Measuring the Efficiency of an FCC Spectrum Auction*

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October 2009

Abstract

FCC spectrum auctions sell licenses to provide mobile phone service in designated geographic territories. We propose a method to structurally estimate the deterministic component of bidder valuations and apply it to the 1995–1996 C-block auction. We base our estimation of bidder values on a pairwise stability condition, which implies that two bidders cannot exchange licenses in a way that increases total surplus. Pairwise stability holds in many theoretical models of simultaneous ascending auctions, including some models of intimidatory collusion and demand reduction. Pairwise stability is also approximately satisfied in data that we examine from economic experiments. The lack of post-auction resale also suggests pairwise stability. Using our estimates of deterministic valuations, we measure the allocative efficiency of the C-block outcome.

*Fox would like to thank the NET Institute, the Olin Foundation and the NSF, grant 0721036, for financial support. Bajari thanks the National Science Foundation, grants SES-0112106 and SES-0122747 for financial support. Thanks to helpful comments from Christopher Adams, Susan Athey, Lawrence Ausubel, Timothy Conley, Peter Cramton, Nicholas Economides, Philippe Fevrier, Matthew Gentzkow, Philip Haile, Ali Hortacsu, Robert Jacques, Jonathan Levin, Paul Milgrom, Harry Paarsch, Ariel Pakes, Robert Porter, Philip Reny, Bill Rogerson, Gregory Rosston, John Rust, Andrew Sweeting, Chad Syverson and Daniel Vincent. Thanks also to seminar participants at Chicago, CIRANO, Duke, the IIOC, Iowa, Maryland, Michigan, MIT, the NBER, the NET Institute, Northwestern, SITE, UCLA and Washington University. Thanks to Peter Cramton for sharing data on license characteristics, to David Porter for sharing data on experimental auctions, to Todd Schuble for help with GIS software, and to Chad Syverson for sharing data on airline travel. Excellent research assistance has been provided by Luis Andres, Wai-Ping Chim, Stephanie Houghton, Dionysios Kaltis, Ali Manning, Denis Nekipelov, David Santiago and Connan Snider.

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1 Introduction

The US Federal Communications Commission (FCC) auctions licenses of radio spectrum for mobile phone service. Based on the recommendations of academic economists, the FCC employs an innovative simultaneous ascending auction. We study data from the 1995–1996 auction of licenses for the C Block of the 1900 MHz PCS spectrum band. The C block divided the continental United States into 480 small, geographically distinct licenses. A mobile phone carrier that holds two geographically adjacent licenses can offer mobile phone users a greater contiguous coverage area. One intent of auctioning small licenses is to allow bidders, rather the FCC, to decide where geographic complementarities lie. Bidders can potentially assemble packages of licenses that maximize the benefits from geographic complementarities. The US practice of dividing the country into small geographic territories differs markedly from European practice, where often nationwide licenses are issued. These nationwide licenses ensure that the same provider will operate in all markets, so that all geographic complementarities are realized.

Economic theory suggests the allocation of licenses in a simultaneous ascending auction need not be efficient. Brusco and Lopomo (2002) and Engelbrecht-Wiggans and Kahn (2005) demonstrate that bidders may implicitly collude through the threat of bidding wars. For example, a bidder might not add an additional license to a package to take advantage of complementarities because of threats of higher, retaliatory bids on the bidder's other licenses. For auctions of multiple homogeneous items, Ausubel and Cramton (2002) demonstrate that bidders may find it profitable to unilaterally reduce demand for licenses, similar to how a monopolist raises prices and profits by reducing supply. Descriptive empirical evidence supports the concern about intimidatory collusion and demand reduction in FCC spectrum auctions. Cramton and Schwartz (2000, 2002) show that bidders in the AB block did not aggressively compete for licenses and in the later DEF block auction used the last digits of numeric bids to signal rivals not to bid on other licenses.

This paper provides the first structural model of bidding in the FCC spectrum auctions, aside from Hong and Shum (2003), who model bidding for each license in a spectrum auctions as a single-unit auction and therefore do not measure complementarities. Our estimator is based on the assumption that the allocation of licenses is pairwise stable in matches, that is, the exchange of two licenses by winning bidders must not raise the sum of the valuations of the two bidders. In our econometric model, bidder valuations are a parametric function of license characteristics, bidder characteristics, and bidder private values. We build a maximum score or maximum rank correlation estimator like Manski (1975) and Han (1987), where the objective function is the number of inequalities that satisfy pairwise stability. We provide sufficient conditions for the consistency of our estimates and discuss the identification of our model under nonparametric assumptions. We estimate the influence of various bidder and license characteristics on bidder valuations. Finally, we measure the efficiency of the observed allocation of licenses and discuss the implications of our estimates for alternative auction designs.

We spend some time justifying the assumption of pairwise stability in matches only with references to the experimental and theoretical literatures on simultaneous ascending auctions. In terms of the experimental literature, we use data from experimental auctions by Banks et al. (2003), where bidder valuations are known, and show that the outcomes typically come close to satisfying pairwise stability in matches only.

In terms of theory, we analyze the outcomes generated by the equilibria to the simultaneous ascending auction in Brusco and Lopomo (2002) and Engelbrecht-Wiggans and Kahn (2005). Under situations with both symmetric and asymmetric bidders and licenses that we exposit in detail below, the equilibrium outcomes in the theory papers will satisfy pairwise stability in matches only. Undiscovered equilibria may be an issue, but we discuss the equilibrium outcome property of interim efficiency as a reason for bidders to coordinate on these collusive equilibria. Additionally, a version of a model of demand reduction, as in Ausubel and Cramton (2002), satisfies pairwise stability in matches only, although the latter result requires the assumption of straightforward bidding and no complementarities, as in Milgrom (2000).

In terms of the resale market, there was very little resale or swaps of licenses between bidders immediately after the auction, even though such activity was legally permissible and presumably had low transactions costs compared to the value of each license. This lack of immediate resale or swapping suggests the auction was pairwise stable in matches. We note that pairwise stability does not imply that the auction is allocatively efficient. Pairwise stability is a weaker condition than efficiency: efficiency implies pairwise stability but not the reverse. One of our contributions will be to show that estimation can rely only on pairwise stability without ruling out the stronger condition of efficiency.

Our research contributes to the literature on spectrum auctions and the empirical analysis of multiple unit auctions in several ways. First, we estimate a structural model of bidding in spectrum auctions. The existing empirical literature on FCC spectrum auctions is primarily descriptive. McAfee and McMillan (1996) provide an early analysis of the AB auction results. Cramton and Schwartz (2000, 2002) report evidence of attempts at coordination through bid signalling. Ausubel, Cramton, McAfee and McMillan (1997) and Moreton and Spiller (1998) present bid regressions showing evidence for complementarities. The structural approach is useful because it allows the researcher to quantitatively measure components of bidder valuations and the efficiency of the allocation of licenses, if the researcher is willing to formally state identifying assumptions.

Second, our estimator contributes to the literature on the structural estimation of multiple-unit auctions. Hortacsu (2002), Fevrier, Préget and Visser (2009), Wolak (2007), Chapman, McAdams and Paarsch (2006), and Kastl (2009) study divisible good auctions, like those for electricity and treasury bills. To our knowledge, Cantillon and Pesendorfer (2006), who study sealed-bid auctions for bus routes in London, is the only other paper to study auctions of multiple heterogeneous items. Cantillon and Pesendorfer have a tremendous data advantage: data on bids for each possible package of bus routes in a sealed-bid auction. Compared to Cantillon and Pesendorfer, we work with an ascending auction without package bidding and allow for implicit collusion.

All of the above papers specify a particular model of equilibrium behavior and invert a bidder's first-order condition to recover its valuation. For single-agent papers using this strategy, see Donald and Paarsch (1993), Donald and Paarsch (1996), Elyakime, Laffont, Loisel and Vuong (1994), Guerre, Perrigne and Vuong (2000), Athey and Levin (2001), Campo (2006), Flambard and Perrigne (2009), Hendricks, Pinkse and Porter (2003), Bajari and Ye (2003), Jofre-Bonet and Pesendorfer (2003), Athey, Levin and Seira (2008) and Krasnokutskaya (2009).[1] This first-order-condition approach and other approaches using bid data (such as Haile and Tamer, 2003) are not possible in our application because bids may be poor reflections of valuations under intimidatory collusion. None of the above estimators

[1]See Athey and Haile (2008) and Paarsch and Hong (2006) for surveys of some of this material.

are consistent in the presence of implicit collusion.

In addition, our paper contributes to the literature on structural estimation by allowing for a fixed effects model of unobserved heterogeneity in bidder valuations. In previous research, a maintained assumption is that the econometrician observes all publicly available information that the bidders observe. This is obviously very strong. Because bidders have more market experience than an econometrician, we would expect them to have better information. Our approach allows for license-specific fixed effects in valuations. To the best of our knowledge, the only previous paper that allows for unobserved heterogeneity is Krasnokutskaya (2009). However, her approach does not extend to our setting because it relies on the fact that the auction is conducted using sealed bids.

Third, previous methods for structural estimation in auctions identify bidder valuations from final bids submitted in the auction. Theorists such as Crawford and Knoer (1981), Kelso and Crawford (1982), Leonard (1983), Demange, Gale and Sotomayor (1986), Hatfield and Milgrom (2005), Day and Milgrom (2007), and Edelman, Ostrovsky and Schwarz (2007), among others, have pointed out that a one-to-many, two-sided matching game is a generalization of an auction of multiple heterogeneous items. We are the first to use this insight in empirical work. We estimate the deterministic component of valuations as a function of recorded license and bidder characteristics, up to a normalization, based on the match between bidder characteristics and license characteristics. We do not use bid data in our preferred estimator. We argue that the link between bids and bidder valuations may be polluted if intimidatory collusion is present, as is likely in our application. We demonstrate that a closely-related estimator that uses bid data does not yield reasonable estimates of bidder valuations, consistent with the potential biases mentioned above. Because we do not use bid data and because we are somewhat agnostic about the particular equilibrium being played, we focus on estimating systematic or deterministic components of payoffs, namely how valuations relate to observed bidder and license characteristics, including the gains from geographic complementarities. We do not estimate the distribution of license- and bidder-specific private values.

Fourth, the effective size of the choice set for bidders in our application is very large. In our application, there are 480 licenses and, as a result, there are more potential packages of licenses than there are atoms in the universe. Any estimator that relies on a direct comparison of the discrete choice between all potential packages will be computationally infeasible. Our estimator, based on pairwise stability in matches, circumvents this computational difficulty by considering a set of necessary conditions for equilibrium behavior.

Fifth, the true data generating process is a noncooperative, dynamic game. This game has multiple equilibria, including implicitly collusive and competitive equilibrium. Our estimator uses necessary conditions that hold across a set of studied equilibria and does not require us to state the equilibrium being played. We also avoid having to numerically solve for the equilibrium to the dynamic game as part of estimation. This would not be possible, given the indeterminacy of the equilibrium, the huge state space in a simultaneous ascending auction, and the massive choice set of bidders.

Finally, this is the first paper to empirically estimate a two-sided, non-search matching game with transferable utility, except for Dagsvik (2000), Choo and Siow (2006), and Weiss (2007), who work with logit-based specifications applied mostly to one-to-one matching, or marriage. We use the matching estimator in Fox (2009a) but modify the asymptotic argument for consistency to address the reality that we have data from one large auction.[2] As a bidder can win more than one license and we focus on complementarities, we are the first paper to estimate a many-to-one matching game where the payoffs of bidders are not additively separable across licenses (unlike, say, Sørensen (2007)).

In our results section, we find mixed evidence concerning the efficiency of the observed allocation of the licenses. At least since Coase (1959), the use of spectrum auctions has been justified on efficiency grounds. We find that bidders strongly value complementarities between licenses and that bidders with larger initial eligibilities value licenses more. Also, we find that awarding each license to a distinct bidder would dramatically reduce allocative efficiency, justifying spectrum auctions as efficiency enhancing over the prior lotteries regime. However, we find evidence that the observed packages of licenses were too small for an efficient allocation given the presence of complementarities between licenses. Consistent with this finding, many of the bidders in our sample failed to pay for the licenses that they won or were acquired by larger firms. While Congress's stated motivation of the FCC spectrum auction design was that the design should maximize efficiency, it is possible the government had other motivations, such as revenue maximization, in mind. Regardless, efficiency is one criterion and an analysis of efficiency should be a major ingredient in any policy reform.

Our results have implications for future spectrum auctions. Our findings suggest that small license territories, together with the possibility of intimidatory collusion, can generate an inefficient allocation of licenses.

2 Background for the C block auction

2.1 FCC spectrum auctions for mobile phones

Wireless phones transmit on the publicly-owned radio spectrum. In order to prevent interference from multiple radio transmissions on the same frequency, the Federal Communications Commission (FCC) issues spectrum users licenses to transmit on specified frequencies. Wireless phones in the United States transmit on two major regions of radio spectrum. The FCC assigned 800 MHz licenses in the 1980's using comparative-worth regulatory hearings, lotteries, and induced partnerships among applicants. In the 1990's, Congress and the Clinton administration decided the mobile-phone industry could support more competitors, and so the FCC allocated additional spectrum in the 1900 MHz (PCS) block to mobile-phone carriers. The FCC assigned the new PCS spectrum licenses using auctions instead of using lotteries.

There were three initial auctions of mobile-phone spectrum between 1995 and 1997. The first auction (the AB blocks) sold 99 licenses for 30 MHz of spectrum for 51 large geographic regions and raised \$7.0 billion for the US Treasury. The second auction (the C block) sold 493 30 MHz licenses in more narrowly-defined geographic regions to smaller bidders that met certain eligibility criteria. The C-block auction closed with winning bids totaling \$10.1 billion, although some bidders were unable to make payments, and their licenses were later re-auctioned. The third auction (the DEF blocks) sold three licenses for 10 MHz in each of the same 493 markets as the C block. The bids totaled \$2.5 billion in the DEF blocks.

[2]The empirical application in Fox (2009a) was added after the paper was initially circulated. Subsequent uses of the estimator in Fox also postdate early versions of our paper.

There are a number of reasons to prefer to use data from the C block auction instead of the AB or DEF blocks. First, the number of observations is much larger in the C block: there are 255 bidders in the C block compared to only 30 in the AB blocks and 155 in the DEF blocks.[3] Furthermore, there were two licenses for sale for every geographic region in the AB blocks, and three licenses for every geographic region in the DEF blocks. An AB or DEF block bidder was thus guaranteed to be competing directly against at least one other winning carrier after the auction ended. This direct externality in the structural payoffs of bidders complicates the analysis of bidding behavior considerably. In the C block, each geographic region had only one license for sale.[4]

The C-block auction took 184 rounds, lasting from December 1995 to April 1996. Incumbent carriers did not participate in the C block because of discounts offered to small businesses.[5] The discounts fulfilled a Congressional mandate to encourage smaller companies to offer wireless-phone service. Bidding for the C block was more aggressive than in the AB block, with bids (for only half the spectrum sold in the AB blocks) totaling \$10.1 billion. Figure 1 is a map of the licenses won by the top-12 winning bidders. Figure 1 shows that the largest winner in the C block auction was NextWave, whose winning bids totaled \$4.2 billion for 56 licenses, including close to \$1 billion for the New York City license.

2.2 After the auction: mergers

C-block bidders were given an extended payment plan of ten years. Many of the bidders planned to secure outside funding for both their license bids and other carrier startup costs after the auction. Securing licenses first and financing later was an extremely important part of the business plan of what was until the late 1990s the most successful American mobile-phone carrier, McCaw Cellular.[6] This strategy was based on McCaw's (correct) forecast of the revenue potential in mobile phones, which was higher than the forecasts of larger companies (Murray, 2001). It is possible that many of the C-block bidders were trying to recreate McCaw's strategy. With a scarce license, a small-business bidder becomes a relevant player in the mobile-phone industry, and can expect to hold serious discussions with financiers.

[3]Moreover, many of the bidders in the AB and DEF blocks were incumbent mobile phone carriers, and for antitrust reasons were ineligible to bid in geographic markets where they already held licenses. In particular, parties owning more than a 40% interest in an existing wireless license in an area could not bid on another license in that area. Imposing the legal choice set of each bidder creates considerable additional complexity in estimation.

The C block, by comparison, featured only potential new entrants, so all bidders could potentially bid on all licenses. The antitrust policy may have lowered competition in the AB auction (Ausubel et al., 1997; Salant, 1997). The FCC limited any one bidder from winning more than 98 total licenses in the C and F entrepreneurs blocks. Only NextWave came close to meeting this limit. Ausubel et al. (1997) point out that because the limit was in total licenses rather than total population, NextWave had incentives to purchase licenses with the highest total population. Our coming pairwise-stability condition will be consistent with NextWave favoring licenses with more population.

[4]After the auction, winning C-block bidders were much more likely to compete against incumbent mobile-phone carriers operating in the same geographic region than against other C-block bidders.

[5]Plans to give additional advantages to women and minorities were dropped because of litigation. Small-business ownership requirements were not overly strict. Two ownerships structures qualify bidders as small businesses. The first structure is a control group must hold 25% of the business's equity. Of that 25%, 15% (or 3/5) of the equity must be held by qualifying entrepreneurs. Of the remaining 75% of equity, no more than 25% can be controlled by any one entity. An alternative structure says the control group can have 50% of the equity, with 30% being qualifying entrepreneurs. This allows the other 50% to be held by one outside entity, which in effect allows the company to partner with a major firm. The most famous case of partnering is Cook Inlet, an Alaskan-native corporation that partnered with the incumbent carrier Western Wireless.

[6]McCaw was purchased by AT&T for \$17.4 billion and renamed AT&T Wireless in 1993. AT&T Wireless was itself purchased by Cingular in 2004. Cingular was renamed AT&T in 2007.

Compared to McCaw, the C-block winners did not have an early-mover advantage. As it turns out, many C-block winners were unable to meet their financial obligations to the FCC. These new carriers were unable to secure enough outside funding to both operate a mobile-phone company and pay back the FCC. Many C-block winners returned their licenses to the FCC, where they were re-auctioned. Others companies merged with larger carriers (forming a large part of the licenses held by T-Mobile USA, for example) or were able to protect their licenses in bankruptcy court. NextWave is the most famous case of bankruptcy protection. NextWave was eventually able to settle with the FCC and sell some of its licenses to other carriers for billions of dollars. Ex-post, the C block bidders, who were accused of bidding too aggressively at the time, underpredicted the eventual market value of the licenses. However, much of this value was to larger carriers, not small-business entrants who could not secure the financing to operate as a mobile-phone carrier. Twelve years later, in 2007, only a few C-block winners, such as GWI/MetroPCS, remain true independent carriers marketing service under their own brand.

Aside from NextWave, the legal form of most license transfers was a merger, not a resale. Many of these acquisitions of C-block carriers took place years afterward, and many involved one firm that was an incumbent / non-C-block bidder. The most important instance of a merger was the creation of T-Mobile USA in 2001 from the takeover of the existing carrier VoiceStream by Deutsche Telekom as well as mergers with the independent carriers Aerial, Omnipoint and PowerTel.

The resale and merger activity suggests that a bidder's post-auction value for winning licenses was not only a function of the package of territories it planned to serve as a mobile-phone carrier. Valuations might be a function of the bidder's beliefs about the expected value from resale of its licenses, from mergers after the auction and the risk of bankruptcy.[7] Valuations also likely reflect the ability to serve traveling customers through roaming agreements as well as signing up new subscribers directly. Therefore, attempting to directly recover a bidder's value from operating a mobile-phone carrier will be quite naive in this setting. We favor a nuanced interpretation of the estimates from our structural model that encompasses the possibilities of both merger and independent operation.[8]

2.3 Auction rules and bidder characteristics

Similar rules govern all FCC auctions for mobile-phone spectrum. FCC spectrum auctions are simultaneous, ascending-bid, multiple-round auctions that can take more than a hundred days to complete. Formally speaking, a simultaneous ascending auction is a dynamic game with incomplete information. Each auction lasts multiple rounds, where in each round all licenses are available for bidding. During a round, bidding on all licenses closes at the same time. These auction rules were explicitly designed by academic economists to allow bidders to assemble packages exhibiting complementarities, while letting the bidders themselves and not the FCC determine where the true complementarities lie. If bidders have finite valuations, they will cease bidding after a finite number of rounds, although the length of the auction is not known at the start. Package bidding is not allowed; bidders place bids on each license separately.

Each bidder makes a payment before the auction begins for initial eligibility. A bidder's eligibility

[7] The FCC's unjust enrichment regulation penalizes resale to carriers that do not qualify as eligible entrepreneurs.

[8] Our policy application focuses on the geographic size of licenses. Larger licenses might actually make resale easier, as carriers need to negotiate with fewer existing owners to secure coverage. The initial 800-MHz licenses that were handed out using lotteries in the 1980s took over twenty years to consolidate.

is expressed in units of total population. A bidder cannot bid on a package of licenses that exceeds the bidder's eligibility. For example, a bidder who pays to be eligible for 100 million people cannot bid on licenses that together contain more than 100 million residents. Eligibility cannot be increased after the auction starts. During the auction, the eligibility of bidders that do not make enough bids is reduced. By the close of the auction, many bidders are only eligible for a population equal to the population of their winning licenses.

The eligibility payments were 1.5 cents per MHz-individual in a hypothetical license for the C block. Compared to the closing auction prices, these payments are trivial. We use eligibility to control for a bidder's willingness to devote financial resources towards winning spectrum. This paper does not consider strategic motives (such as intimidating rivals) for choosing eligibility levels.

Table 1 lists characteristics of the 85 winning and 170 non-winning bidders in the continental United States.[9] The average winning bidder paid fees to be eligible to bid on licenses covering 10 million people, while the average losing bidder was eligible to bid on licenses covering only 5 million people. Bidders also had to submit financial disclosure forms (the FCC's Form 175) in order to qualify as entrepreneurs for the C block, which was limited to new entrants only. Table 1 shows that the financial characteristics of winners and non-winners are similar, which leads us to believe that these disclosure forms did not represent the true resources of bidders. Hence, in our structural estimator, we use initial eligibility as an individual bidder characteristic instead of assets or revenues.

Table 1 lists the mean number of licenses bid on and won by winners and non-winners. The mean winning bidder won 5 licenses and entered at least one bid on 39 licenses. Although not listed in the table, the top-15 winning bidders, in terms of number of licenses, were active bidders on many licenses. The top-15 winners won an average of 16 licenses and bid on an average of 123 (out of 493) licenses.[10] Most of the major winners and some of the non-winners were investors operating on a national scale. The role of idiosyncratic valuations of licenses due, for instance, to local knowledge seems relatively minor, as the bidding in the C block auction was dominated by national investors that were competing for licenses over the entire country.

2.4 Prices and winning packages

Despite the many potential complications, the C-block auction generated closing bids where the underlying characteristics of licenses explain much of the variation in prices across licenses. The most important characteristic of a license is the number of people living in it, who represent potential subscribers to mobile-phone service. The population-weighted mean of the winning prices per resident is \$40. The second most important characteristic in determining the closing prices was population density. Radio-spectrum capacity is more likely to be binding in more densely populated areas. A regression of a license's winning price divided by its population on its population density gives an R^2 of 0.33.[11] However, prices per resident varied widely across the AB, C and DEF auctions. It is difficult to reconcile

[9] The C block also contains licenses for Alaska and Hawaii as well as Puerto Rico and several other island territories of the United States. The potential for complementarities between these licenses and licenses in the continental United States seems limited, so we restrict attention to the contiguous 48 states.

[10] One of the losing bidders submitted bids on all licenses. This bidder withdrew from the auction because it felt that the prices were too high for its business plan.

[11] Ausubel et al. (1997) use proprietary consulting data on the population density of the expected build-out areas for C block mobile phone service. They have provided us the same data, which we use here.

this across-auction variation with a view that the final bids closely reflect bidder valuations (Ausubel et al., 1997).

Table 2 lists characteristics of winning packages. Only licenses in the continental United States are included in the packages summarized in Table 2. The average winning bidder agreed to pay $116 million and won a license covering 2.9 million people. The largest winner, NextWave, bid $4.2 billion for a package covering 94 million people.

Figure 2 plots the log of a bidder's initial eligibility on the horizontal axis and the log of the package's winning population on the vertical axis. A quadratic fit to the data is also included. The R^2 of the quadratic is quite high, at 0.67. Initial eligibility is predictive of cross-bidder differences in acquired spectrum.

2.5 Suggestive evidence on complementarities

A major justification for the simultaneous ascending auction is that it allows bidders to assemble packages of geographically-adjacent licenses. Such adjacent licenses are said to exhibit complementarities or synergies.

One's prior might be that complementarities are not important in the spectrum auctions. The FCC chose market boundaries to be in sparsely settled areas in order to minimize complementarities across markets.[12] Furthermore, 1900 MHz PCS wireless phone service is mainly deployed in urban areas and along major highways, so there might not even be PCS service along the boundaries of two markets.[13] Finally, companies can coordinate with contracts (roaming agreements) if the same company does not own the adjacent licenses.[14]

However, an initial inspection of our data is compatible with the existence of geographic complementarities. The map of the top-12 winners (by the number of licenses) in Figure 1 shows several bidders win licenses in markets adjacent to each other.[15] For example, NextWave, the largest winner, purchases clumps of adjacent licenses in different areas of the country. GWI/MetroPCS fits the cluster pattern well, winning licenses in the greater San Francisco, Atlanta and Miami areas.

On the other hand, the majority of winning bidders win only a few licenses. Figure 1 emphasizes this by also plotting the 26 licenses in the continental United States that were the only license won by their winning bidders. We calculate that only 20 out of 89 C-block winning bidders won packages of licenses where the population in adjacent licenses within the package was more than 1 million.[16] Aer Force

[12] Bajari, Fox and Ryan (2008) estimate that consumers do have high willingnesses to pay to avoid roaming surcharges while traveling. So there is evidence that economic primitives do justify complementarities in bidders' structural profit functions.

[13] To some extent, PCS licenses are primarily built out in urban areas because the FCC requires build outs to cover a certain fraction of the population of the market, rather than a fraction of the market's land area. 800-MHz carriers tend to cover both urban and rural areas because the FCC requires coverage as a large fraction of the land area of those licenses.

[14] The Coase Theorem suggests that, in a frictionless world, such contracts will implement the efficient outcome. Our paper uses revealed preference to investigate whether bidders thought the Coase Theorem would be operative in the post-auction mobile-phone-service industry. Because we find estimates of strong complementarities, our estimates suggest bidders did not believe the Coase Theorem would allow them to offer low-cost service to consumers through contracts such as roaming agreements.

[15] Ausubel et al. (1997) study in part the earlier AB auction and show several bidders win licenses adjacent to markets where the bidder is a mobile-phone incumbent, or a landline telephone carrier. For example, Pacific Bell, at the time a California telephone company, won AB block licenses in California. Other bidders, such as the forerunners of Sprint PCS and AT&T Wireless, embarked on a strategy of winning licenses in as many markets as allowed.

[16] This complementarity measure is calculated over pairs of licenses. If a license is adjacent to two others in a package,

is the prime example of a top-12 bidder that did not seem overly concerned with complementarities. Figure 1 shows that Aer Force won 12 licenses in the continental United States, but that none of them are adjacent to each other. From the maps alone, it appears some winning bidders cared more about geographic complementarities than others.

Previous researchers have generally concluded that complementarities were important. Ausubel, Cramton, McAfee and McMillan (1997) and Moreton and Spiller (1998) examine whether adjacent licenses exhibited complementarities by regressing the log of winning bids on market and bidder characteristics. Ausubel et al. study the AB and C block auctions and find that the log of winning bids are positively related to whether the runner-up bidders won adjacent licenses, as one might expect in an ascending-bid auction. However, the coefficient in the C-block auction is economically small, meaning that prices do not seem to strongly reflect any value of complementarities. Moreton and Spiller have better measures of incumbency, and also find that winning bids are positively related to the runner-up bidder's measures of complementarities. The results are the most statistically significant for the C block auction. Ausubel et al. and Moreton and Spiller do not claim their price regressions correspond to hedonic estimates of bidder valuations. Rather, they specify descriptive or in-sample prediction regressions designed to summarize facts about the closing bid prices.

The previous authors also discuss scale economies, the notion that a wireless network involves fixed costs that can be spread out among more customers in a larger carrier. Scale economies can be represented by allowing valuations to be a convex function of package characteristics such as total population. However, because bidders with higher valuations (more initial eligibility) win packages with higher populations, it may be hard to empirically distinguish operating scale economies from heterogeneities in bidder valuations.

The map of winners, Figure 1, suggests that the clusters of nearby licenses in winning packages are possibly too small. If bidder valuations were primarily a function of complementarities, we might expect to see the entire southeast won by one bidder, for example.

The fact that many bidders win clusters of licenses, as seen in the map in Figure 1, is suggestive evidence that complementarities matter to some degree. An alternative explanation is that a bidder has correlated license-specific values across licenses in a geographic cluster. Any deterministic measure such as complementarities can be explained away by bidders having correlated private values for nearby licenses. However, the evidence suggests that the largest winners were not local businessmen with special attachments to particular, large regions. Many of the the largest winners, such as NextWave, Omnipoint and GWI/MetroPCS, won small clusters in many regions of the country. MetroPCS has its headquarters in Dallas, but won licenses only near Atlanta, Miami and San Francisco. DCR/Pocket won licenses stretching from Detroit to Dallas, an oddly-shaped region to be a regional specialist in. PCS2000 won mainly a cluster of licenses in the West, but had its headquarters far away in Puerto Rico. Further, Table 1 shows that the typical winning bidder bid on 39 licenses and won 5, for a ratio of around 8. The largest winners bid on many more licenses than that.

Note that we do not view the price regressions of Ausubel et al. (1997) and Moreton and Spiller as a consistent estimator of bidder valuations, for at least two reasons. First, the auction induces an econometric selection problem in the final allocation of licenses to bidders. Winning packages have

its population will be counted twice. The 89 winners include four bidders who won licenses only outside of the continental United States.

high payoffs for some observed or unobserved reasons; otherwise they would not win. As both bidder- and license-specific valuations and complementarities across licenses contribute to total payoffs, those packages with relatively low complementarities will have relatively high bidder- and license-specific valuations. As the bidder- and license-specific valuations are typically not observed and are related to the error term in the price regression, there will be correlation between the complementary proxies and the error terms in the price regression. Linear regression will thus be inconsistent.

Even if winning packages' complementarities were somehow uncorrelated with winning packages' bidder- and license-specific valuations, the estimator would still be inconsistent. Under intimidatory collusion as discussed in the next subsection, prices will not reflect valuations and so price regressions will not identify structural parameters. In order to interpret price regression as estimates of structural parameters, one would need to assume that the outcome to the auction is equivalent to a competitive equilibrium to a decentralized auction. We formally define a competitive equilibrium below as Definition 3.7. We will base estimation on a weaker assumption than Definition 3.7.

2.6 Suggestive evidence about intimidatory collusion

Milgrom (2000, Theorems 2,3) proves that a simultaneous ascending auction is equivalent to a tatonnement process that finds a competitive equilibrium of the economy, under two assumptions: 1) the licenses are mutual substitutes for all bidders, and 2) all bidders bid straightforwardly. Unfortunately, neither one of the assumptions needed to prove that a simultaneous ascending auction is efficient appear to hold in the C-block data. We have already discussed suggestive evidence that there may be complementarities in bidders' valuations for multiple licenses.

Bidding straightforwardly means that a bidder submits new bids each period in order to maximize its structural profit function, rather than some other continuation value in a dynamic game. One violation of straightforward bidding is jump bidding. When making a jump bid, a bidder enters a bid that exceeds the FCC's minimum bid for that round. We define a jump bid to be any bid that is 2.5% greater than the FCC's minimum bid for that license and round. Figure 3 shows that there was a non-trivial level of jump bidding during the C block auction.

When jump bidding, a bidder risks the chance that the jump bid will exceed the valuation of rival bidders, and be the final price. A jump bidder therefore has a nonzero probability of overpaying for a license.[17] However, there are possible strategic advantages from jump bidding. In a single unit, affiliated values model, Avery (1998) demonstrates that jump bidding may signal the jump bidder's intentions to bid aggressively throughout the auction. Because other bidders fear the winner's curse, they may discontinue bidding in order to avoid overpaying conditional on winning the item.[18]

Figure 3 shows jump bidding was prevalent towards the beginning of the auction, where the risk of overpaying is much lower. The number of total new bids dramatically slowed during the second

[17] Daniel and Hirshleifer (1998) study a model of an ascending auction of a single item where bidding is costly. An equilibrium involves jump bidding to speed the conclusion of the auction. As bids in the C block auction totalled $10.1 billion, it is unlikely that bidders had a high opportunity cost of time and placed jump bids to speed the conclusion of the auction, at least towards the end of the auction. Serious bidders employed teams of professionals to manage bidding activity.

[18] Theorists have shown that jump bidding can happen for non-collusive reasons. For example, in a simultaneous ascending auction where the bidding on each license can close at different times, Zheng (2006) shows that jump bidding can alleviate the exposure problem mentioned below. The FCC auction rules have a more direct withdrawal mechanism to mitigate the exposure problem.

half of the auction, and this slowdown is especially severe for jump bids. The presence of jump bids might represent signals that are attempts at intimidation, but jump bids are not evidence the signals successfully caused other bidders to withdraw. There are anecdotes of actual retaliation. In round 3, Pocket (DCR) placed a large jump bid of 60% more than the minimum for Las Vegas. In round 70, MetroPCS (GWI) outbid Pocket for Las Vegas and PCS2000 for Reno. In round 71, Pocket outbid MetroPCS on Reno and Salt Lake City, the only time Pocket bid on either of those licenses. Further, PCS2000 outbid MetroPCS on Las Vegas, the only time since round 12 PCS2000 had bid on Las Vegas. In round 72, after seeming to retaliate against MetroPCS, Pocket enters the winning bid for Las Vegas, meaning the bid stands until the end of the auction, round 184.

There are other anecdotes of intimidation that do not involve jump bids. Towards the end of the auction, NextWave and AerForce were competing for Fredericksburg, Virginia. NextWave needed Fredericksburg to complete a regional cluster around Washington, DC. In round 162, NextWave outbid AerForce for Fredericksburg. In round 163, AerForce responded not only by bidding on Fredericksburg but also by bidding on Lakeland, Florida. Lakeland is a small population territory that AerForce had not bid on in a long while and that NextWave had been winning. In round 164, NextWave bid again and retook Lakeland, but never bid again on Fredericksburg. By challenging AerForce on Fredericksburg, NextWave only succeeded in paying 10% (two bid increments) more to win Lakeland.

Cramton and Schwartz (2000, 2002) provide many more examples of signalling and implicit collusion through intimidation, especially in the auctions for the AB and DEF blocks. We feel the evidence is strong enough that any estimation method for spectrum auction data must be based on conditions that hold in the presence of this type of implicitly collusive behavior.

3 Pairwise Stability in Matches Only

In this section, we define the economic environment, the equilibrium allocation and prices, and a necessary condition that will hold in the most relevant theoretical models of spectrum auctions in the literature. Our goal is to motivate the coming pairwise stability in matches only condition through experimental evidence, evidence about resale after the auction, and theoretical analysis based on the results in several papers. Unlike the usual approach in the empirical auctions papers cited in the introduction, we will not restrict to attention to a single data generating process because of the complexity of FCC spectrum auctions. Rather, we will investigate to what extent the pairwise stability in matches only condition holds across several possible data generating processes. Along the way, we will discuss many complexities in real-life FCC spectrum auctions that might be assumed away in a purely theoretical econometrics paper about identification and estimation in simultaneous ascending auctions of multiple heterogeneous items. We will not claim that our estimation strategy is robust against all possible real-world complexities in spectrum auctions. Rather, we will argue that it is robust against some of these complications and performs relatively well in the presence of other complications.

3.1 Bidders' profit functions

There are $a = 1, ..., N$ bidders and $j = 1, ..., L$ licenses for sale. We will abuse notation and let N be the set of all bidders and L the set of all licenses. Our environment is a multiple-unit auction where bidders may win a package of licenses. We let $J \subset L$ denote such a package of licenses. In the C block, the licenses are permits to transmit mobile-phone signals in specified geographic territories and there is only one license per territory. There were $N = 255$ registered bidders in the C block and 493 licenses for sale. We will limit attention to the $L = 480$ licenses for sale in the continental United States and mostly to the $H = 85$ winning bidders in the continental US.

In the model, bidder a maximizes its profit

$$\pi_a(J) - \sum_{j \in J} p_j$$

from winning package J at prices $\{p_j\}_{j \in J}$. Bidder a's profit is comprised of two parts. The term $\pi_a(J)$ is a's **valuation** for the package of licenses J and $\sum_{j \in J} p_j$ is the price that a pays for this package. In our application, we will parameterize the valuation $\pi_a(J)$ as

$$\pi_a(J) = \bar{\pi}_\beta(w_a, x_J) + \sum_{j \in J} \xi_j + \sum_{j \in J} \epsilon_{a,j}. \tag{1}$$

The scalar function $\bar{\pi}_\beta(w_a, x_J)$ takes as arguments the characteristics w_a of bidder a and the characteristics x_J of the package of licenses J. The function $\bar{\pi}_\beta$ is parameterized by a finite vector of parameters β. Later β will be the object of estimation.

Let y_j be the vector of characteristics of license j. The characteristics x_J of a package J are formed by $x_J = \zeta(Y)$ from the $J < \infty$ license characteristics in $Y = \{y_1, \ldots, y_J\}$. The function ζ is known to the researcher. In our application, the vector x_J will include the total population covered by the licenses J and measures of geographic or travel complementarities between the licenses in package J. The scalar w_a will be bidder a's initial eligibility, as discussed in Section 2.3.[19] x_J is public information to the bidders and observed by the econometrician.

We now list a series of assumptions. These assumptions will be referenced in a series of remarks about the robustness of the theoretical results in the papers Brusco and Lopomo (2002) and Engelbrecht-Wiggans and Kahn (2005), below. We first assume w_a is private information.

Assumption 3.1. *The scalar bidder heterogeneity w_a is private information: known only to bidder a. Each w_a is in the data.*

Making w_a private simplifies some of the analysis below, as Remark 3.13 argues. Privately observed values is the natural case to start with in developing an estimator for simultaneous ascending auctions of multiple heterogeneous items under implicit collusion. However, the assumption of private information is strong for the C block because w_a is in the data and was disclosed prior to the auction. In an alternative assumption, we can allow w_a to be commonly observed information if we make the following functional form assumption about how w_a enters profits.

Assumption 3.2. *In contrast to Assumption 3.1, the scalar bidder heterogeneity w_a is public information. Further, $\bar{\pi}_\beta(w_a, x_J) = h_\beta(w_a) \cdot \bar{\pi}^1_\beta(x_J) + \bar{\pi}^2_\beta(x_J)$, where $h_\beta(\cdot)$ is a monotone function, and $\bar{\pi}^1_\beta$*

[19] Each individual territory is a heterogeneous item. In spectrum auctions, bidders cannot bid on packages, and all bidders do not necessarily place bids on all territories. Therefore, our only feasible estimation strategy is to make $\bar{\pi}_\beta(w_a, x_J)$ a function of the observable characteristics of package J and bidder a. Our maximum score estimation strategy will not permit us to estimate a random-effect distribution of unobserved bidder valuations; we need data on w_a.

and $\bar{\pi}_\beta^2$ are unrestricted functions of x_J. Each w_a is in the data.

A model where w_a is public information is a model with bidders with known asymmetries. See Remarks 3.13, 3.18 and 3.19 below for discussion about the need for this assumption, which relates to implicit collusion in particular models. Note that regardless of whether w_a is private information or public information, w_a is a private value in the sense of Milgrom and Weber (1982) given that rival bidders would not update their own valuations if they learned the value w_a.

We next turn to the ξ_j terms.

Assumption 3.3. *The term ξ_j is a license j fixed effect, which we assume is publicly observed by the bidders. ξ_j may be statistically dependent with y_j and hence x_J, for $j \in J$. Each ξ_j is not in the data.*

The fixed effect enters bidders' valuations additively and is meant to capture the characteristics of license j that are observed by the bidders, but which are unobserved to the econometrician. For example, we lack controls for the incumbent mobile phone companies as well as the winners of the earlier AB auctions and potential merger and roaming partners. However, including ξ_j allows us to account for this license-specific unobserved heterogeneity. As is standard in fixed effect models, we cannot identify the effects of elements of x_J that are collinear with the fixed effects ξ_j. We can, however, identify β in $\bar{\pi}_\beta(w_a, x_J)$, which captures the interaction between the bidder and license characteristics observed by the econometrician.[20] We will not estimate the fixed effects or their distribution, but our estimator is consistent in their presence. Specifying a random effect distribution for the ξ_j would be yield an inconsistent estimator if ξ_j is statistically dependent with y_j in the true data generating process. Further, another estimator that, say, modeled a bidder deciding which licenses to bid on at the current round's prices, would be inconsistent if the current prices were correlated with the fixed effect.

The unobservables, $\epsilon_{a,j}$, reflect bidder a's private information about license j. We use the independent-private-values framework, which is the standard in both the empirical auctions literature (for single or divisible items) and in the theoretical literature on simultaneous ascending auctions.

Assumption 3.4. *The $\epsilon_{a,j}$ are i.i.d. across bidders and licenses and are independent of all w's, x's and ξ's. Each $\epsilon_{a,j}$ is privately observed by bidder a, but the distribution of $\epsilon_{a,j}$ is common knowledge among the bidders. Each $\epsilon_{a,j}$ is not in the data.*

These reflect bidder-specific costs and benefits from operating in a particular territory. As we will briefly discuss, our approach of using necessary conditions for estimation will not allow us to identify the distribution of the $\epsilon_{a,j}$'s. For the C block, the trade press and the number of licenses bid on by each bidder suggest that many winning bidders were willing to operate in any region of the country. This suggests the variance of $\epsilon_{a,j}$ is small. A small variance of $\epsilon_{a,j}$ contrasts with the AB blocks, where many bidders were incumbents trying to win territories near their existing service areas.

[20] The term ξ_j plays a similar role to the omitted product attributes in the model of Berry, Levinsohn and Pakes (1995). The prior empirical auctions literature typically assumes that there is no unobserved heterogeneity about the objects for sale, i.e. $\xi_j = 0$ for all j. Ignoring ξ_j in this fashion will likely generate biased estimates of bidder valuations. We anticipate that the valuation for elements of x_J that are positively (negatively) correlated with ξ_j will be biased upwards (downwards). The only prior paper to account for unobserved item heterogeneity in auctions is Krasnokutskaya (2009), who considers the case of first price, asymmetric auctions and shows that correcting for unobserved heterogeneity changes the results substantially. We extend the literature by accounting for unobserved item heterogeneity in a multiple unit auction.

Almost all estimators for even auctions of a single item are for the case of private values in the sense of Milgrom and Weber (1982). Models with common values are fundamentally unidentified. Further, the theoretical literature on the simultaneous ascending auction uses the assumption of private values. Because both econometric estimators and theoretical analyses use only private values, we focus on the case of private values as well. Under private values, bidders would not revise their own valuations if they were to observe the private information of rivals. For the C block, there is some evidence that some bidders stuck to their private evaluations of the value of wireless service and did not update their valuations. The bidder with the second-highest initial eligibility won no licenses because the prices exceeded that bidder's evaluation of the profit potential from wireless services.

In a common values model, ξ_j might be unobserved to the bidders as well. If a bidder were able to learn ξ_j, they would revise their own valuations. Again, common values are usually not part of formal models of spectrum auctions because of technical complexity. However, part of the benefit of a simultaneous ascending auction and the eligibility rules is that bidders disclose information about the value of licenses through bidding. Therefore, at the end of the auction a lot of information about ξ_j has been disclosed, possibly mitigating any winner's curse (Hong and Shum, 2003).

3.2 Pairwise stability and other properties of auction outcomes

A spectrum auction is a data generating process M that takes as arguments the valuations $\{\pi_a(J)\}_{a=1,\ldots,N}^{J\subseteq L}$ and bidder characteristics $\{w_a\}_{a=1,\ldots,N}$.[21] This data generating process produces a vector of prices for each license $p^L = (p_1, \ldots p_L)$ and an allocation of licenses $A = \{J_1, \ldots, J_N\}$. We let $J_a \subset L$ denote the package of licenses won by bidder a. We denote the econometric data generating process M by:

$$\{p^L, A\} = M\left(\{\pi_a(J)\}_{a=1,\ldots,N}^{J\subseteq L}, \{w_a\}_{a=1,\ldots,N}\right).$$

Below, we discuss several pieces of evidence for why simultaneous ascending auctions satisfy one set of necessary conditions. We first present the necessary condition, as well as several alternatives for comparison.

As discussed in the introduction, an auction of multiple heterogeneous items can be viewed as a one-to-many, two-sided matching game. The two sides of the market are bidders and licenses. An item for sale can be won by only one bidder, but a bidder can win multiple items. The exclusivity of each item in an auction makes bidders rivals to match with the item. The results from Section 2 suggest there is important information about valuations that is contained in which bidders win which licenses. For example, the clustering of licenses in Figure 1 suggests that complementarities in licenses may be important. Table 1 and Figure 2 show that bidders with higher initial eligibilities win more licenses. This is consistent with bidders with higher eligibilities having higher valuations for licenses (recall that price per unit of population was fairly uniform across licenses in our sample). We will now define sets of conditions on the match between bidders and licenses that are necessary conditions for some models of spectrum auctions.

[21] In a full specification of a Bayesian Nash game, one would specify all agents' beliefs and possibly other objects as entering the data generating process. We exclude those belief distributions as arguments for conciseness: to focus on the objects of econometric analysis.

Definition 3.5. The outcome $\{p^L, A\} = \{p^L, \{J_1, \ldots, J_N\}\}$ is a **pairwise stable outcome in both prices and matches** if, for each bidder $a = 1, \ldots, N$, corresponding winning package $J_a \subset L$, and licenses $i \in J_a$ and $j \notin J_a$, $j \in L$,

$$\pi_a(J_a) - p_i \geq \pi_a((J_a \backslash \{i\}) \cup \{j\}) - p_j. \tag{2}$$

In the above definition, at the closing prices p^L, bidder a must not want to swap one of its winning licenses i for some other bidder's license j.[22] Keep in mind that private values $\epsilon_{i,j}$ are included in the definition of $\pi_a(J_a)$.

Pairwise stability in prices and matches may not hold under common models of simultaneous ascending auctions. We also consider the following definition for a stable outcome in matches only. A statistical version of this property, to be defined below, will be the main condition we use for identification and estimation.

Definition 3.6. An allocation of bidders to licenses $A = \{J_1, \ldots, J_N\}$ is a **pairwise stable outcome in matches only** if, for each pair of winning bidders $a \in N$ and $b \in N$, corresponding winning packages $J_a \subset L$ and $J_b \subset L$, $J_a \cap J_b = \emptyset$, as well as licenses $i_a \in J_a$ and $i_b \in J_b$,

$$\pi_a(J_a) + \pi_b(J_b) \geq \pi_a((J_a \backslash \{i_a\}) \cup \{i_b\}) + \pi_b((J_b \backslash \{i_b\}) \cup \{i_a\}). \tag{3}$$

Pairwise stability in matches only considers swapping licenses: the total surplus of two bidders must not be increased by an exchange of one license each.[23] Unlike Definition 3.5, the above definition does not involve the prices of the licenses. Again, this definition of pairwise stability in matches only includes econometric unobservables such as private values in the definition of bidder valuations.

Pairwise stability in both prices and matches implies pairwise stability in matches only. Adding the inequality

$$\pi_b(J_b) - p_j \geq \pi_b((J_b \backslash \{j\}) \cup \{i\}) - p_i$$

to (2) cancels the license prices and gives (3). Thus, we will focus on the weaker of the two conditions. One way to motivate pairwise stability in matches only is to say that bidders would not want to exchange licenses along with side payments, at the end of the auction. This is a true mathematical interpretation of pairwise stability in matches only, and we will use the lack of resale after the auction to suggest that the outcome may have been pairwise stable in matches. However, when examining the output of theoretical models of bidding, we will not rely on bidders exchanging licenses with side payments.

[22] A key difference from our definition of pairwise stability in prices and matches and the most common definition in the matching literature is that we do not impose a nonnegative profit condition:

$$\pi_a(J_a) - \sum_{i \in J_a} p_i \geq 0.$$

This nonnegativity condition provides little information about geographic complementarities. Further, the condition may be violated because of the exposure problem in simultaneous ascending auctions. We discuss the exposure problem below.

[23] One could strengthen Definition 3.6 to consider exchanges of bundles of two or more licenses between each of two bidders. This notion could be called "two bidders, two bundles stability." This is a stronger condition as it implies Definition 3.6. The rest of the section focused on motivating Definition 3.6 and not "two bidders, two bundles stability." Given the lack of motivation and the desire to use weaker rather than stronger assumptions in estimation, we focus on Definition 3.6. In a previous draft with a slightly different specification for the profit function, we did estimate a model using inequalities derived from exchanges of bundles of two licenses for each bidder, and found that the point estimates were quite similar to the estimates based on Definition 3.6.

Rather, certain noncooperative equilibria to dynamic games will end up being pairwise stable in matches only, even if they are not pairwise stable in both prices and matches.

For comparison purposes, we also define two conditions that are stronger than the conditions above.

Definition 3.7. The outcome $\{p^L, A\} = \{p^L, \{J_1, \ldots, J_N\}\}$ is a **competitive equilibrium** whenever

$$J_a = \arg\max_{J \subseteq L} \left\{ \pi_a(J) - \sum_{j \in J} p_j \right\}$$

for all bidders $a \in N$.

By competitive equilibrium, we mean the prices in the data generating process are such that, in a hypothetical decentralized market for licenses that uses posted prices, the posted prices would assign licenses to bidders in the same way as the auction does. A competitive equilibrium is a stronger condition than pairwise stability in both prices and matches. Milgrom (2000) gives strong assumptions under which a simultaneous ascending auction gives an outcome that is a competitive equilibrium. His conditions are not satisfied in our standard setup.

Definition 3.8. An assignment of bidders to licenses $A = \{J_1, \ldots, J_N\}$ is **efficient** whenever

$$\sum_{a \in N} \pi_a(J_a) \geq \sum_{a \in N} \pi_a(J'_a)$$

for all other partitions $\{J'_1, \ldots, J'_N\}$ of L, where a partition satisfies $L = \bigcup_{a=1}^{N} J'_a$ and $J_a \cap J_b = \emptyset \; \forall \; a, b \in N$.

This notion of allocative efficiency exploits the additive separability of profits in bids. Efficiency is a stronger condition than pairwise stability in matches only. Let a and b be two winning bidders. It may be efficient for a to win all the licenses, $J_a \cup J_b$. Pairwise stability in matches only simply says an equal exchange of one license each does not raise the sum of valuations for the two bidders.

The definition of efficiency uses knowledge of the private values $\epsilon_{a,j}$ and, if some licenses are not allocated to bidders, fixed effects ξ_j. Our estimation strategy will not recover estimates of the distributions of these unobservables, as we discuss below.[24] When we turn to measuring efficiency at the end of the paper, we will use the following measure of efficiency, which focuses on the contribution to valuations arising from observed license (package) and bidder characteristics.

Definition 3.9. An allocation of bidders to licenses $A = \{J_1, \ldots, J_N\}$ is **deterministically efficient** whenever

$$\sum_{a \in N} \bar{\pi}_\beta(a, J_a) \geq \sum_{a \in N} \bar{\pi}_\beta(a, J'_a)$$

for all other partitions $\{J'_1, \ldots, J'_N\}$ of L, where a partition satisfies $L = \bigcup_{a=1}^{N} J'_a$ and $J_a \cap J_b = \emptyset \; \forall \; a, b \in N$. Likewise, $\sum_{a \in N} \bar{\pi}_\beta(a, J'_a)$ for some partition $\{J'_1, \ldots, J'_N\}$ is a cardinal (non-ordinal) **measure of deterministic efficiency**.

[24] If all bidders have symmetric bidder- and license-specific private-value distributions, auctioning smaller licenses would maximize the values of the private values. In Section 2.5, we argued that we believe these private values have a relatively small dispersion as most winning bidders were national bidders.

Note that we could base our empirical work on efficiency. Often observers casually study the map of winning bidders' packages in a spectrum auction, Figure 1 for the C block, and notice a clustering pattern. If one moves from the map to thinking about complementarities being important, often one is thinking about efficiency being the reason these complementarities are realized. Our estimator will use Figure 1, the winning packages, as its dependent variable. Intuitively, our estimator will measure the importance of clustering patterns and other patterns on the map. Our insight is that this way of looking at the map can yield a consistent estimator under pairwise stability in matches only, which is weaker than efficiency. Indeed, Fox (2009b) proves that nonparametric identification of $\bar{\pi}(w_a, x_J)$ can occur equally as well with the conditions from pairwise stability as the conditions from efficiency (or the core). We return to nonparametric identification in Section 5.5. First we will discuss examples of important models of spectrum auctions where the outcome is inefficient but does satisfy pairwise stability in matches only.

Efficiency implies pairwise stability in matches only. Pairwise stability in matches only is a relatively weak condition that allows but does not impose the stronger conditions of efficiency, a competitive equilibrium and pairwise stability in both prices and matches. The rest of this section motivates why the spectrum auction outcome may be pairwise stable in matches only.

3.3 Experimental evidence on pairwise stability

Banks, Olson, Porter, Rassenti and Smith (2003) conducted experimental evaluations of the FCC simultaneous ascending auction, at the behest of Congress and the FCC. The authors assigned profit functions to subject bidders and let the winning subjects keep their profits. A key advantage of experimental data is that the profit functions of bidders are experimentally induced and hence observed by an econometrician. Hence we can test directly whether the auction outcome satisfied pairwise stability in matches only. Bajari and Hortaçsu (2005) previously used experimental evidence to evaluate the outputs of structural estimators in auctions of a single item.

Banks et al. consider 52 auctions, each with 10 licenses for sale and between 6 and 8 bidders. In some cases, bidder profit functions exhibited complementarities between some subset of the 10 licenses, and other times bidder valuations were additive across licenses. The valuations of bidders in the same auction varied greatly. The experiments we analyze used standard FCC eligibility rules. The authors generously shared the data from the experiments with us. We looked at the final allocation of licenses to bidders and well as the closing prices. Given knowledge of the bidder profit functions in each experiment, we calculate whether the outcomes satisfied pairwise stability in matches only as well as pairwise stability in both licenses and matches.

Table 3 gives the results of our analysis of the experimental evidence. Within each auction, we analyzed each pair of licenses won by different bidders. We checked whether Definition 3.6, pairwise stability in matches only, holds for each pair of licenses. We calculate the percentage of the inequalities that are satisfied within each auction; the first column in Table 3 reports the mean of this percentage across the 52 auctions. The mean auction had 95.1% of its inequalities formed by the exchange of licenses between two winning bidders satisfy Definition 3.6. No economic model is a perfect approximation of reality; we feel that the approximation of Definition 3.6 to outcomes to these experimental auctions is high.

More ambitiously, one might be interested in the fraction of auctions where the restrictions fit the data perfectly: 100% of theoretically valid inequalities are satisfied. Table 3 shows that this number is 55.8%, or 29 out of the 52 auctions, for pairwise stability in matches only. In more than half of the experiments, the restrictions of pairwise stability in matches only are completely satisfied. We feel that Table 3 validates the use of Definition 3.6 in structural empirical work, as no model can fit the data perfectly.

We repeat the same exercises for pairwise stability in both matches and prices, which is Definition 2. Not surprisingly, the mean percentage of satisfied inequalities across the 52 auctions is lower than before, at 88%. Also, only 9.6% (5 out of 52) of the auctions satisfy Definition 2 perfectly: prices are such that bidders would prefer the licenses they won over alternative licenses. Thus, pairwise stability in matches only has more experimental evidence in its favor than pairwise stability in both matches and prices.[25]

3.4 Lack of Resale After the Auction

After the auction, reports in the trade media and government records indicate there was very little resale activity. Few if any bidders swapped licenses or sold licenses to other bidders. These transfers or exchanges would have been legal: the FCC's unjust enrichment rules penalized transfers only to bidders that were not qualified for the C block, not transfers to other C-block bidders. If the outcome was not pairwise stable in matches only, we believe bidders would have exchanged licenses (and perhaps money) in order to restore pairwise stability in matches only. While any negotiation is costly, the total bids in the C-block auction were more than $10 billion, suggesting that negotiation time would be a small cost to incur in order to improve the assignment of bidders to licenses.

Cramton (2006) interprets the lack of immediate, post-auction resale as evidence that the C-block auction's outcome is efficient, a stronger condition than pairwise stability in matches only. We think Cramton's interpretation is too strong. During the ten-year period after the auction, many of the C-block bidders were involved in mergers to create the large, national mobile phone carriers of today. Most of these mergers were with companies that did not directly bid in the C-block auction. Fox and Perez-Saiz (2006) describe some of these mergers and show that they were primarily designed to expand the geographic coverage area of providers. The revealed preference of C-block bidders to participate in mergers to increase scale is evidence that the winning packages may have been too small. Mergers are a much costlier form of license adjustment than exchanges, and it is possible an outcome could be pairwise stable in matches only but inefficient, due to an inefficiently small scale for most winning bidders. Consolidation may increase structural profits, but swapping licenses does not.

3.5 The models of Brusco and Lopomo (2002) and Engelbrecht-Wiggans and Kahn (2005)

Section 2.6 presented suggestive evidence that bidders might have been implicitly colluding through the auction mechanism. We referred to both bidding anecdotes in the C-block and previous, non-structural

[25] In related experiments, Kwasnica and Sherstyuk (2007) conduct experiments with the simultaneous ascending auction. They find that bidders are able to collectively select equilibria that share many of the features of the Brusco and Lopomo (2002) and Engelbrecht-Wiggans and Kahn (2005) equilibria.

empirical work by other authors on other FCC spectrum auctions. Based on this evidence, we are not ready to conclude that there was definitely collusion. We would not want to impose in estimation that the outcome was inefficient, and we will not. However, we believe the evidence in favor of implicit collusion is strong enough that any structural estimator for spectrum auction data should be consistent under the only models of implicit collusion in simultaneous ascending auctions in the literature.

Brusco and Lopomo (2002), or BL, and Engelbrecht-Wiggans and Kahn (2005), or EK, present models of simultaneous ascending auctions that in many cases allow for implicit collusion between bidders. A common theme will be that BL's and EK's examples of auctions with private values that are independent across items for sale will often satisfy pairwise stability in matches only.

3.5.1 Base analysis

Let there be two bidders indexed by a and two licenses. Let each bidder a have a (privately observed) payoff π_a^1 for license 1, π_a^2 for license 2, and $\pi_a^{1,2} = \pi_a^1 + \pi_a^2 + k_a$ for licenses 1 and 2 for some $k_a > 0$. The vector $\left(\pi_a^1, \pi_a^2, k_a\right)$ is drawn independently across the two bidders from the support $[0,1]^2 \times \left[\underline{k}, \overline{k}\right]$. Further, the marginal distributions of π_a^1 and π_a^2 are identical. This assumption will be relaxed. Note that there is not any additional restrictions on the joint distribution of π_a^1 and π_a^2. The equilibrium notion used in both BL and EK is symmetric, perfect Bayesian equilibrium.

BL's paper is motivated by the simultaneous ascending auction used by the FCC and the empirical evidence suggestive of collusion in that auction. The auction rules approximate those of the FCC: each discrete round t a bidder either remains silent or can raise the highest bid of the previous round by any positive amount. There is no minimum bid increment. Let $-\infty$ represent not bidding for one of the two objects. BL first study the case of $\underline{k} = \overline{k} = 0$, or no complementarities. The main equilibrium for the case without complementarities is their Proposition 2, which we now restate.

Theorem 3.10. *Under BL's Condition A on the marginal distributions of π_a^1 and π_a^2, the following strategy, together with some consistent belief system, forms a symmetric, perfect Bayesian equilibrium:*

In the first round, all types $\left(\pi^1, \pi^2\right)$ such that $\pi^1 \geq \pi^2$ open with bids of $\{0, -\infty\}$ on the two licenses. All types such that $\pi^1 < \pi^2$ open with bids of $\{-\infty, 0\}$ on the two licenses.

In the second and subsequent rounds, if the initial bids are $(\{0, -\infty\}, \{-\infty, 0\})$ or $(\{-\infty, 0\}, \{0, -\infty\})$, no types place more bids. If the initial bids are $(\{0, -\infty\}, \{0, -\infty\})$, then all types $\left(\pi^1, \pi^2\right)$ such that $\Delta\pi = \pi^1 - \pi^2$ keep raising their bid on license 1 while refraining from bidding on license 2 until either i) the opponent stops, or ii) the bids reach the value $\Delta\pi$. In case i), the bidders places no more bids for the next two rounds. In case ii), the bidder bids $\{-\infty, 0\}$ on license 2, winning that license. If the initial bids are $(\{-\infty, 0\}, \{-\infty, 0\})$, a symmetric bidding for license 2 ensues.

If any behavior not in accordance to the above strategy is observed, competitive bidding commences, where each license is won by the bidder with the highest value at a price equal to the rival bidder's value.

BL's result describes an implicitly collusive equilibrium. In the first round, the bidders see whether they can each walk away with the license they value the most at a price of 0. Thus, they try to split the licenses. However, they cannot be split arbitrarily, because bidders have privately observed values. The bidders need to bid in the first round to signal the other bidder the item they value the most. If it turns out that both bidders value license 1 the most, then they bid until one bidder decides to switch to license 2. It becomes profitable to switch to license 2 when $\Delta\pi$ no longer exceeds the price, i.e. when the profits from winning license 2 at a price of 0 exceed the profits from winning license 1 at a price of p_1. This type of implicitly collusive equilibrium is sustained through threats of resorting to competitive bidding if out-of-equilibrium behavior is observed.

The bidder behavior in Theorem 3.10 is somewhat but not overly sophisticated. More or less, bidders just bid on the one item at a time that would maximize their valuations minus their profits. If a bidder gets greedy and tries to win both items, the rival bidder punishes the greedy bidder by bidding on both items as well. This type of implicitly collusive behavior is similar to behavior children learn in recess at school: trying to take everything will result in a confrontation. The equilibrium in Theorem 3.10 does not require overly sophisticated analysis on the part of the bidders.

It is the use of $\Delta\pi$ to decide when to switch that ensures that this implicitly collusive outcome satisfies pairwise stability in matches only. There are two reasons why this use of $\Delta\pi$ is not arbitrary. First, a bidder switching to the non-preferred license before $\Delta\pi$ would be leaving money on the table: the other bidder might drop out at $\Delta\pi - \eta$ for $\eta > 0$. The second reason is the notion of interim efficiency in Remark 3.16 below.

Corollary 3.11. *In the BL equilibrium in Theorem 3.10, the outcome always satisfies pairwise stability in matches only.*

Proof. There are two sets of outcomes. First, the bidders a and b may split the licenses after the first round. Without loss of generality, this happens when $\pi_a^1 \geq \pi_a^2$ and $\pi_b^1 < \pi_b^2$. Addition gives

$$\pi_a^1 + \pi_b^2 > \pi_a^2 + \pi_b^1,$$

and Definition 3.6 is satisfied. Second and again without loss of generality, bidder a may win license 1 after bidder b deviates to win license 2 when the price of license 1 exceeds $\Delta\pi_b = \pi_b^1 - \pi_b^2$. We thus know $\Delta\pi_b < \Delta\pi_a$. Rearranging the inequality gives

$$\pi_a^1 + \pi_b^2 > \pi_a^2 + \pi_b^1,$$

and again Definition 3.6 is satisfied. $\square$

Therefore, the BL model of bidder behavior in the simultaneous ascending auction satisfies pairwise stability in matches only. There are many extensions to these results. In a series of remarks, we now explore the robustness of the equilibrium in Theorem 3.10 and hence Corollary 3.11. Note that we follow the expositional style of the theory papers BL and EK, where formal theorems are given for simple examples and extensions are discussed less formally.

Remark 3.12. In the remarks that follow, often the analysis of BL and EK is worried that implicit collusion, as in Theorem 3.10, is not sustainable. In these cases, bidders may find the expected value (over the private values of rivals) for competing for all the licenses to be higher than implicitly colluding. Competitive bidding does not provide a concern for the estimator, under this section's assumptions. BL and EK show competitive bidding is a perfect Bayesian equilibrium that will result in an efficient outcome. Efficient outcomes are automatically pairwise stable in matches only.

Remark 3.13. Theorem 3.10 requires an assumption on the marginal distribution of π^1 and (because they are identically distributed) π^2. However, little is assumed about the joint distribution. Thus, Theorem 3.10 and Corollary 3.11 allow a bidder's private values to be correlated across licenses. There can be ex-post high-value or low-value bidders. In this case, the identities of the high-value or low-value bidders are not common knowledge. Also see Theorem 4 in EK, which also studies the case of joint dependence between π^1 and π^2, or ex-post high and low private-value bidders. Note that if our bidder heterogeneity measure w was private information and entered π^1 and π^2, it would just induce correlation between π^1 and π^2. So a private w is nested in the above analysis. This explains our earlier Assumption 3.1.

Remark 3.14. Theorem 3.10 studies the case of two bidders and two licenses, or more accurately, the same number of bidders and licenses. The discussion following Theorem 4 in EK states that simple implicitly collusive equilibria are possible whenever the number of licenses exceeds the number of bidders. Further, Proposition 4 in BL finds a collusive equilibrium where there are more bidders than licenses. In this equilibrium, high-value bidders raise the price to weed out weak bidders, before attempting to signal and implicitly collude. All implicitly-colluding bidders must win an item for collusion to be successful. Because of the need to weed out the weak bidders, we would not necessarily expect to see very low prices in intimidatory-collusive equilibria. Indeed, the prices in the C block were not particularly low. Altogether, combining the results in BL and EK, the restriction to two bidders and two licenses in Theorem 3.10 is not substantive.

Remark 3.15. Theorem 3.10 studies the case where each license has an identical marginal distribution. Remark 3 in BL explores the case where each of the private values for licenses 1 and 2 has a known, bidder-invariant marginal distribution and $E\left[\pi^1\right] > E\left[\pi^2\right]$: license 2 on average has a lower private value π^2. This is the case in our profit specification: package observables (to the bidders) x_J and $\sum_{j=1}^{J} \xi_j$ shift around the mean of profits. The statement of Theorem 3.10 refers to BL's Condition A, which we do not formally state for conciseness. It is a condition on the marginal distribution that ensures that even a high-value type would find it profitable to implicitly collude and win one item for a low price rather than competing and winning both items. As a high-value type a does not know the privately observed values of a rival b, this is a condition on the rival b's mean private value. Remark 3 in BL states collusion can take place under different assumptions about the private value distribution, i.e. when π^1 and π^2 have different marginal distributions and $E\left[\pi^1\right] > E\left[\pi^2\right]$. A stronger condition on the distribution of each π^j is needed because the bidder who bids on the item with a lower mean private value must be induced to stick with that item and not also compete for the other license in competitive bidding. If collusion is sustainable, it follows the form in Theorem 3.10 and hence the equivalent of Corollary 3.11 holds. In any case, Remark 3 in BL is encouraging because FCC spectrum auctions are auctions of heterogeneous licenses. Further, if these conditions for implicit collusion fail to hold, Remark 3.12 states competition ensues, under which the pairwise stability in matches only condition still holds.

Remark 3.16. Theorem 3.10 presents just one symmetric, perfect Bayesian equilibrium to the game in question. Straightforward bidding is always a symmetric equilibrium, as Remark 3.12 discusses, for example. Under straightforward bidding, pairwise stability in matches only is satisfied because the outcome is efficient. However, neither BL or EK claim to find all possible symmetric equilibria to the game. It is possible that symmetric equilibria that do not satisfy pairwise stability in matches only exist (without changing the assumptions of Theorem 3.10), although they are currently unknown. However, BL use an equilibrium property known as *interim efficiency.* Proposition 3 in BL suggests that, under additional conditions, that the outcome in Theorem 3.10 maximizes a "weighted sum of all types' expected surplus", where the maximization is taken over all incentive compatible allocations such that each bidder always receives one object. Thus, Proposition 3 in BL uses the property of interim efficiency to suggest that, if possible, bidders would want to coordinate on the equilibrium Theorem 3.10, rather than some arbitrary, undiscovered equilibrium. Thus, the existence of other symmetric, perfect Bayesian equilibria that have yet to be found should not dissuade us from considering the equilibrium in Theorem 3.10.

Remark 3.17. As we have stated, there can be multiple equilibria to a simultaneous ascending auction. BL and EK describe several such equilibria, including competitive and implicitly collusive equilibria. All of the outcomes in these equilibria satisfy pairwise stability in matches. We will not write down a likelihood because, in part, a likelihood would require the researcher to state the true equilibrium being played. We are agnostic as to the specific equilibrium: we require only a property, pairwise stability in matches, that will hold across many equilibria.

Remark 3.18. Consider a case where there are ex ante, commonly observed high and low-type bidders. For simplicity, say the payoff to bidder a from winning license j is $\pi_a^j = w_a \cdot y_j + \epsilon_{a,j}$, where here the scalar y_j is a characteristic that raises profits, such as the population of the license. Let the standard deviation and support of the mean-zero, i.i.d. private values $\epsilon_{a,j}$ be small and let $w_a \gg w_b$ and $y_1 \gg y_2$, so that $\pi_a^1 + \pi_b^2 > \pi_a^2 + \pi_b^1$ for all realizations of $\epsilon_{a,j}$. Implicit collusion without signalling could occur: the high-type bidder a is assigned license 1 and the low-type bidder b is assigned license 2, both at prices of 0. This equilibrium could be sustained through threats of competitive bidding if π_a^1 is always sufficiently higher than π_a^2 and the loss to a from competitive bidding, the valuations of the rival $\pi_b^1 + \pi_b^2$, is sufficiently large. This type of equilibrium does not involve signalling and is based on public information (here w_a, w_b, y_1 and y_2) rather than private information, so the equilibrium is not mathematically similar to the equilibrium in Theorem 3.10, although the outcomes are quite similar. The equilibrium outcome still satisfies pairwise stability in matches only, as the bidders assortatively match to licenses: high w with high y and $\pi_a^1 + \pi_b^2 > \pi_a^2 + \pi_b^1$. Thus, ex ante, commonly observed high- and low-type bidders can be compatible with pairwise stability in matches only, even without signalling. Further, a failure for this type of equilibrium to hold leads to a competitive equilibrium and Remark 3.12. Altogether, this discussion explains Assumption 3.2, which says that if profits are monotone in the scalar w_a, w can be publicly observed. This assumption was motivated by the desire to satisfy pairwise stability in matches only in the simultaneous ascending auction.

Remark 3.19. Another case is when there are known asymmetries in the values for bidders a and b for licenses 1 and 2. For example, bidder a may be known to on average have a high private value for license 1, while bidder b may be known to on average have a high private value for license 2. Notationally, bidder-and-license-specific private values $\pi_j^a = \epsilon_{a,j}$ have commonly observed, bidder and license specific distributions $F_{a,j}$. In this case, a lot of the information on bidder idiosyncratic valuations is public and, definitionally, no longer privately observed information. Indeed, Theorem 5 in EK allows the distribution of private values to vary across bidder/item pairs. In that case, there is less need to signal using the bidding mechanism to coordinate and pairwise stability in matches only may not occur. Intuitively, this type of result just applies a folk-theorem-like result to the publicly observed part of payoffs. For an equivalent of Corollary 3.11 to hold, valuations must be private or valuations must be monotone in the observed heterogeneity, as in Remark 3.18. Most conditions under which Corollary 3.11 does not hold, in our experience, involve aspects of valuations that are asymmetric across licenses and are not privately observed. However, Section 2.5 argued based on institutional details that this type of extreme asymmetry was not common in the C-block auction. As we have discussed, there is little evidence that the major winning bidders were local businessmen with pre-announced, bidder- and license-specific valuations for particular licenses. Further, colluding based on ex ante known bidder-and-license-specific asymmetries would not require signalling via the bidding mechanism, as in Theorem 3.10. Section 2.6 and the previous, descriptive empirical literature argue that there is a lot of evidence in favor of bidders signalling each other using the bidding mechanism, rather than relying on known asymmetries in bidder valuations.

Remark 3.20. Related to the previous remark is the possibility of *asymmetric*, perfect Bayesian equilibria.[26] Asymmetric equilibria are not discussed in BL and EK. Consider a case where $E\left[\pi^j\right]$ is high, π^j has a small, bounded support relative to $E\left[\pi^j\right]$, and π^1 and π^2 are independent. Then the equilibrium outcome where bidder a wins license 1 at a price of 0 and bidder b wins license 2 at a price of 0 is sustainable by the threat of resorting to competitive bidding. The gain from colluding is so high and the cost of competition so high that the expected value of colluding is high for both bidders, even if a has a higher private value for 2 and b has a higher private value for license 1. Here the equilibrium is asymmetric because bidder a takes an action regardless of its license values. Note that this equilibrium requires no signalling: bidders divide up the items before private values are realized. The empirical evidence in the previous literature and in Section 2.6 is strongly suggestive that signalling took place.[27] Thus, relying on the equilibrium refinement of symmetry as in symmetric, perfect Bayesian equilibria, seems logical for a first estimator for simultaneous ascending auctions with complementarities given the empirical evidence. By restricting attention to symmetric equilibria, we follow the theory on the simultaneous ascending auction.

[26] Note the two uses of the word "asymmetric": asymmetric equilibria here and asymmetric bidder valuations in Remark 3.19.

[27] One could possibly write down an asymmetric equilibrium with signalling. In that case, the empirical evidence of signalling would not be evidence in favor of the symmetry refinements used in BL and EK. Pairwise stability in matches only will not hold under all possible data generating processes for simultaneous ascending auctions. Any asymmetric equilibrium is one such data generating process.

Remark 3.21. The outcome in Theorem 3.10 does not satisfy pairwise stability in both matches and prices, if not winning an item is considered a potential match. One outcome is that bidder a wins license 1 at a price of 0 and bidder b wins license 2 at a price of 0. In this case, both bidders would prefer to win an additional item at a price of 0. They are prevented from doing so only by a threat of competitive bidding, which lowers their expected profits.

Remark 3.22. Bulow, Levin and Milgrom (2009) have emphasized the role of budget constraints in a much more recent spectrum auction than the C block. Pairwise stability in matches only respects one version of a budget constraint: the number of matches of each bidder is the same on the left and right sides of the inequality. Pairwise stability in matches only does not ask why one bidder did not win more licenses at the expense of a rival, only why license j_1 was won by bidder 1 and license j_2 was won by bidder 2 and not the reverse. Thus, the inequalities in pairwise stability in matches only capture some of the spirit of budget constraints. We note that almost all other estimators for auctions of a single item do not respect budget constraints at all; bids are suggested to be informative of valuations, not budget constraints.[28]

Remark 3.23. FCC spectrum auctions have eligibility rules. At the end of the C-block auction, all but two bidders, who were competing for a single license, had settled on their final, winning packages. Their final-round eligibilities were only slightly above the populations of their winning packages. The condition of pairwise stability in matches only is not a statement about the behavior of bidders at the auction's final round, when they had no free eligibility. Rather, pairwise stability in matches only is a condition on the entire data generating process (all the rounds of the auction) and the final allocation that results from the data generating process. The interesting signalling behavior in the BL and EK models arise at the start of the auction, when bidders' eligibilities are above the populations of their final winning packages.

Remark 3.24. An additional concern in simultaneous ascending auctions is the exposure problem, where a bidder fails to secure additional licenses to complete a package and therefore prefers to not to win a license it did win at the end of the auction. Cramton (2006) argues that the price discovery advantages of and the withdrawal options in the FCC's simultaneous ascending auction design mitigate any exposure problem. Pairwise stability in matches will still hold under an exposure problem if total structural profit would not be increased by swapping licenses. Given the exposure problem, pairwise stability holds if the bidders are exposed on the "best of a bad menu" of licenses.

The above sequence of remarks, using references to explicit results in BL and EK, suggests that the existence of symmetric equilibria that produce pairwise stability in matches is relatively robust to the number of bidders, the number of licenses, correlation in private values for each bidder (ex post

[28] One estimation approach would be to impose pairwise stability in matches only for exchanges of licenses with similar closing prices. Our experiments show that this reduces the empirical power (increases the width of the confidence intervals) of the estimator considerably.

high and low types), commonly observed correlation in private values for each bidder (ex ante high and low types), ex ante asymmetries in the distribution of private values for each license, and the concern about unstudied equilibria to the model. Under these assumptions, the analysis of BL and EK suggests that the outcome is likely to satisfy pairwise stability in matches only. We believe pairwise stability in matches, by focusing on exchanges instead of additions and subtractions of licenses, satisfies some of the spirit behind budget constraints.

There are sufficient conditions for the equilibrium to satisfy pairwise stability in matches only. As the first structural estimator for spectrum auctions that allows for complementarities and implicit collusion, we cannot address every possible complication that might arise in some model of a spectrum auction. No structural estimator for even single-unit auctions without collusion allows for flexible bidder- and license-specific distributions for private values, unless covariates parameterizing the entirety of the asymmetric distributions are observed and used in estimation. Despite this, collusive equilibria to models of simultaneous ascending auctions may satisfy pairwise stability in matches only under a large but not exhaustive set of ex-ante and ex-post asymmetries across licenses and bidders.

3.5.2 Analysis with complementarities

We now study the case with nonzero complementarities. BL study only the case of large complementarities, or $\underline{k} > 1$.[29] EK do not study complementarities. We present the full analysis of BL for their Proposition 7. Define $\Theta^1 = \left\{\left(\pi^1, \pi^2, k\right) \in [0,1]^2 \times \left[\underline{k}, \overline{k}\right] \mid \pi^1 > \pi^2\right\}$ and $\Theta^2 = \left\{\left(\pi^1, \pi^2, k\right) \in [0,1]^2 \times \left[\underline{k}, \overline{k}\right] \mid \pi^2 \leq \pi^1\right\}$. Then BL's statement of Proposition 7 is:

Theorem 3.25. *There exist two sets $\tilde{\Theta}^1 \subset \Theta^1$ and $\tilde{\Theta}^2 \subset \Theta^2$ such that the following strategy, together with some belief system, forms a symmetric perfect Bayesian equilibrium:*

Types $\left(\pi^1, \pi^2, k\right) \in \left\{[0,1]^2 \times \left[\underline{k}, \overline{k}\right]\right\} \setminus \left\{\tilde{\Theta}^1 \cup \tilde{\Theta}^2\right\}$ open the auction with $\{0, 0\}$ (0 bids) and compete for both licenses. Types $\left(\pi^1, \pi^2, k\right) \in \tilde{\Theta}^1$ open with $\{0, -\infty\}$. Types $\left(\pi^1, \pi^2, k\right) \in \tilde{\Theta}^2$ open with $\{-\infty, 0\}$.

If the initial bids are $\{\{0, -\infty\}, \{-\infty, 0\}\}$ or $\{\{-\infty, 0\}, \{0, -\infty\}\}$ then bidders do not place any further bids. For all other opening bids having positive probability in accordance with the strategy described above, the bidders play an equilibrium where the two licenses are allocated to the bidder with the highest $\pi_a^{1,2}$ at a price equal to the other bidder's $\pi_{-a}^{1,2}$.

If, at any stage, a bidder makes a bid which cannot be observed if the strategy described above is followed, then the bidders play an equilibrium where the two licenses are allocated to the bidder with the highest $\pi_a^{1,2}$ at a price equal to the other bidder's $\pi_{-a}^{1,2}$.

The sets $\tilde{\Theta}^1$ and $\tilde{\Theta}^2$ are symmetric in the sense that $\left(\pi^1, \pi^2, k\right) \in \tilde{\Theta}^1$ if and only if $\left(\pi^2, \pi^1, k\right) \in \tilde{\Theta}^2$.

This equilibrium splits hypothetical bidders into three groups. The first group has a low valuation for each of the two licenses if won separately and will never be intimidated to implicitly collude. The second group will settle for winning license 1 if the other bidder will settle for license 2, here at prices of 0. The third group will settle for license 2 if the other bidder will settle for license 1. This implicit-collusion outcome is inefficient because complementarities are large, $\underline{k} > 1$, and the outcome assigns the licenses to different bidders. The implicitly-collusive equilibrium is sustained by threats of resorting

[29] Sherstyuk (2003) studies the case of moderate complementarities and finds competitive equilibria to a decentralized economy may exist. However, she presents an example where an auction is unable to achieve the competitive outcome. The paper does not study any form of collusion.

to competitive bidding. BL discuss that if $\underline{k} = \overline{k}$, so that both bidders have the same value for the complementarities, then the value of the complementarities will always be competed away in competitive bidding, so that there will be no first group of bidders that refuse to implicitly collude. Note that our empirical specification for the component $\bar{\pi}_\beta(w_a, x_J)$ of the profit function (1) (see (5) below) will have $\underline{k} = \overline{k}$, as all complementarities will arise from the x_J term and the complementarities between different licenses for the same bidder will not be interacted with the potentially privately observed (recall the alternative Assumptions 3.1 and 3.2) bidder characteristic w.[30]

The implicitly-collusive outcome of splitting the licenses in Theorem 3.25 will not satisfy pairwise stability in prices and matches because the bidders would prefer to win a second item at a price of 0. However, the equilibrium outcome in Theorem 3.25 does satisfy pairwise stability in matches only, even though the outcome may not be efficient because of collusion.[31]

Corollary 3.26. *In the BL equilibrium, the outcome always satisfies pairwise stability in matches only.*

Proof. There are two sets of outcomes. First, competitive bidding may be triggered and the bidder with the highest value, say a, for the package of both licenses will win both licenses. In this case, inequality (3) becomes, for $i_a = 1$ and for i_b being the outcome of not winning a license,

$$\pi_a^{1,2} + \pi_b(\emptyset) = \pi_a^1 + \pi_a^2 + k_a + 0 > \pi_a^1 + \pi_b^2,$$

because $\pi_a^2 \geq 0$ and BL only study the case with $\underline{k} > 1$, so $k_a > \pi_b^2$. In the other outcome, bidders a and b may split the items so that, without loss of generality, a wins 1 and b wins 2. By the definition of the sets Θ^1 and Θ^2, this happens only when $\pi_a^1 > \pi_a^2$ and $\pi_b^1 \leq \pi_b^2$. So, (3) becomes, for $i_a = 1$ and for $i_b = 2$,

$$\pi_a^1 + \pi_b^2 > \pi_a^2 + \pi_b^1.$$

□

The equilibrium BL find is quite natural. In a game with symmetric private values, having a high private-value realization for a license tells the bidder little about the valuations of its rivals. There is little to gain from bidding on a subset of licenses that are not the highest private-value realizations of the bidder. Because agents have private information, they must signal through bids to find sustainable, implicitly-collusive equilibria. Over all, we think that the theoretical evidence from the BL and EK models in favor of the outcome satisfying pairwise stability is matches only is reasonably strong. Certainly, as this is the first estimator for simultaneous ascending auctions that allows for complementarities and implicit collusion, there are improvements that could be made in future research.

[30] BL also mention that complementarities might break implicit collusion. Counterintuitively, it is not the level of complementarities that prevents collusion, but the variability of complementarities across bidders. So our specification for $\bar{\pi}_\beta(w_a, x_J)$ below may make it easier for a collusive equilibrium to be sustained.

[31] The proof of the corollary requires pairwise stability in matches only to allow a losing bidder to exchange the status of not winning a license with a winning bidder. The actual empirical work will not use inequalities involving losing bidders. Any equilibrium outcome that satisfies pairwise stability in matches only for both losing and winning bidders will satisfy pairwise stability in matches only for only winning bidders.

3.6 Demand reduction

Demand reduction is when bidders unilaterally choose to not compete for all units they have positive valuations for. This can be profitable if they know rival bidders have decreasing returns to scale in their valuations, which can include the case of constant marginal valuations for a finite number of homogeneous items that is lower than the number of items for sale (Ausubel and Cramton, 2002). This type of inefficiency happens for a different strategic reason than the implicit collusion discussed above.

Pairwise stability is an implication of the tatonnement conditions for the spectrum auction model of Milgrom (2000). In Appendix A, we use the tatonnement conditions of Milgrom to demonstrate that both Definitions 3.5 and 3.6 are satisfied in a simultaneous-ascending-auctions model of demand reduction, without complementarities. The analysis of Milgrom requires straightforward bidding; strategic bidding during the auction itself is not allowed. The analysis of Milgrom should not be seen as an equilibrium to a Bayesian Nash game, but we note that both BL and EK prove that competitive bidding is a Bayesian Nash equilibrium to the simultaneous ascending auction.

3.7 Existence of a pairwise stable allocation under complementarities

While the true data generating process is likely a dynamic Nash game, we rely on the conditions of pairwise stability in matches only for estimation. Milgrom (2000) and Hatfield and Milgrom (2005) give a key condition under which a competitive equilibrium, Definition 3.7, and so, a pairwise stable in matches only allocation, Definition 3.6, is guaranteed to exist in a many-to-one matching environment like a spectrum auction, where one bidder matches to many licenses but each license is matched to only one bidder. The key condition is that preferences of bidders for items exhibit substitutes, not complementarities, across multiple licenses in the same package. Therefore, there is no general existence theorem for a pairwise stable allocation in a many-to-one matching environment with complementarities across multiple licenses in the same package.

The lack of a general existence theorem does not mean that economists should not conduct theoretical analyses or structural empirical work for markets that might have complementarities. Markets with complementarities are of critical empirical interest, as this paper shows.

Even if a model lacks a general existence theorem, it is certainly possible that the actual data are generated from a valid pairwise stable allocation: there is no existence problem for this market. This is the maintained assumption for this paper. This assumption can be informally checked after estimation. Looking ahead to the structural estimates, column 2 of Table 5 will indicate that 95% of the potential inequalities from the estimation analog of Definition 3.6, pairwise stability in matches only, are satisfied at the converged estimates. Computationally, the maximum score estimation analog to Definition 3.6 drops the bidder- and license-specific private values $\epsilon_{a,j}$, even though the estimator allows their presence, to some degree. So the 95% fit, which strikes us as high, comes without relying on private values at all. By making private values $\epsilon_{a,j}$ for the observed matches between bidders a and licenses j high, and keeping $\epsilon_{a,j} = 0$ for matches that are not part of the final allocation, we can easily raise the fit from 95% to 100% of the inequalities from Definition 3.6. Thus, there exists a continuum of private-value realizations where the C block satisfies pairwise stability in matches only, when $\beta = \hat{\beta}$, the estimated parameters.

3.8 Pairwise stability in other models of auctions

Here we briefly mention that pairwise-stability conditions may be applicable to other auction settings as well. Some collusive models will satisfy Definition 3.6. In single-unit auctions, many models of collusion are efficient in the sense that bidders with higher values win more often. A pooled dataset of many auctions with the same set of bidders in each can satisfy pairwise stability in matches only. Graham and Marshall model bidder rings in second-price auctions. In equilibrium, the ring member with the highest value always wins if any ring member does. McAfee and McMillan (1992) present similar results for an all-inclusive bidder ring in a first-price auction. Similarly, Athey, Levin and Seira (2008) assume that, in auctions of a single item with an explicitly-organized bidding ring, the highest-value bidder will represent the ring and compete for the item for sale. Previous estimation methods for an ascending auction of a single item that rely on bids being informative of values, such as Haile and Tamer (2003), need not generate consistent estimates under these models of collusion.

In a spectrum auction, a collusive equilibrium would satisfy Definition 3.6 if the bidders coordinate on an allocation of licenses that maximizes joint surplus. However, in the C block, we have no evidence to suggest that an explicit bidding ring was active. The empirical evidence in Cramton and Schwartz (2000, 2002) suggests that bidders signaled each other through the bidding mechanism, rather than through illegal back-channel communications. There is no evidence that any firms exchanged transfers after the auction as bribes to compensate losing bidders for not winning licenses.

Under some conditions, pairwise stability will hold in the Google keyword auction studied by Edelman, Ostrovsky and Schwarz (2007).[32]

4 The estimator

4.1 Estimator

We will later formally assume that the outcome to the C-block auction satisfies a statistical version of pairwise stability in matches, based upon the experimental evidence, lack of resale after the auction, theoretical evidence about implicit collusion, and theoretical evidence about demand reduction. Recall equation (1), the total profit equation. To make the statistical objective functions more readable, we will sometimes write $\bar{\pi}_\beta(a, J) \equiv \bar{\pi}_\beta(w_a, x_J)$ for bidder a and package J. Let H be the number of winning bidders.

Fox (2009a) introduces a semiparametric maximum score estimator for many-to-many matching games with transferable utility. Maximum score was first introduced by Manski (1975). In our application, this estimator may be more appropriately labeled a maximum rank correlation estimator (Han, 1987), because of the asymptotic argument discussed below. The estimator is semiparametric as no parametric distributions for the unobservables $\epsilon_{a,j}$ and the fixed effects ξ_j are imposed. The estimator is based on forming the empirical analog of the inequalities in Definition 3.6, which uses data on matches but not prices.

[32] Day and Milgrom (2007) study the normative problem of developing core-selecting auctions. These auction produce allocations that satisfy the two definitions of pairwise stability, with and without prices, according to the *stated* preferences of bidders. Our goal in studying the simultaneous ascending auction is positive: to examine whether it is likely that an existing mechanism produces an assignment that satisfies Definition 3.6 according to the *actual* preferences of bidders.

Consider a simple auction with two bidders a and b and two licenses 1 and 2. In the data, a wins 1 and b wins 2. The estimator $\hat{\beta}$ is any vector that satisfies the inequality

$$\bar{\pi}_{\beta}(a,\{1\})+\bar{\pi}_{\beta}(b,\{2\})\geq\bar{\pi}_{\beta}(a,\{2\})+\bar{\pi}_{\beta}(b,\{1\}).$$

The inequality is satisfied whenever, for any pair of licenses i and j and bidders a and b, the sum of the deterministic parts of bidder valuations is not increased by an exchange of licenses. With only two bidders and two licenses, typically many such parameters β will satisfy the inequality. Further, the confidence interval for β will be large. We need to use all of the data to produce an estimate of β with a reasonably small confidence interval.

For the full sample, the estimator $\hat{\beta}$ is any vector that maximizes the objective function

$$Q^{\text{match}}(\beta)=\frac{2}{H(H-1)}\sum_{a=1}^{H-1}\sum_{b=a+1}^{H}\sum_{i=1}^{|J_a|}\sum_{j=1}^{|J_b|} 1\left[\bar{\pi}_{\beta}(a,J_a)+\bar{\pi}_{\beta}(b,J_b)\geq\bar{\pi}_{\beta}(a,(J_a\backslash\{i\})\cup\{j\})+\bar{\pi}_{\beta}(b,(J_b\backslash\{j\})\cup\{i\})\right]. \quad (4)$$

The objective function $Q^{\text{match}}(\beta)$ considers all combinations of two licenses won by different bidders, a and b. If an inequality is satisfied, the count or score of correct predictions increases by 1. The estimator's inequalities include only the deterministic portion of structural profits, $\bar{\pi}_{\beta}(w_a,x_J)$. Many inequalities will remain unsatisfied, even at the true parameter vector, because of the unobserved realizations of private values $\epsilon_{a,j}$, which also affect matches. Because not all inequalities can be satisfied, changing the score objective to squaring the deviations from deterministic pairwise stability makes the estimator inconsistent. The estimate $\hat{\beta}$ maximizes the score of correct predictions. The estimator is relatively simple to compute.[33]

The complete profit term (1) includes fixed effects. If instead we worked with $\bar{\pi}_{\beta}(a,J_a)+\sum_{j\in J}\xi_j$ in (4), the license-specific unobservables ξ_i would enter into both sides of (2) and difference out. For example, ξ_i for $i\in J_a$ enters both $\bar{\pi}_{\beta}(a,J_a)+\sum_{k\in J}\xi_k$ and $\bar{\pi}_{\beta}(b,(J_b\backslash\{j\})\cup\{i\})+\sum_{k\in(J_b\backslash\{j\})\cup\{i\}}\xi_k$. As each ξ_j appears on the left and right sides, it differences out. Therefore, we do not need to directly estimate these parameters as fixed effects, just like one does not estimate fixed effects in a panel-data linear regression model when the fixed effects are differenced out. Indeed, the fixed effects are not identified because there is one fixed effect for each license; this is the well-known incidental parameters problem. However, the estimator for β will be consistent in the presence of fixed effects, because of the differencing.

The maximum rank correlation or maximum score approach only estimates the parameters β in $\bar{\pi}_{\beta}(w_a,x_J)$, not the distribution of any error terms. Parameters in a function of observables have always been the object of interest in maximum score (Manski, 1975; Han, 1987; Horowitz, 1992; Matzkin, 1993). There are computational and economic theory reasons why we focus on β and use a maximum rank correlation estimator. These are discussed in the next subsection.

[33] $Q^{\text{match}}(\beta)$ is a step function and as a result, in a finite sample there can be a continuum (or multiple continua) of parameters that maximize $Q^{\text{match}}(\beta)$. Any maximizer is a consistent estimator. In practice, reporting a 95% confidence region for each element of β removes this ambiguity. We use the global optimization routine known as differential evolution to maximize $Q^{\text{match}}(\beta)$ (Storn and Price, 1997). We find that differential evolution is more likely to find the global optimum than simulated annealing.

Readers may be wondering about identification: how can a researcher use qualitative data on which bidder is allocated which license to estimate parameters in a valuation function? The question of identification using matching data is studied formally in Fox (2009b). We will postpone a formal discussion of identification until Section 5.5.

4.2 Maximum likelihood as an alternative

One way to motivate our estimator is to compare it to maximum likelihood, a standard estimator in structural work. Maximum likelihood is an efficient estimator, if the researcher is willing to make a lot of assumptions. Consider the alternative of writing down a likelihood for the C-block auction. First, the researcher would have to pick a particular game and a parametric distribution for all of the model unobservables, ϵ and ξ. The likelihood would be an integral over the market unobservables: the $N \cdot L = 255 \cdot 480 = 122,400$ private values $\epsilon_{a,j}$ and the $L = 480$ unknown (to the researcher) ξ values. So there would be a roughly 123,000 dimensional integral. The integrand in the integral is the solution to the auction game. As a simultaneous ascending auction is a dynamic game, this would involve dynamic programming. The state variable would be the current standing bid and winning bidder on each of the 480 items, or 960 state variables, if the past history of play is excluded from the state variable. Estimating a model with 10 state variables is considered a computational challenge. Presumably this dynamic game would have many multiple equilibria, as we acknowledged previously. The researcher would have to choose one type of equilibrium in order to search for it numerically for each guess of the parameter values and each guess of the 123,000 unobservables. The dynamic game would be complex as off-equilibrium (mostly) threats of punishment sustain implicit collusion, if an implicit collusively equilibrium was chosen by the researcher to numerically search for. Finally, the likelihood estimator makes ξ_j a random effect instead of a fixed effect. The estimator would be inconsistent if ξ_j was statistically dependent with y_j.

One advantage of the maximum score estimator is that it involves only a small number of counterfactual packages. Computing a likelihood would require evaluating all possible packages. Several papers in the collection Cramton, Shoham and Steinberg (2006) explore how even computing a winning bid in an alternative combinatorial auction is an active area of research in computer science. Cramton (2006) argues that a major motivation for using the simultaneous ascending auction over a package-bidding combinatorial auction is the computational challenge in evaluating all packages. Evaluating all possible packages is not a tractable estimation strategy in the C-block environment that has more packages than the atoms in the universe.

The previous arguments also explain why we do not try to estimate the distribution of private values $\epsilon_{a,j}$. It would be computationally difficult and the researcher would have to take a stand on the particular equilibrium being played.

We do not estimate the distribution of the fixed effects ξ_j because the distribution is not identified from data on who matches with whom. All bidders value each ξ_j equally and its presence will not affect equilibrium allocations of licenses to bidders, under many equilibria we previously considered, including competitive and some collusive equilibria. Further, because each ξ_j is a fixed effect and not a random effect, its distribution is allowed to depend on y_j, the recorded characteristics of license j. Identifying such a conditional distribution is not possible with one fixed effect per license.

4.3 Empirical specification for the deterministic valuation function

In our application, we let $w_a = \{\text{elig}_a\}$ be the initial (before the auction begins) eligibility of bidder a. Also, let

$$x_J = \left\{ \{\text{pop}_j\}_{j=1}^{J}, \text{complem.}_J \right\}$$

be equal to the population of all licenses in the package J as well as a vector complem._J of constructable empirical proxies for the complementarities in the package. Our choice of $\bar{\pi}_\beta(w_a, x_J)$ is

$$\bar{\pi}_\beta(w_a, x_J) = \pm 1 \cdot \text{elig}_a \cdot \left(\sum\nolimits_{j \in J} \text{pop}_j \right) + \beta' \text{complem.}_J \tag{5}$$

The interaction $\text{elig}_a \cdot \left(\sum_{j \in J} \text{pop}_j \right)$ captures the fact in Table 2 that bidders with more initial eligibility won more licenses. The scalar $w_a = \{\text{elig}_a\}$ is our main measure of bidder characteristics, given that Table 2 shows financial measures were uncorrelated with winning a license. The coefficient on $\text{elig}_a \left(\sum_{j \in J} \text{pop}_j \right)$ has been normalized to ± 1 because dividing both sides of the inequality in (3) by a positive constant will not change the inequality. The term $\beta' \text{complem.}_J$ provides the total contribution of the several complementarity measures in the vector complem._J. Each element of complem._J is a nonlinear construction from the characteristics of the underlying licenses in the package J, so complem._J does not cancel in (3). The parameters β describe the relative importance of each complementarity measure in terms the units of $\text{elig}_a \cdot \left(\sum_{j \in J} \text{pop}_j \right)$.[34] Recall that (1) has license-specific unobservables, ξ_j. These unobservables capture the common element to the valuation of licenses, such as the base contribution of population, the fact that spectrum is more scarce in more densely-populated territories and the fact that competition from incumbent carriers may be stronger in some territories than others.

We choose a simple functional form for $\bar{\pi}_\beta(w_a, x_J)$ in order to demonstrate that a parsimonious model is able to fit the data quite well. A more complicated functional form would have little benefit in terms of the overall fit of the model and would obscure the interpretation of the parameters.

4.4 Three proxies for potential complementarities

We construct proxies for geographic economies of scope and use them as our primary measure of complementarities. Three alternative measures are used in order to examine the robustness of our results. Table 4 presents descriptive statistics about the three measures as well as the correlation matrix for the three measures. The measures are highly but not perfectly correlated with each other.

[34] Eligibility is the initial eligibility of a bidder, as seen in Table 2. Population is just the number of residents (in the 1990 census) of the license. To aid interpretation, we divide both measures by the population of the continental United States, so that an eligibility or population of 1 corresponds to a true value of 253 million people. By the fractional normalization, the mean population $\sum_{j \in J} \text{pop}_j$ among the 85 winning packages is 0.012 (standard deviation of 0.044), and the mean $\text{elig}_a \left(\sum_{j \in J} \text{pop}_j \right)$ is 0.004 (standard deviation 0.029).

4.4.1 Geographic distance

Our first proxy for geographic scope is based on the geographic distance between pairs of licenses within a package.[35] For a package J in the set L of all licenses, potential complementarities are

$$\text{geocomplem.}_J = \sum_{i \in J} \text{pop}_i \frac{\left(\sum_{j \in J, j \neq i} \frac{\text{pop}_i \text{pop}_j}{\text{dist}_{i,j}^{\delta}}\right)}{\left(\sum_{j \in L, j \neq i} \frac{\text{pop}_i \text{pop}_j}{\text{dist}_{i,j}^{\delta}}\right)}, \tag{6}$$

where population is measured in fractions of the US total population and distance is measured in kilometers.[36] The distance, $\text{dist}_{i,j}$, between licenses i and j is raised to a power $\delta = 4$ to make this measure overweight nearby territories.[37] The measure geocomplem._J proxies for short-distance travel and cost and marketing synergies across nearby territories. Also, geocomplem._J is similar to the well-known gravity equation in international trade. The measure geocomplem._J has the desirable feature that any firm's complementarities cannot decrease by adding licenses to a package.

4.4.2 Two travel measures

Geographic measures of distance may not capture the returns to scope that carriers are concerned about. Mobile-phone customers may travel by means other than ground transportation. For example, many business users travel by air between Los Angeles and New York. In fact, the C-block bidder NextWave won both the New York and Los Angeles licenses. We have two complementarity proxies based upon travel between two licenses. The first measure, from the 1995 American Travel Survey (ATS), is proportionate to the number of trips longer than 100 km between major cities. All forms of transportation are covered. The downside of this measure is that for privacy reasons the ATS does not provide enough information about rural origin and destinations to tie rural areas to particular mobile phone licenses. Our second measure, from the Airline Origin and Destination Survey for the calendar year 1994, is the projected number of passengers flying between two mobile phone license areas.[38] The drawback of the air travel measure is that it assumes all passengers stay in the mobile-phone license area where their destination airport is located.[39] Both travel measures for a package J are population-

[35] We measure distance between two licenses using the population-weighted centroid of each license. The population-weighted centroid is calculated using a rasterized smoothing procedure using county-level population data from the US Census Bureau.

[36] This geographic-complementarity proxy can be motivated as follows. Consider a mobile phone user in a home market i. That mobile phone user potentially wants to use his phone in all other markets. He is more likely to use his phone if there are more people to visit, so his visit rate is increasing in the population of the other license, j. The user is less likely to visit j if j is far from his home market i, so we divide by the distance between i and j. We care about all users equally, so we multiply the representative user in i's travel experience by the population of i.

[37] The choice of $\delta = 4$ is arbitrary and was chosen to make the clusters of licenses seen in Figure 1 have non-trivial levels of complementarities.

[38] Intermediate stops are not counted for either dataset. For both datasets, geographic information software (GIS) was used to match origins and destinations with mobile phone licenses. For airports, the origin and destination license areas are easy to calculate. For the MSAs (Metropolitan Statistical Areas) used in the ATS, the equivalent C-block license area was found using the centroid of the origin or destination MSA. The C-block license boundaries for urban areas roughly follow MSAs.

[39] We effectively code that there are zero potential complementarities between rural licenses for both travel measures.

weighted means across licenses, and take the form

$$\text{travelcomplem.}_J = \sum_{i \in J} \text{pop}_i \frac{\sum_{j \in J, j \neq i} \text{trips (origin is } i, \text{destination is } j)}{\sum_{j \in L, j \neq i} \text{trips (origin is } i, \text{destination is } j)}. \tag{7}$$

Our ATS measure uses the count of raw trips in the survey, and the air travel count is inflated to approximate the total number of trips during 1994.[40] As with geographic distance, if $J = L$, $\text{travelcomplem.}_J = \sum_{i \in L} \text{pop}_i = 1$. Here again, adding a license to a package cannot take away complementarities between other licenses, so travelcomplem._J only increases as licenses are added to J.[41]

5 Consistency and asymptotic inference

There is a lot of information one can learn from an auction market with 85 winning bidders and 480 licenses. However, as the asymptotics in Fox (2009a) are in the number of statistically independent matching markets, it is less clear how to construct an asymptotically-valid approximation to the finite-sample distribution of the estimator, which is needed for statistical inference. This section introduces an asymptotic argument that lets the number of recorded winning bidders grow large. To the extent that any asymptotic approximation to the finite-sample distribution is a good one, asymptotics in the number of winning bidders will likely allow more accurate inference using data on a single, large matching market.

We assume the econometrician observes some finite number of recorded agents from an aggregately deterministic auction. Let $H \leq N$ represent the number of recorded winning bidders; we analyze the asymptotics as $H \to \infty$. Readers may already be thinking of the complications in such an asymptotic argument. As new bidders are added to an auction, they may compete and win items that were previously won by existing bidders. Asymptotics where the existing dependent-variable data change with the addition of new data are beyond the scope of our asymptotic approximation.[42] Instead, we introduce the fiction that the real-life matching market or auction with H winning bidders is a subset of some very large auction. As H gets larger in the asymptotic approximation, the researcher collects more data on a single auction. The fiction is not to be taken literally: there were only 85 winning bidders in the C block.

5.1 Asymptotic model

Let there be one very auction. This auction is so large that there are an uncountable number of winning bidders and items for sale in it. Therefore, it is necessary to model each winning bidder and license as existing in characteristic space, rather than using a countable number of indices. We focus on notation

[40] Our airline passenger measure does not distinguish between origins and destinations, so we simply divide the formula for the complementarity proxy by 2. If all airline travel is round-trips during the same calendar year, this measure should be exactly correct.

[41] For all geographic-complementarity proxies, some fraction of the winning packages has a value of 0. For example, 26 out of the 85 winning packages contain only one license in the continental United States.

[42] An asymptotic distribution is used as approximation to the finite-sample distribution, which often cannot be derived analytically. Normally, a researcher does not model the real-world process of collecting more observations to construct an asymptotic approximation for inference. If an observation is a past US president, the researcher does not model presidential elections in the year 3000.

appropriate to many-to-one, two-sided matching model, where the two sides are winning bidders and licenses.

Let each winning bidder have characteristics w. Likewise, each license has characteristics y. The function $\zeta(Y)$ take a set $Y = \{y_1, \ldots, y_J\}$ of $J < \infty$ license characteristics and forms a package characteristic $x = \zeta(Y)$. The function ζ is known to the researcher. The deterministic valuation of a bidder w from a package x is $\bar{\pi}_\beta(w, x)$. Let the characteristic vectors have continuous support and let them have densities. Let the bidder characteristics and the license characteristics be i.i.d. The exogenous features of the matching market include $g_w(w)$, the density of bidder characteristics w, as well as $g_y(y)$, the density of license characteristics y. The terms w, x and y can all be vectors. The equilibrium outcome in this auction includes a density $g_{x,w}^{\beta,S}(\langle w, x\rangle)$, which gives the frequency of the ordered pair $\langle w, x\rangle$, representing a winning package with characteristics x for a bidder with characteristics w. The density $g_{x,w}^{\beta,S}(\langle w, x\rangle)$ is an endogenous outcome, and so it is a function of the finite-vector of unknown parameters β and the densities $S = (g_\epsilon(\epsilon), g_\xi(\xi \mid y))$ of both the license- and bidder-specific private values and the license fixed effects.[43] We discuss S in more detail below; it is not an object of estimation.[44] Let $\beta \in \mathcal{B}$ and $S \in \mathcal{S}$. We assume the density $g_{x,w}^{\beta,S}(\langle w, x\rangle)$ is deterministically computed as part of the equilibrium; there are no stochastic elements affecting $g_{x,w}^{\beta,S}(\langle w, x\rangle)$, although each individual bidder w's winning package is a random draw from the derived conditional density, $g_{x|w}^{\beta,S}(x \mid w)$.[45]

5.2 Statistical analog to pairwise stability in matches only

In our asymptotic world, the statistical notion of pairwise stability in matches only that we need is as follows.

Assumption 5.1. *Let $\langle w_1, x_1\rangle$ and $\langle w_2, x_2\rangle$ be two hypothetical winning packages and let $x_1 = \zeta(Y_1)$ for $Y_1 = \{y_{1,1}, \ldots, y_{1,J_1}\}$ and $x_2 = \zeta(Y_2)$ for $Y_2 = \{y_{2,1}, \ldots, y_{2,J_2}\}$. Let $y_{1,i_1} \in Y_1$ and $y_{2,i_2} \in Y_2$. Let $x_3 = \zeta((Y_1 \setminus \{y_{1,i_1}\}) \cup \{y_{2,i_2}\})$ and $x_4 = \zeta((Y_2 \setminus \{y_{2,i_2}\}) \cup \{y_{1,i_1}\})$. Assume, for any $\beta \in \mathcal{B}$ and $S \in \mathcal{S}$,*

$$\bar{\pi}_\beta(w_1, x_1) + \bar{\pi}_\beta(w_2, x_2) > \bar{\pi}_\beta(w_1, x_3) + \bar{\pi}_\beta(w_2, x_4)$$

if and only if

$$g_{x,w}^{\beta,S}(\langle w_1, x_1\rangle) \cdot g_{x,w}^{\beta,S}(\langle w_2, x_2\rangle) > g_{x,w}^{\beta,S}(\langle w_1, x_3\rangle) \cdot g_{x,w}^{\beta,S}(\langle w_2, x_4\rangle).$$

The statistical version of pairwise stability in matches is a condition on the equilibrium sorting pattern. It says that if an exchange of licenses produces a lower sum of deterministic payoffs, then the

[43] The density for the fixed effect for a license ξ $g_\xi(\xi \mid y)$ takes as conditioning arguments license characteristics y, as fixed effects ξ can be distributed dependently with y.

[44] We do not use data on non-winning bidders in our main set of estimates. There were no unsold licenses in the C block. However, losing bidders and unsold licenses can be introduced. The special notation $g_{x,w}^{\beta,S}(\langle w, 0\rangle)$ gives the density for losing bidders and $g_{x,w}^{\beta,S}(\langle 0, x\rangle)$ gives the density for unsold single licenses, where the density is only positive if $x = \zeta(\{y\})$ for a singleton package $\{y\}$. By the physical feasibility of the auction equilibrium, $g_w(w) = \int g_{x,w}^{\beta,S}(\langle w, x\rangle)\, dx$ and $g_x(x) = \int g_{x,w}^{\beta,S}(\langle w, x\rangle)\, dw$.

[45] The literature has some examples of aggregately-deterministic matching models with a continuum of agents. Sattinger (1979) studies a perfect information, one-to-one, two-sided matching game with a continuum of agents. Each agent is distinguished by a scalar type. Choo and Siow (2006) analyze a model with a deterministic aggregate assignment, where each agent has errors $\epsilon_{\langle a,i\rangle}$ for each match $\langle a, i\rangle$.

frequency of observing winning packages with the same characteristics as the exchange of licenses must be lower than observing winning packages with characteristics that give higher payoffs. Note that the same number of licenses is won by a bidder on both sides of the inequalities in Assumption 5.1. We do not ask why a single bidder did not win more licenses, because bidders may split the licenses among them because of intimidatory collusion, not efficiency.

Assumption 5.1 is not formally motivated with a model where each bidder has license-specific private values $\epsilon_{a,j}$. However, it is easy to algebraically show that aggregately-deterministic, logit-based matching model in Choo and Siow (2006), which has match-specific shocks, satisfies Assumption 5.1.[46]

We view Assumption 5.1 as a primitive assumption that takes the spirit of pairwise stability in matches only and makes it into an operational statistical assumption. The statement of Assumption 5.1 compares the deterministic portion of match payoffs, $\bar{\pi}_\beta(w,x)$, to match frequencies: it allows the spirit behind license- and bidder-specific private values $\epsilon_{a,j}$ to also affect matches. Otherwise, there would be a statement that the observed matches have probability 1 rather than a comparison of the product of two densities. In common with the earlier literature on single-agent maximum score models (Manski, 1975), we do not specify or estimate a parametric functional form for the distribution S of both the license- and bidder-specific private values $\epsilon_{a,j}$ and the license-specific fixed effects, ξ_j. Thus, our approach is semiparametric.

We should be clear that we have not shown that Assumption 5.1 is derivable from any model, other than Choo and Siow (2006). Nevertheless, Fox (2009a) presents Monte Carlo evidence that the maximum score / maximum rank correlation estimator, the estimator used in this paper, has a low bias and root mean-squared error (RMSE) for β when using data from a single large matching market with 300 agents on each side and i.i.d. error terms $\epsilon_{a,j}$ for all matches, with a distribution S. This Monte Carlo result holds even if S is a mixture of normals.

Assumption 5.1 rules out problems involving multiple equilibria. By using data from only one auction, we condition on the equilibrium being played in that market. Also, the assumption implicitly assumes an equilibrium assignment $g^{\beta,S}_{x,w}(\langle w,x\rangle)$ exists.

5.3 Consistency

The estimator is any parameter that maximizes (4). Han (1987) introduced a similar estimator for single-agent ordered choice and showed that it was consistent. He called it a maximum rank correlation estimator. It is easy to adapt results in Han (1987) to show semiparametric point identification for β if $\bar{\pi}_\beta(w,x)$ is a linear index and some or all elements of w and x have continuous support, as they do in our application. Here we assume point identification in order to focus on the asymptotic theory.[47] The following assumption is sufficient for the consistency of the semiparametric estimator in (4).

Assumption 5.2.

1. *Each $\beta \in \mathcal{B} \subseteq \mathbb{R}^{|\beta|}$,$|\beta| < \infty$. $\mathcal{B}$ is compact.*

[46] In Choo and Siow (2006), Assumption 5.1 is satisfied with match probabilities replacing the density $g^{\beta,S}_{x,w}(\langle w,x\rangle)$, as Choo and Siow allow only characteristics with discrete support in w and x.

[47] In a finite sample, multiple parameter vectors will typically maximize the objective function. Any of these will be a consistent estimator. A unique parameter vector, the true parameter β^0, will maximize the probability limit of the objective function.

2. *Identification: Let $\beta^0 \in \mathcal{B}$ and $S^0 \in \mathcal{S}$ be the true parameters that generate the data. For any $\beta^1 \neq \beta^0$, $\beta_1 \in \mathcal{B}$, and for any $S^1 \in \mathcal{S}$, there exists a set of pairs of characteristics $\mathcal{X}$ with positive probability such that, for any $(w_1, x_1, w_2, x_2) \in \mathcal{X}$, $g_{x,w}^{\beta^0,S^0}(\langle w_1, x_1\rangle) \cdot g_{x,w}^{\beta^0,S^0}(\langle w_2, x_2\rangle) > g_{x,w}^{\beta^0,S^0}(\langle w_1, x_3\rangle) \cdot g_{x,w}^{\beta^0,S^0}(\langle w_2, x_4\rangle)$ while $g_{x,w}^{\beta^1,S^1}(\langle w_1, x_1\rangle) \cdot g_{x,w}^{\beta^1,S^1}(\langle w_2, x_2\rangle) < g_{x,w}^{\beta^1,S^1}(\langle w_1, x_3\rangle) \cdot g_{x,w}^{\beta^1,S^1}(\langle w_2, x_4\rangle)$, and also $x_1 = \zeta(Y_1)$ for some $Y_1 = \{y_{1,1}, \ldots, y_{1,J_1}\}$, $y_{1,i_1} \in Y_1$, $x_2 = \zeta(Y_2)$ for some $Y_2 = \{y_{2,1}, \ldots, y_{2,J_2}\}$, $y_{2,i_2} \in Y_2$, $x_3 = \zeta((Y_1 \backslash \{y_{1,i_1}\}) \cup \{y_{2,i_2}\})$ and $x_4 = \zeta((Y_2 \backslash \{y_{2,i_2}\}) \cup \{y_{1,i_1}\})$.*

3. *All characteristics have continuous support.*

4. *Each match $\langle x, w\rangle$ is an independent draw from $g_{x,w}^{\beta^0,S^0}(\langle w, x\rangle)$.*

As the maximum score and maximum rank correlation literatures show for linear-index functional forms of $\bar{\pi}_\beta(w, x)$, for consistency, only one characteristic of either w or x needs to have continuous support. Continuous support for all elements of w and x simplifies the statement of Assumption 5.1.[48] The estimator is consistent as more data on winning bidders are collected from an underlying very large spectrum auction.

Theorem 5.3. *As $H \to \infty$, any parameter vector $\hat{\beta}_H \in \mathcal{B}$ that maximizes the maximum rank correlation objective function* (4) *is a consistent estimator for $\beta^0 \in \mathcal{B}$, the parameter vector in the data generating process.*

The proof is omitted because showing the consistency of the maximum rank correlation estimator is not an original contribution. Readers can refer to the original proof in Han (1987) or consult general theorems for the consistency of extremum estimators in Newey and McFadden (1994).

5.4 Inference

Sherman (1993) shows that the maximum rank correlation estimator is $\sqrt{H}$-consistent and asymptotically normal. The objective function at a given β is a U-statistic of second order. As H grows, the terms in the double summation grow proportionately to H^2. Intuitively, the inner summation acts like a smoother without requiring an explicit kernel and bandwidth. The derivation relies on a general set of results for the asymptotic distribution of U-processes in Sherman (1994).

The asymptotic variance matrix derived in Sherman (1993) is slightly complex to use in that it requires additional nonparametric estimates of components that appear in the variance matrix. To avoid this complexity we use a resampling procedure. The bootstrap is one resampling procedure, although the validity of the bootstrap had not been established for the maximum rank correlation estimator of Han (1987) until after the empirical work was performed (Subbotin, 2007). We instead turn to a procedure known as subsampling, which is consistent under fairly weak conditions. As Politis, Romano and Wolf (1999) state, essentially the only assumption needed for the validity of subsampling is that (their Assumption 2.2.1) there exists a limiting distribution such that the finite-sample distribution of the normalized (by a factor $\tau_H = \sqrt{H}$) estimator converges weakly to that distribution. Sherman (1993) establishes this condition: the maximum rank correlation estimator is $\sqrt{H}$-consistent and asymptotically normal.

[48] Otherwise the statement would involve a probability measure over inequalities, rather than a density evaluated at a point.

5.5 Nonparametric identification of matching models

Fox (2009b) proves a sequence of theorems about the nonparametric identification of $\bar{\pi}(w_a, x_J)$, for situations where the number of markets is large, not the number of winning bidders in one market. However, the proofs could be easily modified to deal with asymptotics in H. Nonparametric means not requiring functional form assumptions on S and well as $\bar{\pi}$. In other words, the profit function $\bar{\pi}$ is not known up to a finite vector of parameters β. Fox (2009b) presents general results in addition to simple examples using marriage with two men and two women. Readers interested in simple examples should see the other paper.

Theorems 5.1–5.3 of Fox (2009b) consider the identification of features of $\bar{\pi}(w_a, x_J)$. The units of profit (dollars) are not identifiable from data on matches. However, because of the transferable utility structure, features of profits can be identified from qualitative match data.[49] Let $z = \{w_a, x_J\}$ be a long vector of bidder and package characteristics. Two variables are said to be complements when $\frac{\partial^2 \bar{\pi}(z)}{\partial z_1 \partial z_2} > 0$, where z_1 and z_2 are two components of z. One theorem shows that the sign of $\frac{\partial^2 \bar{\pi}(z)}{\partial z_1 \partial z_2}$ is identified. Let z_1 and z_2 be characteristics from two different licenses, or one bidder and one license characteristic. Let z_3 and z_4 be two characteristics from different entities as well.[50] Another theorem shows the ratio $\frac{\partial^2 \bar{\pi}(z)}{\partial z_1 \partial z_2} / \frac{\partial^2 \bar{\pi}(z)}{\partial z_3 \partial z_4}$ is identified. Therefore, we can identify the relative importance of sorting on different types of characteristics, such as bidder eligibility, license population and geographic complementarities.[51]

Two very different functions may have the same $\frac{\partial^2 \bar{\pi}(z)}{\partial z_1 \partial z_2}$. Let $z = (z_1, z_2)$. For both $\bar{\pi}(z) = -(z_1 - z_2)^2$ and $\bar{\pi}(z) = 2z_1 z_2$, $\frac{\partial^2 \bar{\pi}(z)}{\partial z_1 \partial z_2} = 2$. Theorems 6.1–6.3 of Fox (2009b) prove that we can identify whether the structural profit $\bar{\pi}(w_a, x_J)$ is higher or lower at any two combinations of bidder and package characteristics. We can distinguish between $-(z_1 - z_2)^2$ and $2z_1 z_2$. This is important for our empirical work because we wish to be able to claim that bidders characteristics, license population and package complementarities are all "goods" that raise output. The formal results show we can identify whether characteristics are "goods", "bads" or something in between, as in $-(z_1 - z_2)^2$.

Note that all of the above identification theorems use only the restrictions embedded in pairwise stability in matches only, Definition 3.6. The proofs do not use extra inequalities that could be added if restrictions from efficiency, Definition 3.8, were in addition employed. So identification in addition to estimation relies only on pairwise stability, not efficiency. In spectrum auctions, the models of implicit collusion in Section 3.5 are good examples of data generating processes that lead to pairwise stability in matches but not efficiency.

In empirical work, we use a semiparametric estimator where we assume $\bar{\pi}_\beta(w_a, x_J)$ is known up to a finite vector of parameters, β.[52] One reason is because numerical optimization of the maximum rank correlation objective function is somewhat challenging (although certainly manageable): we wish to keep a reasonable number of unknown parameters to do as best we can in ensuring we have found the global maximum to the objective function. Another reason is counterfactuals: identification theorems can only

[49] To express the identifiable features as derivatives, these theorems work with only continuous characteristics.

[50] The identities of the entities in the two pairs can be the same or not.

[51] Our measures of geographic complementarities are actually package and not license characteristics, and fall under Theorems 5.3 and 6.1 in Fox (2009b).

[52] Our functional form for profits will be chosen so that objects that are nonparametrically identified, such as $\frac{\partial^2 \bar{\pi}(z)}{\partial z_1 \partial z_2} / \frac{\partial^2 \bar{\pi}(z)}{\partial z_3 \partial z_4}$, pin down the parameters β.

nonparametrically identify features of $\bar{\pi}(w_a, x_J)$ in the support $\{w_a, x_J\}$ found in the population data. Our efficiency counterfactuals, like many counterfactuals in other papers, use our estimates to construct welfare measures for levels of $\{w_a, x_J\}$ not in the support of the data. For extrapolating outside the support of the data, one needs some parametric assumptions. On the other hand, we should emphasize that there were quite large winners in the C-block auction, so the extrapolation from the support of the data to the counterfactuals in our paper in less than counterfactuals in other settings.

6 Estimates of profit functions

Table 5 lists estimates of β in the structural profit function, (5), from the maximum rank correlation estimator.[53] The numbers in parentheses are 95% confidence intervals from subsampling.[54]

6.1 Main estimates

Columns 1 and 2 report estimates using the pairwise stability inequalities in the objective function with matches only (4). As in (5), because matches are qualitative outcomes, we normalize the coefficient on $\text{elig}_a\left(\sum_{j\in J}\text{pop}_j\right)$ to be ± 1. Computationally, we estimate the other parameters β separately for the $+1$ and -1 normalizations and pick the vector with the highest number of satisfied inequalities. The results in columns 1 and 2 of Table 5 show that $+1$ is the correct point estimate. This fits the fact in Figure 2 that bidders with more initial eligibility win packages with more total population. Bidders with higher values win more.

Column 1 includes only one proxy for geographic complementarities: geographic distance, (6). The coefficient of $\beta_{\text{geo.}} = 0.69$ means, at the furthest extrapolation, that if one bidder with the maximum eligibility of 1 were to win the entire United States (population of 1), then the also maximized complementarities (value of $1 \cdot \beta_{\text{geo.}}$) would give a total package value of $1 \cdot 1 + 0.69 \cdot 1 = 1.69$. The value from complementarities corresponds to $0.69/1.69 = 41\%$ of the total package value. This extrapolation is not representative of the variation in the data. Across the 85 winning packages, the standard deviation of $\text{elig}_a\left(\sum_{j\in J}\text{pop}_j\right)$ is 0.03 and the standard deviation of geocomplem._J is 0.024. The means of the two explanatory variables, $\text{elig}_a\left(\sum_{j\in J}\text{pop}_j\right)$ and geocomplem._J , are also nearly identical. Given the similar means and standard deviations, the coefficient estimate $\beta_{\text{geo.}} = 0.69$ implies that variation in the geographic location of licenses, geocomplem._J, is roughly $0.69/1 = 69\%$ as important in explaining the sorting pattern we see as variation in the match between bidders with more eligibility and packages with more population, $\text{elig}_a\left(\sum_{j\in J}\text{pop}_j\right)$. Recall the discussion in Section 2.5 about why

[53] The objective function was numerically maximized using the global optimization algorithm known as differential evolution (Storn and Price, 1997). More than ten runs were performed for all specifications. Not every run converges to a true global maximum. The reported point estimates are the best found maxima, although care was taken to ensure that runner up computed maxima were qualitatively the same as the best found values. While there is an indeterminacy in the point estimates that maximize the step-function objective function, any global maximum is a consistent estimator for the population parameter. We report the first parameter estimate that gives the highest objective function value.

[54] We use 150 replications. For each replication, we use subsets equal to a random sample (without replacement) of 25 of the 85 packages. In unreported results, we take subsets of the data by using only the inequalities corresponding to 120 out of the 480 licenses in the United States. For each license, we evaluate the structural profit functions using the full winning package, whether all of the package's licenses are among the subset of 120 or not. The confidence regions from drawing random licenses are similar to the regions found by drawing packages. Subsampling has not been extended to allow for spatial autocorrelation, so we do not adjust for such correlation, although see Politis and Romano (1993) for related results on the bootstrap.

correlated bidder- and license-specific private values are unlikely to be a good alternative explanation for complementarities, in the C block.

Column 2 adds the two travel based complementarity measures to the specification. Now, not only do we measure the relative importance of $\text{elig}_a \left(\sum_{j \in J} \text{pop}_j\right)$ and complementarities in sorting, we see which measure of complementarities is most important. Total trips using all forms of travel has a coefficient of 0.32, while the coefficient on geocomplem._J, 0.65, remains roughly the same as in column 1. One interpretation is that the geographic pattern of clustering reflects more than just customers wishing to make calls while traveling. Other forms of complementarities include marketing and cost-of-service synergies. The second travel measure, air travel, has a positive coefficient of 0.234, and is statistically insignificant. The point estimate of 0.234 does show an important role for air travel synergies.[55] The standard deviation of air travel complementarities is 0.017, which is only a little smaller than, say, the standard deviation of geographic distance complementarities of 0.024. Given the similar standard deviations, the point estimate shows air travel is important but not as important as geographic distance or the composite measure of travel.

Table 5 lists the percentage of satisfied inequalities at the point estimates, which is a measure of statistical fit. 95% of the inequalities are satisfied. Simply, vertical differences in bidder valuations for licenses and complementarities across licenses in the same package can explain most of the sorting patterns at the pair of licenses level.

Column 2 is our preferred, final set of estimates. The estimates use the pairwise stability in matches inequalities. As previously argued, such inequalities are satisfied under some forms of intimidation.

6.2 Estimators not consistent under intimidation

This section explores two alternative estimators that are inconsistent under intimidation, or implicit collusion. We show that alternative estimators that abstract from intimidatory collusion generate bizarre estimates of bidder valuations.

6.2.1 Estimates with forced transfers of licenses

Columns 3 and 4 of Table 5 consider a variant of the maximum score estimator where bidder a adds a license j to its package J without swapping the license for another. Let $\eta(j)$ be the bidder who wins license j. A corresponding inequality for a's decision not to win j involves an increase in the number of a's licenses by 1 and a decrease in the number of $\eta(j)$'s licenses by 1. Let H be the set of 85 winning bidders. The estimator is any parameter value that maximizes

$$Q^{\text{addmatch}}(\beta) = \sum_{a=1}^{H} \sum_{j=1}^{L} 1\left[a \neq \eta(j)\right] \cdot 1\left[\bar{\pi}_\beta(a, J_a) + \bar{\pi}_\beta\left(\eta(j), J_{\eta(j)}\right) \geq \bar{\pi}_\beta(a, J_a \cup \{j\}) + \bar{\pi}_\beta\left(\eta(j), J_{\eta(j)} \setminus \{j\}\right)\right],$$

where $J_{\eta(j)}$ is the complete package won by the bidder that won license j. The estimator imposes the condition that a did not increase its package by one license because the total structural profit of a and

[55] The point estimate on air travel is a lower bound on the complementarities from air travel, as air travel also appears in the ATS survey and is being double counted. Roughly 75% of trips in the ATS are by car, but the fraction by air increases with distance.

$\eta(j)$ would go down from doing so: it would be less efficient. This condition may be untenable because a may instead have not added the license j to a's package because of a fear of suffering retaliation from bidder $\eta(j)$. Therefore, maximizing $Q^{\text{addmatch}}(\beta)$ produces an inconsistent estimator under the intimidatory equilibria in Brusco and Lopomo (2002) and Engelbrecht-Wiggans and Kahn (2005).

Columns 3 and 4 report a priori unreasonable estimates. In column 3, the coefficient on $\text{elig}_a\left(\sum_{j\in J}\text{pop}_j\right)$ is the wrong sign: negative. Figure 2 shows that bidders with higher values (proxied by initial eligibility) win packages with greater populations, which is inconsistent with this conclusion. Also, the coefficients on complementarities are implausibly large. For example, in column 3 the point estimate of 44.9 shows the structural profit from complementarities is 45 times the (negative) profit from winning an equivalent amount of population (times eligibility). The coefficient in column 4 is an even-larger 168.

6.2.2 Estimates with prices

Columns 5 and 6 of Table 5 report estimates using both matches and prices data. The maximum score objective function is based on Definition 3.5, pairwise stability in both matches and prices. When using price in addition to matches data, the estimator $\hat{\beta}$ is any vector that maximizes the objective function

$$Q^{\text{price}}(\beta) = \frac{1}{L^2}\sum_{i=1}^{L}\sum_{j=1}^{L} 1\left[\eta(i) \neq \eta(j)\right] \cdot 1\left[\bar{\pi}_\beta\left(\eta(i), J_{\eta(i)}\right) - \bar{\pi}_\beta\left(\eta(i), \left(J_{\eta(i)}\setminus\{i\}\right)\cup\{j\}\right) \geq p_i - p_j\right], \tag{8}$$

where p_i is the final, closing price of license i and $\eta(i)$ is defined above. Here, we impose the restriction that bidder $\eta(i)$ prefers to win its package $J_{\eta(i)}$ instead of winning $\left(J_{\eta(i)}\setminus\{i\}\right)\cup\{j\}$, or license j instead of i, at the closing prices to the auction. In other words, we impose the condition that the closing prices explain why bidder $\eta(i)$ won license i instead of j. Rearranging the inequality gives the inequality in Definition 3.5, except that like the other estimators, the private-value terms $\epsilon_{a,j}$ are not included, as is standard for maximum rank correlation estimators. Fixed effects ξ_j cannot be allowed in this type of estimator; for consistency using prices, we must assume the fixed effects are always zero.

Akkus and Hortacsu (2007) were the first to use the estimator with prices and perform a Monte Carlo study. In all of our Monte Carlo experiments (see Appendix B for some) with i.i.d. private-value terms $\epsilon_{a,j}$, the estimator performs extremely well.[56]

Section 3.5 discusses how pairwise stability in both prices and matches does not necessarily hold under intimidatory equilibria. Therefore, if prices are determined by an intimidatory equilibria, then the estimator with prices will likely be inconsistent. In columns 5 and 6, we have included price, measured in *trillions* of dollars. The coefficient on price is normalized to -1.[57] Taken literally, the coefficient on 0.372 on $\text{elig}_a\left(\sum_{j\in J}\text{pop}_j\right)$ in column 5 says that the value of a bidder with eligibility equal to the entire US's population winning the entire US is \$372 billion (although it is not statistically distinct

[56] In the Monte Carlos, we draw the error terms and then compute the fake-data prices using a linear program (Roth and Sotomayor, 1990). Empirically, there is no need to instrument for the endogenous prices because of the simultaneous determination of prices and matches in a tatonnement mechanism. The dependent variable in a maximum score inequality enters the inequality. In (8), the dependent variable in an inequality is the collection $\{J_{\eta(i)}, J_{\eta(j)}, p_i, p_j\}$, even though every element of $J_{\eta(j)}$ does not appear in the inequality. There is no analog to regressing a discrete choice on endogenous prices, unless one worries about omitted variable bias from ξ_j. See Berry, Levinsohn and Pakes (1995) for one approach to dealing with the correlation of p_j and ξ_j.

[57] We estimate the model for a coefficient of $+1$ and find the objective function for -1 is lower.

from zero). Likewise, the value of complementarities from a nationwide license is \$418 billion. These estimates are absurdly high, given that the bids for the C block totalled \$10.1 billion.[58]

How is the model fitting the outcome data? Only the ratio of two parameters that enter structural payoffs linearly, say $\beta_{\text{geo}}/\beta_{\text{price}}$, is identified from an inequality. A high dollar value for non-price package and bidder characteristics is equivalent to saying the estimated coefficient on license price β_{price} would be economically quite small in magnitude if some other characteristic's coefficient was normalized to ± 1. A small coefficient on price is consistent with the finding in Section 2.4 that population and population density, characteristics mostly subsumed into ξ_j, explain most price variation.[59]

As we discussed in Section 2.5, Ausubel, Cramton, McAfee and McMillan (1997) included measures of the runner-up bidder's potential complementarities in a license-level price regression, and found a nonzero but economically-small coefficient. Together, the estimates from (8) and the price regressions suggest that prices may not clear the market in the sense of sorting price-taking bidders to different packages in a competitive market. Pairwise stability in prices and matches, Definition 3.5, may not be satisfied. Definition 3.7, competitive equilibrium, is likely not satisfied for the same reason.

We have suggested intimidation as a non-competitive behavior that is possible in the simultaneous ascending auction. Previous descriptive empirical evidence and our own anecdotes about jump bidding and actual retaliation suggest, at least, that intimidation through signalling is empirically common (Cramton and Schwartz, 2000). Our results suggest that the link between prices and valuations have been contaminated by jump bids and the possibility of retaliation and that an estimator, as in column 2, which is robust to these possibilities is preferable.

7 Policy implications

In this section, we discuss the policy implications of our estimates, including efficiency and the design of future auctions.

7.1 Actual and counterfactual surplus

In this section, we compare the surplus from the observed allocation of licenses to several counterfactual license allocations. The results from the previous section suggest that various measures of complementarities are important determinants of bidder valuations. However, the auction allocated licenses to 85 different bidders, which suggests that an efficiency improvement is possible by grouping licenses into larger winning packages. Furthermore, our earlier results suggest that demand reduction and intimidatory collusion may be present in the auction, which causes or exacerbates this inefficiency.

[58] The annual revenue for the wireless phone industry in 2006, with nine or more active licenses per territory (not just the C block), was \$113 billion. It is unlikely that bidders in 1996 felt the C block had 7–8 times the *profit* potential as the *revenue* from all blocks combined 10 years later.

[59] In Definition 3.5 and the objective function (8), ξ_j does not difference out of the inequality, like it does in Definition 3.6. While the specifications in columns 5 and 6 in Table 5 emphasize comparisons with the columns that do not use price data, we have also estimated specifications including population and population density. The point estimates on the covariates that affect the efficiency of alternative assignments of licenses to bidders are then \$886 billion for winning the entire US's population for $\text{elig}_a\left(\sum_{j\in J}\text{pop}_j\right)$ and \$743 billion for the geographic complementarities geocomplem._J for winning the entire US. These estimates dramatically reinforce the finding that the coefficient β_{price} is to estimated to be economically small.

We use the point estimates from column 2 of Table 5 and the definition of deterministic efficiency in Definition 3.9. For a given allocation of licences, Table 6 reports the value of $\sum_{a \in N} \bar{\pi}_\beta(a, J_a)$. It is easiest to look at the last row of the table first. The last row considers the largest winner (and bidder with the highest initial eligibility), NextWave, winning all 480 licenses in the continental United States. NextWave was initially eligible for 176 million people, or 0.697 of the 1990 population.[60] Therefore, the contribution to total value from NextWave's differential use for licenses is 0.697, or around 0.7. For the three geographic-complementarity proxies, NextWave winning all licenses would maximize these, at a value of 1. So the total differential value (excluding the ξ_j's) of a nationwide license is $1 \cdot 0.697 + \beta_1 \cdot 1 + \beta_2 \cdot 1 + \beta_3 \cdot 1$, where the three β's are the complementarity parameters estimated in column 2 of Table 5. The total value of a nationwide license is then 1.91.[61]

Now consider the other four efficiency evaluations. The first row considers the actual allocation of bidders to licenses in the C block auction. The total surplus generated by the C block is 0.784, quite a bit less than the 1.91 from the nationwide license. The terms in all four columns are considerably smaller than in the bottom column, suggesting that the C block failed to maximize the potential benefits from complementarities. The lost surplus from complementarities due to air travel and ATS trips is particularly large compared to a single nationwide license.

The second row considers an extreme where all 480 licenses are won by separate bidders. There can be no across-license complementarities. We impose that Basic Trading Area licenses (BTAs, those auctioned in the C block) are the lowest level of disaggregation possible. There are 255 C-block bidders (losers and winners). We assortatively match bidders to licenses by initial eligibility for bidders and population for licenses, so that NextWave wins New York, for example. For the $480 - 255 = 225$ licenses with the smallest populations, we say they are won by bidders with the lowest (255th) level of initial eligibility. The results show that the contribution from the $\text{elig}_a \left(\sum_{j \in J} \text{pop}_j \right)$ term is 0.171, smaller than the actual allocation's value of 0.345 by about half. This reflects bidders with low valuations winning licenses.[62]

The third row considers grouping the 480 BTA licenses into 47 packages reflecting the 47 Major Trading Areas (MTAs) in the continental United States used for the 1995 AB spectrum auction. No BTA belongs to more than one MTA. The MTAs are natural groupings centered around large metropolitan areas, but including lots of rural territory as well. MTAs do not correspond exactly to American states. Again, we assortatively match winning bidders to licenses based on initial eligibility and population, so again NextWave wins New York. However, in the C-block auction NextWave won New York and a lot more, so here the contribution from differential bidder values is quite low, at 0.182. However, the design of the MTA boundaries ensures that most local, geographic-distance complementarities are captured. The measure of geographic-distance complementarities rises from 0.47 to 0.72. On the other hand, the MTAs are only local areas, and so a great deal of travel between regions occurs across MTAs.

[60] Under the auction rules, NextWave would have been ineligible to bid on a national license unless it raised its initial eligibility. In our empirical work, eligibility is not used to define a budget set, as it is in the auction rules. Rather, we use initial eligibility as an observed proxy for bidder heterogeneity in valuations. Even if NextWave changed its initial eligibility, its true structural profit function would remain the same.

[61] Our functional forms are used to measure the efficiency of the counterfactuals. Much of the increase in geocomplem._J, (6), arises from combining nearby or even adjacent licenses in the same package. This is especially true as we set $\delta = 4$ in (6) to emphasize nearby licenses in geocomplem._J. Nearby licenses are observed in the winning packages, so estimating the value of a nationwide package uses less out-of-sample extrapolation than may be apparent at first glance.

[62] Many small licenses may maximize the benefits from i.i.d. bidder- and license-specific private values.

The values of the travel geographic-complementarity measures are very small under the MTA scenario. The total value of this allocation is 0.72, not too much lower than the outcome of the actual C block.

The fourth row considers splitting the United States into four large regions: the northeast, midwest, south and west. We assign each of the 47 MTAs to one of these groupings. The midwest is roughly from Pittsburgh to Wichita, and Washington, DC is in the north. We take the four largest winners by initial eligibility and assortatively match them to the four regions by population. NextWave's package is the midwest; it is still slightly smaller in population than the package NextWave won in the C block. The fourth row shows that the contribution from differential bidder valuations is now higher, the measure of geographic-distance complementarities is close to 1, and the two travel measures are about twice as high as the C block values. Thus, a system of four large regions doubles the value from complementarities compared to the C block, and significantly raises the amount of the US population won by high-value bidders. The United States is much bigger than a typical Western European nation; auctioning four licenses is workable plan that captures a large fraction of the maximum possible value, 1.42 out of 1.91.[63]

7.2 Policy implications for bidder anonymity

In 2006, the FCC requested comments on a proposed policy change to make the bidder identities of submitted bids anonymous. The intention of this rule change is to limit intimidation and signalling. A previous draft of this paper addressed one mechanism of signalling other bidders (jump bids) more explicitly. Here, our policy counterfactuals suggest that the simultaneous ascending auction produced inefficiently small winning packages. If bidder anonymity is one way of reducing the scope of intimidation, then it may make the final allocation of licenses to bidders more efficient.

7.3 Competitive scale-reducing economic forces

Intimidation and demand reduction reduce the size of winning packages and make the resulting mobile-phone industry lack true national players. At least three other economic forces that are compatible with competitive bidding work in the same direction. First, bidders may have budget constraints, so that financial constraints from outside of the auction make the auction outcome inefficient. See also Remark 3.22. Second, bidders may run down eligibility by focusing on a smaller license and be unable to switch to a license with a larger population once the price of the smaller license becomes too expensive. Path dependence may lock a bidder into considering only substitute licenses with relatively small populations. See also Remark 3.23. Third, the FCC's rules prevented one bidder from winning more than 98 licenses in the C and F auctions. Only the largest bidder, NextWave, was anywhere close to bumping up against this constraint.

The previous descriptive literature and our bidding anecdotes in Section 2.6 show that bid signalling did go on during the C-block auction (Cramton and Schwartz, 2000). However, measuring the extent or effectiveness of signalling seems difficult when these other factors operate in the same direction. We note all three of the competitive reasons for inefficiently-small winning packages are consistent with larger licenses raising efficiency.

[63] By efficiency, we mean the total structural profit of bidders. Bajari, Fox and Ryan (2008) use demand estimation to measure the willingness of consumers to pay for larger coverage areas.

8 Conclusions

We measure the efficiency of the outcome of an FCC spectrum auction using a structural model of the deterministic portion of bidder valuations. A spectrum auction is a complex dynamic game, with many bidders and many items for sale. The simultaneous ascending auction is potentially susceptible to intimidatory collusion. Intimidation may result in winning packages that are inefficiently small, as bidders split the market to coordinate on paying less to the seller.

Our approach to estimation uses a statistical version of a necessary condition: pairwise stability in matches. Pairwise stability in matches says the sum of structural profit functions from two winning bidders must not be increased by swapping licenses. There are four pieces of evidence suggesting that this condition is likely to hold in simultaneous ascending auctions: experimental evidence, the lack of post-auction resale, theoretical analysis of implicit collusion, and theoretical analysis of demand reduction. Intimidatory collusion under symmetric private values is sustained by bid signalling and threats of retaliation by reverting to straightforward bidding.

We employ a matching maximum score or maximum rank correlation estimator, which maximizes the number of inequalities that satisfy pairwise stability. A previous version of this paper was the first empirical application of the estimator. While our application focuses on a spectrum auction, the pairwise-stability condition behind our estimator is applicable to some other auctions, including those with efficient collusion.

There are more potential packages than the atoms in the universe in the C block. The estimator is computationally simple as it avoids evaluating all possible counterfactual packages, which would be needed for a likelihood-based approach. Also, the estimator controls for additive, license-specific unobservables. We estimate valuations using pairwise stability, which uses data on only the matches between bidders and licenses, not the closing prices. Indeed, we show that two alternative maximum score estimators, including one with prices, are not consistent under intimidation. These estimators produce bizarre estimates using the C-block data.

Our estimates empirically validate the FCC's focus on complementarities when designing the mechanism for allocating radio spectrum. Also, the spectrum auction itself produces a much higher surplus than awarding licenses through the FCC's prior practices, such as lotteries. However, we find that the final allocation of licenses was allocatively inefficient before considering private values. Total surplus is nearly doubled by awarding four large, regional licenses to the four highest-value bidders. A nationwide license would capture even more of the total surplus. To some degree, our findings validate the European approach of offering nationwide licenses and hence capturing all geographic complementarities.

A Demand reduction and pairwise stability, without complementarities

Demand reduction is studied by Ausubel and Cramton (2002) for the case of sealed bid auctions of multiple homogeneous items. In a simultaneous ascending auction, demand reduction is consistent with straightforward bidding by forward-looking agents. Kagel and Levin (2001) and List and Lucking-Reiley (2000) find substantial demand reduction in experiments. This section considers demand reduction, but in a world without complementarities because of a need to refer to a Milgrom (2000) theorem. Because complementarities are the focus of our empirical work, we place this material in an appendix, although we feel the results are interesting for the estimation method. Also, Milgrom requires bidders to bid straightforwardly, so this analysis does not distinguish between publicly and privately observed information and so does not work with Bayesian Nash equilibria. On the other hand, Brusco and Lopomo (2002) and Engelbrecht-Wiggans and Kahn (2005) show that competitive bidding is a Bayesian Nash equilibrium to simultaneous ascending auctions.

Consider bidders a and b competing for two licenses 1 and 2. Use the shorthand notation $\pi_a^{1,2}$ for $\pi_a(\{1,2\})$. Let the profits of bidders a and b for the three possible packages be as listed in Table 7, case 1. Bidder a has a higher value for all packages. Bidder b has decreasing returns to scale: there is no incremental value from winning both licenses.

If both bidders bid straightforwardly in a simultaneous ascending auction, and ignoring minimum bid increments, a will win both licenses at prices equal to b's values: $p_1 = \pi_b^1$ and $p_2 = \pi_b^2$. However, if a reduces its demand and lets b win item 2 at $p_2 = 0$, a can win item 1 at $p_1 = 0$. Bidder b accepts this because it has a demand for only one license and prefers 2 to 1. The demand reduction outcome is inefficient: total structural profit is maximized by having a win both items. However, when a wins 1 and b wins 2, $\pi_a^1 + \pi_b^2 > \pi_a^2 + \pi_b^1$, so that the total structural profit cannot be increased with license swaps. Definition 3.6 is satisfied as the bidders disagree on the profit ranking of the licenses. One can use the zero prices to show Definition 3.5 is satisfied as well.

Now we will argue that the example does not rely on bidder disagreement over the profit ranking. Case 2 in Table 7 changes b's structural profits so that a and b agree on the profit ranking of licenses 1 and 2: $\pi_b^1 \geq \pi_b^2$. At the beginning of the auction, with $p_1 = p_2 = 0$, bidder b will bid on item 1 as b prefers 1 and has a demand for only one item. Only at a price $p_1^\star$ such that $\pi_b^1 - p_1^\star = \pi_{b,2}$ will b accept winning license 2 instead of 1. If $\pi_a^1 - p_1^\star > \pi_a^2$, then substituting in $p_1^\star = \pi_b^1 - \pi_b^2$ to $\pi_a^1 - p_1^\star > \pi_a^2$ again gives $\pi_a^1 + \pi_b^2 > \pi_a^2 + \pi_b^1$. Definitions 3.5 and 3.6 are satisfied.

What if in case 2, $\pi_a^1 - p_1^\star = \pi_a^1 - \left(\pi_b^1 - \pi_b^2\right) < \pi_a^2$? If a finds it profitable to reduce its demand, a will reduce its demand on license 1 and win 2, leaving $\pi_a^2 + \pi_b^1 > \pi_a^1 + \pi_b^2$. Again, $p_1^\star$ is set, by straightforward bidding, to make a and b coordinate on a pairwise stable outcome. Definitions 3.5 and 3.6 are satisfied. The points made in this example are more general.[64]

Theorem A.1. *Consider straightforward bidding in a simultaneous ascending auction with demand reduction. Under the tatonnement conditions of Milgrom (2000), the outcome is a pairwise stable outcome to a matching game where the maximum number, or quota, of licenses that a bidder can win is the number of licenses the bidder won in the outcome. Both Definitions 3.5 and 3.6 are satisfied.*

Proof. Let the allocation portion of the demand reduction outcome be A, and let bidder a's wining package be J_a. For all bidders a, redefine a's profits for a package J to be negative infinity if J has more licenses than J_a. $\pi_a(J) = -\infty$ for $|J| > |J_a|$. Then Milgrom's tatonnement process theorems (Theorems 2 and 3 in Milgrom) show that the simultaneous ascending auction will find a competitive equilibrium (core outcome) of the economy with the truncated profit functions. Pairwise stability,

[64] The conditions for Milgrom's tatonnement process theorem rule out complementarities, in part to avoid the exposure problem. Definition 3.6 requires only that total structural profit not be raised by swapping licenses. It is compatible with many forms of the exposure problem. See Remark 3.24.

Definition 3.5, is implied by being in the core of the economy with truncated profit functions. As the swaps considered in Definition 3.5 do not change the number of licenses won by any bidder, the profits under the swaps are the same as under the pre-truncated profit functions. So the outcome is pairwise stable under a matching game where bidders cannot add additional licenses to their package. □

Under demand reduction, the outcome may not be efficient, but there is no reason to believe that there exist swaps of licenses that would raise total structural profits.[65] The theorem does not explain how much demand reduction will go on: the unilateral incentive to reduce demand requires knowledge that another bidder has strong decreasing returns to scale.[66] Given unilateral demand reduction and straightforward bidding, the theorem shows that pairwise stability will result. Straightforward bidding under private values is a perfect Bayesian Nash equilibria (Brusco and Lopomo, 2002). If rivals are bidding competitively, only unilateral demand reduction followed by straightforward bidding can raise profits.[67]

B Monte Carlo for estimator with both matches and price data

Fox (2009a) presents Monte Carlo studies showing that the finite-sample performance of the matches-only maximum score estimator is pretty good. However, for a small number of bidders and licenses and a high variance of the error term, the estimator uses data only on matches can have high bias and root mean squared error (RMSE) in a finite sample, as random noise from the $\epsilon_{a,j}$ terms dominates the matching, leaving little signal in the sorting pattern seen in the data. Like similar results in Akkus and Hortacsu (2007), Table 8 reports results from a Monte Carlo study from a one-to-one, two-sided matching market. Each bidder a matches to at most one license j, and the payoff of a bidder is $\bar{\pi}_\beta(a,j)+\epsilon_{a,j} = x_{1,a}x_{1,j}+\beta x_{2,a}x_{2,j}$. There are two characteristics for bidders and two for licenses, with characteristics for each side distributed as a bivariate normal with means $(10,10)$, variances $(1,1)$ and covariance 0.5. The errors are i.i.d. normal with standard deviations listed in the table. For each auction we draw observable characteristics and unobservable error terms and compute an equilibrium assignment and vector of prices using the primal and dual linear programs for two-sided matching (Koopmans and Beckmann, 1957; Shapley and Shubik, 1972). The true β is 1.5. The example is chosen to make using only matches look bad: there is not much signal about $\bar{\pi}_\beta$ in the sorting patterns if the realized matches are visually plotted in characteristic space, especially in the second half of the table where the standard deviation of $\epsilon_{a,j}$ is five times higher than in the upper part of the table. Note that for the C block the map in Figure 1 shows that there are clear sorting patterns; this Monte Carlo study makes using match data bad to show the potential advantages of using price data. The finite-sample bias and RMSE are always much lower with continuous transfer data, even though the data on matches alone are uninformative. For all four cases the bias is small on an absolute scale for small samples, and for three of the four cases the RMSE is low compared to the true value of 1.5.

Table 8 shows a major advantage of using price data: the finite-sample performance is much better if prices are generated from a tatonnement process. There are several advantages to using only match data, even if the prices are generated by a tatonnement process. This first is transparency: there is only one type of dependent variable, so inferring parameters from the US map of winning bidders is straightforward. With two types of dependent variables, it is not as clear where identification arises from. The second is robustness. In this paper,

[65] Under demand reduction bidders bid as if they are constrained to only win at most some fixed quota of licenses. This is similar to the constraints on licenses already present in a matching game: each license can be won by only one bidder. Having quotas on both licenses and bidders has analogs in other matching markets such as marriage, where both men and women can have only one spouse at a time.

[66] The initial eligibilities of other bidders are known before bidding starts. Therefore, some forms of decreasing returns are public knowledge. Further, Cramton (2006) interprets the purchase of spectrum in a small, quick, post-auction sale (a bidder did not make its payments) by NextWave as evidence that NextWave was reducing its demand during the auction.

[67] Ausubel and Schwartz (1999) use a Nash concept to compute the unique subgame perfect equilibrium to a complete information, alternating-bid, ascending auction of a single divisible good. The equilibrium involves demand reduction. Bidders have a common valuation for all units of the divisible good, so pairwise stability is satisfied.

we review models where prices are not generated by a tatonnement process, but the matches are still robust to pairwise swaps.

References

Akkus, Oktay and Ali Hortacsu, "The Determinants of Bank Mergers: A Revealed Preference Analysis," March 2007. University of Chicago working paper.

Athey, Susan and Jonathan Levin, "Information and Competition in U. S. Forest Service Timber Auctions," *Journal of Political Economy*, April 2001, *109* (2), 375–417.

_ **and Philip A. Haile**, "Nonparametric Approaches to Auctions," *Handbook of Econometrics*, 2008, *6.*

_ **, Jonathan Levin, and Enrique Seira**, "Comparing Open and Sealed Bid Auctions: Theory and Evidence from Timber Auctions," December 2008. MIT working paper.

Ausubel, Lawrence M. and Jesse A. Schwartz, "The Ascending Auction Paradox," 1999. University of Maryland working paper.

_ **and Peter Cramton**, "Demand Reduction and Inefficiency in Multi-Unit Auctions," July 2002. working paper.

_ **,** _ **, R. Preston McAfee, and John McMillan**, "Synergies in Wireless Telephony: Evidence from the Broadband PCS Auctions," *Journal of Economics and Management Strategy*, Fall 1997, *6* (3), 497–527.

Avery, Christopher, "Strategic Jump Bidding in English Auctions," *Review of Economic Studies*, 1998, *65*, 185–210.

Bajari, Patrick and Ali Hortaçsu, "Are structural estimates of auction models reasonable? Evidence from experimental data," *Journal of Political Economy*, 2005, *113* (4), 703–741.

_ **and Lixin Ye**, "Deciding Between Competition and Collusion," *Review of Economics and Statistics*, 2003, *85* (4), 971–989.

_ **, Jeremy T. Fox, and Stephen Ryan**, "Evaluating Wireless Carrier Consolidation Using Semiparametric Demand Estimation," *Quantitative Marketing and Economics*, 2008, *6* (4), 299–338.

Banks, Jeffrey, Mark Olson, David Porter, Stephen Rassenti, and Vernon Smith, "Theory, experiment and the federal communications commission spectrum auctions," *Journal of Economic Behavior and Organization*, 2003, *51*, 303–350.

Berry, Steven T., James Levinsohn, and Ariel Pakes, "Automobile Prices in Market Equilibrium," *Econometrica*, July 1995, *63* (4), 841–890.

Brusco, Sandro and Giuseppe Lopomo, "Collusion via Signalling in Simultaneous Ascending Bid Auctions with Heterogenous Objects, with and without Complementarities," *Review of Economic Studies*, April 2002, *69* (2), 407–436.

Bulow, Jeremy, Jonathan Levin, and Paul Milgrom, "Winning Play in Spectrum Auctions," February 2009. working paper.

Campo, Sandra, "Asymmetry and Risk Aversion within the Independent Private Paradigm: The Case of the Construction Procurements," 2006. University of North Carolina working paper.

Cantillon, Estelle and Martin Pesendorfer, "Combination Bidding in Multiple Unit Auctions," 2006. Université Libre de Bruxelles working paper.

Chapman, James T.E., David McAdams, and Harry J. Paarsch, "Bounding Best-Response Violations in Discriminatory Auctions with Private Values," 2006. Duke University working paper.

Choo, Eugene and Aloysius Siow, "Who Marries Whom and Why," *The Journal of Political Economy*, November 2006, *114* (1), 175–201.

Coase, Ronald H, "The Federal Communications Commission," *Journal of Law and Economics*, 1959, *2*, 1–40.

Cramton, Peter, "Simultaneous Asending Auctions," in Peter Cramton, Yoav Shoham, and Richard Steinberg, eds., *Combinatorial Auctions*, MIT Press, 2006.

_ **and Jesse A. Schwartz**, "Collusive Bidding: Lessons from the FCC Spectrum Auctions," *Journal of Regulatory Economics*, May 2000, *17* (3), 229–252.

_ **and** _ , "Collusive Bidding in the FCC Spectrum Auctions," *Contributions to Economic Analysis & Policy*, 2002, *1* (1), Article 11.

_ **, Yoav Shoham, and Richard Steinberg**, *Combinatorial Auctions*, MIT Press, 2006.

Crawford, Vincent P. and Elsie Marie Knoer, "Job Matching with Heterogeneous Firms and Workers," *Econometrica*, 1981, *49* (2), 437–450.

Dagsvik, John K., "Aggregation in Matching Markets," *International Economic Review*, February 2000, *41* (1), 27–57.

Daniel, Kent and David Hirshleifer, "A Theory of Costly Sequential Bidding," 1998. University of Michigan Business School Working Paper No. 98028.

Day, Robert and Paul Milgrom, "Core-Selecting Package Auctions," *International Journal of Game Theory*, June 2007.

Demange, Gabrielle, David Gale, and Marilda A. Oliveira Sotomayor, "Multi-Item Auctions," *The Journal of Political Economy*, August 1986, *94* (4), 863–872.

Donald, Stephen G. and Harry J. Paarsch, "Piecewise Pseudo-maximum Likelihood Estimation in Empirical Models of Auctions," *International Economic Review*, 1993, *34* (1), 121–148.

— **and** — , “Identification, Estimation, and Testing in Parametric Empirical Models of Auctions within the Independent Private Values Paradigm,” *Econometric Theory*, 1996, *12* (3), 517–567.

Edelman, Benjamin, Michael Ostrovsky, and Michael Schwarz, “Internet Advertising and the Generalized Second Price Auction: Selling Billions of Dollars Worth of Keywords,” *American Economic Review*, March 2007, *97* (1).

Elyakime, Bernard, Jean-Jaques Laffont, Patrice Loisel, and Quang Vuong, “First-Price Sealed-Bid Auctions with Secret Reservation Prices,” *Annales d’Economie et de Statistique*, 1994, *34* (0), 115–141.

Engelbrecht-Wiggans, Richard and Charles M. Kahn, “Low revenue equilibria in simultaneous auctions,” *Management Science*, 2005.

Fevrier, Philippe, Raphaelle Préget, and Michael Visser, “Econometrics of Share Auctions,” 2009. CREST working paper.

Flambard, Véronique and Isabelle Perrigne, “Asymmetry in Procurement Auctions: Some Evidence from Snow Removal Contracts,” *Economic Journal*, 2009.

Fox, Jeremy T., “Estimating Matching Games with Transfers,” August 2009. University of Chicago working paper.

— , “Identification in Matching Games,” June 2009. University of Chicago working paper.

— **and Hector Perez-Saiz**, “Mobile Phone Mergers and Market Shares: Short Term Losses and Long Term Gains,” September 2006. NET Institute Working Paper 06-16.

Graham, Daniel A. and Robert C. Marshall, “Collusive Bidder Behavior at Single-Object Second-Price and English Auctions,” *The Journal of Political Economy*, 1987, *95* (6), 1217–1239.

Guerre, Emmanuel, Isabelle Perrigne, and Quang Vuong, “Optimal Nonparametric Estimation of First-Price Auctions,” *Econometrica*, May 2000, *68* (3), 525–574.

Haile, Philip A. and Elie Tamer, “Inference with an Incomplete Model of English Auctions,” *Journal of Political Economy*, February 2003, *111* (1), 1–51.

Han, Aaron K., “Nonparametric Analysis of a Generalized Regression Model: The Maximum Rank Correlation Estimator,” *Journal of Econometrics*, 1987, *35*, 303–316.

Hatfield, John William and Paul R. Milgrom, “Matching with Contracts,” *The American Economic Review*, September 2005, *95* (4), 913–935.

Hendricks, Kenneth, Joris Pinkse, and Robert H. Porter, “Empirical Implications of Equilibrium Bidding in First-Price, Symmetric, Common Value Auctions,” *Review of Economic Studies*, 2003, *70*, 114–145.

Hong, Han and Matthew Shum, “Econometric models of asymmetric ascending auctions,” *Journal of Econometrics*, 2003, *112*, 327–358.

Horowitz, Joel L., "A Smoothed Maximum Score Estimator for the Binary Response Model," *Econometrica*, May 1992, *60* (3), 505–551.

Hortacsu, Ali, "Mechanism Choice and Strategic Bidding in Divisible Good Auctions: An Empirical Analysis of the Turkish Treasury Auction Market," February 2002. University of Chicago working paper.

Jofre-Bonet, Mireia and Martin Pesendorfer, "Estimation of a Dynamic Auction Game," *Econometrica*, September 2003, *71* (5), 1443–1489.

Kagel, John H. and Dan Levin, "Behavior in Multi-Unit Demand Auctions: Experiments with Uniform Price and Dynamic Vickrey Auctions," *Econometrica*, 2001, *69* (2), 413–454.

Kastl, Jakub, "Discrete Bids and Empirical Inference in Divisible Good Auctions," 2009. Stanford University working paper.

Kelso, Alexander S. and Vincent P. Crawford, "Job Matching, Coalition Formation, and Gross Substitutes," *Econometrica*, November 1982, *50* (6), 1483–1504.

Koopmans, Tjalling C. and Martin Beckmann, "Assignment Problems and the Location of Economic Activities," *Econometrica*, January 1957, *25* (1), 53–76.

Krasnokutskaya, Elena, "Identification and Estimation of Auction Models with Unobserved Heterogeneity," August 2009. University of Pennsylvania working paper.

Kwasnica, Anthony M. and Katerina Sherstyuk, "Collusion and Equilibrium Selection in Auctions," *The Economic Journal*, 2007, *117* (516), 120–145.

Leonard, Herman B., "Elicitation of Honest Preferences for the Assignment of Individuals to Positions," *The Journal of Political Economy*, June 1983, *91* (3), 461–479.

List, John A. and David Lucking-Reiley, "Demand Reduction in Multiunit Auctions: Evidence from a Sportscard Field Experiment," *The American Economic Review*, 2000, *90* (4), 961–972.

Manski, Charles F., "Maximum Score Estimation of the Stochastic Utility Model of Choice," *Journal of Econometrics*, 1975, *3*, 205–228.

Matzkin, Rosa L., "Nonparametric identification and estimation of polychotomous choice models," *Journal of Econometrics*, 1993, *58*, 137–168.

McAfee, R. Preston and John McMillan, "Bidding Rings," *The American Economic Review*, 1992, *82* (3), 579–599.

_ **and** _ , "Analyzing the Airwaves Auction," *Journal of Economic Perspectives*, Winter 1996, *10* (1), 159–175.

Milgrom, Paul, "Putting Auction Theory to Work: The Simultaneous Ascending Auction," *Journal of Political Economy*, 2000, *108* (2), 245–272.

Milgrom, Paul R. and Robert J. Weber, "A Theory of Auctions and Competitive Bidding," *Econometrica*, 1982, *50* (5), 1089–1122.

Moreton, Patrick S. and Pablo T. Spiller, "What's In the Air: Interlicense Synergies in the Federal Communications Commission's Broadband Personal Communication Service Spectrum Auctions," *Journal of Law and Economics*, October 1998, *41* (2, Part 2), 677–725.

Murray, James B., *Wireless Nation*, Perseus Publishing, 2001.

Newey, Whitney K. and Daniel McFadden, "Large Sample Estimation and Hypothesis Testing," in "Handbook of Econometrics," Vol. IV, Elsevier, 1994, chapter 36, pp. 2111–2245.

Paarsch, Harry J. and Han Hong, *An Introduction to the Structural Econometrics of Auction Data*, MIT Press, 2006.

Politis, Dimitris N. and Joseph P. Romano, "Nonparametric Resampling for Homogeneous Strong Mixing Random Fields," *Journal of Multivariate Analysis*, 1993, *47*, 301–328.

_ , _ , and Michael Wolf, *Subsampling*, Springer, 1999.

Roth, Alvin E. and Marilda A. Oliveira Sotomayor, *Two-Sided Matching: A Study in Game-Theoretic Modeling and Analysis* Econometric Society Monographs, Cambridge University Press, 1990.

Salant, David J., "Up in the Air: GTE's Experience in the MTA Auction for Personal Communication Services Licenses," *Journal of Economics and Management Strategy*, Fall 1997, *6* (3), 549–572.

Sattinger, Michael, "Differential Rents and the Distribution of Earnings," *Oxford Economic Papers*, March 1979, *31* (1), 60–71.

Shapley, Lloyd S. and Martin Shubik, "The assignment game I: the core," *International Journal of Game Theory*, 1972, *1*, 111–130.

Sherman, Robert P., "The Limiting Distribution of the Maximum Rank Correlation Estimation," *Econometrica*, January 1993, *61* (1), 123–137.

_ , "Maximal Inequalities for Degenerate U-Processes with Applications to Optimization Estimators," *The Annals of Statistics*, March 1994, *22* (1), 439–459.

Sherstyuk, Katerina, "On competitive equilibria with common complementarities," *Mathematical Social Sciences*, August 2003, *46* (1), 55–62.

Sørensen, Morten, "How Smart is Smart Money? A Two-Sided Matching Model of Venture Capital," *Journal of Finance*, December 2007, *LXII* (6), 2725–2762.

Storn, Rainer and Kenneth Price, "Differential Evolution - A Simple and Efficient Heuristic for Global Optimization over Continuous Spaces," *Journal of Global Optimization*, 1997, *115*, 341–359.

Subbotin, Viktor, "Asymptotic and bootstrap properties of rank regressions," November 2007. Northwestern University working paper.

Weiss, Yoram, "Sharing Rules," 2007. unpublished book chapter.

Wolak, Frank A., "Quantifying the Supply-Side Benefits from Forward Contracting in Wholesale Electricity Markets," *Journal of Applied Econometrics*, November 2007, *22*, 1179–1209.

Zheng, Charles Z., "Jump Bidding and Overconcentration in Decentralized Simultaneous Ascending Auctions," 2006. Iowa State University working paper.

Figure 1: Map of the licenses won by the top 12 winning bidders and bidders who won only one license

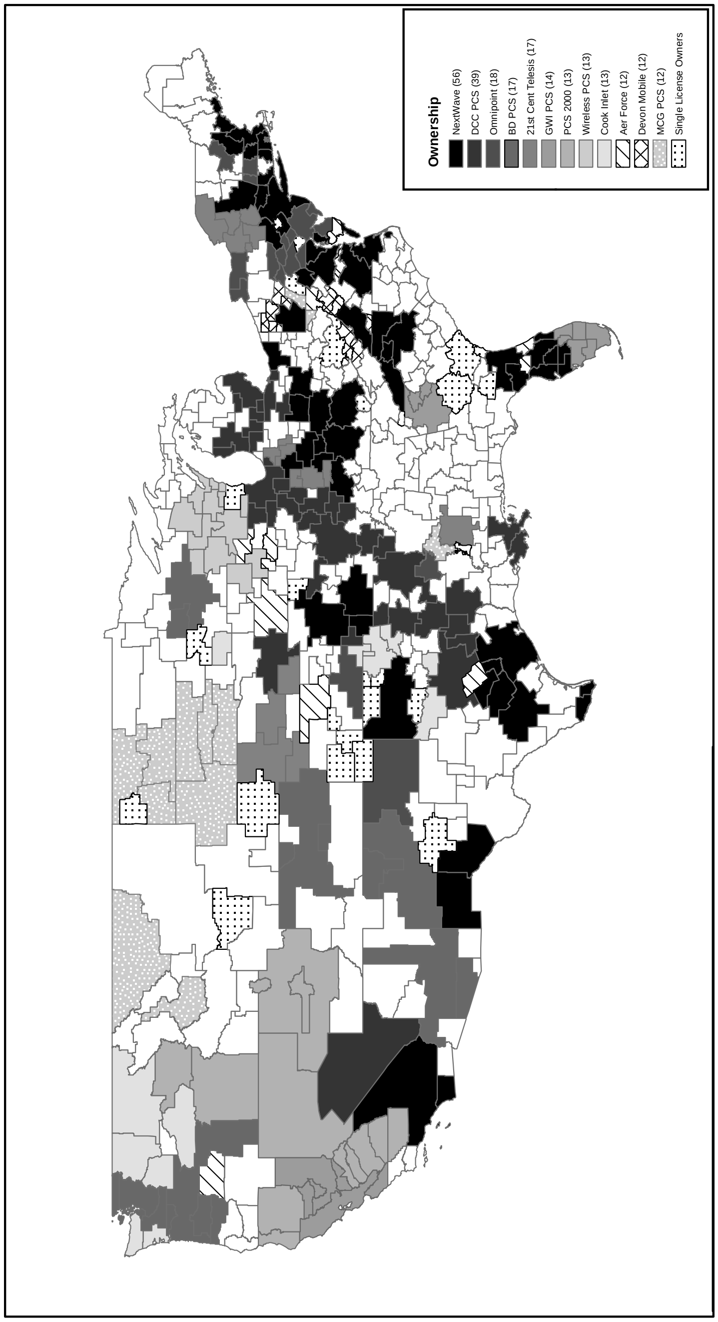

Figure 2: Log of a winning package's population by the log of the winning bidder's initial eligibility

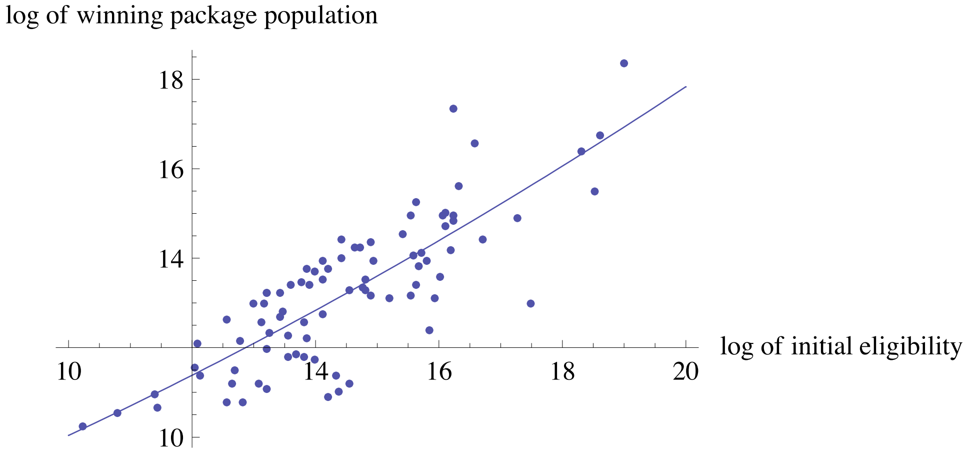

Figure 3: The Number of Jump Bids per Round

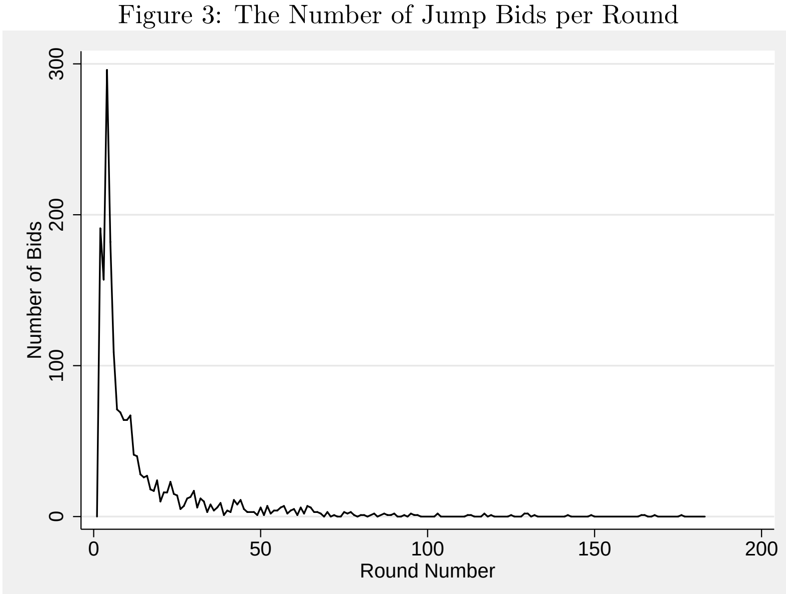

Table 1: Characteristics of winners and non-winners of packages in the continental United States

	Winners		non-Winners	
Characteristic	Mean	Stand. dev.	Mean	Stand. dev.
initial eligibility (millions of residents)	9.77	27.2	5.15	18.5
assets ($ millions)	13.1	21.8	12.3	18.8
revenues ($ millions)	40.7	67.8	39.9	72.3
# of licenses won	5.3	7.1	0	0
# of licenses ever bid on	38.5	70.6	14.8	44.2
# of bidders	85		170	

Table 2: Total closing prices and population of the 85 winning packages

Characteristic	Mean	Standard deviation	Min	Max
Total price ($millions)	116.2	496.1	0.102	4,201
Total population in 1994 (millions)	2.91	10.93	0.027	93.8

Table 3: Experimental evidence on pairwise stability from Banks et al. (2003)

Pairwise Stability in Matches Only, Def. 3.6		Pairwise Stability in Matches and Prices, Def. 2	
% of Inequalities Satisfied Mean Across Auctions	% of 52 Auctions Where 100% Inequalities Satisfied	% of Inequalities Satisfied Mean Across Auctions	% of 52 Auctions Where 100% Inequalities Satisfied
95.1% std. dev. 11.1%	55.8% N/A	88.4% std. dev. 11.6%	9.6% N/A

Banks et al. (2003) conducted 52 simultaneous ascending auctions with varying numbers of bidders and varying true profit functions. We observe the profit functions and use the data to check the fraction of times Definition 3.6 and Definition 3.5 are satisfied. We also report the percentage of the 52 auctions where the restrictions were satisfied by 100% of the inequalities.

Table 4: Winning-package-population weighted means of and correlation matrix for geographic-complementarity proxies

Characteristic	Mean	Standard Deviation	Min	Max
Population / distance two markets in a package	0.0055	0.00232	0	0.198
Trips between markets in a package in the American Travel Survey	0.0032	0.0201	0	0.150
Total trips between airports in markets in a package (thousands)	0.0023	0.0166	0	0.182

Correlation matrix	Geo dist.	ATS Trips	Air travel
Population / distance two markets in a package	1	0.827	0.564
Trips between markets in a package in the American Travel Survey	0.827	1	0.354
Total trips between airports in markets in a package (thousands)	0.564	0.354	1

The sample is the 85 winning packages in the continental United States. The formulas for these measures are equations (6) and (7).

Table 5: Maximum score estimates of profit function parameters

Characteristic	(1)	(2)	(3)	(4)	(5)	(6)
Type of inequalities	Swaps of 1 license each		Transfer of 1 license		Swaps of 1 license w/ prices	
Consistent under demand reduction & intimidation?	Y	Y	N	N	N	N
Population * bidder eligibility	+1	+1	-1	+1	0.372 (-0.143,0.476)	0.357 (-0.156,0.446)
	superconsistent					
Population / distance two markets in a package	0.687 (0.511,0.917)	0.654 (0.464,0.826)	44.9	168 (139,247)	0.418 (0.133,0.569)	0.364 (0.047,0.465)
Trips between markets in a package in the American Travel Survey		0.321 (0.138,0.460)		-8.15 (-13.2,-7.60)		0.126 (-0.133,0.180)
Total trips between airports in markets in a package (thousands)		0.234 (-0.081,0.331)		-1.05 (-1.79,2.17)		0.201 (0.069,0.353)
Price (in *trillions*)					-1	-1
					superconsistent	
# possible inequalities	111,192		40,320		222,384	
% inequalities correct	0.944	0.950	0.930	0.940	0.911	0.918

Eligibility, population and all three complementarity proxies range from 0 to 1. These estimates include licenses only from the continental United States. The maximum score / maximum rank correlation estimators are described in the text. The parentheses are 95% confidence intervals computed using subsampling. Subsampling uses 150 replications, 25 packages per replication and a convergence rate of $\sqrt{\#\text{ packages}}$, as found by Sherman (1993). For each 25 packages, we use only the inequalities where all licenses are from the sampled packages. Subsampled confidence regions are not necessarily symmetric around the point estimate. Parameters that can take on only a finite number of values (here ± 1) converge at an arbitrarily fast rate; they are superconsistent.

Table 6: Counterfactual (differential) efficiency from five allocations

Allocation	$\text{elig}_a \left(\sum_{j \in J} \text{pop}_j \right)$	Geographic distance	Air travel	ATS trips	Total
C block: 85 winning packages	$1 \cdot 0.345 =$ 0.345	$0.645 \cdot 0.470 =$ 0.307	$0.234 \cdot 0.197 =$ 0.046	$0.321 \cdot 0.268 =$ 0.086	0.784
All 480 licenses won by different bidders	$1 \cdot 0.171 =$ 0.171	$0.645 \cdot 0 =$ 0	$0.234 \cdot 0 =$ 0	$0.321 \cdot 0 =$ 0	0.171
Each 47 MTAs separate package	$1 \cdot 0.182 =$ 0.182	$0.645 \cdot 0.722 =$ 0.472	$0.234 \cdot 0.037 =$ 0.009	$0.321 \cdot 0.168 =$ 0.054	0.717
Four large, regional licenses (top four of the 85 actual winners win)	$1 \cdot 0.513 =$ 0.513	$0.645 \cdot 0.964 =$ 0.631	$0.234 \cdot 0.379 =$ 0.088	$0.321 \cdot 0.586 =$ 0.188	1.42
Nationwide license for entire United States (NextWave wins)	$1 \cdot 0.697 =$ 0.697	$0.645 \cdot 1 =$ 0.645	$0.234 \cdot 1 =$ 0.234	$0.321 \cdot 1 =$ 0.321	1.91

Eligibility, population and all three complementarity proxies range from 0 to 1. These counterfactuals use the point estimates from column 2 of Table 5. Only licenses in the continental United States are considered. For the 47 MTAs in the continental United States as well as the four large regions, the top winners in the actual auction are assortatively matched to the counterfactual packages in order of population. For example, NextWave always wins the package with the highest population.

Table 7: Payoffs for two-bidder examples of demand reduction

	Bidder a	Bidder b, case 1	Bidder b, case 2
License 1	$\pi_a^1 \geq \pi_a^2$	$\pi_b^1 \leq \pi_a^1, \pi_b^1 \leq \pi_b^2$	$\pi_b^1 \leq \pi_a^1, \pi_b^1 \geq \pi_b^2$
License 2	$\pi_a^2 \leq \pi_a^1$	$\pi_b^2 \leq \pi_a^2, \pi_b^2 \geq \pi_b^1$	$\pi_b^2 \leq \pi_a^2, \pi_b^2 \leq \pi_b^1$
Both 1 & 2	$\pi_a^{1,2} = \pi_a^1 + \pi_a^2$	$\pi_b^{1,2} = \max\left\{\pi_b^1, \pi_b^2\right\}$	$\pi_b^{1,2} = \max\left\{\pi_b^1, \pi_b^2\right\}$

Table 8: Maximum score Monte Carlo: comparing using data on only matches to data on both matches and prices under tatonnement assumptions with noise dominating matches, true value is 1.5

				Matches		Matches + Prices	
# bidders	# licenses per auction	# spectrum auctions	error std. dev.	bias	RMSE	bias	RMSE
30	30	1	1	0.587	1.93	0.005	0.03
10	10	10	1	0.330	1.05	0.009	0.07
30	30	1	5	1.22	4.22	0.02	0.09
10	10	10	5	1.69	7.36	-0.02	0.446