Mergers as Reallocation

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Abstract

We model merger waves as reallocation waves, and argue that mergers spread new technology in a way that is similar to that of entry and exit of firms. We focus on two periods: 1890-1930 during which electricity and the internal combustion engine spread through the U.S. economy, and 1971-2001 – the Information Age.

1 Introduction

The Q-theory of investment implies that reallocation waves should be times when the dispersion in Q's among firms is high. Capital flows from low-Q firms to high-Q firms. Eisfeldt and Rampini (2002) have recently made this explicit in a business cycle model and found that reallocation of capital and the dispersion of Q are both pro-cyclical. Jovanovic and Rousseau (2002) have shown that the reallocation of capital via merger also responds to the dispersion in Q. Here we formulate a theory of economy-wide merger waves as reallocation episodes prompted by the arrival of major technologies that raise the dispersion in Q among existing and potential new firms. Through the lens of our model, we study the 20th Century and argue that of the five major merger waves, all but the middle wave came about because of the pressure to reallocate capital, pressure that came from two general-purpose technologies – electricity and information technology.

When adopting a new technology, a firm may re-train some of its workers and replace others; it can re-fit its buildings and equipment, where possible, and replace the rest. If it fails in the attempt to reorganize internally, the firm will probably disappear and its assets will be reorganized *externally*. In that case the firm will either liquidate, or it will be taken over. Either way, however, the existing human and physical capital simply changes management. New technology spreads faster if such reallocation works smoothly. This paper studies these mechanisms.

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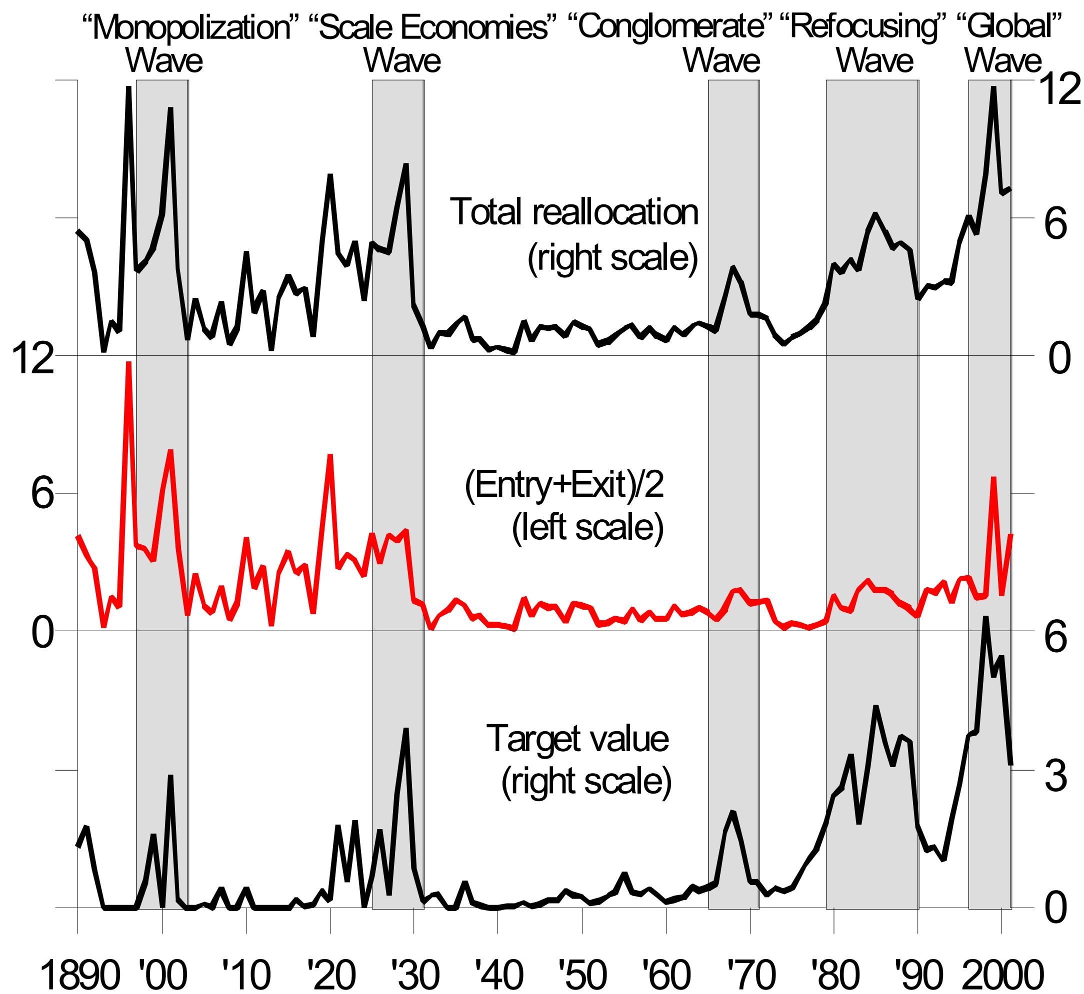


Figure 1: Reallocated capital and its components as percentages of stock market value, with merger waves shaded, 1890-2001.

We study and measure, in particular, the two external adjustment mechanisms – mergers and entry-and-exit (E&E) – using the stock-market capitalization of the firms involved. In Figure 1, the U-shaped top line is our estimate of the total amount of capital that has been reallocated on the U.S. stock market over the past 112 years. Its components are the stock market capitalization of entering and exiting firms divided by two, and the value of merger targets. E&E, given by the center line, is a rough measure of how much capital exits from the stock market and comes back in under different ownership, or at least under a different name.[1] The lower line is

[1] Entries and exits for 1926-2001 are defined as firms that enter or leave the stock files distributed by the University of Chicago's Center for Research in Securities Prices (CRSP). Delisting from CRSP can occur due to liquidation, bankruptcy, financial distress, or lack of investor interest.

the stock-market value of merger targets.[2]

The bottom panel shows the five merger waves and at the very top we list the names commonly given to these waves.[3] This paper will argue, however, that the first two waves represent a form of external reallocation of resources in response to the simultaneous arrival of two general purpose technologies (GPTs) – electricity and (to a lesser extent) internal combustion – and that the last two represent reallocation in response to the arrival of the microcomputer and information technology (IT). The middle "Conglomerate" wave, which is sometimes attributed to "managerial hubris," does not seem to fit our story. Two specific points emerge in Figure 1:

1. Each merger wave was accompanied by a rise in E&E. The deviations from trend are positively related – the correlation is 0.46.

2. Total reallocation has no significant trend, but mergers have grown relative to E&E – the ratio rises by a factor of 9, from 0.18 in the 1890's to 1.63 in the 1990's.

Fact 1 arises, we argue, because society will use both margins of external adjustment in response to a technological shock. Fact 2 arises, we believe, because of the increased importance of teamwork and organization capital, which also has caused market values of companies to rise relative to their book values.

Our contrast of two periods of major technological change – electrification (1890-1930) and IT (1970-2002) is in the spirit of David (1991).

Before assigning a firm as an "exit" we check the list of hostile takeovers from Schwert (2000) for 1975-1996 and individual issues of the *Wall Street Journal* for 1997-2001 to ensure that we record firms taken private under hostile tender offers as mergers. For 1885-1925, we identify entries and disappearances from the New York Stock Exchange (NYSE) using contemporary newspapers. The stock market capitalizations used to form the ratios in Figure 1 are also from CRSP for 1925-2001 and our backward extension of CRSP for earlier years. For the pre-1925 period, prices and par values are from the *The Commercial and Financial Chronicle,* which is also the source of firm-level data for the price indexes reported in the Cowles Commission's *Common Stock Prices Indexes* (1939), and book capitalizations are from *Bradstreet's*, *The New York Times*, and *The Annalist*. The resulting dataset, though limited to annual observations, actually includes more common stocks than the CRSP files in 1925. Coverage for the NYSE begins in 1885, the AMEX in 1962, and NASDAQ in 1972.

[2]We identify targets for 1926-2001 as the 9,236 firms that exited CRSP due to merger. For 1895-1930 we use the original worksheets for mergers in the manufacturing and mining sectors from Nelson (1959), and for 1885-1894 we use the financial news section of weekly issues of the *Commercial and Financial Chronicle.* The target series in Figure 1 includes the market values of exchange-listed firms in the year prior to their acquisition.

[3]We define the shaded merger "waves" as starting when the series for target value stays above a tightly-specified HP trend (λ=100 in the RATS filter program) for two or more consecutive years. The wave "ends" when the series falls below trend for two consecutive years.

2 Model

First we describe a standard one-technology "Ak" model; we then add a second technology with its own capital that suddenly and unexpectedly becomes available.

2.1 One-technology model

Preferences are

$$\frac{1}{1-\sigma}\int_0^\infty e^{-\rho t}c_t^{1-\sigma}dt,$$

aggregate output is

$$y = zk,$$

capital evolves as

$$\dot{k} = -\delta k + x,$$

and the income identity is

$$y = c + x.$$

Equating the marginal product of capital, z, to the user cost of capital, $r + \delta$, and substituting into the consumer's first-order conditions for optimal consumption $\dot{c}/c = (r - \rho)/\sigma$ gives us the constant-growth-rates of income and consumption

$$\frac{\dot{y}}{y} = \frac{\dot{c}}{c} = \frac{z - \delta - \rho}{\sigma}.$$

This model has no transitional dynamics because it is linear in a single state variable, k, and there are no adjustment costs.

2.2 A second technology arrives

Starting from a state in which all its capital embodies a technology z_1, how does the economy transit to a state in which all its capital embodies technology z_2? If the arrival of z_2 at $t = 0$ was unexpected, the growth rate before the transition would have been $(z_1 - \delta - \rho)/\sigma$, and after the transition is over at date T the growth rate will be $(z_2 - \delta - \rho)/\sigma$. For the intervening T periods, two kinds of capital coexist, k_1 and k_2. This is the era of reallocation.

De novo investment and upgrading New capital can be produced from the consumption good, or from old capital.

De novo entry of k_2.—As is usual in one-sector growth models, the production function for new capital (not counting depreciation) is

$$\dot{k}_2 = x_2, \tag{1}$$

where x_2 is the consumption foregone for the purpose of creating k_2.

Two technologies for converting k_1 into k_2 or into c.—We shall model these upgrading costs as convex costs of adjustment. We assume two distinct upgrading activities, one of which involves only k_1 while the other requires both k_1 and k_2. The intuition is easiest if we imagine that k_1 and k_2 must reside in different firms – call these z_1-firms and z_2-firms.

1. *Conversion via "Exit".* Let Δ_1 be amount of k_1 that the z_1-firms retire and convert into an equal number, Δ_1, of units of the consumption good. In so doing, they forego
$$\psi\left(\frac{\Delta_1}{k_1}\right)k_1$$
units of output. Assume that ψ is increasing, convex and differentiable with $\psi'(0)=0$. This adjustment cost is homogeneous of degree 1 in (Δ_1, k_1).

2. *Conversion via "Acquisition".* Let Δ_2 be the total amount of k_1 that the z_2-firms acquire from z_1-firms and convert into Δ_2 units of k_2. In so doing, they forego
$$\phi\left(\frac{\Delta_2}{k_2}\right)k_2$$
units of output. Assume that ϕ is increasing, convex and differentiable with $\phi'(0)=0$. This adjustment cost is homogeneous of degree 1 in (Δ_2, k_2).

Output and the evolution of k_1 and k_2. During the transition, $t \in [0, T]$, and both k_1 and k_2 are used. Net of upgrading costs, output is
$$y = (z_1 - \psi[\varepsilon])\,k_1 + (z_2 - \phi[m])\,k_2, \tag{2}$$
where
$$\varepsilon \equiv \frac{\Delta_1}{k_1}$$
is the exit rate of k_1, and
$$m = \frac{\Delta_2}{k_2}$$
is the acquisitions rate relative to k_2. Consumption is
$$c = y - x_1 - x_2.$$
The two capital stocks evolve as follows:
$$\dot{k}_1 = -\delta k_1 + x_1 - (\varepsilon k_1 + m k_2) \tag{3}$$
$$\dot{k}_2 = -\delta k_2 + x_2 + \varepsilon k_1 + m k_2. \tag{4}$$
These two laws of motion are standard but for the reallocation term $\varepsilon k_1 + m k_2$, which is subtracted from the right-hand side of (3) and added back in (4).

2.3 Equilibrium

Equilibrium consists of m, ε, x_1, and x_2 such that firms maximize and the representative agent consumes optimally. The initial conditions are $k_{1,0} = 1$, $k_{2,0} = 0$, and the aggregate laws of motion (3) and (4) hold with the added restriction that $k_{1,t} \geq 0$. The model has neither external effects nor monopoly power and the Appendix uses the planner's problem to derive the equilibrium formally. In this section we shall give the market-economy interpretation.

Upgrading.—Let q be the price of k_1, and Q the price of k_2. Optimal upgrading by z_1-firms implies that

$$\psi'(\varepsilon) = Q - q \tag{5}$$

and optimal upgrading by z_2-firms implies that

$$\phi'(m) = Q - q. \tag{6}$$

In both cases the replacement cost for k_1 is q, and the upgraded capital has a price of Q. The difference between the two is equated, in (5) and (6), to the marginal cost of adjustment.[4]

Investment.—We assume that $x_2 > 0$. Then

$$Q = 1.$$

On the other hand, it will turn out that $q < 1$ for all $t \in [t, T)$, and therefore $x_1 = 0$ throughout the transition.

Output and upgrading rents.— k_1 and k_2 play a dual role here. Each produces output, and each assists in the upgrading process. Upgrading is subject to increasing marginal costs and so, in equilibrium, entails a rent. The per-unit upgrading rent that k_1 draws is

$$\pi^{\varepsilon}(q) \equiv \max_{\varepsilon} \{\varepsilon - (q\varepsilon + \psi[\varepsilon])\},$$

and the per-unit rent that k_2 draws is

$$\pi^{m}(q) \equiv \max_{m} \{m - (qm + \phi[m])\}.$$

Consumption growth during the transition is

$$\frac{\dot{c}}{c} = \frac{1}{\sigma}(z_2 + \pi^m(q) - \rho - \delta) \tag{7}$$

and the rate of interest is

$$r = z_2 - \delta + \pi^m(q).$$

Output in (2) rises monotonically because, by (5) and (6), ε and m both decline monotonically. This is driven by the monotonic rise in q that we are about to show.

[4] In our partial-equilibrium treatment of takeovers as an investment (Jovanovic and Rousseau 2002), the equivalent of (6) is eq. (8). That paper also assumes adjustment costs on x which we have suppressed here in order to keep the analysis manageable.

The monotonic rise in q during the transition If we can solve for q, we shall be able to infer ε, m, $\pi^{\varepsilon}(q)$, $\pi^{m}(q)$, $\dot{c}/c$, and r. The price of k_1 must be such that the marginal product of k_1 equals its user cost:

$$z_1 + \pi^{\varepsilon}(q) = (r + \delta) q - \dot{q}.$$

Since $\dot{Q} = 0$, the corresponding condition for k_2 is

$$z_2 + \pi^{m}(q) = r + \delta.$$

Combining these two conditions and eliminating r we are left with[5]

$$\frac{\dot{q}}{q} = z_2 + \pi^{m}(q) - \frac{(z_1 + \pi^{\varepsilon}[q])}{q}. \tag{8}$$

Let q^* be the largest value of q at which

$$z_2 + \pi^{m}(q) = \frac{(z_1 + \pi^{\varepsilon}[q])}{q}$$

for all $t \in [0, T]$. Since $\pi^{m}(q) = \pi^{\varepsilon}(q) = 0$ when $q \geq 1$, we have $0 < q^* < 1$. This rest-point q^* is unstable from above:

$$q > q^* \quad \Longrightarrow \quad \dot{q} > 0.$$

But q must approach unity as $t \to T$ because as of date T, $k_{1,t}$ becomes zero and ε_t and m_t must both become zero. That is, since $\phi'(0) = 0$, a unit of k_1 is at date T as valuable as a unit of k_2 because it can be upgraded costlessly. It must therefore be that

$$q_0 \in (q^*, 1) \text{ and } q_T = 1$$

and, from (8), that $\dot{q} > 0$ throughout the transition. Finally, $\dot{q}_T = z_2 - z_1$. Figure 2 illustrates the solution for q_t.

2.4 Summary of implications

The qualitative implications are as follows:

1. At $t = 0$, output falls from $z_1 k_1$ to $(z_1 - \psi[\varepsilon_0]) k_1$ and then starts to rise monotonically.

2. The value of capital also falls from 1 to q_0. Wealth falls from $k_{1,0}$ to $q_0 k_{1,0}$.

3. Thereafter, q_t rises monotonically to 1, and k_1 falls monotonically to zero at date T, as do ε and m.

[5] This equation is derived for the planner's shadow price of k_1 in (17) of the Appendix.

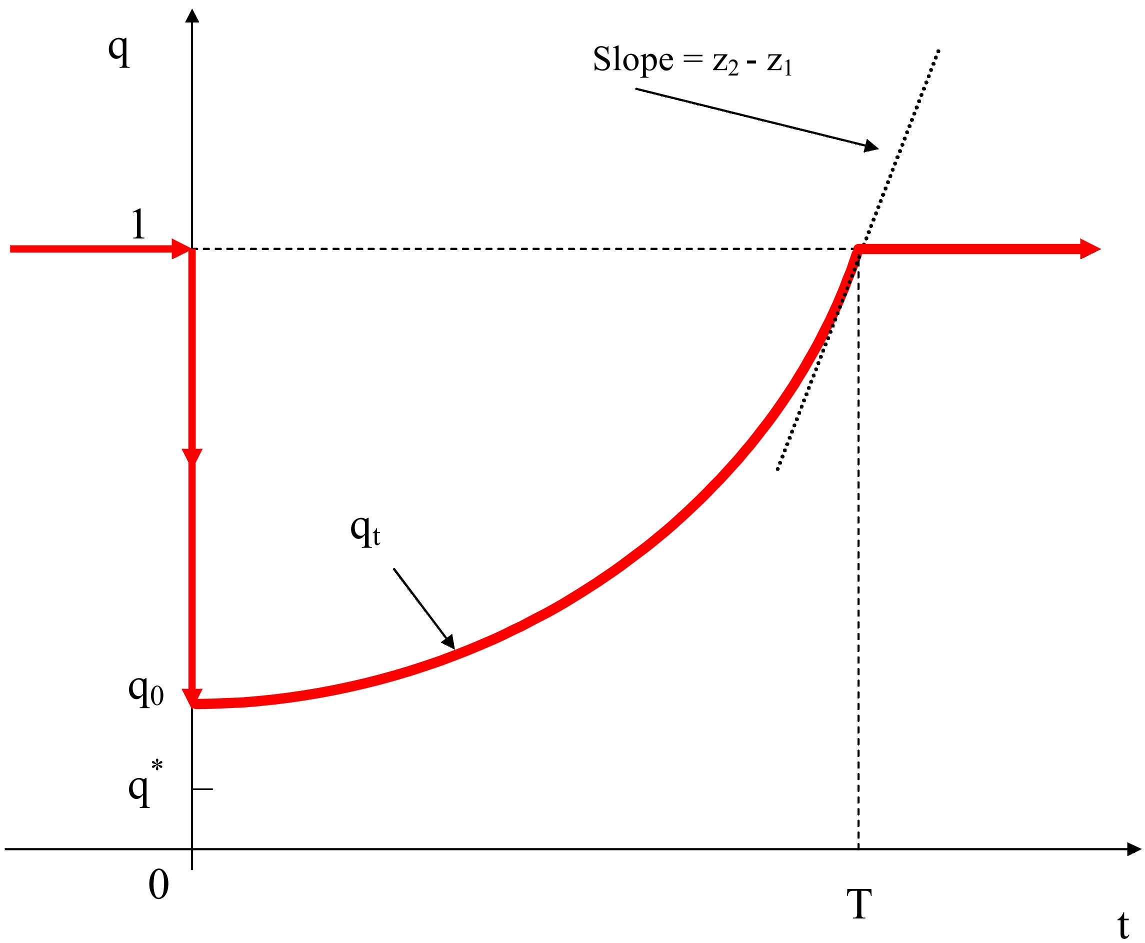


Figure 2: The solution for q_t.

4. Total exits, $q\varepsilon k_1$, decline monotonically, whereas total acquisitions, qmk_2, start and end at zero and are, essentially, inverted U-shaped during the transition.

5. The rate of interest jumps from $z_1 - \delta$ to $z_2 - \delta + \pi^m(q_0)$ and then declines monotonically to $z_2 - \delta$ where it remains thereafter.

6. Consumption falls at date zero. After that consumption growth declines monotonically. More precisely,

$$g_c = \begin{cases} \frac{z_1 - \delta - \rho}{\sigma} & \text{for } t < 0 \\ \frac{z_2 + \pi^m(q_t) - \delta - \rho}{\sigma} & \text{for } t \in (0, T) \\ \frac{z_2 - \delta - \rho}{\sigma} & \text{for } t \geq T. \end{cases}$$

3 Simulations

We now simulate the model. We assume that $\sigma = 1$ and that adjustment costs are quadratic:

$$\phi(m) = \frac{m^2}{2\mu} \quad \text{and} \quad \psi(\varepsilon) = \frac{\varepsilon^2}{2\nu}. \tag{9}$$

The date-0 initial conditions are

$$k_1 = 1 \quad \text{and} \quad k_2 = 0,$$

and the other boundary conditions are

$$k_{1,T} = 0, \tag{10}$$

and

$$q_T = 1. \tag{11}$$

Finally, because the shock is unforeseen, the present value of consumption as of date zero (just *after* the shock) equals wealth, which is just $q_0 k_{1,0}$. Since $k_{1,0} \equiv 1$, the last condition is

$$q_0 = \int_0^T \exp\left(-\int_0^t r_s ds\right) c_t dt + \exp\left(-\int_0^T r_s ds\right) \frac{c_T}{\rho} \tag{12}$$

because $t \geq T$, $r - g = (z_2 - \delta) - (z_2 - \delta - \rho) = \rho$.

Parameter choices are reported in Figures 3 and 4. We will focus the empirical discussion on the three parameters that summarize the gains to reallocation $(z_2 - z_1)$ and the costs (μ, ν). We will discuss the micro evidence on these parameters in detail later.

Parameter	Assumed Value		Source
	Electricity	IT	
$z_2 - z_1$	0.015		Maksimovic and Phillips (2001), Harris, Siegel, and Wright (2002)
ν	2.7,	0.06	Physical k: Ramey and Shapiro (2001), Human k: Neal (1995)
μ	2.7,	0.6	Jovanovic and Rousseau (2002), Ravenscraft and Scherer (1987)

If we assume that discounting is at a rate of five percent per year, the value of $\rho = 0.10$ determines the period-length at 2 years. In both of the simulations we have assumed that the new technology doubles the rate of growth from $z_1 - \rho = 0.015$, or three-quarters of a percent per year before the transition, to $z_2 - \rho = 0.03$, or 1.5 percent per year after the transition. The adjustment-cost parameters, μ and ν, are set much higher implying a far smaller adjustment cost than the micro evidence (see Sections 4.2 and 4.3 below) suggests. Our main interest is in how the diffusion of the new technology is implemented. The three ways in which k_2/k_1 grows are:

1. *Acquisitions.* Relative to market capitalization, acquisitions are

$$M = \frac{qmk_2}{k_2 + qk_1}. \tag{13}$$

2. *Exits.* Relative to market capitalization, exit is

$$E = \frac{q\varepsilon k_1}{k_2 + qk_1}, \tag{14}$$

   and E must decline on average from ε at $t = 0$ to zero at date T.

3. *De novo investment.*

$$X = \frac{x_2}{k_2 + qk_t}.$$

These three series are plotted in the upper left panels of Figures 3 and 4. During the electricity period, exits were several times as important as acquisitions, and this is the main result that we obtain in Simulation 1 (Figure 3), along with a transition that lasts 32 years. If teamwork and organization capital have indeed become more important and the cost of destroying them has risen, this implies a fall in ν. Simulation 2 (Figure 4) raises the ratio μ/ν by a factor of 10, and although it achieves the needed substitution of acquisitions for exits, the transition still takes 32 years.

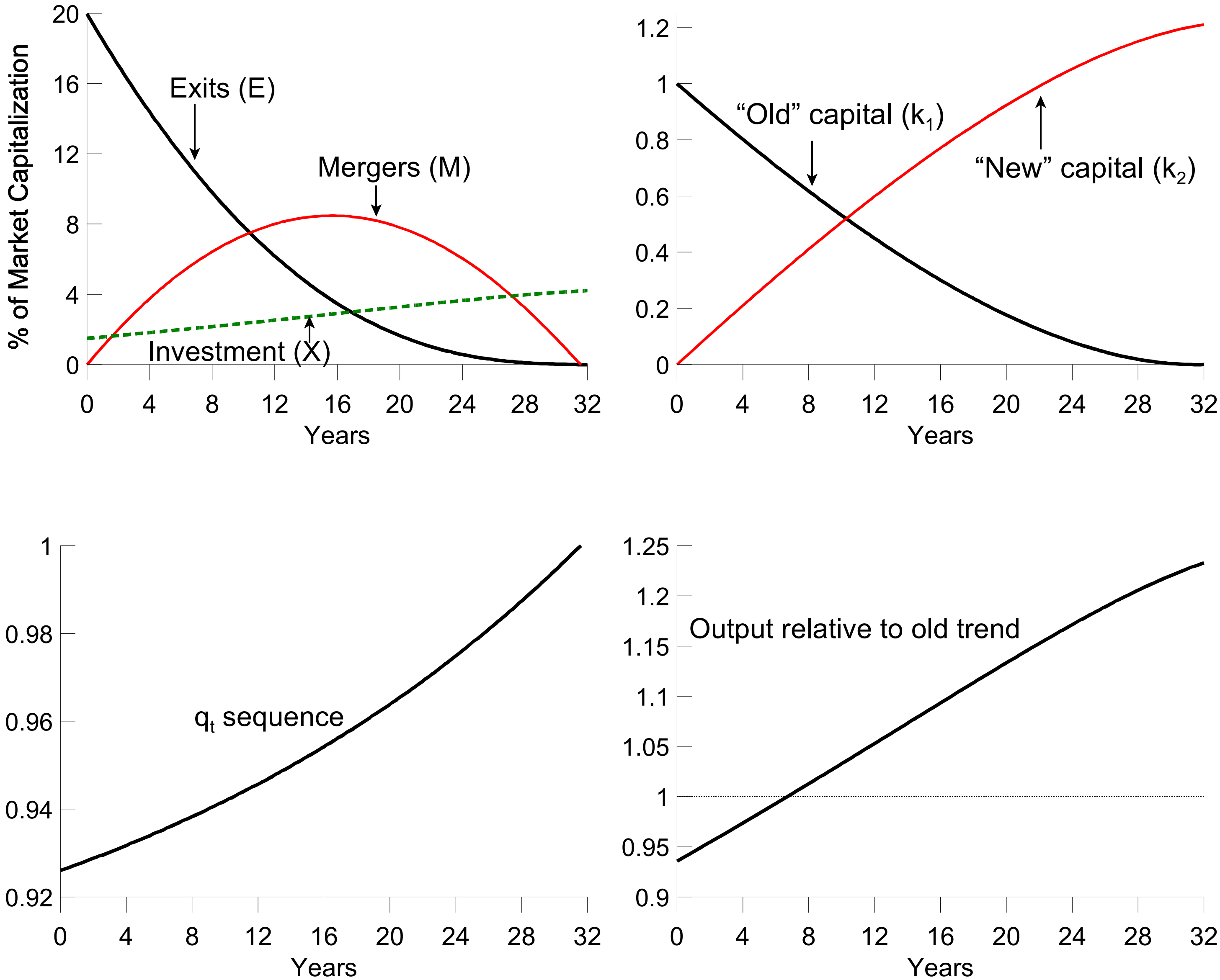
Simulation 1 shows k_2 overtaking k_1 after 10 years. When we raise the adjustment costs in simulation 2, Figure 4 shows that overtaking does not occur until the 17th year. Since, in Simulation 2, ϕ and ψ are much higher than in Simulation 1, it is surprising that we do not see more substitution towards X. Exits lead acquisitions in both simulations. Finally in the last panel we plot the new productivity relative to old trend and find that the productivity slowdown lasts about 7 years in both simulations.

Now we compare the simulations with the aggregate data. In the upper left panels of Figures 3 and 4 we simulated M, E, and X, and now we look at their actual behavior. Figure 5 is the empirical counterpart.

Acquisitions should be inverted-U in that a merger wave must begin and end at zero. Figure 5 shows that mergers crest during the second half of each transition.

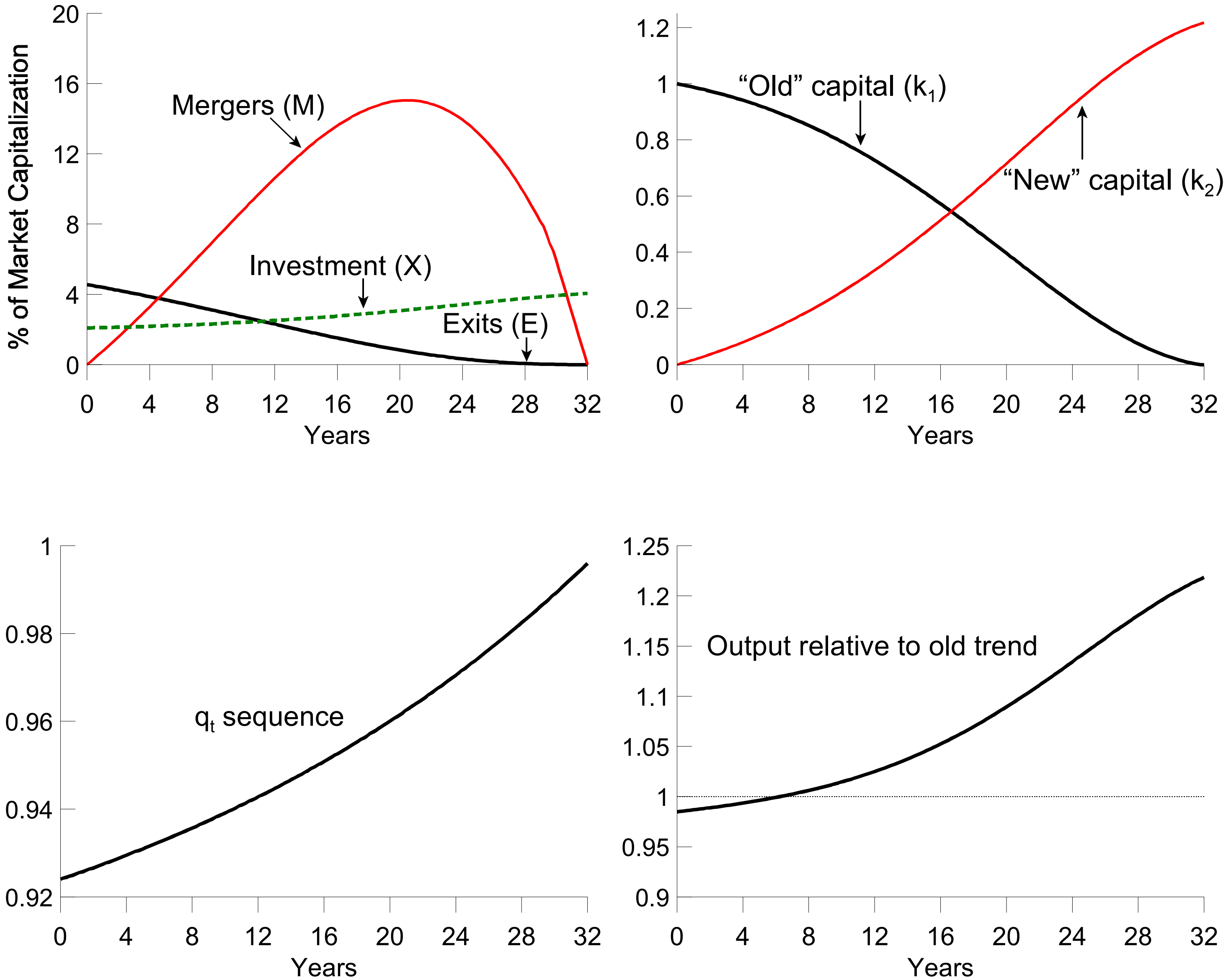
Since k_1 is decreasing, total exits should fall over the transition. Figure 5 shows that exits have a slight negative trend, though the T-statistics in regressions of exits on trend are only 1.27 for the electricity era and 0.90 for the IT era.

We also simulated X in Figures 3 and 4, but in practice we do not know the investment for firms that actually traded on the stock market. For the economy as a whole, investment net of residential structures averaged 10.5% of GDP for 1890-1930



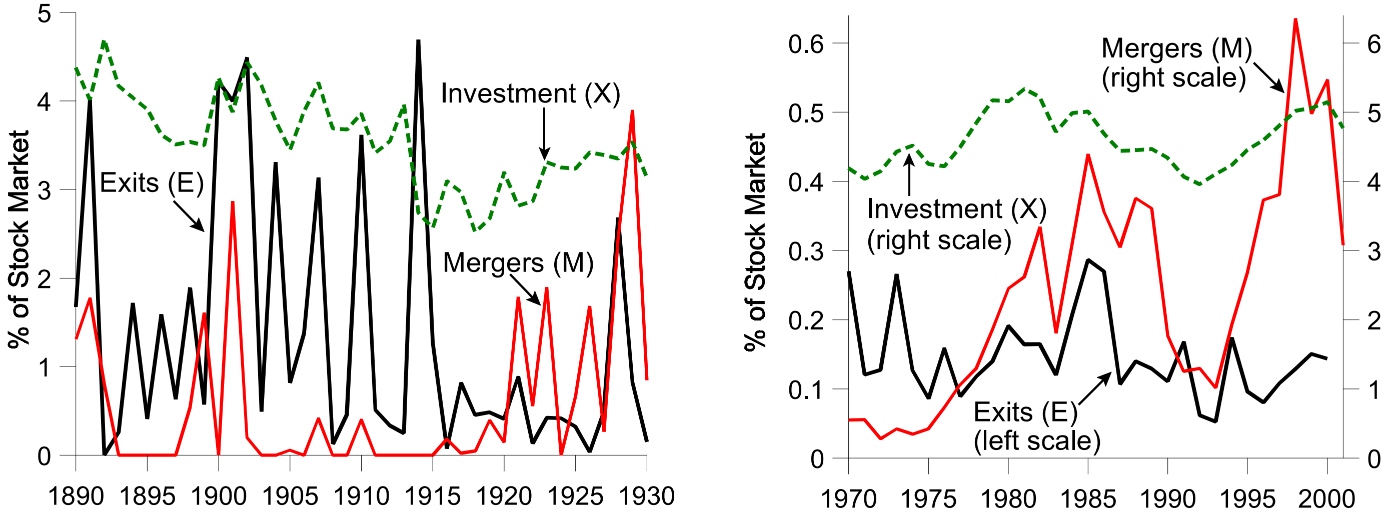
Model settings:
$z_1 = 0.115$, $z_2 = 0.130$, $\mu = 2.7$, $\nu = 2.7$, $\rho = 0.1$, $\sigma = 1$, $\delta = 0$.

Figure 3. Transitional dynamics for "Electricity" model.



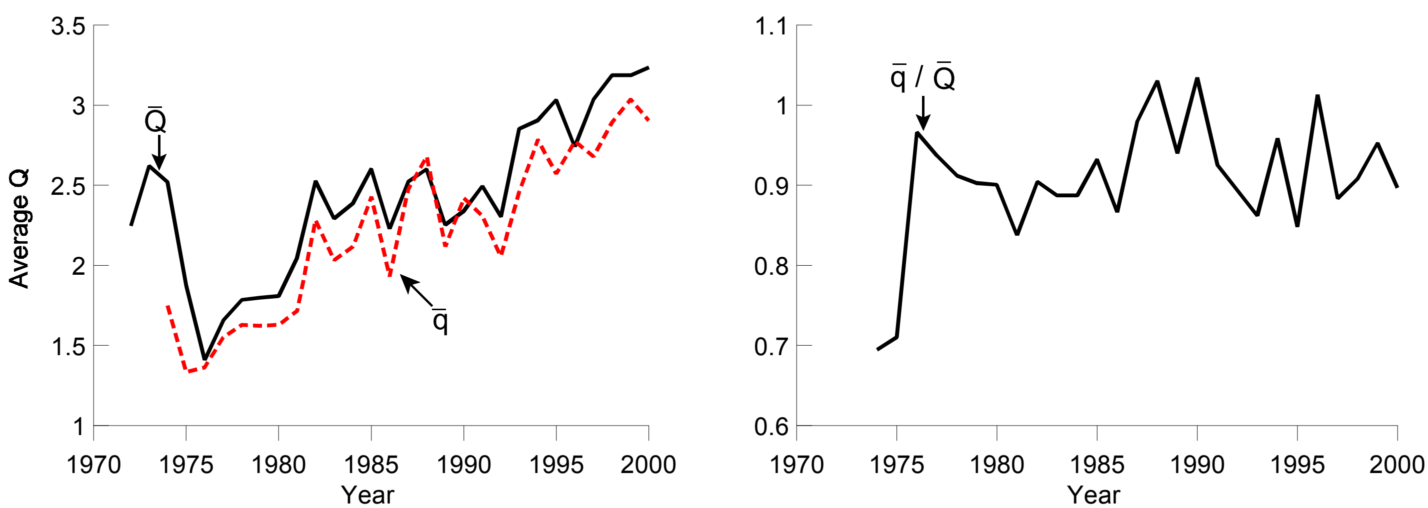
Model settings:
$z_1 = 0.115$, $z_2 = 0.130$, $\mu = 0.60$, $\nu = 0.06$, $\rho = 0.1$, $\sigma = 1$, $\delta = 0$.

Figure 4. Transitional dynamics for "IT" model.



(a) electricity revolution (b) IT revolution

Figure 5. The values of exiting firms and merger targets in two technological epochs.



(a) Q's by investment subgroup (b) the ratio of exiting and target firm q's to acquirer Q's

Figure 6. Prices of the two types of capital in the IT transition.

and 11.5% for 1970-2001.[6] These shares are much higher than in our simulations, but the units are not the same. If the aggregate capital stock was about three times nominal output from 1890-1930 and about two and a half times output from 1970-2001, we can divide each average by these multiples to express investment as shares of stock market capitalization, assuming of course that listed firms form their capital stocks in the same way as unlisted ones. The resulting investment shares of 3.5% for 1890-1930 and 4.6% for 1970-2001 are much closer to the simulations. Panel (b) of Figure 5 shows the upward trend in investment that the model predicts for the transitions, but panel (a) does not.

Using the average market-to-book ratios of exiting and target firms as a proxy for q, panel (a) of Figure 6 shows that q has been rising during the IT episode. But so has Q when measured as the average market-to-book values of acquirers, and this flatly contradicts the implication that $Q = 1$.[7] The model could explain values of Q in excess of unity if we put in adjustment costs for de novo investment, but this complicates the algebra and probably would not affect the implications about the time path of q/Q. Moreover, part of the rise in both q and Q may be due to the rising importance of unmeasured components of k_2 that are not on the firms' books. It is better, therefore, to concentrate on the ratio q/Q. In the theory, Q is unity and so

$$q = \frac{q}{Q}.$$

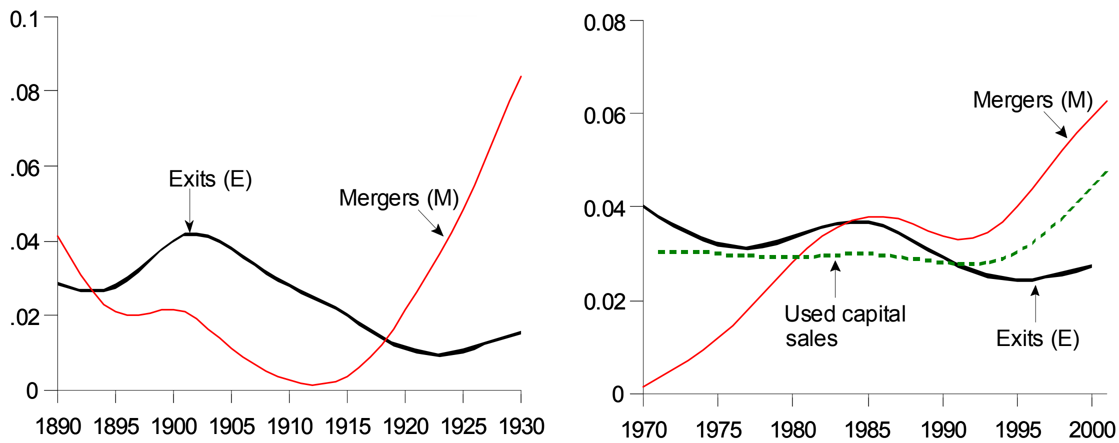
The theory predicts a monotonic rise in this ratio. Panel (b) of Figure 6 shows that the ratio has indeed risen, but much faster than the simulation in the third panel of Figure 4.

Contrasting the 2 ways converting k_1 into k_2.—A key implication of the model is that exits should lead mergers. This is indeed so, but the series for exits and mergers during our two technological eras in Figure 5 are quite variable at high frequencies, which makes this implication more difficult to observe. In Figure 7 we apply the Hodrick-Prescott filter to these series and normalize the area under each curve to unity. For the electrification era in panel (a) exits are indeed downward sloping as

[6] We obtain private domestic fixed investment and its deflator for 1970-2001 from the August 2002 issue of the *Survey of Current Business* (Table 1, pp. 123-4, and Table 3, pp. 135-6) and exclude non-farm residential investment. We use Kendrick (1961, Table A-IIa, column 7) for 1890-1930, and subtract residential nonfarm construction from worksheets underlying Kuznets (1961, Table T-11).

[7] We use the Compustat files to compute firm q's, and define market value as the sum of common equity at current share prices (the product of items 24 and 25), the book value of preferred stock (item 130), and short- and long-term debt (items 34 and 9). Book values are computed similarly, but use the book value of common equity (item 60) rather than the market value.

Since the company coverage within Compustat is very thin before 1972, we begin to compute Q's at this time. We count firms that disappear from Compustat as targets or exits, but only if the firm has been on the files for at least two years. Thus, the series for $\bar{q}$ and $\bar{q}/Q$ begin in 1974. We omitted q's for firms with negative values for net common equity from the plot since they imply negative market-to-book ratios, and eliminated observations with market-to-book values in excess of 100, since many of these are likely to be serious data errors.



(a) electricity revolution

(b) IT revolution

Figure 7: HP-filtered and normalized distributions of exits, mergers, and used capital sales.

our simulations suggest, and mergers dominate later in the reallocation wave. The mean of the exit series before filtering occurs in 1903, about one-third of the way into the reallocation wave, and that of mergers in 1910, at about the halfway point. For the IT revolution, exits also fall, with mergers growing in intensity as the wave progresses. In this case, the mean of the series before filtering occurs halfway into the wave in 1985 for exits, and two-thirds of the way into the wave in 1989 for mergers. Thus, the means of these distributions suggest that exits led mergers in the two reallocation waves, and that reallocation in general occured later in the IT revolution than for the electrification era. Both of these facts are consistent with our simulations.

The trading of used *physical* capital is also correlated with mergers. The dashed line in panel (b) of Figure 7 shows this fact. It plots, after H-P filtering, direct sales of used capital among exchange-listed firms as percentages of their total investment from 1971 to 2001. Since this disinvestment is like partial exit, we expect it to be correlated with exits, and it does move with exits, at least early on. But this capital is also purchased mainly by incumbent firms, and in that sense it should behave like acquisitions. It does so, especially later on in the wave.[8] We derive the series using

[8] In the model such trades between incumbents would not take place because they would involve two adjustment costs: the total adjustment cost of selling Δ units of capital by k_1 firms to k_2 firms would be $\psi\left(\frac{\Delta}{k_1}\right)k_1 + \phi\left(\frac{\Delta}{k_2}\right)k_2$. The data indicate that the total real value of used capital transactions, cumulated from 1971-2000, is only 21.3 percent of real target capitalization over this same period.

all firms common to CRSP and Standard and Poor's Compustat.[9] The correlation coefficient between the series for mergers and used capital purchases before filtering is 0.44.

4 Evidence on z_1, z_2, ϕ, and ψ

The values of z_1 and z_2 were the same in the two simulations, whereas ϕ and ψ were chosen to be lower in the IT era than in the Electrification era. We now discuss some of the evidence on these parameters.

4.1 Estimates of z_1, and z_2

The growth of consumption over the century was about 0.015, and z_1 (together with the other parameters) was chosen to deliver this. For $z_2 - z_1$, which we set at 0.015, we consulted various studies of productivity-enhancing effects of takeovers. For the United States, Maksimovic and Phillips (2001, p. 2053) find a productivity enhancement of 2%. For the United Kingdom, Harris, Siegel, and Wright (2002) find that, before the takeover, the target plants were 1.6%-2% less productive than other plants in the same industry. Thus the gains to reallocation are roughly in the range of these estimates.

4.2 Micro-estimates of ϕ

Our only reliable estimates of ϕ come from Q-regressions of acquisition investment. Like physical investment, acquisitions are not very sensitive to $Q - q$, and this gives us implied adjustment costs that are three or four times larger than what the macro data from the corresponding period seem to imply.

If $\phi(m) = \frac{m^2}{2\mu}$, (6) reads $m = \mu(Q - q)$. In Jovanovic and Rousseau (2002) we fit this equation to Compustat data on acquired capital. We took Q to be the acquirer's Q, and q to be that of the target. Table 1 of that paper reports the estimate of $\hat{\mu} = 0.022$ – after being divided by 100 in order to get it into the present units.

Although larger than the estimated response of physical investment, this estimate for μ is low and implies huge adjustment costs. It must, however, be adjusted for the tendency for firms to operate in several different markets. The diversification of firms' activities means that their relative Q's reflect their average efficiency over all the divisions and plants that they own, and not necessarily their competence in the area where they are buying the capital in question. This presents us with a classical

[9] Capital sales include property, plant, and equipment (Compustat item 107). Investment is the sum of acquired capital (item 129) and direct capital expenditures (item 128). We compute the series for used capital sales shown in panel (b) of Figure 7 after summing each data item across active firms in each year.

errors-in-variables inference problem so that $\hat{\mu}$ is biased down. How much downward bias we should expect depends on how many divisions the acquiring firm has and how much of variation in the division-specific "Q"s (these are theoretical, not empirical concepts) is common and explained by the firm-specific effect. If the efficiencies of the various divisions of the acquiring firm were independent random variables, and if these divisions were equal in size, then the firm's Q would be an average of the Q's of its division:

$$Q = \frac{1}{N} \sum_{i=1}^{N} Q_i.$$

Our estimates are for the acquirers in Compustat. Ravenscraft and Scherer (1987) report that in a sample of 471 large manufacturing firms in 1975, the number of four-digit manufacturing categories in which the firms operated was 7.5. We shall assume that $N = 7.5$ and that the Q_i are independent so that there are no firm-specific effects on efficiency, only division-specific effects. This gives us the adjusted estimate

$$\hat{\mu}^* = 7.5\,(0.022) = 0.17.$$

Two opposing biases on this number are the following: The bulk of the target capital was acquired by firms that were larger than the mean size in this sample. This suggests that the relevant N and, hence, relevant $\hat{\mu}$ should be larger than this. On the other hand, the presence of firm-specific influences on efficiency suggests that the errors-in-variables bias on the coefficient of $Q - q$ should be smaller.

In spite of this adjustment, $\hat{\mu}^*$ is lower, and the ϕ that it implies is higher than will give the best fit to the macro data for the IT episode. The simulations that fit the best for the IT episode take $\mu = 0.6$, implying that adjustment costs were 3.2 times smaller than implied by $\hat{\mu}^*$. On the other hand, the simulations for the electrification episode take $\mu = 2.7$, which is much higher and implies a far smaller adjustment cost. In that sense, $\hat{\mu}^*$ is, at least, between the two values that the simulations assume. We shall return to the issue of the secular decline in μ in section 5.4.

4.3 Micro-estimates of ψ

We do better in this dimension, probably because we do not use $Q-$ investment regressions which are known to deliver large adjustment costs. We have other kinds of evidence on both the physical-capital and human-capital side. This evidence is that the salvage value of physical capital is significantly lower than that for human capital.

Physical capital.—Evidence on physical capital's salvage value is in Ramey and Shapiro (2001). Consider the resources lost when a z_1-firm retires some of its capital. Let p_i be the sales price divided by the purchase price of machine i. Table 3 of Ramey and Shapiro reports the data. Per dollar spent on the machine, the firm's cost of retiring machine i is $C_i \equiv 1 - p_i$. We imagine that if the firm were to retire some of its capital, it would first sell off those machines for which p_i was closest to unity, and so on in order of descending p_i. Suppose the firm has k_1 machines on hand, $i = 1, 2, ...k_1$. Let $G\left(\frac{i}{k_1}\right)$ be the cumulative distribution of C_i among the stock of machines:

$$C_i = G\left(\frac{i}{k_1}\right).$$

The total cost to the firm of retiring εk_1 machines is $\psi(\varepsilon) k_1$, where

$$\begin{aligned}\psi(\varepsilon) &= \int_0^{\varepsilon k_1} G\left(\frac{s}{k_1}\right) ds \\ &= \int_0^{\varepsilon} G(s')\, ds',\end{aligned}$$

after the change of variables $s' = s/k_1$.

Now suppose that the C_i are distributed uniformly on the interval $[0, \nu]$, so that $G(s) = \frac{1}{\nu}s$. Then $\psi(\varepsilon) = \varepsilon^2/2\nu$. The age-aggregated data underlying Figure 3 of Ramey and Shapiro's paper were kindly supplied us by Valerie Ramey, and we plot them in Figure 8. Indeed, there are more C_i values close to unity than to zero. But the opposite is true on the human capital side (below), and we shall argue for these reasons that $\nu = 1.0$ is a good approximation.

Human capital.—Evidence on human capital's salvage value is in Neal (1995). The impact effect (which depends a lot on occupation and whether or not the worker finds a new job in the same industry and occupation) of a displacement is roughly 15% of wages. Wages do recover within a few years, but if we assume that the recovery is because of further investment on the job, then we can take the 15% number to be a permanent effect. This estimate comes from a 14% loss among switchers and a 6% loss among stayers at time of interview. However, stayers' wages grew at 2% per year from time of displacement to re-interview and switcher wages grew at 1% per year during this interval. Since average time from displacement to re-interview is 3 years, this gives a 17% loss for switchers and a 12% loss for stayers. There are more switchers than stayers by almost 2 to 1, thus 15% seems about right. On the other hand, experienced workers with long tenure who switch following displacement lose much more than 15% on average.

This is a mean over all displaced workers, comparable to the mean discount on the average machine sold by the Ramey-Shapiro defense-contracting firm. Now we interpret p_i as the fraction of human capital salvaged. The analog of the data in Figure 8, if they were plotted, would be much like a mirror image of the data in the Figure, with most observations lying in the left portion of the graph. This reinforces the appeal of the uniform distribution on the unit interval for the distribution of salvage values of all the capital put together. In that case the appropriate value is

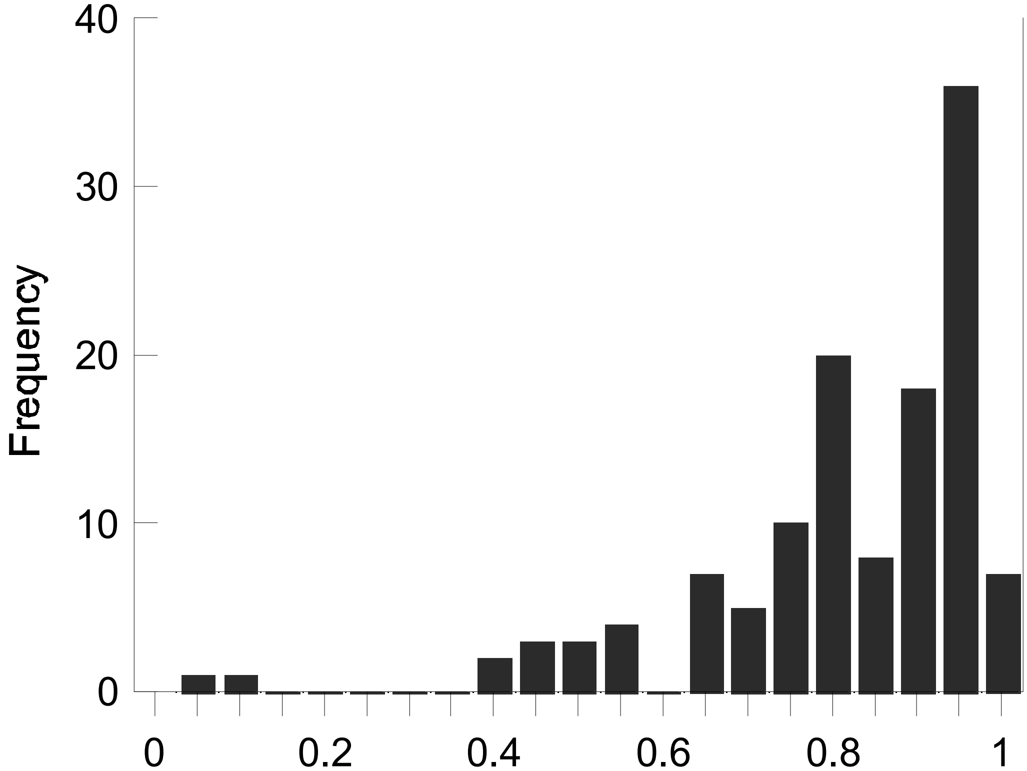


Figure 8: Frequency distribution of 1-p_i from the Ramey and Shapiro (2001) data.

$\nu = 1.0$. This is roughly half-way between the values of 2.7 and 0.06 that we used in the first and second simulations respectively.[10]

5 Other evidence

In this section we report evidence of a more general nature, but still helpful for evaluating the model.

5.1 Acquisitions and sectoral exposure to GPTs

Andrade *et al* (2001) argue that technological and deregulation shocks are behind the merger waves in the high-merger sectors in the '80s and '90s. If that is so, then a new GPT, which applies to most sectors, should prompt an economy-wide merger wave. But if 1890-1930 and 1970-2002 are indeed GPT diffusion eras, then we should

[10] We have simplified the algebra by assuming that ϕ and ψ are both convex in spite of evidence that investment is lumpy and, as we found in Jovanovic and Rousseau (2002, Figure 5) that acquisitions are even more lumpy than investment. Similarly, exit is also likely to involve fixed costs.

have seen more upgrading and reallocation in the sectors that were absorbing more of the two GPT's.

To show this, we run a "value-weighted" least squares regression of real target values as a percent of sectoral market value on a measure of sectoral absorption of the two GPTs at the tail end of the two episodes. For electrification, the measure of sectoral absorption is the ratio of the share of sectoral horsepower that is electrified in 1929 to the share in 1919, and the data are from David (1991). For IT, the absorption measure is the ratio of the share of IT capital (equipment and software) in each sector's capital stock in 2000 to the share in 1990, and the data are from the fixed assets tables of the Bureau of Economic Analysis (2002). The acquisitions that we report are for 1925-30 and 1997-2000 (the merger waves as defined in Figure 1).[11] That is, we look at the growth of the GPT shares over 10-year periods and then report acquisitions during the end-of-period wave. The value-weighted least squares regression is simply generalized least squares with each moment condition weighted by the corresponding sector's share in total GPT capital.[12]

Figure 9 illustrates the regression results, with the areas of the circles proportional to the weighting factors. The two panels of the figure are comparable, and are constrained by the sectoral investment data that we could find for the first epoch. The relation is positive in both epochs, but more so for the electrification era.

We also ran the regression with standard (i.e., unweighted) OLS. For the electrification era, the results were

$$M = \underset{(2.9)}{4.801} + \underset{(1.9)}{1.371}\ Share_{1929/1919} \quad N = 14,\ R^2 = .50,$$

with t-statistics in parentheses. For the IT era, we got

$$M = \underset{(-1.6)}{-7.449} + \underset{(3.3)}{7.592}\ Share_{2000/1990} \quad N = 62,\ R^2 = .06,$$

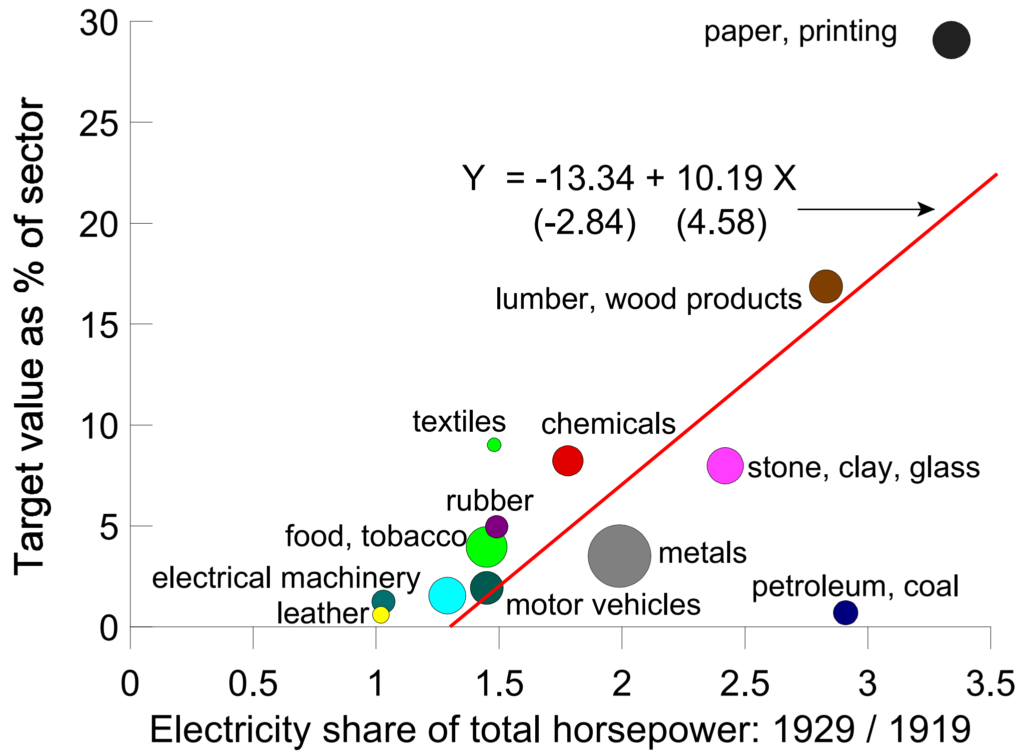
which are weaker, but qualitatively similar to our findings with value-weighted OLS.

5.2 Acquisitions, exits and IPOs by sector

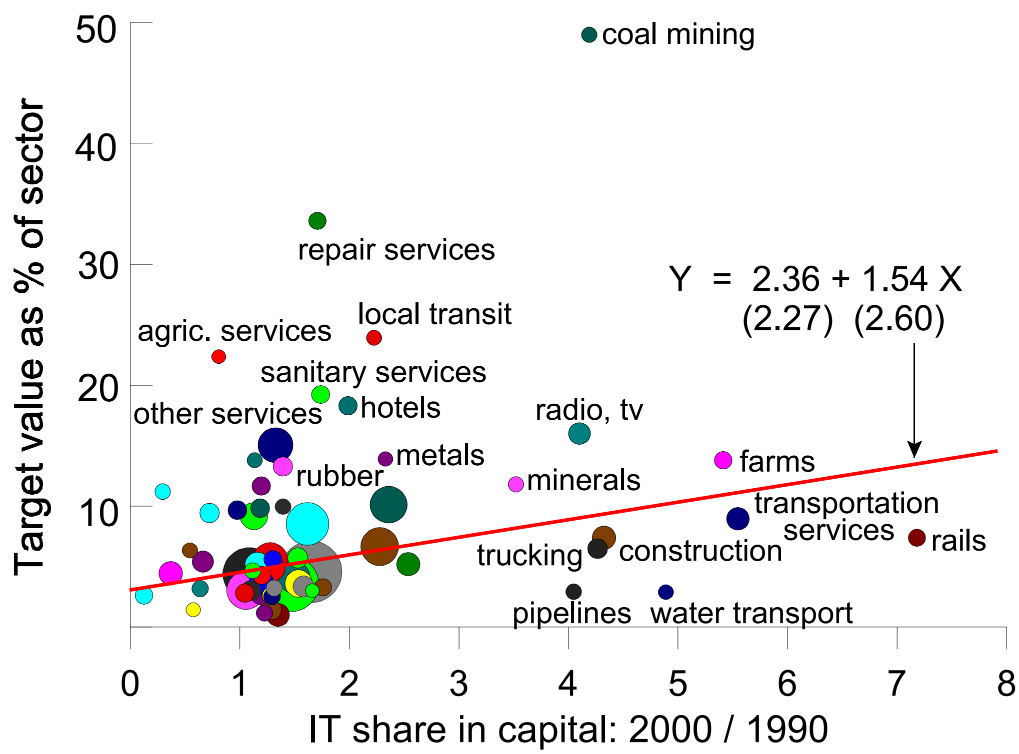
If m and ε are performing the same sort of reallocative function, then they should be positively correlated over sectors. It turns out that they are. The rank correlations

[11] A good deal of U.S. merger activity took place outside of the stock exchange over the 1890-1930 period, and a sectoral breakdown would not be possible unless we use these off-exchange transactions. Panel (a) of Figure 9 therefore uses all targets and sector designations recorded in the worksheets underlying Nelson (1959), and then divides by the total value of exchange-listed firms belonging to a given sector to form the vertical axis quantities. Panel (b) of Figure 9 reflects activity among exchange-listed firms only.

[12] In other words, for the electrification era, we weight the observations by the share of total electrical horsepower that resides in each sector, whereas for the IT era we weight by the share of IT-capital (computer equipment and software) that resides in each sector.



(a) electricity revolution



(b) IT revolution

Figure 9. Target values vs. changes in GPT shares over 10-year periods by sector.

between IPOs and exits on the one hand and acquisitions on the other, with ranks based upon the percentage of each in total sector value (with the merger samples as defined in fn. 11) are given below.

Period	rank correlation	significance	# of sectors
	Mergers and IPOs		
1925-1930	0.718	1%	15
1997-2000	0.480	1%	62
	Mergers and Exits		
1925-1930	0.343	10%	15
1997-2000	0.847	1%	62

These results fit the model well, with all three forms of reallocation highly correlated across sectors.

5.3 The stock-market drop

Initial stock-market capitalization is k_1. Right after the shock, it falls to qk_1. With $k_1 = 1$,the stock market thus exhibits an immediate drop at $t = 0$, from 1 to q.[13] Figure 10 shows that the stock market declined in 1973-74.[14] No such sudden drop is visible for stock prices in the early 1890's. Maybe this is because the market was thin and unrepresentative in those days, with railway stocks absorbing a large chunk of market capitalization. More likely, the realization that the new technology would work well was more gradual and was not prompted by any single event like the completion of the Niagara Falls dam in 1894.

5.4 The secular rise of acquisitions relative to exit and entry

Figure 1 shows a nine-fold increase in the ratio of acquisitions to E&E. We do not explain the trend here, but we can re-formulate the puzzle in terms of our two adjustment costs. Assume they are quadratic as in (9). Then (6) and (5) read $m = \mu(1-q)$ and $\varepsilon = \nu(1-q)$. Note that

$$\frac{m}{\varepsilon} = \frac{\mu}{\nu}.$$

If, for some reason, the ratio ν/μ were to fall, ε would fall relative to m. The nine-fold rise in the ratio of mergers to E&E over the past century suggests that the ratio μ/ν has risen by an order of magnitude over this period (which is also the difference between the first and the second simulations in Figures 3 and 4). "Team capital" or "organization capital" may today be more important than it was in 1900, and this

[13] The drop is in this model due entirely to the jump in r. Hobijn and Jovanovic (2001) get a bigger stock-market drop by assuming that the output produced by the old capital falls in price when new capital is introduced – i.e., through the obsolescence of old capital.

[14] We obtain the composite stock price index from Wilson and Jones (2002), and deflate using the CPI.

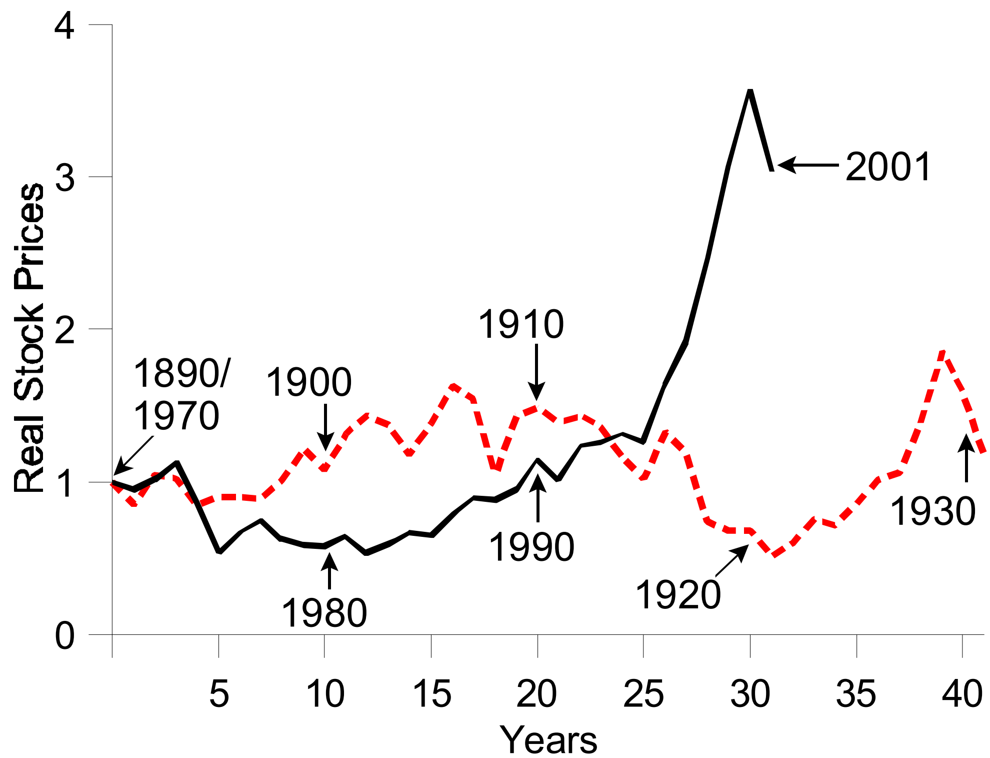


Figure 10: The real Cowles/S&P stock price index across the the transition periods, 1890-1931 and 1970-2001.

makes it worthwhile to preserve the healthy parts of an under-performing firm and fix only the part that works poorly. If a firm is taken over, its teams and its organization can remain intact, whereas if it were to exit through bankruptcy its assets and people will disperse, and this will destroy its team-specific capital.

The M&A sector is much more developed now. One approach to modeling it is as a market-supplied input into the technology for searching for targets. The input presumably gets cheaper over time relative to other goods. Matsusaka (2001) and Faria (2002) are models in which this issue could be studied in more depth.

6 Related work

We mentioned David (1991) earlier. Boldrin and Levine (2001) also have a technology for converting old capital to new. Since they do not allow goods to be converted into new capital one for one, their results are different. Holmes and Schmitz (1990) discuss the selling of capital by inventors to managers which resembles the two conversion activities that we emphasize here. Mortensen and Pissarides (1998) look at constant growth, not at transitions, and they focus on the labor market, but their work is similar in that they have two modes of job improvement that are similar to the two that we have modeled. Caballero and Hammour (1994) study transitions at business-cycle frequencies. Finally, Atkeson and Kehoe (2001), Greenwood and Yorukoglu (1997) and Hornstein and Krusell (1996) study transitions, but they do not focus on adjustment costs like we do.

Shleifer and Vishny (1992) argue that merger waves are driven by liquidity which allows the re-assignment of capital among owners to proceed more smoothly. This suggests that one may augment the adjustment-cost functions ϕ and ψ to include a financial factor. Faria (2002) fits a model related to ours to the Telecom merger wave of the '90s. Toxvaerd (2002) models a merger wave as arising as firms rush to buy so as not to be left without a target.

On the productivity-enhancing role of takeovers – i.e., on the question of why merged capital and exiting and re-entering capital experiences an efficiency rise from z_1 to z_2 – we know that exiting plants are less productive than the average plant and less productive than the average entering plant (Baily, Hulten, and Campbell 1992). A multi-plant firm is likely to sell off its least productive plants (Maksimovic and Phillips 2001). Takeovers do seem to have beneficial real effects. Martin and McConnell (1991) find that managers of takeover targets are more than four times more likely to be replaced than those same managers before the firm had been selected as a target. After a takeover their turnover rate jumps from 10% to 40% or so. McGuckin and Ngyen (1995) and Schoar (2000) find that the productivity of acquiring firms' plants falls and that the productivity of the targets' plants rises following a takeover. Lichtenberg and Siegel (1987) find that plants changing owners had lower initial levels of productivity and higher subsequent productivity growth than plants that did not change hands. The above evidence is for the United States. In the United Kingdom things work roughly the same way, as Harris, Siegel, and Wright (2002) found in their study of 36,000 manufacturing plants of which nearly 5000 were involved in a takeover from 1994 to 1998.

Lang, Stultz, and Walkling (1989) and Servaes (1991) find that the mergers that create the most value are those between high-Q bidders and low-Q targets. Merger announcements do tend to lead to declines in the prices of acquirer shares. But Jovanovic and Braguinsky (2002) show that when firms have private information about the quality of the capital that they own, the bidder discount is consistent with takeovers being constrained efficient, as they are in the present model.

7 Conclusion

While mergers are probably motivated by a variety of factors, one role that they play, this paper argues, is that of reallocation of assets toward the more efficient firms. If this argument is correct, major technological change should lead to merger waves.

We studied two GPT eras – electricity and IT – and found that this seems to have been the case. Our model gets some support from historical data.

On the other hand, the fit of the model is far from perfect. We have no explanation for the conglomeration wave of the 60's, and the wave of 1900 occurs earlier than our model suggests that it should have. We have focused on the reallocation role alone and, overall, it seems to explain merger waves surprisingly well.

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8 Appendix: The planner's solution

The economy is convex, competitive and there are no external effects. We derive the optimal solution for the planner here, whereas in the text we reinterpret the optimum in terms of prices. We use optimal control. The Hamiltonian is

$$H = e^{-\rho t} \left\{ \begin{array}{c} U\left[\left(z_1 - \psi\left[\varepsilon\right]\right) k_1 + \left(z_2 - \phi\left[m\right]\right) k_2 - x_2\right] + q^* \left(-\left[\delta + \varepsilon\right] k_1 - m k_2\right) \\ +Q^* \left(\left[m - \delta\right] k_2 + \varepsilon k_1 + x_2\right) + \lambda^* k_1 \end{array} \right\}$$

where $e^{-\rho t} q^*$ is the multiplier on the $\dot{k}_1$ constraint, $e^{-\rho t} Q^*$ is the multiplier on the $\dot{k}_2$ constraint, and $e^{-\rho t} \lambda^*$ is the multiplier on the non-negativity of k_1. To save on notation, we have assumed that $x_1 = 0$. This is valid if $Q^* > q^*$ so that the planner values k_2 more than k_1. We also ignore the nonnegativity constraint on x_2. The FOCs are

$$\frac{\partial H}{\partial m} = 0 = -U'(c)\, \phi'(m) - q^* + Q^* \tag{15}$$

$$\frac{\partial H}{\partial \varepsilon} = 0 = -U'(c)\, \psi'(\varepsilon) - q^* + Q^* \tag{16}$$

$$\frac{\partial H}{\partial x_2} = 0 = -U'(c) + Q^*$$

$$-\rho q^* + \dot{q}^* = -\frac{\partial H}{\partial k_1} = -U'(c)\left(z_1 - \psi\left[\varepsilon\right]\right) + \left(\delta + \varepsilon\right) q^* - \varepsilon Q^* + \lambda^*$$

$$-\rho Q^* + \dot{Q}^* = -\frac{\partial H}{\partial k_2} = -U'(c)\left(z_2 - \phi\left[m\right]\right) + m q^* - \left(m - \delta\right) Q^*.$$

Now define

$$Q = \frac{Q^*}{U'(c)} \quad \text{and} \quad q = \frac{q^*}{U'(c)} \text{ and } \lambda = \frac{\lambda^*}{U'(c)}.$$

Then the equations become

$$\phi'(m) = Q - q$$

$$\psi'(\varepsilon) = Q - q$$

$$Q = 1$$

$$\frac{-\rho q U' + \dot{q} U' + q \dot{U}'}{U'} = -(z_1 - \psi[\varepsilon]) + (\delta + \varepsilon) q - \varepsilon Q + \lambda$$

$$\frac{-\rho Q U' + \dot{Q} U' + Q \dot{U}'}{U'} = -(z_2 - \phi[m]) + mq - (m - \delta) Q,$$

because

$$-\rho q^* + \dot{q}^* = -\rho q U' + \dot{q} U' + q \dot{U}'$$

and

$$-\rho Q^* + \dot{Q}^* = -\rho Q U' + \dot{Q} U' + Q \dot{U}'.$$

Since $Q = 1$, and since $k_1 > 0$ on $[0, T]$, these conditions simplify to

$$\phi'(m) = 1 - q$$

$$\psi'(\varepsilon) = 1 - q$$

$$\frac{\dot{q} U' + q \dot{U}'}{U'} = -(z_1 - \psi[\varepsilon]) - \varepsilon(1 - q) + (\rho + \delta) q$$

and

$$\frac{\dot{U}'}{U'} = -(z_2 - \phi[m]) + m(1 - q) + \rho + \delta,$$

or,

$$\frac{\dot{q}}{q} + \frac{\dot{U}'}{U'} = -\frac{(z_1 + \pi^{\varepsilon}[q])}{q} + \rho + \delta$$

$$\frac{\dot{U}'}{U'} = -(z_2 + \pi^m[q]) + \rho + \delta.$$

This reduces to a single differential equation for q:

$$\frac{\dot{q}}{q} = (z_2 + \pi^m[q]) - \frac{(z_1 + \pi^{\varepsilon}[q])}{q}. \tag{17}$$

The only stationary solution would be a value q^* at which

$$(z_2 - \pi^m[q]) = \frac{(z_1 + \pi^{\varepsilon}[q])}{q}$$

for all $t \in [0, T]$. Under mild conditions (e.g., if ϕ and ψ are the same function),

$$0 < q^* < 1,$$

and the steady state is unstable. That is,

$$q \gtrless q^* \quad \Longrightarrow \quad \frac{\dot{q}}{q} \gtrless 0.$$

Therefore we must have

$$q_0 > q^*,$$

or else q_t could not converge to unity. Now, if this were so, (17) would imply that

$$\lim_{t \to T} \frac{\dot{q}_t}{q_t} = z_2 - z_1$$

because $\lim_{q \to 1} \pi^i (q) = 0$.

One caveat to the above is that it ignores the constraint $x_2 > 0$. If the upgrading technology is efficient enough, the planner may prefer to set not just x_1 (which we have set equal to zero) but also x_2 equal to zero for a while. We have ignored this constraint, and the solution we derived would not be valid if ψ and especially ϕ were low for relatively large values of ε or m.