_k}{\sum_{k=1}^{\infty} E_0 M_k c_k} \frac{\frac{E_1 M_k c_k}{E_0 M_k c_k}}{\frac{1}{E_0 M_1}} \\ &= \sum_{k=1}^{\infty} \omega_k \frac{E_0 R_{0,1}\left[c_k\right]}{R_{0,1}\left[1\right]} = \sum_{k=1}^{\infty} \omega_k E_0 R^e_{0,1}\left[c_k\right], \end{aligned}$$

with weights

$$\omega_k = \frac{E_0 M_k c_k}{\sum_{k=1}^{\infty} E_0 M_k c_k}.$$

The second term in the sum is the expected return on a period k consumption strip $E_0 R_{0,1}\left[c_k\right]$ in excess of the risk-free rate $R_{0,1}\left[1\right]$. The weights can be interpreted as the value of the period k consumption strip relative to the total value of all consumption strips. M_k is the pricing kernel in period k. It is linked to the stochastic discount factor m by $M_k = m_1 \times \cdots \times m_k$.

Value stocks can then be modelled as a claim to a weighted stream of aggregate consumption $\{f^v(k)c_k\}_{k=1}^{\infty}$, where the function $f(\cdot)$ puts more weight on the consumption realizations in the near future. For example, $f^v(k) = Ce^{ak}$, where a is a negative number and C is a normalization constant, $C = \frac{\sum_{k=1}^{\infty} c_k}{\sum_{k=1}^{\infty} e^{ak} c_k}$. Likewise, growth stocks can be thought of as a claim to a weighted stream of aggregate consumption $\{f^g(k)c_k\}_{k=1}^{\infty}$, where the function $f(\cdot)$ puts more weight on the consumption realizations in the far future. For example, $f^g(k) = Ce^{ak}$, where a is a positive number. The multiplicative equity premium on such a claim is

$$1 + \tilde{\nu}_0 = \sum_{k=1}^{\infty} \tilde{\omega}_k \frac{E_0 R_{0,1}[c_k]}{R_{0,1}[1]},$$

with modified weights

$$\tilde{\omega}_k = \frac{f(k) E_0 M_k c_k}{\sum_{l=1}^{\infty} f(l) E_0 M_l c_l}.$$

The following proposition shows that the properties of the pricing kernel determine the sign of the value spread. In particular, if the pricing kernel has no permanent component, then the model generates a *growth premium*.

Proposition 4. *If* $\gamma > 1$ *and* $f(k) = Ce^{ak}, a > 0$ *then*

$$\lim_{k \to \infty} \frac{E_{t+1} M_{t+k}}{E_t M_{t+k}} = 1 \Rightarrow \lim_{a \to \infty} 1 + \widetilde{\nu}_0 = \lim_{k \to \infty} R^c_{t+1,k} \geq 1 + \nu_0,$$

for any other sequence of weights $\{\omega_k\}$

Proof: see appendix. The proof relies on insights in Alvarez and Jermann (2001).

This implies that the highest equity premium is the one on the farthest out consumption strip. In the absence of a permanent component in the pricing kernel, there is a growth premium. The pricing kernel in our model contains a permanent component through the multiplicative component stemming from the risk of binding solvency constraints: The aggregate weight shock $(\xi^a)^\gamma$ is a non-decreasing stochastic process. This is a necessary condition for generating a value premium.

In the representative agent economy, the equity premia on consumption strips do not change with the horizon. This is easy to show for additive preferences that are separable in both commodities and aggregate endowment growth that is i.i.d with mean $\bar{\lambda}$. The pricing kernel is simply a function of the aggregate consumption growth rate between period 1 and period k: $M_k = \lambda_k^{-\gamma} \lambda_{k-1}^{\gamma} \cdots \lambda_1^{-\gamma}$. Because the aggregate endowment grows every period at the rate λ,

$M_1 c_1 = \lambda_1^{-\gamma} \lambda_1 c_0$. For the period k strip, $M_k c_k = \lambda_k^{1-\gamma} \lambda_{k-1}^{1-\gamma} \cdots \lambda_1^{1-\gamma} c_0$. Hence, the expected return on a period k strip is:

$$E_0 R_{0,1}[c_k] = \frac{E_1(M_k c_k)}{E_0(M_k c_k)} = \frac{E_1\left(\lambda_k^{1-\gamma} \lambda_{k-1}^{1-\gamma} \cdots \lambda_1^{1-\gamma} c_0\right)}{E_0\left(\lambda_k^{1-\gamma} \lambda_{k-1}^{1-\gamma} \cdots \lambda_1^{1-\gamma} c_0\right)} = \left(\frac{\lambda_1}{\bar{\lambda}}\right)^{1-\gamma}$$

The expression does not depend on the horizon k. This shows that equity premia are constant across strips of different horizons in the representative agent economy. The term structure of consumption strips is flat.[7]

2. Computation

To solve the model numerically, we rely on an approximation of g, the growth rate of the aggregate weight process using a truncated history of aggregate shocks. This is discussed in section 2.1. In 2.2, we fully calibrate the model. We simulate the model and discuss the results in section 3.

2.1. Approximating Stationary Equilibria

In general, the aggregate weight process depends on the entire history of shocks z^∞. To avoid the curse of dimensionality, we truncate aggregate histories (Lustig (2003)). Households do not keep track of the entire aggregate history, only the last k lags: $z_t^k = (z_t, z_{t-1}, \cdots, z_{t-k})$ and the current expenditure ratio $r_t(z^t)$. The current expenditure ratio r_t contains additional information not present in the truncated history z^k, namely r_{t-k}.

For a household starting the period with weight $\xi \in L$, the policy function $l(y', z'; \xi, r, z^k) : L \times R \times Z^k \to \mathbb{R}$ produces the new individual weight in state (y', z'). There is one policy function $l(\cdot)$ for each pair $(y', z') \in Y \times Z$. The policy function $g^*(z'; r, z^k) : R \times Z^k \to \mathbb{R}$ forecasts the aggregate weight shock when moving to state z' after history (z^k, r).

Definition. *A stationary stochastic equilibrium is a joint distribution over individual weights, individual endowments, current housing - endowment ratio, truncated aggregate histories, a time invariant distribution $\Phi^*_{(r,z^k)}(\xi, y)$, and updating rules $l(\cdot)$ and $g^*(\cdot)$. For each $\left(z^{k\prime}, z^k\right)$ with $z^{k\prime} =$*

[7] A similar result obtains if preferences are non-separable and aggregate expenditure share growth is i.i.d., even when aggregate expenditure share growth is correlated with aggregate consumption growth.

$\left(z', z^k\right)$

$$\Phi^*_{(r,z^{k\prime})} = \sum_{z^k} \pi(z^{k\prime}|z^k) \int Q\left(\xi, y, r, z^k\right) \Phi^*_{(r,z^k)}(d\xi \times dy)$$

where $Q\left(\xi, y, r, z^k\right)$ *is the transition function induced by the policy functions.*

The forecast of the aggregate weight shock satisfies

$$g^*(z'; r, z^k) = \sum_{y' \in Y} \int l(y', z'; \xi, r, z^k)^{\frac{1}{\gamma}} \Phi^*_{r,z^k}(d\xi \times dy) \frac{\pi(y', z'|y, z)}{\pi(z'|z)}, \tag{11}$$

for each z'. Prices are determined using the stochastic discount factor in equation (7), and using $g^*(\cdot)$ as an approximation to the actual $g(\cdot)$.

For any given realization $\{z\}$, the actual aggregate weight shock $g(\cdot)$ differs from the forecast $g^*(\cdot)$ because the distribution over individual weights and endowments $\Phi^*(\cdot)$ differs from the actual distribution $\Phi(\cdot)$, which depends on z^∞. The definition of stationary equilibrium implies that, *on average across aggregate histories*, $\Phi^*(\cdot) = \Phi(\cdot)$, and markets clear. That is, for every aggregate state z', the allocation error

$$c^a(z'; r, z^k) - c^a(z'; z^\infty) = \frac{g^*(z'; r, z^k) - g(z'; r, z^\infty)}{g^*(z'; r, z^k)} \tag{12}$$

is *on average* zero.[8] As k increases, the approximation error decreases because market clearing holds on average in long histories.

Algorithm We compute the approximating equilibrium as follows. The aggregate weight shock process is initialized at the full insurance value ($g^* = 1$) and the corresponding stochastic discount factor is computed. The cutoff rule for the individual weight shocks ensure that the solvency constraints holding with equality. Then the economy is simulated by drawing $\{z_t\}_{t=1}^T$ for $T = 10,000$ and $\{y_t\}_{t=1}^T$ for a cross-section of $5,000$ households. For each truncated history, we compute the sample mean of the aggregate weight shock $\{g_t^*(z', r, z^k)\}_{t=1}^T$ and the resulting stochastic discount factor $\{m_t^*(z', r, z^k)\}_{t=1}^T$. A new cut-off rule is computed with these new forecasts. These two steps are iterated on until convergence.

Throughout we use $k = 5$ and report percentage allocation errors as a measure of closeness to the actual equilibrium. The average error in equation 12 in a simulation of 10,000 periods is 0.0011

[8] There is an *exact* aggregation result if aggregate uncertainty is i.i.d., with k=0. See Lustig (2003) for a proof in a model without housing.

with standard deviation .0035. The largest error in absolute value is 0.0282.

2.2. Calibration

In this section we fully calibrate the model and report the approximation errors. Table 2 summarizes our choices for benchmark and alternative parameter values.

Income Process The first driving force in the model is the Markov process for the non-durable endowment. It contains an aggregate and an idiosyncratic component.

The aggregate endowment growth process is taken from Mehra and Prescott (1985) and replicates the several moments of aggregate consumption growth in the 1871-1975 data. The growth rate of the aggregate endowment, λ, follows an autoregressive process:

$$\lambda_t(z_t) = \rho_\lambda \lambda_{t-1}(z_{t-1}) + \varepsilon_t,$$

with $\rho_\lambda = -.14, E(\lambda) = .0183$ and $\sigma(\lambda) = .0357$. We discretize the AR(1) process with two aggregate growth states $z = (z^{exp}, z^{rec}) = [1.04, .96]$ and an aggregate state transition matrix $[.83, .17; .69, .31]$. The implied ratio of the probability of a high aggregate endowment growth state to the probability of a low aggregate endowment growth state is 2.65. The unconditional probability of a low endowment growth state is 27.4 percent.

The calibration of a heteroscedastic labor income process is taken from Storesletten, Telmer, and Yaron (2004). They conclude that the volatility of idiosyncratic labor income shocks in the US more than doubles in recessions. Log labor income shares follow an AR(1) with autocorrelation of .92 and a conditional variance of .181 in low and .0467 in high aggregate endowment growth states. Again the AR(1) process is discretized into a two-state Markov chain. The resulting individual income states are $\left(\eta^{hi}, \eta^{lo}\right) = [.6578, .3422]$ in the high and $[.7952, .2048]$ in the low aggregate endowment growth state.[9] We refer to the counter-cyclical labor income share dispersion as the Constantinides and Duffie (1996) effect.

Expenditure Ratio Following Piazzesi, Schneider, and Tuzel (2004), we specify an autoregressive process for the expenditure ratio r. The aggregate expenditure depends on the aggregate

[9] The only difference with the Storesletten, Telmer, and Yaron (2004) calibration is that recessions are shorter in our calibration. In their paper the economy is in the low aggregate endowment growth state 50 percent of the time. That implies that the unconditional variance of our labor income process is lower.

endowment growth:

$$\log r_{t+1} = \bar{r} + \rho_r \log r_t + b_r \lambda_{t+1} + \sigma_r \nu_{t+1}, \tag{13}$$

where ν_{t+1} is an i.i.d. standard normal process with mean zero, orthogonal to λ_{t+1}. In our benchmark calibration we set $\rho_r = .96$, $b_r = .93$ and $\sigma_r = .03$. We discretize the process for $\log(r)$ as a five-state Markov process.

The parameter values are close to the estimates of (13) we find in US National Income and Products Accounts Data. Panel A of table 1 shows estimates for ρ_r and b_r that are consistent across samples and data sources. In periods of high aggregate consumption growth, the expenditure ratio increases.[10]

A second calibration switches off the effect of λ on $\log(r)$: $\rho_r = .96$, $b_r = 0$. Both calibrations fix $\sigma_r = .03$. We choose the constant $\bar{r}$ to match the average housing expenditure share of 19 percent in the National Income and Product Accounts data for 1929-2002.

Average Housing Collateral ratio We scale up aggregate income. This scaling allows us to simultaneously have an average expenditure share of housing services of 19 percent and an average ratio of housing wealth to total wealth 5 percent (benchmark). In the model, the ratio of the aggregate non-durable endowment e_t to the aggregate non-durable consumption c_t^a is 1. The empirical counterpart to e_t is compensation of employees. The empirical counterpart to c_t^a is consumption expenditures on non-durables and services excluding housing services. On average between 1929-2003, the ratio of the former to the latter is 1.17. We use this factor to scale up labor income η in the model. The rescaling implies an average housing collateral ratio of 5 percent. We investigate the sensitivity of our results by also considering an economy with ten percent collateral.

Preference Parameters In the benchmark calibration we use additive utility with $\delta = .95$, $\gamma = \{5, 8\}$, $\varepsilon = .05$. We fix $\psi = 1$ throughout.[11] We also compute the model for $\gamma \in [2, 10]$ and $\varepsilon \in [.05, .75]$. A choice for the parameter ε implies a choice for the volatility of rental prices:

$$\sigma(\Delta \log \rho_{t+1}) = \left| \frac{1}{\varepsilon - 1} \right| \sigma(\Delta \log r_{t+1})$$

[10] Alternatively, we could have calibrated a persistent process for the rental price $\log(\rho_t)$. Panel B shows that rental prices increase in response to a positive aggregate consumption growth in the post-war sample.

[11] Note that $\frac{-cu_{cc}}{u_c} = \alpha_t \gamma + (1 - \alpha_t)\frac{1}{\varepsilon}$. It is a linear combination of γ and ε with weights depending on the non-durable expenditure share $\alpha_t = \frac{c_t}{\rho_t h_t + c_t}$. In all calibrations $\alpha_t = .81$ on average.

In NIPA data (1930-2002), the left hand side is .046 and the right-hand side is .041. The implied ε is .098. A choice for ε too close to one implies excessive rental price volatility. We take $\varepsilon = .05$ as our benchmark and explore parameter values $\varepsilon \leq .75$.

Our benchmark parametrization, as well as the other parameters we consider for sensitivity analysis are summarized in table 2.

Market Return We assume that financial assets are in zero net supply.[12] Because of complete markets we price them as redundant securities. The define the market return as the return on a levered claim to the aggregate consumption process $\{c_t^a\}$. In the data, dividends are more volatile than aggregate consumption. For the period 1930-2001, the leverage parameter κ in $\Delta \log d_{t+1} = \kappa \Delta \log c_{t+1}^a$, is 4.4. We denote the return on a levered claim to aggregate consumption growth R^l and choose leverage parameter $\kappa = 3$. We also price a non-levered claim on the aggregate consumption stream. We denote the corresponding return R^c.

3. Results

Our empirical targets are twofold: (1) to quantitatively assess the variation in *conditional* asset pricing moments, conditional on the housing collateral ratio and compare it to the data (section 3.2), and (2) to generate a return spread between value and growth portfolio returns of the magnitude observed in the data (section 3.3). We also match the *unconditional* moments for the equity premium, its unconditional volatility and Sharpe ratio, and the risk-free rate (section 3.1). Throughout, we contrast the results with those of a representative agent economy with non-separable preferences.

3.1. Unconditional Asset Pricing Moments

Collateral Model Table 3 summarizes the *unconditional* first and second moments of asset returns for the collateral model with non-separable preferences. Table 4 reports unconditional asset pricing moments for the representative agent model with non-separable preferences.

Because consumption growth is less volatile in the data than dividend growth, the relevant comparison of the excess stock market return in the data is with a leveraged consumption claim

[12] Allowing for financial assets in positive net supply will not qualitatively affect the dynamics of our model. It will merely alter the ratio of collateralizeable wealth to total wealth. In the data, mortgages and home equity lines of credit represent 75 percent of all collateral assets (household sector balance sheet, Flow of Funds data). For these two reasons, the omission of financial wealth as collateral does not seem critical.

in the model. The benchmark model with five percent collateral is able to generate a high and volatile levered equity risk premium. The calibration with $\gamma = 5$ and $\varepsilon = .05$ in panel 1 of table 3 generates a 5.1 percent equity premium. The standard deviation is 21.3 percent. In the data (panel 6), the excess return on the market portfolio is 7.5 percent with a volatility of 19.8 percent. To understand the effect of the leverage, we also price a non-levered consumption claim. Its equity premium is 3.0 percent for $\gamma = 5$ and 7.1 percent for $\gamma = 8$. The Sharpe ratio in the model with $\gamma = 8$ is 0.42, equal to the Sharpe ratio observed for 1927-2002.

The model with $\gamma = 8$ ($\gamma = 5$) generates an average risk free rate of 2.9 (5.7) percent, close to the 3.9 percent in the data. For a higher γ, the aggregate weight shocks are bigger on average. This increases the conditional expectation of the stochastic discount factor and pushes down the risk-free rate. The one failure of the model with additive utility is the unconditional volatility of the risk-free rate. It is 7.3 percent in the economy with $\gamma = 5$, 12.5 percent in the economy with $\gamma = 8$, but only 3.2 percent in the data.[13]

Panel 2 of table 3 explores the effect of varying the value of the intratemporal elasticity parameter ε. The effect of a higher intratemporal elasticity of substitution is to increase the equity premium and the market price of risk and to lower the risk-free rate. In the case of $\gamma = 8$, the equity premium for $\varepsilon = .75$ is 4.5 percent higher than for $\varepsilon = .05$. This is largely due to a decline in the risk-free rate of 4.4 percent. Apart from making the collateral ratio more volatile, changing ε mainly changes the representative agent component m^a of the stochastic discount factor. Because the average amount of collateral is held fixed at 5 percent, this is largely a 'representative agent economy' effect.

Increasing the coefficient of relative risk aversion γ from 2 to 10 in panel 3 increases the equity premium on a levered (non-levered) consumption claim from 1.3 to 15.1 (.5 to 11) percent. The unconditional Sharpe ratio increases from .08 to 0.53. The increase in γ *decreases* the risk-free rate from 7 to 0 percent. The first reason for this risk-free rate effect is that households cannot borrow as much as they would like because of binding collateral constraints. Second, the risk-free asset provides insurance against binding constraints, and this lowers the risk-free rate even further. This is a precautionary savings effect coming from the constraints. If households are more risk averse, they want to insure better against the risk of binding constraints. The main effect of a higher

[13]Risk-free rates were more volatile prior to 1927. The unconditional standard deviation of the real risk-free rate is 6.5 percent for the 1889-1979 period (data from Shiller's web site). The model with recursive preferences, described in appendix A.4, generates a volatility of the risk-free rate of 6.5 percent. It still generates a large equity premium (6 percent), a low risk-free rate, and volatile stock market returns.

coefficient of relative risk aversion is to amplify the collateral effect, coming through the second part of the stochastic discount factor, g_{t+1}^{γ}.

The economy with ten percent collateral is closer to the representative agent economy because the collateral constraints are not as tight. The expected excess return on a levered consumption claim is still high. For ($\gamma = 8, \varepsilon = .05$), the equity premium is 7.8 percent (panel 4). With 10 percent collateral, the risk-free rate is higher on average, but less volatile.

Lastly, panel 5 shows that the results are not very sensitive to a change in the expenditure share process. This table displays the results for an AR(1) specification for $\log r_t$ without consumption growth term on the right-hand side ($b_r = 0$). Under this specification, the housing collateral ratio is less volatile. The equity premium and the risk-free rate are one percent lower on average. The volatility of $R^{l,e}$ is 5 percent lower than in the benchmark economy. Changes in ε have a smaller effect on the unconditional asset pricing moments.

Representative Agent Model We contrast the results from the collateral model with the unconditional asset pricing moments in the representative agent economy. Preferences are non-separable between non-durable and housing services consumption, but the collateral affect is shut down. The equity premium is compensation for aggregate consumption growth risk (as in Lucas (1978)) and aggregate composition risk (as in Piazzesi, Schneider, and Tuzel (2004)).

Table 4 shows that the equity premium in the representative agent economy is substantially smaller than in the collateral model. The levered consumption claim has an expected excess return of 2.4 percent for ($\gamma = 5, \varepsilon = .05$) and 4.5 percent for ($\gamma = 8, \varepsilon = .05$). This is less than half as big as for the collateral model with five percent collateral. The equity premium on a non-levered claim is one-third the magnitude of the collateral model. In addition, the levels of the risk-free rate and the stock return are much too high in the representative agent economy. For $\gamma = 8$, the risk-free rate is 15.8 percent and the stock return is 20.3 percent on average. This compares to 4 percent and 11.4 percent in the data for 1927-2002. Moreover, the risk-free rate *increases* with γ. A more risk-averse representative agent wants to borrow increasingly against her labor income and this drives up the risk-free rate. Note that this is the opposite risk-free rate effect as in the collateral model.

An increase in the intratemporal elasticity of substitution ε from .05 to .75 increase the equity premium by 4 percent (1.7) percent in a representative agent economy with $\gamma = 8$ ($\gamma = 5$). The risk-free rate goes down by 6 (1.9) percent. The average Sharpe ratio for the representative agent economy with $\varepsilon = .75$ is .47 (.27), in line with the historical average. However, the empirically plausible equity premium, risk-free rate and Sharpe ratio for $\varepsilon = .75$ come at the expense of an implausibly high rental price growth volatility. For $\varepsilon = .75$, the unconditional standard deviation of rental price growth is 19 percent per annum. In the data, rental price growth volatility is below 5 percent. Driving ε even closer to one leads to exponentially increasing rental price growth volatility.[14] This is the reason we choose $\varepsilon = .05$ as our benchmark economy. In addition, the volatility of the risk-free rate increases sharply with ε.

Housing Market Statistics The model also has predictions for the return on housing in ownership. In the collateral model, the expected excess return on a claim to the aggregate housing dividend stream is similar to the return on the aggregate consumption stream. Panel 1 of table 5 shows that the housing equity premium is 6.8 (2.6) percent for $\gamma = 8$ ($\gamma = 5$). The standard deviation of this return is 19.2 (12.7) percent, leading to a Sharpe ratio of .35 (.20). The Sharpe ratio on a non-levered consumption claim in table 3 was .40 (.23). Using PSID data for 1968-1992, Flavin and Yamashita (2002) estimate the expected excess return and its standard deviation on home ownership to be 6.6 percent and 14.2 percent. This implies a Sharpe ratio for housing of .46. Our benchmark model with $\gamma = 8$ generates expected returns and Sharpe ratios for the housing market that are broadly consistent with the Flavin and Yamashita (2002) numbers. In the representative agent economy (panel 3 of 5), the expected excess return on home ownership is too low, the expected return too high and not sufficiently volatile to match the data.

No Conditional Heteroscedasticity in Income To gauge the relative importance of the two sources of time-variation in risk premia, we shut down the Constantinides and Duffie (1996) mechanism. The labor income share in the low idiosyncratic income state is the same whether the economy experiences low or high aggregate consumption growth. More precisely, we set the income share η equal to [.6935,.6935,.3065,.3065] instead of [.6578,.7952,.3422,.2048] in the benchmark calibration. This implies the same unconditional labor income volatility, but the labor income share process is conditionally homoscedastic. It is constant across aggregate growth states.

Table 6 shows that the housing collateral mechanism alone generates a sizeable equity premium of 7.4 percent ($\gamma = 8$, 5 percent collateral). The standard deviation of the excess return is 23.5 percent and the Sharpe ratio is 0.31. This amounts to 70 percent of the magnitudes we found for

[14]For $\varepsilon > 1$ the representative agent model generates a negative equity premium.

our benchmark model in table 3. The mean risk-free rate is slightly higher, but less volatile.

3.2. Conditional Asset Pricing Moments

The first main prediction of the model is that asset prices behave differently in episodes of high collateral and in periods with collateral scarcity. Expected excess returns are low and the risk-free rate stable in periods when my is high. However, when collateral is scarce, the equity premium and the Sharpe ratio are high. Movement in the housing collateral ratio induces substantial variation in the Sharpe ratio. In this section we do a quantitative assessment of this time-variation and show that is consistent with the data. The unconditional asset pricing moments obscure this time-variation because they average over different collateral regimes. The failure of the representative agent model goes beyond unconditional asset pricing moments; it misses the time-variation in the Sharpe ratio completely.

Shocks to the Risk Sharing Technology The shocks to the collateral ratio come from shocks to the housing endowment. Panel 1 of figure 1 shows the housing collateral ratio my together with the ratio of housing services consumption to total consumption $1 - \alpha = \frac{1}{1+r}$. It is a typical two hundred period window of a long simulation of the benchmark model. The housing collateral ratio is the closely correlated with the housing expenditure share. It is also a persistent process.

Panel 2 illustrate the collateral mechanism. It plots the cutoff consumption share, which is the consumption share at which the solvency constraint holds with equality (dotted line). This is the consumption share of a constrained household. The household's average income share is normalized to one. The consumption share jumps to the cutoff level when the household runs into a binding constraint. This happens when its income share switches from the low to the high idiosyncratic state. The graph shows that the consumption share for constrained households is bigger when collateral is scarce (my is low, full line). For example, in period 195, the consumption share is 15 percent above its mean of one, whereas in period 50, the consumption share is only 7 percent above its mean.

When collateral is scarce, the solvency constraints bind more severely. The consumption share of the constrained households takes a large jump up, while the unconstrained households' consumption share decreases precipitously. As a result, the cross-sectional standard deviation of consumption growth increases. In times of collateral scarcity, there is less risk-sharing. Panel 3 of figure 1 plots the consumption growth dispersion (dashed line, left axis) against the housing collateral ratio my

(full line, right axis).

The aggregate weight shock $g_{t+1} = \Delta \log \xi^a_{t+1}$, which we refer to as the liquidity shock, governs the rate at which the consumption share of the unconstrained agents decreases. Panel 4 plots the aggregate weight shock g^γ (dotted line, left axis) against the housing collateral ratio (full line, right axis). In times of collateral scarcity, the constraints bind more tightly and this is reflected in a large liquidity shock. For example, in period 50 or 110, the liquidity shock is close to one, whereas in period 195 it is 1.07. The stochastic discount factor is high and more volatile in such periods. When housing collateral is abundant, the aggregate weight shock is close to 1 and our model's stochastic discount factor reduces to the one in the representative agent economy. We turn to the effects on asset prices next.

Conditional Asset Pricing Moments The expected return on stocks in excess of the risk-free rate is higher in periods of collateral scarcity. Zooming in on the same 200 simulation periods of the benchmark calibration, the first panel of figure 2 displays an expected excess return on a non-levered claim to aggregate consumption (dotted line, left axis). The equity premium is below 4 percent when the housing collateral ratio is high (for example in period 110), and almost 11 percent when my is low (for example in period 190).

Likewise, the second panel of figure 2 shows that the conditional volatility of the excess return on the consumption claim (dotted line, left axis) is 10 percent when collateral is abundant (period 110) and doubles to 20 percent when collateral is scarce (period 195). Excess returns are much more stable when collateral is abundant.

The net result of the collateral mechanism is a Sharpe ratio that is higher in times of collateral scarcity. The third panel of figure 2 plots the Sharpe ratio on the stock return against the housing collateral ratio (dotted line, left axis). It is 0.3 in period 110 and almost 0.6 in period 195.

We estimate the Sharpe ratio on annual data from 1927-1992 and compare it to the variation in the Sharpe ratio generated by the model. The conditional mean return is the projection of the excess return on the housing collateral ratio, the dividend yield and the ratio of aggregate labor income to consumption, all of which have been shown to forecast annual returns. Likewise, the conditional volatility is the projection of the standard deviation of intra-year monthly returns on the same predictors. Using the projection coefficient estimates we form the Sharpe ratio as the ratio of the predicted excess returns and predicted volatility. Table 7 shows the estimation results for 1 year returns (column 1), but also for 5 year and 10-year cumulative excess returns. The last three lines indicate the mean and standard deviation of the Sharpe ratio as well as its correlation with the housing collateral ratio. In the estimation, the standard deviation of the Sharpe ratio on 1, 5 and 10 year cumulative excess returns is .10, .18, and .20. Lettau and Ludvigson (2003) do a similar exercise for quarterly excess returns between 1952:4 and 2000:4.[15] Their estimate of the unconditional standard deviation of the Sharpe ratio is .45. In the model, the unconditional standard deviation of the Sharpe ratio is .40, .42, .40 for 1, 5 and 10 year cumulative excess returns on a non-levered consumption claim. Our model easily generates a highly volatile Sharpe ratio. Other models have a hard time generating this volatility. The unconditional standard deviation of the Sharpe ratio is .29 in the collateral model, compared to .09 for the Campbell and Cochrane (1999) model and the consumption volatility model of Lettau and Ludvigson (2003). The volatility of the Sharpe ratio in the representative agent model is even smaller.

Furthermore, the correlation between the Sharpe ratio and the measure of collateral scarcity $\widetilde{my}$ is positive in the data and equal to .25, .32, and .50 for 1, 5 and 10 year cumulative excess returns. The corresponding correlations in the model are large and positive (.50, .59 and .39). Figure 3 plots the Sharpe ratio on 5-year and 10-year cumulative returns for the collateral model. Figure 4 does the same for the 5-year cumulative return in the data.

Figure 5 summarizes the findings for conditional asset pricing moments. The top row plots the conditional mean, standard deviation and Sharpe ratio on a claim to aggregate consumption, averaged over histories of the aggregate state z^k, against the housing collateral ratio. In each graph, the solid line plots the conditional moments, conditional on observing a low aggregate consumption growth rate tomorrow ($\lambda(z') = .96$), the dashed line is conditional on observing a high aggregate consumption growth rate ($\lambda(z') = 1.04$). On average, the equity premium is 9 percent higher when collateral is scarce (my = .04) than when it is abundant (my = .10), conditional on being in a boom tomorrow. It is 6 percent higher conditional on being in a recession. Stock returns are up to ten percent more volatile when collateral is scarce. The conditional Sharpe ratio, conditional on being in a boom tomorrow, is .6 when collateral is scarce and .3 when collateral is abundant. The bottom row displays the conditional market price of risk $\sigma_t[m_{t+1}]/E_t[m_{t+1}]$, an upper bound on the Sharpe ratio, the conditional price dividend ratio and the risk-free rate. The price-dividend ratio is high when collateral is scarce. The demand for insurance against binding solvency constraints drives

[15]They find that the dividend yield, the default spread, the term spread, the relative risk-free rate and the consumption wealth ratio plus two lags of volatility jointly explain roughly 30 percent of the time series variation in the conditional second moment and 9 percent of the time series variation in the conditional first moment of the excess stock market return. They using this set of variables to form conditional moments and to compute the Sharpe ratio.

up the price stocks. So, the model simultaneously generates a high equity premium and a high price-dividend ratio because the risk-free rate is very low when collateral is scarce (last panel). The model with recursive preferences shows the same conditional asset pricing patterns (see appendix A.4).

Long Horizon Predictability One of the implications of the time-variation in the equity premium is that the housing collateral model should predict returns. We explore this predictability in depth in our empirical paper (Lustig and VanNieuwerburgh (2004b)). Panel 1 of table 8 summarizes the predictability results of the housing collateral ratio for 1 to 8 year ahead cumulative excess returns (data for 1927-2003 and 1945-2003). The housing collateral ratio we use in this table is based on the outstanding value of mortgage debt (see appendix A.5 for a detailed description of its construction). The data are supportive of the collateral effect: excess returns are higher when collateral is scarce. The effect becomes larger and statistically more significant with the horizon. The R^2 increases.[16]

Our model replicates the pattern of predictability for the housing collateral ratio. The second panel of table 8 reports regression results *inside* the model of excess returns on our measure of housing collateral ratio scarcity. When housing collateral is scarce (my is low), the excess return is high. The magnitude of the slope coefficients is close to the one we find in the data. Moreover, the R^2 of the predictability regression increase with the predictability horizon, just as in the data. We find this negative relationship between my_t and the excess return on a non-levered claim to aggregate consumption, as well as for a levered claim to aggregate consumption.

Other variables, such as the price-dividend ratio and the risk-free rate have also been shown to predict excess returns. In the data, a higher dividend yield and a lower risk-free rate forecast higher future excess returns. The former becomes more important at longer horizons, whereas the latter is more important at short horizons. We also explore the predictability of the price-dividend ratio for returns inside the model. The theory predicts a negative sign for returns and a positive sign for *excess* returns, whereas the data show a negative sign in both regressions.[17] Lastly, the model predicts a negative relationship between current risk-free rate and future excess returns but a positive relationship with future returns. The data show a negative relationship in either case.

[16] Predictability results for the other two collateral measures we consider are reported in Lustig and VanNieuwerburgh (2004b).

[17] Because the risk-free rate is not very volatile in the data, the empirical results for regressions where the left-hand side variable is the real returns instead of the excess return are very similar.

This pattern of predictability of the price-dividend ratio and the risk-free rate is mainly driven by the risk-free rate dynamics. When collateral is scarce, the price-dividend ratio is high and the risk-free rate is low. The theory predicts that the equity premium is high (see figure 5). Because of the persistence in the housing collateral ratio, future equity premia are also high. This must mean that future realized *excess* returns are high on average. However, the high excess returns are the result of *lower* realized returns and even lower future risk-free rates.[18]

3.3. Value Premium

Value firms, with a high ratio of book equity to market equity, historically pay higher returns than growth firms, with a low book-to-market ratio. We use annual return data for 1927-2003 for the US for 10 value portfolios from Fama and French (1992). The decile portfolios are formed every year by sorting the universe of stocks on the ratio of book value to market value of equity. Table 9 reports sample means for the excess return, its unconditional volatility and the Sharpe ratio on the ten book-to-market deciles. The annual excess return on a zero-cost investment strategy that goes long in the highest book-to-market decile and short in the lowest decile is 5.5 percent for 1927-2003. The value premium is 5.7 percent for quintile portfolios. Similar value premia are found for monthly and quarterly returns. Using quarterly data for 1951-2002, Lettau and Wachter (2004) document that the unconditional Sharpe ratio for value stocks (.64) is twice as large as for growth stocks (.32). In table 9, we find a similar increase for annual data from 1945-2003 (from .37 to .56), but a smaller increase over the entire period 1927-2003 (.32 to .42).

Lustig and VanNieuwerburgh (2004b) show that value stocks command higher expected returns because their returns covary more strongly with aggregate consumption growth when collateral is scarce. This mechanism accounts for more than 80 percent if the cross-sectional variation in the book-to-market portfolio returns *in the data.* The second main exercise of this paper is to show the collateral model can endogenously generate a value premium of the magnitude observed in the data.

Decile Return Processes in Data In a first step, we use the data on the decile value portfolio returns to describe the return-generating process for each of the book-to-market decile portfolios. We specify return processes for value and growth stocks as linear functions of the state variables in our model. These are aggregate consumption growth, aggregate expenditure share growth, the

[18] Results for the data and model are available upon request.

housing collateral ratio and the interaction terms of the housing collateral ratio with aggregate consumption growth and expenditure share growth. The return in excess of a risk-free rate on the j^{th} book-to-market decile portfolio is:

$$\begin{aligned} R^{e,j}_{t+1} &= \beta_0 + \beta^j_{my}\widetilde{my}_{t+1} + \beta^j_c \Delta \log c^a_{t+1} + \beta^j_{c,my}\widetilde{my}_{t+1}\Delta \log c^a_{t+1} + \\ & \beta^j_\alpha \Delta \log \alpha^a_{t+1} + \beta^j_{\alpha,my}\widetilde{my}_{t+1}\Delta \log \alpha^a_{t+1} + \nu^j_{t+1}, \end{aligned} \tag{14}$$

where $\widetilde{my}_{t+1} = \frac{my^{max} - my_{t+1}}{my^{max} - my^{min}}$ is rescaled so that all realizations are between 0 and 1. It is a measure of collateral scarcity. The beta vector in equation (14) is estimated by ordinary least squares and reported in table 10. The first panel uses a housing collateral ratio based on the market value of outstanding mortgages, the second panel uses a measure based on residential real estate wealth and the third panel employs a measure based on residential fixed asset values.

Returns on value stocks (decile 1) are high in recessions, i.e. they covary negatively with aggregate consumption growth. Growth stocks are much less sensitive to aggregate consumption growth; $|\beta_c|$ increases monotonically from decile 1 to decile 10. This pattern is robust across different collateral measures. As for the collateral effect, value stocks covary more strongly with aggregate consumption growth when collateral is scarce ($\beta_{c,my} > 0$). For growth stocks this effect is much less pronounced; $\beta_{c,my}$ increases monotonically from decile 1 to decile 10. This pattern is robust across different collateral measures. Lastly, value stocks are more sensitive to aggregate expenditure share shocks; β_α increases monotonically for all three collateral measures.

Decile Return Processes in Model In a second step, we generate ten excess return processes as the product of the factor loadings $\left(\beta^j_{my}, \beta^j_c, \beta^j_{c,my}, \beta^j_\alpha, \beta^j_{\alpha,my}\right)$ estimated from the data and the aggregate state variables (factors) generated in the model. For each excess return, the intercept β^j_0 is determined such that the Euler equation $E_t[m_{t+1}R^{j,e}_{t+1}] = 0$ is satisfied. This is a consistency requirement that the SDF of the model prices these assets correctly. We then simulate the model for 10,000 periods and compute unconditional mean and standard deviation of each of the decile portfolio returns.

Table 11 reports the excess returns on the ten value portfolios predicted by the collateral model, ordered from growth (B1) to value (B10) for $\gamma = 5$ and $\gamma = 8$. In each panel we use three sets of empirical factor loadings, corresponding to the three housing collateral measures. These are the three sets of betas from table 10.

For the fixed asset measure, the value spread is 6 percent for $\gamma = 8$ and 4 percent for $\gamma = 5$. For the other two collateral measure, the value spread is 4 percent for $\gamma = 8$ and 3 percent for $\gamma = 5$. For $\gamma = 8$, the model is able to replicate the 5.2 percent value spread in the data. Furthermore, the model predicts a noticeable increase the Sharpe ratio between the first decile (growth) and the tenth decile (value). The Sharpe ratio doubles for all three collateral measures and $\gamma = 8$. In the data there is similar increase in the higher Sharpe ratio for post-war data (see table 9).

In contrast, the representative agent economy generates no value premium. We estimate the consumption betas from an equation like 14, but with aggregate consumption growth and aggregate expenditure share growth as the factors. These are the only two risk factors in the representative agent economy. There is much less of a pattern in the betas than what we found for the collateral model. The estimated betas are used together with aggregate consumption growth shocks in the model to generate 10 excess return processes. There is no pattern in the excess returns. The value premium is zero. The Sharpe ratios on growth stocks are higher than the ones on value stocks, the opposite pattern as found in the data.[19]

Duration Value stocks are stocks whose cash-flow profiles are tilted towards the present whereas growth stocks' payoffs are farther in the future. Dechow, Sloan, and Soliman (2002) estimate a 6 year duration for value stocks and a 20 year duration for growth stocks.

Lettau and Wachter (2004) provide insight into the properties that an asset pricing model must satisfy to generate a lower expected return for long duration assets (growth stocks) than for short duration assets (value stocks). The correlation between aggregate consumption growth and the Sharpe ratio on equity must be non-negative. Then, long duration assets are less risky because a negative consumption growth innovation leads to lower risk premia in the future. They point out that many asset pricing models, such as the Campbell and Cochrane (1999), generate a negative correlation instead.

Our model endogenously generates this *positive correlation.* Figure 6 plots the Sharpe ratio for a simulation of the model and indicates periods with negative consumption growth by shaded bars. The Sharpe ratio increases during periods of high aggregate consumption growth and falls when consumption growth is negative.[20] There is history dependence. The decline is bigger the longer was

[19]Detailed results available upon request.

[20]We find the same positive correlation in the data. We form a conditional Sharpe ratio as the ratio of predicted excess returns and predicted volatility, based on regressions with an aggregate consumption growth dummy, a lagged aggregate consumption growth dummy and the housing collateral ratio as independent variables. The dummy variables capture boom times, they are 1 when aggregate consumption growth is above the sample mean minus one

the preceding boom. The left picture on the second row of figure 5 shows the same relationship. The conditional market price of risk is higher in periods of high aggregate consumption growth (dashed line) than in periods of low aggregate consumption growth (full line).

The key to this positive correlation is the combination of history dependence in the wealth dynamics and the persistence of the housing collateral ratio. In every successive period of high aggregate consumption growth, the wealth distribution becomes more condensed because fewer households are constrained and the unconstrained households' consumption share drifts down. A low consumption growth shock after a long period of expansion leads to a very large aggregate weight shock and high SDF (figure 7). Many households are constrained and their consumption share jumps up. This shock wipes out the left tail of the wealth distribution. In anticipation of such large shock, households demand a large excess return per unit of standard deviation. This generates the positive correlation between aggregate consumption growth and the Sharpe ratio. The wealth dynamics are not enough. The persistence of the housing collateral ratio is essential to generate enough persistence in the Sharpe ratio so as to make the correlation positive.[21] The collateral model's ability and the representative agent model's failure to price the value portfolios goes back to the properties of their respective stochastic discount factors.

Consumption Strips As we showed in section 1.6, the building blocks of the equity premia on value and growth stocks are equity premia on consumption strips. Ultimately, the collateral model generates a higher expected return and a higher Sharpe ratio for value stocks than growth stocks because short term assets are more risky than long term assets. Therefore, we compute risk premia on consumption strips of various maturities.

Figure 8 plots expected excess returns and figure 9 Sharpe ratios on consumption strips of horizons 2 to 30 years for the benchmark model with additive utility ($\gamma = 3 - 8$). Risk premia and Sharpe ratios on consumption strips are lower for long horizon strips. The duration effect is even stronger in the collateral model with recursive utility (see appendix A.4).

Table 12 reports equity premia on claims to $\{Ce^{ak}c_k\}$ (see section 1.6). These are baskets of consumption strips of different maturities, where the constant a governs the duration of the basket. We vary a from -.5 to .5. The corresponding baskets have a duration between 2.3 years and 43 years. We think of the basket with duration of 5 years as the value stock and the basket with

sample standard deviation.

[21]We computed the model without the collateral effect and found that the term structure of risk premia was flat.

duration of 40 years as the growth stock (Dechow, Sloan, and Soliman (2002)). The value spread is 3.9 percent for $\gamma = 5$ and 5.7 percent for $\gamma = 8$. The latter matches the value spread in the data of 5.7 percent. In addition, Sharpe ratios on value portfolios are much higher than on growth portfolios. For $\gamma = 8$, the Sharpe ratio on the 5-year duration portfolio is .44 and the Sharpe ratio on the 40-year duration portfolio is .08.

4. Conclusion

This paper specifies, calibrates and solves a general equilibrium asset pricing model with housing collateral. Agents write state-contingent promises backed by the value of the housing stock. Time variation in the price of housing induces time variation in the economy's ability to share labor income risk. When the ratio of housing wealth to total wealth is low, households with binding collateral constraints experience a larger change in the consumption share. In such periods of limited risk-sharing possibilities, agents demand a higher risk premium on financial assets. The housing collateral mechanism endogenously generates time-varying volatility in the Sharpe ratio on equity. This is a novel feature of the model. This quantitatively matches the dynamics of the Sharpe ratio on equity in US data: It is high in periods of collateral scarcity and volatile. In contrast, the representative agent model delivers virtually no variation in the Sharpe ratio. Other equilibrium models have similar difficulties generating enough volatility in the Sharpe ratio. The model also explains cross-sectional variation in excess returns along the value dimension. Excess returns on short duration assets (such as value stocks) are higher than returns on long duration assets (such as growth stocks). The reason for this duration effect is that a negative consumption growth shock leads to lower future Sharpe ratios. Lastly, the model quantitatively matches key unconditional asset pricing moments. To the best of our knowledge, this is one of the few models that starts from first principles and quantitatively matches the unconditional and conditional moments of aggregate asset prices, and generates a meaningful variation in the cross-section of returns.

Why does the collateral model work better than the standard consumption CAPM? The model suggest that the answer lies in allowing for time-variation in risk-sharing among heterogeneous agents. The standard CCAPM implies that risk-sharing is always perfect. In Lustig and Van-Nieuwerburgh (2004a), we provide direct empirical support for the underlying time-variation in risk-sharing. Using US metropolitan area data, we find that the degree of insurance between regions decreases when the housing collateral ratio is low. This is consistent with evidence from

Blundell, Pistaferri, and Preston (2002), who find evidence for time-variation the economy's risk sharing capacity. Regional income and consumption data provide direct support for the existence and importance of the collateral mechanism, whose asset pricing implications where the focus of this paper.

In future work, we plan to further exploit the link between asset prices and risk-sharing by using asset prices to measure the cost of consumption uncertainty, along the lines of Alvarez and Jermann (2003).

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5. Figures and Tables

Figure 1. Risk Sharing and Collateral Ratio

The average collateral share is 5 percent, the discount factor is .95 and the coefficient of risk aversion is 8. The first panel plots the non-durable expenditure share α, the ratio of non-durable consumption to non-durable plus housing services consumption (dotted line). The second panel is the cutoff level consumption share at which the solvency constraints hold with equality (dotted line). The third panel is the cross-sectional standard deviation of consumption growth across households (dotted line). The fourth panel is the aggregate weight shock g_{t+1} (dotted line). The full line in each panel is the collateral ratio my, the ratio of housing wealth to total wealth (right axis). The graphs display a two hundred period model simulation.

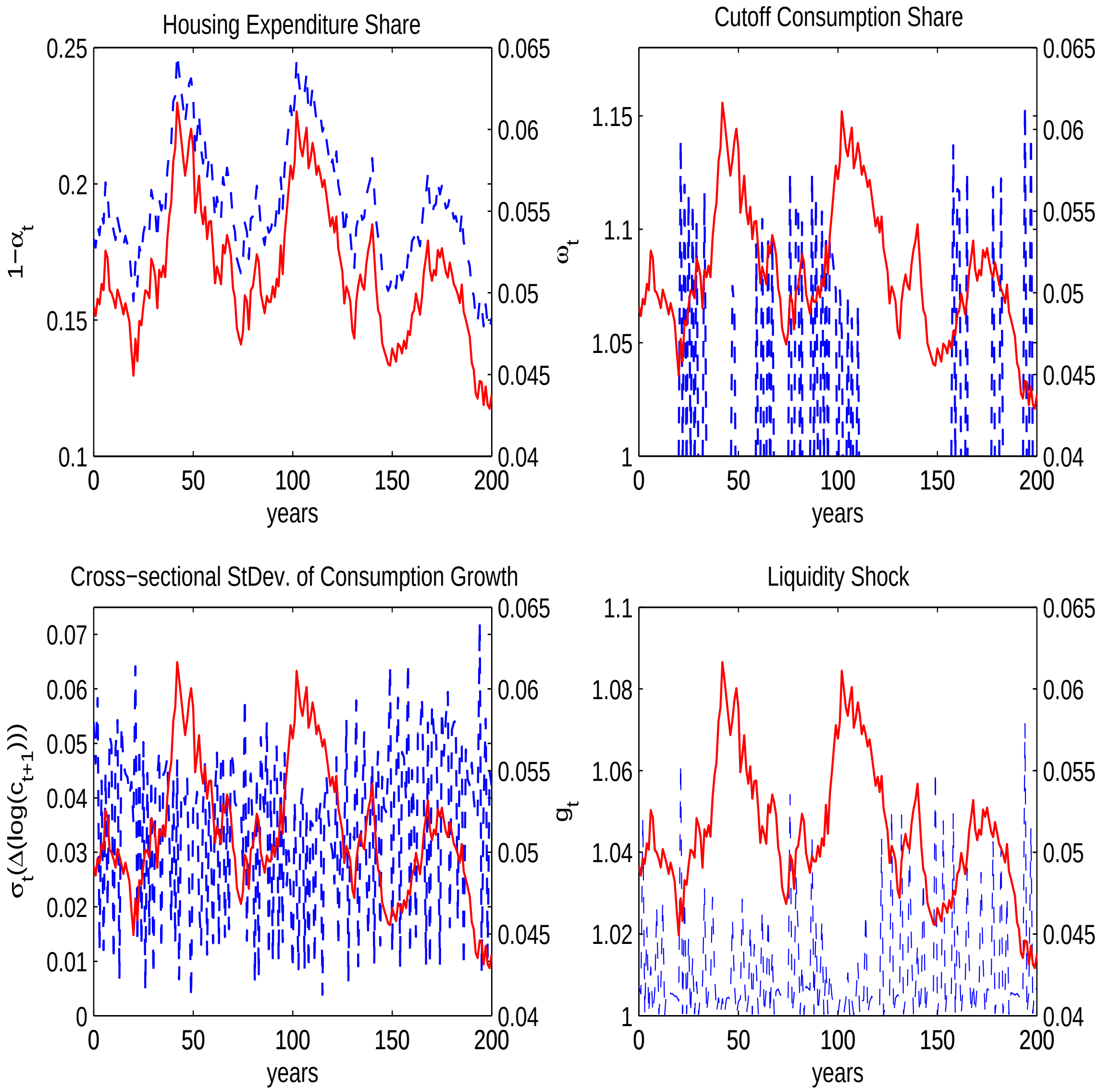


Figure 2. Conditional Asset Pricing Moments and Collateral Ratio

The average collateral share is 5 percent, the discount factor is .95 and the coefficient of risk aversion is 8. The first panel plots the expected excess return $R_t^{c,e}$ on a non-levered claim to aggregate consumption (dotted line), the second panel is the conditional standard deviation of the excess return $R_t^{c,e}$ (dotted line), and the third panel is the Sharpe ratio on a non-levered claim to aggregate consumption (dotted line). The full line in each panel is the collateral ratio my, the ratio of housing wealth to total wealth (right axis). The graphs display a two hundred period model simulation.

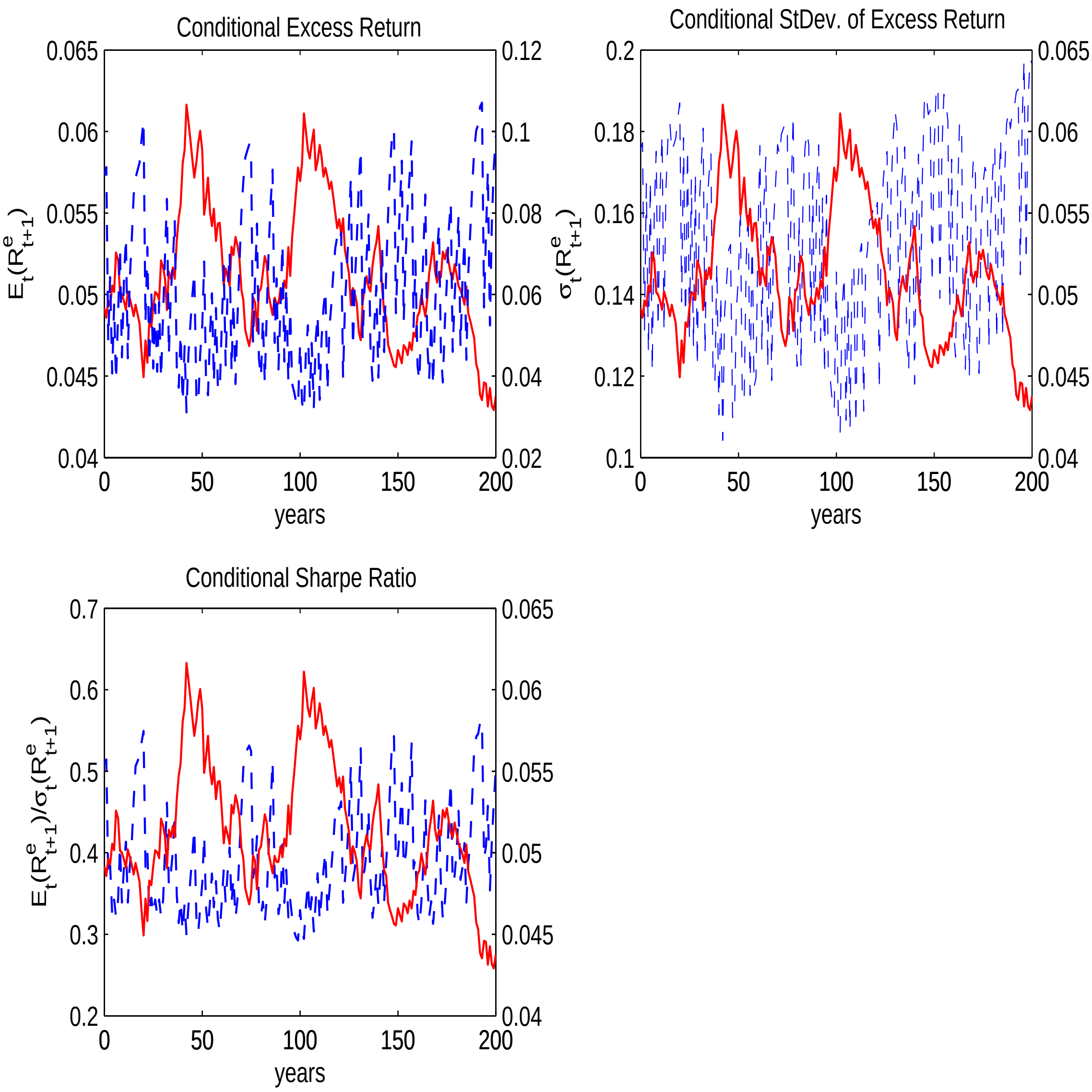


Figure 3. Housing Collateral Ratio and Long-Horizon Sharpe Ratio in Model.
The average collateral share is 5 percent, the discount factor is .95 and the coefficient of risk aversion is 8. This the Sharpe ratio on a 10 year and 5 year cumulative excess return on a non-levered consumption claim (dotted line), and the collateral ratio my is the ratio of housing wealth to total wealth (full line) for a one hundred period model simulation.

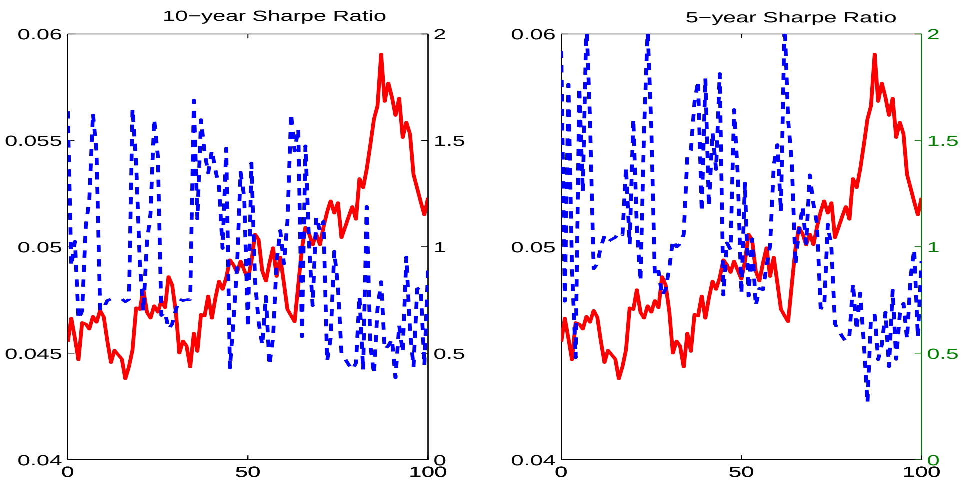


Figure 4. Housing Collateral Ratio and Long- Horizon Sharpe Ratio in Data.
This is the Sharpe ratio on 5-year cumulative stock market returns in the data for 1928-1997. See table 7 for details on its construction. The housing collateral ratio my is zero on average by construction (see appendix A.5 for the estimation procedure).

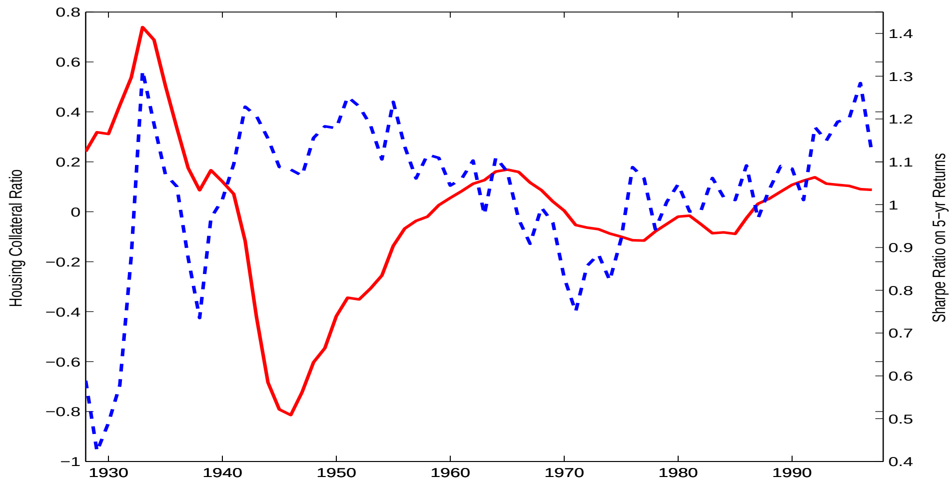


Figure 5. Summary Conditional Asset Pricing Moments.

The average collateral share is 5 percent, the discount factor is .95 and the coefficient of risk aversion is 8. This the expected excess return on a claim to aggregate consumption, its conditional standard deviation and its Sharpe ratio (top row), and the conditional market price of risk, conditional price-dividend ratio and the risk-free rate, averaged over histories and plotted against the housing collateral ratio. The full line denotes the conditional moments, conditional on observing a low aggregate consumption growth rate tomorrow, whereas the dotted line denotes the moments conditional on observing a high aggregate consumption growth rate.

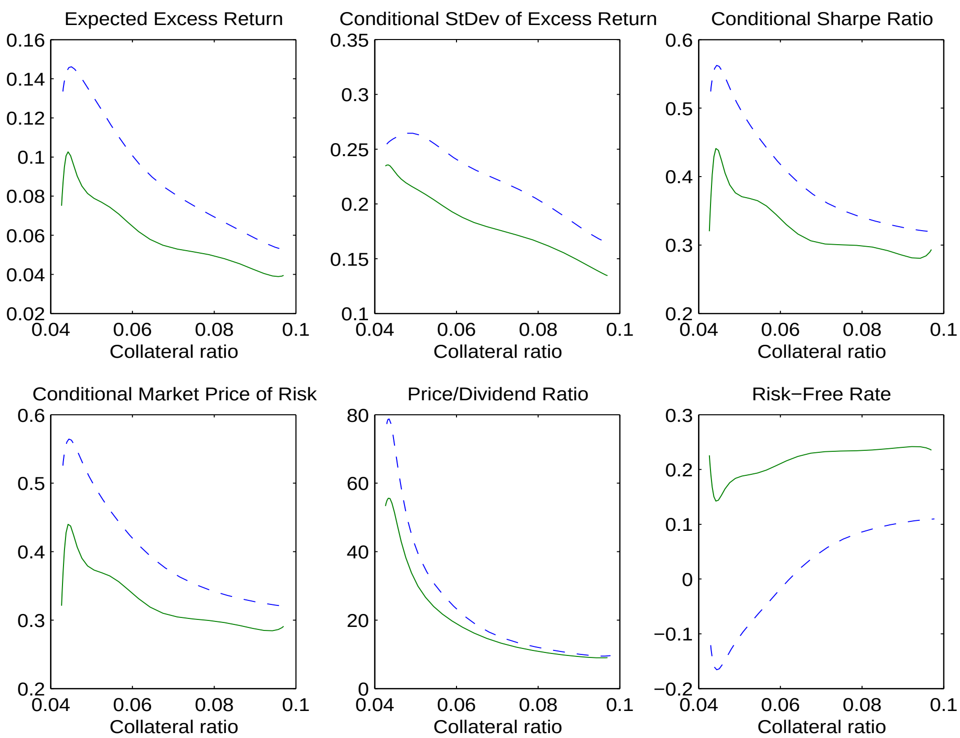


Figure 6. Aggregate Consumption Growth Shocks and Sharpe Ratio on Equity.
Benchmark model calibration with risk aversion 8 and 5 percent collateral. One hundred period model simulation. Shaded bars indicate periods with low aggregate consumption growth. The dotted line denotes the Sharpe ratio on a non-levered claim to aggregate consumption $E_t[R^{c,e}]/\sigma_t[R^{c,e}]$.

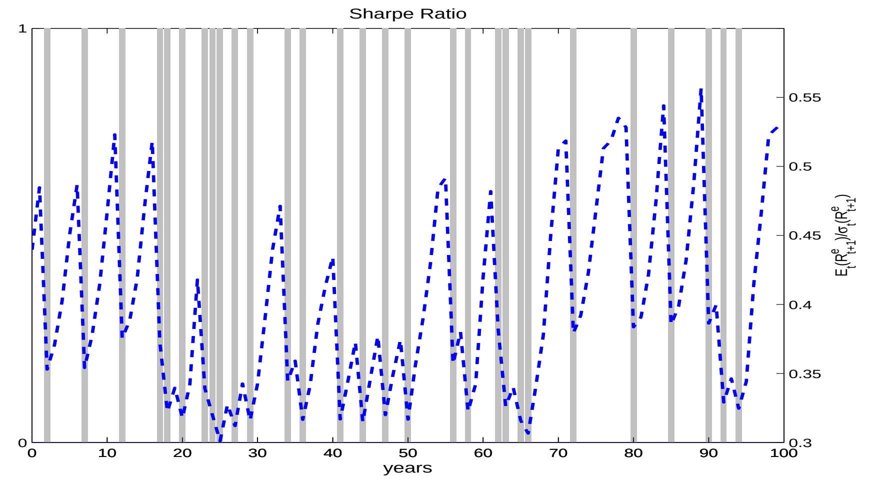


Figure 7. Aggregate Consumption Growth Shocks and The Stochastic Discount Factor.
Benchmark model calibration with risk aversion 8 and 5 percent collateral. One hundred period model simulation. Shaded bars indicate periods with low aggregate consumption growth. The dotted line denotes the Stochastic Discount Factor m_{t+1}.

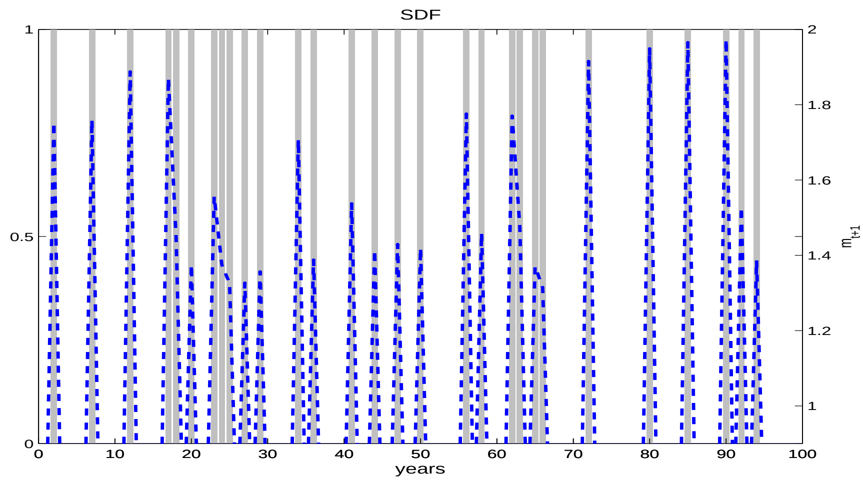


Figure 8. Term Structure of Risk Premia on Consumption Strips.
The average collateral share is 5 percent, the discount factor is .95 and the coefficient of risk aversion γ is between 3 and 8. The figure plots the expected excess return on a claim to aggregate consumption k periods from now, $k = 2, 3, ..., 30$.

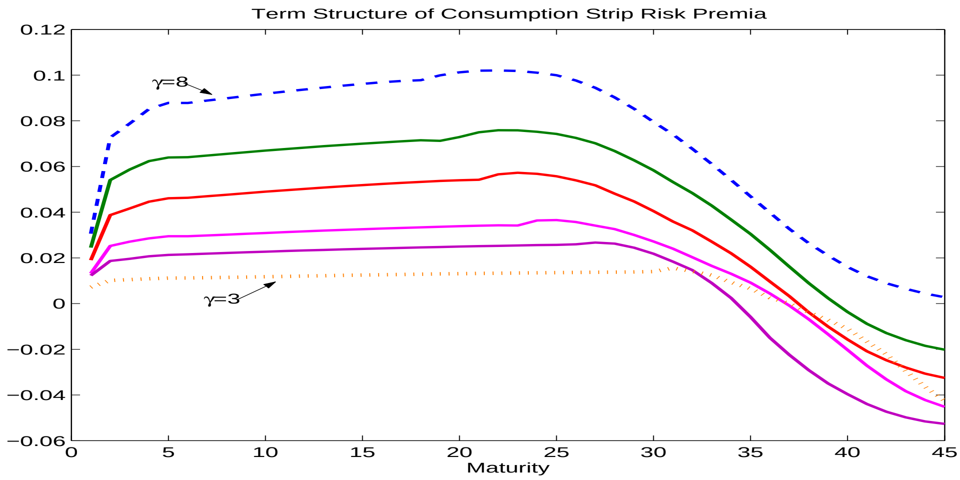


Figure 9. Term Structure of Sharpe Ratios on Consumption Strips.
The average collateral share is 5 percent, the discount factor is .95 and the coefficient of risk aversion γ is between 3 and 8. The figure plots the conditional Sharpe ratio on a claim to aggregate consumption k periods from now, $k = 2, 3, ..., 30$.

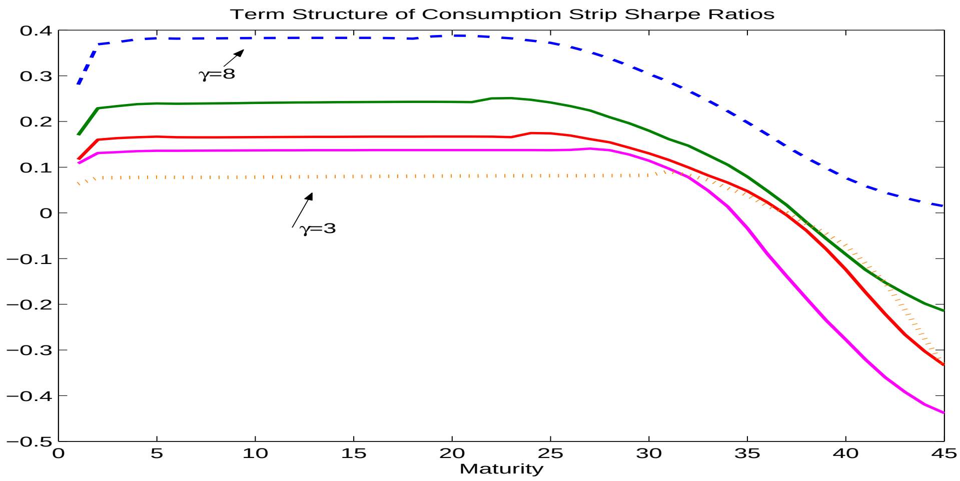


Table 1
Expenditure Share and Rental Price Regression Results.

Panel A reports regression results for $log(r_{t+1}) = \theta log(r_t) + \lambda\Delta \log(c_{t+1}) + \epsilon_{t+1}$, where r is the expenditure share of nondurable consumption. Panel B reports results for the regression $log(\rho_{t+1}) = \rho - rlog(\rho_t) + b_r\Delta \log(c_{t+1}) + \epsilon_{t+1}$, where ρ is the rental price. Below the OLS point estimates are HAC Newey-West standard errors. The left panel reports the results for the entire sample, while the right panel reports the results for the post-war sample. The variables with superscript 1 are available for 1926-2002. The variables with superscript 2 are only available for 1929-2002. The data appendix contains detailed definitions and data sources for these variables.

Expl. Var.	ρ_r	b_r	ρ_r	b_r
	1926/9-2002		*1945-2002*	
	Panel A: Expenditure Share			
$log(r^1)$	.925		.950	
	(.039)		(.033)	
$log(r^1)$	.890	.824	.957	.824
	(.033)	(.141)	(.033)	(.180)
$log(r^2)$	.940		.936	
	(.037)		(.026)	
$log(r^2)$	.940	.816	.952	.816
	(.032)	(.159)	(.027)	(.181)
	Panel B: Rental Price			
$log(\rho^1)$	.953		.851	
	(.027)		(.056)	
$log(\rho^1)$	.955	.102	.817	.261
	(.027)	(.181)	(.054)	(.240)
$log(\rho^2)$	.941		.911	
	(.023)		(.046)	
$log(\rho^2)$	.932	-.321	.896	.259
	(.023)	(.158)	(.047)	(.172)

Table 2
Parameter Calibration

Parameter	Benchmark	Sensitivity Analysis
γ	8	[2,3,...,9,10]
ε	.05	[.15,.25,...,.65,.75]
ψ	1	$\times$
$\bar{r}$	4.26	$\times$
ρ_r	.96	$\times$
b_r	.93	0
σ_r	.03	$\times$
$E[my]$	19 (5%)	9 (10 %)
λ	[1.04,.96]	$\times$
η	[.6578,.7952,.3422,.2048]	[.6935,.6935,.3065,.3065]
κ	1	3

Table 3
Unconditional Asset Pricing Moments for Collateral Model.

Averages from a simulation of the model for 5,000 agents and 10,000 periods. In the first column, $R^{l,e}$ denotes the excess return on a levered claim to aggregate consumption growth, with leverage parameter $\kappa = 3$. $R^{c,e}$ denotes the excess return on a non-levered claim to aggregate consumption growth. The third column reports the unconditional mean of the risk-free rate. Columns four to six report unconditional standard deviations of levered and non-levered consumption claims and risk-free rate. The last two columns report Sharpe ratios on levered and non-levered consumption claims. Panel 1 is the benchmark calibration with 5 percent collateral. The expenditure share process is an AR(1) with an aggregate consumption growth term: $\log r_t = \bar{r} + \rho_r \log r_{t-1} + b_r \Delta(\log(c_{t+1})) + \sigma_r \nu_t$. Panel 2 reports results for different parameters of intratemporal elasticity of substitution between non-durable and housing services consumption ε. All other parameters are held constant at their benchmark level and there is 5 percent collateral. Panel 3 varies the coefficient of relative risk aversion γ (5 percent collateral). Panel 4 reports the moments for an economy with 10 percent collateral. Panel 5 is the benchmark calibration with 5 percent collateral but with an expenditure share process is an AR(1) without consumption growth term: $\log r_t = \bar{r} + \rho_r \log r_{t-1} + \sigma_r \nu_t$. Panel 6 reports historical averages for 1927-2001 (data from Kenneth French) and for 1889-1979 (data from Shiller).

(γ, ε)	$E(R^{l,e})$	$E(R^{c,e})$	$E(r^f)$	$\sigma(R^{l,e})$	$\sigma(R^{c,e})$	$\sigma(r^f)$	$\frac{E(R^{l,e})}{\sigma(R^{l,e})}$	$\frac{E(R^{c,e})}{\sigma(R^{c,e})}$
Panel 1: Benchmark								
$(5, 0.05)$	0.051	0.030	0.057	0.213	0.129	0.073	0.238	0.230
$(8, 0.05)$	0.108	0.071	0.029	0.257	0.179	0.125	0.421	0.398
Panel 2: Varying ε								
$(8, 0.15)$	0.110	0.091	0.024	0.258	0.175	0.128	0.425	0.516
$(8, 0.35)$	0.114	0.097	0.016	0.256	0.176	0.128	0.443	0.550
$(8, 0.55)$	0.125	0.110	0.001	0.261	0.186	0.134	0.479	0.593
$(8, 0.75)$	0.153	0.123	−0.015	0.274	0.188	0.136	0.558	0.657
Panel 3: Varying γ								
$(2, 0.05)$	0.013	0.005	0.069	0.162	0.067	0.022	0.081	0.079
$(4, 0.05)$	0.036	0.019	0.067	0.194	0.105	0.052	0.183	0.176
$(6, 0.05)$	0.068	0.042	0.050	0.227	0.145	0.089	0.300	0.293
$(10, 0.05)$	0.151	0.110	−0.003	0.285	0.213	0.159	0.530	0.514
Panel 4: 10 percent Collateral								
$(5, 0.05)$	0.033	0.028	0.101	0.159	0.090	0.051	0.207	0.318
$(5, 0.75)$	0.053	0.050	0.068	0.177	0.118	0.077	0.301	0.427
$(8, 0.05)$	0.078	0.056	0.075	0.207	0.150	0.107	0.376	0.373
$(8, 0.75)$	0.127	0.098	0.025	0.233	0.189	0.137	0.547	0.517
Panel 5: AR(1) process for $\log(r)$								
$(5, 0.05)$	0.041	0.026	0.048	0.165	0.108	0.071	0.252	0.239
$(5, 0.75)$	0.042	0.028	0.048	0.166	0.112	0.069	0.255	0.247
$(8, 0.05)$	0.091	0.066	0.012	0.207	0.154	0.119	0.440	0.427
$(8, 0.75)$	0.104	0.080	−0.001	0.220	0.176	0.124	0.475	0.456
Panel 6: Data								
1927-2002	0.075		0.039	0.198		0.032	0.419	
1889-1979	0.060		0.014	0.192		0.065	0.313	

Table 4
Unconditional Asset Pricing Moments for Representative Agent Model.

Averages from a simulation of the representative agent model with non-separable preferences for 10,000 periods. The rows in each block are for different values for the coefficient of relative risk aversion γ and the intratemporal elasticities of substitution between non-durable and housing services consumption ε. All other parameters are held constant at their benchmark level. The expenditure share process is an AR(1) with an aggregate consumption growth term: $\log r_t = \bar{r} + \rho_r \log r_{t-1} + b_r \Delta(\log(c_{t+1})) + \sigma_r \nu_t$. In the first column, $R^{l,e}$ denotes the excess return on a levered claim to aggregate consumption growth, with leverage parameter $\kappa = 3$. $R^{c,e}$ denotes the excess return on a non-levered claim to aggregate consumption growth. The third column reports the unconditional mean of the risk-free rate. Columns four to six report unconditional standard deviations of levered and non-levered consumption claims and risk-free rate. The last two columns report Sharpe ratios on levered and non-levered consumption claims.

(γ, ε)	$E(R^{l,e})$	$E(R^{c,e})$	$E(r^f)$	$\sigma(R^{l,e})$	$\sigma(R^{c,e})$	$\sigma(r^f)$	$\frac{E(R^{l,e})}{\sigma(R^{l,e})}$	$\frac{E(R^{c,e})}{\sigma(R^{c,e})}$
$(5, 0.05)$	0.024	0.010	0.129	0.127	0.057	0.031	0.186	0.177
$(5, 0.35)$	0.023	0.012	0.127	0.132	0.061	0.034	0.173	0.189
$(5, 0.75)$	0.041	0.023	0.110	0.150	0.084	0.049	0.271	0.275
$(8, 0.05)$	0.045	0.023	0.158	0.146	0.074	0.055	0.309	0.310
$(8, 0.35)$	0.051	0.026	0.151	0.152	0.080	0.061	0.334	0.325
$(8, 0.75)$	0.085	0.055	0.098	0.179	0.114	0.085	0.472	0.480

Table 5
Unconditional Asset Pricing Moments for Housing Market.

Averages from a simulation of the collateral model with 5,000 agents for 10,000 periods. In the left panel the coefficient of relative risk aversion γ is 5; in the right panel it is 8. The rows are for different intratemporal elasticities of substitution between non-durable and housing services consumption ε. All other parameters are held constant at their benchmark level. The expenditure share process is an AR(1) with an aggregate consumption growth term: $\log r_t = \bar{r} + \rho_r \log r_{t-1} + b_r \Delta(\log(c_{t+1})) + \sigma_r \nu_t$. In the first column, $\sigma(\Delta\log(\rho_t))$ denotes the volatility of rental price growth. In the second column $R^{h,e}$ denotes the return on a claim to the aggregate housing endowment in excess of the risk-free rate. The third column denotes the unconditional standard deviation of $R^{h,e}$. The fourth column gives the average housing collateral ratio, the ratio of housing wealth to total wealth my. Panel 1 is the collateral model with 5 percent collateral, panel 2 is the collateral model with 10 percent collateral and panel 3 is the representative agent economy.

ϵ	$\sigma(\Delta\log(\rho_t))$	$E(R^{h,e})$	$\sigma(R^{h,e})$	my_t	$\sigma(\Delta\log(\rho_t))$	$E(R^{h,e})$	$\sigma(R^{h,e})$	my_t
Panel 1: 5 percent Housing Collateral								
	Risk Aversion 5				**Risk Aversion 8**			
.05	0.050	0.026	0.127	0.056	0.050	0.068	0.192	0.057
.15	0.055	0.026	0.123	0.056	0.056	0.070	0.196	0.058
.35	0.073	0.027	0.128	0.056	0.073	0.072	0.195	0.058
.55	0.105	0.032	0.134	0.056	0.105	0.082	0.206	0.059
.75	0.190	0.044	0.151	0.058	0.189	0.103	0.221	0.062
Panel 2: 10 percent Housing Collateral								
	Risk Aversion 5				**Risk Aversion 8**			
.05	0.050	0.016	0.088	0.105	0.050	0.048	0.156	0.106
.15	0.056	0.016	0.089	0.104	0.055	0.048	0.157	0.108
.35	0.073	0.015	0.085	0.056	0.073	0.056	0.168	0.108
.55	0.105	0.019	0.095	0.107	0.105	0.060	0.167	0.109
.75	0.189	0.028	0.112	0.108	0.189	0.088	0.201	0.114
Panel 3: representative Agent Economy								
	Risk Aversion 5				**Risk Aversion 8**			
0.05	0.050	0.006	0.053	0.055	0.050	0.015	0.079	0.056
0.15	0.056	0.006	0.054	0.055	0.056	0.015	0.080	0.056
0.35	0.073	0.006	0.057	0.056	0.073	0.018	0.085	0.057
0.55	0.105	0.009	0.061	0.056	0.104	0.023	0.095	0.058
0.75	0.188	0.016	0.078	0.058	0.189	0.041	0.122	0.059

Table 6
Unconditional Asset Pricing Moments - No CD Effect.

Same as Table 3, but without conditional heteroscedasticity in the individual income process (Constantinides and Duffie (1996) effect). In particular, the vector of income shares η is [.6935,.6935,.3065,.3065] as opposed to [.6578,.7952,.3422,.2048] in table 3.

γ	$E(R^{l,e})$	$E(R^{c,e})$	$E(r^f)$	$\sigma(R^{l,e})$	$\sigma(R^{c,e})$	$\sigma(r^f)$	$\frac{E(R^{l,e})}{\sigma(R^{l,e})}$	$\frac{E(R^{c,e})}{\sigma(R^{c,e})}$
Panel 1: 5 percent Housing Collateral								
5	0.036	0.022	0.078	0.193	0.136	0.066	0.187	0.165
8	0.074	0.055	0.059	0.235	0.181	0.111	0.315	0.305
Panel 2: 10 percent Housing Collateral								
5	0.024	0.011	0.129	0.126	0.056	0.031	0.193	0.194
8	0.057	0.036	0.123	0.179	0.116	0.080	0.318	0.306

Table 7
Long-Term Sharpe Ratios in Data.

Parameter estimates for $R^e_{t+1} = b_0 + b_1 R_t + b_2 dp_t + b_3 lc_t + b_4 \widetilde{my}_t + \varepsilon_{t+1}$ and $Vol_{t+1} = a_0 + a_1 dp_t + a_2 lc_t + a_3 \widetilde{my}_t + a_4 Vol_t + a_5 Vol_{t-1}$. The variables dp, lc and $\widetilde{my}$ are the dividend yield, the labor-consumption ratio and the housing collateral ratio (based on value of mortgages). R^e denotes the value weighted market return in excess of a 1 month T-bill return. Vol_t is the standard deviation of the 12 monthly returns in year t. $R1, R5, R10$ denote the 1-year, 5-year and 10-year ahead cumulative excess returns. The estimation is by GMM with the OLS normal conditions as moment conditions. Standard errors are Newey-West with lag length 3. The estimation period is the longest common sample: 1927-1992. The last three rows indicate the sample mean, sample standard deviation of the predicted Sharpe ratio and sample correlation between the Sharpe ratio and the scarcity of housing collateral. The predicted Sharpe ratio is the predicted mean excess return divided by its predicted standard deviation.

	R1	Vol1	R5	Vol5	R10	Vol10
constant	-.24	.09	.71	-.004	.76	.05
(s.e.)	(.34)	(.05)	(0.28)	(.04)	(.42)	(.04)
lag ret	.04		.74		.76	
(s.e.)	(.13)		(.13)		(.08)	
dp	1.07	.33	-1.20	.40	.26	.12
(s.e.)	(2.00)	(.27)	(3.16)	(.22)	(3.00)	(.21)
lc	.22	-.07	-.76	.03	-.92	-.03
(s.e.)	(.32)	(.05)	(.33)	(.03)	(.45)	(.04)
$\widetilde{my}$	.02	-.01	.48	-.04	.70	.01
(s.e.)	(0.20)	(.02)	(.20)	(.02)	(.23)	(.02)
lag vol		.51		.96		.80
(s.e.)		(.20)		(.12)		(.12)
2 lag vol		-.18		-.19		.03
(s.e.)	(.13)	(.17)		(.11)		(.10)
E[Sharpe]	.40		.73		1.12	
σ[Sharpe]	.10		.18		.20	
$\rho[Sharpe, \widetilde{my}]$	.26		.32		.50	

Table 8
Predictability of K-Year Excess Returns.

Results of regressing log K-horizon excess returns on the housing collateral ratio. The first panel reports the results in the data. The t-stats in brackets are computed using the Newey West covariance matrix with K lags. The returns are cum-dividend returns on the value-weighted CRSP index. The housing collateral measure is based on the market value of outstanding mortgages $\tilde{my}_t$ and is rescaled to lie between 0 and 1: $\tilde{my}_t = max(my_t) - my_t/(min(my_t) - my_t)$. The risk-free rate is the average 3-month yield from CRSP's Fama-Bliss risk-free rates and inflation is constructed from the CPI (BLS). The long sample contains annual data from 1930-2003. The post-war sample is from 1945-2003. The second panel reports the same regressions inside the model. The regressions were obtained by simulating the model for 10,000 periods under the benchmark parametrization with risk aversion 5. The expenditure share process is an AR(1) with an aggregate consumption growth term: $\log r_t = \bar{r} + \rho_r \log r_{t-1} + b_r \Delta(\log(c_{t+1})) + \sigma_r \nu_t$. The housing collateral ratio my_t is scaled to lie between 0 and 1 for all t.

	b_0	b_1	R^2	b_0	b_1	R^2
	Panel 1: Data					
Horizon	**Entire Sample**			**Post-War Sample**		
1	0.01	0.15	0.02	0.02	0.15	0.04
	[0.24]	[1.25]		[0.63]	[1.73]	
2	0.07	0.25	0.03	0.06	0.31	0.07
	[0.66]	[1.11]		[0.83]	[1.75]	
3	0.16	0.31	0.03	0.12	0.46	0.09
	[1.02]	[0.97]		[1.04]	[1.80]	
4	0.29	0.32	0.02	0.22	0.56	0.09
	[1.50]	[0.80]		[1.42]	[1.82]	
5	0.38	0.47	0.03	0.34	0.66	0.08
	[1.83]	[1.16]		[1.67]	[1.77]	
6	0.41	0.79	0.07	0.43	0.89	0.10
	[1.70]	[1.73]		[1.58]	[1.86]	
7	0.41	1.20	0.12	0.49	1.26	0.14
	[1.32]	[2.11]		[1.30]	[2.02]	
8	0.37	1.80	0.18	0.50	1.79	0.19
	[0.91]	[2.71]		[1.01]	[2.26]	
	Panel 2: Model					
Horizon	**Leverage =1**			**Leverage=4**		
1	-0.00	0.05	0.00	-0.00	0.09	0.00
2	-0.07	0.18	0.02	-0.06	0.23	0.01
3	-0.15	0.33	0.04	-0.14	0.41	0.02
4	-0.26	0.51	0.06	-0.24	0.62	0.03
5	-0.39	0.72	0.09	-0.37	0.85	0.04
6	-0.55	0.96	0.11	-0.51	1.10	0.06
7	-0.72	1.22	0.14	-0.69	1.40	0.07
8	-0.92	1.51	0.17	-0.88	1.73	0.09

Table 9
Annual Value-Weighted Excess Returns in B/M deciles

Excess return in excess of a risk-free rate, volatility and Sharpe ratio. The risk-free rate is the average yield on a 3-month T-Bill. The stock returns on the book-to-market deciles are from Kenneth French's web site, annual data for 1930-2003 and 1945-2003 (source CRSP).

Decile	1	2	3	4	5	6	7	8	9	10
				Panel 1: Sample 1930-2003						
$E(R^e)$	0.071	0.084	0.080	0.081	0.100	0.099	0.108	0.127	0.131	0.139
$\sigma(R^e)$	0.222	0.194	0.196	0.229	0.228	0.238	0.250	0.274	0.291	0.332
$E(R^e)/\sigma(R^e)$	0.321	0.431	0.411	0.355	0.437	0.417	0.433	0.464	0.449	0.419
				Panel 2: Sample 1945-2003						
$E(R^e)$	0.078	0.087	0.086	0.084	0.106	0.107	0.110	0.130	0.126	0.143
$\sigma(R^e)$	0.209	0.175	0.175	0.178	0.182	0.178	0.194	0.214	0.212	0.257
$E(R^e)/\sigma(R^e)$	0.372	0.497	0.492	0.473	0.580	0.599	0.566	0.604	0.592	0.558

Table 10
Beta Estimates for Book-to-Market Decile Returns in Data - 5 Factors.

OLS regression of excess returns of the 10 book-to-market deciles on a constant, a scaled version of the collateral measure $\widetilde{my}_t$, the aggregate consumption growth rate $\Delta(\log(c_{t+1}))$, the interaction term $\widetilde{my}_t\Delta(\log(c_{t+1}))$, the aggregate expenditure share growth rate $\Delta\log(\alpha_{t+1})$, and the interaction term $\widetilde{my}_t\Delta(\log(c_{t+1}))$. These are the five risk factors in the collateral model. The first line of each panel is for the lowest book-to-market decile (growth), the last line for the highest book-to-market decile (value). The number is brackets are OLS t-statistics. In the first panel the housing collateral ratio is based on the value of outstanding mortgages, in the second panel, the housing collateral ratio is based on the value of residential real estate wealth, and in the third panel it is based on the value of residential fixed assets. The data are annual for the period 1930-2003.

	β_0	β_{my}	β_c	$\beta_{c,my}$	β_α	$\beta_{\alpha,my}$
Panel 1: Mortgage-Based Collateral						
1	0.61	12.94	−0.50	3.40	15.63	−34.80
	[5.44]	[12.38]	[1.70]	[3.89]	[6.16]	[15.09]
2	3.59	10.09	−0.86	3.20	10.85	−22.54
	[4.90]	[11.16]	[1.53	[3.51]	[5.55]	[13.60]
3	4.74	6.76	−1.83	5.16	13.62	−26.97
	[4.89]	[11.13]	[1.52]	[3.50]	[5.54]	[13.56]
4	3.64	10.73	−2.69	6.44	24.46	−40.95
	[5.35]	[12.18]	[1.67]	[3.83]	[6.06]	[14.85]
5	3.00	16.99	−3.16	7.51	22.36	−40.71
	[5.41]	[12.31]	[1.69]	[3.87]	[6.12]	[15.00]
6	0.32	18.41	−0.90	5.49	21.52	−37.33
	[5.45]	[12.41]	[1.70]	[3.90]	[6.18]	[15.12]
7	6.22	9.06	−3.55	9.47	20.90	−33.64
	[6.03]	[13.72]	[1.88]	[4.31]	[6.82]	[16.71]
8	3.90	19.38	−3.08	8.67	24.21	−42.34
	[6.55]	[14.92]	[2.04]	[4.69]	[7.42]	[18.18]
9	5.94	18.50	−4.41	9.45	23.25	−33.85
	[7.07]	[16.10]	[2.21]	[5.06]	[8.01]	[19.62]
10	4.78	21.63	−4.36	10.45	25.72	−37.27
	[8.04]	[18.29]	[2.51]	[5.75]	[9.10]	[22.29]
Panel 2: Residential Wealth-Based Collateral						
1	1.24	11.30	−0.78	4.34	15.88	−36.85
	[5.64]	[12.75]	[1.84]	[4.17]	[6.23]	[15.75]
2	6.05	3.78	−0.89	3.42	9.61	−19.25
	[5.12]	[11.57]	[1.67]	[3.79]	[5.65]	[14.30]
3	6.77	1.92	−1.76	4.95	12.23	−23.24
	[5.11]	[11.55]	[1.67]	[3.78]	[5.64]	[14.27]
4	7.22	1.93	−2.65	6.43	22.09	−34.30
	[5.65]	[12.76]	[1.84]	[4.18]	[6.24]	[15.77]
5	5.70	10.28	−3.23	7.71	20.58	−36.54
	[5.69]	[12.86]	[1.86]	[4.21]	[6.28]	[15.89]
6	2.66	12.08	−1.28	6.62	19.70	−32.80
	[5.72]	[12.93]	[1.87]	[4.23]	[6.32]	[15.98]
7	9.74	0.18	−3.99	10.43	18.55	−26.68
	[6.27]	[14.18]	[2.05]	[4.64]	[6.93]	[17.52]
8	5.72	14.60	−3.54	9.79	22.61	−38.87
	[6.82]	[15.41]	[2.23]	[5.05]	[7.53]	[19.04]
9	8.86	10.78	−4.89	10.54	20.70	−26.95
	[7.35]	[16.62]	[2.40]	[5.44]	[8.12]	[20.53]
10	7.85	13.46	−4.94	11.81	23.54	−31.84
	[8.34]	[18.85]	[2.72]	[6.17]	[9.21]	[23.30]
Panel 3: Fixed Assets-Based Collateral Measure						
1	−3.96	20.12	−0.75	4.15	15.29	−33.73
	[10.03]	[21.13]	[2.20]	[5.08]	[7.84]	[20.58]
2	6.82	0.43	−0.92	3.57	5.82	−7.92
	[8.83]	[18.60]	[1.94]	[4.47]	[6.90]	[18.11]
3	5.59	2.33	−2.17	5.97	10.53	−17.72
	[8.96]	[18.89]	[1.97]	[4.54]	[7.01]	[18.39]
4	4.84	3.68	−3.67	9.07	22.76	−35.40
	[9.85]	[20.75]	[2.16]	[4.99]	[7.70]	[20.20]
5	1.15	15.52	−4.44	10.66	21.69	−38.43
	[10.03]	[21.14]	[2.20]	[5.08]	[7.85]	[20.58]
6	−1.21	16.16	−2.14	8.72	20.36	−33.43
	[10.12]	[21.33]	[2.22]	[5.13]	[7.92]	[20.77]
7	8.97	−2.04	−5.32	13.78	17.53	−23.52
	[10.95]	[23.08]	[2.40]	[5.55]	[8.57]	[22.47]
8	1.76	17.16	−5.10	13.68	24.15	−41.89
	[12.06]	[25.40]	[2.65]	[6.11]	[9.43]	[24.73]
9	7.38	8.51	−6.67	15.12	21.10	−27.91
	[12.84]	[27.06]	[2.82]	[6.51]	[10.05]	[26.35]
10	−3.95	32.00	−7.74	18.53	29.21	−48.55
	[14.23]	[29.99]	[3.12]	[7.21]	[11.13]	[29.20]

Table 11
Model-Generated Value Premium in Collateral Model

The table reports expected returns, standard deviation and Sharpe ratio on an artificial asset generated with a set of betas listed in Table 10, but with intercept β_0 chosen so that the Euler equation is satisfied for this asset in the model. Each panel corresponds to a set of betas for a different collateral measure. The parametrization is the benchmark one with expenditure share process is an AR(1) without aggregate consumption growth term.

Decile	1	2	3	4	5	6	7	8	9	10
Panel 1: Risk Aversion 5										
Mortgage-Based Collateral Measure										
$E(R^e)$	0.01	0.01	0.01	0.01	0.02	0.03	0.03	0.03	0.02	0.03
$\sigma(R^e)$	0.06	0.04	0.05	0.07	0.07	0.10	0.10	0.10	0.09	0.12
$E(R^e)/\sigma(R^e)$	0.18	0.23	0.23	0.23	0.23	0.28	0.26	0.26	0.26	0.27
Residential Wealth-Based Collateral Measure										
$E(R^e)$	0.01	0.01	0.01	0.02	0.02	0.03	0.03	0.03	0.03	0.04
$\sigma(R^e)$	0.07	0.04	0.05	0.07	0.07	0.11	0.12	0.11	0.11	0.14
$E(R^e)/\sigma(R^e)$	0.15	0.23	0.23	0.24	0.24	0.28	0.26	0.27	0.26	0.27
Fixed Assets-Based Collateral Measure										
$E(R^e)$	0.01	0.02	0.02	0.02	0.02	0.04	0.04	0.04	0.04	0.05
$\sigma(R^e)$	0.07	0.06	0.07	0.09	0.10	0.13	0.17	0.15	0.16	0.19
$E(R^e)/\sigma(R^e)$	0.17	0.26	0.25	0.24	0.23	0.26	0.25	0.25	0.25	0.25
Panel 2: Risk Aversion 8										
Mortgage-Based Collateral Measure										
$E(R^e)$	0.01	0.01	0.02	0.02	0.02	0.04	0.04	0.03	0.03	0.05
$\sigma(R^e)$	0.06	0.04	0.05	0.06	0.07	0.10	0.10	0.10	0.09	0.12
$E(R^e)/\sigma(R^e)$	0.16	0.24	0.30	0.31	0.26	0.38	0.40	0.35	0.36	0.38
Residential Wealth-Based Collateral Measure										
$E(R^e)$	0.01	0.01	0.02	0.02	0.02	0.04	0.05	0.04	0.04	0.05
$\sigma(R^e)$	0.07	0.04	0.05	0.07	0.07	0.11	0.12	0.11	0.11	0.14
$E(R^e)/\sigma(R^e)$	0.15	0.33	0.35	0.38	0.31	0.40	0.42	0.37	0.39	0.40
Fixed Assets-Based Collateral Measure										
$E(R^e)$	0.01	0.03	0.03	0.04	0.03	0.05	0.07	0.06	0.07	0.07
$\sigma(R^e)$	0.07	0.06	0.07	0.10	0.10	0.14	0.17	0.15	0.16	0.19
$E(R^e)/\sigma(R^e)$	0.15	0.43	0.41	0.40	0.34	0.40	0.43	0.38	0.41	0.36

Table 12
Risk Premia on Portfolios of Consumption Strips.

This table reports the expected excess return, the conditional standard deviation and the Sharpe ratio on baskets of consumption strips. The consumption strips are computed for the baseline model with additive utility and $\gamma = 5, 8$. The first row denotes the duration of each basket in years. The baskets are weighted combinations of consumption strips of different horizons k, with weights governed by Ce^{ak}. Each column reports the basket for a value of parameter a ranging from -.5 to .5.

	Panel 1: Risk Aversion 5										
Duration	2.4	3.0	4.2	5.6	8.2	14.3	25.6	35.2	39.5	41.4	43.4
$E[R^e]$	0.031	0.032	0.034	0.035	0.036	0.035	0.025	0.008	−0.004	−0.011	−0.020
$\sigma[R^e]$	0.106	0.112	0.119	0.123	0.128	0.132	0.131	0.125	0.120	0.117	0.113
$E[R^e]/\sigma[R^e]$	0.290	0.288	0.285	0.285	0.283	0.266	0.190	0.065	−0.033	−0.095	−0.180
	Panel 1: Risk Aversion 8										
Duration	2.3	2.9	4.1	5.4	7.9	13.9	25.7	35.7	40.0	41.7	43.4
$E[R^e]$	0.059	0.063	0.068	0.071	0.074	0.072	0.052	0.027	0.014	0.007	0.002
$\sigma[R^e]$	0.131	0.141	0.154	0.161	0.168	0.174	0.172	0.166	0.161	0.159	0.157
$E[R^e]/\sigma[R^e]$	0.451	0.447	0.443	0.442	0.438	0.411	0.304	0.164	0.084	0.047	0.013

A. Appendix

A.1. Arrow-Debreu Equilibrium

This appendix spells out the household problem in an economy where all trade takes place at time zero.

Household Problem A household of type (θ_0, s_0) purchases a complete contingent consumption plan $\{c(\theta_0, s_0), h(\theta_0, s_0)\}$ at time-zero market state prices $\{p, p\rho\}$. The household solves:

$$\sup_{\{c,h\}} U(c(\theta_0, s_0), h(\theta_0, s_0))$$

subject to the time-zero budget constraint

$$\Pi_{s_0}\left[\{c(\theta_0, s_0) + \rho h(\theta_0, s_0)\}\right] \leqslant \theta_0 + \Pi_{s_0}\left[\{\eta\}\right],$$

and an infinite sequence of collateral constraints for each t

$$\Pi_{s^t}\left[\{c(\theta_0, s_0) + \rho h(\theta_0, s_0)\}\right] \geq \Pi_{s^t}\left[\{\eta\}\right], \forall s^t.$$

Dual Problem Given Arrow-Debreu prices $\{p, \rho\}$ the household with label (θ_0, s_0) minimizes the cost $C(\cdot)$ of delivering initial utility w_0 to itself:

$$\begin{aligned} C(w_0, s_0) \quad &= \quad \min_{\{c,h\}} \left(c_0(w_0 s_0) + h_0(w_0, s_0)\rho_0(s_0)\right) \\ &\quad + \sum_{s^t} p(s^t|s_0)\left(c_t(w_0, s^t|s_0) + h_t(w_0, s^t|s_0)\rho_t(s^t|s_0)\right) \end{aligned}$$

subject to the promise-keeping constraint

$$U_0(\{c\}, \{h\}; w_0, s_0) \geq w_0$$

and the collateral constraints

$$\Pi_{s^t}\left[\{c(w_0, s_0) + \rho h(w_0, s_0)\}\right] \geq \Pi_{s^t}\left[\{\eta\}\right], \forall s^t.$$

The initial promised value w_0 is determined such that the household spends its entire initial wealth:

$$C(w_0, s_0) = \theta_0 + \Pi\left[\{\eta\}\right].$$

There is a monotone relationship between θ_0 and w_0.

The above problem is a convex programming problem. We set up the saddle point problem and then make it recursive by defining cumulative multipliers (Marcet and Marimon (1999)). Let ℓ be the Lagrange multiplier on the promise keeping constraint and $\gamma_t(w_0, s^t)$ be the Lagrange multiplier on the collateral constraint in history s^t. Define a cumulative multiplier at each node: $\zeta_t(w_0, s^t) = 1 - \sum_{s^t} \gamma_t(w_0, s^t)$. Finally, we rescale the market state price $\hat{p}_t(s^t) = p_t(z^t)/\delta^t \pi_t(s^t|s_0)$. By using Abel's partial summation formula and the law of iterated expectations to the

Lagrangian, we obtain an objective function that is a function of the cumulative multiplier process ζ^i :

$$D(c,h,\zeta;w_0,s_0)=\sum_{t\geq 0}\sum_{s^t}\left\{\delta^t\pi(s^t|s_0)\left[\begin{array}{c}\zeta_t(w_0,s^t|s_0)\hat{p}_t(s^t)\left(c_t(w_0,s^t)+\rho_t(s^t)h_t(w_0,s^t)\right)\\+\gamma_t(w_0,s^t)\Pi_{s^t}\left[\{\eta\}\right]\end{array}\right]\right\}$$

such that

$$\zeta_t(w_0,s^t)=\zeta_{t-1}(w_0,s^{t-1})-\gamma_t(w_0,s^t),\ \zeta_0(w_0,s_0)=1$$

Then the **recursive dual** saddle point problem is given by:

$$\inf_{\{c,h\}}\sup_{\{\zeta\}}D(c,h,\zeta;w_0,s_0)$$

such that

$$\sum_{t\geq 0}\sum_{s^t}\delta^t\pi(s^t|s_0)u(c_t(w_0,s^t),h_t(w_0,s^t))\geq w_0$$

To keep the mechanics of the model in line with standard practice, we re-scale the multipliers. Let

$$\xi_t(\ell,s^t)=\frac{\ell}{\zeta_t(w_0,s^t)},$$

The cumulative multiplier $\xi(\ell,s^t)$ is a non-decreasing stochastic sequence (sub-martingale). If the constraint for household (ℓ,s_0) binds, it goes up, else it stays put.

First Order Necessary Conditions The f.o.c. for $c(\ell,s^t)$ is :

$$\hat{p}(s^t)=\xi_t(\ell,s^t)u_c(c_t(\ell,s^t),h_t(\ell,s^t)).$$

Upon division of the first order conditions for any two households ℓ' and ℓ'', the following restriction on the joint evolution of marginal utilities over time and across states must hold:

$$\frac{u_c(c_t(\ell',s^t),h_t(\ell',s^t))}{u_c(c_t(\ell'',s^t),h_t(\ell'',s^t))}=\frac{\xi_t(\ell'',s^t)}{\xi_t(\ell',s^t)}. \tag{15}$$

Growth rates of marginal utility of non-durable consumption, weighted by the multipliers, are equalized across agents:

$$\frac{\xi_{t+1}(\ell',s^{t+1})}{\xi_t(\ell',s^t)}\frac{u_c(c_{t+1}(\ell',s^{t+1}),h_{t+1}(\ell',s^{t+1}))}{u_c(c_t(\ell',s^t),h_t(\ell',s^t))}=\frac{\hat{p}_{t+1}(s^{t+1})}{\hat{p}_t(s^t)}=\frac{\xi_{t+1}(\ell'',s^{t+1})}{\xi_t(\ell'',s^t)}\frac{u_c(c_{t+1}(\ell'',s^{t+1}),h_{t+1}(\ell'',s^{t+1}))}{u_c(c_t(\ell'',s^t),h_t(\ell'',s^t))}.$$

There is a mapping from the multipliers at s^t to the equilibrium allocations of both commodities. We refer to this mapping as the risk-sharing rule.

$$c_t(\ell,s^t)=\frac{\xi_t(\ell,s^t)^{\frac{1}{\gamma}}}{\xi_t^a(z^t)}c_t^a(z^t)\text{ and }h_t(\ell,s^t)=\frac{\xi_t(\ell,s^t)^{\frac{1}{\gamma}}}{\xi_t^a(z^t)}h_t^a(z^t). \tag{16}$$

This rule flows from the optimality conditions and the market clearing conditions.

The time zero ratio of marginal utilities is pinned down by the ratio of multipliers on the promise-keeping constraints. For $t>0$, it tracks the stochastic weights ξ. From the first order condition w.r.t. $\xi_t(\ell,s^t)$ we obtain a reservation weight policy:

$$\xi_t = \xi_{t-1} \text{ if } \xi_{t-1} > \ell^c(y_t, z^t), \tag{17}$$

$$\xi_t = \ell^c(y_t, z^t) \text{ otherwise.} \tag{18}$$

and the collateral constraints hold with equality at the bounds:

$$\Pi_{s^t}\left[\left\{c_t(\ell, s^t; \underline{\xi}_t(\ell, s^t)) + \rho h^i(\ell, s^t; \underline{\xi}(\ell, s^t))\right\}\right] = \Pi_{s^t}\left[\{\eta\}\right].$$

A.2. Collateral Effect

Proof of Proposition 1 Denote the price of a claim under perfect risk-sharing by $\Pi^*[\{\cdot\}]$. Perfect risk sharing can be sustained if and only if

$$\Pi_z^*\left[\left\{c^a\left(1+\frac{1}{r}\right)\right\}\right] \geq \Pi_{z,y}^*\left[\{\eta(y,z)\}\right] \text{ for all } (y,z,r)$$

If this condition is satisfied, each household can get a constant and equal share of the aggregate non-durable and housing endowment at all future nodes. Perfect risk-sharing is possible. q.e.d.

Proof of Proposition 2 Assume utility is separable. Let $C(\ell, y_t, z^t)$ denote the cost of claim to consumption in state (y_t, z^t) for a household who enters the period with weight ξ . The cutoff rule $\ell^c(y_t, z^t)$ is determined such that the solvency constrain binds exactly: $\Pi_{y,z^t}[\{\eta\}] = C(\xi, y_t, z^t)$, where $C(\xi, y_t, z^t)$ is defined recursively as:

$$C(\xi, y_t, z^t) = \frac{\ell^c(y_t, z^t)}{\xi_t^a(z^t)}\left(1+\frac{1}{r_t}\right) + \beta\sum_{z_{t+1}}\pi(z_{t+1}|z_t)\sum_{y'}\frac{\pi(y_{t+1}, z_{t+1}|y_t, z_t)}{\pi(z_{t+1}|z_t)}m_{t+1}(z^{t+1})C(\xi', y_{t+1}, z^{t+1}),$$

and ξ' is determined by the cutoff rule. Note that the stochastic discount factor $m_{t+1}(z^{t+1})$ does not depend on $r_t(z^t)$ because we assumed that utility is separable. This also implies that the cost of a claim to labor income $\Pi_{y,z^t}[\{\eta\}]$ does not depend on r.

We first proof the result for a finite horizon version of this economy. In the last period T, the cutoff rule is determined such that:

$$\eta(y_{T-1}, z^{T-1}) = \frac{\ell^c(y_{T-1}, z^{T-1})}{\xi_T^a(z^{T-1})}\left(1+\frac{1}{r_{T-1}}\right) + \beta\sum_{z_{t+1}}\pi(z_T|z_{T-1})\sum_{y'}\frac{\pi(y_T, z_T|y_{T-1}, z_{T-1})}{\pi(z_T|z_{T-1})}m_T(z^T|z_{T-1})\frac{\xi'^{1/\gamma}}{\xi_T^a(z^T)},$$

where $\frac{\xi'^{1/\gamma}}{\xi_T^a(z^T)} \geq \eta(y_T, z^T)$. Given $r_{T-1}^1 < r_{T-1}^2$, this implies that $\ell^{1,c}(y_{T-1}, z^{T-1}) < \ell^{2,c}(y_{T-1}, z^{T-1})$ for all (y_{T-1}, z^{T-1}). By backward induction we get that, for a given sequence of $\{\xi_t^a(z^t)\}$, $\ell^{1,c}(y_t, z^t) < \ell^{2,c}(y_t, z^t)$ for all nodes (y_t, z^t) in the finite horizon economy. This in turn implies that $\{\xi_t^{a,1}(z^t)\} \leq \{\xi_t^{a,2}(z^t)\}$ for all z^t. This follows directly from the definition of

$$\begin{aligned}
\xi_t^a(z^t) &= \sum_{y^t} \int \xi_t(\ell, y^t, z^t) \frac{\pi(z^t, y^t | z_0, y_0)}{\pi(z^t | z_0)} d\Phi_0 & (19) \\
&= \sum_{y^t} \int_{\ell^c(y_t, z^t)} \xi_{t-1}(\ell, y^t, z^t) \frac{\pi(z^t, y^t | z_0, y_0)}{\pi(z^t | z_0)} d\Phi_0 & (20) \\
&\quad + \sum_{y^t} \int^{\ell^c(y_t, z^t)} \ell^c(y_t, z^t) \frac{\pi(z^t, y^t | z_0, y_0)}{\pi(z^t | z_0)} d\Phi_0 & (21)
\end{aligned}$$

$\xi_t^a(z^t)$ is non-decreasing in $\ell^c(y_t, z^t)$.

The proof extends to the infinite horizon economy if the transition matrix has no absorbing states. The reason is that $lim_{T\to\infty} E_t\left[\beta^{T-t} m_T(z^T|z_t)\pi_{z^T, y_T}\right]$ does not depend on the current state (y_t, z_t). q.e.d.

Proof of Corollary 1 Follows from the definition of the cutoff level in the previous proof. For a given sequence of $\left\{\xi_t^a(z^t)\right\}$, it is obvious that $\ell^{1,c}(y_t, z^t) < \ell^{2,c}(y_t, z^t)$ for all nodes (y_t, z^t). This in turn implies that $\left\{\xi_t^{a,1}(z^t)\right\} \leq \left\{\xi_t^{a,2}(z^t)\right\}$. This follows directly from the definition of the aggregate weight shock (21). As a result, $\xi_t^a(z^t)$ is non-decreasing in $\ell^c(y_t, z^t)$. q.e.d.

Proof of Proposition 3 The proof follows Alvarez and Jermann (2000). Appendix A.3 explains the relationship between the static and sequential budget constraints and solvency constraints.

Proof of Proposition 4 Following the definition of Alvarez and Jermann (2001), the pricing kernel M has no permanent component if

$$\lim_{k\to\infty} \frac{E_{t+1} M_{t+k}}{E_t M_{t+k}} = 1.$$

If, in our model, this condition is satisfied, then it is also true that

$$\lim_{k\to\infty} \frac{E_{t+1} M_{t+k} c_{t+k}}{E_t M_{t+k} c_{t+k}} = 1$$

since $M_{t+k} = \beta^{t+k} c_{t+k}^{-\gamma} (\xi_{t+k}^a)^\gamma$ and if

$$\lim_{k\to\infty} \frac{E_{t+1} \beta^{t+k} c_{t+k}^{-\gamma} (\xi_{t+k}^a)^\gamma}{E_t \beta^{t+k} c_{t+k}^{-\gamma} (\xi_{t+k}^a)^\gamma} = 1$$

then it has to be true for $\gamma > 1$ that

$$\lim_{k\to\infty} \frac{E_{t+1} \beta^{t+k} c_{t+k}^{1-\gamma} (\xi_{t+k}^a)^\gamma}{E_t \beta^{t+k} c_{t+k}^{1-\gamma} (\xi_{t+k}^a)^\gamma} = 1$$

because $0 \leq \beta^{t+k} c_{t+k}^{1-\gamma} (\xi_{t+k}^a)^\gamma \leq \beta^{t+k} c_{t+k}^{-\gamma} (\xi_{t+k}^a)^\gamma$ for $\gamma > 1$. As a result the new random variable $\beta^{t+k} c_{t+k}^{1-\gamma} (\xi_{t+k}^a)^\gamma$ is bounded above by the old random variable, for each state of nature. In addition each realization of the new random variable is drawn from the same probability measure as the old random variable, both at t and $t+1$.

Let the one period holding return on a period-k consumption strip be given by:

$$R_{t+1,k}^c = \frac{M_t}{M_{t+1}} \frac{E_{t+1} M_{t+k} c_{t+k}}{E_t M_{t+k} c_{t+k}},$$

then we know, from the derivation above, that

$$\lim_{k\to\infty} R^c_{t+1,k} = \frac{M_t}{M_{t+1}}.$$

Furthermore, for any return $E_t[\frac{M_{t+1}}{M_t}R_{t+1}] = 1$, we know that $E_t[\log(\frac{M_{t+1}}{M_t}R_{t+1})] \leq \log E_t[\frac{M_{t+1}}{M_t}R_{t+1}] = 0$ by Jensen's inequality. This implies that $E_t \log(\frac{M_t}{M_{t+1}}) \geq E_t \log(R_{t+1})$ or

$$E_t \log \lim_{k\to\infty} R^c_{t+1,k} = \log \frac{M_t}{M_{t+1}} \geq E_t \log(R_{t+1}) \text{ for any asset return } R_{t+1}$$

This implies that the expected log excess return exceeds that any other asset:

$$E_t \log \lim_{k\to\infty} \frac{R^c_{t+1,k}}{R_{t+1,1}} \geq E_t \log \frac{(R_{t+1})}{R_{t+1,1}}$$

Let $f(k) = Ce^{ak}$ with $a > 0$ for growth stocks. In the absence of a permanent component in the pricing kernel:

$$\lim_{a\to\infty} 1 + \widetilde{\nu}_0 = \lim_{k\to\infty} R^c_{t+1,k} \geq 1 + \nu_0 \text{ for any other sequence of weights } \{\omega_k\}$$

This implies that the highest equity premium is the one on the farthest out consumption strip. In the absence of a permanent component in the pricing kernel, there is a growth premium. q.e.d.

A.3. Sequential versus Time-Zero Constraints

We show under which conditions the sequence of budget constraints and collateral constraints in the sequential market setup can be rewritten as one time-zero budget constraint and the collection of collateral constraints shown in equation (10). The proof strategy follows Sargent (1984) (Chapter 8).

Budget Constraint First, we show how the Arrow-Debreu budget constraint obtains from aggregating successive sequential budget constraints. The sequential budget constraint is:

$$c_t(\ell, s^t) + \rho_t(z^t)h^r_t(\ell, s^t) + \sum_{s'} q_t(s^t, s')a_t(\ell, s^t, s') + p^h_t(s^t)h^o_{t+1}(\ell, s^t) \leq W_t(\ell, s^t).$$

Next period wealth is:

$$W_{t+1}(\ell, s^t, s') = \eta_{t+1}(s^t, s') + a_t(\ell, s^t, s') + h^o_{t+1}(\ell, s^t)\left[p^h_{t+1}(s^t, s') + \rho_{t+1}(s^t, s')\right].$$

Multiply the second equation by $q_{t+1}(s')$ and sum over states. Then substitute the expression for $\sum q_{t+1}(s')a_{t+1}(s')$ into the first equation.

$$c_t + \rho_t h^r_t + \sum_{s'} q_{t+1}(s')W_{t+1}(s') \leqslant W_t + \sum_{s'} q_{t+1}(s')\eta_{t+1}(s') + \\ h^o_{t+1}\left(\sum_{s'} q_{t+1}(s')\left[p^h_{t+1}(s') + \rho_{t+1}(s')\right] - p^h_t\right).$$

Similarly, for period $t+1$:

$$
\begin{aligned}
c_{t+1}+\rho_{t+1}h^r_{t+1}+\sum_{s''}q_{t+2}(s'')W_{t+2}(s'') \quad &\leqslant \quad W_{t+1}+\sum_{s''}q_{t+2}(s'')\eta_{t+2}(s'')+ \\
& h^o_{t+2}\left(\sum_{s''}q_{t+2}(s'')\left[p^h_{t+2}(s'')+\rho_{t+2}(s'')\right]-p^h_{t+1}\right).
\end{aligned}
$$

Substituting the expression for $t+1$ into the expression for t by substituting out W_{t+1}, we get:

$$
c_t+\rho_t h^r_t+\sum_{s'}q_{t+1}(s')\left[c_{t+1}+\rho_{t+1}h^r_{t+1}\right]+\sum_{s'}\sum_{s''}q_{t+1}(s')q_{t+2}(s'')W_{t+2}(s'')\leqslant
$$

$$
W_t+\sum_{s'}q_{t+1}(s')\eta_{t+1}(s')+\sum_{s'}\sum_{s''}q_{t+1}(s')q_{t+2}(s'')\eta_{t+2}(s'')+h^o_{t+1}\left(\sum_{s'}q_{t+1}(s')\left[p^h_{t+1}(s')+\rho_{t+1}(s')\right]-p^h_t\right)+
$$

$$
\sum_{s'}q_{t+1}(s')h^o_{t+2}(s')\left(\sum_{s''}q_{t+2}(s'')\left[p^h_{t+2}(s'')+\rho_{t+2}(s'')\right]-p^h_{t+1}\right).
$$

Let Π_{s^t} be the value of a dividend stream $\{d\}$ starting in history s^t priced using the market state prices $\{p\}$:

$$
\Pi_{s^t}\left[\{d\}\right]=\sum_{j\geq 0}\sum_{s^{t+j}|s^t}p_{t+j}(s^{t+j})d_{t+j}(s^{t+j}),
$$

where for a given path s^{t+j} following history s^t, p is defined as

$$
p_{t+j}(s^{t+j}|s^t)=q_{t+j}\left(s^{t+j}|s^{t+j-1}\right)q_{t+2}(s^{t+2}|s^{t+1})...q_{t+1}(s^{t+1}|s^t).
$$

Repeating the successive substitutions, the budget set is given by

$$
\Pi_{s^t}\left[\{c+\rho h^r\}\right]\leqslant W_t-\eta_t+\Pi_{s^t}\left[\{\eta\}\right] \tag{22}
$$

under 2 assumptions: (1) the transversality condition

$$
\lim_{j\to\infty}\sum_{s^{t+j}}p_{t+j}(s^{t+j})W_{t+j}(s^{t+j})=0, \tag{23}
$$

is satisfied and (2) there are no arbitrage opportunities:

$$
p^h_{t+j-1}(s^{t+j-1})=\sum_{s^{t+j}|s^{t+j-1}}q_{t+j}(s^{t+j})\left[p^h_{t+j}(s^{t+j})+\rho_{t+j}(s^{t+j})\right],\qquad \forall j\geq 0,\forall s^{t+j} \tag{24}
$$

If the latter condition were not satisfied, a household could achieve unbounded consumption by investing sufficiently high amounts in housing shares h^o and financing this by borrowing. This is a feasible strategy because ownership shares in the housing tree are collateralizable.

Because $W_0=\eta_0+\theta_0$, and relabelling $h^r_t=h_t$, we obtain the Arrow-Debreu budget constraint

$$
\Pi_{s^0}\left[\{c+\rho h\}\right]\leqslant\theta_0+\Pi_{s^0}\left[\{\eta\}\right]
$$

Collateral Constraints Second, we show the equivalence between the collateral constraints of the sequential markets setup and the solvency constraint in the static economy. The sequential collateral constraints are:

$$\left[p_t^h(z^t) + \rho_t(z^t)\right] h_{t-1}^o(s^{t-1}) + a_{t-1}(s^{t-1}, s_t) \geq 0,$$

and the collateral constraints in a history s^t:

$$\Pi_{s^t}\left[\{c + \rho h\}\right] \geq \Pi_{s^t}\left[\{\eta\}\right]. \tag{25}$$

The equivalence follows if and only if

$$a_{t-1}(s^{t-1}, s_t) + h_{t-1}^o(s^{t-1})\left[p_t^h(z^t) + \rho_t(z^t)\right] = \Pi_{s^t}\left[\{c + \rho h - \eta\}\right].$$

But this follows immediately from the budget constraint (22) holding with equality and the definition of W:

$$W_t(s^t) - \eta_t(s) = a_{t-1}(s^{t-1}, s_t) + h_{t-1}^o(s^{t-1})\left[p_t^h(z^t) + \rho_t(z^t)\right].$$

Under conditions (23) and (24) an allocation that is feasible and immune to the threat of default in sequential markets is feasible and immune to the threat of default in time-zero markets.

The equivalence implies that the allocation of home-ownership h^o is indeterminate in the sequential economy.

A.4. Recursive Preferences

In the main text, we assume additive utility. In this section, we show how the model's stochastic discount factor changes when preferences are of the Kreps and Porteus (1978) type. We show that this type of preferences is important to generate low risk premia on long horizon assets.

Preferences The household's utility at time t, V_t, is given by a composite of the utility it derives from current consumption and its future expected utility:

$$V_t = \left[(1-\delta)\left(c_t^{\frac{\varepsilon-1}{\varepsilon}} + \psi h_t^{\frac{\varepsilon-1}{\varepsilon}}\right)^{\frac{(1-\phi)\varepsilon}{\varepsilon-1}} + \delta\left(\mathcal{R}_t V_{t+1}\right)^{1-\phi}\right]^{\frac{1}{1-\phi}},$$

where future expected utility is defined by

$$\mathcal{R}_t V_{t+1} = \left(E[V_{t+1}^{1-\gamma}]\right)^{\frac{1}{1-\gamma}}$$

The coefficient ϕ is the inverse of the intertemporal elasticity of substitution, γ measures the risk aversion and ε measures the intratemporal elasticity of substitution between non-durable and housing services consumption. Additive utility is a special case with $\gamma = \phi$.

Risk-Sharing Rule The risk-sharing rule with recursive preferences takes the same form as equation (16), but the stochastic consumption weights are different:

$$c_t(\ell, s^t) = \frac{\hat{\xi}_t(\ell, s^t)^{\frac{1}{\phi}}}{\hat{\xi}^a_t(z^t)} c^a_t(z^t) \text{ and } h_t(\ell, s^t) = \frac{\hat{\xi}_t(\ell, s^t)^{\frac{1}{\phi}}}{\hat{\xi}^a_t(z^t)} h^a_t(z^t). \tag{26}$$

The new stochastic consumption weight $\hat{\xi}_t(\ell, s^t)$ equals the old stochastic consumption weight $\xi_t(\ell, s^t)$ multiplied by the utility gradient between period 0 and period t, $\mathcal{M}_{0,t}(\ell, s^t)$:

$$\mathcal{M}_{0,t}(\ell, s^t) = \prod_{s^\tau \preceq s^t} \mathcal{M}_\tau, \qquad \mathcal{M}_\tau = \left(\frac{V_\tau}{\mathcal{R}_{\tau-1} V_\tau}\right)^{\phi-\gamma}.$$

The new consumption weights still have a recursive structure:

$$\hat{\xi}_{t+1} = \left(\frac{\xi_{t+1}}{\xi_t} \mathcal{M}_{t+1}\right) \hat{\xi}_t.$$

While the individual consumption weights $\{\xi\}$ were non-decreasing processes, this is no longer true for the new stochastic weight shocks $\{\hat{\xi}\}$, because $\mathcal{M}_{t+1}$ may be less than one. Furthermore, even if the solvency constraints never bind, the new consumption weights change over time.

As before, the aggregate weight shock is the cross-sectional average of the individual stochastic consumption weights: $\hat{\xi}^a_t(z^t) = E[\hat{\xi}_t^{\frac{1}{\phi}}(\ell, s^t)]$.

Stochastic Discount Factor The stochastic discount factor still equals the marginal utility growth of the unconstrained households. It is of the same form as in the additive utility case

$$m_{t+1} = \delta \left(\frac{c^a_{t+1}}{c^a_t}\right)^{-\phi} \left(\frac{\alpha^a_{t+1}}{\alpha^a_t}\right)^{\frac{\varepsilon\phi-1}{\varepsilon-1}} \left(\frac{\hat{\xi}^a_{t+1}}{\hat{\xi}^a_t}\right)^{\phi}. \tag{27}$$

The first two terms reflect aggregate consumption growth risk and composition risk. The last term however embodies both long-run consumption growth risk and the risk of binding solvency constraints. The long-run consumption growth risk is the last term of the representative agent's SDF:

$$m^a_{t+1} = \delta \left(\frac{c^a_{t+1}}{c^a_t}\right)^{-\phi} \left(\frac{\alpha^a_{t+1}}{\alpha^a_t}\right)^{\frac{\varepsilon\phi-1}{\varepsilon-1}} \left(\frac{V^e_{t+1}}{\mathcal{R}_t V^e_{t+1}}\right)^{\frac{\phi-\gamma}{\phi}},$$

where V^e_t denotes the continuation utility of a representative agent who consumes the aggregate non-durable and housing services endowment. Epstein and Zin (1991) show that the representative agent SDF can be rewritten as a function of the gross return on an asset paying the aggregate non-durable and the aggregate housing endowment stream, $R^{c,h}$.

$$m^a_{t+1} = \delta^{\frac{1-\gamma}{1-\phi}} \left(\frac{c^a_{t+1}}{c^a_t}\right)^{-\phi\left(\frac{1-\gamma}{1-\phi}\right)} \left(\frac{\alpha^a_{t+1}}{\alpha^a_t}\right)^{\frac{\varepsilon\left(\frac{1-\gamma}{1-\phi}\right)-1}{\varepsilon-1}} \left(R^{c,h}_{t+1}\right)^{\frac{\phi-\gamma}{1-\phi}},$$

The SDF of the economy is still the product of the representative agent economy's SDF and a liquidity shock.

$$m_{t+1} = m^a_{t+1} \tilde{g}^{\phi}. \tag{28}$$

Table 13
Unconditional Asset Pricing Moments - Recursive Utility.

Same as Table 3, but preferences are of the recursive utility type with inverse intertemporal elasticity of substitution $\phi = 5$ and $\epsilon = .05$. The risk aversion parameter γ varies between 6 and 8.

γ	$E(R^{l,e})$	$E(R^{c,e})$	$E(r^f)$	$\sigma(R^{l,e})$	$\sigma(R^{c,e})$	$\sigma(r^f)$	$\frac{E(R^{l,e})}{\sigma(R^{l,e})}$	$\frac{E(R^{c,e})}{\sigma(R^{c,e})}$
6	0.044	0.028	0.061	0.178	0.121	0.067	0.248	0.235
7	0.050	0.036	0.062	0.178	0.120	0.070	0.284	0.298
8	0.060	0.041	0.068	0.179	0.118	0.061	0.337	0.347

Contrary to the aggregate weight shock $g = \frac{\xi^a_{t+1}}{\xi^a_t}$ in the case of additive utility, the new liquidity shock $\tilde{g}$ is no longer theoretically restricted to be greater than or equal to one.

Calibration For the economy with recursive preferences, we use $\phi = 5$, where ϕ is the inverse of the intertemporal elasticity of substitution.

Unconditional Asset Pricing Moments Table 13 shows the unconditional asset pricing statistics for the collateral model under recursive preferences. The equity premium on a levered consumption claim for the economy with 5 percent collateral and $\gamma = 8$ is 6 percent, excess stock returns have a volatility of 18 percent and the Sharpe ratio of the stock return is 0.33. The risk-free rate is 6.8 percent on average. These moments are of the same magnitude as the ones we found for additive preferences. However, the volatility of the risk-free rate is only half as large: 6.1 percent versus 12.5 percent under additive preferences.

Conditional Asset Pricing Moments All relationships between the housing collateral ratio and the conditional asset pricing moments carry over to the model with recursive preferences. Figure 10 shows a similar pattern for conditional moments as figure 5.

Consumption Strip Risk Premia Figure 11 and 12 plot the equity premia and Sharpe ratios on the same strips for an inverse intertemporal elasticity of substitution $\phi = 5$. We recall that when $\phi = \gamma$, we are back in the case of additive preferences. If γ is sufficiently above ϕ, the decline in equity premia and Sharpe ratios for long horizon strips is large. The equity premium on a claim to aggregate consumption two periods from now is 4 percent, while the equity premium on a claim to aggregate consumption 30 years from now is -2 percent. Claims on payoffs far in the future have a lower expected excess return than claims on payoffs in the near future. Likewise, the Sharpe ratio falls from 0.4 to -0.1. Claims on payoffs far in the future have a lower conditional Sharpe ratios than claims on payoffs in the near future.

Betas on Consumption Strips Finally, the model generates the same pattern of betas for consumption strips as was found in the data for value portfolios. A regression *inside the model* of excess returns on the consumption strips (in excess of the zero coupon bonds of the same maturity) on the asset pricing factors in equation (14) is consistent with the pattern in the betas as the one estimated for the 10 value decile portfolios in the data.

Figure 10. Summary Conditional Asset Pricing Moments - Recursive Utility
The average collateral share is 5 percent, the discount factor is .95, the coefficient of risk aversion is 7, the inverse intertemporal elasticity of substitution is 5. This the expected excess return on a claim to aggregate consumption, its conditional standard deviation and its Sharpe ratio (top row), and the conditional market price of risk, conditional price-dividend ratio and the risk-free rate, averaged over histories and plotted against the housing collateral ratio. The full line denotes the conditional moments, conditional on observing a low aggregate consumption growth rate tomorrow, whereas the dotted line denotes the moments conditional on observing a high aggregate consumption growth rate.

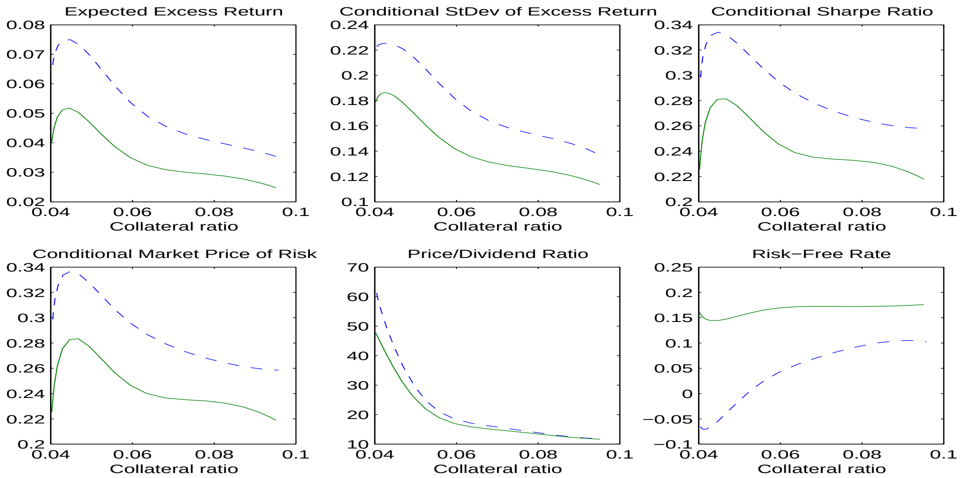
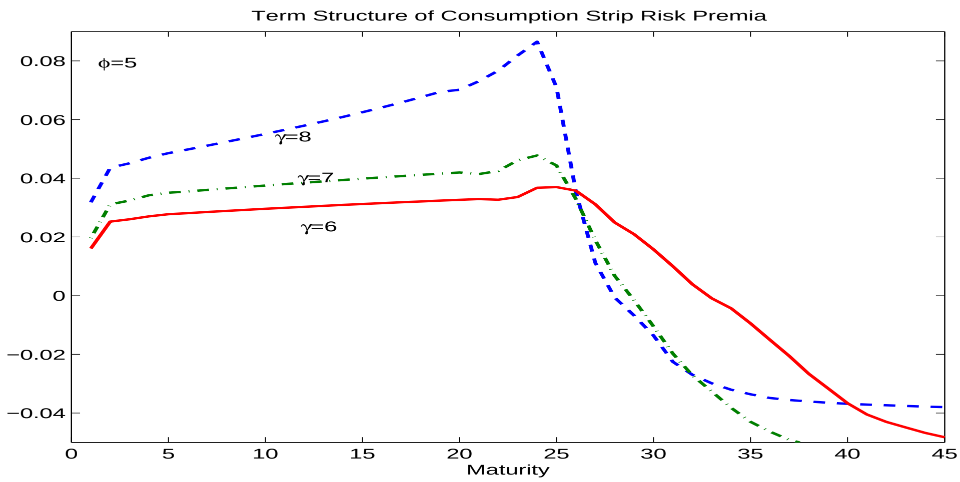
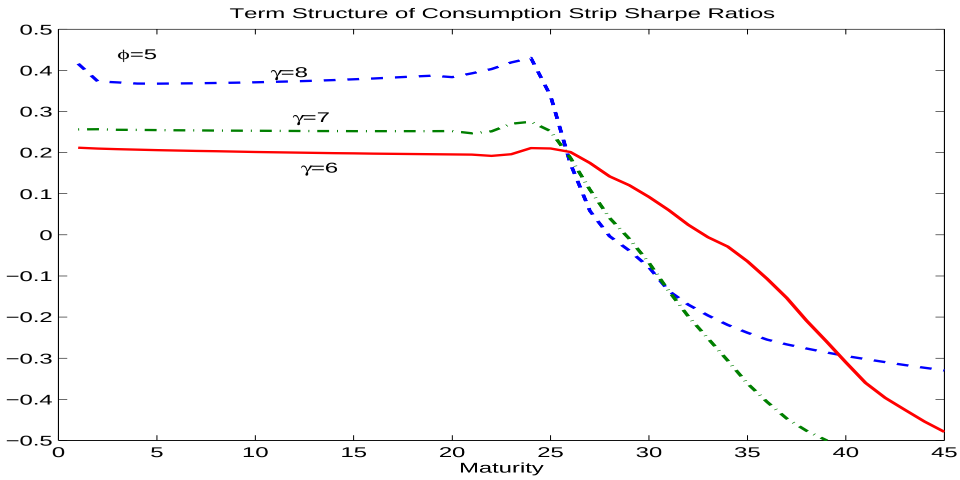


Figure 11. Term Structure of Risk Premia on Consumption Strips - Recursive Utility
The average collateral share is 5 percent, the discount factor is .95 and the coefficient of risk aversion $\gamma \in \{6, 7, 8\}$. Preferences are of the Epstein-Zin type with inverse intertemporal elasticity of substitution of $\phi = 5$. The figure plots the expected excess return on a claim to aggregate consumption k periods from now, $k = 2, 3, ..., 30$.



In the data (table 10), we found that excess returns on value stocks (short duration) had a higher correlation with consumption growth than growth stocks (long duration). The consumption growth beta, β_c, is higher for value stocks. Figure 13 shows the same pattern for the consumption strips in the model with recursive preferences. The consumption beta for a 2 year consumption strip is higher than the consumption beta for a 30 year consumption strip.

Figure 12. Term Structure of Sharpe Ratios on Consumption Strips - Recursive Utility
The average collateral share is 5 percent, the discount factor is .95 and the coefficient of risk aversion $\gamma \in \{6, 7, 8\}$. Preferences are of the Epstein-Zin type with inverse intertemporal elasticity of substitution of $\phi = 5$. The figure plots the conditional Sharpe ratio on a claim to aggregate consumption k periods from now, $k = 2, 3, ..., 30$.



In the data (table 10), we also found strong evidence that excess returns on value stocks (short duration) have a much higher correlation with consumption growth when collateral is scarce than growth stocks. I.e. the conditional correlation, conditional on my is higher. The composite consumption beta, or 'collateral beta' $\beta_c + my\beta_{c,my}$ measures this conditional correlation. When collateral is scarce ($\widetilde{my} = 1$ or $my = my_{min}$), the collateral beta for value stocks is 6.09 and 2.90 for growth stocks in the data. The model with recursive preferences generates a higher collateral beta for short duration assets ('value stocks') than for long duration assets ('growth stocks'). The full line in figure 14 illustrates how the conditional correlation decreases with rising maturity for the model with $\phi = 5$ and $\gamma = 8$.

Figure 13. Consumption Growth Betas - Recursive Utility
The average collateral share is 5 percent, the discount factor is .95 and the coefficient of risk aversion $\gamma \in \{6, 7, 8\}$. The inverse elasticity of substitution is $\phi = 5$. The figure plots the consumption growth betas β_c, obtained from a regression inside the model of excess returns of consumption strips of different horizons on the risk factors of the model (equation **??**).

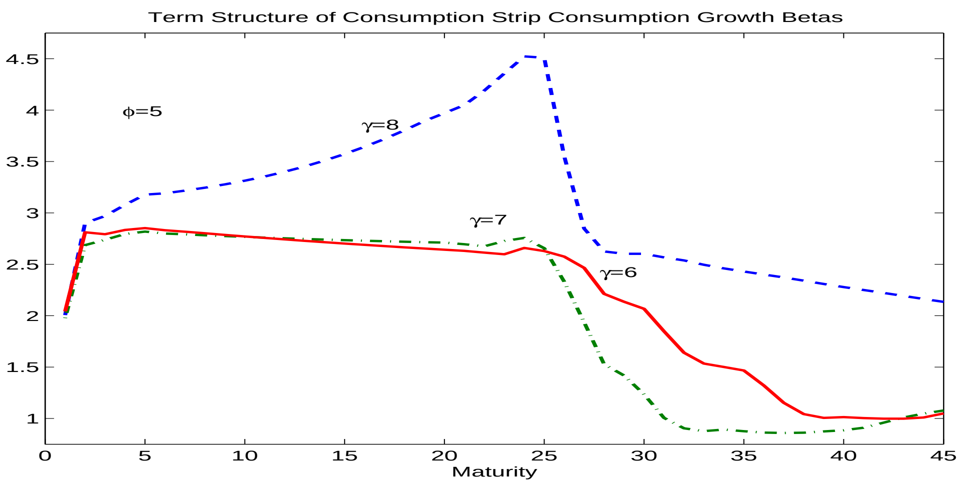
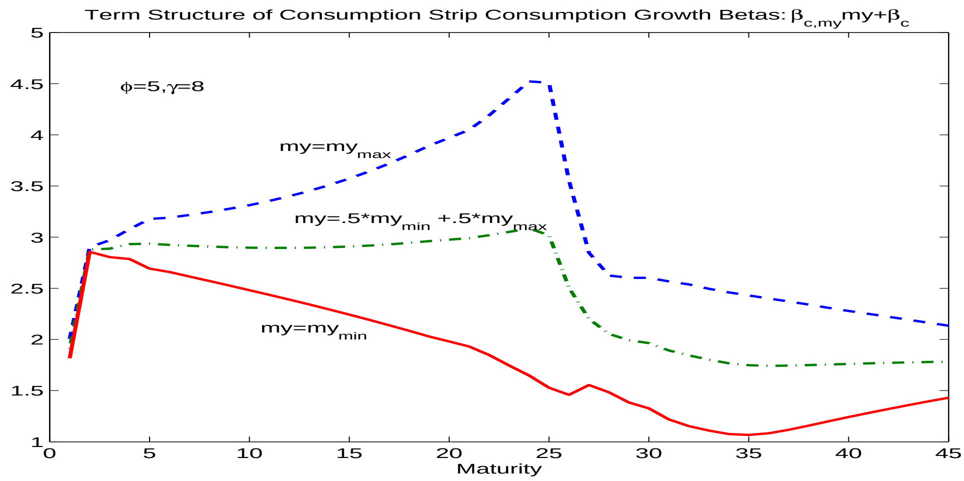


Figure 14. Collateral Betas - Recursive Utility

The average collateral share is 5 percent, the discount factor is .95 and the coefficient of risk aversion γ is 8. The inverse elasticity of substitution is $\phi = 5$. The figure plots the consumption growth betas $\beta_c + my\beta_{c,my}$, obtained from a regression inside the model of excess returns of consumption strips of different horizons on the risk factors of the model (equation **??**). The betas are plotted for periods in which the housing collateral ratio is high (full line), average (dash-dotted line) and low (dashed line).



A.5. Data Appendix

We use two sets of variables: financial variables and aggregate macroeconomic variables. All variables are annual and for the United States.

Financial Data

Aggregate Dividends Aggregate dividends are the dividends on the Standard and Poor's composite stock price index. The data are available for the period 1889-2001 from Robert Shiller's web site. The standard deviation of real dividend growth is .11 for 1930-2001.

Market Return We also include the market return R^{vw}, the value-weighted return on all NYSE, AMEX and NASDAQ stocks.

Size and Book-to-Market Portfolios We use ten portfolios of NYSE, NASDAQ and AMEX stocks, grouped each year into ten value (book-to-market ratio) bins. Book-to-market is book equity at the end of the prior fiscal year divided by the market value of equity in December of the prior year. Portfolio returns are value-weighted. All returns are expressed in excess of an annual return on a one-month Treasury bill rate (from Ibbotson Associates). The returns are available for the period 1926-2002 from Kenneth French's web site and are described in more detail in Fama and French (1992).

Aggregate Macroeconomic Data

Consumption and Income Consumption is non-durable consumption C, measured by total expenditures minus apparel and minus rent and imputed rent. The housing expenditure ratio, r, is the ratio of non-durable expenditures to rent expenditures.

The income endowment in the model corresponds to an after-government income concept; it includes net transfer income. Aggregate income Y is labor income plus net transfer income. Nominal data are from the National Income and Product Accounts for 1930-2002. Consumption and income are deflated by the consumer price index and divided by the number of households N.

Price Indices Aggregate rental prices ρ_t are constructed as the ratio of the CPI rent component p_t^h and the CPI food component p_t^c. Data are for urban consumers from the Bureau of Labor Statistics for 1926-2001. The price of rent is a proxy for the price of shelter and the price of food is a proxy for the price of non-durables. We use the rent and food components because the shelter and non-durables components are only available from 1967 onwards. Two-thirds of consumer expenditures on shelter consists of owner-occupied housing. The BLS uses a rental equivalence approach to impute the price of owner-occupied housing. Because ρ_t is a relative *rental* price, our theory is conceptually consistent with the BLS approach. We also use the all items CPI, p_t^a, which goes back to 1889. All indices are normalized to 100 for the period 1982-84.

Housing Collateral We use three distinct measures of the housing collateral stock HV: the value of outstanding home mortgages (mo), the market value of residential real estate wealth (rw) and the net stock current cost value of owner-occupied and tenant occupied residential fixed assets (fa). The first two time series are from the Historical Statistics for the US (Bureau of the Census) for the period 1889-1945 and from the Flow of Funds data (Federal Board of Governors) for 1945-2001. The last series is from the Fixed Asset Tables (Bureau of Economic Analysis) for 1925-2001.

We use both the value of mortgages HV^{mo} and the total value of residential fixed assets HV^{rw} to be robust to changes in the extent to which housing can be used as a collateral asset. We use both HV^{rw}, which is a measure of the value of housing owned by households, and HV^{fa} which is a measure of the value of housing households live in, to be robust to changes in the home-ownership rate over time. Real per household variables are denoted by lower case letters. The real, per household housing collateral series $hv^{mo}, hv^{rw}, hv^{fa}$ are constructed using the all items CPI from the BLS, p^a, and the total number of households, N, from the Bureau of the Census.

Housing Collateral Ratio Log, real, per household real estate wealth ($\log hv$) and labor income plus transfers ($\log y$) are non-stationary. According to an augmented Dickey-Fuller test, the null hypothesis of a unit root cannot be rejected at the 1 percent level. This is true for all three measures of housing wealth ($hv = mo, rw, fa$).

If a linear combination of $\log hv$ and $\log y$, $\log(hv_t) + \varpi \log(y_t) + \chi$, is trend stationary, the components $\log hv$ and $\log y$ are said to be stochastically cointegrated with cointegrating vector $[1, \varpi, \chi]$. We additionally impose the restriction that the cointegrating vector eliminates the deterministic trends, so that $\log(hv_t) + \varpi \log(y_t) + \vartheta t + \chi$ is stationary. A likelihood-ratio test (Johansen and Juselius (1990)) shows that the null hypothesis of no cointegration relationship can be rejected, whereas the null hypothesis of one cointegration relationship cannot. This is evidence for one cointegration relationship between housing collateral and labor income plus transfers. We estimate the cointegration coefficients from vector error correction model:

$$\left[\begin{array}{c}\Delta\log\left(hv_t\right)\\ \Delta\log\left(y_t\right)\end{array}\right]=\alpha\left[\log\left(hv_t\right)+\varpi\log\left(y_t\right)+\vartheta t+\chi\right]+\sum_{k=1}^{K}D_k\left[\begin{array}{c}\Delta\log\left(hv_{t-k}\right)\\ \Delta\log\left(y_{t-k}\right)\end{array}\right]+\varepsilon_t. \tag{29}$$

The K error correction terms are included to eliminate the effect of regressor endogeneity on the distribution of the least squares estimators of $[1,\varpi,\vartheta,\chi]$. The housing collateral ratio my is measured as the cointegration relationship:

$$my_t=\log\left(hv_t\right)+\hat{\varpi}\log\left(y_t\right)+\hat{\vartheta}t+\hat{\chi}.$$

The OLS estimators of the cointegration parameters are superconsistent: They converge to their true value at rate $1/T$ (rather than $1/\sqrt{T}$). The superconsistency allows us to use the housing collateral ratio my as a regressor without need for an errors-in-variables standard error correction.