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# Paid Placement: Advertising and Search on the Internet\*

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## Abstract

Paid placement, where advertisers bid payments to a search engine to have their products appear next to keyword search results, has emerged as a predominant form of advertising on the Internet. This paper studies a product-differentiation model where consumers are initially uncertain about the desirability of and valuation for different sellers' products, and can learn about a seller's product through a costly search. In equilibrium, a seller bids more for placement when his product is more relevant for a given keyword, and the paid placement of sellers by the search engine reveals information about the relevance of their products. This results in efficient (sequential) search by consumers and increases total output.

**Keywords:** Paid placement, Advertising, Auction, E-commerce, Search.

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## 1. INTRODUCTION

Paid placement, online advertising in which links to advertisers' products appear next to keyword search results, has emerged as a predominant form of Internet advertising, generating \$8.2 billion ad revenues in 2005 (Satagopan et al. 2005). Under paid placement advertising, sellers (advertisers) bid payments to a search engine to be placed on its "recommended" list for a keyword search. A group of advertisers who bid more than the rest are selected for placement, and their positions of placement reflect their order of bids, with the highest bidder placed at the top position. The rapid growth of paid placement advertising has made it one of the most important Internet institutions, and has led to enormous commercial successes for search engines. For example, Google, which derives most of its revenue from paid placement advertising, has a market capitalization of \$123.24 billion; by contrast, the combined market capitalization of the big three US auto manufacturers is \$83.54 billion.<sup>1</sup> Despite the popularity and importance of the phenomenon, the economics of the online market with paid placement advertising has received little formal study. How do sellers form their bidding strategies? How does paid placement advertising affect consumer search and welfare? And what determines the revenues of a search engine in equilibrium? We develop a market equilibrium model that addresses these questions in this paper.

We consider a game in which differentiated sellers first bid payments to a search engine to be placed on its list of search outcomes associated with a particular keyword (product). Only a small number of sellers are listed due to limited number of positions available on the list. Sellers differ in their "relevance," which we model as the probability that any consumer will find a seller's product to be her desired variety. Each consumer is ex ante uncertain about which seller's product will match her preference and how much she is willing to pay for the product. By searching (inspecting) a seller's website, the consumer will learn about the seller's product and price. But there are search costs to inspect a seller's website; hence a consumer needs to form a search strategy, and, if a search yields a match, a purchase

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<sup>1</sup>These figures are based on stock quotes on September 21, 2006. The market capitalizations for General Motors, Ford, DaimlerChrysler AG are \$17.52 billion, \$14.75 billion, and \$51.27 billion, respectively.

strategy. On the other hand, sellers take into account consumers' search and purchase behavior when choosing pricing and bidding strategies. In equilibrium, a seller bids more for placement when his product is more relevant for a given keyword, and the paid placement of sellers by the search engine reveals information about the relevance of different sellers. This results in efficient (sequential) search by consumers and increases total output.

A distinctive feature of our model is that firms sell differentiated products online. As such our approach is very different from those in the literature of Internet market, where firms are assumed to sell homogeneous goods (e.g., Baye and Morgan, 2001; Iyer and Pazgal, 2003; He and Chen, 2006). In our model, consumers search for their desired product varieties, and a search engine serves as a useful intermediary that provides information about the relevance of different sellers' products.<sup>2</sup> This view of the role played by search engines is consistent with industry observations. It has been noticed that consumers are increasingly turning to search engines for their information needs, with traffic rising from 133 billion searches in 2004 to a projected 162 billion in 2010 (Satagopan et al. 2005). According to WebSideStory, a San Diego research firm, 90% of shopping searches originate at the top four search sites, Google, Yahoo, AOL and MSN, all of which offer paid placement advertising (Frangos 2002). Unlike other studies in the literature on Internet search and pricing, in our model sellers use pure strategies in setting prices, and, when sellers differ in their marginal costs, there is equilibrium price dispersion under pure strategies.

The auction of ad placement by search engines has been studied in a recent paper by Edelman et al. (2005), which demonstrates that the auction mechanism for paid-placement advertising is one of generalized second price auction.<sup>3</sup> We also model the auction as a second price auction, where a winning bidder for an ad position pays the next highest bid;

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<sup>2</sup>In addition to the papers just mentioned, other studies on Internet retailing and Internet institutions include Lal and Sarvary (1999)'s investigation of the conditions under which Internet may soften price competition, and the examination by Zettelmeyer et al. (2001) on how Internet intermediary such as Autobyte.com and Carpoint.com affects consumer and firm behaviors.

<sup>3</sup>An interesting anecdote is that when Edelman et al. (2005) show that paid-placement search engines adopt second price auction in the bidding process, Hal Varian, who consulted for Google, acknowledged the validity of their finding (Coy 2006).

but a major difference in our analysis is that we embed the bidding process in a market game where consumers' search and purchase decisions, as well as sellers' pricing decisions, are all determined endogenously. Consequently, the values of sellers in being placed on the ad list and being placed at different positions are endogenous.<sup>4</sup> Interestingly, once the equilibrium values of different ad slots for different sellers are computed, a winning bidder for an ad slot bids his value from obtaining that slot, same as in a regular second price auction.

Our model is also related to the literature on advertising. Advertising in our model conveys product information, as, for instance, in Nelson (1974), Grossman and Shapiro (1984), Meurer and Stahl (1994), and Anderson and Renault (2006). The information conveyed by the ads through paid placement, however, is about the relevance of a seller's product relative to a particular keyword search and is thus unique to the Internet institution. Advertising by the sellers acts as a device to coordinate consumer search, and the more consumers a seller can attract in turn enable the seller to bid more payment to be placed by the search engine. Since it's the more relevant seller who can benefit more from attracting more consumers to visit its website, in equilibrium the more relevant sellers indeed bid more and are placed higher on the search engine's list, and it would indeed be rational for a consumer to search based on paid-placement advertising to find her desired product. This is related to the result in Bagwell and Ramey (1994), where advertising coordinates consumers to search stores that have lower marginal costs, and hence lower prices; and expecting more consumers, these stores indeed have the incentive to invest in reducing marginal costs.

The rest of the paper is organized as follows. Section 2 lays out the basic model. Section 3 studies market equilibrium. We characterize consumers' equilibrium search and purchase decisions, and firms' equilibrium pricing and bidding strategies. Section 4 extends the basic model to allow sellers to have different costs, which generates price dispersion under pure strategies. Section 5 concludes.

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<sup>4</sup>For studies of auctions with endogenous valuations, see, for instance, Lewis (1983), Krishna (1993), and Chen (2000).

## 2. BASIC MODEL

### 2.1 Institutional Characteristics

Google AdWords and Yahoo Search Marketing are leaders in paid-placement advertising. Microsoft recently joined the foray and launched MSN adCenter in October 2005. A common trait of the paid-placement search engines is the emphasis on relevance of advertisement to the keyword consumers use. If Internet users suddenly see a large amount of irrelevant search results pages that have nothing to do with their search, they might leave that search engine for another. This is exactly what paid-placement search engines are trying to avoid at all costs. For instance, Overture rejects close to 30% of keyword applications or ad listings that are submitted to them over a period of time (Thibodeau 2004).<sup>5</sup>

Google is thus far the champion of paid-placement advertising. It handles paid advertisement listings for its own site and online behemoth AOL, as well as AskJeeves and Earthlink. These paid listings are links that appear not within the search results, but rather as a colored box to the right of the page. Ad buyers do not get guaranteed placement. The listings appear somewhere, but they might not be at the top of the list of advertisements. They will not appear at all on the non-Google sites in the ad network if enough other advertisers have paid more or have more popular sites. That is because Google determines listing order based on the amount the advertiser is willing to pay per click-through and the number of clicks the ad gets. The idea is to promote the most relevant ads—as Google sees it—to its users.

### 2.2 Preliminaries

Consider a consumer who wants to buy a necklace. She needs to choose among a number of features: style, color, material, to name just a few. The choices the consumer needs to make can be mind boggling. Fortunately, she is computer savvy. Like many other consumers, she searches Google to find her desired product. Depending on her level of

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<sup>5</sup>Overture has been acquired by Yahoo in 2005 and is now Yahoo Search Marketing.

product knowledge and how well-defined her preferences are, she can use keywords such as “pearl necklace,” “freshwater pearl necklace,” “white cultured freshwater pearl necklace” to initiate her search. Hundreds of online sellers offer pearl necklace, although the number and type of varieties they carry differ. For instance, a seller may carry more varieties so that a consumer has a better chance finding her desired product from it. The problem is that the consumer may not know which seller is more relevant for her interests; and without the help of paid-placement advertising, a seller’s chance of reaching this consumer is rather small. If, however, a seller appears in the colored box to the right of the consumer’s search results, its chance of reaching the consumer is much higher. The seller can realize a sale if the consumer finds her desired product among the seller’s offering, although she does not know exactly which pearl necklace suits her the best before searching. In this environment, sellers need to figure out how to place bids to maximize their profits, a search engine (such as Google) needs to choose a profit-maximizing method to arrange the ad placement, and consumers need to form an optimal search strategy. All of these have implications on the properties of paid placement as an internet institution, in its effects on firm profits, consumer welfare, and efficiency. In what follows, we construct a parsimonious model to capture the essence of the paid-placement mechanism, to illuminate the strategic interactions between the players involved, and to shed light on the managerial and efficiency implications of the mechanism.

### 2.3 Assumptions and Model Setup

There are  $m \geq 3$  differentiated sellers, selling to a unit mass of consumers at a constant marginal cost  $c$ . Later, we relax this assumption and allow marginal costs to differ across sellers in Section 4. Given a particular keyword, the  $m$  sellers’ products have different “relevance” for the consumers. With probability  $\beta_i$  seller  $i$ ’s product matches the preference of any randomly chosen consumer, in which case the consumer’s valuation for the seller’s product is  $v$ , which is the realization of a random variable with cdf  $F(v)$  and pdf  $f(v)$  on  $[\underline{v}, \bar{v}]$ , where  $0 \leq \underline{v} < \bar{v}$ ; with probability  $1 - \beta_i$  seller  $i$ ’s product does not match the preference of the consumer, in which case the consumer’s valuation for the seller’s product

is zero. A consumer learns about her  $v$  only when she finds the desired product. We call  $\beta_i$  the match (or relevance) probability of seller  $i$ , and make the technical assumption that  $\beta_i$  is independent of  $F(v)$ , and is independent and identical for every consumer.

Each seller's match probability is her private information. Without loss of generality, let

$$\beta_1 \geq \beta_2 \geq \dots \geq \beta_m,$$

and refer seller  $i$  as seller type  $i$ . Thus each seller has private information about his type  $i$ , although the distribution of seller types and possible values of  $\beta_i$  are common knowledge. For convenience, we shall assume

$$\beta_i = \begin{cases} \gamma^{i-1}\beta & \text{for } i = 1, 2, \dots, I \\ \gamma^I\beta & \text{for } i = I + 1, \dots, m \end{cases},$$

where  $\beta, \gamma \in (0, 1)$  and  $2 \leq I \leq m$ . Thus the match probability decreases among the sellers at a constant rate  $\gamma$  for  $I$  sellers, then it becomes constant for the rest of the sellers. We denote seller  $i$  by  $S_i$ .

Each consumer is ex ante uncertain about which seller's product is desirable for her preference. She also does not know the match probability of any particular seller. But she can find out whether the seller's product is what she desires by visiting the seller's website. She can also decide which sellers' websites to visit by first searching through a search engine with a keyword for the product, and the search engine then shows a list of paid advertising sellers. Note that there is horizontal differentiation between different product varieties; but sellers are differentiated by their different relevance (matching probabilities). A seller could be more relevant simply because he carries a higher number of product variety. A seller can choose to pay the search engine, denoted as  $E$ , to be included in the list.  $E$  has  $n \leq m$  positions in the list,  $E_1, E_2, \dots, E_n$ , that it can auction to the sellers in a second price auction, where the seller who bids the most gets listed the highest (at  $E_1$ ) and pays the second highest bid, the seller who bids the second highest gets listed the second highest (at  $E_2$ ) and pays the third highest bid, and so on. In other words, let the bids of the sellers in descending order be  $b_j$ ,  $j = 1, \dots, m$ . Then sellers  $S_j$  will be included in the list with the order  $j = 1, 2, \dots, n$ . For convenience, we assume  $n = 3 = I$ , although it is straightforward



to extend our analysis to any arbitrary  $n$  and  $I$ . Thus, by assumption  $E$  has three positions on its list for paid placement,  $E_1$ ,  $E_2$ , and  $E_3$ ; and  $\beta_i = \gamma^{i-1}\beta$  for  $i = 1, 2, 3$  but  $\beta_i = \gamma^3\beta$  for  $i \geq 4$ .

The timing of the game is as follows. Sellers, having learned their private  $\beta_i$ , first bid to be listed on  $E$ . The chosen sellers are listed on  $E$ . Sellers then simultaneously and independently choose their prices, which are not observed by any consumer until the consumer searches the seller's website. Consumers then decide whether and how to search the websites, and they may possibly use information from  $E$ 's list. There are costs for consumers to search the websites of sellers. The cost for each consumer to conduct her  $j$ 'th search is  $t_j$ ,  $j = 1, \dots, m$ . A consumer makes a unit purchase if and when she finds her desired product, the price does not exceed her realized  $v$ , and searching further does not yield a higher expected surplus for her. All players are risk neutral. We make the following technical assumptions:

**A1.** There is a unique  $p^o$  such that

$$p^o = \arg \max_{p \in [c, \bar{u}]} (p - c) [1 - F(p)]. \quad (1)$$

**A2.**

$$t_j = \begin{cases} t & \text{for } j = 1, 2, 3, 4 \\ t^h & \text{for } j > 4 \end{cases}, \quad (2)$$

where

$$t < \gamma^3 \beta \int_{p^o}^{\bar{v}} (v - p^o) f(v) dv < t^h. \quad (3)$$

A sufficient, but not necessary, condition for **A1** is that the hazard rate  $\frac{f(p)}{1-F(p)}$  is monotonically increasing. This monotonic hazard rate condition is satisfied for many familiar distributions, such as uniform, exponential, and normal distributions. We define

$$\pi^o \equiv (p^o - c) [1 - F(p^o)]. \quad (4)$$

**A2** captures the idea that a consumer's marginal search cost becomes higher after some searches, perhaps due to "capacity constraint" in her time that can be used for search; this simplifies the analysis of consumer search, but is otherwise not essential for our results.

### 3. EQUILIBRIUM ANALYSIS

A profile of strategies in our model consists of a search and purchase strategy by each consumer, a bidding strategy by seller  $S_i$ , and a pricing strategy by seller  $S_i$ . After observing the placement of sellers, buyers have beliefs about the relevance (type) of different sellers. An equilibrium (perfect Bayesian equilibrium) is a profile of strategies, together with a system of beliefs by buyers, such that each player is optimizing, and buyers' beliefs are consistent with the strategies and placement of sellers.

We start our analysis with consumers' search strategies. Suppose that the sellers placed on  $E$ 's list are in the order of their relevance, namely that  $S_i$  takes the positions of  $E_i$  for  $i = 1, 2, 3$ . Suppose further that all sellers set their prices equal to  $p^o$ . Then, a consumer's expected return from searching  $E_i$  is

$$\gamma^{i-1}\beta \int_{p^o}^{\bar{v}} (v - p^o) f(v) dv, \text{ for } i = 1, 2, 3,$$

and her expected return from searching any randomly selected seller not listed on  $E$  is

$$\gamma^3\beta \int_{p^o}^{\bar{v}} (v - p^o) f(v) dv.$$

Since

$$t < \gamma^3\beta \int_{p^o}^{\bar{v}} (v - p^o) f(v) dv < t^h.$$

from **A2**, it is optimal for each consumer to search sequentially, in the order of  $E_1, E_2, E_3$ , and then one randomly selected seller not listed on  $E$ . She stops searching either if she finds her desired product or if she has conducted these four searches without finding her desired product. When the consumer finds that a seller's product matches her needs, she purchases the product if  $v \geq p^o$ ; and does not purchase if  $v < p^o$ . Since her  $v$  is the same for the desired product from any seller, she will not conduct additional search once her search has yielded a match.

We therefore have:

**Lemma 1** *Suppose that  $S_1, S_2, S_3$  are placed on  $E$ 's list in descending order and other sellers are not placed on the list. Suppose further that each seller's price is  $p^o$ . Then it is optimal for each consumer to search sequentially  $E_1, E_2, E_3$  and then one randomly selected seller not listed on  $E$ . She stops searching either when she finds her desired product, in which case she purchases if and only if  $v \geq p^o$ , or when she has conducted these four searches without finding her desired product.*

We next consider sellers' pricing strategies, given consumers' search and purchase behavior described in Lemma 1. If a seller's product matches a consumer's needs, then the seller's price that maximizes his expected profit from this consumer, without knowing the consumer's realized  $v$ , is  $p^o$ . Since a consumer will purchase the seller's product if  $v \geq p^o$ ,  $p^o$  must be the optimal price for the seller,<sup>6</sup> independent of whether the seller is listed on  $E$  or what his position on  $E$  is.

Therefore, given consumers' search and purchase behavior described in Lemma 1, if  $S_1, S_2$ , and  $S_3$  are placed at  $E_1, E_2$ , and  $E_3$ , the expected profits of  $S_i$ , excluding their payments to  $E$ , are

$$\begin{aligned}
\pi_1 &= \beta\pi^o \\
\pi_2 &= (1 - \beta)\gamma\beta\pi^o = (1 - \beta)\gamma\pi_1 \\
\pi_3 &= (1 - \gamma\beta)(1 - \beta)\gamma^2\beta\pi^o = (1 - \gamma\beta)\gamma\pi_2 \\
\pi_k &= \frac{1}{m-3}(1 - \gamma^2\beta)(1 - \gamma\beta)(1 - \beta)\gamma^3\beta\pi^o = \frac{1-\gamma^2\beta}{m-3}\gamma\pi_3, \text{ for } k = 4, \dots, m.
\end{aligned} \tag{5}$$

We notice that the analysis of bidding strategies here differs from the usual second price auction, since there are multiple positions to be auctioned, and the values of  $E_2, E_3$  and not winning the bid are endogenous for the bidders, depending on who will be placed at the different positions. To determine how each seller will bid to be placed on  $E$ , we look for an equilibrium where  $b_1 > b_2 > b_3 > b_k$  for  $k = 4, \dots, m$ , and  $S_i$  ( $i = 1, 2, 3$ ) bids the

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<sup>6</sup>This is a familiar result in the search literature, following the seminal work of Diamond (1971). Our model captures the situation where consumers' search for relevance dominates search for price. Our analysis does not depend crucially on each firm charging  $p^o$ . The qualitative nature of our results will be the same as long as firms' optimal price is a constant, or, as we show in Section 4, the distribution of firms' prices are within a sufficiently small interval relative to consumers' search costs.

value of being placed at  $E_i$ . In such a possible equilibrium, given the placement rule and consumers' search behavior,  $S_4$ 's expected profit from not being placed on  $E$ 's list is  $\pi_4$ . If  $S_4$  is placed at  $E_3$  to replace  $S_3$ 's position, his expected profit would be

$$(1 - \beta)(1 - \gamma\beta)\gamma^3\beta\pi_o = \gamma\pi_3.$$

Therefore  $S_4$  is willing to bid

$$\Delta_4 \equiv \gamma\pi_3 - \pi_4 = \gamma\pi_3 - (1 - \gamma^2\beta)\frac{\gamma}{m-3}\pi_3 = \left(1 - \frac{1 - \gamma^2\beta}{m-3}\right)\gamma\pi_3$$

to be placed at  $E_3$ . On the other hand, to keep his current position,  $S_3$  is willing to bid

$$\Delta_3 = \pi_3 - (1 - \beta)(1 - \gamma\beta)(1 - \gamma^3\beta)\frac{\gamma^2\beta}{m-3}\pi_o = \left(1 - \frac{1 - \gamma^3\beta}{m-3}\right)\pi_3.$$

We have

$$\Delta_3 - \Delta_4 = \left(1 - \frac{1 - \gamma^3\beta}{m-3}\right)\pi_3 - \left(1 - \frac{1 - \gamma^2\beta}{m-3}\right)\gamma\pi_3 = \frac{(1 - \gamma)(m - 4)}{(m - 3)}\pi_3 \geq 0,$$

where the inequality holds strictly if  $m > 4$ . Thus, if  $S_3$  bids  $\Delta_3$ , the increase of his profit from not on  $E$  to at  $E_3$ , or the value of  $E_3$  to him, is  $\Delta_3$ . Taking as given the proposed equilibrium placement,  $S_3$  outbids  $S_4$  for  $E_3$ . The expected payoff for  $S_3$  at this proposed equilibrium would be  $\pi_3 - \Delta_4$ .

For  $S_2$ , his expected payoff to be placed at  $E_3$  would be

$$(1 - \beta)(1 - \gamma^2\beta)\gamma\beta\pi_o - \Delta_4.$$

To keep his position at  $E_2$ ,  $S_2$  is thus willing to bid

$$\begin{aligned} \Delta_2 &= \pi_2 - [(1 - \beta)(1 - \gamma^2\beta)\gamma\beta\pi_o - \Delta_4] \\ &= (1 - \beta)\gamma\beta\pi_o - (1 - \beta)(1 - \gamma^2\beta)\gamma\beta\pi_o + \Delta_4 \\ &= (1 - \beta)\gamma\beta[1 - (1 - \gamma^2\beta)]\pi_o + \left(1 - \frac{1 - \gamma^2\beta}{m-3}\right)\gamma\pi_3 \\ &= (1 - \beta)\gamma^3\beta^2\pi_o + \left(1 - \frac{1 - \gamma^2\beta}{m-3}\right)\gamma\pi_3. \end{aligned}$$

For  $S_1$ , his expected payoff to be placed at  $E_2$  would be

$$(1 - \gamma\beta)\beta\pi_o - \Delta_3.$$

To keep his position at  $E_1$ ,  $S_1$  is willing to bid

$$\begin{aligned}\Delta_1 &= \pi_1 - ((1 - \gamma\beta)\beta\pi^o - \Delta_3) = \beta\pi^o - (1 - \gamma\beta)\beta\pi^o + \Delta_3 = \gamma\beta^2\pi^o + \Delta_3 \\ &= \gamma\beta^2\pi^o + \left(1 - \frac{1 - \gamma^3\beta}{m - 3}\right)\pi_3.\end{aligned}$$

Theorem 1 below establishes that bidding  $\Delta_i$  is indeed an equilibrium strategy for  $S_i$ ,  $i = 1, 2, 3, 4$ .

**Theorem 1** *Assume  $\beta \geq \max\left\{2 - \frac{1}{\gamma}, \frac{1-\gamma}{2-\gamma}\right\} \equiv \underline{\beta}(\gamma)$ . Then, the basic model has an equilibrium in which seller  $S_i$  bids to pay  $E$*

$$\begin{aligned}b_1 &= \gamma\beta^2\pi^o + \left(1 - \frac{1-\gamma^3\beta}{m-3}\right)\pi_3, \\ b_2 &= (1 - \beta)\gamma^3\beta^2\pi^o + \left(1 - \frac{1-\gamma^2\beta}{m-3}\right)\gamma\pi_3, \\ b_3 &= \left(1 - \frac{1-\gamma^3\beta}{m-3}\right)\pi_3; \\ b_k &= \left(1 - \frac{1-\gamma^2\beta}{m-3}\right)\gamma\pi_3, \quad k = 4, \dots, m,\end{aligned}\tag{6}$$

$S_1, S_2, S_3$  are placed at  $E_1, E_2, E_3$  and pay  $b_2, b_3$ , and  $b_4$ , respectively. Each seller's price is  $p^o$ , and each consumer searches and purchases as described in Lemma 1.

The proof for Theorem 1 is contained in the appendix. Basically, one needs to show that, given the bids of other sellers, no seller can benefit by bidding differently from his equilibrium bid. This involves showing that  $S_k$ ,  $k = 4, \dots, m$  would not want to bid sufficiently more to be placed at  $E_1, E_2$ , or  $E_3$ ; that  $S_3$  neither would want to bid sufficiently more to be placed at  $E_2$  or  $E_1$ , nor would want to lower its bid to be not placed on  $E$ ; and similarly for  $S_2$  and  $S_1$ . The additional parameter restriction provides a sufficient, but not necessary condition (when  $m > 4$ ), for such an equilibrium. Notice that this parameter restriction is satisfied if  $\beta \geq \max\left\{\frac{1}{2}, \gamma\right\}$ .

In the equilibrium characterized in Theorem 1, the search engine provides information about the relevance of the products to consumers. It turns out that this is also the unique equilibrium of the game, under a mild condition on consumers' search behavior; namely that consumers will search in the order of  $E_1, E_2$ , and  $E_3$  if they are indifferent between alternative orders of search on  $E$ . The argument is as follows: First, there can be no

equilibrium in which  $S_1, S_2, S_3$  are placed on  $E$  but not in the order of  $E_1, E_2, E_3$ . Suppose to the contrary that there is such an equilibrium. Then, if a less relevant seller, say  $S_3$ , bids more and is placed at a higher position on  $E$ , consumers would optimally search the lower placed but more relevant seller(s) before  $S_3$  at such an equilibrium (because consumers have correct beliefs in equilibrium), which means that  $S_3$  could benefit by lowering its bid and its placement position on  $E$ , contradicting the equilibrium assumption. If, on the other hand, all three sellers bid the same amount and are placed on  $E$  in random order, consumers would have the same expected payoff from any order of search on  $E$ . But if in this case consumers will search in the order of  $E_1, E_2, E_3$ , any of the  $S_i$ ,  $i = 1, 2, 3$ , will have the incentive to deviate by bidding a little more in order to be placed at the top, again contradicting the equilibrium assumption. Next, there can be no equilibrium in which some  $S_k$  with  $k > 3$ , say  $S_4$ , is placed on  $E$  and  $S_4$  bids differently from the other two sellers placed on  $E$ . Suppose to the contrary that there is such an equilibrium. Then,  $S_4$  must bid at least as high as the highest bidder not listed on  $E$ . But at such an equilibrium buyers would search randomly from the sellers not on  $E$ , before searching  $S_4$ , since the expected match probability from sellers not listed on  $E$  would be higher than that of  $S_4$ . This implies that  $S_4$  would benefit from a deviation that lowers his bid (or refrains from bidding) so that he will be placed on  $E$ , contradicting the equilibrium assumption. Finally, it is straightforward to show that there can also be no equilibrium in which some  $S_k$  with  $k > 3$ , say  $S_4$ , is placed on  $E$  and  $S_4$  bids the same amount as at least one other seller placed on  $E$ . We thus have:

**Remark 1** *Assume that consumers will search in the order of  $E_1, E_2$ , and  $E_3$  if they are indifferent between alternative orders of search on  $E$ . Then the equilibrium characterized in Theorem 1 is also the unique equilibrium of the game.*

The restriction on consumer search behavior ensures the equilibrium uniqueness. If consumers would search in random order on  $E$  when they are indifferent between alternative orders of search on  $E$ , then it appears possible to have a “partially pooling” equilibrium, where  $S_1, S_2, S_3$  bid the same amount, are placed with equal chance at  $E_1, E_2, E_3$ , and consumers search with equal chance of alternative orders on  $E$ . In the rest of the paper, we

shall maintain the assumption on consumer search behavior in Remark 1, and focus on the “separating” equilibrium characterized in Theorem 1.

One way to evaluate the efficiency property of paid-placement advertising is to see how it impacts consumer search costs to achieve a given probability of finding a match. With paid-placement advertising, for the match probability to be  $\beta$ ,  $1 - (1 - \beta)(1 - \gamma\beta)$ ,  $1 - (1 - \beta)(1 - \gamma\beta)(1 - \gamma^2\beta)$ , and  $1 - (1 - \beta)(1 - \gamma\beta)(1 - \gamma^2\beta)(1 - \gamma^3\beta)$ , the consumer needs to incur respectively  $t$ ,  $2t$ ,  $3t$ , and  $4t$ . Without paid-placement advertising, and for large  $m$ , the probability of a match from each search is approximately

$$\frac{1}{m} (1 + \gamma + \gamma^2 + (m - 3)\gamma^3) \beta \approx \gamma^3 \beta.$$

The probability of achieving a match from  $\tau$  searches is approximately

$$1 - (1 - \gamma^3 \beta)^\tau.$$

Thus, to achieve any particular probability of match, the expected search times, or the expected search cost, is lower under paid-placement advertising. In fact, one can easily see that paid-placement advertising leads to an efficient search procedure for the consumers.

Another way to evaluate the efficiency property of paid-placement advertising is to see how it impacts expected output. The expected output under paid-placement advertising is

$$q_h = [1 - (1 - \beta)(1 - \gamma\beta)(1 - \gamma^2\beta)(1 - \gamma^3\beta)] [1 - F(p^o)], \quad (7)$$

while the expected output without paid-placement advertising is approximately

$$q_l = [1 - (1 - \gamma^3\beta)^4] [1 - F(p^o)], \quad (8)$$

which is less than  $q_h$ . We therefore have:

**Corollary 1** *Paid-placement advertising leads to an efficient search procedure for consumers, and to higher total output.*

In equilibrium, the search engine’s profit is

$$\begin{aligned} \pi_E &= b_2 + b_3 + b_4 \\ &= (1 - \beta) \gamma^3 \beta^2 \pi^o + \left[ \left(1 - \frac{1 - \gamma^2 \beta}{m - 3}\right) \gamma + \left(1 - \frac{1 - \gamma^3 \beta}{m - 3}\right) + \left(1 - \frac{1 - \gamma^2 \beta}{m - 3}\right) \gamma \right] \pi_3, \end{aligned}$$

or

$$\pi_E = (1 - \beta) \gamma^3 \beta^2 \pi^o + \frac{(m - 4)(1 - 2\gamma) + 3\beta\gamma^3}{m - 3} (1 - \beta)(1 - \gamma\beta) \gamma^2 \beta \pi^o. \quad (9)$$

Therefore, treating  $m$  as a continuous variable, we have:

$$\frac{\partial \pi_E}{\partial m} = (1 + 2\gamma - 3\gamma^3 \beta)(1 - \beta)(1 - \gamma\beta) \gamma^2 \beta \frac{\pi^o}{(m - 3)^2} > 0, \quad (10)$$

Also,

$$\begin{aligned} \lim_{m \rightarrow \infty} \frac{\partial \pi_E}{\partial \beta} &= (4\beta\gamma - 2\gamma - 2\beta + 4\beta\gamma^2 - 6\beta^2\gamma^2 + 1) \gamma^2 \pi^o \\ &= [-6\beta^2\gamma^2 + 2(2\gamma + 2\gamma^2 - 1)\beta - 2\gamma + 1] \gamma^2 \pi^o. \end{aligned}$$

Solving

$$-6\beta^2\gamma^2 + 2(2\gamma + 2\gamma^2 - 1)\beta - 2\gamma + 1 = 0,$$

we obtain

$$\hat{\beta}(\gamma) = \frac{1}{6\gamma^2} \left( 2\gamma + 2\gamma^2 - 1 + \sqrt{(2\gamma + 2\gamma^2 - 1)^2 - 6\gamma^2(2\gamma - 1)} \right), \quad (11)$$

which increases in  $\gamma$ , with  $\lim_{\gamma \rightarrow 0} \hat{\beta}(\gamma) = \frac{1}{2}$  and  $\lim_{\gamma \rightarrow 1} \hat{\beta}(\gamma) = \frac{3 + \sqrt{3}}{6}$ .

Thus  $\pi_E$  has an inverted U-shape with respect to  $\beta$ : it increases in  $\beta$  for  $\beta < \hat{\beta}(\gamma)$  and decreases in  $\beta$  for  $\beta > \hat{\beta}(\gamma)$ .

**Corollary 2** *The search engine's profit,  $\pi_E \equiv b_2 + b_3 + b_4$ , is strictly increasing in the number of firms,  $m$ . Furthermore, when  $m$  is large and  $\underline{\beta}(\gamma) < \hat{\beta}(\gamma)$ ,  $\pi_E$  is increasing in the match probability  $\beta$  for  $\beta \in (\underline{\beta}(\gamma), \hat{\beta}(\gamma)]$  but is decreasing in  $\beta$  for  $\beta \in (\hat{\beta}(\gamma), 1)$ .*

As more sellers are present in the market, a seller is less likely to be selected randomly by a buyer, and thus placement on the search engine's recommended list is more valuable. This motivates the sellers to bid more for placement, increasing the search engine's revenue. To see the non-monotonic relationship between the search engine's revenue and seller's relevance, notice that an increase in  $\beta$  has a positive effect on the value of being placed at  $E_1$ , but has two opposite effects on the value of being placed at  $E_2$  and  $E_3$ : while it increases the probability of match when a consumer visits the seller's website, it also



reduces the probability that the consumer will visit  $E_2$  or  $E_3$ , since the consumer is more likely to purchase at  $E_1$ . The balance of these effects results in the search engine's revenue being first increasing and then decreasing in  $\beta$ .<sup>7</sup>

In sum, Section 3 analyzes the fundamental properties of the paid-placement mechanism. We show that it is optimal for the search engine to place the most relevant firms on its list in descending order. As such, paid-placement advertising improves consumer welfare and enhances firms' profits. Our analysis also sheds light on the search engine's profit. Specifically, the search engine should recruit as many firms as possible into the paid-placement mechanism. In addition, it is optimal for the search engine to restrict the precision of keywords such that the resulting match probability is not too high. Our model is most applicable to products characterized by a high degree of horizontal differentiation and limited price dispersion, i.e., product categories where product variety is large and similar items do not differ substantially in price, such as watches, jeweleries, computers, etc.

#### 4. HETEROGENEOUS COSTS

Our basic model has the property that in equilibrium all sellers charge the same price,  $p^o$ . In this section, we make a simple modification to our model that would allow us to generate price dispersion in equilibrium. The modification is that, instead of assuming the same cost for all sellers, we now assume that sellers may have different costs. More specifically, we assume that each seller's constant marginal cost  $c_i$  is the realization of a random variable distributed on  $[\underline{c}, \bar{c}]$ , with cdf and pdf  $G(\cdot)$  and  $g(\cdot)$ , respectively; and each seller learns its cost realization after bidding on  $E$ .<sup>8</sup>

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<sup>7</sup>Our model assumes that  $\beta$  is given exogenously. If the search engine can affect the value of  $\beta$  by, say, the selection of key words, our analysis suggests that the search engine should choose  $\beta$  at some intermediate level to maximize its revenue.

<sup>8</sup>This way, bidding by sellers does not signal sellers' costs (prices), allowing us to focus on the role of paid-placement advertising in signaling product relevance. We are not aware of evidence suggesting that sellers with paid-placement advertising have systematically higher or lower costs (prices), and under our formulation all sellers have the same expected price in equilibrium.

For any  $c_i \in [\underline{c}, \bar{c}]$ , let

$$p^o(c_i) = \arg \max_{p \in [c, \bar{u}]} (p - c_i) [1 - F(p)], \quad (12)$$

$$\tilde{\pi}^o = \int_{\underline{c}}^{\bar{c}} (p^o(c) - c) [1 - F(p^o(c))] dG(c). \quad (13)$$

Then, if seller  $i$  is listed as  $E_1$ , its expected profit is  $\beta_i \tilde{\pi}^o$ , provided that consumers first visit  $E_1$ .

We modify assumption **A2** to assume

**A2'**.

$$\gamma \beta \int_{\underline{c}}^{\bar{c}} [p^o(\bar{c}) - p^o(c)] g(c) dc < t < \gamma^3 \beta \int_{\underline{c}}^{\bar{c}} \left[ \int_{p^o(c)}^{\bar{v}} [v - p^o(c)] f(v) dv \right] g(c) dc < t^h.$$

**A2'** requires that the cost dispersion is not too large, so that in equilibrium a consumer stops searching once she finds her desired product, and she does not search more than four times. Notice that **A2'** becomes **A2** when  $[\underline{c}, \bar{c}]$  converges to a constant  $c$ .

We again look for the equilibrium in which paid-placement conveys information about the sellers' product relevance.

First, suppose that  $S_1, S_2, S_3$  are placed at  $E_1, E_2, E_3$ , respectively, and  $S_k$  are not placed on  $E'$ 's list,  $k = 4, \dots, m$ . Suppose further that each seller prices at  $p^o(c_i)$ . Then, it is optimal for consumers to search sequentially, in the order of  $E_1, E_2, E_3$ , and then randomly chosen non-listed sellers. If a consumer finds her desired product at a particular seller, her expected return from having another search cannot exceed

$$\gamma \beta \int_{\underline{c}}^{\bar{c}} [p^o(\bar{c}) - p^o(c)] g(c) dc,$$

which is less than  $t$  by assumption. On the other hand, conditional on having not found a match, a consumer's expected return from searching a non-listed seller is

$$\gamma^3 \beta \int_{\underline{c}}^{\bar{c}} \left[ \int_{p^o(c)}^{\bar{v}} [v - p^o(c)] f(v) dv \right] g(c) dc,$$

which is larger than  $t$  but less than  $t^h$  by assumption. Therefore, given the suggested placement of sellers and their prices, it is optimal for each consumer to search sequentially at most four sellers, in the order of  $E_1$ ,  $E_2$ ,  $E_3$ , and a randomly chosen non-listed seller; she stops searching either when she finds a match or when she has searched four times; and she makes a purchase if she finds a match and her  $v$  is at or above the seller's price.

Next, given the search and purchase behavior of consumers, it is optimal for  $S_i$  to set  $p^o(c_i)$ . Hence, at the time of bidding for placement, the expected profit of  $S_i$  from any consumer who visits  $S_i$  is simply  $\tilde{\pi}^o$ .

Finally, to establish the equilibrium, we need to show that each seller bids optimally and the bidding by the sellers indeed results in the proposed order of placement under the second price auction. At the proposed equilibrium, the expected profits of  $S_i$ , excluding their payments to  $E$ , are

$$\begin{aligned}
\tilde{\pi}_1 &= \beta \tilde{\pi}^o, \\
\tilde{\pi}_2 &= (1 - \beta) \gamma \beta \tilde{\pi}^o = (1 - \beta) \gamma \tilde{\pi}_1, \\
\tilde{\pi}_3 &= (1 - \gamma \beta) (1 - \beta) \gamma^2 \beta \tilde{\pi}^o = (1 - \gamma \beta) \gamma \tilde{\pi}_2, \\
\tilde{\pi}_k &= \frac{1}{m-3} (1 - \gamma^2 \beta) (1 - \gamma \beta) (1 - \beta) \gamma^3 \beta \tilde{\pi}^o = \frac{1 - \gamma^2 \beta}{m-3} \gamma \tilde{\pi}_3, \text{ for } k = 4, \dots, m.
\end{aligned} \tag{14}$$

If  $S_4$  is placed at  $E_3$ , his expected profit would be

$$(1 - \beta) (1 - \gamma \beta) \gamma^3 \beta \tilde{\pi}^o.$$

Thus  $S_4$  is willing to bid

$$\tilde{\Delta}_4 = (1 - \beta) (1 - \gamma \beta) \gamma^3 \beta \tilde{\pi}^o - \frac{(1 - \beta) (1 - \gamma \beta) (1 - \gamma^2 \beta)}{m - 3} \gamma^3 \beta \tilde{\pi}^o = \left(1 - \frac{1 - \beta \gamma^2}{m - 3}\right) \tilde{\pi}_3$$

to be placed at  $E_3$ . On the other hand, to keep his position at  $E_3$ ,  $S_3$  is willing to bid

$$\tilde{\Delta}_3 = \tilde{\pi}_3 - (1 - \beta) (1 - \gamma \beta) (1 - \gamma^3 \beta) \frac{\gamma^2 \beta}{m - 3} \tilde{\pi}^o = \left(1 - \frac{1 - \gamma^3 \beta}{m - 3}\right) \tilde{\pi}_3.$$

Similarly, as in our earlier analysis where all sellers have the same constant marginal cost, to keep their positions at  $E_2$  and  $E_1$ ,  $S_2$  and  $S_1$  are willing to bid, respectively,

$$\begin{aligned}
\tilde{\Delta}_2 &= (1 - \beta) \gamma^3 \beta^2 \tilde{\pi}^o + \left(1 - \frac{1 - \gamma^2 \beta}{m - 3}\right) \gamma \tilde{\pi}_3, \\
\tilde{\Delta}_1 &= \gamma \beta^2 \tilde{\pi}^o + \left(1 - \frac{1 - \gamma^3 \beta}{m - 3}\right) \tilde{\pi}_3.
\end{aligned}$$

Therefore, analogous to Theorem 1, we have:

**Theorem 2** *Assume that  $\beta \geq \max\left\{2 - \frac{1}{\gamma}, \frac{1-\gamma}{2-\gamma}\right\}$ . Then, the game with heterogeneous seller costs has an equilibrium in which seller  $S_i$  bids to pay  $E$*

$$\begin{aligned}
b_1 &= \gamma\beta^2\tilde{\pi}^o + \left(1 - \frac{1-\gamma^3\beta}{m-3}\right)\tilde{\pi}_3, \\
b_2 &= (1-\beta)\gamma^3\beta^2\tilde{\pi}^o + \left(1 - \frac{1-\gamma^2\beta}{m-3}\right)\gamma\tilde{\pi}_3, \\
b_3 &= \left(1 - \frac{1-\gamma^3\beta}{m-3}\right)\tilde{\pi}_3, \\
b_k &= \left(1 - \frac{1-\gamma^2\beta}{m-3}\right)\gamma\tilde{\pi}_3, \quad k = 4, \dots, m,
\end{aligned} \tag{15}$$

$S_1, S_2, S_3$  are placed at  $E_1, E_2, E_3$  and pay  $b_2, b_3,$  and  $b_4,$  respectively.  $S_i$  charges price  $p^o(c_i)$ . Each consumer searches sequentially, in the order of  $E_1, E_2, E_3,$  and then a randomly selected non-listed seller; stops searching further when she finds a match or has searched four sellers; and purchases if the price of the product that matches her needs does not exceed her valuation for the product.

The proof of Theorem 2 is entirely the same as the proof of Theorem 1 in the appendix, except replacing  $\pi^o$  and  $\pi_3$  there by  $\tilde{\pi}^o$  and  $\tilde{\pi}_3$ . Notice that in equilibrium sellers tend to have different prices, depending on the realization of their costs, and the expected price of each seller is

$$\tilde{p} = \int_{\underline{c}}^{\bar{c}} p^o(c) dG(c).$$

In the literature, price dispersion is often generated in models with mixed strategies, where some consumers purchase only from particular sellers (due to loyalty or imperfect information) while other consumers purchase only from the seller with the lowest prices (e.g., Baye and Morgan, 2001; Janseen and Moraga-González, 2004; Rosenthal, 1980; and Varian, 1980). An important exception is Reinganum (1979), where a price distribution is generated by a set of firms with different marginal costs choosing pure strategies. Our model in this section has followed the approach of Reinganum in considering possibly different marginal costs for different firms. Unlike her model, where in equilibrium each consumer only searches once, consumers engage in sequential search here because firms sell differentiated products and each consumer searches for the variety matching her preference.

## 5. CONCLUSION

One of the great promises of the Internet is its efficiency in disseminating information. More information, however, can be a mixed blessing for consumers, as evidenced by, for instance, the intrusion of junk e-mails to our lives. For the Internet to be a beneficial medium, therefore, the information it delivers should go to consumers who exhibit such information needs. More specifically, efficiency requires that consumers who search for information receive information from the most relevant sources. Indeed, the ability to deliver relevant information to consumers who search for information is unique to the Internet. Such characteristics may exist in other media but are far more costly.

Paid-placement advertising, where a search engine acts as an intermediary between firms and consumers, facilitates the transmission of information from firms to consumers and has enjoyed phenomenal commercial successes. This paper has developed a market equilibrium model that uncovers the economic forces behind the success of this important Internet institution. When consumers must engage in costly search to find their desired product variety, they face the issue of how to search various sellers, who carry different product varieties. Advertising through paid placement enables sellers to reveal information about their product relevance to consumers: A seller with a more relevant product expects a higher probability of a sale from the visit of a consumer to the seller's website, and hence a higher expected profit from attracting such a visiting consumer; this motivates the seller to bid more and to receive a higher ad placement position. Moreover, since consumers do not learn a seller's price until visiting the seller's website, in equilibrium the expected price from each seller is the same. Therefore, it is optimal for consumers to search sellers sequentially, according to their placement on the search engine's list. In equilibrium, paid placement advertising leads to efficient search by consumers and higher total output. Our analysis also sheds light on the search engine's strategies. In particular, we demonstrate that there is an inverted U-shaped relationship between the search engine's profit and relevance, implying that the search engine's profit is maximized when the keyword relevance is set at some intermediate level.

As is well-known in the literature (Diamond, 1971), with costly consumer search a market with many firms may nevertheless sustain a single monopoly price. This is also the case in our model when all sellers have the same marginal cost. When sellers ex post have different marginal costs, there is price dispersion in the equilibrium of our model, even though each seller still sets a monopoly price based on his realized marginal cost. The fact that the next seller a consumer searches may not offer the product match she desires diminishes her expected return from searching further for a lower price. Consequently, under the assumption that the cost (price) dispersion is relatively small, a consumer will not search further once she has found her desired product.

There are several directions for future research. One possibility is to allow competition among search engines, which could affect the bidding incentives of sellers and could address issues such as whether competition will lead to the adoption of efficient information dissemination mechanisms. A strong assumption in our model when we allow sellers to have different marginal costs is that sellers learn their costs after they place bids to the search engine. It would be desirable to relax this assumptions in future research. While we find a model with equilibrium price dispersion under pure strategies interesting, future research could explore other models of price dispersion, possibly with mixed strategies. Furthermore, it would also be interesting to empirically evaluate the assumptions and implications of our analysis. For instance, our analysis predicts that sellers placed higher by a search engine for a keyword search will have higher expected sales for the product, which is empirically testable.

## APPENDIX

**Proof of Theorem 1.** Given the placement of  $S_1, S_2, S_3$  at  $E_1, E_2, E_3$  and each seller's price  $p^o$ , each consumer's search and purchase behavior is optimal. Given consumer behavior, each seller's price is optimal. Thus, our proof will be complete if it is shown that no seller can benefit from bidding differently. Since it is a second price auction, we need only be concerned with deviations by  $S_i$  that would change the placement of  $S_i$ . Let  $\pi_i^j$  be seller  $i$ 's payoff at position  $E_j$ , including  $i$ 's payment to  $E$ .

First consider  $S_4$ . In order to be placed at  $E_3$ ,  $S_4$  needs to bid at least  $b_3$ , and his expected payoff at  $E_3$ , after paying  $b_3$ , is

$$\pi_4^3 = (1 - \beta)(1 - \gamma\beta)\gamma^3\beta\pi^o - \left(1 - \frac{1 - \gamma^3\beta}{m - 3}\right)\pi_3 = \gamma\pi_3 - \left(1 - \frac{1 - \gamma^3\beta}{m - 3}\right)\pi_3.$$

On the other hand, the expected profit of  $S_4$  not to be on  $E$  is  $\pi_4 = \frac{1 - \gamma^2\beta}{m - 3}\gamma\pi_3$ . We have

$$\pi_4^3 - \pi_4 = \gamma\pi_3 - \left(1 - \frac{1 - \gamma^3\beta}{m - 3}\right)\pi_3 - \frac{1 - \gamma^2\beta}{m - 3}\gamma\pi_3 = -\frac{(1 - \gamma)(m - 4)}{(m - 3)}\pi_3 \leq 0$$

where the inequality holds strictly if  $m > 4$ . Hence,  $S_4$  has no incentive to switch position with  $S_3$ .

Similarly, we have

$$\begin{aligned} \pi_4^2 - \pi_4 &= (1 - \beta)\gamma^3\beta\pi^o - \left[(1 - \beta)\gamma^3\beta^2\pi^o + \left(1 - \frac{1 - \gamma^2\beta}{m - 3}\right)\gamma\pi_3\right] - \frac{1 - \gamma^2\beta}{m - 3}\gamma\pi_3 \\ &= (1 - \beta)^2\gamma^3\beta\pi^o - \gamma\pi_3 = (1 - \beta)^2\gamma^3\beta\pi^o - \gamma(1 - \gamma\beta)(1 - \beta)\gamma^2\beta\pi^o \\ &= -(1 - \beta)(1 - \gamma)\gamma^3\beta^2\pi^o < 0, \end{aligned}$$

and

$$\begin{aligned}
\pi_4^1 - \pi_4 &= \gamma^3 \beta \pi^o - \left[ \gamma \beta^2 \pi^o + \left( 1 - \frac{1 - \gamma^3 \beta}{m - 3} \right) \pi_3 \right] - \frac{1 - \gamma^2 \beta}{m - 3} \gamma \pi_3 \\
&= (\gamma^2 - \beta) \gamma \beta \pi^o - \left( 1 - \frac{1 - \gamma^3 \beta}{m - 3} + \frac{1 - \gamma^2 \beta}{m - 3} \gamma \right) \pi_3 \\
&= (\gamma^2 - \beta) \gamma \beta \pi^o - \frac{m + \gamma - 4}{m - 3} (1 - \gamma \beta) (1 - \beta) \gamma^2 \beta \pi^o \\
&= (\gamma^2 - \beta) \gamma \beta \pi^o - (1 - \gamma \beta) (1 - \beta) \gamma^2 \beta \pi^o + \frac{1 - \gamma}{m - 3} (1 - \gamma \beta) (1 - \beta) \gamma^2 \beta \pi^o \\
&= \gamma \beta \pi^o \left\{ (\gamma^2 - \beta) - (1 - \gamma \beta) (1 - \beta) + \frac{1 - \gamma}{m - 3} (1 - \gamma \beta) (1 - \beta) \gamma \right\} \\
&= \gamma \beta \pi^o \left\{ -\gamma (1 - \gamma) \left[ 1 - \frac{(1 - \gamma \beta) (1 - \beta)}{m - 3} \right] + \beta [\gamma + \gamma^2 (1 - \beta) - 1] \right\} \\
&< 0
\end{aligned}$$

if

$$1 \geq \gamma [1 + \gamma (1 - \beta)],$$

which holds if  $\beta \geq \max \left\{ 2 - \frac{1}{\gamma}, \frac{1 - \gamma}{2 - \gamma} \right\}$ . It follows that  $S_4$  has no incentive to switch position with  $S_2$  or  $S_1$ . Thus  $S_4$  cannot benefit from any deviation.

Next, consider  $S_3$ . If  $S_3$  switches position with  $S_4$ , its payoff would be  $\pi_3^4$ ; while its payoff at  $E_3$ , after paying  $E$ , is  $\pi_3 - b_4$ . We have:

$$\begin{aligned}
&\pi_3^4 - [\pi_3 - b_4] \\
&= \frac{1}{m - 3} (1 - \gamma^3 \beta) (1 - \gamma \beta) (1 - \beta) \gamma^2 \beta \pi^o - \left[ \pi_3 - \left( 1 - \frac{1 - \gamma^2 \beta}{m - 3} \right) \gamma \pi_3 \right] \\
&= \frac{1}{m - 3} (1 - \gamma^3 \beta) (1 - \gamma \beta) (1 - \beta) \gamma^2 \beta \pi^o \\
&\quad - \left[ 1 - \left( 1 - \frac{1 - \gamma^2 \beta}{m - 3} \right) \gamma \right] (1 - \gamma \beta) (1 - \beta) \gamma^2 \beta \pi^o \\
&= -\frac{1}{m - 3} (m - 4) (1 - \beta \gamma) (1 - \gamma) (1 - \beta) (\pi^o) \beta \gamma^2 < 0,
\end{aligned}$$

Hence,  $S_3$  has no incentive to switch position with  $S_4$ . Similarly, we have



$$\begin{aligned}
\pi_3^2 - [\pi_3 - b_4] &= \left[ (1 - \beta) \gamma^2 \beta \pi^o - (1 - \beta) \gamma^3 \beta^2 \pi^o - \left( 1 - \frac{1 - \gamma^2 \beta}{m - 3} \right) \gamma \pi_3 \right] \\
&\quad - \left[ \pi_3 - \left( 1 - \frac{1 - \gamma^2 \beta}{m - 3} \right) \gamma \pi_3 \right] \\
&= (1 - \beta) \gamma^2 \beta \pi^o - (1 - \beta) \gamma^3 \beta^2 \pi^o - (1 - \gamma \beta) (1 - \beta) \gamma^2 \beta \pi^o = 0,
\end{aligned}$$

and

$$\begin{aligned}
&\pi_3^1 - [\pi_3 - b_4] \\
&= \gamma^2 \beta \pi^o - \gamma \beta^2 \pi^o - \left( 1 - \frac{1 - \gamma^3 \beta}{m - 3} \right) \pi_3 - \left[ \pi_3 - \left( 1 - \frac{1 - \gamma^2 \beta}{m - 3} \right) \gamma \pi_3 \right] \\
&= \gamma^2 \beta \pi^o - \gamma \beta^2 \pi^o - \left[ 1 - \frac{1 - \gamma^3 \beta}{m - 3} + 1 - \left( 1 - \frac{1 - \gamma^2 \beta}{m - 3} \right) \gamma \right] \pi_3 \\
&= \gamma^2 \beta \pi^o - \gamma \beta^2 \pi^o + \frac{m\gamma - 4\gamma - 2m + 7}{m - 3} (1 - \gamma \beta) (1 - \beta) \gamma^2 \beta \pi^o \\
&= \gamma^2 \beta \pi^o - \gamma \beta^2 \pi^o + \frac{-(m - 3)(2 - \gamma) + 1 - \gamma}{m - 3} (1 - \gamma \beta) (1 - \beta) \gamma^2 \beta \pi^o \\
&= \gamma^2 \beta \pi^o - \gamma \beta^2 \pi^o - (2 - \gamma) (1 - \gamma \beta) (1 - \beta) \gamma^2 \beta \pi^o + \frac{1 - \gamma}{m - 3} (1 - \gamma \beta) (1 - \beta) \gamma^2 \beta \pi^o \\
&\leq \gamma^2 \beta \pi^o - \gamma \beta^2 \pi^o - (2 - \gamma) (1 - \gamma \beta) (1 - \beta) \gamma^2 \beta \pi^o + (1 - \gamma) (1 - \gamma \beta) (1 - \beta) \gamma^2 \beta \pi^o \\
&= -\gamma \beta^2 (\pi^o) (\beta \gamma^2 - \gamma^2 - \gamma + 1) \leq 0
\end{aligned}$$

if

$$1 \geq \gamma [1 + \gamma (1 - \beta)].$$

It follows that  $S_3$  has no incentive to switch position with  $S_2$  or  $S_1$ . Thus  $S_3$  cannot benefit from any deviation.

Next, consider  $S_2$ . If  $S_2$  switches position with  $S_4$ , its payoff would be  $\pi_2^4$ ; while its payoff at  $E_2$ , after paying  $E$ , is  $\pi_2 - b_3$ . We have:

$$\begin{aligned}
\pi_2^4 - [\pi_2 - b_3] &= \frac{1}{m - 3} (1 - \gamma^3 \beta) (1 - \gamma^2 \beta) (1 - \beta) \gamma \beta \pi^o \\
&\quad - \left[ (1 - \beta) \gamma \beta \pi^o - \left( 1 - \frac{1 - \gamma^3 \beta}{m - 3} \right) (1 - \gamma \beta) (1 - \beta) \gamma^2 \beta \pi^o \right] \\
&= \frac{-1}{m - 3} ((m - 4) (1 - \gamma) + (m - 3) \beta \gamma^2 + \beta \gamma^3 (1 - \gamma)) (1 - \beta) \beta \gamma \pi^o < 0.
\end{aligned}$$

Hence,  $S_2$  has no incentive to switch position with  $S_4$ . Similarly, we have

$$\begin{aligned}
& \pi_2^3 - [\pi_2 - b_3] \\
&= (1 - \gamma^2\beta)(1 - \beta)\gamma\beta\pi^o - b_4 - [\pi_2 - b_3] \\
&= (1 - \gamma^2\beta)(1 - \beta)\gamma\beta\pi^o - \left(1 - \frac{1 - \gamma^2\beta}{m - 3}\right)\gamma\pi_3 - \left[(1 - \beta)\gamma\beta\pi^o - \left(1 - \frac{1 - \gamma^3\beta}{m - 3}\right)\pi_3\right] \\
&= (1 - \gamma^2\beta)(1 - \beta)\gamma\beta\pi^o - (1 - \beta)\gamma\beta\pi^o - \left[\left(1 - \frac{1 - \gamma^2\beta}{m - 3}\right)\gamma - \left(1 - \frac{1 - \gamma^3\beta}{m - 3}\right)\right]\pi_3 \\
&= (1 - \gamma^2\beta)(1 - \beta)\gamma\beta\pi^o - (1 - \beta)\gamma\beta\pi^o + \frac{m - 4}{m - 3}(1 - \gamma)(1 - \gamma\beta)(1 - \beta)\gamma^2\beta\pi^o \\
&< (1 - \gamma^2\beta)(1 - \beta)\gamma\beta\pi^o - (1 - \beta)\gamma\beta\pi^o + (1 - \gamma)(1 - \gamma\beta)(1 - \beta)\gamma^2\beta\pi^o \\
&= \gamma^2\beta(1 - \beta)(\beta\gamma^2 - 2\beta\gamma - \gamma + 1)\pi^o \leq 0
\end{aligned}$$

if

$$\gamma[1 + \beta(2 - \gamma)] \geq 1,$$

which holds if  $\beta \geq \max\left\{2 - \frac{1}{\gamma}, \frac{1 - \gamma}{2 - \gamma}\right\}$ . Furthermore,

$$\begin{aligned}
& \pi_2^1 - [\pi_2 - b_3] \\
&= \gamma\beta\pi^o - \gamma\beta^2\pi^o - \left(1 - \frac{1 - \gamma^3\beta}{m - 3}\right)\pi_3 - \left[(1 - \beta)\gamma\beta\pi^o - \left(1 - \frac{1 - \gamma^3\beta}{m - 3}\right)\pi_3\right] \\
&= \gamma\beta\pi^o - \gamma\beta^2\pi^o - (1 - \beta)\gamma\beta\pi^o = 0
\end{aligned}$$

It follows that  $S_2$  has no incentive to switch position with  $S_3$  or  $S_1$ . Thus  $S_2$  cannot benefit from any deviation.

Finally, consider  $S_1$ . If  $S_1$  switches position with  $S_4$ , its payoff would be  $\pi_1^4$ ; while its payoff at  $E_1$ , after paying  $E$ , is  $\pi_1 - b_2$ . We have:

$$\begin{aligned}
& \pi_1^4 - [\pi_1 - b_2] \\
&= \frac{1}{m-3} (1 - \gamma^3\beta) (1 - \gamma^2\beta) (1 - \gamma\beta) \beta\pi^o - \beta\pi^o + (1 - \beta) \gamma^3\beta^2\pi^o + \left(1 - \frac{1 - \gamma^2\beta}{m-3}\right) \gamma\pi_3 \\
&= -\beta\pi^o + (1 - \beta) \gamma^3\beta^2\pi^o + \gamma (1 - \gamma\beta) (1 - \beta) \gamma^2\beta\pi^o + \\
&\quad \left[ \frac{1}{m-3} (1 - \gamma^3\beta) (1 - \gamma^2\beta) (1 - \gamma\beta) \beta\pi^o - \frac{1 - \gamma^2\beta}{m-3} \gamma (1 - \gamma\beta) (1 - \beta) \gamma^2\beta\pi^o \right] \\
&= \beta (\pi^o) (\gamma^3 - \beta\gamma^4 - \beta^2\gamma^3 + \beta^2\gamma^4 - 1) + \frac{(1 - \beta\gamma^2) (1 - \beta\gamma) (\gamma + \gamma^2 + 1) (1 - \gamma) \pi^o \beta}{m-3} \\
&\leq \beta (\pi^o) (\gamma^3 - \beta\gamma^4 - \beta^2\gamma^3 + \beta^2\gamma^4 - 1) + (\beta\gamma^2 - 1) (\beta\gamma - 1) (\gamma + \gamma^2 + 1) (1 - \gamma) \pi^o \beta \\
&= -\gamma\beta^2 (\pi^o) (\gamma - \gamma^4 + 1 - \beta\gamma^3 + \beta\gamma^5) < 0.
\end{aligned}$$

Hence,  $S_1$  has no incentive to switch position with  $S_4$ . Similarly, we have

$$\begin{aligned}
\pi_1^3 - [\pi_1 - b_2] &= (1 - \gamma^2\beta) (1 - \gamma\beta) \beta\pi^o - b_4 - [\pi_1 - b_2] \\
&= (1 - \gamma^2\beta) (1 - \gamma\beta) \beta\pi^o - \left(1 - \frac{1 - \gamma^2\beta}{m-3}\right) \gamma\pi_3 - \beta\pi^o \\
&\quad + (1 - \beta) \gamma^3\beta^2\pi^o + \left(1 - \frac{1 - \gamma^2\beta}{m-3}\right) \gamma\pi_3 \\
&= -\gamma\beta^2 (\pi^o) (1 + \gamma - \gamma^2) < 0
\end{aligned}$$

$$\begin{aligned}
& \pi_1^2 - [\pi_1 - b_2] \\
&= (1 - \gamma\beta) \beta\pi^o - b_3 - [\pi_1 - b_2] \\
&= (1 - \gamma\beta) \beta\pi^o - \left(1 - \frac{1 - \gamma^3\beta}{m-3}\right) \pi_3 - \beta\pi^o + (1 - \beta) \gamma^3\beta^2\pi^o + \left(1 - \frac{1 - \gamma^2\beta}{m-3}\right) \gamma\pi_3 \\
&= (1 - \gamma\beta) \beta\pi^o - \beta\pi^o + (1 - \beta) \gamma^3\beta^2\pi^o - \left(1 - \frac{1 - \gamma^3\beta}{m-3}\right) \pi_3 + \left(1 - \frac{1 - \gamma^2\beta}{m-3}\right) \gamma\pi_3 \\
&= -\gamma\beta^2 (\pi^o) (\beta\gamma^2 - \gamma^2 + 1) - \frac{m-4}{m-3} (1 - \gamma) (1 - \gamma\beta) (1 - \beta) \gamma^2\beta\pi^o < 0.
\end{aligned}$$

It follows that  $S_1$  has no incentive to switch position with  $S_3$  or  $S_2$ . Thus  $S_1$  cannot benefit from any deviation.

In sum, none of the sellers can benefit from any deviation when  $\beta \geq \max \left\{ 2 - \frac{1}{\gamma}, \frac{1-\gamma}{2-\gamma} \right\}$ .  
Our proof is thus complete.  
Q.E.D.

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