Priced Risk and Asymmetric Volatility in the Cross-Section of Skewness

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Abstract

We investigate the sources of skewness in aggregate risk-factors and the cross-section of stock returns. In an ICAPM setting with conditional volatility, we find theoretical time series predictions on the relationships among volatility, returns, and skewness for priced risk factors. Market returns resemble these predictions; however, size, book-to-market, and momentum factor returns show alternative behavior, leading us to conclude these factors are not priced risks. We link aggregate risk and skewness to individual stocks and find empirically that the risk aversion effect manifests in individual stock skewness. Additionally, we find several firm characteristics that explain stock skewness. Smaller firms, value firms, highly levered firms, and firms with poor credit ratings have more positive skewness.

Elementary porfolio theory states that individuals evaluating a broadly diversified portfolio are concerned only with systematic risk. Risk idiosyncratic to a particular stock is not relevant. This notion has been ingrained into the finance field; however, the nature of systematic risk is still not thoroughly understood.

Considerable recent research in the sources of systematic risk has focused on negative skewness in individual stocks and the stock market as a whole. A number of papers including Bae, Kim, and Nelson (2007), Bekaert and Wu (2000), and Campbell and Hentschel (1992) have documented a "volatility feedback" effect where future high volatility is associated with low returns and high future expected returns. Berd, Engle, and Voronov (2005) show that this volatility feedback effect induces negative skewness in the market. Furthermore, Dittmar (2002), Harvey and Siddique (2000), Kraus and Litzenberger (1976), and others have studied how preference for positive skewness can generate a risk premium on negatively skewed assets. Harvey and Siddique in particular claimed that skewness preference may explain anomalies such as the momentum effect.

Aside from papers focusing on empirically observed stock returns, a growing literature has also documented skewness of individual stocks as it manifests in option prices. Bakshi, Kapadia, and Madan (2003) examined risk-netural skewness of the market and 30 individual stocks and found negative skewness in both, with less negative skewness in individual stocks. Dennis and Mayhew (2002) found that risk-neutral skewness of individual stocks is more negative for stocks with higher β and in times when the market is more negatively skewed or has higher volatility. Duan and Wei (2006) studied implied volatility smiles and found that systematic risk is priced in the options smile, which is indicative of aggregate negative skewness manifesting in individual stocks.

In this paper we study negative skewness in priced risk factors and individual stocks. The goal of this study is twofold. First, we establish the basis for asymmetric volatility and negative skewness in risk factor returns with an intertemporal capital asset pricing model setting. We use this as a basis for testing which of the Fama-French-Carhart factors are priced risk factors. We find that provided our risk aversion hypothesis is true, only market risk appears to be priced.

Next, we examine the sources of skewness in individual stocks. We find that the market asymmetric volatility effect manifests in individual stock returns and in option-based risk-neutral density skewness. Stocks with higher systematic risk exhibit more negative skews. We also find several idiosyncratic firm characteristic effects on skewness. Smaller firms have more positively skewed returns than larger firms. Consistent with Hong, Wang, and Yu (2007) we also find that firms with fewer financial constraints and whose stock price has declined have more positively skewed returns.

This paper differs from previous literature in the following. First, we use aversion to systematic risk and time-varying volatility to study the question of whether covariation in returns results from underlying priced risk or a statistical artifact. Second, we study skewness using a much larger universe of firms as well as both physical and risk-neutral measures of skewness. Bakshi, Kapadia, and Madan (2003) used options on the 30 largest stocks. Dennis and Mayhew (2002) used options traded on the Chicago Board Options Exchange. We use the entire CRSP/Compustat universe to analyze physical measure stock return skewness and the Optionmetrics' IvyDB database, which contains all US listed options, to analyze risk-neutral measure skewness. The broader sample offers us further insight into the cross-sectional variation of skewness.

This paper is organized as follows. Section 1 outlines the theoretical link between time-varying volatility in priced risk factors and time series properties of returns. Section 2 studies the time series properties of the Fama-French-Carhart factors for evidence of priced risk. Section 3 links aggregate risk and asymmetric volatility to individual stock skewness, tests this relationship empirically, and identifies idiosyncratic sources of individual stock skewness.

1 Risk Aversion, Time-Varying Volatility, and Skewness

Consider the setting of Merton's (1973) Intertemporal Capital Asset Pricing Model (ICAPM). An investor holds a portfolio which is a linear combination of the market portfolio and m portfolios that have the highest correlation with m state variables that determine the investment opportunity set. In this setting suppose that investment opportunities are good when a state variable is high. Then an asset whose return is positively correlated with this state variable will be a poor hedge against adverse shocks to investment opportunities and hence will command a positive risk premium.

Suppose volatility of the market and of several state variables is itself time varying. In this scenario, if market volatility is expected to increase rational investors will demand less of assets subject to market risk. This results in a contemporaneous drop in market returns and a higher future market risk premium.

If volatility of a state variable increases the covariance of returns with that state variable will increase. Hence, assets which command a risk premium will be even less desirable as investors will be fearful of the increased probability of a large adverse shock to the investment opportunity set. Rational investors will choose to hold less of such assets, which will lead to low contemporaneous returns and high future expected returns on the assets that have the highest premium on the risk of the state variable.

More formally, in the ICAPM assets can be priced by a stochastic discount factor that is linear in a set of state variables or mimicking portfolios f_{it}.

$$m_{t+1} = a_t + b_{1t}\, f_{1,t+1} + \cdots + b_{mt}\, f_{m,t+1} \tag{1}$$

All assets obey the one period pricing equation

$$E_t\big[m_{t+1}(1 + r_{t+1})\big] = 1 \tag{2}$$

which can be rewritten as

$$E_t\big[1 + r_{t+1}\big] = \frac{1 - Cov_t[m_{t+1}, 1 + r_{t+1}]}{E_t[m_{t+1}]} \tag{3}$$

Assuming there exists a riskfree asset with return r^f_{t+1}, combining with (1) and rewriting, we get

$$E_t[r_{t+1}] = r^f_{t+1} - (1 + r^f_{t+1})\Big\{b_{1t}\, Cov_t(f_{1,t+1}, r_{t+1}) + \cdots + b_{mt}\, Cov_t(f_{m,t+1}, r_{t+1})\Big\} \tag{4}$$

By defining betas in the usual fashion, this can be written as

$$E_t[r_{t+1}] = r^f_{t+1} - (1 + r^f_{t+1})\Big\{b_{1t}\beta_{1t}\, Var_t(f_{1,t+1}) + \cdots + b_{mt}\beta_{mt}\, Var_t(f_{m,t+1})\Big\} \tag{5}$$

Assume that the b_i and β_i coefficients are constant across time. Then, if volatility of a factor increases, the risk premium on that factor will widen. Hence following a positive shock to volatility of a factor, assets that experience a positive risk premium on that factor (such as a factor mimicking portfolio) will see a positive shock to future discount rates, leading to a contemporaneous decrease in returns. In other words, the model predicts a negative contemporaneous correlation between shocks to volatility of a factor and returns on the factor mimicking portfolio.

This volatility-return correlation has been observed for the market factor in many papers dating back to Black (1976) and Christie (1982). Whether a factor is priced risk can potentially be tested by examining evidence of asymmetric volatility in factor returns. According to the above argument, priced risk factors should exhibit a contemporaneous negative relation between returns and volatility. Other returns may not display this phenomenon. In fact, Dennis, Mayhew, and Stivers (2006) found greater evidence of asymmetric volatility in the systematic component of stock returns than in stock idiosyncracies.

The asymmetric volatility effect also produces a particular time aggregation pattern in returns, as documented by Berd, Engle, and Voronov (2005). The skewness of returns grows more negative for longer horizons. Panel (e) of Figure 1 shows the skewness of returns at varying horizons simulated using an asymmetric GARCH model with parameters estimated from market returns. The one-day return is slightly negatively skewed; however, skewness becomes more negative as the return horizon expands to quarterly returns. As the return horizon expands further, by the central limit theorem returns converge to a symmetric normal distribution. We would expect such a pattern for the returns of any priced risk factor.

Although we presented the intuition in terms of the ICAPM, the resulting link between returns and volatility of a factor is more general than the ICAPM setting. All that is needed for the above analysis is a representative agent economy where the stochastic discount factor is linear in a set of variables. The Ross (1976) arbitrage pricing theory model would predict the same results.

2 Asymmetric Volatility in the Fama-French-Carhart Factors

Fama and French (1992) proposed a three factor pricing model of stock returns including the standard CAPM market factor, along with factors whose mimicking portfolios are constructed based on size and book-to-market ratios of the cross-section of stock returns. Carhart (1997) added a factor constructed based on stock return momentum. These studies argued the factor returns were priced risk by examining historical returns of portfolios isolating that risk.

The first task of this paper is to revisit the question of whether the Fama-French-Carhart factors are priced risk factors. Rather than looking at expected returns, however, we examine to what extent these factors exhibit asymmetric volatility and the previously discussed pattern of time aggregation in return skewness as evidence of priced risk.

Figure 1 reports the observed empirical skewness of each factor's returns at horizons of 1 to 250 days. We calculated skewness using overlapping returns from 1988 to 2005. The market factor appears to fit the pattern. One day returns are slightly negative and grow more negative with time aggregation. The skewness does not seem to converge to zero at longer horizons, however, this may be an artifact of relatively few yearly return observations in the 18 year sample.

The other factors, however, show little to no resemblance to the theoretical prediction. One day size factor returns are negatively skewed, but quarterly returns are actually positively skewed. The book-to-market and momentum factors also exhibit more positive skewness with time aggregation in returns.

[Figure 1 about here.]

The same results manifest in parameter estimation of the asymmetric GARCH model on each of the factors. Table 1 reports the estimation of the following specification on each of the Fama-French-Carhart factors.

$$r_t = \mu + \varepsilon_t \ , \ \varepsilon_t = \sqrt{h_t}\eta_t \ , \ \eta_t \sim N(0,1) \tag{6}$$

$$h_t = \omega + \alpha\varepsilon_{t-1}^2 + \gamma\varepsilon_{t-1}^2 I_{t-1} + \beta h_{t-1} \tag{7}$$

[Table 1 about here.]

According to our hypothesis of volatility feedback, the coefficient on the asymmetric term $\varepsilon_{t-1}^2 I_{t-1}$ should be positive, thus encapsulating the negative relation between returns and volatility. The parameter estimate is significantly positive for the market factor. For the other factors, however, the parameter is negative and less significant. The coefficient of asymmetry on the size factor in particular is an order of magnitude smaller than its market factor counterpart.

These results seem to suggest that, contrary to the ICAPM prediction, as variance of a state variable increases and hence covariance of returns with that state variable increases, the risk premium on that factor decreases. If the risk premium arises from investor desire to hedge against adverse shocks to the investment opportunity set, this would imply that investors demand more of assets that become worse hedges.

An alternative explanation is that shocks to volatility of size, book-to-market, and momentum factors are negatively correlated with shocks to market volatility. In this case, a positive shock to a factor such as size would have two effects on the size factor mimicking portfolio. First, through the volatility feedback effect, returns on that portfolio would decrease. However, if the portfolio has a positive market beta, market volatility would cause returns to increase. Theoretically, this second effect could dominate, yielding the results shown Table 1.

In unreported results, we checked for this possibility by calculating the correlations of the estimated GARCH variance series from (6). All pairwise correlations were positive. Furthermore, innovations in the four variance series were also positively correlated. Hence, a positive shock to size factor volatility is also associated with a positive shock to market volatility, and hence the contemporaneous return on the size mimicking portfolio should decrease, a result which does not appear in the data.

3 Systematic and Idiosyncratic Sources of Firm Skewness

The preceding discussion focused on analyzing aggregate risk factors for evidence of asymmetric volatility. Next, we link individual stock skewness to aggregate risk factor skewness and show that the asymmetric volatility effect that produces negative skewness in the market appears in the cross section of individual stock returns. Stocks which have higher component of aggregate risk have more negative skewness.

Since we have argued that the market factor is the only priced risk factor, consider a one factor model. Assume that individual stock idiosyncracies are uncorrelated with market variance and that idiosyncratic stock variance is uncorrelated with market returns. In the appendix we show that this implies the skewness of individual stock returns can be decomposed as

$$SKEW_i = \left(\frac{\beta_i \sigma_m}{\sigma_i}\right)^3 SKEW_m + \left(\frac{\sigma_{\varepsilon_i}}{\sigma_i}\right)^3 SKEW_{\varepsilon i} \tag{8}$$

where $SKEW_i$ is the skewness of stock i, $SKEW_m$ is market skewness, and $SKEW_{\varepsilon i}$ is the skewness of stock i's idiosyncratic returns. Equation (8) states that individual stock skewness is a linear function of market skewness. Note that the multiplier for the market skewness is equal to $\text{sign}(\beta_i) \times (R^2)^{3/2}$, where R^2 is the population R^2 of a CAPM regression, ie. the proportion of return variance that is systematic. We will denote this quantity as R_i^3. We know that the market skewness component is negative. Hence, we would expect to find that stocks with higher R_i^3 tend to have more negative skewness.

It should be noted that the downside risk implied by this relationship is aggregate risk. Hence, aggregate risk impacts the pricing of individual stock options, a prediction that is consistent with the results of Duan and Wei (2006), who found evidence that systematic risk is priced in options in that stocks with higher systematic risk have steeper smiles and higher implied volatilties. We will show that stocks with a higher component of systematic risk have more negative skewness in the risk-neutral distribution implied by option prices.

In addition, we examine the idiosyncratic sources of firm skewness through our cross-sectional study. Previous studies have found such sources to be important in determining firm skewness. Hong, Wang, and Yu (2007) argued that firms which have relatively few financing constraints have more positive skewness due to their ability to repurchase shares following a price decline. They additionally found that firms with large market capitalization, high leverage, high market-to-book ratios, or whose stock has increased tend to have more negative skewness. Dennis and Mayhew (2002) also found that larger stocks have more negative skewness; however, in their sample risk-neutral skewness was more positive for firms with higher leverage. We will attempt to revisit these effects with our broader sample.

3.1 Data

We conduct our analysis using stock returns from CRSP, accounting data from COMPUSTAT, analyst data from IBES, and option data from Optionmetrics. The stock return-based skewness data are from 1988-2005, and the option-based skewness data span 1996-2005. The firm characteristic variables are used as lagged predictors, hence they span 1987-2004.

3.1.1 Physical measure skewness

For each year and each stock from 1988-2005, we calculate daily, monthly, and quarterly overlapping continuously compounded returns within the year. We then calculate skewness of the returns at daily, monthly, and quarterly horizons for each year. For stocks with missing observations, we use all available observations within the year. To remove outliers, we discard any skewness values greater than three in magnitude.

3.1.2 Firm characteristics

Firm characteristics are computed as follows. Using CRSP daily returns for each stock, we calculate the Amihud (2002) illiquidity measure *Illiq* yearly. Firm β is estimated as the sum of β_1, β_2, and β_3 in the following regression based on Scholes and Williams (1977), calculated for each stock within each year. The lead and lag terms account for nonsynchronicity in trading.

$$r_{it} = \alpha + \beta_1 r_{m,t-1} + \beta_2 r_{mt} + \beta_3 r_{m,t+1} + \varepsilon_{it} \tag{9}$$

To remove outliers, we discard values of β larger than 10 in magnitude. We also use the R^2 of this regression to compute firm R^3, defined as $\text{sign}(\beta) \times (R^2)^{3/2}$ to match the relationship between firm and market skewness in equation (8).

We use year end data from CRSP and COMPUSTAT for the following characteristics. *Logsize* is the log of year end price multiplied by the number of shares outstanding. *Leverage*, the debt to assets ratio, is computed using the book value of debt and market value of equity. *Credit* is the S&P credit rating, scaled so that 0 is AAA, 1 is AA+, and so on. For firms that are missing a credit rating, we assign $Credit = 8$, which corresponds to the median rating of BBB. We also create an indicator variable *Missingcredit* which is one if the credit ring was not available. *BM*, the book to market ratio, is calculated using the accounting value of equity and the end of year market price. This variable is windsorized at the 99th percentile.

Lastly, we retrieve the average number of analysts *Numanalyst* covering each firm in each year from IBES. For firms which have no data, we set *Numanalyst* to zero.

3.1.3 Risk-netural measure skewness

We use the Bakshi, Kapadia, and Madan (2003) approach to computing skewness of the risk-neutral distribution using option prices. There are several difficulties with this approach.

First, we do not observe a continuum of strikes; there are discretely spaced strike prices. Second, observed option prices are generally on American options, whereas the BKM technique relies on European option prices. Third, and most important, there are often several put options and few call options or few put options with multiple call options on a single day. This is especially true after larger stock price movements. For example, after a large price drop there may only be one or two out-of-the-money put options and many such call options. Dennis and Mayhew (2002) mitigate this third bias by discarding some options so that the domain of integration is symmetric. A difficulty with this approach is that risk-neutral skewness is heavily dependent on the tails of the distribution. By discarding far out-of-the-money options, the measure of risk-neutral skewness becomes significantly less accurate.

Our approach is instead to fit a curve for the implied volatility smile as a function of moneyness on each date. The approach is as follows. From Optionmetrics, we obtain the implied volatilities for each option on a given stock and date. We average the implied volatilities for put and call options on the same maturity, underlying, and strike. For each option we calculate the moneyness, defined as $\frac{\ln(K/S)-rT}{\sigma\sqrt{T}}$, using the linearly interpolated zero coupon interest rate from Optionmetrics, the current stock price, and the historical 22 trading day volatility calculated from returns.

Next we fit a quadratic spline with a knot at 0 to the implied volatilities as a function of moneyness. We discard any options with $|moneyness| > 2$ or maturity less than 90 days and any observations where all options have only positive or only negative moneyness[1]. Assuming European options with equivalent strike and maturity have the same implied volatilities as their American counterparts, we then calculate European call option prices for one month maturity options on a continuum of strike prices with moneyness up to ± 20. In this calculation we assume that the implied volatility slope is constant for $|moneyness|$ below

[1]Two standard deviations were chosen as the cutoff point due to a lack of quality data on deep out-of-the-money observations which appeared to be noisy and unreliable.

-2 and above 2.[2] We then calculate the relevant integrals and apply the BKM formula to obtain the risk-neutral skewness for each stock on each date. Lastly, we average the skewness for each firm across each year to match the firm-year observations of our other data.

3.2 Preliminary Analysis

Table 2 reports descriptive statistics for the firm skewness and characteristics observations. Our common sample comprises 108,520 firm-year observations of daily, monthly, and quarterly return skewness and end-of-previous year firm characteristics. There are between 4,632 and 7,410 firms within each year. In analyzing risk-neutral skewness, our sample consists of 21,146 firm-year observations spanning 1996-2005. All statistics and analysis done involving risk-neutral skewness is done for this subsample; other statistics are reported for the full sample.

[Table 2 about here.]

Note from the table that average skewness of daily individual stock is positive with a value of 0.1561. The average skewness measures display the previously discussed pattern of time aggregation. Monthly return skewness is less positive, and quarterly return skewness is actually slightly negative, with an average skewness of -0.0037.

Using the method described above, we found an average risk-neutral skewness of -0.0037. It is important to note that the level of skewness varied depending on the cutoff point of moneyness chosen. We found that the implied volatility smile is left skewed; however, due to the asymmetry in curvature between the left and right tails, deep OTM call volatilities were often higher than deep OTM put volatilties even when less OTM call volatilities were lower

[2] We also tried extrapolating based on the quadratic spline and based on a linear slope on each tail of the implied volatility smile. Both approaches failed to produce option prices that converge to zero. The implied volatilities in each case rose so quickly that options further out of the money would have higher prices, which is a violation of no arbitrage.

than equivalently OTM put volatilties. Due to this shape of the smile, a higher moneyness threshold produces more positive skewness. The main interest of this paper is to analyze cross-sectional variation and not the level of skewness itself. Hence, we chose a low threshold where option data is more abundant to ensure less noisy measures of risk-neutral skewness.

We are missing credit rating data on 80% of our sample, hence we do not expect great predictive power of the *Credit* variable. Our analyst coverage data fares much better. Roughly 53% of the observations have positive analyst coverage.

Our measure of β appears to be highly noisy. The average β is considerably less than one, which implies that in our sample we have many small stocks whose returns appear not to covary greatly with the market. Furthermore, as reported in Table 3 rank correlations between β and other firm characteristics are much larger than Pearson correlations. For example, the Spearman correlation between β and R^3 is 0.69, whereas the Pearson correlation is only 0.38. These effects may be due to mismeasured βs stemming from measurement error in returns.

[Table 3 about here.]

The difficulty in estimating β carries into our key firm characteristic variable, firm R^3. In particular, R^3 is highly correlated with firm size. The Pearson correlation between R^3 and *Logsize* is 0.54. This high correlation renders difficulty in separating the effects of systematic risk and idiosyncratic firm size on firm skewness. Both variables, however, have considerable explanatory power. Table 4 reports skewness for sorts based on firm size and by R^3. In Panel A ten deciles were assigned in each year based on firm size. For each decile we calculated average R^3, daily, montly, and quarterly return skewness, and risk-neutral skewness. Note that there are relatively few observations for risk-neutral skewness in the smallest three size deciles.

[Table 4 about here.]

Discarding the smallest three size deciles, firm skewness is more positive for smaller firms. For over half of the observations, empirical skewness was positive at all horizons. For the largest stocks, however, skewness is negative at monthly and quarterly horizons as well as in the risk-neutral measure. These findings paint a picture broader than but consistent with Bakshi, Madan, and Kapadia (2003). In their study of the 30 largest stocks, they found that the stocks had negative risk-neutral skewness, albeit less negative than the market. Here we find that the largest stocks tend to be negatively skewed, but the cross-section of stocks shows considerable variation.

The effect of size may be due primarily to a correlation of firm size and R^3. In Panel B of Table 4, we report firm skewness for R^3 sorted deciles. In each year we assign ten deciles based on firm R^3 and calculate average firm size, daily, monthly, and quarterly return skewness, and risk-neutral skewness. Clearly, higher R^3 is associated with larger firm size. Although R^3 is a weaker predictor of daily return skewness, with time aggregation the effect of R^3 is strong. Discarding the lowest two R^3 deciles, monthly and quarterly skewness are monotonically decreasing in firm R^3. The risk-neutral skewness exhibits a similar pattern aside from the lower R^3 deciles.

3.3 Regression Results

Lastly, we perform a regression analysis of the sources of skewness in individual stock returns. Central to our analysis is the asymmetric volatility effect which produces negative skewness in market returns. We estimate the following three specifications to see this effect in the cross section of returns and also analyze idiosyncratic sources of skewness.

$$SKEW_{it} = \beta_0 + \beta_1 R^3_{it} + \beta_2 Illiq_{it} + \varepsilon_{it} \tag{10}$$

$$SKEW_{it} = \beta_0 + \beta_1 R^3_{it} + \beta_2 Illiq_{it} + \beta_3 Vol_{it} + \beta_4 BM_{it} + \varepsilon_{it} \tag{11}$$

$$\begin{aligned} SKEW_{it} = &\beta_0 + \beta_1 R^3_{it} + \beta_2 Illiq_{it} + \beta_3 Vol_{it} + \beta_4 BM_{it} + \beta_5 Logsize_{it} \\ &+ \beta_6 Leverage_{it} + \beta_7 Credit_{it} + \text{Year Dummies} + \varepsilon_{it} \end{aligned} \tag{12}$$

We estimate the panel regression coefficients by ordinary least squares. OLS errors are biased in the presence of correlation across firm or across time. In a regression of various firm characteristics, we would expect to find significant time series correlation. To account for the correlation within firm across time, we use Rogers (1993) clustered standard errors. Previous studies such as Dennis and Mayhew (2002) and Bakshi, Madan, and Kapadia (2003) used the Fama and MacBeth (1973) procedure. Petersen (2007) has shown that with firm serial correlation, Fama-MacBeth standard errors are considerably biased and often worse than OLS standard errors. Clustered standard errors, on the other hand, approximate the true errors much better.

This accounts for correlation across time. To check robustness to correlation across firm within time periods, we add year dummies to the regression specification. Table 5 presents the results of this estimation.

[Table 5 about here.]

The univariate regressions show a very strong and consistent effect of systematic risk in individual stock skewness. As predicted, the coefficient on R^3 is negative, reflecting the negative skewness of the market. At the monthly horizon, it is more negative than at the daily horizon. At the quarterly horizon, however, it is less negative. This appears to defy the time aggregation pattern predicted earlier; however, it could be an artifact of few quarterly observations within each year. In the risk-neutral measure, the systematic risk of a stock has considerable effect on skewness. The coefficient on R^3 for the risk-neutral skewness regression is nearly three times as large (-0.8656) than it is for daily skewness (-0.2533).

This effect is generally robust to controls for liquidity, volatility, and book-to-market. Again, size captures much of the explanatory power of R^3. When firm size is added to the specification, the coefficent on R^3 is no longer significant at the monthly and quarterly horizons and is much less significant at the daily horizon or in the risk-neutral measure. We regard size as a proxy for β, which is measured poorly. Indeed, as shown above, the two variables are highly correlated and in a regression setting it is difficult to distinguish the effects of each variable.

Coefficients on firm characteristics also seem to have considerable effect on skewness. One theory for firm idiosyncratic skewness, advanced by Hong, Wang, and Yu (2007) is that irrational investors may drive own the price of a stock temporarily. Rational managers that are not financial constrained will then repurchase shares, thus creating more positive skewness for firms that have experienced declines and are less constrained.

We find evidence consistent with the Hong, Wang, and Yu hypothesis. The coefficient on the book-to-market ratio is generally positive and significant, which implies that value stocks are more positively skewed than growth stocks. This is consistent with a hypothesis of overselling of stocks that have experienced a decline. Additionally, our estimated coefficients on *Leverage* and *Credit* are negative or zero. This implies that less levered firms and firms with better credit ratings are more positively skewed, consistent with the notion that firms with lower financial constraints have more positive skewness.

The coefficients on illiquidity and volatility are more puzzling. Higher illiquidity implies more negative skewness at daily horizons but more positive skewness in the risk-neutral distribution and at longer horizons. Higher volatility implies negative daily return skewness and in the risk-neutral distribution but positive skewness at longer horizons. We do not have a compelling explanation of these effects.

4 Conclusion

We study the sources of skewness in individual stock returns. Theoretically in a linear factor model, an increase in future volatility of a priced risk factor is associated with a contemporaneous drop in returns on that factor. This gives rise to an asymmetric volatility model which has been documented for the market factor. We estimate this model on the Fama-French-Carhart factors and conclude that only market risk is priced.

We then link individual stock skewness to skewness of the market and conclude that firms with higher systematic risk as measured by R^3 have more negatively skewed returns. We measure skewness of individual stocks under the physical and risk-neutral densities using stock returns and option prices and find supporting evidence in both measures. In addition, larger firms, value firms, levered firms, and financially constrained firms have more positive skewness.

Future research could investigate further the link between skewness and various firm characteristics. Although Hong, Wang, and Yu have proposed a model where stock skewness is related to credit quality, we have no such explanation for the potential link between stock liquidity and skewness documented in our regression results. More work could be done to understand the idiosyncratic sources of skewness.

A Individual Stock Skewness Decomposition

We decompose skewness of individual stock returns as follows. Write

$$r_t = \beta r_{mt} + \varepsilon_t \tag{13}$$

where β is defined as $\frac{Cov(r_t, r_{mt})}{Var(r_{mt})}$. Denote the mean and variance of r_t, r_{mt}, and ε_t by $\mu, \sigma^2, \mu_m, \sigma_m^2, \mu_\varepsilon$, and σ_ε^2 respectively, where all expectations in this derivation may be taken conditionally or unconditionally. Note that

$$\mu = \beta\mu_m + \mu_\varepsilon \tag{14}$$

and that

$$Cov(r_{mt}, \varepsilon_t) = Cov(r_{mt}, r_t - \frac{Cov(r_t, r_{mt})}{Var(r_{mt})} r_{mt}) = 0 \tag{15}$$

Assume that $Cov(r_{mt}^2, \varepsilon_t) = 0$ and $Cov(r_{mt}, \varepsilon_t^2) = 0$. Now decompose skewness of the stock return by

$$\begin{aligned} E\left[\left(\frac{r_t - \mu}{\sigma}\right)^3\right] =& E\left[\left(\frac{\beta r_{mt} + \varepsilon_t - \mu}{\sigma}\right)^3\right] \\ =& E\left[\left(\frac{\beta r_{mt} - \beta\mu_m + \varepsilon_t + \beta\mu_m - \mu}{\sigma}\right)^3\right] \end{aligned}$$

separating and using (14), we get

$$\begin{aligned} =& E\left[\left(\beta\frac{r_{mt} - \mu_m}{\sigma} + \frac{\varepsilon_t - \mu_\varepsilon}{\sigma}\right)^3\right] \\ =& \beta^3 E\left[\left(\frac{r_{mt} - \mu_m}{\sigma}\right)^3\right] + 3\beta^2 E\left[\left(\frac{r_{mt} - \mu_m}{\sigma}\right)^2\left(\frac{\varepsilon_t - \mu_\varepsilon}{\sigma}\right)\right] \\ &+ 3\beta E\left[\left(\frac{r_{mt} - \mu_m}{\sigma}\right)\left(\frac{\varepsilon_t - \mu_\varepsilon}{\sigma}\right)^2\right] + E\left[\left(\frac{\varepsilon_t - \mu_\varepsilon}{\sigma}\right)^3\right] \\ =& \frac{\beta^3\sigma_m^3}{\sigma^3} E\left[\left(\frac{r_{mt} - \mu_m}{\sigma_m}\right)^3\right] + \frac{3\beta^2}{\sigma^3} E[r_{mt}^2(\varepsilon_t - \mu_\varepsilon)] \\ &+ \frac{3\beta}{\sigma^3} E[\varepsilon_t^2(r_{mt} - \mu_m)] + \frac{\sigma_\varepsilon^3}{\sigma^3} E\left[\left(\frac{\varepsilon_t - \mu_\varepsilon}{\sigma_\varepsilon}\right)^3\right] \end{aligned}$$

By our assumptions, the middle two terms drop and we are left with

$$E\left[\left(\frac{r_t - \mu}{\sigma}\right)^3\right] = \left(\frac{\beta\sigma_m}{\sigma}\right)^3 E\left[\left(\frac{r_{mt} - \mu_m}{\sigma_m}\right)^3\right] + \left(\frac{\sigma_\varepsilon}{\sigma}\right)^3 E\left[\left(\frac{\varepsilon_t - \mu_\varepsilon}{\sigma_\varepsilon}\right)^3\right] \tag{16}$$

Equation (8) says that the skewness of an individual stock is the sum of the market skewness and the skewness of the stock's idiosyncratic component, weighted by the proportion of stock volatility due to the market and idiosyncratic components.

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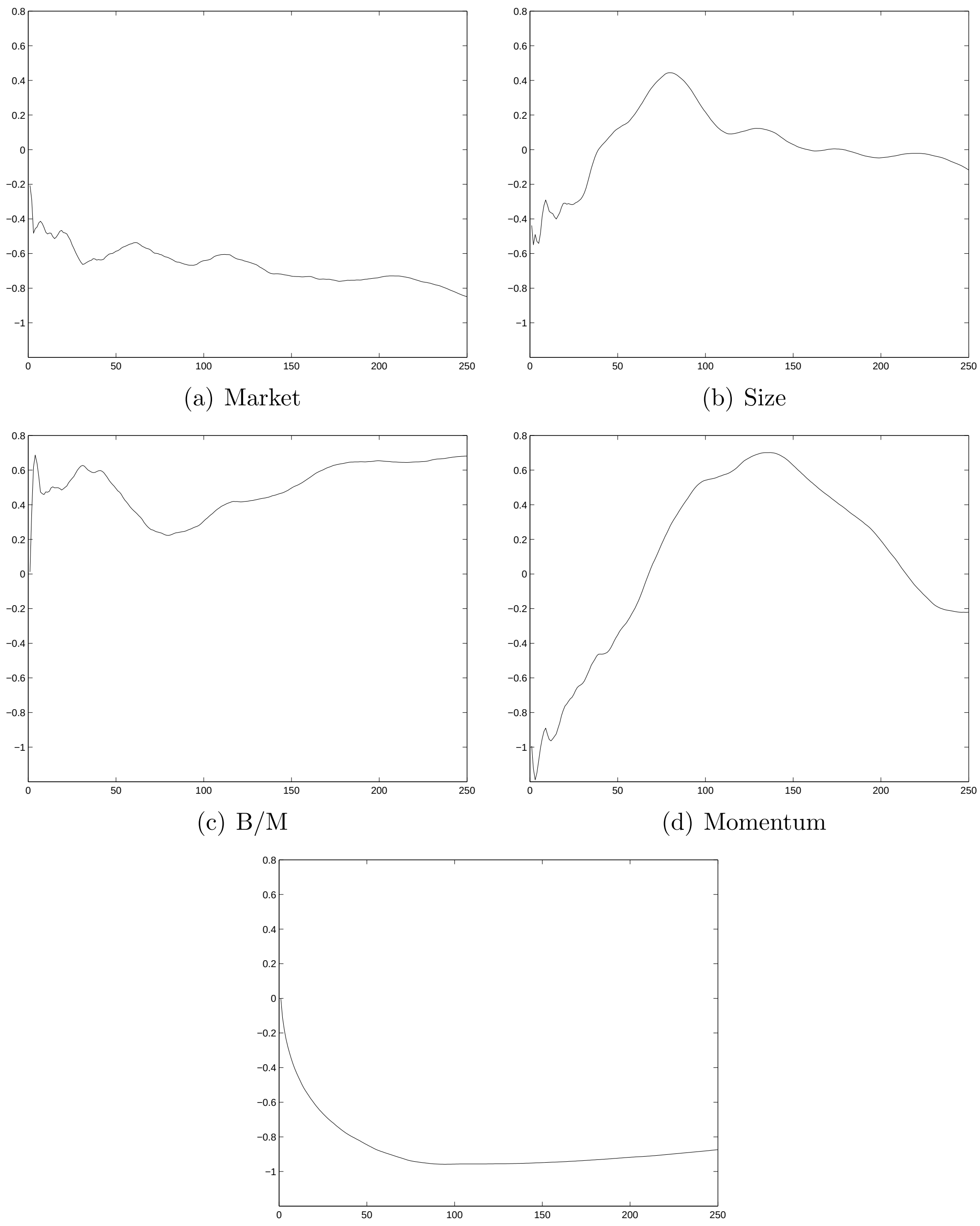

(a) Market

(b) Size

(c) B/M

(d) Momentum

(e) Simulated Market

Figure 1: Average empirical skewness of Fama-French-Carhart factors at varying horizons. Panels (a)-(d) show the skewness of each factor calculated for 1-day up to 250-day returns from 1988-2005. Overlapping returns were used in the calculation. Panel (e) displays the pattern of skewness from a simulated sample of 1 million days using the asymmetric GARCH model of equation (6) and parameters estimated in Table 1.

Table 1: Factor asymmetric volatility estimation. Maximum likelihood estimates are given for estimating equation (6) on daily factor returns from 1988-2005 as provided on Kenneth French's website. The indicator variable I_t is positive if $\varepsilon_t < 0$.

	Market	Size	B/M	Momentum
Mean Equation				
μ	0.0315	0.0013	0.0114	0.0482
t	(2.71)	(0.19)	(2.11)	(8.13)
Variance Equation				
ω	0.0154	0.0045	0.0016	0.0012
t	(10.92)	(6.16)	(5.17)	(4.01)
ε_{t-1}^2	0.0166	0.0605	0.0994	0.1215
	(2.73)	(13.05)	(14.89)	(14.87)
$\varepsilon_{t-1}^2 I_{t-1}$	0.1065	-0.0167	-0.0344	-0.0514
	(13.00)	(-2.52)	(-4.52)	(-6.20)
h_{t-1}	0.9099	0.9318	0.9120	0.9079
	(147.70)	(172.30)	(140.01)	(172.62)
$\log L$	-5582.6	-3371.7	-2508.9	-3321.9

Table 2: Descriptive statistics for firm skewness and characteristics. The sample includes 108,520 yearly observations of firm skewness and characteristics spanning the CRSP/Compustat universe from 1988-2005. Risk-neutral skewness is available only for a sample of 21,146 firm-year observations spanning the Optionmetrics data of 1996-2005. The risk-neutral skewness statistics reported are calculated for this smaller sample.

	Mean	Std Dev	Min	Median	Max
Skewness Measures					
Daily	0.1562	0.5592	-2.9992	0.1348	2.9948
Monthly	0.0658	0.5473	-2.9989	0.0605	2.9689
Quarterly	-0.0037	0.4969	-2.9895	-0.0014	2.8789
*Risk-neutral**	-0.2062	0.3765	-2.8467	-0.2150	6.3440
Firm Characteristics					
R^3	0.0338	0.0629	-0.3092	0.0080	0.6820
Illiq	1.30E-05	3.30E-05	1.18E-10	1.88E-06	0.0028
Volatility	0.5550	0.2701	0.0311	0.5080	1.6389
B/M	0.7164	0.6617	0.0000	0.5612	4.6090
Leverage	0.1342	0.1620	0.0000	0.0697	0.9813
Credit	8.1259	1.5893	0.0000	8.0000	21.0000
Beta	0.7467	0.7508	-8.4952	0.6540	9.0861
Logsize	18.5868	2.1235	9.8255	18.4656	27.1242
MissingCredit	0.8009	0.3993	0.0000	1.0000	1.0000
Numanalyst	2.9226	4.9454	0.0000	1.0000	43.3750

Table 3: Correlation of skewness and firm characteristics. Pearson correlation coefficients are reported below the diagonal whereas Spearman coefficients are reported above. Correlations with risk-neutral skewness are calculated for the subsample for which we have risk-neutral skewness data. All other correlations are calculated for the full sample.

	Daily	*Month*	*Quart*	*RN**	R^3	*Illiq*	*Vol*	*B/M*	*Lev*	*Credit*	*Beta*	*Logsize*	*Miss*	*Num*
Daily	1.000	0.301	0.118	0.081	0.019	-0.002	0.006	0.027	-0.003	0.016	0.047	-0.019	0.042	-0.011
Monthly	0.320	1.000	0.415	0.046	-0.104	0.159	0.074	0.069	-0.017	-0.003	-0.048	-0.166	0.097	-0.155
Quarterly	0.119	0.448	1.000	-0.013	-0.034	0.060	0.048	0.013	-0.021	-0.001	-0.008	-0.050	0.037	-0.051
*RN**	0.065	0.046	-0.009	1.000	-0.227	0.211	-0.089	0.027	0.013	0.035	-0.068	-0.165	0.077	-0.139
R^3	-0.028	-0.073	-0.017	-0.203	1.000	-0.664	-0.286	-0.171	0.024	-0.047	0.688	0.645	-0.349	0.411
Illiq	-0.024	0.068	0.031	0.053	-0.188	1.000	0.304	0.274	-0.037	0.035	-0.448	-0.799	0.420	-0.620
Volatility	-0.004	0.060	0.043	-0.086	-0.216	0.216	1.000	-0.084	-0.202	0.110	0.116	-0.523	0.356	-0.198
B/M	0.033	0.064	0.008	0.017	-0.122	0.187	0.077	1.000	0.256	0.042	-0.236	-0.255	0.019	-0.163
Leverage	0.012	-0.004	-0.015	0.018	-0.023	0.012	-0.102	0.265	1.000	0.118	-0.101	0.072	-0.244	0.054
Credit	0.012	-0.005	-0.001	0.029	-0.123	-0.017	0.074	0.038	0.175	1.000	0.033	-0.063	-0.083	0.011
Beta	0.021	-0.043	-0.002	-0.060	0.383	-0.233	0.087	-0.190	-0.111	0.044	1.000	0.349	-0.109	0.252
Logsize	-0.040	-0.146	-0.041	-0.156	0.539	-0.376	-0.510	-0.307	-0.006	-0.090	0.305	1.000	-0.485	0.620
MissingCredit	0.051	0.089	0.035	0.065	-0.340	0.156	0.332	0.059	-0.175	-0.159	-0.074	-0.507	1.000	-0.321
Numanalyst	-0.050	-0.122	-0.033	-0.123	0.376	-0.205	-0.207	-0.140	0.026	-0.074	0.195	0.645	-0.392	1.000

Table 4: Skewness by firm size decile and by firm R^3 decile. Reported for each decile are mean firm size, R^3, risk-neutral skewness, and realized return skewness at daily, monthly, and quarterly horizons.

Panel A: Skewness by Size Decile

Decile	*Logsize*	R^3	*Daily*	*Monthly*	*Quarterly*	*Risk-neutral**
1	15.2958	0.0027	0.0791	0.1149	0.0004	0.2922
2	16.4537	0.0042	0.1640	0.1569	0.0284	-0.0247
3	17.1150	0.0066	0.1938	0.1593	0.0283	-0.0847
4	17.6665	0.0105	0.2217	0.1397	0.0271	-0.1518
5	18.1857	0.0172	0.2137	0.1076	0.0174	-0.1575
6	18.7247	0.0254	0.1978	0.0682	0.0031	-0.1530
7	19.2952	0.0367	0.1693	0.0224	-0.0218	-0.1877
8	19.9304	0.0490	0.1534	-0.0121	-0.0289	-0.1874
9	20.7692	0.0667	0.1211	-0.0357	-0.0398	-0.1995
10	22.4310	0.1187	0.0478	-0.0630	-0.0514	-0.2602

Panel B: Skewness by R^3 Decile

Decile	R^3	*Logsize*	*Daily*	*Monthly*	*Quarterly*	*Risk-neutral**
1	-0.0041	17.0304	0.1229	0.1215	0.0205	-0.1380
2	0.0006	16.9429	0.1406	0.1382	0.0196	-0.2052
3	0.0022	17.2797	0.1720	0.1287	0.0215	-0.1593
4	0.0049	17.6481	0.1703	0.1095	0.0120	-0.1979
5	0.0093	18.0858	0.1794	0.0877	0.0034	-0.1748
6	0.0164	18.5832	0.1813	0.0642	-0.0050	-0.1966
7	0.0271	19.0573	0.1750	0.0387	-0.0095	-0.1858
8	0.0438	19.5741	0.1607	0.0147	-0.0227	-0.1894
9	0.0734	20.2110	0.1547	-0.0157	-0.0371	-0.2057
10	0.1640	21.4555	0.1050	-0.0292	-0.0397	-0.2484

Table 5: Regression analysis of firm skewness. There are N=108,520 firm-year observations spanning 1988-2005 for the daily, monthly, and quarterly realized skewness regressions. There are N=21,146 firm-year observations spanning 1996-2005 for the risk-neutral skewness regressions. The t statistics reported are based on Rogers (1993) clustered standard errors.

	Daily Skewness			Monthly Skewness			Quarterly Skewness			Risk-neutral Skewness		
Variable	(1)	(3)	(4)	(5)	(7)	(8)	(9)	(11)	(12)	(13)	(15)	(16)
Intercept	0.1647	0.1595	0.4330	0.0873	0.0037	0.6815	0.0009	-0.0450	0.1454	-0.1379	-0.0425	0.6483
t	(73.10)	(27.08)	(11.89)	(40.04)	(0.72)	(21.66)	(0.48)	(-10.24)	(5.27)	(-34.63)	(-4.56)	(11.87)
R^3	-0.2533	-0.2864	-0.1350	-0.6366	-0.4504	-0.0547	-0.1357	-0.0389	-0.0594	-0.8656	-0.9037	-0.1970
	(-8.66)	(-9.42)	(-3.55)	(-24.99)	(-16.63)	(-1.67)	(-5.30)	(-1.46)	(-1.81)	(-28.75)	(-30.06)	(-6.08)
Illiq		-605.11	-778.91		700.28	160.39		334.82	271.40		12261.1	-4584.9
		(-9.85)	(-10.81)		(6.13)	(1.79)		(5.65)	(4.44)		(3.26)	(-1.62)
Vol		-0.0134	-0.0574		0.0723	-0.0113		0.0688	0.0364		-0.1927	-0.1609
		(-1.85)	(-6.10)		(9.88)	(-1.28)		(10.95)	(4.79)		(-15.48)	(-10.00)
B/M		0.0303	0.0229		0.0391	0.0200		0.0000	-0.0057		0.0029	0.0235
		(9.93)	(6.96)		(12.40)	(5.98)		(0.01)	(-2.07)		(0.33)	(2.46)
Logsize			-0.0138			-0.0313			-0.0061			-0.0362
			(-8.47)			(-22.71)			(-5.06)			(-16.26)
Leverage			0.0028			-0.0373			-0.0398			-0.0017
			(0.21)			(-3.02)			(-3.75)			(-0.07)
Credit			0.0022			-0.0040			-0.0012			-0.0012
			(1.67)			(-3.71)			(-1.29)			(-0.90)
Year Dummies	No	No	Yes	No	No	Yes	No	No	Yes	No	No	Yes
R^2	0.0008	0.0030	0.0157	0.0054	0.0118	0.0388	0.0003	0.0024	0.0201	0.0410	0.0545	0.1771