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Abstract

Over the postwar, the U.S., Europe and Japan have experienced what may be thought of as medium frequency oscillations between persistent periods of robust growth and persistent periods of relative stagnation. These medium frequency movements, further, appear to bear some relation to the high frequency volatility of output. That is, periods of stagnation are often associated with significant recessions, while persistent booms typically are either free of recessions or are interrupted only by very modest downturns. In this paper we explore the idea of medium term cycles, which we define as reflecting the sum of the high and medium frequency variation in the data. We develop a methodology for identifying these kinds of fluctuations and then show that a number of important macroeconomic time series exhibit significant medium term cycles. The cycles feature strong procyclical movements in both disembodied and embodied technological change, research & development, and the efficiency of resource utilization. We then develop a model to explain the medium term cycle that features both disembodied and embodied endogenous technological change, along with countercyclical markups and variable factor utilization. The model is able to generate medium term fluctuations in output, technological change, and resource utilization that resemble the data, with a non-technological shock as the exogenous disturbance. In particular, the model offers a unified approach to explaining both high and medium frequency variation in aggregate business activity.

Keywords: Business Cycles, Endogenous Technological Change.
JEL Classification: E3, O3.

1 Introduction

Contemporary analyses of business fluctuations have tended to focus on relatively high frequency movements in economic activity. A standard way to detrend output, for example, is to apply a Hodrick-Prescott filter to the raw times series. As noted by Baxter and King (1995), the Hodrick-Prescott filter with conventional smoothing weights produces fluctuations about trend that are roughly equivalent to what obtains from a band pass filter that isolates frequencies between 2 and 32 quarters. A virtue of the approach is that it produces stationary output oscillations that coincide well with identified business cycle peaks and troughs.

Yet there are several related respects in which this high frequency characterization of fluctuations may be quite limited. First, over the post war, the economies of the U.S., Europe, and Japan have displayed what may be thought of as medium frequency fluctuations. In the U.S., for example, the early to late 1960s was a period of robust growth. The early 1970s to the early 1980s was time of relative stagnation on average. For most of the 1990s there was a return to strong growth. Similar oscillations between periods of robust growth to periods of stagnation that last beyond the time frame we normally think of for conventional business cycles have occurred in both Europe (see, e.g. Blanchard (1997)) and Japan.

Second, on the surface it appears there may be some relation between the high frequency output fluctuations and these medium frequency oscillations.[1] The long U.S. expansion of the 1960s was free of any significant high frequency downturn. By contrast, the stagnation of the 1970s and early 1980s was accompanied by a number of major recessions, including the two considered the worst of the post war. On the other hand, the high growth period over the last eight years has been interrupted by only one recession, still considered modest by postwar standards. The possibility of a connection between the high frequency oscillations and the medium frequency behavior of output has important implications for how we think about and model business fluctuations.

The purpose of our paper is twofold. First, we develop a simple methodology for characterizing medium term business cycles, which we define to include the overall

[1]We are not the first to make this observation. For example, Rotemberg (1999) notes that, with the conventional detrending methods, the cycle appears correlated with the trend.

high and medium frequency variation in business activity.[2] We then present evidence based on postwar U.S. time series to suggest that a number of important macroeconomic variables exhibit significant medium term cycles. Second, we develop a simple quantitative model of medium term business fluctuations. A particular goal is to provide a unified approach to understanding high and medium frequency business cycle dynamics.

In section 2 we present evidence on medium term fluctuations. Roughly speaking, we construct measures of medium term cycles by running a much smoother trend through the data than is typically used in the conventional high frequency analysis. We then show that a number of key variables exhibit a significant medium term cycle: i.e., in each case the medium frequency component differs significantly from zero and its variation is at least of the same magnitude of the high frequency component, and often is greater. These findings, as we show, are statistically significant and not an artifact of a small sample size.

In addition, there are a number of interesting patterns. For example, over the medium term there is a strong co-movement between output and both disembodied and embodied technological change. Total factor productivity (TFP) moves procyclically over the medium term while the relative price of capital (a measure of embodied technology change) moves countercyclically. In addition, research and development (R&D) moves procyclically. It is also worth emphasizing that the co-movements of both the relative price of capital and R&D with output are significantly more pronounced over the medium term than at the high frequency alone. In this regard, there are aspects of the medium term cycle that are distinct from the conventionally analyzed high frequency cycle. The medium term cycle, however, is not only a productivity phenomenon. Measures of the degree of efficiency of resource utilizations also display medium term cycles. For example, the markup of price over (social) marginal cost moves countercyclically over the medium term. Capacity utilization moves procyclically.

[2]While in a similar spirit, this definition is nonetheless somewhat different from Blanchard's (1997) and Caballero and Hammour's (1998) notion of the medium term. These authors have in mind mainly the medium frequency variation in the data, while we include both the high and medium frequency variation. We do so because we want to emphasize the interrelation between high and medium frequency fluctuations.

In section 3, we develop a quantitative framework, using the facts uncovered in section 2 as guidance. In particular, the model features both endogenous productivity along with countercyclical markups and endogenous factor utilization. We incorporate endogenous productivity in order to provide a unified explanation for the combination of procyclical TFP and a countercyclical relative price of capital over the medium term. That R&D is procyclical over the medium term provides some additional support for this approach. Another important advantage of endogenous technological change is that, as we elaborate below, it provides a way to connect the high frequency cycle with the strong medium frequency movements in productivity observed in the data. The alternative would be to have exogenous high frequency technology shocks as the main source of both high frequency and medium frequency variation.[3] Many recent papers, however, have argued that much of the high frequency variation in measured productivity reflects endogenous factor utilization and not true productivity changes (e.g., Burnside, Eichenbaum and Rebelo, 1995). With endogenous productivity, it is possible to have non-technological shocks as the main driving force at the high frequency but have the model generate significant medium term movements in both disembodied and embodied technological change. Endogenous productivity alone, however, is not enough. As we show, it is the interaction of endogenous productivity, countercyclical markups and endogenous factor utilization that introduces a kind of medium term business cycle propagation mechanism whereby non-technology shocks can generate the kind of medium term movements in output, productivity and resource utilization that are observed in the data.

Broadly speaking, our approach is similar to Evans, Honkapohja, and Romer (1998) who also develop an endogenous growth model to study business fluctuations that take place over medium term horizons. They emphasize how the various complementarities within their framework may give rise to sunspot fluctuations in the growth rate. In addition to differing considerably in detail, our model has a unique steady state growth rate: The complementarities in our model then work to magnify the effects of business cycle shocks. Another key difference is that we examine the model against the data and, in the process, try to explain both the high and medium frequency

[3]In addition, a model of exogenous productivity variation would require the arbitrary assumption that shocks to TFP and the relative price of capital are correlated in a way to match the strong negative correlation of these variables over the medium term.

variation.

This evaluation of the model against the data is conducted in section 4. A number of recent papers have suggested that shifts in the household's labor-leisure choice are the most important source of cyclical variation (e.g. Hall (1997), Neville and Ramey (2002)). We accordingly take this shock as the exogenous driving force, though we do not take literally the interpretation that it reflects preference shifts. As discussed in Gali, Gertler and Lopez-Salido (2002) (GGLS), this kind of shock is isomorphic to a variation in the wage markup within a model where labor market frictions are present. Rather than introducing labor market frictions explicitly (to an already complex model), we simply explore the implications of labor supply shifts, keeping in mind that alternative interpretations are possible, as we discuss later on.

Overall, our model does a reasonably good job in capturing the key features of the medium term cycle. Even though the main driving force is a non-technological shock, the model captures most of the cyclical variation in productivity, both at the high and medium frequencies. Endogenous utilization and countercyclical markups help generate the high frequency co-movement of output with TFP and the relative price of capital. Endogenous productivity then takes over to help account for the medium term co-movements. For comparison, we also explore how a conventional real business cycle model with exogenous productivity shocks as the driving force can account for medium term cycles. Concluding remarks are in section 5.

2 Methodology and Facts

In this section we develop some facts about medium term business fluctuations for quarterly post war data. To identify medium term fluctuations, we first remove from the data a long term trend. We assume that the long term corresponds to frequencies of fifty years and below. The rough idea is that we allow for the possibility that over our fifty year sample there has been one long cycle that has been heavily influenced by factors important at the very low frequencies, such as demographics, and so on. We then remove this trend from the raw data to construct a series for medium term business cycles based on frequencies between 2 and 200 quarters.

We next split the medium term cycle in two: into a high frequency component defined to include frequencies between 2 and 32 quarters (the standard representation of fluctuations)[4] and a medium frequency component, consisting of the frequencies between 32 and 200 quarters. Note that the medium frequency component is effectively the difference between the conventional trend (obtained from removing frequencies above 32 quarters) versus our smoothed trend (obtained removing frequencies above 200 quarters). As we show below, it is in general not correct to think of the medium frequency variation in the data as orthogonal to the high frequency variation. For this reason we focus on understanding the overall fluctuations in the data between frequencies of 2 and 200 quarters and not simply the medium frequency component.

We use a band pass filter to isolate different frequencies. Most of the series are non-stationary. For these series, we first apply the filter to log differences of the data to obtain smoothed growth rates for different frequencies of interest. We then level up the smoothed growth rates to obtain measures of trends in log levels. For robustness, we also filter the data in log levels (with a simple linear trend removed) and construct trends based directly on the smoothed level data. Because the band pass filter provides only an approximation for finite series, we "pad" the series by forecasting and backcasting, to minimize biases that are likely to arise, especially at sample endpoints. We also perform several other robustness checks, including a simple Monte Carlo exercise with our model.

The data is quarterly from 1948:1 - 2001:2, except noted otherwise. We focus on a set of variables that are useful for characterizing the movements in both productivity and resource utilization over the medium term. The seven series we consider are: GDP per capita (specifically, non-farm private business output per person between the ages sixteen and sixty-five), total factor productivity (TFP), labor productivity measured by output per hour in the non-farm private business sector (the BLS measure), the relative equipment price (the Gordon series), private R&D, the markup, measured as the difference between the marginal product of labor and the representative household's marginal rate of substitution between consumption and leisure, and capacity utilization. The series on private R&D and TFP are only available at an annual frequency.

[4]Examples include the Hodrick and Prescott (1997) filter, and the approximations of Baxter and King (1995) and Christiano and Fitzgerald (2001) to the band pass filter. Closer to our goal is Rotemberg (1999), who proposes a heuristic method to detrend macro time series that makes the trend as smooth as possible while also producing cycles with plausible properties.

Figure 1 plots per capita non-farm private business output. The solid line is the medium term cycle and the line with crosses is the medium frequency component. The difference between the two lines is the high frequency component. This latter component closely resembles what is obtained from applying a Hodrick-Prescott filter to the data. Overall the medium frequency component appears to dominate the movement in the series. As one might expect, the medium frequency component exhibits sustained upward movement in the 1960s, reverses course in the 1970s, and does begin a sustained upward trend again until the 1990s. It is also interesting that the periods absent major recessions are the periods of medium term growth, whereas the medium term decline coincides with the major recession of the 1970s and early 1980s.[5]

Perhaps more than any other point in time, the 1980-82 period illustrates how considering the medium term cycle can affect perspective of the condition of the economy. This recession is generally considered the worst of the postwar period by a significant margin. However, the high frequency component indicates a significant but not huge drop in output relative to trend. The medium term cycle, however, suggests a much larger and more protracted drop relative to trend, one that seems consistent with conventional wisdom. It is not always the case that the medium term filter makes recessions look worse; the 1974-75 recession, which was a sharp but protracted decline looks about the same in magnitude under either filter. Nonetheless, the medium frequency component appears quantitatively important to overall postwar fluctuations.

Figure 2 plots the medium term fluctuation in labor productivity along with the medium frequency component. Again, the difference is the high frequency component. This series similarly exhibits large medium term swings that, for the most part coincide with the medium term movement in output. This series exhibits about two thirds the volatility of output. It appears to be an important factor in the medium term cycle, but not the only factor.

The medium term movement in labor productivity is not simply associated with variation in capital. In figure 3 we plot the medium term cycle in TFP. The move-

[5] The correlation between the medium frequency variation and output and the high frequency (as defined by the HP filtered data) is 0.25.

ments closely match the medium term movement in labor productivity. An interesting departure, though, occurs in the late 1990s, as the surge in labor productivity outpaces the growth in TFP, consistent with the view that capital deepening played an important role in productivity growth over this period. Otherwise, the variation in the two productivity series coincides closely.

Could the medium term cycle in output and productivity be an outcome of our procedure identifying strange looking long term trends due to the finite sample? Figure 4 plots the long term trends for both output growth and labor productivity growth. It shows series that are highly consistent with conventional wisdom. There is a steady modest decline in the long term growth rate in each series over the post war until the mid 1990s. Our decomposition thus suggests that part of the sustained boom during the 1960s and reversal in the early 1970s was the result of medium term factors. We emphasize further that the change in the long term growth rate of either variable is quite slow relative to the medium term oscillations. Thus it is unlikely that reasonable amounts of measurement error in the long term trend growth rate could significantly affect our measures of medium term fluctuations.

Another way to look at this issue is to construct standard error bands for the medium frequency component. To do so, for each series we lengthen the effective sample by creating artificial time series. We build these by estimating a fourth order autoregressive process and then use the fitted process to extend the actual time series both forward and backwards periods. We create 500 time series by randomly shocking the estimated process, assuming that the random term is a draw from an i.i.d. normal. Each time series represents a possible sample path for the economy. For each we construct the associated medium frequency component. We can then construct standard error bands for this component by computing the distribution obtained from the 500 time series. Figure 5 portrays the medium frequency component along with ninety five percent confidence intervals for the case of output. It is clear that the medium frequency component differs statistically from zero and is not simply an artifact of a small sample.[6] We obtain similar results for all the other variables, though we do not report them here.

Figure 6 next portrays the medium term cycle and its medium frequency compo-

[6] Put differently, the difference between the trend based on removing frequencies above 200 quarters is statistically different than the trend based on removing frequencies above 32 quarters.

nent for the Gordon price of capital (which includes both equipment and structures).[7] Interestingly, this series also displays a strong medium term movement, consistent with the patterns in the output and productivity data. There is a sustained decline during the 1960s, a reversal of course in the 1970s, and then eventually a sustained decline in the latter 1990s. It seems hard to argue that the medium term co-movement with output and the price of capital is simply a coincidence. Next, figure 7 plots the medium term cycle in private research and development. The cycle is positively correlated with the medium term output cycle and exhibits a larger amplitude. Finally, figures 8 and 9 plots the medium term cycles in the markup and in capacity utilization, respectively. The markup moves inversely with output, as we would expect, and displays a medium term cycle that is comparable in amplitude and duration to the output cycle.[8] Capacity utilization is procyclical over the medium term. While the series exhibits cyclical variation at the medium frequency, it is less pronounced than for output, as one might expect.

In Table 1 we present some formal statistics on the volatility in the medium term cycle for each variable, along with the relative volatilities of the respective high frequency and medium term components. In addition, in the last column we construct a confidence interval for the ratio of the medium term to high frequency components using 500 artificially padded time series constructed in the manner described earlier. In each case the medium term component has a larger unconditional volatility than does the high frequency component.

The next several figures present the cross correlations of a set of variables with output both over the medium term cycle and over the conventionally measured cycle. Given our interest in the medium term, we consider correlations of the annual data.

[7]Greenwood, Hercowitz and Krusell (1997) emphasize the secular decline in the price of capital to argue that embodied technological change has been important for overall long term growth. Here we observe that there is also an interesting medium term cycle in the relative price of capital.

[8]We measure the markup as the gap between the marginal product of labor and the household's labor-leisure choice. That is, in measuring marginal cost, we treat as the true social cost of labor the household's marginal rate of substitution between consumption and leisure. We assune Cobb-Douglas production, log utility and a unitary Frisch labor supply elasticity. See GGLS and Chari, Kehoe and McGrattan (2002) for a rationale for using the labor market wedge to compute the markup. In addition these papers as well as Hall (1997) and Mulligan (2001) show that this wedge/markup is countercyclical at both high and low frequencies, consistent with Figure 8.

To do so we first take annual averages of all the data that is quarterly. In each figure, the dark line reflects cross-correlation that corresponds to the medium term cycle, while the gray line reflects the cross-correlation for the conventionally measure cycle (which corresponds to the high frequency component of the medium term cycle). The parallel lines indicate ninety-five percent confidence bands.

Figure 10 plots the autocorrelation for output. Interestingly, the high frequency component shows no persistence: Movements in output at time t are uncorrelated with movements at $t-1$ or $t+1$, and slightly negatively correlated with output at $t-2$ and $t+2$. Put differently, conventional detrending methods yield output fluctuations at the annual frequency that exhibit virtually no persistence. In sharp contrast, the medium cycle exhibits significantly greater persistence, with significant positive correlation of output at t with both output two years earlier and output two year later.

Figure 11 plots the cross correlation of labor productivity with output. As with output, the cross-correlation suggests greater persistence over the medium term than over the high frequency. While at the high frequency, the contemporaneous relationship is positive, there is little correlation at leads and lags. By contrast, the medium term movement in productivity displays a significant lead of at least two years over its respective series for output and a slightly more persistent significant lag. If anything, the medium term movement in output appears to lead movements in labor productivity on average, suggested by the slightly asymmetric pattern of the cross correlations. Not surprisingly, given Figures 2 and 3, the cross correlations of output with TFP are similar to those with labor productivity, as can be observed in Figure 12.

Figure 13 plots the cross correlation of output with the relative price of capital. Again, the high frequency suggests a weak relation, while the medium term suggest a strong inverse connection. Both lags and leads of the price of capital are significantly correlated with output at t over the medium term. There does appear to be a slight lead in output over the price of capital, similar (though of opposite sign) to the lead of output over productivity at the medium term.

Figure 14 next examines R&D. Again, the co-movement over the medium term with output is much stronger than at the high frequency. Indeed, the correlation is not significant at the high frequency. Interestingly, there appears to be a strong lead of R&D over output at the medium frequency, as suggested by the pattern of the cross correlation.

Finally, figure 15 plots the cross-correlation of output with the markup. Observe that there is a strong and persistent negative correlation at the medium term frequency, especially as compared with the relatively transitory relation at the high frequency. To the extent that the movements in the markup reflect (inversely) the degree of resource utilization, the figure suggests that this kind of cyclical factor may be operational over the medium term, as well as at the high frequency.[9]

3 Model

In this section we develop a model of medium term business fluctuations. The model allows for endogenous productivity growth, countercyclical markups and endogenous factor utilization. Endogenous productivity offers a unified way to explain the strong medium term procyclical fluctuations of total factor productivity along with the strong countercyclical movement of the relative price of capital. The pronounced countercyclical movement in the markup and procyclical movement in utilization over the medium term motivates incorporating these phenomena, as well.

By incorporating endogenous productivity, we have in mind allowing for mechanisms where investment of resources today (and or enhanced economic activity today) leads to subsequent increases in productivity tomorrow. The main examples include R&D leading to the development of new technologies, technology adoption, and learning by doing. We opt for a simple model of R&D based on Romer (1990) partly for tractability and partly because, as we noted earlier, the procyclical movement in R&D provides some support for this approach. But we do not mean to suggest that these other mechanisms are unimportant.

The model has both final output goods firms and intermediate output goods firms. In addition, there are both final capital and intermediate capital goods producers. Following Romer (1990), introduction of new intermediate goods is the source of sustained productivity growth in each sector. Creation of new intermediate products depends on research and development. We distinguish between productivity growth

[9]Though we do not report the results here, capacity utilization displays a strong positive correlation with output over the medium term.

in goods production and in the production of new capital goods, in order to capture medium terms fluctuations in the price of capital as well as TFP growth.[10] In addition, procyclical entry and exit of final goods firms introduces countercyclical markup behavior. The particular formulation is based on Gali and Zilibotti (1995.) As we discuss, a medium term simultaneous feedback relation between markup variation and R&D and productivity growth emerges. This mechanism tends to enhance medium term fluctuations.

We first discuss the final output production sector, then turn to the capital goods sector. We next characterize the research and development process. We then turn to households and finally characterize the complete equilibrium.

3.1 Final Goods Output

Final Output Composite

There is a final output composite that may be used for a variety of different purposes described below. This composite Y_t is a CES aggregate of M_t differentiated final goods, where $Y_t(j)$ is the output of final good producer j:

$$Y_t = [\int_0^{M_t} Y_t(j)^{\frac{1}{\mu_t}} dj]^{\mu_t} \tag{1}$$

with $\mu_t > 1$. After normalizing the index of final goods price P_t at unity and letting $P_t(j)$ denote the price of final good j, cost minimization yields:

$$Y_t(j) = (P_t(j))^{-\frac{\mu_t}{\mu_t - 1}} Y_t \tag{2}$$

with

$$P_t = 1 = [\int_0^{M_t} P_t(j)^{\frac{1}{1-\mu_t}} dj]^{1-\mu_t}. \tag{3}$$

[10]Some other ways to model medium term dynamics based on capital accumulation include Gilchrist and Williams (2001) and Benhabib and Hobyjn (2003).

We allow the number of final goods firms, M_t, and the markup, μ_t, to be time varying. For now we take the paths of these variables as exogenous. We will subsequently endogenize these variables in a simple way that introduces procyclical net entry and countercyclical markups.

Final Goods Firms:

Each final good firm packages together Z_t differentiated intermediate goods (indexed by n) to produce a final good, where $Y_t^{nj}(k)$ is the output of intermediate good producer k supplied to final good firm j:

$$Y_t(j) = \left(\int_0^{Z_t} (Y_t^{nj}(k)^{\frac{1}{\vartheta}} dk \right)^{\vartheta} \tag{4}$$

with $\vartheta > 1$. Within our framework, endogenous development of new intermediate goods is the source of TFP growth. Note that the CES production technology implies that the average final good product of intermediate goods input is increasing in Z_t. In section 3.3 we will characterize the process of new intermediate goods product development.

In the meantime, from cost minimization,

$$Y_t^{nj}(k) = (\frac{P_t^n(k)}{P_t^n})^{-\frac{\vartheta}{\vartheta-1}} Y_t(j) \tag{5}$$

where $P_t^n(k)$ is the price of intermediate good k and where P_t^n is the associated price index, given by

$$P_t^n = [\int_0^{Z_t} P_t^n(k)^{\frac{1}{1-\vartheta}} dk]^{1-\vartheta}. \tag{6}$$

In addition, profit maximization implies that each final goods producer sets price $P_t(j)$ as a constant markup μ_t over the price index for the intermediate composite P_t^n :

$$P_t(j) = \mu_t P_t^n \tag{7}$$

All final goods firms set the same price (and thus produce the same quantity of output).

Intermediate Goods Firms

Each intermediate firm k produces a specialized good using labor input $L_t(k)$ and capital services, where $K_t(k)$ is the quantity of capital and $U_t(k)$ is the capital utilization rate. The technology for each firm is Cobb-Douglas, as follows:

$$Y_t^n(k) = (U_t(k)K_t(k))^\alpha L_t(k)^{1-\alpha} \tag{8}$$

Following Greenwood, Hercowitz and Huffman (1988), we assume that the depreciation rate of capital is increasing in the utilization rate, i.e., for each firm the depreciation rate is given by $\delta(U_t(k))$, with $\delta'(\cdot) > 0$.

Each firm maximizes profits given the demand by final goods producers, the wage rate W_t, the rental price of capital net of depreciation, D_t and the price of capital P_t^K.[11] At the optimum, each firm adjusts capital, labor and capital utilization to the point where the respective marginal product equals the markup ϑ over the factor price, i.e., so that

$$P_t^n(k)\alpha\frac{Y_t^n(k)}{K_t^n(k)} = \vartheta[D_t + \delta(U_t(k))P_t^K], \tag{9}$$

$$P_t^n(k)(1-\alpha)\frac{Y_t^n(k)}{L_t(k)} = \vartheta W_t \tag{10}$$

and

$$P_t^n(k)\alpha\frac{Y_t^n(k)}{U_t(k)} = \vartheta\delta'(U_t(k))P_t^K K_t. \tag{11}$$

3.2 Capital Goods Output

Final Capital Good Composite:

Capital goods are produced using final output goods. As with output, there is both a retail and a wholesale stage to the production of final capital goods. In particular, new capital is a composite J_t that combines the capital produced by M_t^K retailers indexed by r $(J_t(r))$ according to the following CES aggregator:

[11] The gross rental that each firm pays is $D_t + \delta(U_t(k))P_t^K$.

$$J_t = \left(\int_0^{M_t^K} J_t(r)^{\frac{1}{\mu_t^K}} dr \right)^{\mu_t^K}, \tag{12}$$

with $\mu_t^K > 1$. From cost minimization

$$J_t(r) = \left(\frac{P_t^K(r)}{P_t^K}\right)^{-\frac{\mu_t^K}{\mu_t^K - 1}} J_t, \tag{13}$$

with

$$P_t^K = \left[\int_0^{M_t^K} P_t^K(r)^{\frac{1}{1-\mu_t^K}} dr\right]^{1-\mu_t^K}. \tag{14}$$

As with the final goods retail market, we will subsequently endogenize M_t^K and μ_t^K (the capital goods markup) in a way that leads to procyclical net entry and a countercyclical capital goods markup. As will become clear, the countercyclical capital goods markup contributes to the countercyclicality of the relative price of capital and to the amplification of its fluctuations.

Final Capital Goods Firms:

Each retailer r produces $J_t(r)$ units of new capital by combining A_t differentiated intermediate capital goods $I_t^r(s)$ according to the following production function:[12]

$$J_t(r) = \left(\int_0^{A_t} I_t^r(s)^{\frac{1}{\theta}} ds \right)^{\theta}, \text{with } \theta > 1. \tag{15}$$

As with final goods, there are efficiency gains in producing new capital from increasing the number of intermediate inputs, A_t. In section 3.3 we characterize the endogenous development of new intermediate capital goods. These efficiency gains are the source of embodied technological change and, as we shall see, are manifested in a fall in the relative price of capital.

[12] We have also experimented with a more flexible formulation that allows for a gradual decline in the demand of the old intermediate capital goods as new intermediate goods are developed. However, the results derived under the current formulation are extremely robust to this variation.

Let $P_t^I(s)$ be the price of intermediate capital good s and P_t^I be the price index for the intermediate good composite. Then from cost minimization

$$I_t^r(s) = (\frac{P_t^I(s)}{P_t^I})^{-\frac{\theta}{\theta-1}} J_t(r), \tag{16}$$

with

$$P_t^I = [\int_0^{A_t} P_t^I(s)^{\frac{1}{1-\theta}} ds]^{1-\theta}. \tag{17}$$

From profit maximization, it is optimal for each retailer to charge a markup μ_t^K over the marginal cost P_t^I :

$$P_t^K(r) = \mu_t^K P_t^I. \tag{18}$$

Note that countercyclical movements in μ_t^K will induce countercyclical movements in $P_t^K(r)$ relative to P_t^I.

Intermediate capital goods producers:

Each intermediate capital producer s uses the final output composite to manufacture a differentiated input $I_t(s)$. Given the demand function, profit maximization implies that each of these firms sets the price as a fixed markup θ over the price of the final output composite:

$$P_t^I(s) = \theta \tag{19}$$

3.3 R&D

Innovators develop new intermediate goods for the production of final output and new capital. They do so by conducting research and development, using the final output composite as input. They then manage the production of these goods, earning the profit stream from the differentiated good. To finance their research activities, they borrow from households. They pay out the remaining profits as dividends to households who own the equity stakes in their respective enterprises.

We first characterize the solution to the problem faced by an innovator of new intermediate goods for final output, and then turn to intermediate capital goods producers.

Creation of new intermediate goods

Given that each intermediate goods producer produces the same level of output, we can express profits for each one at time t, π_t^z, as

$$\pi_t^z = (\vartheta - 1) P_t^n Y_t^n / Z_t. \tag{20}$$

Given that R_{t+1} is the one period discount rate between $t+1$ and t, we can express the present value of profits, v_t^z, as

$$v_t^z = \pi_t^z + R_{t+1}^{-1} E_t v_{t+1}^z. \tag{21}$$

The above equation reflects the marginal gain from inventing a new intermediate capital good.

We assume that the flow of new products generated by each innovator p (*i.e.* $Z_{t+1}(p) - Z_t(p)$) depends linearly on the research and development funds $S_t^z(p)$ invested by innovator p, as follows

$$Z_{t+1}(p) - Z_t(p) = \varphi_t^z S_t^z(p) \tag{22}$$

where φ_t^z is the productivity of the R&D as perceived by the individual innovator.[13] As in Romer (1990), the linear formulation permits a simple decentralization of the innovation process. We differ from Romer, however, by having the innovation technology use as input a final good composite of capital and labor, as opposed to just labor.[14] Doing so enhances the ability of the model to generate procyclical R&D, as is consistent with the evidence.

[13]These new projects should not be thought of as major new general purpose technologies such as the computer or the internet, whose development and adoption takes many years and is likely exogenous to the cycle. Rather, they are best thought of as marginal improvements, such as new machines or new software that are produced and adopted with a reasonably short lag.

[14]An alternative approach might be to introduce imperfect substitutability between scientists and other form of labor, along with some rigidity of scientists salaries. These two factors would reduce the incentive to substitute to scientists in bad times.

We assume that φ_t^z depends on the aggregate values of the stock of innovations, Z_t, the wholesale value of the capital stock $P_t^I K_t$, and research and development S_t^z, and the stock of innovations as follows:

$$\varphi_t^z = \chi^z Z_t (\frac{S_t^z}{P_t^I K_t})^{\rho-1} (P_t^I K_t)^{-1} \tag{23}$$

with $0 < \rho \leq 1$ and where χ^z is a scale parameter. This formulation for the productivity of the R&D technology satisfies three interesting properties. First, it accommodates a positive spillover of the current stock of innovations on the creation of new products, i.e. φ_t^z increases linearly in Z_t. Second, the productivity of the R&D technology falls with the wholesale value of the capital stock (i.e. $P_t^I K_t$). This is the scaling factor that ensures that the equilibrium growth rate of new projects is stationary. Finally, we introduce an aggregate congestion to R&D conducted through the factor $(S_t^z / P_t^I K_t)^{\rho-1}$.

The linearity of the R&D technology as perceived by the individual researchers together with a free entry assumption imply that each new product developer p must break even. As a result, the resources invested in R&D by the p^{th} innovator satisfy the following condition:

$$R_{t+1}^{-1} E_t v_{t+1}^z \cdot (Z_{t+1}(p) - Z_t(p)) - S_t^z(p) = 0.$$

Capital Embodied Innovations:

Innovators of new intermediate capital goods similarly engage in research and development to make new products that can be patented to gain market power and then sold at a monopolistic price. The process is entirely symmetric to the development of new intermediate goods for output. The profit maximizing price charged by the inventor of an intermediate good is θ. This implies that the instantaneous profits of an innovator (in a symmetric equilibrium) are

$$\pi_t^a = (\theta - 1) I_t / A_t, \tag{24}$$

and that the value of the innovation at time t, v_t^a, is given by

$$v_t^a = \pi_t^a + R_{t+1}^{-1} E_t v_{t+1}^a. \tag{25}$$

The technology for producing new innovations is given by

$$A_{t+1}(q) - A_t(q) = \varphi_t^a S_t^a(q) \tag{26}$$

where $S_t^a(p)$ is the R&D expenditure by the pth capital goods innovator, with

$$\varphi_t^a = \chi^a A_t (\frac{S_t^a}{P_t^I K_t})^{\rho-1} (P_t^I K_t)^{-1} \tag{27}$$

and where χ^a is a scale parameter.

It is interesting to note that, with the timing adopted in the model, current R&D investments in the intermediate capital goods sector come into place in production only after two periods (i.e. years). It takes one year for R&D to yield a new intermediate capital good, and then another year for the new production process to produce new capital. This lag is in line with the evidence presented by Pakes and Schankerman (1984).[15]

Finally, the free entry condition implies

$$R_t^{-1} E_t v_{t+1}^a \cdot (A_{t+1}(q) - A_t(q)) - S_t^a(q) = 0.$$

3.4 Endogenous Countercyclical Markups.

We now characterize the joint determination of the quantity of firms, M_t and M_t^K, and the corresponding markups, μ_t and μ_t^k, for both the retail output and retail capital goods sectors. We use a simple approach based on Gali and Zilibotti (1995) (GZ) where procyclical competitive pressures associated with endogenous procyclical net entry induce countercyclical movements in the markup.[16]

Within the GZ framework competitive pressures are increasing in the number of firms in the market. In particular, the elasticity of substitution among final goods is

[15] The development and comercialization lag for disembodied innovations is assumed to be shorter than for the embodied innovations (one vs. two years). We think that this is a reasonable assumption given the higher complexity and more difficult implementation of new forms of capital as compared to new services, or mangerial techniques.

[16] It is also possible to generate countercyclical markups by allowing for money and nominal price rigidities (see, e.g., the discussion in GGLS.) .To keep the model tractable, however, we abstract from money and nominal rigidities.

increasing in the number of active firms. Because the markup varies inversely with the elasticity of substitution, we can write:[17]

$$\mu_t = \mu(M_t);\ \mu'() < 0. \tag{28}$$

$$\mu_t^K = \mu^K\left(M_t^K\right);\ \mu^{K\prime}() < 0 \tag{29}$$

Entry takes place until at the margin gross profits equal operating costs. We assume that retail output firms must pay a per period operating cost Ψ_t and that retail capital goods firms must pay Ψ_t^k. The free entry condition for each sector is given by, respectively:

$$\frac{\mu_t - 1}{\mu_t} P_t^f(j)\, Y_t^f(j) = \Psi_t. \tag{30}$$

$$\frac{\mu_t^K - 1}{\mu_t^K} P_t^K(r) J(r_t) = \Psi_t^K \tag{31}$$

where the left side of each equation is gross profits. We assume that firms take operating costs as given. The operating costs drift up over time proportionately with the wholesale value of the capital stock to ensure balanced growth:

$$\Psi_t = b P_t^I K_t \tag{32}$$

$$\Psi_t^K = b_k P_t^I K_t \tag{33}$$

One possible interpretation of this formulation is that operating costs are proportionate to the sophistication of the economy, as measured by the wholesale value of the capital stock.

Note that under our formulation, given operating costs, the markup in each sector will vary inversely with the market value of firm output. As we show later, in the general equilibrium the markup varies inversely with the output/capital ratio.

[17] Gali and Zilibotti (1995) provide micro-foundations for this formulation, based on a model of Cournot competition.

3.5 Households

There is a representative household that consumes, supplies labor and saves. It may save by either accumulating capital or lending to innovators. The household also has equity claims in all monopolistically competitive firms. It makes one period loans to innovators and also rents capital that it has accumulated directly to firms.

Let C_t be consumption and μ_t^w a preference shifter. Then the household maximizes the present discounted utility as given by the following expression:

$$E_t \sum_{i=0}^{\infty} \beta^{t+i} \left[\ln C_t - \mu_t^w \frac{(L_t)^{\phi+1}}{\phi+1} \right] \tag{34}$$

The budget constraint is as follows.

$$C_t = W_t L_t + \Pi_t + [D_t + P_t^k] K_t - P_t^k K_{t+1} + R_t (S_{t-1}^z + S_{t-1}^a) - S_t^z - S_t^a - T_t \tag{35}$$

where Π_t reflects the profits of monopolistic competitors paid out fully as dividends to households and T_t reflects lump sum taxes.

The household's first order conditions are standard:

$$\frac{W_t}{C_t} = \mu_t^w L_t^{\phi}, \tag{36}$$

$$\frac{1}{C_t} = R_{t+1} E_t \{ \beta \frac{1}{C_{t+1}} \}, \tag{37}$$

$$R_{t+1} \cdot E_t \{ \frac{1}{C_{t+1}} \} = E_t \{ \frac{D_{t+1} + P_{t+1}^k}{P_t^k} \frac{1}{C_{t+1}} \}. \tag{38}$$

Note that the preference shifter μ_t^w is observationally equivalent to a "wage markup" that distorts the equality between the wage and the households' leisure consumption tradeoff.

Government

Government spending is financed with lump sum transfers:

$$G_t = T_t. \tag{39}$$

3.6 Symmetric Equilibrium Conditions

The output composite is divided among five uses: consumption, investment, government consumption, research and development, and operating costs. Accordingly, the aggregate resource constraint is given by

$$Y_t = C_t + I_t + G_t + S_t^z + S_t^a + M_t\Psi_t + M_t^K\Psi_t^K. \tag{40}$$

where $M_t\Psi_t$ and $M_t^K\Psi_t^K$ are total operating costs in the retail output and retail capital goods sectors, respectively.

Given that all intermediate goods firms produce the same level of output, as do all final goods firms, the link between the final output composite and total intermediate goods production, Y_t^n, is given by

$$Y_t = M_t^{\mu_t - 1} Z_t^{\vartheta-1} Y_t^n, \tag{41}$$

with Y_t^n given by

$$Y_t^n = (U_t K_t)^{\alpha} L_t^{1-\alpha} \tag{42}$$

Over time efficiency gains in final output production arises with the introduction of new intermediate products, i.e with growth in Z_t. There may also be procyclical efficiency gains associated with cyclical fluctuation in final goods firms (represented by cyclical fluctuations in M_t).

Capital accumulation obeys the following law of motion:

$$K_{t+1} = (1 - \delta(U_t)) K_t + J_t. \tag{43}$$

Symmetry in the production of new intermediate capital goods implies that new capital, J_t, bears the following relation to investment I_t.

$$J_t = \left(M_t^K\right)^{\mu_t^K - 1} A_t^{\theta-1} \cdot I_t. \tag{44}$$

As with the production of final output goods, there are efficiency gains from having a variety of intermediate capital goods for the production of final goods: New capital (J_t) per unit of input (I_t) increases with A_t.

Note further that after taking into account the markup of the intermediate input over the final output composite, the price of J_t in units of final output is given by

$$P_t^I = \theta A_t^{1-\theta} \tag{45}$$
$$P_t^K = \mu_t^K \left(M_t^K\right)^{1-\mu_t^K} P_t^I. \tag{46}$$

Thus, the introduction of new intermediate capital goods raises the efficiency of production of new capital and reduces its price. In addition, countercyclical movements in the capital goods markup enhance the countercyclical variation in the final capital goods price.

The labor market equilibrium satisfies

$$(1-\alpha)\frac{Y_t^n}{L_t} = \mu_t \mu_t^w \vartheta L_t^\phi C_t. \tag{47}$$

Observe that under the interpretation of μ_t^w as a wage markup, the total markup corresponds to the product $\mu_t \mu_t^w$ This latter expression, in turn, corresponds to the labor market wedge that we used earlier to compute the time series for the markup. Next, capital utilization satisfies

$$\alpha\frac{Y_t^n}{U_t} = \mu_t \vartheta \delta' \left(U_t\left(k\right)\right) \cdot P_t^K K_t.$$

We may express the intertemporal Euler equation as

$$C_t^{-1} = E_t \beta C_{t+1}^{-1} [\frac{\frac{1}{\mu_t \vartheta}\alpha\frac{Y_t}{K_t} + P_{t+1}^K}{P_t^K}]. \tag{48}$$

Next, the growth rate of innovation in the output and capital goods sectors depends positively on R&D in the respective sector and negatively on the wholesale value of the capital stock, as follows:

$$\frac{Z_{t+1}-Z_t}{Z_t}=\chi^z(\frac{S_t^z}{P_t^I K_t})^\rho \tag{49}$$

$$\frac{A_{t+1}-A_t}{A_t}=\chi^a(\frac{S_t^a}{P_t^I K_t})^\rho \tag{50}$$

There is a corresponding free entry condition for each sector that in equilibrium require that the value of the gain from innovation (the left side of each equation) equal the cost of R&D (the right side in each case):

$$E_t\left[\frac{(V_{t+1}^z/Z_{t+1})}{R_{t+1}}\right](Z_{t+1}-Z_t)=S_t^z, \tag{51}$$

$$E_t\left[\frac{(V_{t+1}^a/A_{t+1})}{R_{t+1}}\right](A_{t+1}-A_t)=S_t^a, \tag{52}$$

where V_{t+1}^z and V_{t+1}^a represent, respectively, the discounted streams of total profits of intermediate goods and intermediate capital goods firms, (i.e. $V_{t+1}^z \equiv v_{t+1}^z Z_{t+1}$, $V_{t+1}^a \equiv v_{t+1}^a A_{t+1}$). The value of the firms in these sectors can be expressed recursively as:

$$V_t^z=(\varphi-1)Y_t^n+E_t\{\frac{Z_t}{Z_{t+1}}\frac{V_{t+1}^z}{R_t}\}, \tag{53}$$

$$V_t^a=(\theta-1)I_t+E_t\{\frac{A_t}{A_{t+1}}\frac{V_{t+1}^a}{R_t}\}, \tag{54}$$

Finally, free entry by final goods producers yields the following inverse link between the markup and output scaled by capital, for each sector:

$$\left[1-\frac{1}{\mu_t}\right] = bM_t^{\mu_t}\frac{P_t^I K_t}{Y_t} \tag{55}$$

$$\left[1-\frac{1}{\mu_t^K}\right] = b_K(M_t^K)^{\mu_t^K}\frac{P_t^I K_t}{J_t} \tag{56}$$

where it is useful to keep in mind that $\mu_t=\mu(M_t)$ and $\mu_t^K=\mu^K(M_t^K)$, with $\mu'(\cdot)<0$ and $\mu^{K\prime}(\cdot)<0$.

4 Model Simulations

In this section we use the model to explore how exogenous shocks to the markup can generate the kind of medium term fluctuations that we see in the data. We take as the period length of the model one year. This allows us to capture gestation lags in the R&D process in a more parsimonious way than if the period length were a quarter. In addition, some of the data we are trying to match is only available at the annual frequency.

We first describe the calibration and then present some model simulations.

4.1 Model Calibration

There are eight reasonably standard parameters: the discount factor β; the depreciation rate δ; the inverse of the labor supply elasticity ϕ; the capital share α; the ratio of government consumption to output, G/Y; the steady state utilization rate U; the elasticity of the change in depreciation with respect to utilization, $(\delta''/\delta')U$, the steady state final goods markup μ, and the steady state wage markup, μ^w. As values for these we use conventional parametrizations: $\beta = 0.95$; $\delta = 0.08$; $\phi = 1$; $\alpha = 1/3$; $G/Y = 0.2$; $U = 0.8$; $(\delta''/\delta')U = 0.5$; $\mu = 1.2$; and $\mu^w = 1.2$.[18] We set the final goods markup equal to 1.2 based on the range of evidence that Rotemberg and Woodford (1995) (RW) describe. In addition, given evidence that markups are higher on average in the capital goods industries, we set $\mu^k = 1.15$, so that the steady state of the cyclical markup in the capital goods sector over intermediate output, $\mu^k\mu$, is approximately 1.4, the upper range of the estimates discussed in RW. To calibrate the remaining parameters, we use a mixture of independent evidence and the requirement that the model match the balanced growth facts.

A key parameter in our model is the elasticity of new intermediate goods with respect to R&D, ρ. Here, there is no precise evidence. Griliches (1990) presents some

[18]We set U equal to 0.8 based on the average capacity utilization level in the postwar period as measured by the Board of Governors. We set $(\delta''/\delta')U$ equal to 0.5. based on the range of values used in the literature that vary from 0.1 used by Rebelo and King (1999) to unity used by Baxter and Farr (2001). We appeal to the range of evidence discussed in Rotemberg and Woodford (1995) to set $\mu = 1.2$. Given that average labor income tax rates have been roughly twenty percent, we set $\mu^w = 1.2$.

estimates using the number of new patents as a proxy for technological change. With cross-sectional data, he obtains an estimate of ρ around unity. With panel data, however, the estimated elasticity is around 0.5, though very imprecise. Indeed, the reciprocal regression (R&D on new patents) leads to point estimates of ρ well above unity. He attributes the difference in these estimates to two factors: first, the noise in the firm-level data (specially in the time series dimension); second, that maybe there are diminishing returns for a narrowly defined product line but not when R&D intensity shifts from one product line to another. Beyond this, there is the general problem that new patents may be a very imprecise measure of technological progress, leading to downward bias in the point estimates of ρ.[19]

These considerations lead us to gather some independent evidence on ρ. In particular, for the capital goods sector, the percent change in the relative price of capital may provide a reasonable good barometer of technological change. In particular, allowing for measurement error, our model implies the following relation between the percent change in the price of intermediate capital goods and R&D in that sector:

$$\frac{P_t^I - P_{t-1}^I}{P_{t-1}^I} = -\xi \left(\frac{S_{t-1}^a}{P_{t-1}^I K_{t-1}} \right)^{\rho} + \epsilon_t. \tag{57}$$

with $\xi = (\theta - 1)\,\chi$. We can estimate ρ in equation (57) using the Gordon (1990) relative price of capital data to measure $\frac{P_t^I - P_{t-1}^I}{P_{t-1}^I}$ and the R&D to capital ratio to measure $\left(\frac{S_{t-1}}{P_{t-1}^I K_{t-1}^I} \right)$. We estimate ρ to be about 1.1 though the estimate is quite imprecise. Given these considerations, we fix ρ at 0.95, an intermediate value in the range of our estimates and Griliches' two sets of estimates (cross-sectional and panel). Note that this parameter value applies as well to the R&D process for intermediate

[19]Using patents to measure technological progress has three important drawbacks. First, though in the model all of the innovations are symmetric, in reality, they clearly are not. Failure to account for a upward trend in the quality of new patents will bias downwards Griliches estimates of ρ. (New patents are likely to be worth more than old patents because the R&D expenses per patent have been trending up). Second, the productivity literature has long recognized that in recent years firms are less prone to patent their innovations. This trend will also introduce an additional downwards bias in the estimate of ρ. Finally, the finite number of workers at the patent office tends to smooth out the number of patents granted in a given year generating additional artificial concavity in the estimation of equation (50).

goods in the final goods sector, given our assumption that the innovation technology for this sector is symmetric to the one for the capital goods sector.

Given information on the share of R&D expenditures in the capital goods sector, we can use the restrictions implied by balanced growth to pin down the following seven parameters: the intermediate goods markups in the output and capital goods sectors, ϑ and θ; the overhead cost parameters in each sector, b and b^K; the slope coefficients on the research technologies in each sector, χ^z and χ^a, and the share of resources devoted to the development of new intermediate goods in output, S^z/Y.[20] Doing so yields the following parameter values (see the appendix for details): $\vartheta = 1.6$; $\theta = 2.0$; $b = 0.1$; $b^K = 0.0093$; $\chi^z = 0.083$; $\chi^a = 1.18$; and $S^z/Y = 0.095$.

Next, we set equal to unity the elasticity of the price markup with respect the number of firms in each sector (*i.e.* $\partial \log \mu / \partial \log M$ and $\partial \log \mu^K / \partial \log M^K$). By doing so, the overall medium term variation in the number of firms is roughly consistent with the data. In the data, the percent standard deviation of the number of firms relative to the standard deviation of the total markup over the medium term cycle is 0.65. With our parametrization, the model produces a ratio of 0.15. However, given that the data does not weight firms by size and given that most of the cyclical variation is due to smaller firms, the true size-weighted variation in the number of firms (relative to the markup) is probably closer to 0.15 than 0.65. As a check, we show that our results are largely unaffected by varying the price markup elasticity over the range from 0.5 to 1.5.

Finally, we fix the autocorrelation of the preference/wage markup shock so that the model generates an autocorrelation that matches that of the overall gross markup, $\mu_t \cdot \mu_t^w$, as measured by Gali, Gertler and Lopez Salido [2002]. This results in a value of 0.7.

[20] We let the model determine S^z/Y because we do not have a good measure of private expenditures for development of new intermediate final goods and services outside of manufacturing. Jones and Williams (2000) also follow this route.

4.2 Preference/Wage Markup Shocks and Medium Term Fluctuations

We now explore how the model economy responds to shocks to the labor supply preference parameter μ_t^w. As we have been emphasizing, movements in μ_t^w are observationally equivalent to movements in the markup of wages over the households consumption/leisure tradeoff. Thus one may interpret this shock either as reflecting preference shifts or some other factor that influences the degree of labor market efficiency. In the latter instance, this factor could include exogenous shifts in the wage markup brought about by varying union pressures, etc. It could also reflect some kind of demand shock which, in conjunction with labor market rigidities, induces countercyclical wage markup behavior.[21] This kind of shock has been shown to be of central importance at the high frequency (e.g. Hall, 1997). Here we show that it is also capable of generating medium term cycles of the type we have described, so long as it is embedded in a framework with endogenous technology change and countercyclical price markups.

Figure 16 illustrates the response of the model economy to a one unit positive shock to μ_t^w. The simple dashed line is our full blown model with endogenous productivity and endogenous markup variation. The dotted line corresponds to the simple benchmark with neither endogenous productivity nor endogenous markup variation. Again, the period length is a year. Not surprisingly, the initial output decline is largest in the full blown model, due to the endogenous markups in both the output and capital goods markets. While the rise in the goods markup μ_t reduces firm factor demands, the rise in the capital goods markup, μ_t^K, increases the relative price of capital, further dampening investment. The utilization rate drops as well, which works to enhance the overall decline in economic activity. Over time, output climbs back toward the initial trend, but it does not make it back all the way due to the endogenous permanent decline in productivity. After fifteen years or so it levels off at a new steady state. The permanent decline in output is about one third of the initial decline. In the benchmark model, of course, output simply reverts back to its initial

[21] As discussed in GGLS, a monetary model with nominal wage rigidities can generate a countercyclical wage markup. To keep our framework tractable, however, we abstract from monetary considerations.

steady state.

The initial decline in measured TFP and labor productivity results mainly from variable factor utilization (in conjunction with overhead costs).[22] Over time, the initial decline in R&D slows the rate of growth of new intermediate products in both the goods and capital sectors, leading to a permanent drop relative to trend in total factor productivity and labor productivity and permanent rise relative to trend in the price of capital. Both the variable markup and the endogenous productivity mechanisms appear to influence heavily the medium term dynamics of output, productivity and the relative price of capital.

Why is R&D procyclical in this model? First observe that R&D involves the same factor intensities as goods production. It is thus not as sensitive to wage behavior as in the Schumpeterian models of Aghion and Howitt (1992) and Aghion and Saint-Paul (1991).[23] Second, a component of R&D goes to development of new capital goods. Fluctuations in investment are not only highly procyclical, but also highly persistent. The persistence arises because declines in the price of capital associated with endogenous R&D lead to subsequent increases in the steady state level of capital. Thus, there are highly persistent procyclical movements in profitability from creating new capital goods. As the figure illustrates the endogenous markup also works to amplify the process. The persistent medium term output movement also stimulates R&D that contributes directly to TFP.

We next explore the ability of the model to generate medium term fluctuations consistent with those we observed in the data by creating an artificial time series. We do so by repeatedly feeding in markup shocks that are drawn from a random number generator. We then filter the artificial time series the same way we filtered the data. We construct two time series: one that corresponds to frequencies between 0 and 50 years and another (the medium frequency component) that corresponds to

[22]The formal definition of TFP that we adopt is the following: $TFP_t = \frac{Y_t A^{\alpha(\theta-1)}}{K_t^{\alpha} L_t^{1-\alpha}}$. This definition assumes that the BEA measures of TFP do not correct properly for capacity utilization and that the variety externality in the creation of new capital is not captured by the linear aggregation methodology used by the BEA to compute the capital stock. Reassuringly, this definition of TFP is consistent with the BLS average growth rate of TFP in the post-war, given our parametrization of the production function.

[23]Barlevy (2003) shows that if productivity shocks in the final goods sector are sufficiently persistent, R&D can be procyclical even in a Schumpeterian model.

frequencies between eight and fifty years. Figure 17 plots both of these series over a fifty year horizon. The solid line plots the overall medium term fluctuation while the line with squares plots the medium term component. As before, the difference between the two series corresponds to the high frequency component of the business cycle. As with the data, the medium frequency component is important to the overall fluctuations. The relative variances of the two components appear roughly similar. Further, the period length of the typical medium term cycle (about fifteen to twenty years) is also consistent with the data.

Next, figure 18 repeats the same exercise for TFP with similar results. The medium term fluctuation in TFP is about half that of output, but with a similar duration, as is roughly consistent with the evidence. To get a sense of how well the model captures of co-movements of key variables, figure 19 plots the medium frequency components of output TFP and the markup. As with the data, the model generates highly procyclical medium run movements in productivity and highly countercyclical movements in the markup. In each instance, the timing and magnitude are in line with the evidence.

We next look a bit more concretely at how the model lines up against the data. Table 2 computes the ratio of the standard deviation of a particular variable to the standard deviation of output over the medium term (2 quarter to 200 quarter frequencies). The first column is the full blown model. The second column is the benchmark with exogenous productivity and an exogenous markup. The third is the model which has just an endogenous markup. The fourth is the model with just endogenous productivity. Overall, the full blown model does best. At the 2-200 quarter frequency, the model captures, most though not all of the variability of labor productivity relative to output. In the data, the ratio of the standard deviation of labor productivity to output is 0.6 and is about the same for the ratio of TFP variability to output variability. In the full blown model, labor productivity variability to output is about 0.41 while TFP variability is about 0.49 percent of output. This model also does a reasonable job of characterizing the relative volatility of the markup: 1.06 versus 1.24 in the data. Capacity utilization is also roughly on the mark: 1.0 versus 0.87 in the data. Investment is a bit too volatile: 3.28 versus 2.47.

The model does however generate too little variability in the relative price of capital and too much variability in R&D. It generates about half the relative variability in the price of capital observed in the data, but about two and a quarter times the amount of R&D observed in the data. One possible way to improve on this dimension might be to allow for diffusion lags and endogenous technology adoption.[24] By doing so, we may be able to reduce the responsiveness of R&D, while still matching the variability of productivity. Another route, with possibly the same effect, would be to introduce learning by doing. Yet another possibility is to consider shocks to the R&D process in the capital goods sector. Overall, however, the full blown model performs considerably better than the other models.

To illustrate the strength of the endogenous propagation mechanisms, the bottom row of table 2 reports the unconditional standard deviation of output in each model, holding constant the variability of the exogenous shock. In the full blown model, the unconditional standard deviation is at least double the size produced by any of the other models. Interestingly, the models with either just endogenous markups or just endogenous productivity do not significantly enhance output volatility relative to simple model that contains neither propagation mechanism. It thus appears that it is the combination of endogenous markups and endogenous productivity that works to enhance volatility over the medium term.

Next, figure 20 compares the model autocorrelations and cross-correlations with the data. Overall, the model does a reasonable job of capturing medium term dynamics. The model captures reasonably well the co-movement of output with TFP and the markup. The contemporaneous model correlation between R&D and output is too strong, suggesting that there may be other sources of cyclical variation in R&D that are not captured by the model. Otherwise the model lead–lag relation between R&D and output roughly matches the data. Broadly speaking, the model also captures the inverse movement of the of the relative price of capital with output. While it captures the lag relationship well, it does not capture the lead. One possibility is that we need to include medium term shocks to the innovation process for the production of new capital goods. These might be associated to the exogenous arrival of general purpose technologies (Helpman (1998) and Aghion and Howitt (1998, ch. 8)) that make

[24]With the diffusion lags, the sensitivity of profits from innovation to the cycle will decline, reducing the sensitivity of R&D. On the other hand, so long as it is procyclical, the endogenous variability in adoption will generate variability in productivity. That is, endogenous adoption will compensate for the weakening of the pure R&D effect on medium term productivity movements.

easier the development of secondary innovations. Including this shock might not only improve the timing of the cyclical movement in the relative price of capital but it might also help the model account for the overall volatility of this variable.

As a final diagnostic, we consider whether the relative importance of the high and medium term frequencies of the medium term cycle generated by the model is similar to the data. In table 3 we summarize the ratio of the standard deviation of the medium frequency to the sum of the standard deviations of the medium and high frequencies of the medium term cycle for the relevant variables. There we can see that, for all the variables, the model approximately captures the relative size of the fluctuations in the high and medium frequencies. In this sense, the model that we have built is broadly consistent with the data at all the different frequencies (above the very long run).

The spirit of our exercise has been to explore how well a model driven by non-technology shocks can account for the medium term cyclical swings in productivity. A natural question to ask, however, is how our model compares with a real business cycle model that has exogenous shocks to total factor productivity as the forcing variable. We thus eliminate endogenous productivity and endogenous markup movements. But we keep variable factor utilization, based on King and Rebelo's (1999) finding that this feature improves performance. We consider two polar cases for the stochastic process that governs the technology shock: where the serial correlation is the same as for our markup, 0.7; the other where the technology shock obeys a unit root. As Table 4 shows, the first RBC model does good a job of capturing the relative volatility of total factor productivity, though not substantially better than does our non-technology shock driven model. This model does come closer to capturing the relative volatility of labor productivity. However, it does so partly by generating much greater relative volatility of investment than is present in the data: 4.24 versus 2.47. The model also doesn't quite capture the persistence of output, 0.59 versus 0.66 in the data, whereas our endogenous productivity/markup model is right on the mark. The RBC model with a unit root in technology captures TFP volatility well, but significantly overstates labor productivity. As well, output is too persistent relative to the data, as the bottom row of the table suggests. The first order autocorrelation is 0.8, again versus 0.66 in the data.

Both the RBC models, of course, are silent about movements in the relative price of capital and the markup. It is possible, for example, to augment the RBC model with shocks to the relative price of capital. However, to match the simultaneous procyclical movement in TFP and countercyclical movement in the relative price of capital over the medium term, it would probably be necessary to arbitrarily assume that the exogenous shocks to these two variables are negatively correlated. In this regard, it may very well be that adding the shock to the relative price of capital reduces the performance of the model in certain dimensions; e.g. investment is likely to become even more volatile relative to the data.[25]

Finally, a well known weakness of the RBC model is that it cannot explain the volatility of hours at conventional business cycle frequencies (using reasonable parametrizations of the Frisch labor supply elasticity). The same limitation applies when the model is matched against the medium term cycle. In the data the ratio of the standard deviation of hours to that of output is 0.69. The RBC models yield 0.39 and 0.26, well below the evidence. Our model, however, closely matches the data (0.73). As Table 3 indicates, further, the model also seems to closely mimic the relative variation of hours over the high versus medium frequencies. Part of this good performance reflects the strong effect of the labor supply shock on hours. Thus, the evidence on hours suggests another good reason to consider non–technology shocks as the driving force (along with appealing to endogenous technological change to capture medium term productivity variation.)

5 Concluding Remarks

A logical initial step in characterizing business fluctuations is to separate the cycle from the trend. A common recent practice is to associate the trend with variation in the data at frequencies 32 quarters and below. Though the resulting cycles correspond well to conventional notions of the business cycle, there is no precise theory to guide this particular detrending method. In this paper we explore the idea that the post war business cycle should not be associated with only high frequency variation in output (as generated by the conventional filter), but rather should also include medium frequency oscillations between periods of robust growth and periods of stagnation.

[25] See Greenwood, Hercowitz and Krusell (2000) for a model of how exogenous shocks to the relative price of capital might generate business fluctuations.

We refer to the combination of high and medium frequency variation in the data as the medium term cycle.

We construct measures of the medium term cycle and then develop a model of medium term fluctuations, treating the high and medium frequency variation as phenomena to be explained within a unified framework. The model features both disembodied and embodied endogenous technological change, along with countercyclical markups and variable factor utilization. We then illustrate how a non-technological shock can generate medium term fluctuations similar to those observed in the data, including the cyclical movements in technological change and resource utilization at both the high and medium term frequencies.

There are, however, several places where the model is off. As in the data, the model generates procyclical movement in R&D over the medium term. The model, however, generates variability in R&D that is about twice what we observe in the data. Conversely, the relative price of capital is only about half as variable as in the data. As we discussed, a possible extension would be to allow for diffusion lags with endogenous technology adoption. So long as adoption is procyclical, endogenous medium term productivity movements can arise that are less reliant on movements in R&D.[26] Learning by doing provides another route. Another possibility is to introduce medium term shocks to the innovation process for the production of new capital goods. That the model does not capture well the lead in the relative price of capital over the medium term cycle is another reason to believe that this type of shock may be important.

Finally, we have described a particular set of mechanisms (e.g. endogenous productivity, etc.) that we argued help explain medium term movements in U.S. data. Surely there other mechanisms that are worth exploring, particularly if one is interested medium fluctuations in other countries, as well. It is likely that labor market considerations are relevant in Europe, for example. Similarly, financial sector considerations may be central to understanding the prolonged stagnation in Japan.

[26] Loosely speaking, under this interpretation one might think of the variation in R&D in our model as also reflecting variation in the investments that lead to the adoption of new technologies.

Appendix

We first use the requirement that the model match the steady state growth rate of output, g_Y, and the steady state rate of decline in the relative price of capital, g_Q. The annual growth rate of non-farm business output per person in working age (16-65 years) for the period 1948-2001 is 2.4 percent. The average rate of decline of the quality adjusted price of capital is 2.6 percent, according to Gordon (1990) and Cummins and Violante (2002). This information is sufficient to compute the steady state growth rate of quality adjusted capital, g_K, and the gross interest rate, R. In the steady state of our model, the investment output ratio is equal to $(\delta + g_K) / \left[\alpha / (\mu\vartheta) \left[\frac{R}{1+g_Q} - (1-\delta)\right]\right]$. The previous information together with the average investment output ratio observed for the U.S. postwar period (12 percent) implies that the product of the final good composite and the intermediate good price markups, $\mu\vartheta$, is approximately equal to 1.9. Given μ to 1.2, we set $\vartheta = 1.6$.

Combining this information with the aggregate production function for the economy permits us to solve for the steady state growth rate of Z, g_Z. Once we have solved for the implied value of g_Z, we can use equation (53) to solve for the steady state ratio of the market value of the intermediate goods firms to output, $\frac{V_t^z}{Y_t}$. Given that we have fixed ρ at 0.95, we can use the free entry condition (51) and the production function for new intermediate goods (49) to identify the productivity parameter in the production function for new intermediate goods, χ^z, and the share of resources devoted to the development of new intermediate goods in output, S^z/Y.

Similarly, we can use the free entry condition (52), the production function for new intermediate capital goods (50), the expression for the price of a unit of new capital good (17), the expression for the market value of the stock of intermediate capital goods (54), and data on the average R&D share in GDP in manufacturing to obtain values for χ^k and θ.

Given a measure of S^a/Y, the R&D intensity in the capital goods sector, and a value for the curvature parameter ρ in the production function for new intermediate capital goods, permits us to calibrate χ^k and θ. Unfortunately, the data on the intensity of resources devoted to develop new intermediate capital goods is not very reliable. The NSF measure of resources devoted to nondefense R&D in manufacturing has averaged about 1 percent of GDP in the postwar period. However, the NSF measures of R&D do not include resources devoted to improving the managerial and personnel practices, or to the development of new services that improve the quality of the output in the manufacturing sector. Work by Comin (2002a, 2002b), suggests that the true number should be substantially larger.[27] We accordingly set S^a/Y to 1.4 percent. This number implies a gross markup for the intermediate capital goods sector of $\theta = 2$, which we find quite reasonable.

Next we address the calibration of the parameters of the overhead cost functions. Without loss of generality, we set number of firms operating in the final output and final capital goods sectors in steady state to unity (*i.e.* $M = M^K = 1$). The size of the operating costs (b and b^K) is then set to match the consumption output ratio in steady state (keep in mind that overhead costs enter the economy-wide resource constraint.)

[27] As shown by Comin (2002a), these investments are responsible for a major fraction of the observed improvements in productivity in the US economy. More specifically, Comin (2002b) has shown that these investments are especially important in the durable manufacturing sector.

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Figure 1: Non-farm Business Output per person 16-65

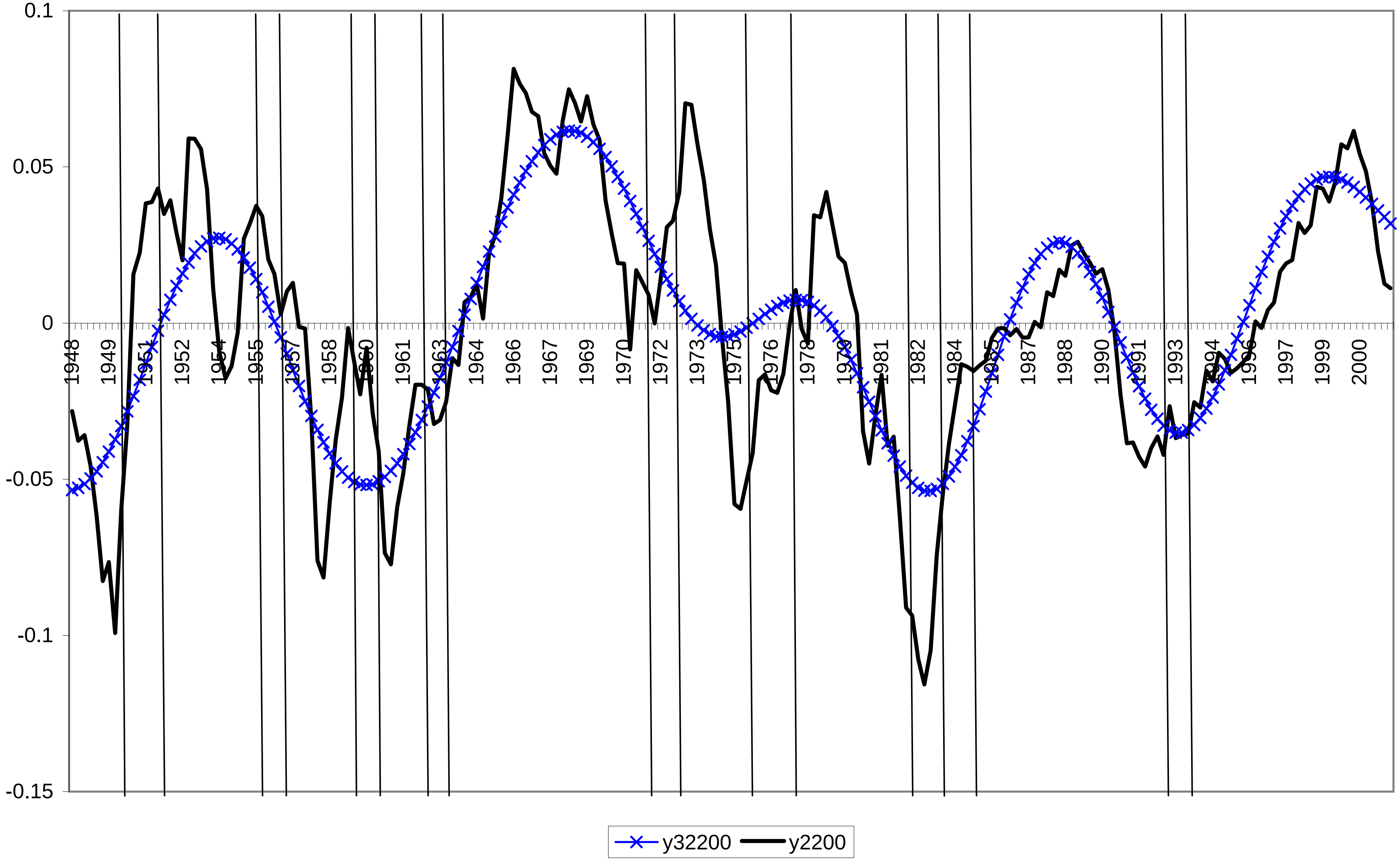


Figure 2: Labor Productivity

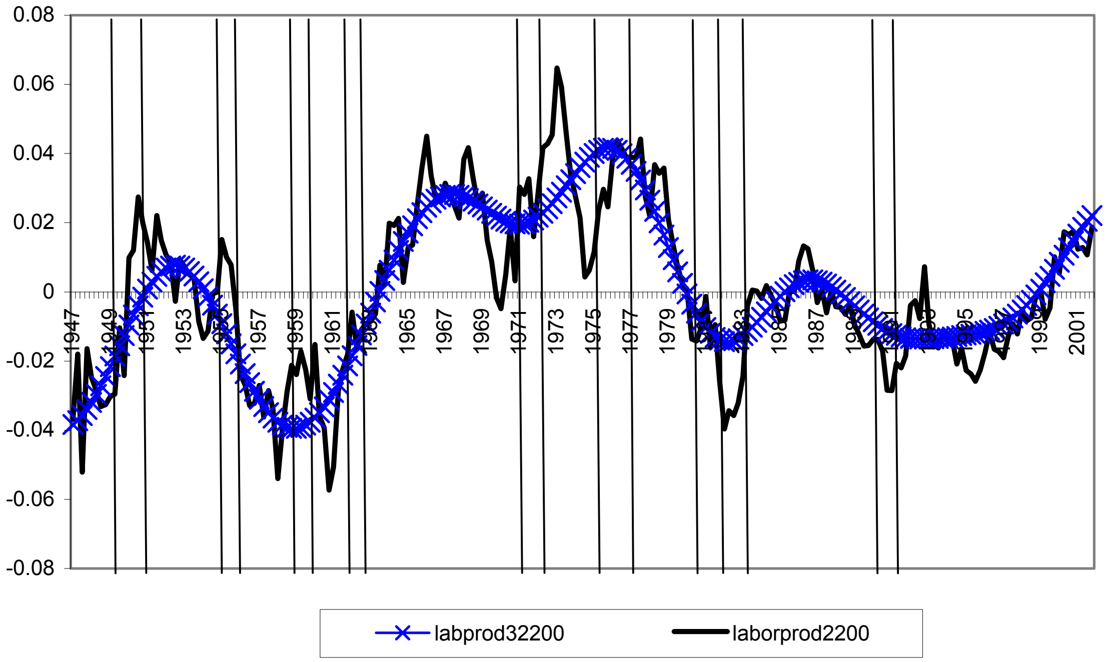


Figure 3: TFP

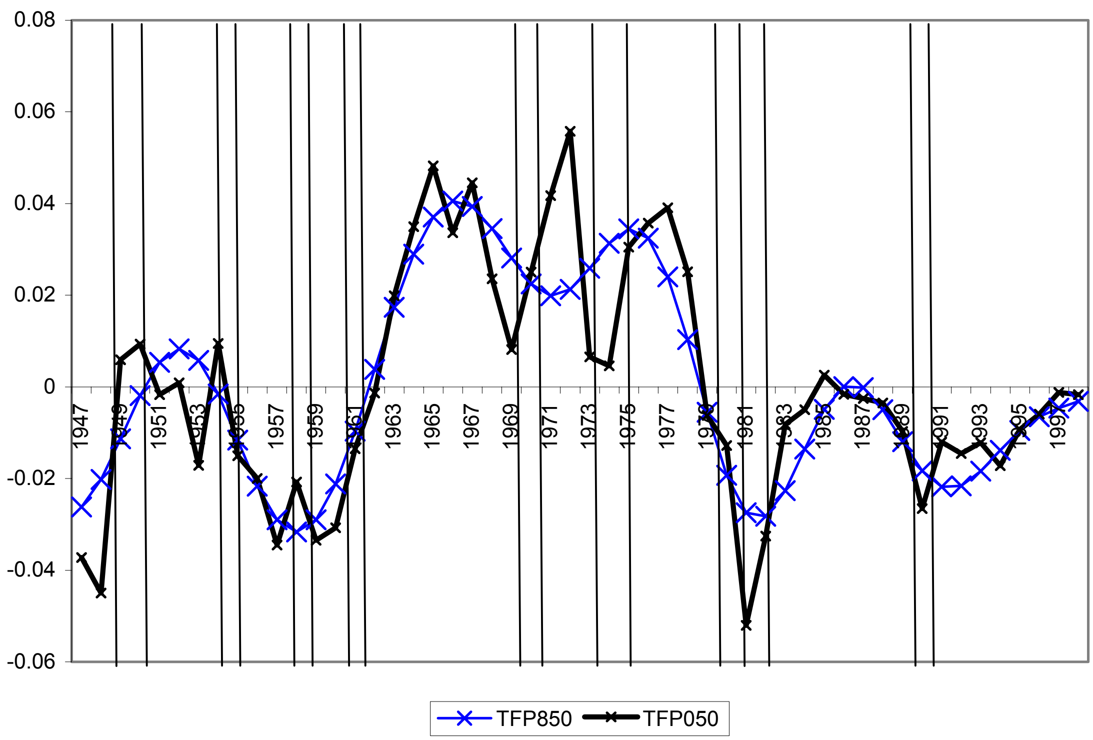
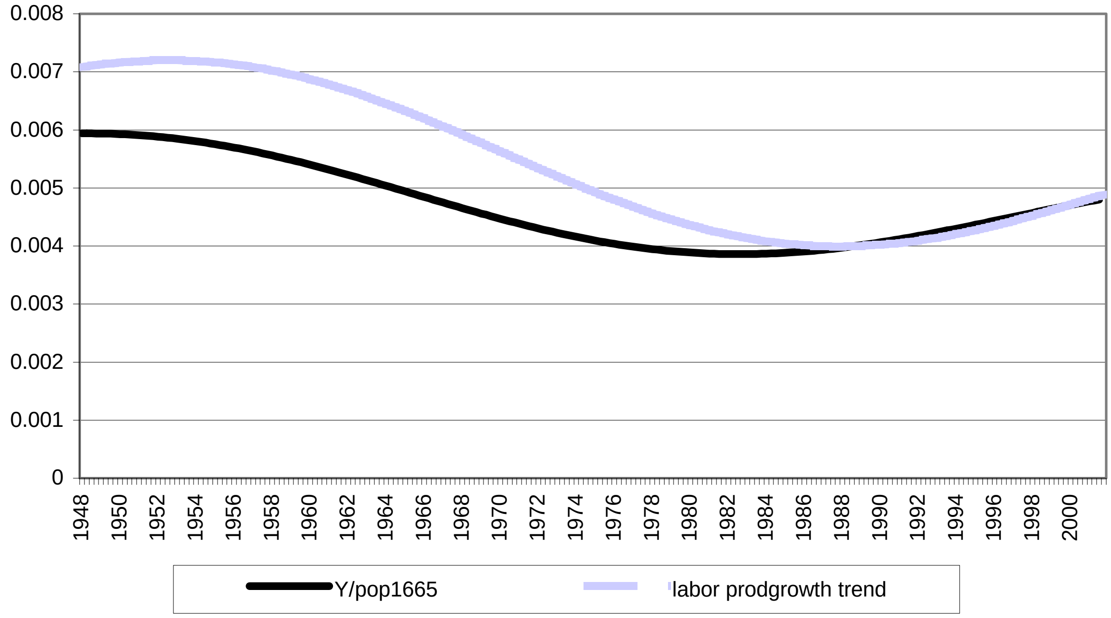


Figure 4: Long run growth rates for Output per person 16-65 and Labor Productivity



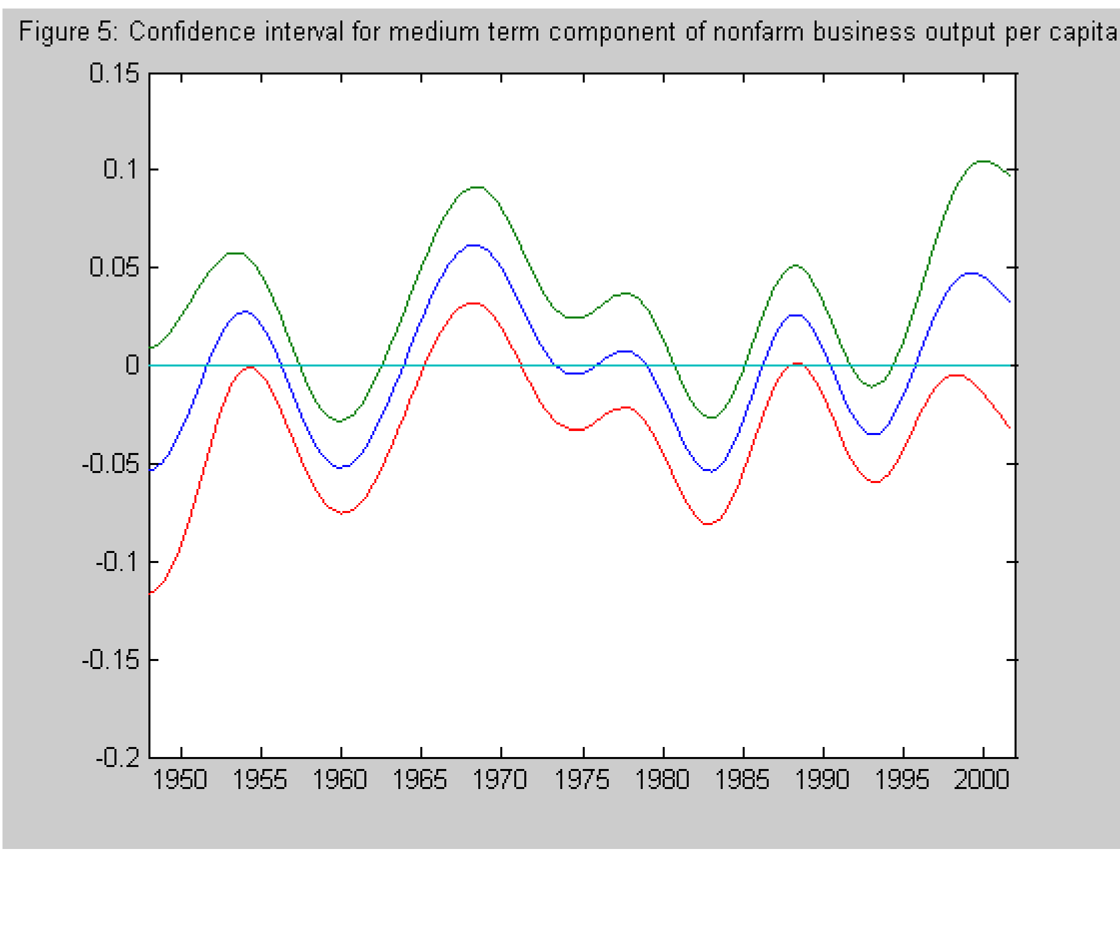


Figure 6: Relative price of capital (Gordon)

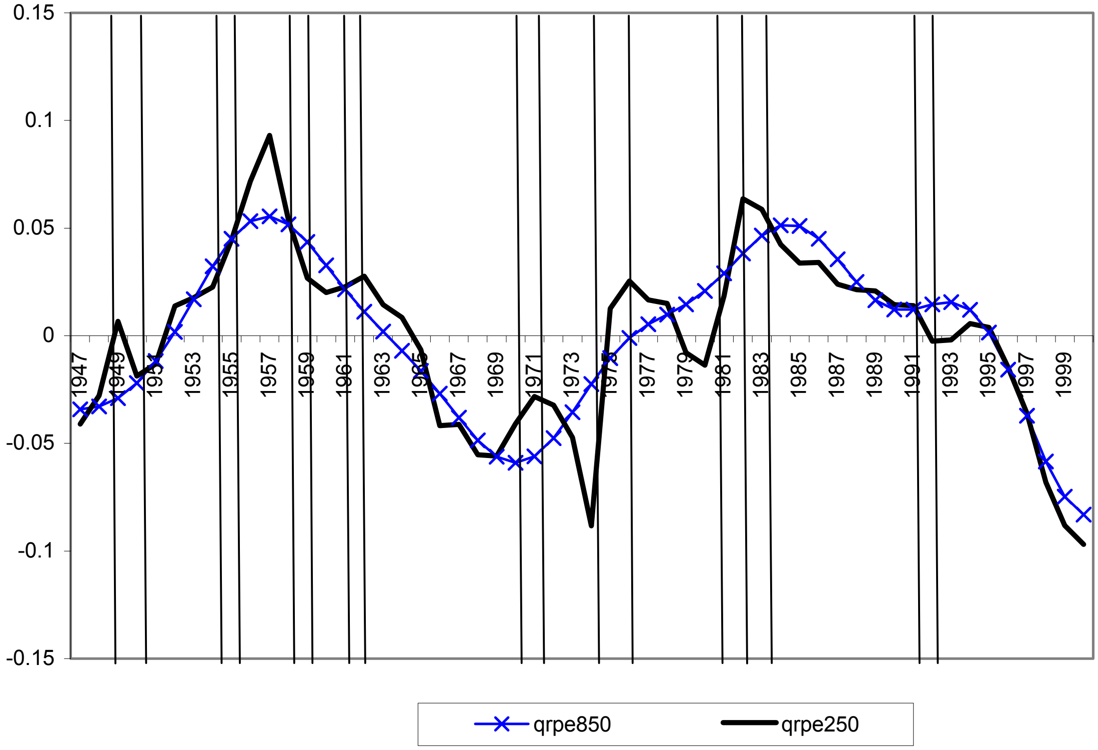


Figure 7: Nonfederally-funded R&D expenditures

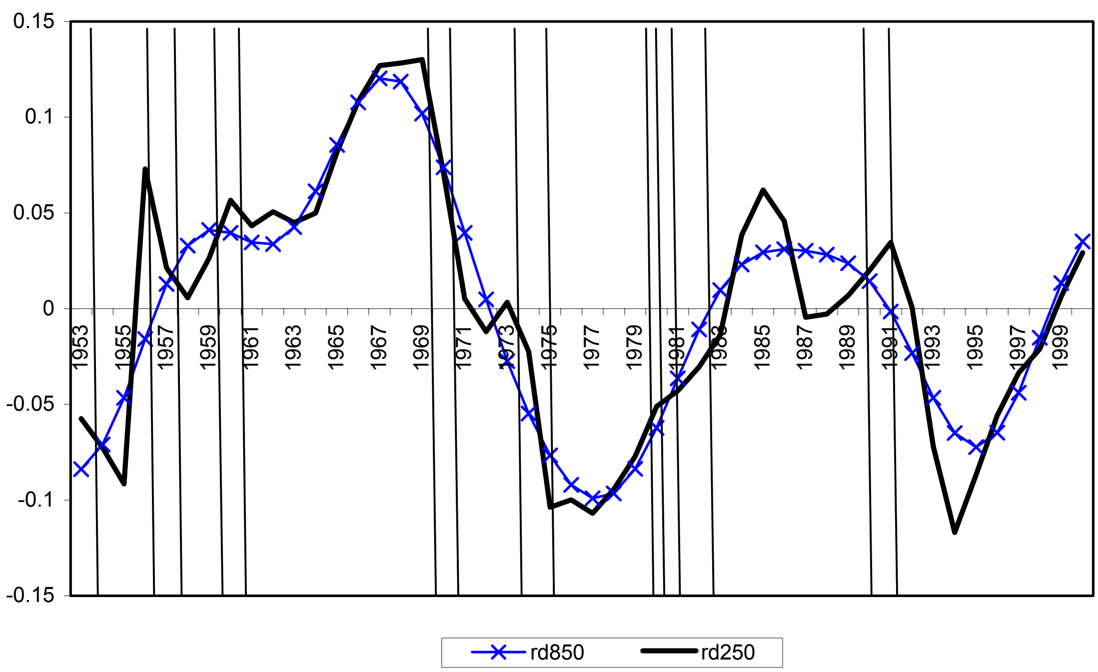


Figure 8: Markup

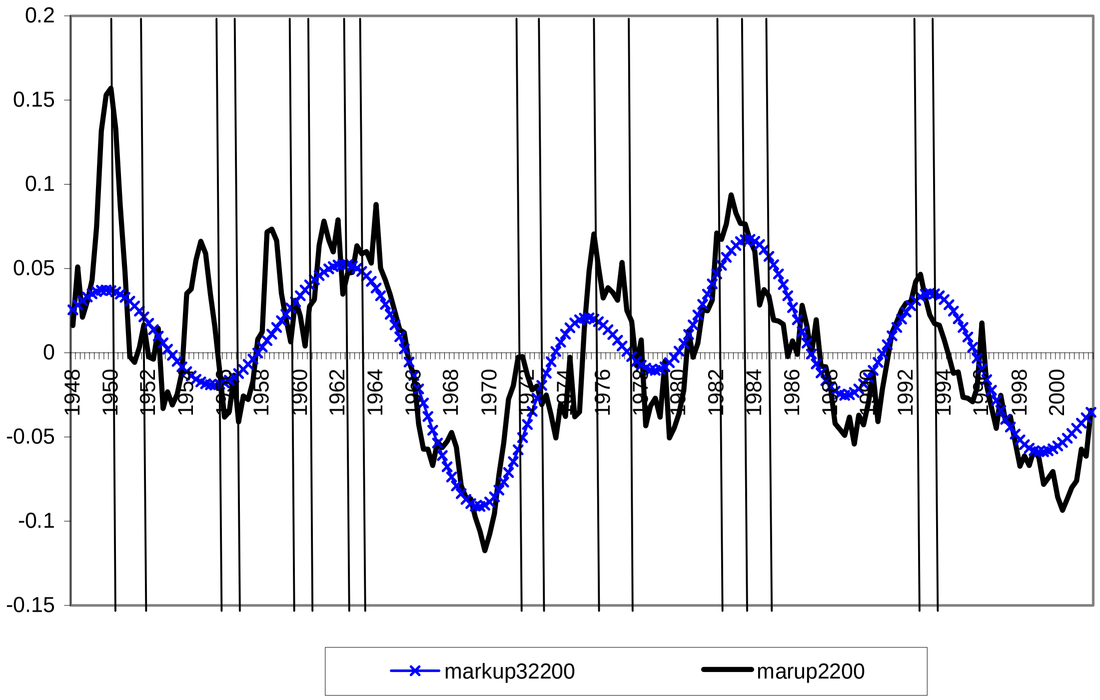


Figure 9: Capacity Utilization (Board measure)

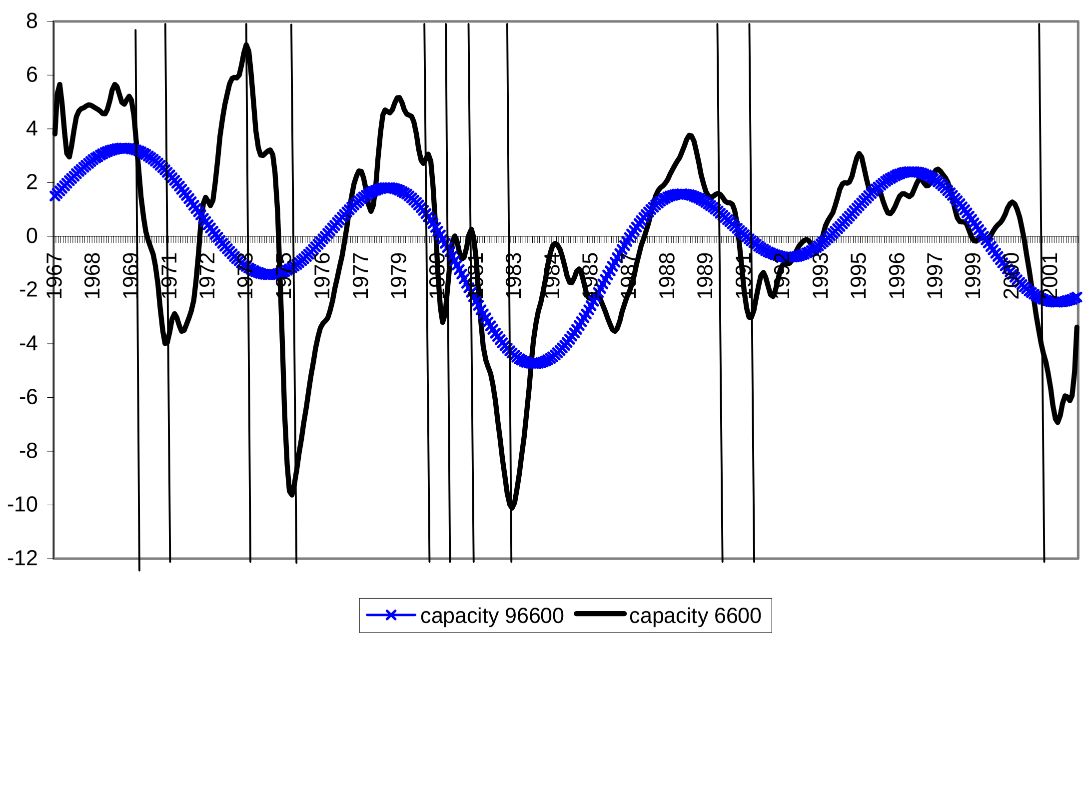


Figure 10: Cross correlation Output vs. Output

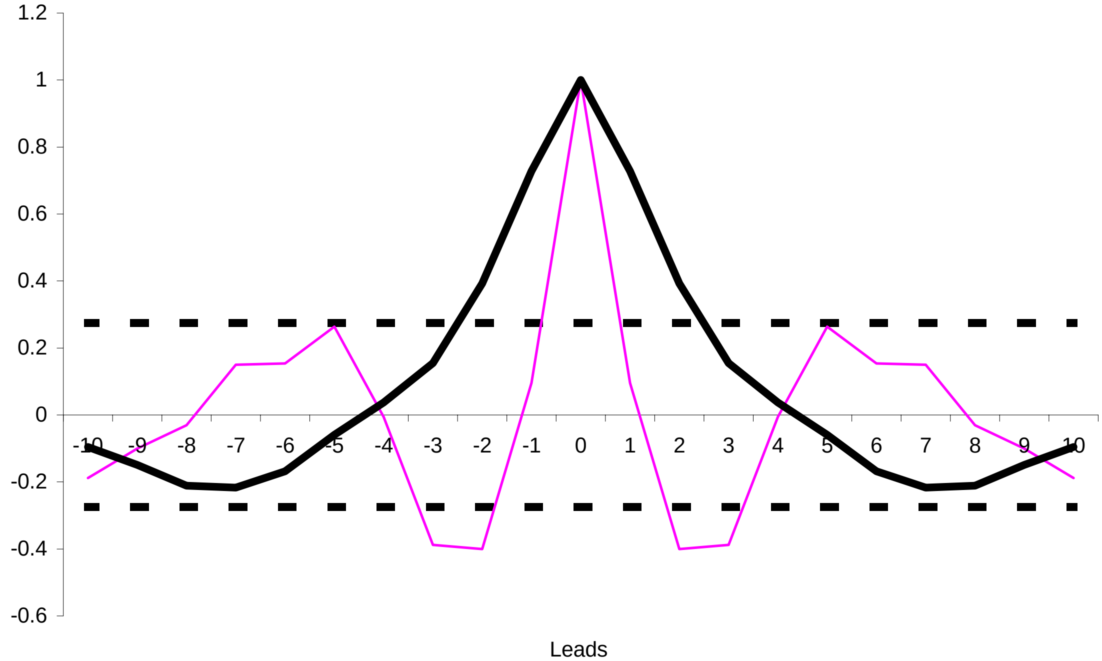


Figure 11: Cross Correlation Output vs labor productivity

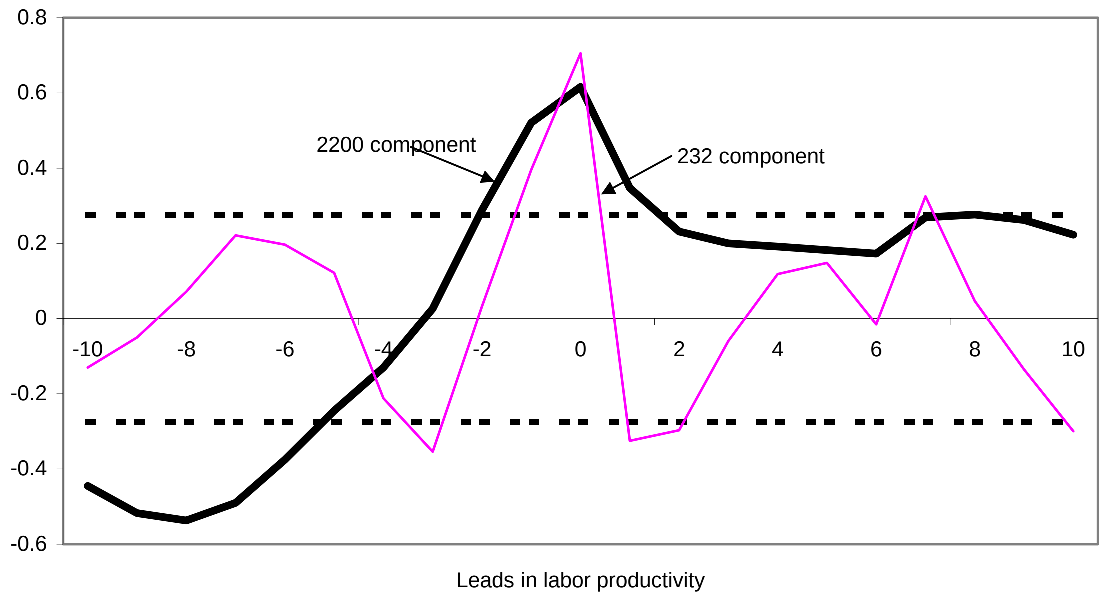


Figure 12: Cross Correlation Output vs. TFP

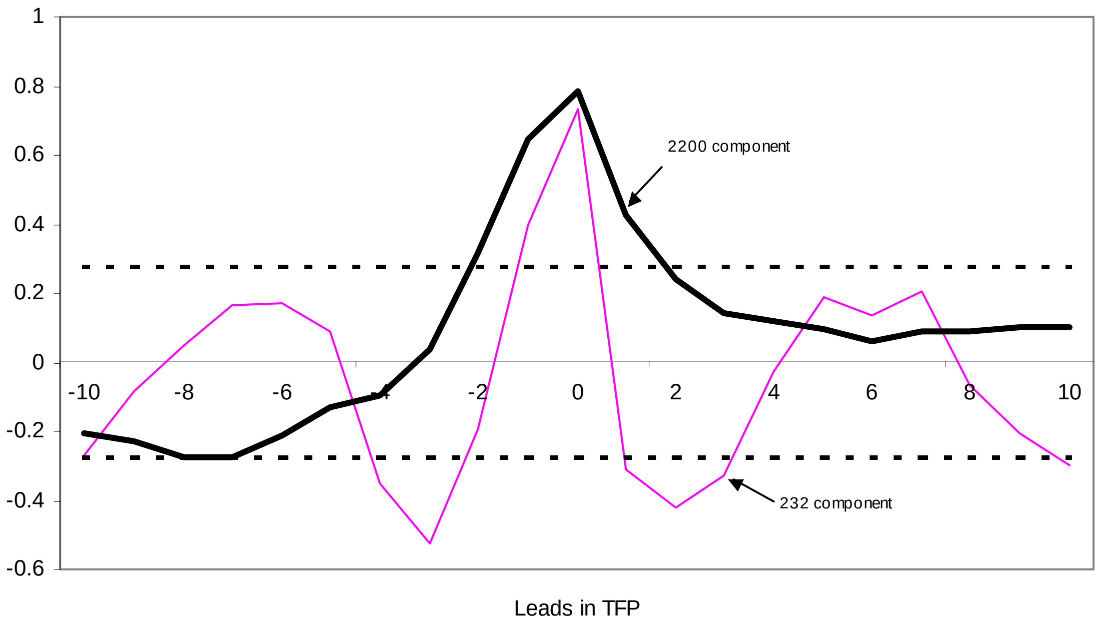


Figure 13: Cross Correlation Output with relative price of capital (quality adjusted)

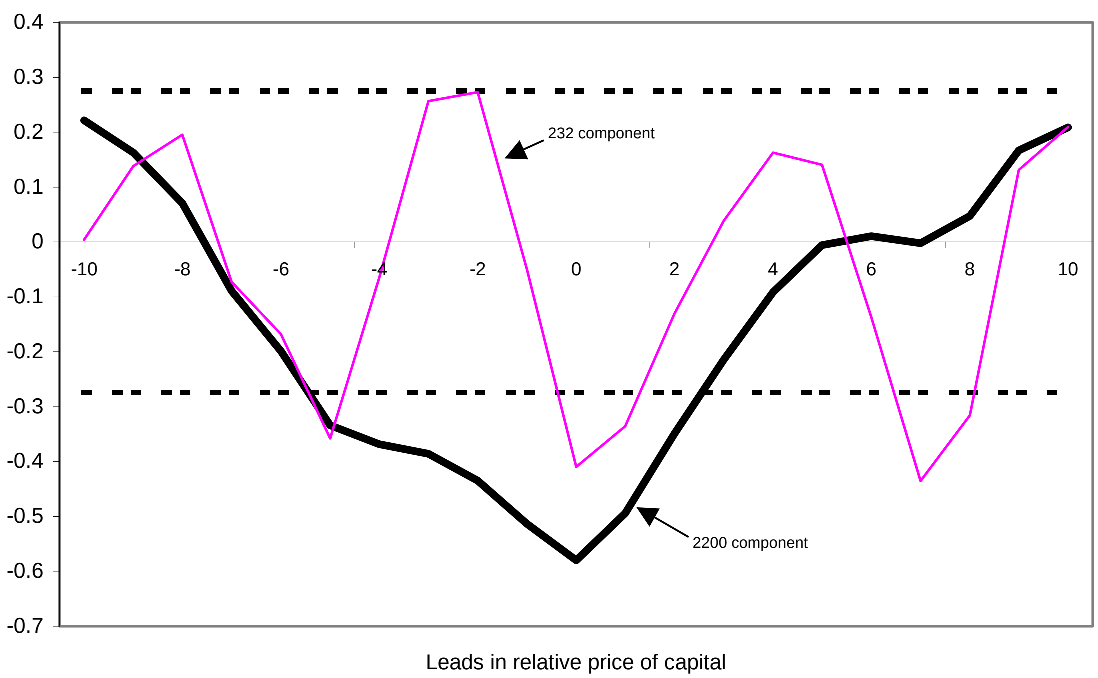


Figure 14: Cross Correlation Output vs. R&D

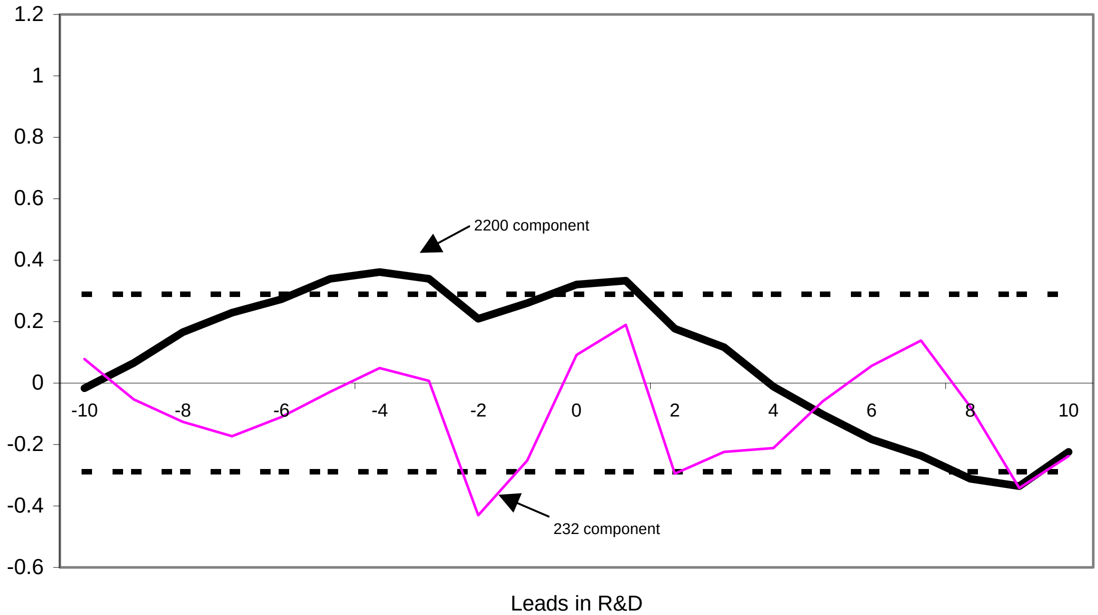


Figure 15: Cross Correlation Output and Markup

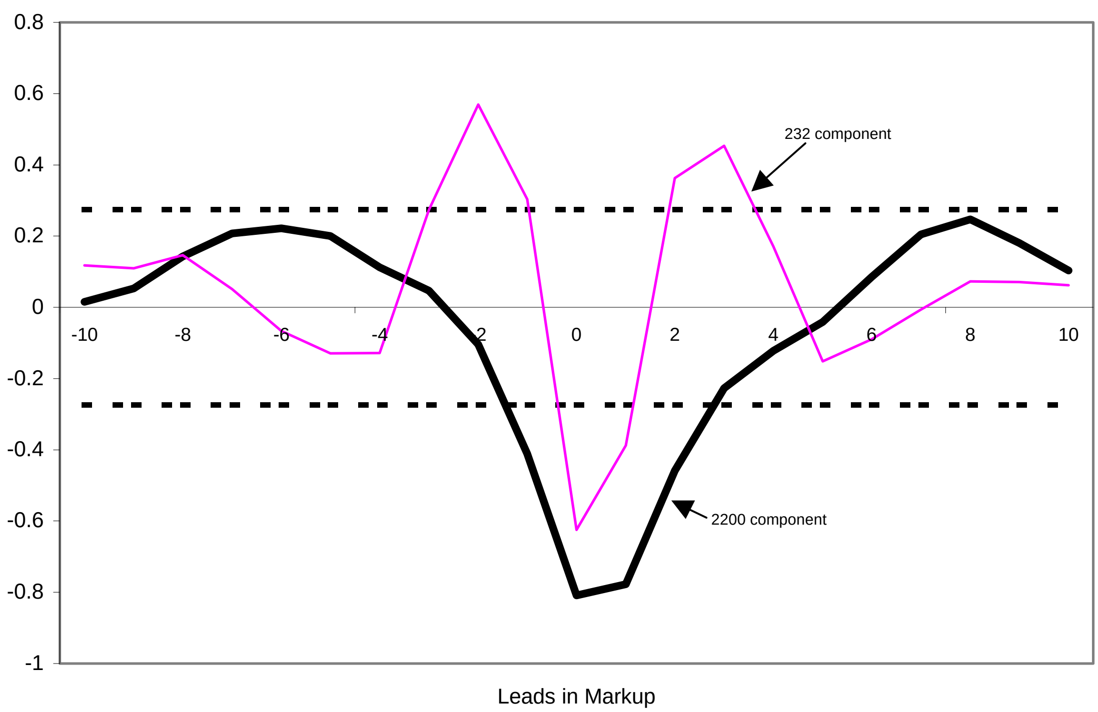


Figure 16: Impulse response functions for a preference shock, baseline 1 (solid) vs. model (dash)

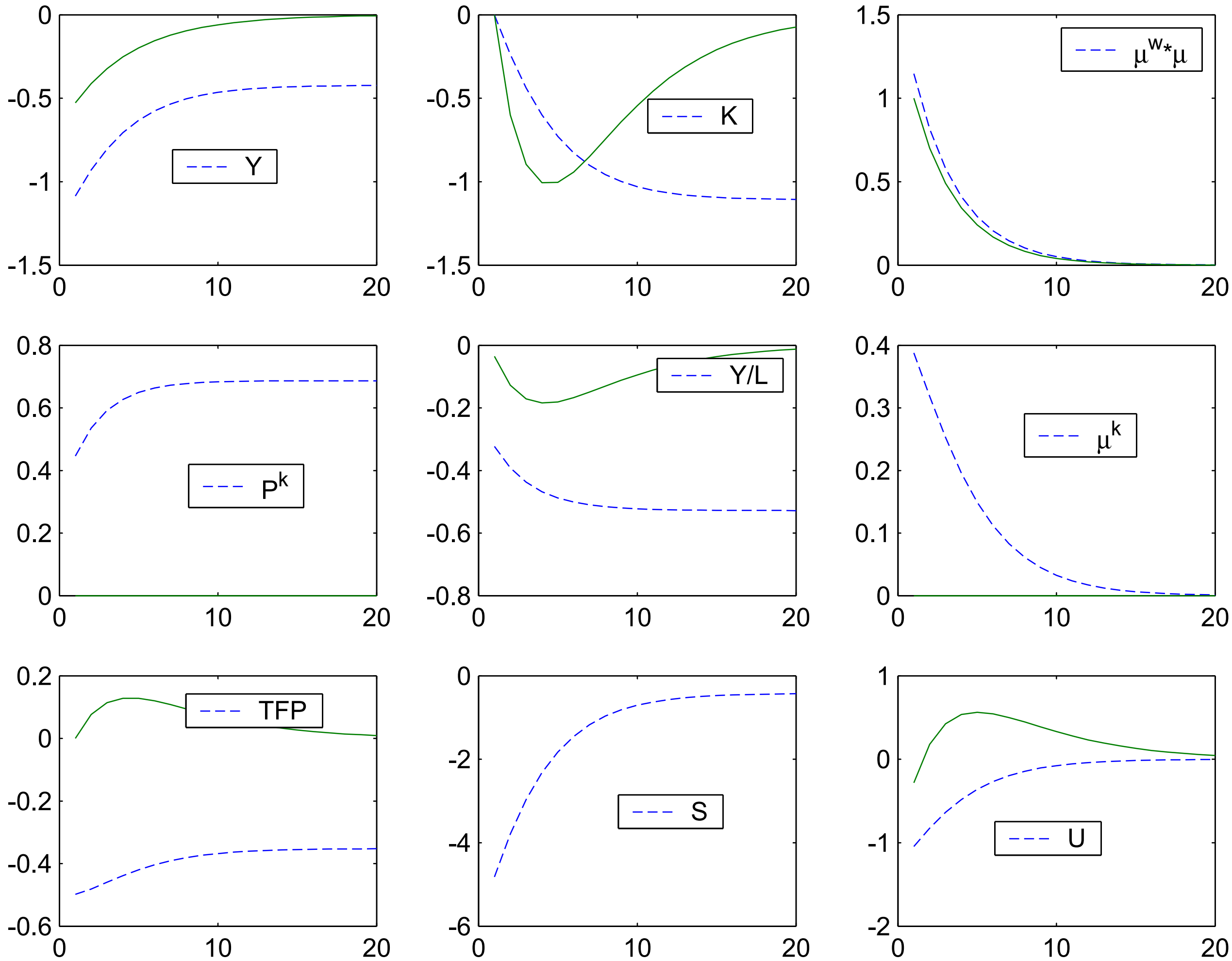


Figure 17: Medium term cycle and medium term component for output in the model

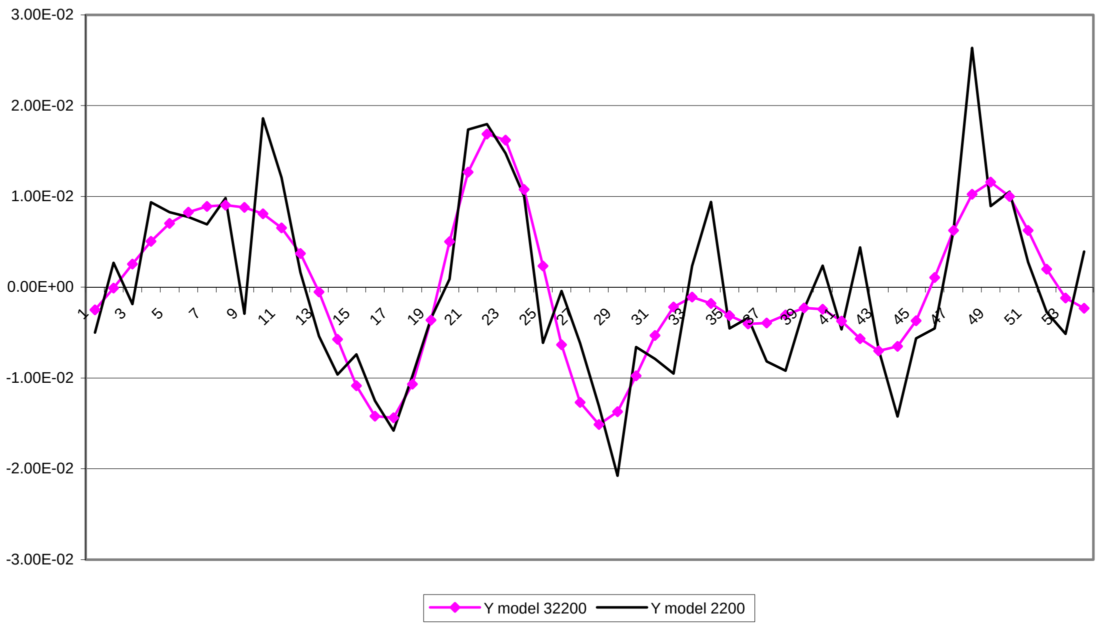


Figure 18: Medium term cycle and medium term component of TFP in the model

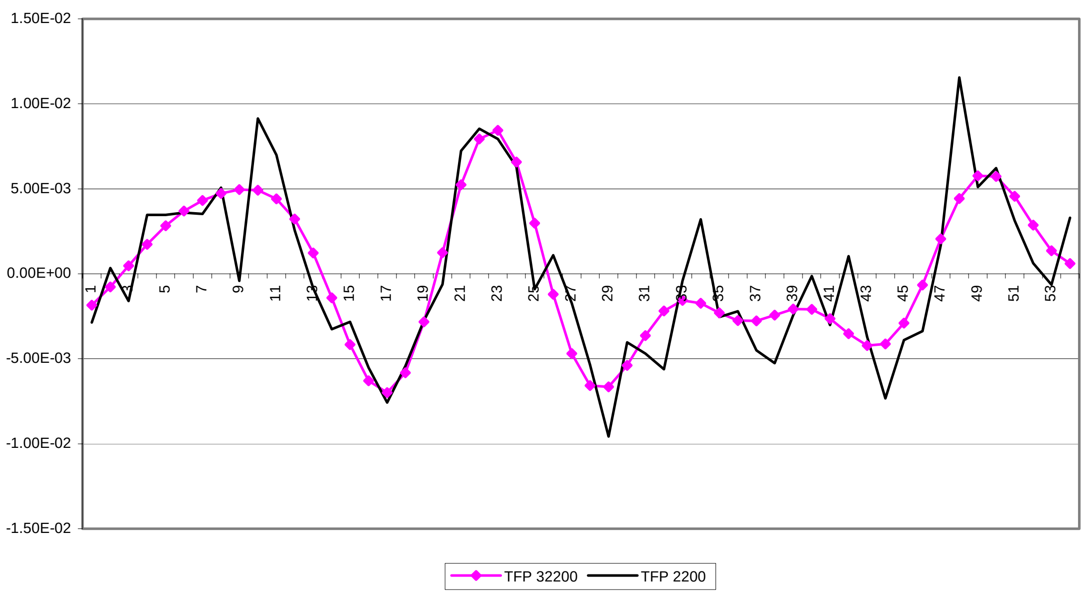


Figure 19: Medium term component for Output, TFP and the Markup in the model

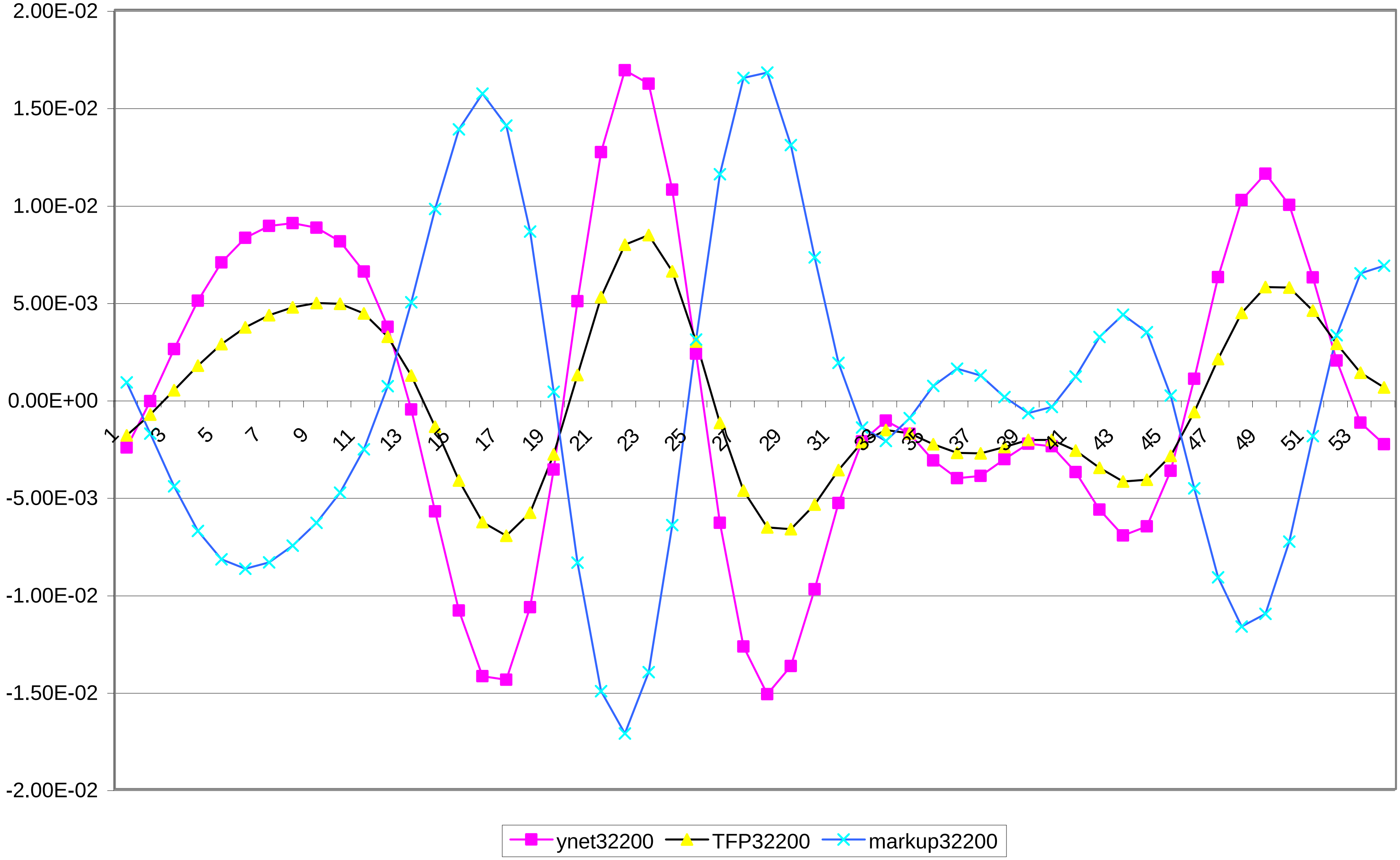


Figure 20: Cross correlation with output for data (dash) and model (solid).

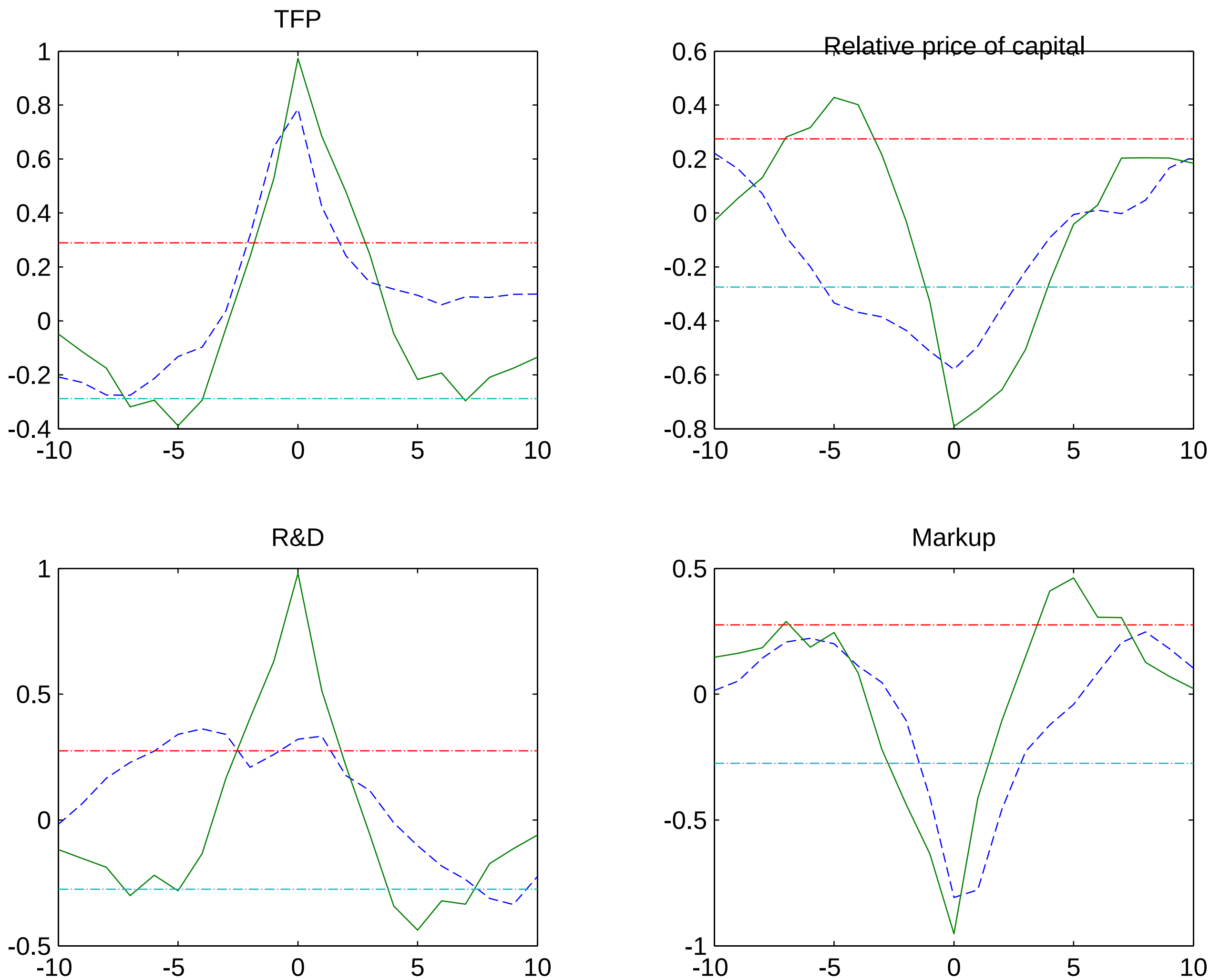


Table 1: Relative Volatilities

Empirical volatilities					
	Sd(2200)	Sd(232)	Sd(32200)	sd(32200)/(sd(232)+sd(32200))	confidence interval for the ratio
Non-Farm Business output per person with 16-65	4.05	2.42	3.24	0.57	0.55 - 0.67
Labor Productivity	2.34	1.16	2.08	0.64	0.6 - 0.72
TFP	2.5	1.22	2.15	0.64	0.59 - 0.75
Real Investment	10	5.62	8.57	0.61	0.59 - 0.69
R&D	7.65	3.4	5.95	0.64	0.69 - 0.78
Markup	5.02	3	3.84	0.56	0.53 - 0.66
Relative price of capital (quality adjusted)	4.34	1.54	4.12	0.73	0.69 - 0.82

Table 2: Relative Volatilities

Statistic	Data	Endogenous Productivity Growth and Endogenous Mark-up	Benchmark 1 exogenous productivity and mark-up	Benchmark 2 exogenous productivity endogenous mark-up	Benchmark 3 endogenous productivity exogenous mark-up
Labor Productivity	0.58	0.41	0.37	0.99	0.25
TFP	0.62	0.49	0.26	2.36	0.35
Investment	2.47	3.28	9.87	16.63	2.05
R&D	1.89	4.52	0.77	0.93	4.39
Markup	1.22	1.06	1.64	7.84	1.61
Capacity utilization	0.87	1.00	0.68	5.07	0.63
Relative price of capital (quality adjusted)	1.07	0.56	0.00	2.43	0.29
Standard Deviation of Output	-	0.97	0.48	0.23	0.53

Standard Deviations Relative to Non-farm business output

Table 3: Relative Volatilities at different frequencies, sd(32200)/(sd(232)+sd(2200))

Statistic	Data	Endogenous productivity growth and Markup
Output	0.57	0.60
Labor Productivity	0.64	0.73
TFP	0.64	0.64
Investment	0.6	0.62
R&D	0.64	0.60
Markup	0.56	0.57
Capacity utilization rate	0.43	0.60
Relative price of capital (quality adjusted)	0.73	0.73
Hours worked	0.56	0.59

Table 4: Relative Volatilities preferred models and RBC

Statistic	Data	Endogenous productivity growth and Markup	RBC with rho=0.7	RBC with rho=1
Labor Productivity	0.58	0.41	0.66	0.82
TFP	0.62	0.49	0.54	0.64
Investment	2.47	3.20	4.24	3.32
R&D	1.89	4.50	0.00	0.00
Markup	1.24	1.06	0.00	0.00
Capacity utilization	0.87	1.00	0.72	0.65
Relative price of capital (quality adjusted)	1.07	0.55	0.00	0.00
Hours worked	0.69	0.73	0.39	0.26
First order autocorrelation for output	0.66	0.67	0.59	0.80

Standard Deviations Relative to Output