Information Systems Research
Vol. 15, No. 3, September 2004, pp. 287–308
ISSN 1047-7047 | EISSN 1526-5536 | 04 | 1503 | 0287

DOI 10.1287/isre.1040.0030

Managing Digital Piracy: Pricing and Protection

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This paper analyzes the optimal choice of pricing schedules and technological deterrence levels in a market with digital piracy where sellers can influence the degree of piracy by implementing digital rights management (DRM) systems. It is shown that a monopolist's optimal pricing schedule can be characterized as a simple combination of the zero-piracy pricing schedule and a piracy-indifferent pricing schedule that makes all customers indifferent between legal usage and piracy. An increase in the quality of pirated goods, while lowering prices and profits, increases total surplus by expanding both the fraction of legal users and the volume of legal usage. In the absence of price discrimination, a seller's optimal level of technology-based protection against piracy is shown to be at the technologically maximal level, which maximizes the difference between the quality of the legal and pirated goods. However, when a seller can price discriminate, its optimal choice is always a strictly lower level of technology-based protection. These results are based on the following digital rights conjecture: that granting digital rights increases the incidence of digital piracy, and that managing digital rights therefore involves restricting the rights of usage that contribute to customer value. Moreover, if a digital rights management system weakens over time due to the underlying technology being progressively hacked, a seller's optimal strategic response may involve either increasing or decreasing its level of technology-based protection. This direction of change is related to whether the DRM technology implementing each marginal reduction in piracy is increasingly less or more vulnerable to hacking. Pricing and technology choice guidelines are presented, and some welfare implications are discussed.

Key words: piracy; digital piracy; piracy deterrence; copyright; digital rights management; DRM; nonlinear pricing; price discrimination; screening; intellectual property; enforcement
History: Barrie Nault, Associate Editor. This paper was received on May 20, 2003, and was with the author 8 months for 2 revisions.

1. Introduction

Over the last decade, sellers of digital products have actively fought the availability of pirated copies of their products. Nevertheless, digital piracy rates are still high and increasing in many markets, despite a continuous increase in the availability and sophistication of copy protection and digital rights management technologies. Studies of piracy trends by the Business Software Alliance indicate that while the worldwide software piracy rate declined between 1994 and 1999, it increased again in 2000 and 2001 and continues to rise in the Asia-Pacific and Eastern European regions (Business Software Alliance 2003). In addition, as recently as 2003, about 36% of all installed desktop software worldwide was pirated (Business Software Alliance 2004). Piracy concerns have further expanded following the emergence of file-sharing networks like Gnutella and Kazaa, which have substantially increased the availability and exchange of high-quality illegal versions of software, music, and digital video. While entertainment industry estimates of lost sales from piracy, pegged at up to $10 billion annually (Murphy 2003), may be overstated, there is evidence that access to file-sharing networks may reduce the probability of legal purchases by up to 30% (Zentner 2003), and that piracy reduces both legitimate revenues and the pricing power of sellers of music (Hui and Png 2003).

The sustained presence of piracy complicates the design of pricing schedules for sellers of digital goods. It also poses the new challenge of choosing an appropriate level of technology-based protection, and of strategically responding to the hacking of existing digital rights management (DRM) systems. These are the issues addressed by the model in this paper.

The first part of the paper studies pricing strategy in the presence of piracy. When faced with rising digital piracy, a seller's pricing power is increasingly limited by the quality and availability of pirated copies that are imperfect substitutes for the legally available product. This effect of piracy is analyzed by developing a model of monopoly price discrimination in which heterogeneous customers can buy variable quantities of a digital good, and different customer types get differing value from the usage of the legal good as well as from the usage of its pirated substitute. A number of examples of digital goods used in varying quantities are discussed in §2. The seller's optimal pricing schedule is shown to be a simple combination of two contracts—the optimal pricing schedule in the absence of piracy (termed the *zero-piracy pricing schedule*) and a *piracy-indifferent pricing schedule* that makes all customers indifferent between legal usage and piracy. While the presence of an inferior pirated substitute lowers prices and profits, it may lead to social benefits realized through an expansion in both the fraction of customers who purchase the product *legally* as well as the volume of legal usage.

The next part of this paper studies technology-based protection against digital piracy, typically achieved by implementing DRM systems. DRM platforms for digitally delivered products include SecureMedia Encryptonite (which is embedded in many digital televisions), Macrovision SafeCast, Apple's FairPlay, and Microsoft Windows Media DRM Series. Other DRM systems aimed specifically at protecting the physical sources of the digital files shared illegally over the Internet include the Windows Media Data Session Toolkit, and Macrovision's Cactus Data Shield.

However, implementing effective DRM-based technological deterrence often necessitates a direct reduction in the value of the *legal product*. For instance, all DRM platforms for digital video and music involve encryption that increases file sizes, thereby lowering value by increasing download times for digitally delivered content. Textual content that is protected by Adobe's DRM partners can be electronically scanned by OCR software that takes PDF files as direct inputs and produces near-perfect scanned versions. Deterring this form of piracy will necessitate degrading fonts in rendered legal files, again lowering quality for legal users.

More importantly, implementing DRM often constrains the flexibility of usage for a legal user. Many online music services implement DRM by limiting the rendering of MP3 files to a single device, and by placing related restrictions on the portability of these files. Highly restricted services like MusicNet and Rhapsody were not especially successful when first introduced, and this is partly attributed to the fact that their protection schemes "...treat everyone like a potential criminal, and they take all the joy out of buying and playing music" (Mossberg 2003). In contrast, the iTunes music service from Apple has chosen to place substantially fewer restrictions on a customer's ability to download, share, and burn purchased MP3 files,[1] at the risk of facing higher levels of piracy. This service has enjoyed early success, capturing an estimated market share of 70% for legal digital music as of mid-2004 (Flynn 2004, Levy 2004). Analogously, restricting printing rights for ebooks prevents the creation of high-quality pirated PDF versions, but discourages ebook adoption by customers used to reading printed pages. The inability to create a backup of a digital movie on a DVD disc reduces illegal secondary sales but deters value to legal digital purchasers.

Managing digital rights therefore involves restricting the rights of usage that contribute to customer value, and reducing this value in the process. Consequently, when choosing the appropriate level of technology-based protection against piracy, the seller of a digital good needs to trade off the effectiveness of *deterring piracy* with the *value reduction* of the legal product that is caused by the implementation of the DRM system. To study this trade-off, the second part of the paper incorporates endogenous choices of technology-based protection into the model of pricing with digital piracy developed in the first part. The *technologically maximal* level of protection, which is the level of protection at which the quality difference between the legal good and the pirated good is maximized, is contrasted with the *profit-maximizing* level of protection. When a seller can price discriminate, its profit-maximizing level of protection is shown to be *strictly lower* than the technologically maximal level. The economic drivers of this result are explored

[1] iTunes allows users to burn their MP3 files to an unlimited number of CDs, copy them to an unlimited number of iPod MP3 players, play them on up to three computers, and stream them over a private LAN.

in some detail because they indicate that even after accounting for quality degradation on legal products, a DRM system that maximizes the quality gap between the legal good and the pirated good is overprotecting the digital product. Additionally, as the effectiveness of a DRM system weakens over time, which typically occurs due to its technology being progressively hacked, a seller's optimal technological and pricing responses are examined. It is shown that the seller may wish to either decrease or increase its level of technology-based protection. Conditions under which each of these responses is optimal are characterized, and the implications for preemptive under- or overprotection are discussed.

The model in this paper views pricing strategy and technological deterrence as alternative instruments a seller can use to manage piracy, which is consistent with the model in Png and Chen (2003) and also with the observation that a software publisher can reduce piracy through increased deterrent controls or by reducing market price (Gopal and Sanders 1998). When allowed to price discriminate, this paper shows that a monopolist chooses a lower (and superior, both profit-wise and socially) level of DRM protection, suggesting that pricing policy and DRM technology can be complementary instruments for piracy deterrence, rather than necessarily being substitutes (Png and Chen 2003). The monopolist's investment in technology-based deterrence is indeed excessive from a welfare-maximizing perspective because any level of DRM protection is socially suboptimal, though it is shown that admitting price discrimination can mitigate the level of overprotection to some extent, as can the threat of DRM hacking in some cases. This paper also expands Png and Chen's observation that subsidies on legal usage are desirable. The threat of piracy causes a price-discriminating monopolist's choice of pricing to subsidize legal purchases, and also results in differential subsidies to different customer types. Apart from increasing total surplus, this has the potentially desirable welfare property of reducing the differences in consumer surplus between different customer types.

Unlike Png and Chen (2003), explicit taxes on copying devices or government subsidies on legal usage are not considered. Moreover, technology-based controls modeled in this paper directly influence the quality of the pirated good; the formation of sharing groups or software clubs (Gopal and Sanders 1998, Bakos et al. 1999) is not explicitly modeled. This focus allows a far richer demand and technology specification—usage in variable quantities by customers who value both legal usage and piracy differentially, simultaneously admitting a combination of second-degree price discrimination and variable technological protection against piracy, and explicitly considering the negative effect that DRM can have on the value from legal usage. A distinguishing aspect of this paper is therefore its deeper analysis of the economic effects of DRM technologies, and their role as technological deterrents to piracy that are controlled explicitly (and strategically) by a seller who can also use pricing policy to manage piracy.

This paper also contributes to the literature on the economics of copying and piracy (Johnson 1985, Liebowitz 1985). Conner and Rummelt's (1991) early model of strategic piracy deterrence establishes that increases in protection technology always increase firm profits, unless the product displays positive network effects. More recently, Belleflamme (2003) studies the interdependence between different producers' incentives to accommodate/deter the presence of a pirated good. Chellappa and Shivendu (2003) model sampling and pricing in the presence of a pirated good derived from an evaluation version of the legal good. The model in this paper builds on the approach of each these papers, by preserving their notion of the pirated good as an inferior (vertically differentiated) substitute for the legal good, a model on which many prior IS papers are based (for instance, Nault 1997). Additionally, it generalizes their pricing analysis significantly, by modeling and deriving a menu of prices, rather than a single variable price or a pair of prices for two quality-differentiated products. It also explicitly takes into account the differing value of pirated products to different customer types. This generalization is important because it substantially alters results relating to the optimal level of technological protection and to post-implementation protection and pricing trends. These are differences that would not be evident in a model with unit consumption and no price discrimination. This approach is also more likely to provide managerially relevant pricing guidelines because the results prescribe a straightforward way to actually design pricing schedules in the presence of digital piracy, as illustrated by a simple example in §5.

Unlike the piracy models in Conner and Rummelt (1991), Takeyama (1994), and Shy and Thisse (1999), positive network externalities are not considered. These externalities are significant in many software markets (as documented for spreadsheet software, for instance, by Brynjolfsson and Kemerer 1996); in the industries more recently threatened by digital piracy—music, video, and content—there may be indirect network effects (from complementary device sales) and word-of-mouth effects. Their presence is likely to directionally strengthen the results of this paper, as discussed briefly in §6.

The rest of this paper is organized as follows. Section 2 provides an overview of the model and describes the seller's optimal pricing schedule in the absence of piracy. Section 3 derives the optimal pricing strategy in the presence of different levels of digital piracy. Section 4 models the economic effects of using digital rights management, derives optimal technology-based protection levels, and characterizes strategic responses to changes in the effectiveness of an implemented DRM system. Section 5 presents a brief example. Section 6 discusses the managerial and welfare implications of the model's results, and concludes with directions for future research.

2. Model

2.1. Seller and Customers

I model a digital good that may be used by consumers in continuously varying quantities. This could either be a homogeneous information good, e.g., a software package that corporations install on a varying number of employee desktops (the unit of variable quantity in this example would be the number of licenses), or an electronic teaching case, like those bought in varying quantities from the Harvard Business School Press by universities, often with substantial volume discounts (the unit of variable quantity for this example would be number of students who are licensed to access the electronic case). Other possible examples that are similar might include corporate purchases of ebooks for employees, purchases of code comprising digital versions of copyrighted entertainment characters (paid for based on the number of instances the code is used), and so on. Alternatively, the model also applies to large libraries of related digital goods[2] from which different customers use different subsets. For instance, a digital music service that offers access to a large bundle of songs (such as the million-plus-song library available to users of Apple's iTunes) of which each user desires and pays for a small fraction—the unit of variable quantity would be the number of songs downloaded in this case. Another example of this kind is a news service that offers access to a large archive of articles (like the *New York Times* archive) from which different readers download different articles, paying per archived article downloaded (which would be the unit of variable quantity in this case). Other related examples include libraries of graphics/clipart, music videos, journal articles, ringtones, games, or film archives. As more goods become digital, the set of examples of this kind is likely to increase.

The seller of the digital good (termed the *legal good*) is assumed to be a monopolist, by virtue of owning a copyright. Any fixed costs of production or IP protection are assumed to be sunk, and variable costs of production are zero. In addition to the legal good, there is also a *pirated good* available, which is a lower-quality substitute for the legal good and is free.

Customers are heterogeneous, indexed by their type $\theta \in [\alpha, \beta]$. The preferences of a customer of type θ for a good of quality z are represented by the multiplicatively separable utility function

$$u(q, \theta, z) = zU(q, \theta), \tag{2.1}$$

where q is the quantity of the good used by the customer. The function $U(q, \theta)$ is assumed to take the following form:

$$U(q, \theta) = \theta q - \tfrac{1}{2}q^2. \tag{2.2}$$

This functional form is chosen for analytical convenience. The paper's results generalize directionally for more abstract functional forms, as discussed in §3.

[2] This might also be viewed as a multiproduct pricing problem, with each of the goods being treated as a separate product. However, for extremely large sets of digital goods (like the millions of songs or news articles mentioned in the examples), precise multiproduct nonlinear pricing is unlikely to be practically viable. A more detailed discussion of this point is available from the author.

Subscripts of functions represent partial derivatives with respect to the corresponding variable. For instance, the partial derivative of $U(q, \theta)$ with respect to q is denoted $U_q(q, \theta)$, and the cross-partial of $U(q, \theta)$ with respect to q and θ is denoted $U_{q\theta}(q, \theta)$. This notation is preserved throughout the paper.

The following properties of the utility function follow from Equation (2.2).

(1) For every θ, $U(q, \theta)$ has a finite maximum usage $\sigma(\theta) = \arg\max_q U(q, \theta) = \theta$.

(2) $U_q(q, \theta) > 0$ for $q < \sigma(\theta)$, and $U_q(q, \theta) < 0$ for $q > \sigma(\theta)$.

(3) $U_{qq}(q, \theta) = -1$, and $U_{q\theta}(q, \theta) = 1$. Therefore, $U(q, \theta)$ is strictly concave in q (diminishing marginal value from usage), and has the Spence-Mirrlees single crossing property.

The quality of the legal good is denoted by the variable v, and the quality of the pirated good is denoted by the variable s. The preferences of a customer of type θ for the legal good and the pirated good are therefore represented by the functions $vU(q, \theta)$ and $sU(q, \theta)$, respectively, where q is the quantity of the good (legal, pirated) used by the customer and $U(q, \theta)$ is as defined in (2.2). The parameter s is related to how much customers value the pirated good, and it is also interpreted as the *threat of piracy* faced by the seller.

The levels of quality v and s are initially exogenous (in §3) and then influenced by the seller's choice of DRM protection (in §4). However, s is always assumed to be strictly less than v, implying that the pirated good is always strictly inferior to the legal good, and therefore the seller can make a nonzero profit. While pirated software is generally considered inferior due to reasons directly related to product value (restrictions on functionality, lack of technical support), the feasibility of digital replication of content might suggest that pirated goods and legal goods may be perceived as being of equal quality by potential buyers; however, this is generally not the case. When attempting to access pirated content on peer-to-peer file-sharing networks like Gnutella and Kazaa, users often cannot find the exact title they are looking for. Even when available, locating this title can be slow and unreliable (due to the way pure peer-to-peer networks operate). Download speeds can be extremely slow. Often, the contents of media files are not what they were supposed to be, partly due to the posting of decoy files by media companies.[3] Furthermore, the resolution of pirated songs and movies is variable (and often poor). Using pirated products opens the user to the threat of litigation. Each of these factors can lead a user to view the legal good as being of higher quality, and the lower value of s relative to v is a simple way of modeling this.

The maximum value that a customer of type θ can get from a pirated good of quality s is denoted $\hat{u}(\theta, s)$:

$$\hat{u}(\theta, s) = sU(\sigma(\theta), \theta) = \frac{s\theta^2}{2}. \tag{2.3}$$

Because the pirated goods are free, $\hat{u}(\theta, s)$ is the *reservation utility* of customer type θ.

The monopolist does not observe the type θ of any customer, but knows $F(\theta)$ (the probability distribution of types in the customer population). For expositional simplicity, and because the hazard rate of the customer-type distribution plays a significant role in subsequent analysis, we define the *inverse hazard rate* function $h(\theta)$:

$$h(\theta) = \frac{1 - F(\theta)}{f(\theta)}, \tag{2.4}$$

and the *cumulative inverse hazard rate* function $H(\theta)$:

$$H(\theta) = \int_{\alpha}^{\theta} \frac{1 - F(x)}{f(x)}\, dx. \tag{2.5}$$

The probability distribution of types is assumed to have the following properties.

(1) $f(\theta) > 0$ for all θ, where $f(\theta)$ is the density corresponding to the distribution $F(\theta)$.

(2) $h_\theta(\theta) \le 0$ for all θ: the inverse hazard rate is nonincreasing in θ.

Each customer knows his or her own type θ. Without any loss in generality, the total number of customers in the market is normalized to one.

2.2. Customer Choice and Pricing Schedules

The seller offers a nonlinear pricing schedule (sometimes referred to as either a *contract* or a *pricing schedule*) that assigns a nonnegative price to each feasible

[3] Ripley (2004) reports that this is an active deterrence strategy by movie companies, and that bogus postings by "bored hackers" contribute further to this issue; for instance, downloads of pirated copies of what was purportedly *The Last Samurai* "turned out to be *Scary Movie 3*, *Santa Clause 2* and a porn flick" (p. 56).

level of usage for the *legal good*. Rather than considering all possible pricing functions, the revelation principle ensures that we can restrict our attention to direct mechanisms—menus of quantity-price pairs $q(t), \tau(t)$, indexed by $t \in [\alpha, \beta]$—that are incentive compatible. The function $\tau(t)$ specifies the total price for a usage level $q(t)$; the variable t has the same domain as the variable θ, and is used for notational clarity in Equations (2.6) and (2.7) below. A pricing schedule $q(t), \tau(t), t \in [\alpha, \beta]$ for the legal good is said to be incentive compatible if it satisfies

$$\theta = \arg\max_t [vU(q(t), \theta) - \tau(t)], \quad \text{for all } \theta. \tag{2.6}$$

Given a pricing schedule $q(t), \tau(t)$ for the legal good, a customer of type θ purchases the legal good if the surplus from doing so is at least as much as the value the customer would derive from the (free) pirated good. Mathematically, if

$$\max_t [vU(q(t), \theta) - \tau(t)] \geq \hat{u}(\theta, s), \tag{2.7}$$

then customers of type θ purchase the legal good. Therefore, an incentive-compatible pricing schedule $q(\theta), \tau(\theta)$ is said to *induce participation* from customer type θ if all customers of this type (weakly) prefer the legal good to the pirated good:

$$[vU(q(\theta), \theta) - \tau(\theta)] \geq \hat{u}(\theta, s). \tag{2.8}$$

The constraint (2.8) above is often referred to as the *piracy constraint* for type θ because it becomes progressively harder to satisfy as s increases. Note that the piracy constraint is type dependent. In the special case of $s = 0$, the pirated good has no value, $\hat{u}(\theta, s) = 0$, and constraint (2.8) above reduces to the standard individual rationality constraint.

Finally, the *optimal pricing schedule* $q^*(\theta, v, s)$, $\tau^*(\theta, v, s)$ is the incentive compatible pricing schedule that maximizes the seller's profits.

In general, the sequence of events is as follows: The seller announces its pricing schedule (and technological choices, if any), the customers make their purchase decisions (whether to use the legal good or the pirated good, and at what usage level) based on the pricing schedule, and each party gets its payoffs. An exact timeline is specified separately in each of the following sections.

2.3. Optimal Pricing Schedule in the Absence of Piracy

The optimal pricing schedule in the absence of piracy, termed the *zero-piracy pricing schedule*, is specified in this section. The zero-piracy pricing schedule benchmarks the analysis of pricing in the presence of piracy, and is also used in constructing the corresponding optimal pricing schedules.

Lemma 1. *The zero-piracy pricing schedule $q^{ZP}(\theta, v)$, $\tau^{ZP}(\theta, v)$, which is the optimal pricing schedule for the seller when $s = 0$, takes one of the following two forms.*

(a) *If $h(\alpha) \leq \alpha$, then the pricing schedule is designed to include all customer types. The optimal contract is*

$$q^{ZP}(\theta, v) = \theta - h(\theta); \tag{2.9}$$

$$\tau^{ZP}(\theta, v) = \frac{v[\alpha^2 - h(\theta)^2]}{2} + vH(\theta), \tag{2.10}$$

for all $\theta \in [\alpha, \beta]$.

(b) *If $h(\alpha) > \alpha$, then a set $[\alpha, \theta_{ZP}]$ of customer types are priced out of the market, where θ_{ZP} is defined as*

$$\theta_{ZP} = \theta\colon h(\theta) = \theta, \quad \theta \in (\alpha, \beta). \tag{2.11}$$

The optimal contract is

$$q^{ZP}(\theta, v) = \theta - h(\theta); \tag{2.12}$$

$$\tau^{ZP}(\theta, v) = \frac{v[h(\theta_{ZP})^2 - h(\theta)^2]}{2} + v[H(\theta) - H(\theta_{ZP})], \tag{2.13}$$

for $\theta \in [\theta_{ZP}, \beta]$, and

$$q^{ZP}(\theta, v) = 0, \quad \tau^{ZP}(\theta, v) = 0, \tag{2.14}$$

for $\theta \in [\alpha, \theta_{ZP}]$.

All proofs are available in the appendix.

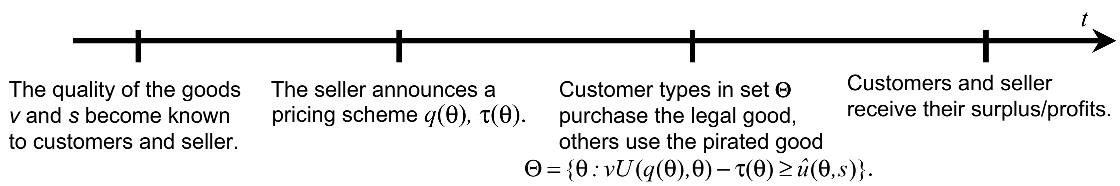
3. Pricing with Digital Piracy

This section analyzes pricing strategy when the seller faces digital piracy. The sequence of events modeled in this section is summarized in Figure 3.1.

3.1. Piracy-Indifferent Pricing Schedule

This section specifies the incentive-compatible pricing schedule that implements *piracy indifference*. Under this pricing schedule, all customer types are indifferent between the legal good and the pirated good. This pricing schedule is important because it often forms a building block for the optimal pricing schedule.

Figure 3.1 Timeline of Events for Section 3



Lemma 2. *The unique incentive compatible, piracy-indifferent pricing schedule* $q^{PI}(\theta, v, s)$, $\tau^{PI}(\theta, v, s)$ *for the legal good takes the following form*:

$$q^{PI}(\theta, v, s) = \frac{s\theta}{v}; \tag{3.1}$$

$$\tau^{PI}(\theta, v, s) = \frac{s[v-s]\theta^2}{2v}. \tag{3.2}$$

Under this pricing schedule, each customer type gets the same surplus from its optimal usage of the legal good of quality v and its maximal usage of the pirated good of quality s.

Lemma 2 establishes that there is a unique piracy-indifferent pricing schedule, under which each customer type gets a net surplus exactly equal to its reservation utility—the value $\hat{u}(\theta, s)$ that the customer would get from his or her maximal usage of the pirated good. From Equation (3.1), all customer types purchase positive quantities of the legal good under this pricing schedule. The usage levels of the legal good are strictly increasing in the quality of the pirated good s. In addition, (3.2) indicates that so long as $s < v$, the total payment $\tau^{PI}(\theta, v, s)$ from each customer type θ is strictly positive, and the piracy-indifferent pricing schedule is therefore profitable for the seller.

3.2. Optimal Price Discrimination in the Presence of Piracy

This section describes how to design the seller's optimal pricing schedule in the presence of piracy. Its main result is presented below.

Theorem 1.

(a) *When the quality of the pirated good is lower—that is, when* $s \le v[\alpha - h(\alpha)]/\alpha$ *—the seller's optimal pricing schedule is a modified version of the zero-piracy pricing schedule, with total prices adjusted downwards by the same amount across all usage levels. The optimal contract is*

$$q^*(\theta, v, s) = q^{ZP}(\theta, v); \tag{3.3}$$

$$\tau^*(\theta, v, s) = \tau^{ZP}(\theta, v) - \frac{s\alpha^2}{2}, \tag{3.4}$$

for all $\theta \in [\alpha, \beta]$, *where* $q^{ZP}(\theta, v)$ *and* $\tau^{ZP}(\theta, v)$ *are as defined in Equations* (2.9) *and* (2.10).

(b) *When the quality of the pirated good is higher—that is, when* $s > v[\alpha - h(\alpha)]/\alpha$*—the seller's optimal pricing strategy is as follows.*

(i) *Customer types are partitioned into two sets,* $[\alpha, \hat{\theta}]$ *and* $[\hat{\theta}, \beta]$, *where the transition type* $\hat{\theta}$ *is defined by*

$$\hat{\theta} = \theta\colon\ vh(\theta) = [v-s]\theta, \quad \theta \in (\alpha, \beta). \tag{3.5}$$

(ii) *The optimal pricing schedule for the lower set of customers is simply the piracy-indifferent pricing schedule*:

$$q^*(\theta, v, s) = q^{PI}(\theta, v, s); \tag{3.6}$$

$$\tau^*(\theta, v, s) = \tau^{PI}(\theta, v, s), \tag{3.7}$$

for $\theta \in [\alpha, \hat{\theta}]$.

(iii) *The optimal pricing schedule for the higher set of customers is an adjusted version of the zero-piracy pricing schedule, with total prices adjusted downward by the same amount across all usage levels*:

$$q^*(\theta, v, s) = q^{ZP}(\theta, v); \tag{3.8}$$

$$\tau^*(\theta, v, s) = \tau^{ZP}(\theta, v) - \left(vH(\hat{\theta}) - \frac{[v-s]\hat{\theta}^2}{2} + \frac{v\alpha^2}{2}\right), \tag{3.9}$$

for $\theta \in [\hat{\theta}, \beta]$, *where* $\tau^{ZP}(\theta, v)$ *is as defined in Part* (a) *of Lemma* 1, *in Equation* (2.10).

Theorem 1(a) establishes that when the quality of the pirated good is lower,[4] the optimal pricing

[4] Clearly, the condition $s \le v[\alpha - h(\alpha)]/\alpha$ of Theorem 1(a) does not just depend on the quality of the pirated good s, but also depends on v, α, and $h(\alpha)$. The statement "when the quality of the pirated good is lower" is meant to indicate that for a fixed distribution, and fixed values of v and α, the proposition is more likely to apply at lower values of s.

schedule is simply the zero-piracy pricing schedule, with a constant reduction in total price across all usage levels. The resulting usage level of each consumer is unaffected by the presence of piracy, and the reduction in total price across all customers is proportional to s. An immediate corollary is that as the quality of the pirated good s increases, prices are strictly lower at all usage levels.

Theorem 1(b) establishes that at higher quality levels for the pirated good, the portion of the optimal pricing schedule that is relevant to a lower set of customer types $[\alpha, \hat{\theta}]$ is simply the piracy-indifferent pricing schedule. Because $q^{PI}(\theta, v, s) > 0$, all these customer types purchase positive quantities of the legal good. It can be shown that $\hat{\theta} > \theta_{ZP}$, and therefore any customer type who did not purchase in the absence of piracy is now a legal user at the piracy-indifferent usage level $q^{PI}(\theta, v, s)$.

The presence of digital piracy can therefore have the socially beneficial effect of inducing *legal* usage from customers who may have otherwise been excluded by the seller's optimal price discrimination. While counterintuitive, this result has a straightforward economic explanation. In the absence of piracy, when the seller is a monopolist and there is no imperfect substitute for the seller's legal good, the seller finds it favorable to price discriminate in a manner that captures a higher level of surplus from the customer types $\theta > \theta_{ZP}$, at the cost of excluding customer types $[\alpha, \theta_{ZP}]$ from the market. The only reason why customer types $\theta \in [\alpha, \theta_{ZP}]$ are excluded is because the seller's optimal surplus extraction from higher customer types would not be feasible if there was any positive usage level affordable to these lower customer types. In the presence of piracy, each customer type who purchases the legal good must be provided with positive surplus of at least $\hat{u}(\theta, s)$—the value from maximal usage of the pirated good. Because the seller is forced to provide this surplus level to the higher set, customer types in the lower set can now be offered positive and affordable usage levels, without affecting incentive compatibility (and the seller's price-discrimination objectives).

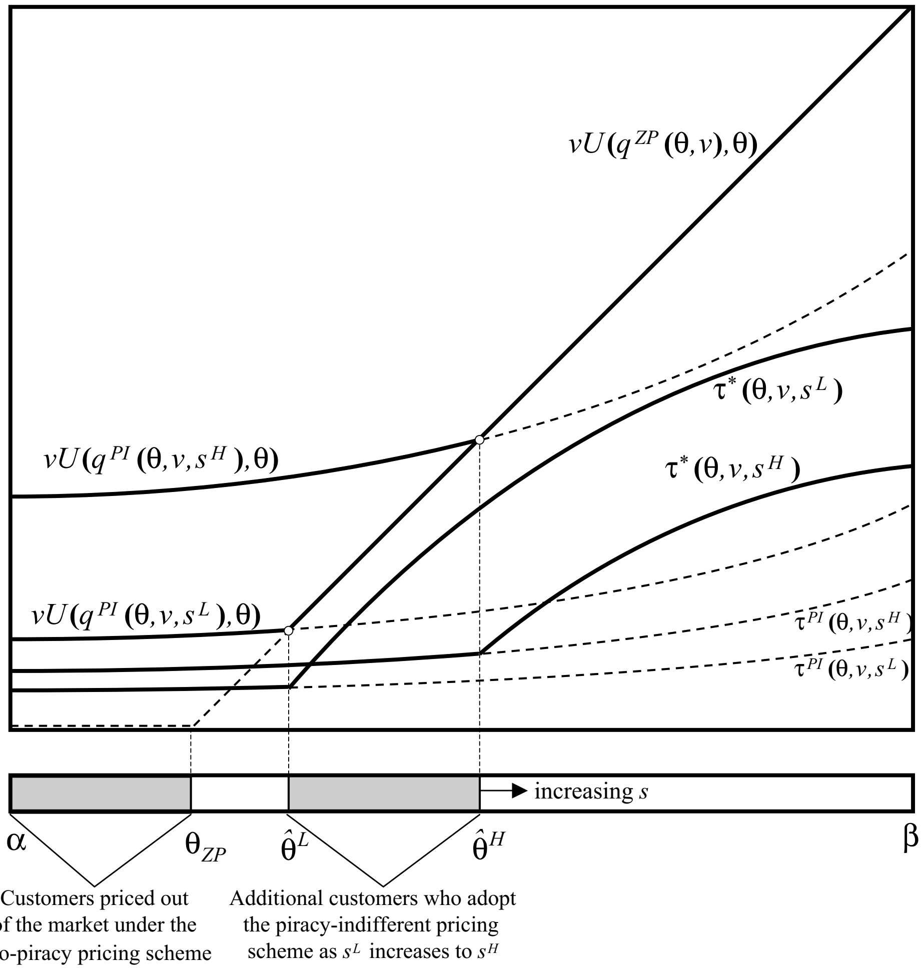
It is straightforward to establish that because $q^{ZP}(\theta, s) < q^{PI}(\theta, s)$ for $\theta \in [\alpha, \hat{\theta}]$, total usage either remains constant or goes up for all customer types, relative to the usage levels under the zero-piracy contract, and this increase is more pronounced at higher values of s. As a consequence, the total value $vU(q^*(\theta, v, s), \theta)$ created by the usage of each customer type also increases, which in turn implies that total surplus is higher at higher levels of s. These observations are illustrated further in Figure 3.2. Additionally, under the optimal pricing schedule, all customer types get a surplus level that is at least as high as their reservation utility $\hat{u}(\theta, s)$; the latter is also the surplus to each customer under the piracy-indifferent pricing schedule. This increase in consumer surplus is due to the seller's desire to increase profits beyond the level obtained under the piracy-indifferent contract, by inducing higher usage across all customer types. Higher usage is necessarily accompanied by an increase in surplus for all types to ensure incentive compatibility.

Together, $\tau^{PI}(\theta)$ and $q^{PI}(\theta)$ may imply a pricing function for which successive marginal prices are increasing for certain ranges of q. However, so long as average price is decreasing in q, the pricing schedule implied by the piracy-indifferent contract can still be implemented. In the event that this is not the case, the schedule can be approximately implemented by using a family of two-part tariffs, as discussed in §6.4 of Wilson (1993) and §9.5 of Laffont and Martimort (2002).

Theorem 1 is proved for a specific utility function, in which value (from both legal and pirated usage) is multiplicatively separable into a quadratic function $U(q, \theta)$ and the quality parameter v (or s). However, the main result—that optimal pricing in the presence of piracy is a combination of the piracy-indifferent pricing schedule and an adjusted version of the zero-piracy pricing schedule with total prices adjusted downward by the same amount across all usage levels—generalizes quite broadly.[5] Loosely, the restrictions necessary for the result to hold are that utility $u(q, \theta, z)$ is strictly concave in usage, has a specific though not unusual kind of curvature (that $u_{q\theta}(q, \theta, z)$ is strictly positive, nondecreasing in q, and nonincreasing in θ), and that the variation in specific marginal rates of change in utility with quality are positive at both $z = v$ and $z = s$.

[5] The mathematical details of the exact conditions under which the result generalizes are available on request.

Figure 3.2 Changes in Pricing and Surplus as the Quality of the Pirated Good s Changes



Notes. For $s = s^L, s^H$, the piracy-indifferent pricing schedules are $q^{PI}(\theta, v, s)$, $\tau^{PI}(\theta, v, s)$, and the total surplus generated by the usage of customer type θ is $vU(q^{PI}(\theta, v, s), \theta)$. The difference between $vU(q^{PI}(\theta, v, s), \theta)$ and $\tau^{PI}(\theta, v, s)$ is the minimum surplus that type θ must be provided to induce them to purchase the legal good, rather than using the pirated good. The thicker curves represent the optimal total prices and total surplus from optimal usage, while the dotted curves correspond to the portions of the "building blocks" that are not part of the optimum. As shown, the optimal pricing schedule always involves the piracy-indifferent contract, for a subset of lower types $[\alpha, \hat{\theta}]$. The set of types $[\alpha, \theta_{ZP}]$ who would have been priced out of the market under the zero-piracy pricing schedule are now included. As s increases, the increase in $U(q^{PI}(\theta, v, s), \theta) - \tau^{PI}(\theta, v, s)$ forces the seller to expand the lower set of customers, and to lower prices for the higher types as well. Moreover, as s increases, the surplus $vU(q^*(\theta, v, s), \theta)$ generated by each customer type's consumption increases, which raises total surplus.

4. Digital Rights Management

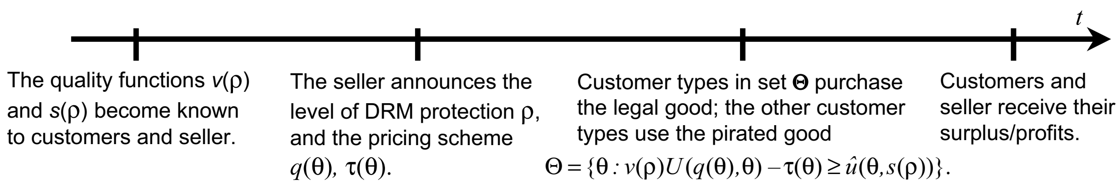
This section studies DRM systems that enable a seller to explicitly control its level of piracy protection. In addition to choosing a pricing schedule, the seller is now assumed to choose a level of technology-based protection $\rho \in [0, 1]$. This choice affects the quality level of both the legal good and the pirated good, as discussed in §1. Specifically, at a level of technology-based protection ρ, the quality of the legal good is denoted $v(\rho)$ and the quality of the pirated good is denoted $s(\rho)$. The functions $v(\rho)$ and $s(\rho)$ are assumed to have the following properties.

(1) $v(\rho) > s(\rho)$ for all ρ: The quality of the legal good is strictly higher than the quality of the pirated good, for all $\rho \in [0, 1]$.

(2) $v_\rho(\rho) < 0$, $s_\rho(\rho) < 0$: The quality of both the pirated good and the legal good are strictly decreasing in the level of technology-based protection ρ.

(3) $s_\rho(0) < v_\rho(0)$: An increase in the level of technology-based protection initially reduces the quality

Figure 4.1 Timeline of Events for Section 4



of the pirated good more rapidly than the quality of the legal good. In other words, the DRM system is effective, at least initially.

(4) $v_{\rho\rho}(\rho) < s_{\rho\rho}(\rho)$. The *rate* of quality degradation of the legal product increases relative to the *rate* of quality degradation of the pirated good as ρ increases. This property implies diminishing returns from increasing technology-based protection ρ and also ensures that $v_\rho(\rho) = s_\rho(\rho)$ at a unique point.

The costs to the seller of changing the level of protection ρ are assumed to be zero. This assumption is made to highlight the strategic and revenue effects of changes in technology-based protection levels. The sequence of events is summarized in Figure 4.1.

4.1. Technologically Maximal Protection Level

Given a pair of quality functions $v(\rho)$ and $s(\rho)$, the *technologically maximal level* of technology-based protection ρ^e is defined as the level of technology-based protection that maximizes the difference in quality between the legal good and the pirated good:

$$\rho^e = \arg\max_{\rho} [v(\rho) - s(\rho)]. \tag{4.1}$$

Under the properties of $v(\rho)$ and $s(\rho)$, the function $v(\rho) - s(\rho)$ is strictly concave in ρ, and therefore ρ^e is unique. The technologically maximal level ρ^e is of interest because it seems like an intuitively natural choice for the seller, particularly when no variable costs are incurred from altering the level of protection. It is also likely to be the level of protection marketed by a DRM vendor who is interested in highlighting the technological effectiveness of its solution (for instance, by advertising lots of reduction in piracy, only a minimal effect on product quality).

Additionally, if the seller does not price discriminate and chooses to charge each customer the same usage-independent fee T, then ρ^e is the optimal level of DRM protection. To see why this is the case: Given values of T and ρ, the customer type θ who is indifferent between the legal good and the pirated good solves:

$$v(\rho)\frac{\theta^2}{2} - T = s(\rho)\frac{\theta^2}{2}, \tag{4.2}$$

which implies that the indifferent type θ at a price T and protection level ρ is

$$\theta = \min\left[\alpha, \sqrt{\frac{2T}{v(\rho) - s(\rho)}}\right]. \tag{4.3}$$

In the former case (when all customers adopt),

$$T = [v(\rho) - s(\rho)]\frac{\alpha^2}{2}, \tag{4.4}$$

and the seller maximizes profits by maximizing $v(\rho) - s(\rho)$. In the latter case, the profit function that the seller maximizes when simultaneously choosing the optimal values of ρ and T is

$$\Pi(\rho, T) = T\left[1 - F\left(\sqrt{\frac{2T}{v(\rho) - s(\rho)}}\right)\right]. \tag{4.5}$$

The first-order condition $\Pi_\rho(\rho, T) = 0$ for the optimal ρ yields

$$\left(T\sqrt{\frac{2T}{(v(\rho^*) - s(\rho^*))^3}} \times f\left(\sqrt{\frac{2T}{v(\rho^*) - s(\rho^*)}}\right)\right) \cdot [v_\rho(\rho^*) - s_\rho(\rho^*)] = 0, \tag{4.6}$$

which implies that

$$v_\rho(\rho^*) - s_\rho(\rho^*) = 0$$

because $T > 0$, $f(x) > 0$ for all x, and $v(\rho) > s(\rho)$.

This choice of technologically maximal DRM protection when the seller charges all customers the same price has a simple intuitive explanation. Without the ability to price discriminate, the seller's profits are driven entirely by the piracy constraint for the customer type θ who, at price T, is indifferent between the legal good and the pirated good. The difference in value between the legal and pirated good for this customer type θ is $[v(\rho)-s(\rho)]U(\sigma(\theta),\theta)$, which is also equal to the fixed fee that the seller charges all customers. Any increase in $v(\rho)-s(\rho)$ is therefore strictly profit improving for the seller.

4.2. Optimal Technology-Based Protection when the Seller Price Discriminates

This subsection characterizes the seller's optimal level of technology-based protection when the seller can price discriminate, and shows that it is always strictly lower than the technologically maximal level defined earlier.

Suppose that the relevant pricing schedule across the entire range $\rho\in[0,\rho^e]$ is as specified by Theorem 1(a). This occurs when $h(\alpha)\le[v(\rho)-s(\rho)]\alpha/v(\rho)$ for all $\rho\in[0,\rho^e]$. Under the optimal pricing schedule, the seller's profits as a function of ρ are

$$\Pi(\rho)=\Pi^L(\rho)\equiv\int_\alpha^\beta\left(\frac{\alpha^2[v(\rho)-s(\rho)]}{2}+v(\rho)\left[H(\theta)-\frac{h(\theta)^2}{2}\right]\right)f(\theta)\,d\theta. \tag{4.7}$$

In contrast, under Theorem 1(b), the piracy constraint is binding for a positive fraction $[\alpha,\hat{\theta}]$ of customers; moreover, the seller's increased pricing power from an increase in $v(\rho)$ only applies to the higher set $[\hat{\theta},\beta]$ of customer types. For a specific ρ, define $\hat{\theta}(\rho)$ as the transition type between the two portions of the optimal pricing schedule derived in Theorem 1(b):

$$\hat{\theta}(\rho)=\theta\colon\ v(\rho)h(\theta)=\theta[v(\rho)-s(\rho)],\quad \theta\in(\alpha,\beta). \tag{4.8}$$

This is identical to the definition of $\hat{\theta}$ in (3.5), simply indexed by ρ. Correspondingly, under the optimal pricing schedule specified by Theorem 1(b), the seller's profits as a function of ρ reduce to

$$\begin{aligned}\Pi(\rho)=\Pi^H(\rho)\equiv{}&\left(\frac{s(\rho)[v(\rho)-s(\rho)]}{v(\rho)}\right)\int_\alpha^{\hat{\theta}(\rho)}\frac{\theta^2}{2}f(\theta)\,d\theta\\&+v(\rho)\int_{\hat{\theta}(\rho)}^\beta\left(H(\theta)-\frac{[h(\theta)]^2}{2}\right)f(\theta)\,d\theta\\&+[1-F(\hat{\theta}(\rho))]\left(\frac{[v(\rho)-s(\rho)][\hat{\theta}(\rho)]^2}{2}-v(\rho)H(\hat{\theta}(\rho))\right).\end{aligned} \tag{4.9}$$

The next result shows that under either set of these conditions, the optimal level of technology-based protection ρ^* is always strictly lower than the technologically maximal level ρ^e.

Theorem 2. *The profit-maximizing level of technology-based protection* ρ^*:

$$\rho^*=\arg\max_\rho\Pi(\rho), \tag{4.10}$$

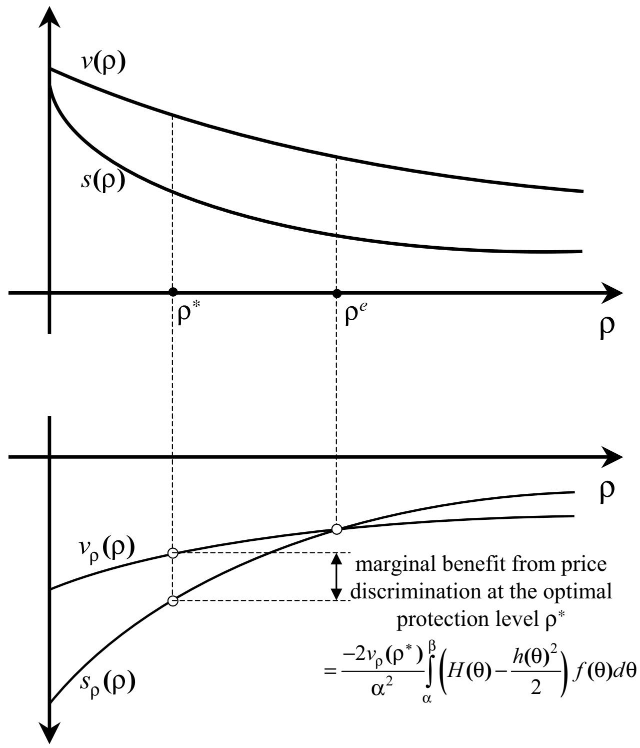
is always strictly ***lower*** *than the technologically maximal level of protection* ρ^e. *That is,* $\rho^*<\rho^e$, *where* ρ^e *is defined in* (4.1).

Theorem 2 is a surprising result because it indicates that at the optimal ρ^* a small increase in technology-based protection would actually degrade the quality of the pirated good *more* than the quality of the legal good. However, it is not profitable for the seller to implement this increase in protection. The result is illustrated graphically in Figure 4.2

This result can be explained intuitively by examining its underlying economic effects. When the digital good is less easily replicable (that is, under the conditions of Theorem 1(a)), the piracy constraints defined in (2.8) are nonbinding for all customer types $\theta>\alpha$. Therefore, the cost to the seller of a marginal increase in $s(\rho)$ is proportionate to the value the *lowest* type $\theta=\alpha$ gets from the pirated good. Simultaneously, the benefits of a marginal increase in $v(\rho)$ are twofold. First, there is an increase in total price across all users due to the weakening of the piracy constraint for $\theta=\alpha$, which is identical in magnitude to the cost of the marginal increase in $s(\rho)$ described above. In addition, there is a revenue change equal to the sum of the different changes in total price for different customer types that arises from optimally readjusting prices to satisfy incentive compatibility. Under the optimal pricing schedule, the latter effect of a marginal increase in $v(\rho)$ is always positive.

Put simply, a small increase in $s(\rho)$ strengthens the piracy constraint, while a small increase in $v(\rho)$ weakens the piracy constraint (in an identical and opposite way). Additionally, this small increase in $v(\rho)$ also improves the seller's ability to price discriminate across all the customer types. Therefore, when the seller can price discriminate, the benefit of a marginal

Figure 4.2 **Result of the Optimal Level of Technology-Based Protection** ρ^*



Notes. ρ^* is strictly lower than ρ^e, the technologically maximal level. ρ^e occurs at the point where the difference between $v(\rho)$ and $s(\rho)$ is maximum, which is when $[v_\rho(\rho) - s_\rho(\rho)] = 0$. However, as illustrated, the first-order conditions for maximizing the seller's profits $\Pi^L(\rho)$ indicate that the optimal level ρ^* is at a point where $[v_\rho(\rho) - s_\rho(\rho)]$ is strictly positive because there is an additional marginal benefit from increased pricing power when $v(\rho)$ is higher.

increase in $v(\rho)$ is more than the cost of a corresponding marginal increase in $s(\rho)$. As a consequence, $\rho^* < \rho^e$.

Correspondingly, under the conditions of Theorem 1(b), a marginal increase in $s(\rho)$ strengthens the piracy constraint across all the customer types in the lower set $[\alpha, \hat{\theta}(\rho)]$, whose usage levels are according to the piracy-indifferent contract. However, a marginal increase in $v(\rho)$ balances this effect exactly for each customer type. The result establishes that in addition, the marginal increase in $v(\rho)$ still has a net positive effect on the seller's pricing power for the higher set $[\hat{\theta}(\rho), \beta]$.

Clearly, Theorem 2 would continue to hold even if there was a direct fixed or variable cost to implementing DRM, so long as this cost was nondecreasing in the level of technology-based protection. Further implications of the result are discussed in §6.

4.3. Strategic Responses to Weakening DRM Technology

Often, a DRM technology weakens over time, largely due to it being hacked by engineers who are trying to "break" the protection scheme. This section investigates how a seller should alter its level of technology-based protection and pricing in response to this progressive weakening of the DRM technology. The discussion in this subsection uses the profit function corresponding to the conditions under which the pricing schedule does not include a piracy-indifferent segment, as derived in Theorem 1(a) and specified by $\Pi^L(\rho)$ in (4.7). Comparable results hold for the case corresponding to Theorem 1(b), or for the profit function $\Pi^H(\rho)$ specified in (4.9), though the corresponding analysis is more involved.

The weakening of the DRM system is modeled as causing a gradual increase in the quality of the pirated good over time. Specifically, if the level of technology-based protection chosen is ρ, the initial quality of the pirated good—immediately after implementing DRM—is denoted $s(\rho, 0)$, and its quality at time t is represented by the function $s(\rho, t)$, where $s_t(\rho, t) > 0$. This is because as DRM protection weakens, the digital product becomes easier to pirate. Additionally, increasing the protection level ρ reduces the quality of the pirated good, as in §4.2.

It is assumed that the quality of the legal good $v(\rho)$ is not directly affected by the weakening of the DRM system. The seller's profit function therefore takes the following form:

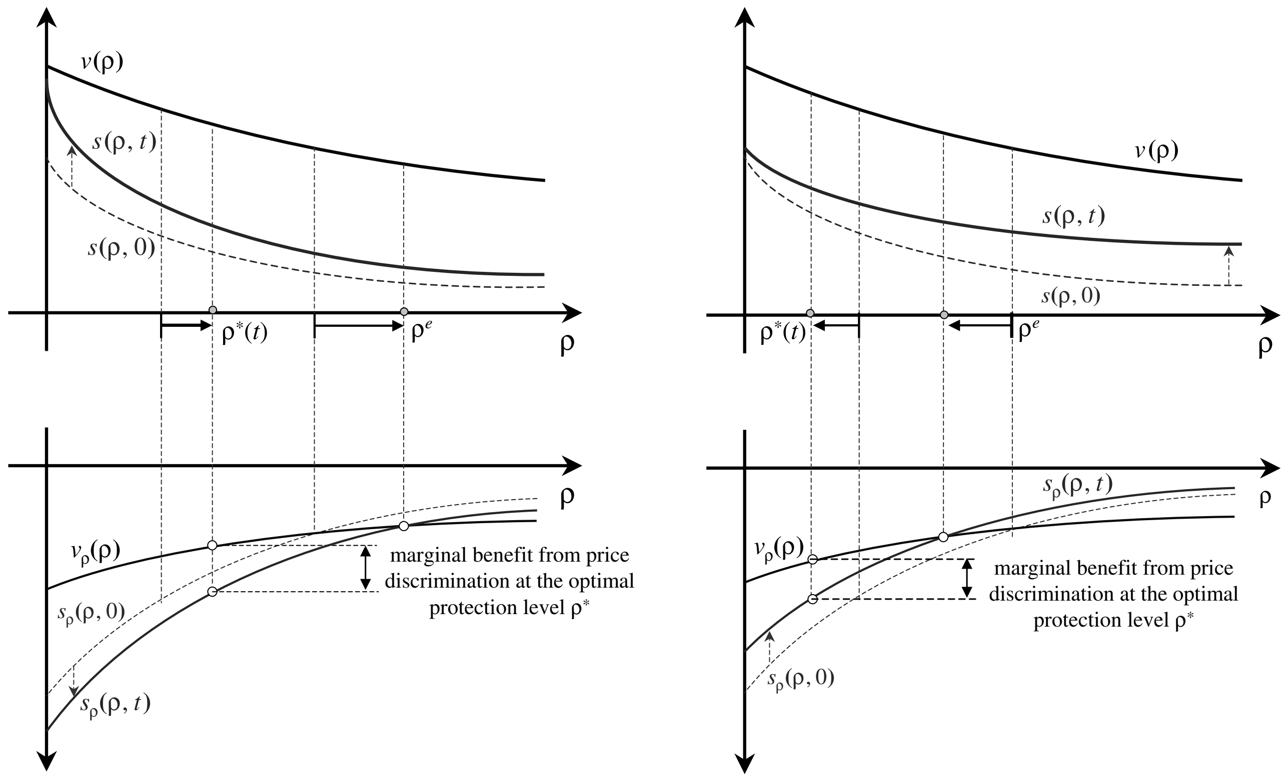
$$\Pi^L(\rho, t) = \int_\alpha^\beta \left(\frac{\alpha^2[v(\rho) - s(\rho, t)]}{2} + v(\rho)\left[H(\theta) - \frac{h(\theta)^2}{2}\right]\right) f(\theta)\, d\theta, \quad (4.11)$$

and the optimal level of protection at time t, denoted $\rho^*(t)$, solves

$$\rho^*(t) = \arg\max_\rho \Pi^L(\rho, t). \quad (4.12)$$

There are many ways in which $s(\rho, t)$ might evolve over time. An important clarification here is that there are no intertemporal demand dependencies in the formulation. The discussion implicitly assumes a new

Figure 4.3 Changes in the Optimal Level of Technology-Based Protection if a DRM System Is Progressively Weakened Over Time



(a) Changes in DRM protection when $s_{\rho t}(\rho, t) < 0$ **(b) Changes in DRM protection when $s_{\rho t}(\rho, t) > 0$**

set of customers at each point in time t and examines changes in protection that are optimal given the changes in the relationship between s and ρ that is determined by $s(\rho, t)$.

Three scenarios are analyzed. The optimal technological and pricing responses prescribed for the seller are directionally different in each case, and these scenarios were chosen to highlight these differences. Under the first scenario, there is a constant upward drift in the quality of the pirated good, across all levels of protection:

$$s_t(\rho, t) > 0, \qquad s_{\rho t}(\rho, t) = 0. \tag{4.13}$$

In this case, the marginal properties of $s(\rho, t)$ with respect to ρ do not change over time. That is, $s_\rho(\rho, t) = s_\rho(\rho, 0)$ for all t. Because $s_\rho(\rho, t)$ is constant over time, the optimal level of technology-based protection $\rho^*(t)$ is constant and unaffected by the weakening of the technology. The seller should consequently maintain the same level of technology-based protection. While $v(\rho^*(t))$ remains unchanged, $s(\rho^*(t), t)$ increases over time because $s_t(\rho, t) > 0$. Therefore, total prices should be lower over time, at each usage level.

Under the second scenario, the weakening of the DRM system leads to *smaller* changes in the quality of the pirated good at higher levels of technology-based protection:

$$s_t(\rho, t) > 0, \qquad s_{\rho t}(\rho, t) < 0. \tag{4.14}$$

This type of change is illustrated in Figure 4.3(a). It is characteristic of a DRM system under which higher levels of protection not only reduce the quality of the pirated good (by making it harder to replicate the digital good), but also make it increasingly difficult to hack the system.

Because $s_{\rho t}(\rho, t) < 0$, the weakening of the DRM technology reduces the slope of $s(\rho, t)$ over time (that is, makes the slope more negative), thereby moving the function $s_\rho(\rho, t)$ downward. As a consequence, the optimal level of protection shifts to the right, and the seller's optimal technological response is to *increase* its level of technology-based protection over time.

The optimal adjustment ρ from $\rho^*(0)$ to $\rho^*(t)$ causes a net decrease in the quality of the legal good to $v(\rho^*(t))$, accompanied by what is typically a net increase in the quality $s(\rho^*(t), t)$ of the pirated good relative to the initial value $s(\rho^*(0), 0)$. The pricing schedule in Theorem 1 indicates that this should lead to a decrease in total prices. Even if there is a net decrease in the quality of the pirated good $s(\rho, t)$, it will be lower in magnitude than the corresponding decrease in the quality of the legal good $v(\rho)$. As discussed in §4.2, this always lowers prices. Therefore, in conjunction with its technological response, the seller's optimal pricing response to the weakening of the DRM system is to reduce prices across all customer types.

The third scenario is where the weakening of the DRM technology leads to *larger* changes in the quality of the pirated good at *higher* levels of protection:

$$s_t(\rho, t) > 0, \qquad s_{\rho t}(\rho, t) > 0. \tag{4.15}$$

This type of change is illustrated in Figure 4.3(b). It is characteristic of a technology for which higher levels of protection reduce the quality level of the pirated good, but the technology that implements every marginal increase in the level of protection is increasingly vulnerable to hacking. Note that the assumption that $s_\rho(\rho, t) < 0$ is maintained, and so quality levels of the pirated good continue to be lower at higher levels of protection, even post-hacking.

In contrast with the earlier scenario, because $s_{\rho t}(\rho, t) > 0$, the weakening of the DRM technology increases the slope of $s(\rho, t)$ over time, thereby moving the function $s_\rho(\rho, t)$ upwards. As a consequence, the optimal level of protection moves to the left, and the seller's optimal technological response is to *reduce* its level of technology-based protection over time.

As the optimal level of protection $\rho^*(t)$ decreases, there is a substantial increase in the quality of the pirated good $s(\rho^*(t), t)$. There is also an increase in the quality of the legal good (because $\rho^*(t)$ decreases, $v(\rho^*(t))$ increases). It is clear that the increase in $s(\rho^*(t), t)$ is more than the increase in $v(\rho^*(t))$—however, as discussed in §4.2, a marginal increase in the quality of the legal good increases prices and profits more than a corresponding marginal increase in the quality of the pirated good. Therefore, the direction of the pricing response cannot be characterized in general. However, because the prices for lower customer types are progressively more affected by the quality of the pirated good, the pricing response will be progressively less favorable for higher customer types, independent of its direction.

Sometimes, implementing frequent changes to its level of technology-based protection ρ is costly for the seller. If the seller anticipates that there will be a weakening of the DRM system over time, it may be in its best interest to start out by overprotecting its legal good under the second scenario and underprotecting it under the third scenario. This issue is discussed further in §6.

5. Example

In this section, the optimal pricing schedule is derived explicitly for a specific family of customer-type distributions. This example, based on Theorem 1, further illustrates the effects of digital piracy on pricing, usage, and welfare, and highlights the effect of varying some properties of the customer-type distribution.

The family of customer-type distributions used in the example have a shifted beta density function[6] $B(\theta; a, b)$, with support $[\alpha, 1+\alpha]$, $a = 1$, and parametrized by $b \geq 1$. When $b = 1$, the distribution is the unit uniform distribution $U[\alpha, 1+\alpha]$. When $b > 1$, the distribution is positively skewed and $f(\theta)$ is strictly decreasing in θ. The beta density function is illustrated in Figure 5.1, for a candidate set of b values.

When $a = 1$ and $b > 0$, the beta distribution has the distribution function $F(\theta) = 1 - (1+\alpha-\theta)^b$ and density function $f(\theta) = b(1+\alpha-\theta)^{b-1}$. Accordingly, the inverse hazard rate function $h(\theta)$ and the cumulative inverse hazard rate function $H(\theta)$ take the following form:

$$h(\theta) = \frac{1-(\theta-\alpha)}{b}; \tag{5.1}$$

$$H(\theta) = \frac{2(\theta-\alpha)-(\theta-\alpha)^2}{2b}. \tag{5.2}$$

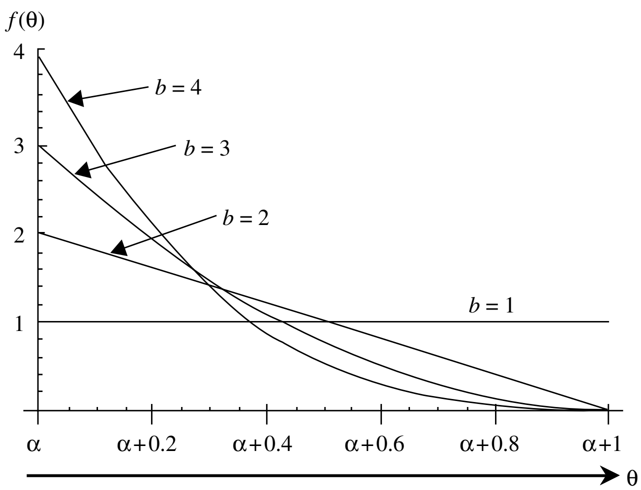
For the purpose of this example, the value of v is normalized to one. The optimal pricing schedules are

[6] The general form of the beta density function is

$$B(x; a, b) = \frac{x^{a-1}(1-x)^{b-1}}{\beta(a, b)},$$

where $\beta(a, b) = \int_0^1 x^{a-1}(1-x)^{b-1}\,dx$ is the beta function with parameters a and b.

Figure 5.1 **Beta Density Function Used to Characterize Different Customer-Type Distributions in the Example, with $a = 1$, and for Different Values of b**



derived based on the results of §3.2. Because v has been normalized to one, it is dropped as an argument of the pricing and usage functions.

Table 5.1 summarizes the relevant consumption and total pricing functions that define the optimal pricing schedule. When the quality of the pirated good is lower, an explicit pricing function can be easily derived:

$$p(q) = \left(\frac{2+b}{2b^2(b+1)} - \frac{\alpha(2+\alpha)}{2(1+b)} - \frac{\alpha^2 s}{2}\right) + \frac{2(1+\alpha)q - q^2}{2(b+1)}. \tag{5.3}$$

The optimal pricing schedule is therefore a *nonlinear two-part tariff*. This is the kind of tariff that would be optimal in the absence of piracy, and (5.3) implies that at low levels of piracy, it continues to remain so, though with a reduction in the fixed part of the two-part tariff that is proportionate to the quality of the pirated good. The properties of the variable portion are better understood by differentiating both sides of (5.3) with respect to q:

$$p_q(q) = \frac{(1+\alpha-q)}{(b+1)}; \tag{5.4}$$

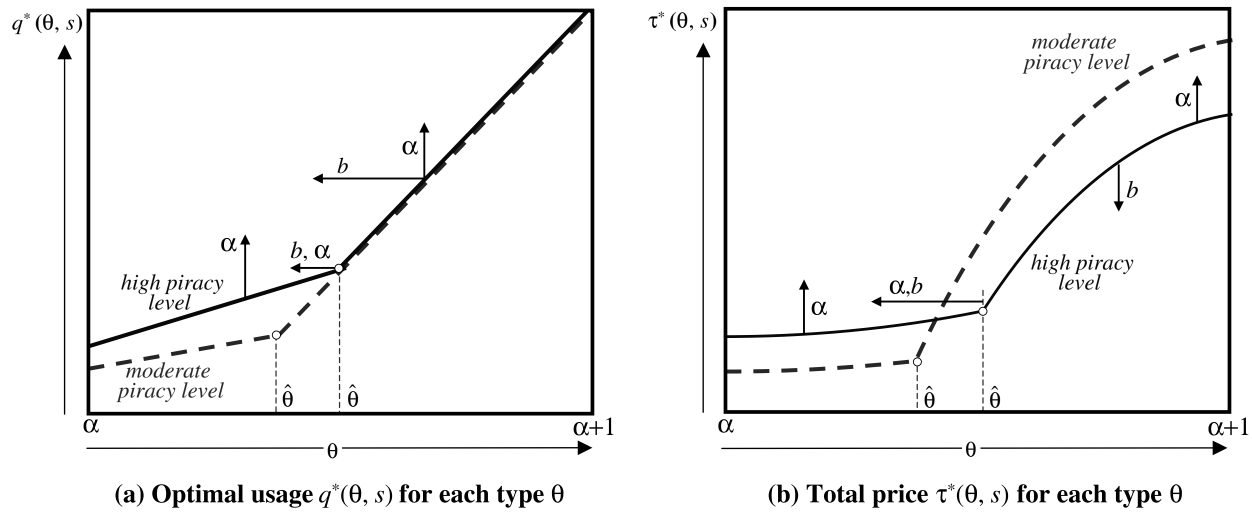
$$p_{qq}(q) = -\frac{1}{(b+1)}. \tag{5.5}$$

Equation (5.5) implies that the variable portion of the optimal pricing schedule is strictly concave in q for any $b > 0$. In addition, (5.4) indicates that the

Table 5.1 **Optimal Contracts and Surplus Expressions for the Example**

Intermediate contracts	
Zero-piracy usage level:	$q^{ZP}(\theta) = \dfrac{\theta[1+b] - [\alpha+1]}{b}$.
Piracy-indifferent pricing schedule:	$q^{PI}(\theta, s) = s\theta$;
	$\tau^{PI}(\theta, s) = \dfrac{s[1-s]\theta^2}{2}$.
Transition type $\hat{\theta}$:	$\hat{\theta} = \alpha + \dfrac{1 - \alpha b[1-s]}{1 + b[1-s]}$.
Optimal pricing schedule	
Lower quality of pirated good	
For $s \le 1 - \dfrac{1}{\alpha b}$	$q^*(\theta, s) = \dfrac{\theta[1+b] - [\alpha+1]}{b}$;
	$\tau^*(\theta, s) = \dfrac{[1-s]b^2\alpha^2 - 1 + [1+b][2(\theta-\alpha) - (\theta-\alpha)^2]}{2b^2}$.
Higher quality of pirated good	
For $s \ge 1 - \dfrac{1}{\alpha b}$, and $\theta \le \hat{\theta}$	$q^*(\theta, s) = s\theta$;
	$\tau^*(\theta, s) = \dfrac{s(1-s)\theta^2}{2}$.
For $s \ge 1 - \dfrac{1}{\alpha b}$, and $\theta \ge \hat{\theta}$	$q^*(\theta, s) = \dfrac{\theta(1+b) - (\alpha+1)}{b}$;
	$\tau^*(\theta, s) = \dfrac{\hat{\theta}[1-s]}{2} + \dfrac{[1+b][2(\theta-\alpha) + (\theta-\alpha)^2]}{2b^2} - \dfrac{2(\hat{\theta}-\alpha) + (\hat{\theta}-\alpha)^2}{2b}$.

Figure 5.2 Usage Levels and Total Price for Different Customer Types in the Example



Notes. The dashed curves depict a moderate piracy level, while the solid curves depict a higher level of piracy. The labeled arrows represent the direction in which the respective curves or points shift when the corresponding parameter increases (and while only one set of curves is labeled, the shift is directionally identical for both s values). For instance, an increase in w raises the usage levels and total price for all types, at all feasible levels of piracy.

marginal price $p_q(q)$ is strictly increasing in α and strictly decreasing in b. This indicates that variable prices increase with an average increase in value and are lower when there is a higher proportion of the lower customer types, but are unaffected by s, as one would expect from Theorem 1(a).

When the quality of the pirated good is higher, Figure 5.2 illustrates the usage function $q^*(\theta, s)$ and total price function $\tau^*(\theta, s)$ for moderate and high piracy levels. The labeled arrows in the figures represent the direction in which the functions shift in response to a change in the corresponding parameter. As the quality of the pirated good s increases, the lower set of customer types expands, and there is a strict increase in each of these customers' (piracy-indifferent) usage levels. The legal usage of the rest of the customers remains the same; however, the total price paid by each of these customer types is strictly lower. Moreover, the variable portion of the tariff is now lowered by an increase in the quality of the pirated good.

An increase in α increases both the usage and total price across all customers, and also shifts a fraction of customers away from the piracy-indifferent contract. An increase in the skewness of the distribution of customer types (an increase in b, as depicted in Figure 5.2) increases the size of the higher customer-type set. This increase in skewness does not affect consumption or pricing for those customers who remain in the lower set—however, for all other customers, there is a strict increase in consumption and a strict reduction in price. This is because as b increases, there is a larger density of customers at the lower end of the market, and it is in the seller's interest to increase usage for lower customer types, thereby increasing the surplus generated by the usage of these customers, as well as the seller's profit potential. This increase is at the expense of lower pricing power on the higher end of the market, which makes sense intuitively because there are fewer high-type customers.

6. Discussion and Conclusion

A number of new results relating to managing digital goods subject to piracy have been derived in §§3 and 4. This section discusses some of these results further, highlighting pricing and technology-based protection guidelines, discussing some welfare issues, and concluding with an outline of open research questions.

6.1. Guidelines for Pricing with Digital Piracy

With a positive but relatively low threat of piracy, Theorem 1(a) shows that piracy affects a seller's pricing power uniformly across its different customer segments. This is despite the fact that each of these segments may value the pirated good differently. Consequently, the seller's pricing strategy should be to design the optimal pricing schedule unconstrained by piracy, and then simply adjust total prices downward across all usage levels, by an amount proportionate to the value its lowest customer type would get from the pirated good.

As piracy levels increase, sellers need to segment their customers more carefully, paying closer attention to the differential value that customers may get from the pirated good. The optimal pricing adjustment in response to a higher threat of piracy often induces legal purchasing by a new set of customers who were previously priced out of the market. The corresponding part of the pricing schedule is based on the piracy-indifferent contract, which is a low-price, low-usage pricing schedule. Additionally, as piracy levels increase, pricing for the higher segment of the market should be lowered further, by the value the lowest customer type in this segment would derive from the pirated good.

As the market for a successful digital product matures, there is often an increase in desired usage levels across all customers in the market (which corresponds to a shift in the type distribution to the right, modeled as an increase in the value of α). In response, the seller should expand the fraction of customers included in its higher segment, and simultaneously increase prices. Alternatively, the seller may observe a net increase in average desired usage because of a progressive upward shift of lower-end customer types, which results in a flattening of the distribution of customer types. In this case, it is optimal for the seller to shrink the higher segment, move more customer types into the piracy-indifferent segment, and raise prices for the higher segment.

6.2. Guidelines for Managing DRM-Based Piracy Deterrence

DRM is a valuable technological deterrent to piracy, and can improve a seller's profitability substantially. The model in §4 highlights the importance of considering the effects that a seller's DRM implementation will have on the value of its legal product, and provides guidelines for how to optimally balance value reduction with piracy deterrence. An immediate consequence of this trade-off is that excessive restrictions on legal usage that aim to deter piracy can result in a failure to create a viable market for the legal good. As discussed in §1, the early success of Apple's iTunes music service that, at the risk of higher piracy levels, placed far fewer restrictions on legal usage than its online predecessors like MusicNet and Rhapsody may be an instructive illustration.

A more subtle result is that if the seller can price discriminate, choosing the level of protection that balances marginal value reduction with marginal piracy deterrence—the technologically maximal level—is never optimal. Instead, the seller is always better off choosing a lower level of technology-based protection. When considering potential DRM solutions, it is natural for sellers to focus primarily on the ability of the technology to deter piracy. However, because the effect that DRM has on the value of the legal good is *more important* for profitability, sellers may need to realign their focus when evaluating these products. Correspondingly, when designing rights management technologies, a vendor should focus more on how effective its solution will be in preserving the value provided to legal users because this is the dominant profit driver for its corporate customers.

Even the best DRM technologies are unlikely to be hacker-proof. The results of §4.3 provide technological and pricing responses for sellers who must deal with this reality, and establish that it is critical that sellers understand the DRM technology before they respond to the threat of hacking. As shown in §4.3, the nature of the interaction between the level of protection and the corresponding difficulty of breaking the protection scheme is what determines the optimal technological response. In implementing each marginal increase in protection, if the DRM system relies on technology that is increasingly fragile, then the seller is likely to need to reduce its protection levels over time. On the other hand, if the successive "pieces" of the system are progressively more robust, the seller's best strategic response to hacking is to increase its level of technology protection over time, and simultaneously reduce its prices.

While pricing responses are easy to implement, continuous variation of technology-based protection levels over time is often expensive, technologically difficult, and sometimes impossible to implement. As a consequence, it may be good strategy to preemptively implement a suboptimal level of protection, based on the appropriate expectation of how future hacking will affect piracy levels. Again, a clear understanding of the details of the technology is crucial—whether an increase in protection levels makes the technology more robust or more vulnerable to hacking is an important determinant of whether to preemptively overprotect or underprotect.

6.3. Welfare and Policy Issues

The analysis in §3 suggests that the presence of digital piracy may lead a seller to alter pricing in a manner that increases the legal usage of existing customers, and that includes lower-end customers who had been priced out of the market earlier. These changes raise total surplus, and the increase is a consequence of higher *legal* usage—welfare benefits from piracy that have not been highlighted in the literature thus far, which has focused on surplus that might be generated from the use of the pirated product. Moreover, there may be a corresponding increase in distributional equity because total surplus is shared more evenly between customer types. These increases in surplus come at the expense of a reduction in seller profits, which may affect incentives for the creation of content. This trade-off needs to be analyzed before concluding that piracy has unilateral welfare benefits. However, in many creative industries (such as music, art, and literature), the ability to capture rents from one's creations may not be the primary driver of innovation.

Digital piracy has also led to a stronger emphasis on protecting intellectual property using technology, rather than the legal system. This has already led to substantial debate about the extent to which copyright owners should be able to control the usage of their products, especially in light of the somewhat overreaching legal protection that the 1998 Digital Millennium Copyright Act provides these owners. The social benefits of digital piracy that are outlined above strengthen recent arguments (Samuelson 2003) that one needs to carefully assess the welfare implications of an enforcement system that increasingly relies on technology, thereby giving sellers a broader set of rights over their content in practice.

6.4. Concluding Remarks

A logical direction for future research would be to extend the model of this paper to a dynamic setting, allowing one to study the optimal technology-protection paths of §4.3 more precisely. An extension of this kind may also admit the possibility of modeling how customers use pirated goods as a way of assessing the quality of a experiential digital good, as explored in a simpler setting by Chellappa and Shivendu (2003).

A natural question that arises in light of the past literature (Conner and Rummelt 1991, Takeyama 1994) relates to how the relationship between network effects and piracy might change when one admits price discrimination in addition to endogenous choices of protection technology. Directionally, the presence of a positive network effect would strengthen the results of §4.2 because the network benefits from increased gross usage would suggest the optimality of even lower levels of technology-based protection. Solving a model that incorporates network effects explicitly would indicate the magnitude of this impact, and is another interesting direction of future investigation.

Another potentially important externality is the *usage externality* induced by piracy. Because most pirated goods are "produced" from legal copies of the product, their quality and availability may be proportionate to the extent of legal usage of the product. For example, pirated software is generally made available by legal users who crack the software's copy protection scheme, and therefore the quality (and availability) of pirated software is likely to be higher when there are more legal users. Correspondingly, the variety of pirated music available on file-sharing networks depends on the variety of music legally purchased by the users who create illegal copies. As the proportion of purchases via digital channels increases, this becomes a particularly important effect, and one which a seller should consider when pricing its product or choosing a level of DRM technology. This is a negative externality from the point of view of the seller, but a positive externality for its customers. Studying how this externality affects pricing and welfare represents another promising line of work.

In conclusion, this paper describes how to design effective pricing policy in the presence of digital piracy, how to simultaneously use price discrimination and technology-based protection to deter piracy, and how to appropriately vary protection levels in response to a weakening of one's DRM technology. I hope these results will deepen our understanding of the economics of digital piracy, help guide managerial decisions in the growing number of industries whose products are being digitized, and motivate further research into this increasingly important area.

Acknowledgments
The author thanks Paul Belleflamme, Annette Boom, Anindya Ghose, Chris Kemerer, Ravi Mantena, Barrie Nault, Roy Radner, Pamela Samuelson, three anonymous referees, seminar participants at New York University, and conference participants at the fourth ZEW Conference on the Economics of Information and Communications Technologies and the second annual congress of the Society for Economic Research on Copyright Issues for their feedback on earlier versions of this paper.

Appendix. Proofs

Proof of Lemma 1. In the absence of piracy, all customer types have equal reservation utility of zero. The utility function $vU(q,\theta)$ and the inverse hazard rate $h(\theta)$ of the customer-type distribution satisfy all the conditions necessary to apply the solution to the standard nonlinear pricing model, where all types with nonzero allocations are separated at the optimum. See Maskin and Riley (1984) for an exposition of the theory or Lemma 1 of Sundararajan (2004), which provides a complete proof of a more general version of this model (in the absence of piracy). Under this solution, the optimal allocation to each type $\theta \in [\alpha, \beta]$ satisfies

$$q^{ZP}(\theta, v) = \max\{q(\theta, v), 0\}, \tag{A.1}$$

where, for each θ, $q(\theta, v)$ is the unique solution to

$$vU_q(q(\theta, v), \theta) = vU_{q\theta}(q(\theta, v), \theta)h(\theta), \tag{A.2}$$

and the optimal total price charged to type $\theta \in [\alpha, \beta]$ is

$$\tau^{ZP}(\theta, v) = vU((q^{ZP}(\theta, v), \theta) - \left(vU(q^{ZP}(\alpha, v), \alpha) + \int_{x\in Q} vU_\theta(q^{ZP}(x, v), x)\,dx\right), \tag{A.3}$$

where

$$Q = \{\theta\colon q(\theta, v) \geq 0\}. \tag{A.4}$$

Recall that $U(q, \theta) = \theta q - \frac{1}{2}q^2$. Substituting the functional forms for $U_q(q, \theta)$ and $U_{q\theta}(q, \theta)$ into (A.2) yields

$$q(\theta, v) = \theta - h(\theta). \tag{A.5}$$

(a) Because $h_\theta(\theta) \leq 0$, if $h(\alpha) \leq \alpha$, it follows that $q(\theta, v) \geq 0$ for all θ. As a consequence, for each θ,

$$q^{ZP}(\theta, v) = \theta - h(\theta). \tag{A.6}$$

Substituting (A.6) into (A.3) and rearranging yields

$$\tau^{ZP}(\theta, v) = v\left(\frac{\theta^2 - [h(\theta)]^2}{2} - \frac{\alpha^2 - [h(\alpha)]^2}{2} - \int_\alpha^\theta [x - h(x)]\,dx\right),$$

which simplifies to the expression in (2.10).

(b) If $h(\alpha) > \alpha$, then $q(\alpha, v) < 0$. However, we know that $h(\beta) = 0$, and therefore that $q(\beta, v) = \beta > \alpha$. Because $h_\theta(\theta) \leq 0$, this implies that there is a unique $\theta_{ZP} \in (\alpha, \beta)$ such that

$$\theta_{ZP} = h(\theta_{ZP}); \tag{A.7}$$

$$q(\theta, v) < 0 \quad \text{for } \theta < \theta_{ZP}; \tag{A.8}$$

$$q(\theta, v) > 0 \quad \text{for } \theta > \theta_{ZP}. \tag{A.9}$$

Consequently, from (A.1), it follows that

$$q^{ZP}(\theta, v) = 0 \quad \text{for } \theta \leq \theta_{ZP}; \tag{A.10}$$

$$q^{ZP}(\theta, v) = \theta - h(\theta) \quad \text{for } \theta \geq \theta_{ZP}. \tag{A.11}$$

Because $U(0, \theta) = 0$ for all θ, the expression for $\tau^{ZP}(\theta, v)$ in (A.3) reduces to

$$\tau^{ZP}(\theta, v) = 0 \quad \text{for } \theta \leq \theta_{ZP}; \tag{A.12}$$

$$\tau^{ZP}(\theta, v) = v\left(\frac{\theta^2 - [h(\theta)]^2}{2} - \int_{\theta_{ZP}}^\theta [x - h(x)]\,dx\right) \quad \text{for } \theta \geq \theta_{ZP}. \tag{A.13}$$

Simplifying (A.13) yields the expression in (2.13). □

Proof of Lemma 2. Recall that $\sigma(\theta) = \arg\max_q U(q, \theta) = \theta$. The conditions for a contract $q(\theta), \tau(\theta)$ to be both incentive compatible and piracy indifferent are

$$\theta = \arg\max_x v(U(q(x), \theta) - \tau(x); \tag{A.14}$$

$$vU(q(\theta), \theta) - \tau(\theta) = sU(\sigma(\theta), \theta) \quad \text{for all } \theta. \tag{A.15}$$

(A.14) ensures incentive compatibility, and (A.15) ensures piracy indifference. First-order conditions for (A.14) for each θ yield

$$vU_q(q(\theta), \theta)q_\theta(\theta) = \tau_\theta(\theta) \quad \text{for all } \theta. \tag{A.16}$$

Now, differentiating (A.15) with respect to θ and using the fact that $U_q(\sigma(\theta), \theta) = 0$, one gets

$$vU_q(q(\theta), \theta)q_\theta(\theta) + vU_\theta(q(\theta), \theta) - \tau_\theta(\theta) = sU_\theta(\sigma(\theta), \theta). \tag{A.17}$$

Substituting (A.16) into (A.17) yields

$$U_\theta(q(\theta), \theta) = \frac{sU_\theta(\sigma(\theta), \theta)}{v}, \tag{A.18}$$

which on substitution of the functional form of $U(q,\theta)$ yields

$$q(\theta)=\frac{s\theta}{v}. \tag{A.19}$$

Substituting (A.19) into (A.15) and rearranging, one gets

$$\tau(\theta)=\frac{s[v-s]\theta^2}{2v}. \tag{A.20}$$

Therefore, the simultaneous requirements of incentive compatibility and piracy indifference yield a unique contract. Consequently, the unique piracy-indifferent contract is

$$q^{PI}(\theta,v,s)=\frac{s\theta}{v}, \tag{A.21}$$

$$\tau^{PI}(\theta,v,s)=\frac{s[v-s]\theta^2}{2v}. \tag{A.22}$$

□

Proof of Theorem 1. The proof introduces some new notation, which follows Jullien (2000) for the most part. Define

$$\hat{h}(\gamma,\theta)=\frac{\gamma-F(\theta)}{f(\theta)}. \tag{A.23}$$

Clearly, $\hat{h}(1,\theta)=h(\theta)$. Next, define

$$l(\gamma,\theta)=\arg\max_q vU(q,\theta)-\hat{h}(\gamma,\theta)U_\theta(q,\theta). \tag{A.24}$$

First-order conditions for (A.24) yield

$$l(\gamma,\theta)=\theta-\hat{h}(\gamma,\theta). \tag{A.25}$$

Finally, define the set Θ

$$\Theta=\{\theta\colon l(1,\theta)\le q^{PI}(\theta,v,s)\le l(0,\theta)\}, \tag{A.26}$$

where $q^{PI}(\theta,v,s)$ is as defined in (3.7) of Lemma 2. It is easily shown that $l(0,\theta)>q^{PI}(\theta,v,s)$ for all θ, and therefore the latter inequality in (A.26) is redundant.

The following two intermediate results—Lemmas 3 and 4—are used in the proof of this theorem.

Lemma 3. *If $h(\theta)$ is nonincreasing for all θ, then $\hat{h}_\theta(\gamma,\theta)\le 0$ for all θ,γ such that $\hat{h}(\gamma,\theta)\ge\alpha$.*

Proof. Assume the converse—that for some γ, $\hat{h}(\gamma,\theta)$ is increasing in some interval $[\theta_1,\theta_2]$. This implies that

$$\frac{\gamma-F(\theta_1)}{f(\theta_1)}<\frac{\gamma-F(\theta_2)}{f(\theta_2)}. \tag{A.27}$$

Because $F(\theta_1)<F(\theta_2)$, this implies that $f(\theta_1)>f(\theta_2)$, which in turn implies that

$$\frac{1-\gamma}{f(\theta_1)}<\frac{1-\gamma}{f(\theta_2)}. \tag{A.28}$$

Adding (A.27) and (A.28) yields $[1-F(\theta_1)]/f(\theta_1)<[1-F(\theta_2)]/f(\theta_2)$, which contradicts the fact that $h(\theta)$ is nonincreasing, and the result follows. □

Lemma 4. *For all γ,θ such that $\hat{h}(\gamma,\theta)\ge\alpha$, $l_\theta(\gamma,\theta)>q^{PI}_\theta(\theta,v,s)$.*

Proof. Differentiating (A.25) with respect to θ yields

$$l_\theta(\gamma,\theta)=1-\hat{h}_\theta(\gamma,\theta), \tag{A.29}$$

which implies that $l_\theta(\gamma,\theta)\ge 1$ because $\hat{h}_\theta(\gamma,\theta)\le 0$. Similarly, differentiating (3.7) with respect to θ yields

$$q^{PI}_\theta(\theta,v,s)=\frac{s}{v}<1, \tag{A.30}$$

and the result follows. □

Under Lemmas 3 and 4, all the conditions for Proposition 3 of Jullien (2000) to apply are met. Given that $l(0,\theta)>q^{PI}(\theta,v,s)$, the optimal contract is therefore specified by

$$q^*(\theta,v,s)=q^{PI}(\theta,v,s)\quad\text{for }\theta\in\Theta, \tag{A.31}$$

$$q^*(\theta,v,s)=l(1,\theta)\quad\text{for }\theta\notin\Theta. \tag{A.32}$$

From (A.25) and (3.7),

$$l(1,\theta)=\theta-h(\theta); \tag{A.33}$$

$$q^{PI}(\theta,v,s)=\frac{s\theta}{v}. \tag{A.34}$$

Part (a): Under the conditions of Part (a) of the theorem, $h(\alpha)\le[v-s]\alpha/v$. Because $h_\theta(\theta)\le 0$, this implies that

$$h(\theta)\le\frac{\theta[v-s]}{v} \tag{A.35}$$

for all θ, which when combined with (A.33) and (A.34) implies that

$$l(1,\theta)>q^{PI}(\theta,v,s), \tag{A.36}$$

and the set Θ is therefore empty. Comparing (A.33) and (2.9) and using (A.32) yields

$$q^*(\theta,v,s)=q^{ZP}(\theta,v)\quad\text{for all }\theta. \tag{A.37}$$

The expression for $\tau^*(\theta,v,s)$ follows from imposing profit maximization, incentive compatibility, and the participation constraint for $\theta=\alpha$.

Part (b): Under the conditions of Part (b) of the theorem, $h(\alpha)>[v-s]\alpha/v$. This implies that $l(1,\alpha)<q^{PI}(\alpha,v,s)$. Recall that the support of $f(\theta)$ is the interval $[\alpha,\beta]$. Now, because $h(\beta)=0$, it is clear that $l(1,\beta)>q^{PI}(\beta,v,s)$. Using the fact that $h_\theta(\theta)\le 0$, it is easily shown that Θ is an interval $[\alpha,\hat{\theta}]$ where

$$\hat{\theta}=\theta\colon l(1,\theta)=q^{PI}(\theta,v,s). \tag{A.38}$$

Substituting the expressions for $l(1,\theta)$ and $q^{PI}(\theta,v,s)$ into (A.38) and rearranging yields

$$\hat{\theta}=\theta\colon \theta[v-s]=vh(\theta). \tag{A.39}$$

Consequently, from (A.31), (A.32), and (A.39), it follows that

$$q^*(\theta,v,s)=q^{PI}(\theta,v,s)\quad\text{for }\theta\le\hat{\theta}, \tag{A.40}$$

$$q^*(\theta,v,s)=q^{ZP}(\theta,v)\quad\text{for }\theta\ge\hat{\theta}, \tag{A.41}$$

where θ is as defined in (A.39). The expressions for $\tau^*(\theta, v, s)$ for $\theta \le \hat{\theta}$ follow immediately from Lemma 2. For $\theta \ge \hat{\theta}$, the $\tau^*(\theta, v, s)$ expressions follow from simultaneously imposing profit maximization and incentive compatibility, and account for the participation constraint of customer type $\hat{\theta}$. □

The proof of Theorem 2 uses Lemma 5, which follows directly from Theorem 1.

Lemma 5. *$\int_{\hat{\theta}}^{\beta}[H(\theta) - H(\hat{\theta}) - h(\theta)^2/2]f(\theta)\,d\theta > 0$, where $\hat{\theta}$ is as defined in* (3.5) *of Theorem* 1(b).

Proof. Assume the converse:

$$\int_{\hat{\theta}}^{\beta}\left[H(\theta) - H(\hat{\theta}) - \frac{h(\theta)^2}{2}\right]f(\theta)\,d\theta \le 0. \tag{A.42}$$

Now, for $\theta \ge \hat{\theta}$, the optimal pricing schedule under Theorem 1(b) can be rearranged as

$$\tau^*(\theta) = \frac{[v-s]\hat{\theta}^2}{2} + v\left[H(\theta) - H(\hat{\theta}) - \frac{h(\theta)^2}{2}\right], \tag{A.43}$$

and the seller's profits from the customer types in $[\hat{\theta}, \beta]$ are

$$\frac{[v-s]\hat{\theta}^2}{2}[1-F(\hat{\theta})] + v\int_{\alpha}^{\beta}\left[H(\theta) - H(\hat{\theta}) - \frac{h(\theta)^2}{2}\right]f(\theta). \tag{A.44}$$

(A.42) and (A.47) imply that the seller's profits from $[\hat{\theta}, \beta]$ are lower than $\frac{1}{2}[v-s]\hat{\theta}^2[1-F(\hat{\theta})]$. However, if the seller were to offer customers in $[\hat{\theta}, \beta]$ the fixed-fee contract

$$T = \frac{[v-s]\hat{\theta}^2}{2}, \tag{A.45}$$

then all of these customer types would participate, yielding profits of $\frac{1}{2}[v-s]\hat{\theta}^2[1-F(\hat{\theta})]$ from the segment $[\hat{\theta}, \beta]$. Moreover, the fixed-fee contract would not affect incentive compatibility for $\theta < \hat{\theta}$. This means that the seller can (weakly) improve the contract derived in Theorem 1(b), which contradicts the fact that this is the unique optimal contract. The result follows. □

Note that as $\hat{\theta}$ tends to α, Theorem 1(a) becomes applicable, and in the limit the lemma also implies that

$$\int_{\alpha}^{\beta}\left[H(\theta) - \frac{h(\theta)^2}{2}\right]f(\theta) > 0. \tag{A.46}$$

(A.46) can also be derived using an argument similar to the proof of this Lemma, but in the context of Theorem 1(a).

Proof of Theorem 2. Recall again that the support of $f(\theta)$ is the interval $[\alpha, \beta]$. First, consider the case when $h(\alpha) \le [v(\rho) - s(\rho)]\alpha/v(\rho)$. From (4.7),

$$\Pi(\rho) = \Pi^L(\rho) = \int_{\alpha}^{\beta}\left(\frac{\alpha^2[v(\rho)-s(\rho)]}{2} + v(\rho)\left[H(\theta) - \frac{h(\theta)^2}{2}\right]\right)f(\theta)\,d\theta. \tag{A.47}$$

Differentiating (A.47) with respect to ρ yields

$$\Pi_\rho^L(\rho) = \frac{\alpha^2[v_\rho(\rho) - s_\rho(\rho)]}{2} + v_\rho(\rho)\int_{\alpha}^{\beta}\left[H(\theta) - \frac{h(\theta)^2}{2}\right]f(\theta)\,d\theta. \tag{A.48}$$

The optimal value of ρ^* must satisfy the first-order condition $\Pi_\rho^L(\rho^*) = 0$. Rearranging (A.48),

$$v_\rho(\rho^*) - s_\rho(\rho^*) = \frac{-2v_\rho(\rho^*)}{\alpha^2} \cdot \int_{\alpha}^{\beta}\left[H(\theta) - \frac{h(\theta)^2}{2}\right]f(\theta)\,d\theta. \tag{A.49}$$

Using the fact that $v_\rho(\rho) < 0$ for all ρ, and Equation (A.46) from Lemma 4, (A.49) implies that

$$v_\rho(\rho^*) - s_\rho(\rho^*) > 0. \tag{A.50}$$

Because $v_{\rho\rho}(\rho) - s_{\rho\rho}(\rho) < 0$, (A.50) implies that $\rho^* < \rho^e$, which completes the proof for lower levels of piracy.

Next, consider the case when $h(\alpha) \ge [v(\rho) - s(\rho)]\alpha/v(\rho)$. From (4.9),

$$\begin{aligned}\Pi^H(\rho) = {} & \left(\frac{s(\rho)[v(\rho)-s(\rho)]}{v(\rho)}\right)\int_{\alpha}^{\hat{\theta}(\rho)}\frac{\theta^2}{2}f(\theta)\,d\theta \\ & + v(\rho)\int_{\hat{\theta}(\rho)}^{\beta}\left(H(\theta) - \frac{[h(\theta)]^2}{2}\right)f(\theta)\,d\theta \\ & + [1-F(\hat{\theta}(\rho))]\left(\frac{[v(\rho)-s(\rho)][\hat{\theta}(\rho)]^2}{2} - v(\rho)H(\hat{\theta}(\rho))\right).\end{aligned} \tag{A.51}$$

Differentiating both sides of (A.51) with respect to ρ, cancelling out common terms, and rearranging substantially yields the following expression:

$$\Pi_\rho^H(\rho) = f^A(\rho) + f^B(\rho) + f^C(\rho) + f^D(\rho) + f^E(\rho), \tag{A.52}$$

where

$$f^A(\rho) = \frac{s_\rho(\rho)[v(\rho)-s(\rho)]^2}{v(\rho)^2}\int_{\alpha}^{\hat{\theta}(\rho)}\frac{\theta^2}{2}f(\theta)\,d\theta; \tag{A.53}$$

$$f^B(\rho) = [v_\rho(\rho) - s_\rho(\rho)] \cdot \left[[1-F(\hat{\theta}(\rho))]\frac{[\hat{\theta}(\rho)]^2}{2} + \frac{s(\rho)^2}{v(\rho)^2}\int_{\alpha}^{\hat{\theta}(\rho)}\frac{\theta^2}{2}f(\theta)\,d\theta\right]; \tag{A.54}$$

$$f^C(\rho) = v_\rho(\rho)\int_{\hat{\theta}(\rho)}^{\beta}\left[H(\theta) - H(\hat{\theta}) - \frac{h(\theta)^2}{2}\right]f(\theta)\,d\theta; \tag{A.55}$$

$$f^D(\rho) = \hat{\theta}_\rho(\rho)[1-F(\hat{\theta}(\rho))][\hat{\theta}(\rho)[v(\rho)-s(\rho)] - v(\rho)h(\hat{\theta}(\rho))]; \tag{A.56}$$

$$f^E(\rho) = \hat{\theta}_\rho(\rho)f(\hat{\theta}(\rho))\left[\frac{v(\rho)^2h(\hat{\theta}(\rho))^2 - [v(\rho)-s(\rho)]^2[\hat{\theta}(\rho)]^2}{2v(\rho)}\right]. \tag{A.57}$$

From the definition of $\hat{\theta}(\rho)$, we know that

$$[v(\rho) - s(\rho)]\hat{\theta}(\rho) = v(\rho)h(\hat{\theta}(\rho)). \tag{A.58}$$

Substituting (A.58) into (A.56) and (A.57) yields $f^D(\rho) = f^E(\rho) = 0$ for all ρ. Also, because $s_\rho(\rho) < 0$, it follows that $f^A(\rho) < 0$ for all ρ. Moreover, Lemma 5 and the fact that $v_\rho(\rho) < 0$ imply that $f^C(\rho) < 0$ for all ρ. Now, the optimal value of ρ^* must satisfy the first-order condition $\Pi^H_\rho(\rho^*) = 0$. Rearranging (A.52), this condition reduces to

$$f^B(\rho^*) = -[f^A(\rho^*) + f^C(\rho^*)]. \tag{A.59}$$

Because we have established that $f^A(\rho) < 0$, $f^C(\rho) < 0$ for all ρ, (A.59) implies that $f^B(\rho^*) > 0$. For any $\hat{\theta}(\rho) < \beta$, the term in square parentheses on the RHS of (A.54) is strictly positive. In conjunction with (A.59), this in turn implies that

$$v_\rho(\rho^*) - s_\rho(\rho^*) > 0. \tag{A.60}$$

Because $v_{\rho\rho}(\rho) - s_{\rho\rho}(\rho) < 0$, (A.60) implies that $\rho^* < \rho^e$. □

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