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Selling and Leasing Software with Network Externality

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Selling and Leasing Software with Network Externality*

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Abstract

Previous studies suggested that a monopoly durable goods seller can use leasing to effectively avoid the time-inconsistent problem raised by Coase Conjecture. This paper extends those previous works by examining the monopoly seller's selling and leasing strategy for a special type of durable good --- software. We look at a software vendor that can sell (at a posted price) or lease his product where as a lesser he guarantees that the lessees will always have the latest version of the software. We address some of the specific issues of implementing the selling and/or leasing policies at the packaged software market, including the impact of network externality, upgrade compatibility, and commitment on pricing in a dynamic environment. We show that by properly defining their pricing structure, software vendors can segment the market and second-degree price discriminate the consumers. We also demonstrate how software vendors can manage the trade-offs of selling and leasing to achieve a higher profit as well as the corresponding welfare effect on the consumers.

Key Words: Software licensing, Coarse Conjecture, Price discrimination, Network externality, Commitment, Upgrade, Compatibility, Risk.

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1. Introduction

Software has been traditionally sold as a property. Users pay a fee for the perpetual proprietary license to use it for the whole life cycle. The development of information technology, especially the Internet, has unleashed unprecedented levels of process innovation as well as product innovation in the software industry. The way software is sold is also changed. Many software publishers such as Sun, Oracle, and Microsoft have expanded their software distribution methods with a new subscription licensing policy, which converts the purchase of software to a subscription to the service.

With the advent of this subscription licensing model, a software vendor has expanded choices to distribute his products, including the traditional perpetual licensing policy in which software is sold as a commodity; a pure subscription model that leases the software as a service; or a third choice that is a hybrid model that sells and leases the software simultaneously. This paper compares the three licensing policies offered in terms of pricing, vendor profit, and consumer surplus.

Software can be used for a period of time without replacement, though its value may depreciate. In this sense, it is a kind of durable goods. There are some special characteristics that differentiate software from the conventional durable goods such as books and automobiles, namely,

(i), software cannot be resold or appropriated because of intellectual property rights. There are no other sources of getting a license except the original vendor, the retailers, or service providers. A second-hand market like that for used cars therefore does not exist for software;

(ii), as an information good, software has strong economy of scale in production—it is costly to create the first copy but has negligible marginal cost to produce additional copies;

(iii), with the development of information technology, it is easy to improve the value of already installed software through upgrades without interfering with the original customization;

(iv), the use of software has a strong network externality effect (Katz and Shapiro, 1986). The value of using particular software increases with the number of its adopters.

Among the above special characteristics of software products, (i) to (iii) can be considered as a simplified version of the traditional durable goods selling or leasing problem; however, characteristic (iv) complicates the problem and has never been formally addressed. Therefore we cannot apply all the established conclusions about the physical durable goods to compare software licensing policies. This paper will fill in the gap and contribute to the theory of durable good by examining the impact of network externality on the monopoly seller's selling and leasing strategies.

Besides the above differences, software compatibility and related varying network effects in software upgrades further complicate the consumers and vendor's decisions. Software is usually designed to be compatible with previous versions (backward compatibility) due to the network effect from users of the old version software, while forward compatibility (compatible with future versions) is hard to achieve before the new version comes out. The stronger a network effect exists in software adoption, the higher incentive it gives the monopolist software vendor to lower the price of the first version software, so that he can boost the market share, increase network externality for future version software, and sell it at a higher price in the future.

By addressing the specific issues related to implementing the licensing policies raised above, such as, compatibility, network externality and commitment problem, this paper investigates how a monopolist software vendor uses different licensing policies and pricing menus to segment consumers based on their sensitivity to product quality and realize second-price discrimination. Considering consumers' self-selection behavior, we will also study the optimal licensing policy of the software vendor—selling or leasing his products exclusively, or adopting a hybrid strategy of both sales and leases and propose managerial insights. Therefore this study extends the previous studies of Padmanabhan et al. (1997), Fudenberg and Tirole (1998) and Ellison and Fudenberg (2000), which focused on a special case of our model for insights on durable goods selling and excessive software upgrade.

The most important result of the study is that in a durable market, leasing strategy can be used to resolve the time-inconsistent problem, as suggested by Coase (1972), Bulow (1982) and the others, but not in all cases. Specifically, in the software market considered in this paper, leasing can still make the seller commit to future prices, by definition, but it does not solve the time-inconsistent problem in the sense that the maximal profit for the seller can be lower than that with commitment. This is not the case in previous studies, which found that leasing "can achieve all the standard results of a nondurable monopolist" (Bulow 1982). This surprising result is due to the strong network effect in software market.

Leasing puts a constraint of constant rent over time, thus it helps the vendor commit on a high future price at a cost of losing some market share in the current period. On the other hand, without the constraint on selling prices for different versions of software, the vendor can use a lower first period price to strengthen the network effect aiming for a higher profit in the future. However, selling lacks the ability of making commitment; therefore the vendor may not be able to reap the benefit of the network effect. Those are the trade-offs between selling and leasing faced with a monopolist software vendor with the presence of network externality. When network externality is large enough, leasing may not be a dominant strategy for the software vendor. Instead, he may adopt a pure selling or hybrid strategy to achieve the optimal profit. That is, when network externality is taken into account, leasing may not always achieve the optimal profit as shown in previous studies (Coase 1972, Swan 1977, Bulow 1982). This hypothesis has been supported by the results from the model. We show that because leasing strategy forgoes some market share for the ability of committing on second period price, with the presence of network externality, profit under pure leasing strategy may be lower than that of a nondurable monopolist. Therefore leasing can only achieve constrained optima, and it addresses the time-inconsistent problem at the cost of losing market share and reducing consumer surplus.

The rest of the paper is organized as follows. Section 2 presents a review of the relevant literature. Section 3 describes the model of consumer choices and firm strategies. Sub-section 3.1 - 3.4 presents the equilibria of pure selling and pure leasing strategies under the five models of different market configurations. Section 4 concludes with suggestions for future research.

2. Literature Review

Many economics and marketing researchers have examined the ways of selling durable goods. Coase (1972) raised the conjecture that a monopolist durable goods seller could not sell the goods at the monopolist price (time-inconsistent problem) because rational and patient consumers would anticipate the future price drop and delay their purchase. Hence Coase proposed leasing (rather than selling) durable goods as a way to solve the commitment problem.

Stokey (1981) and Gul et al. (1986) have rigorously verified this conjecture, followed by Bulow (1982)'s conclusion that leasing can achieve the optimal profit for a monopolist non-durable good seller, therefore it dominates selling. Some recent works, however, challenged the assumptions of the Coase model and proposed conditions under which leasing can co-exist with selling as the optimal strategy for a monopolist durable good seller. For example, Bucovetsky and Chilton (1986), proposed threat of entry as a reason for the monopolist to mix selling and leasing; Desai and Purohit (1998) find that leasing does not dominate selling in all cases if the depreciation of the durable goods is taken into account, and that a mixed strategy is optimal when the depreciation rates differ between selling and leasing. We adopt a setting similar to the traditional Coase Conjecture where a monopolist software vendor sells or leases his product to the market. We want to examine the impact of network externality alone on the vendor's selling and leasing strategies ruling out the impact of competition and product depreciation.

Among the studies address the distribution strategy of packaged software, Choudhary et al. (1998) study the problem of renting software from a different point of view, arguing that renting software in the first period to otherwise later adopters can increase the seller's profit. Another related stream of literature concerns product upgrades. This paper discusses the upgrade problem from a different perspective from that of Padmanabhan et al. (1997) and Fudenberg and Tirole (1998), both of which suggested that product upgrade as a way to deal with demand uncertainty. Padmanabhan et al. (1997) suggest that when a monopolist seller can choose both prices and qualities over two periods but is uncertain about the initial exogenous demand, upgrades come from the underprovision of introductory quality, which signals high externality in the market. Fudenberg and Tirole (1998) study the pricing of a new generation of durable goods depending on the information the monopolist has about its past customers. Our paper endogenizes the demand for goods of different qualities and distributions. The current paper studies intertemporal product quality improvement, which differs from vertical product differentiation in a static state, as studied by Mussa and Rosen (1978), Moorthy (1984), and Bhargava and Choudhary (2001), among others.

Our work also draws on the literature on markets that involve either direct or indirect network effects. Katz and Shapiro (1994) categorize such markets and identify the issues firms and consumers face while dealing with these markets. Farrell and Saloner (1986) investigate how the installed base for such products interacts with a firm's incentive to innovate and evaluate the welfare implications of certain strategies that firms might adopt. Similar issues are also considered by Katz and Shapiro (1985), Farrell and Saloner (1985), and Choi (1994). Ellison and Fundenberger (2000) examined software issues incorporating network externality and they found that the network effect causes excessive upgrades above the social optimal level. That paper is the closest to our work, but they did not consider leasing or selling, which is the traditional Coase Conjecture problem and also has great practical implications.

We investigate whether leasing can solve the time-inconsistent problem in "Coase conjecture" for special durable goods like software, that is, whether web-based leases will become the dominant way of delivering software. We discuss leasing and selling software in a monopolist context, trying to explain the motivation for different strategies. We analyze the optimal software distribution strategies and discuss the impact on users: (1) to provide insights into Independent Software Vendors (ISVs); (2) to explain some of the empirical observations regarding software licensing policies; (3) to examine the differences in strategies regarding software and other durable products; and (4) to understand the effect of the selling and leasing strategies on the consumer welfare and social welfare.

3. Model

We model the intertemporal consumer behavior and the firm's strategic licensing policy with a two-period model. We introduce our model by first detailing the assumptions about the players—consumers and the monopoly software vendor (the ISV).

Assume that the consumers of the software product have the following form of net utility:

$$U(q, x, p; \theta) = \theta q + e x - p, \tag{1}$$

where p and q are the price paid and the quality of the software product, respectively; x represents the population of the adopters of the product; and θ and e are the intensity of the quality preference and the network externality, respectively. Software "quality" includes such dimensions as speed, compatibility with available operating systems, functionalities, user interface, ease of learning, warranty, service and support and other characteristics that affect the users' valuation of the product. Consumers are heterogeneous in their quality preference θ but are homogeneous in their sensitivity to network externality e. Consumers are indexed by their quality preference θ and are uniformly distributed on the support [0, 1]. Similar utility functional forms can be found in Mussa and Rosen (1978), Salant (1989), and Ellison and Fudenberg (2000). Measures of quality include speed, compatibility, functionalities, user interface, ease of learning, warranty, service and support, etc.

The ISV can provide the software to the market through one-time sales, leases, and sales of upgrades. Assume that there are two versions of the software. Version I, with quality q_1, is released at the beginning of period 1. Because of ongoing development, an upgraded version with quality q_2 is released at the beginning of period 2, and $q_2 > q_1$. In order to simplify the problem, we assume there the second version software quality is certain and leave out the discussion about a stochastic q_2 to other studies.

To cut down on the number of cases under consideration we assume that $q_1 \geq \beta \cdot (q_2 - q_1)$, so that it is optimal for consumers with higher quality preference θ to consume in period 1 as opposed to waiting until period 2. Similar to Assumption 1 in Fudenberg and Tirole (1998), this assumption only excludes very large improvements between the two versions, but considerably simplifies the analysis.

We study the most common case of software production in terms of compatibility: the new version software is backward compatible but not forward compatible with the previous version. That is, this software system can successfully use interfaces and data format from earlier versions of the system, but it is not designed in such a way that it fits with planned future versions of itself. Hence, consumers who upgrade or buy the new version software can use new features of the latest version and enjoy the network externality from the installed base—the users of the old version of the software as well as adopters of the new version; but those consumers who continue to use the old version software cannot use the new features and their network externality will only come from the users of the same version.

The ISV sets the selling price p_1, the upgrade price p_u, the second-period selling price p_2 for the new version, and the per period rent p_r at the beginning of period 1. In the lease contract, the vendor is committed to keep the rent, p_r, the same during the two periods. Since the software vendor would like to attract the users to purchase in the first period and can not tell whether a buyer in the second period owns the first version, he can not charge an upgrade price p_u higher than the sale price of version II, p_2; that is, $p_u \leq p_2$.

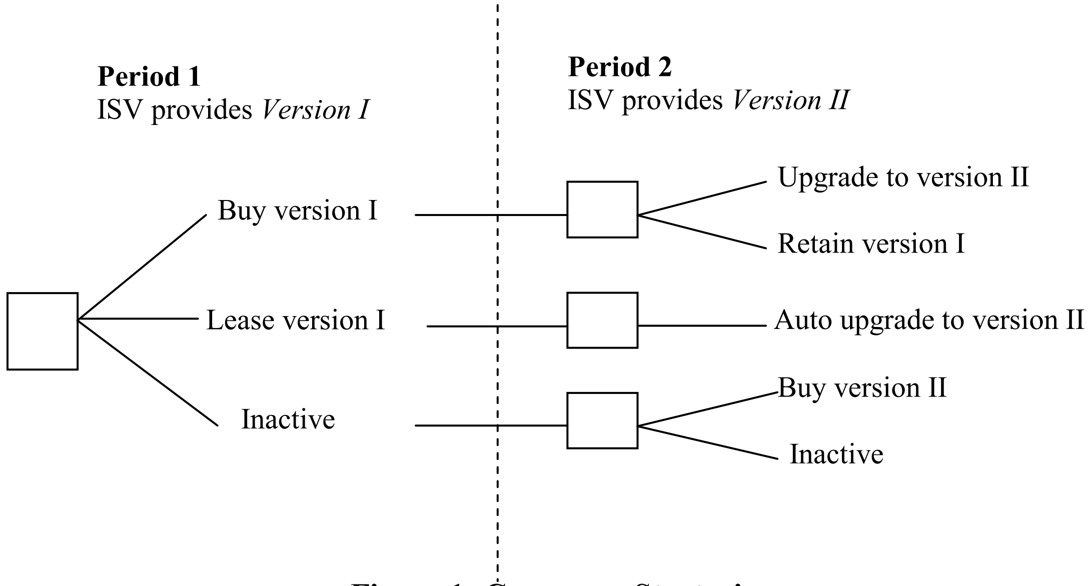

Figure 1: Consumer Strategies

Figure 1 depicts all the available consumer choices. Those consumers who adopt version I in the first period have the option of upgrading to the new version in period 2 at a cost of p_u, but the consumers who enter the lease contract in the first period will use the new version software without any additional charge besides the per period rent. Those consumers who are inactive in the first period will have the choice of purchasing version II or not in the second period.

For a consumer of type θ who purchases the version I software in the first period, the total discounted value can be expressed as

$$V_B(\theta) = U(q_1, x_0, p_1; \theta) + \beta \max\{U(q_2, x_2, p_u; \theta), U(q_1, x_1, 0; \theta)\} \tag{2}$$

where $q_B(\theta)$ is solved from $U(q_B(\theta), x_2, p_u; \theta) = U(q_1, x_1, 0; \theta)$.

The value of the buyer as in Equation (2) is given by the sum of the net utility for the users in period 1, as defined in (1), and the discounted value from the second period, when the buyer can decide to upgrade the software or to keep using version I. The discount factor is $\beta \in [0, 1]$. Here x_0 denotes the adoption population of version I of the software in the first period (including both the buyers and the lessees), x_1 denotes the adoption population that continue using version I in period 2 (those buyers in period 1 who do not upgrade to new version), and x_2 is the number of users using version II in period 2 (including both period 1buyers who upgrade and the lessees).

After entering a lease contract, the lessee receives continuous streams of product with updates for a fixed annual payment. The value for entering the lease contract in the first period is

$$V_L(\theta) = U(q_1, x_0, p_r; \theta) + \beta U(q_2, x_2, p_r; \theta) \,. \tag{3}$$

This value is the sum of her net utility of using the latest version of the software over the two periods. According to the contract, she has to pay the rent p_r every period during the lease[1].

Finally, the expected discounted value for a consumer inactive in the first period is

$$V_I(\theta) = \beta \max\{\theta q_2 + e x_2 - p_2, 0\} \,. \tag{4}$$

where $q_I(\theta)$ is solved from $U(q_I(\theta), x_2, p_2; \theta) = 0$.

If a consumer cannot get a positive net utility from buying the first version of the software or entering the lease contract, she will be inactive in the first period and wait to buy the second version of the software in period 2 if her utility of buying q_2 would be higher than the price p_2.

For a consumer with quality preference θ, the expected total discounted value from using the software over the two periods is

$$V(\theta) = \max\{V_B(\theta), V_L(\theta), V_I(\theta)\} \,. \tag{5}$$

Here we focus our attention on the consumers' buy-or-lease decision given the vendor's licensing policy.

To compare the licensing strategies for the software vendor, consumers and social impact under the various combinations of market conditions, we will solve the market equilibria under the pure selling, pure leasing and hybrid strategies for each of the following eight models (detailed model setup will be presented below). From the comparisons of the models, we will show the impact of network externality on firm licensing policy selection, its market share, profit and consumer decision and surplus.

The software vendor may be able to use the hybrid strategy to better price discriminate the consumers and extract their surplus. Yet consumers may also benefit from more choices to realize the value of using the software product. Thus it is unclear what effect it will bring to consumer surplus and social welfare.

	With Commitment	Without commitment
Without network externality	3.1: Model 1 Benchmark	3.2: Model 2 without commitment
With network externality	3.3 Model 3 With externality	3.4 Model 4 without commitment and with externality

Table 1. Model Structure of the Paper.

3.1 Benchmark Model - No externality, with commitment

In the benchmark model, we consider the case where there is no externality ($e = 0$), and the seller can commit to pre-announced second period prices, i.e., p_u and p_2.

3.1.1 Pure selling

If the software vendor decides not to lease the software but only sell it, a consumer of type θ decides to buy it or not in period 1. A consumer evaluates her benefits from each of the two choices to make the first period decision: $V(\theta) = \max\{V_B(\theta), V_I(\theta)\}$. There exists a tradeoff between buying and waiting. If the consumer buys the software, she will use it from period 1 and keeps the option of upgrading or not in period 2, while a consumer who waits to buy in period 2 cannot use it in period 1 but will keep her option of buying the new version or not in the next period open.

Based on (2) and (4), the discounted total value of a first period buyer is:

$$V_B(\theta) = U(q_1, x_0, p_1; \theta) + \beta[\max\{U(q_2, x_2, p_u; \theta), U(q_1, x_1, 0; \theta)\}]$$
$$= \begin{cases} \theta(1+\beta)q_1 - p_1 & \theta \le p_u/(q_2 - q_1) \\ \theta(q_1 + \beta q_2) - (p_1 + \beta p_u) & \theta \ge p_u/(q_2 - q_1) \end{cases},$$

and that of a waiter in period 1 is:

[1] Assume the penalty for breaching the contract is large enough to prohibit deviators.

$$V_I(\theta) = \beta \max\{U(q_2, x_2, p_2; \theta), 0\} = \max\{\theta q_2 - p_2, 0\}$$
$$= \begin{cases} \beta(\theta q_2 - p_2) & \theta \geq p_2 / q_2 \\ 0 & \theta \leq p_2 / q_2 \end{cases} .$$

Therefore a user have four choices over the two periods, buy version I in period 1 and upgrade to version II in period 2 (BU), buy version I in period 1 and keep using it in period 2 (BH), wait to buy version II in period 2 (IB), and inactive in both periods (II). Since $q_1 \geq \beta \cdot \Delta q$, we have the consumer market segmentation as described in Lemma 1 and shown in Figure 2.

Lemma 1: *Consumers with quality preference* $\theta \in [0, \theta_0]$ *are inactive; those with* $\theta \in (\theta_0, \theta_I]$ *will wait to buy version II in the second period; those with* $\theta \in (\theta_1, \theta_2]$ *buy version I in the first period buy do not upgrade in the second period when version II is available; consumers with* $\theta \in (\theta_2, 1]$ *will always use the latest version of the software during the two periods.*

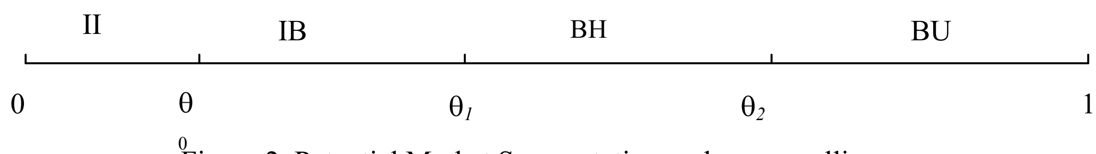

Figure 2. Potential Market Segmentation under pure selling.

The cutoff values are:

$$\theta_0 = \frac{p_2}{q_2}, \theta_1 = \frac{p_1 - \beta p_2}{q_1(1+\beta) - \beta q_2}, \theta_2 = \frac{p_u}{q_2 - q_1}. \tag{6}$$

Taking the consumers' choices into account, the software vendor optimizes discounted total profit over the two periods:

$$\max_{p_1, p_2, p_u} \Pi(p_1, p_2, p_u) = x_{BU} \cdot (p_1 + \beta p_u) + x_{BH} \cdot p_1 + x_{IB} \cdot \beta p_2$$
$$s.t. \quad \begin{cases} x_{BU} \geq 0, x_{BH} \geq 0, x_{IB} \geq 0, x_{II} \geq 0 \\ p_2 \geq p_u \end{cases} \tag{7}$$

where x_i (i = BU, BH, and IB) represents the demand from each consumer segment, and can be calculated by the difference of the cutoff values in each segment:

$$x_{BU} = 1 - \theta_2, x_{BH} = \theta_2 - \theta_1, \text{ and } x_{IB} = \theta_1 - \theta_0. \tag{8}$$

Plug (6) and (8) into (7), the optimal prices are

$$p_1^* = \frac{(1+\beta)q_1}{2}, p_2^* = \frac{q_2}{2}, p_u^* = \frac{(q_2 - q_1)}{2} \tag{9}$$

Given (6)-(9), the optimal total profit of the software vendor in Model 1 is:

$$\Pi^* = \frac{(q_1 + \beta q_2)}{4}, \tag{10}$$

The equilibrium consumer segmentation is given by $x_{BU} = \frac{1}{2}$.

Therefore it is optimal for the software vendor to sell only to high-end users and to price out buy-and –holder users as well as the opportunistic consumers who wait to buy in period 2. Although consumers who purchase the software in the first period do have the option of not upgrading in period 2, the option has no value here since the prices are set such that all of them will chose to upgrade.

Consumer surplus and social welfare are

$$CS^* = \frac{(q_1 + \beta q_2)}{8}, \ \ W^* = \frac{3(q_1 + \beta q_2)}{8}. \tag{11}$$

respectively.

3.1.2 Pure leasing

If the software vendor chooses not to sell the software but only allows leasing the product, a consumer has only two choices over the two periods, that is, to lease or not to lease. Her total value will be $V(\theta) = \max\{V_L(\theta), V_I(\theta)\}$. From (3), a lessee's total value over the two periods is:

$$V_L(\theta) = U(q_1, x_0, p_r; \theta) + \beta U(q_2, x_2, p_r; \theta) = \theta(q_1 + \beta q_2) - (1+\beta)p_r,$$

and the value for someone who does not lease is $V_I(\theta) = 0$.

The consumers are segmented into two groups, consumers with $\theta \geq \frac{p_r}{(q_1 + \beta q_2)/(1+\beta)}$ will lease, otherwise will stay inactive.

The vendor will choose an optimal leasing price to maximize total profit:

$$\max_{p_r} \Pi(p_r) = x_L \cdot (1+\beta)p_r$$
$$s.t. \quad x_L \geq 0$$

Solving the above maximization problem, we can see the optimal leasing price is $p_r^* = \frac{(q_1 + \beta q_2)}{2(1+\beta)}$, which is the same as the total expenses of the buy-and-upgrade users equally allocated to each period.

The software vendor receives the same profit as in the pure selling scenario $\Pi^* = \frac{(q_1 + \beta q_2)}{4}$.

Half of the consumers will lease and the other half will not.

Compare the equilibria under pure selling and pure leasing, we can see that the ISV is indifferent to either strategy. Consumer segmentations are also identical. For any consumer, the total discounted payment to the ISV is also the same as in the pure selling scenario. The only difference is that under pure leasing, a consumer pays same leasing price in the two periods, while under pure selling, the first period purchasing price and the upgrade price can be different. In particular, when the quality increase is not too large $\Delta q < (1+\beta)q_1$, a myopic consumer can be easily enticed by the lease contract with a lower per-period rent than the purchase price in period 1.

Since the ISV is indifferent to the pure selling and pure leasing strategy, all the prices, allocations and profits will be the same in a hybrid market. Consumers are segmented into two parts, with half of the consumers indifferent with respect to lease or buy and upgrade and the other half is inactive in both periods.

3.2 Model 2 --- No externality, no commitment

The above benchmark model assumes that the seller is able to commit to the second period upgrade and selling prices. To see the effect of this assumption, we look at the software vendor's decision on whether to deviate from the committed prices, i.e., p_u and p_2 at period 2. At period 2, the software vendor's profit function is:

$$\max_{p_2, p_u} \Pi(p_2, p_u) = x_{IB} p_2 + x_{BU} p_u)$$

and the optimal prices are:

$$p_2^* = \frac{q_2}{4} < \frac{q_2}{2}, p_u^* = \frac{(q_2 - q_1)}{2}.$$

The upgrade price is the same as the commitment price but the selling price is lower than the committed one. It is in this sense that the optimal decision in the benchmark model is not time consistent: given the chance to reconsider its decision, the seller would be better off to change the second period sell price.

Therefore, the seller's commitment is not credible, which is consistent with the conjecture by Coase (1972) that rational consumers would anticipate that prices should fall down. In this section, we consider the Nash Equilibrium without commitment, that is, the seller cannot commit to pre-announced second period prices p_u and p_2, unless there is a binding contract such as a lease. Since leasing is a way of committing on the second period price, by introducing an external constraint, the market equilibrium for leasing will be the same as in the benchmark model.

3.2.1 **Pure selling**

Given the price of version 1 software, consumers decide whether to buy or not. The software vendor sets price of version 2 software and upgrade price at the beginning of period II. Then consumers choose to upgrade or not if they have already bought the first version in period I, otherwise they choose to buy version 2 or not.

We solve this two-stage Stackleberg game backward, starting from the consumers and software vendor's decision at the second period, taking the first period outcome as given. In the second period, a consumer who has bought the software may or may not upgrade depending on her type, the upgrade price and the quality improvement. The utility of the consumer of type θ can be expressed as

$$\begin{aligned} V_B(\theta) &= \max\{U(q_2, x_2, p_u;\theta), U(q_1, x_1, 0;\theta)\} = \max\{\theta q_2 - p_u, \theta q_1\} \\ &= \begin{cases} \theta q_1 & \theta \le p_u/(q_2 - q_1) \\ \theta q_2 - p_u & \theta \ge p_u/(q_2 - q_1) \end{cases} . \end{aligned}$$

A consumer who has been inactive in period 1 will buy the new version software in the second period when her type θ is greater than the price and quality ratio of the new software.

$$V_I(\theta) = \max\{U(q_2, x_2, p_2; \theta), 0\} = \max\{\theta q_2 - p_2, 0\}$$
$$= \begin{cases} \theta q_2 - p_2 & \theta \geq p_2 / q_2 \\ 0 & \theta \leq p_2 / q_2 \end{cases}$$

The potential market segmentation over the two periods is as depicted in Figure 3, as well as in (6) and (8). Thus, a consumer's strategy in the second period depends on the relative value of her type θ and the two cutoff values $\theta_2 = p_u / (q_2 - q_1)$ and $\theta_0 = p_u / q_2$. Given the consumers' strategies, we study the second-period profit maximization problem under the constraints that each of the four segments is nonnegative:

$$\max_{p_2, p_u} \Pi'(p_2, p_u) = x_{BU} p_u + x_{IB} p_2$$
$$s.t. \quad \begin{cases} x_{II} \geq 0, x_{IB} \geq 0, x_{BH} \geq 0, x_{BU} \geq 0 \\ p_2 - p_u \geq 0 \end{cases},$$

which yields the optimal upgrade price given p_1

$$p_u^* = \frac{(q_2 - q_1)}{2}, \; p_2^* = \frac{q_2}{2(1+\beta)q_1} p_1, \; x_{BU} = \theta_2 = \frac{1}{2}, x_{IB} = \frac{p_1}{2[(1+\beta)q_1 - \beta q_2]}, x_{II} = \frac{p_1}{2(1+\beta)q} \quad (12)$$

The other two constraints require:

$$(1+\beta)q_1(1 - \frac{q_1}{q_2}) \leq p_1 \leq \frac{(1+\beta)q_1}{2}(1 - \frac{\beta q_2}{2(1+\beta)q_1 - \beta q_2}). \quad (13)$$

Taking the second period equilibrium into consideration, the seller maximizes the total profit by deciding the first period price p_1

$$\Pi = \max_{p_1}\{(x_{BU} + x_{BH})p_1 + \beta \Pi'\}.$$

The optimal price p_1 is subject to inequalities (13) to make sure the assumed market segmentation is valid. Since the global optimum falls out of the range, the constrained optimal solution is

$$p_1 = \frac{(1+\beta)q_1}{2}(1 - \frac{\beta q_2}{2(1+\beta)q_1 - \beta q_2}), \text{ and } x_{BH} = 0. \quad (14)$$

Thus the optimal profit under this pure selling strategy is $\Pi^S = \Pi^L - \frac{\beta(1+\beta)^2 q_1^2 q_2}{4(2(1+\beta)q_1 - \beta q_2)^2}$. This profit is less than the optimal profit in the commitment case, which is also the profit under the pure leasing strategy. This is due to the vendor's inability of making a credible commitment. Therefore, leasing is believed to solve this time-inconsistent problem, as suggested in Coarse (1967) and other literature.

Compared with the pure leasing strategy, consumers are better off since some lower value consumers have the opportunity to buy in the second period. The increase in consumer surplus is even greater than the loss in the software vendor's profit, thus social welfare is also higher than in the complete commitment case:

$$CS^S = CS^L + \frac{\beta(1+\beta)q_1 q_2 (5(1+\beta)q_1 - 2\beta q_2)}{8(2(1+\beta)q_1 - \beta q_2)^2},\ W^S = W^L + \frac{\beta(1+\beta)q_1 q_2 (3(1+\beta)q_1 - 2\beta q_2)}{8(2(1+\beta)q_1 - \beta q_2)^2}.$$

3.2.2 Hybrid

A consumer receives the same benefit from leasing and buy version 1 software and upgrade to version 2 in period II, but at different cost structures. Thus, when the total cost of leasing the software is slightly lower than that of buy it and upgrade later, all consumers will strictly prefer leasing to buying and upgrade the software. Consequently, when the software provides both selling and leasing options, those consumers who enter the lease contract will cannibalize with the buy-and-upgrade type of consumers. The potential market segmentation for the hybrid case is shown in Figure 3.

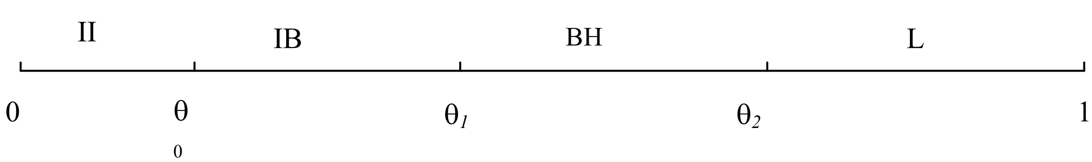

Figure 3. Potential Market Segmentation under hybrid strategy.

The cutoff values are:

$$\theta_0 = \frac{p_2}{q_2}, \theta_1 = \frac{p_1 - \beta p_2}{q_1(1+\beta) - \beta q_2}, \theta_2 = \frac{(1+\beta)p_r - p_1}{\beta(q_2 - q_1)}. \tag{15}$$

When the vendor cannot commit on second period prices, he will set per-period rent p_r and price for version I software p_I at the beginning of period 1. Consumers then choose to lease, buy or remain inactive. At period 2, after the vendor sets price for version II software p_2, those consumers who have not purchased version I decide to buy new software or not, while those who already entered the lease contract receive automatic upgrade at the cost of the rent and those who adopters of version I but non-lessees keep using the old version software.

Solving the two-stage game backward, we consider the vendor's second period profit maximization problem first.

$$\max_{p_2} \Pi'(p_2) = x_L p_r + x_{IB} p_2$$
$$s.t. \quad x_{II} \geq 0, x_{IB} \geq 0, x_{BH} \geq 0, x_L \geq 0$$

which yields the optimal price given p_I and p_r

$$p_2^* = \frac{q_2}{2(1+\beta)q_1} p_1, x_L = 1 - \frac{p_r(1+\beta) - p_1}{\beta(q_1 - q_2)}, x_{IB} = \frac{p_1}{2[(1+\beta)q_1 - \beta q_2]}, x_{II} = \frac{p_1}{2(1+\beta)q} \quad (16)$$

Taking the second period equilibrium into consideration, the seller maximizes the total profit by deciding the first period price p_1

$$\max_{p_1, p_r} \Pi(p_1, p_r) = x_L p_r + x_{BH} p_1 + \beta \Pi'$$
$$s.t. \quad \begin{cases} x_L \geq 0 \\ x_{BH} \geq 0 \end{cases} .$$

The constrained optimal solution is

$$\left. \begin{aligned} p_1 &= \frac{(1+\beta)q_1((1+\beta)q_1 - \beta q_2)(2(1+\beta)q_1^2 + \beta^2 q_1 q_2 - \beta^2 q_2^2)}{4(1+\beta)^2 q_1^3 - \beta(3+2\beta-\beta^2)q_1^2 q_2 - \beta^2(1+2\beta)q_1 q_2^2 + \beta^3 q_2^3} \\ p_r &= \frac{(2(1+\beta)q_1^2 + \beta^2 q_1 q_2 - \beta^2 q_2^2)^2}{2(1+\beta)(4(1+\beta)^2 q_1^3 - \beta(3+2\beta-\beta^2)q_1^2 q_2 - \beta^2(1+2\beta)q_1 q_2^2 + \beta^3 q_2^3)} \end{aligned} \right., \ x_{BH} = 0. \quad (17)$$

$$\Pi^H = \Pi^L - \frac{\beta(1+\beta)^2 q_1^3 q_2}{4(4(1+\beta)^2 q_1^3 - \beta(3+2\beta-\beta^2)q_1^2 q_2 - \beta^2(1+2\beta)q_1 q_2^2 + \beta^3 q_2^3)}$$

This profit is smaller than that under the pure leasing strategy. This is due to the vendor's inability of making a credible commitment.

Compared with the pure leasing strategy, consumers are better off and social welfare is also higher:

$$
\begin{aligned}
CS^S &= CS^L + \frac{3\beta(1+\beta)^2 q_1^3 q_2}{8(4(1+\beta)^2 q_1^3 - \beta(3+2\beta-\beta^2)q_1^2 q_2 - \beta^2(1+2\beta)q_1 q_2^2 + \beta^3 q_2^3)} \\
W^S &= W^L + \frac{\beta(1+\beta)^2 q_1^3 q_2}{8(4(1+\beta)^2 q_1^3 - \beta(3+2\beta-\beta^2)q_1^2 q_2 - \beta^2(1+2\beta)q_1 q_2^2 + \beta^3 q_2^3)}
\end{aligned}
. \tag{18}
$$

Comparing equilibria generated by models 1 and 2, we can see being unable to make commitment does decrease the software vendor's profit and leasing is a way of making commitment of second period price.

3.3 Model 3 –with externality, with Commitment

Software is the type of product that has demonstrated strong network effect through file sharing, skill exchange among users, as well as availability of complementary products. Therefore a software product with a greater network effect will be valued more by the users. In this model, we will consider the network externality effect in addition to the benchmark case, that is, the seller can commit to pre-announced second period prices. The market equilibrium under the pure leasing policy is expected to be the same as in the benchmark case in Section 3.1.1.

3.3.1 Pure leasing

Suppose that the ISV offers a take-it-or-leave-it lease contract over two periods. If a user takes it, she will pay rent p_r at the beginning of each period and enjoy the latest version of the software without any additional charge.

The expected discounted value for a consumer who takes the lease is given in (3). Consumers with $\theta \geq \theta_L$ will get positive utility from the contract. The level of θ_L can be obtained by solving $V_L(\theta_L) = 0$.

$$
\theta_L = \frac{(1+\beta)p_r - e(x_0 + \beta x_2)}{q_1 + \beta q_2} \tag{19}
$$

Replacing x_0 and x_2 by 1 - θ_L, we have

$$\theta_L = \frac{(1+\beta)(p_r - e)}{q_1 + \beta q_2 - e(1+\beta)} \tag{20}$$

Given the consumers' choice, the ISV sets the optimal rent in the lease contract to maximize his discounted total profit from the two-period contract:

$$\begin{aligned} &\max_{P_r} \Pi(p_r) = (1+\beta) p_r (1-\theta_L) \\ &s.t. \quad \theta_L \le 1 \end{aligned} \; . \tag{21}$$

When the network effect does not exceed the quality effect, the value of the parameters β, ρ, *e, q_1, q_2* $\in \{ q_1 + \beta q_2 \ge (1+\beta)e \}$, we have the optimal price schedule for the ISV, which is

$$p_r^l = \frac{q_1 + \beta q_2}{2(1+\beta)} \, . \tag{22}$$

Here we use the superscript *l* of the price to represent the equilibrium under the pure leasing strategy. Consumers with quality preference $\theta \ge \theta_L = \frac{1}{2} - \frac{(1+\beta)e}{2(q_1 + \beta q_2 - (1+\beta)e)}$ will enter the lease contract. The ISV gets profit $\Pi^l(p_r^l) = \frac{(q_1 + \beta q_2)^2}{4(q_1 + \beta q_2 - e(1+\beta))}$. Total consumer surplus is $CS^l = \frac{(q_1 + \beta q_2)^3}{8(q_1 + \beta q_2 - e(1+\beta))^2}$.

3.3.2 Pure selling

The ISV announces the selling prices of version I software p_1 and version II software p_2, and the upgrade price p_u, that rational consumers in period 1 should expect to pay to get version II in the next period. At the beginning of period 1, consumers make the decision of whether to buy version I. In period 2, those users who have bought version I can choose to upgrade to version II at a cost of p_u or to keep using version I. Consumers have the same four types of choices as in the benchmark case:

(1) Buy version I in period 1 and upgrade to version II in period 2 — (BU) and gain total expected discounted value

$$V_{BU}(\theta) = \theta(q_1 + \beta q_2) + e(x_0 + \beta x_2) - (p_1 + \beta p_u) \, . \tag{23}$$

(2) Buy version I in period 1 and do not upgrade in period 2 — (BH). The total expected discounted value over the two periods is

$$V_{BH}(\theta) = \theta q_1 + e x_0 - p_1 + \beta(\theta q_1 + e x_1) . \tag{24}$$

(3) Do not buy in period 1 but buy version II in period 2 with a total value

$$V_{IB}(\theta) = \beta(\theta q_2 + e x_2 - p_2) \tag{25}$$

(4) Stay inactive in both periods and gain $V_{II} = 0$.

Therefore the potential market segmentation is also the same as described in Lemma 1. The cutoff values for the four segments are $\theta_0, \theta_1, \theta_2$ and θ_3 as in Figure 2.

Taking into account the consumers' self-selection behavior, the ISV sets prices p_1 and p_u to maximize his discounted total profit over the two periods:

$$\begin{aligned} &\max_{p_1, p_u, p_2} \Pi(p_1, p_u, p_2) = p_1(x_{BU} + x_{BH}) + \beta(p_u x_{BU} + p_2 x_{IB}) \\ &s.t. \quad \begin{cases} x_{BU} \geq 0, x_{BH} \geq 0, x_{IB} \geq 0, x_{II} \geq 0, \\ p_2 \geq p_u \end{cases} \end{aligned} . \tag{26}$$

Solving this optimization problem, there are two possible equilibria.

(1) When the ratio of the second period quality to the first period quality q_2/ q_1 is high but still satisfies our assumption made at Section 3, the software vendor will charge a higher price p_1 in period 1 which cannibalizes with those relatively low value consumers who buy in period 1 but do not upgrade. Therefore it is optimal for the vendor to price the software products so that those medium value consumers have to either buy and upgrade or do not buy the software at either period. The resulting market segmentation is (II, BU). The price schedule is

$$\begin{aligned} p_1 &= \frac{(1+\beta)q_1(q_1 + \beta q_2) + e(\beta q_2 - (2\beta^2 + 4\beta + 1)q_1)}{2(q_1 + \beta q_2 - (1+\beta)e)} \\ p_2 &= \frac{q_2(q_1 + \beta q_2) - e((2+\beta)q_2 - q_1)}{2(q_1 + \beta q_2 - (1+\beta)e)} \\ p_u &= \frac{(q_2 - q_1)(q_1 + \beta q_2) - e((2+\beta)q_2 - (3+2\beta)q_1)}{2(q_1 + \beta q_2 - (1+\beta)e)} \end{aligned} \tag{27}$$

In this equilibrium, the market segmentation, profit of the vendor and consumer surplus are identical to those under the pure leasing strategy.

(2) Another equilibrium exists when the ratio of q_2/ q_1 is relatively low. Under this equilibrium, the software will give up those consumers who wait to buy in period 2 but engage in expanding the market share from the first period adopters by lowering the first period selling price p_1. The resulting market segmentation is (II, BH, BU).

This equilibrium is characterized by the following

$$
\begin{aligned}
p_1 &= \frac{(1+\beta)(q_1-e)(2(1+\beta)q_1(q_2-q_1)-e(2+\beta)q_1-\beta q_2))}{(4+3\beta)e^2+4(1+\beta)(q_1(q_2-q_1)-eq_2)} \\
p_2 &= \frac{(2+\beta)e^2(2q_2-q_1)+2(1+\beta)q_1q_2(q_2-q_1)-e(2(1+\beta)q_1^2+(4+3\beta)q_2(q_2-q_1))}{(4+3\beta)e^2+4(1+\beta)(q_1(q_2-q_1)-eq_2)} \\
p_u &= \frac{2(1+\beta)q_1(q_2-q_1)^2+e^2((2+\beta)q_2-(3+2\beta)q_1)-e(1+\beta)(2q_2-q_1)(q_2-q_1)}{(4+3\beta)e^2+4(1+\beta)(q_1(q_2-q_1)-eq_2)} \\
x_{IB} &= 0 \qquad\qquad (28) \\
x_{BH} &= \frac{(2(1+\beta)q_1-(4+3\beta)e)(q_2-q_1-e)}{(4+3\beta)e^2+4(1+\beta)(q_1(q_2-q_1)-eq_2)} \\
x_{BU} &= \frac{(1+\beta)(2q_1(q_2-q_1)-e(2q_2-q_1))}{(4+3\beta)e^2+4(1+\beta)(q_1(q_2-q_1)-eq_2)} \\
\Pi^s &= \frac{(1+\beta)(q_1(q_2-q_1)(q_1+\beta q_2)-e(1+\beta)q_1^2+\beta q_1q_2-\beta q_2^2))}{(4+3\beta)e^2+4(1+\beta)(q_1(q_2-q_1)-eq_2)}
\end{aligned}
$$

Compared with the pure leasing strategy, the vendor can have a greater market share, generate *more* profits and consumers are also better off through the pure selling strategy, as shown in the following Figures 4-7. Those medium value consumers who are forced to enter the leasing contract have the choice of buying period 1 without the obligation to upgrade in period 2 under the selling policy. Their cost of using the software is reduced significantly (Figure 6). Selling also help those low value consumers who cannot afford the leasing contract to buy version 1 software and enjoy using it for both periods. With the increase of market share, the network externality effect becomes stronger, enabling the software vendor to charge a higher upgrade price to the high value consumers. Therefore high value consumers who always use the latest version software product incur a higher total cost. However, the loss of those high value consumers is outweighed by the gain of the medium and low value consumers. Consumer surplus is increased under the selling policy.

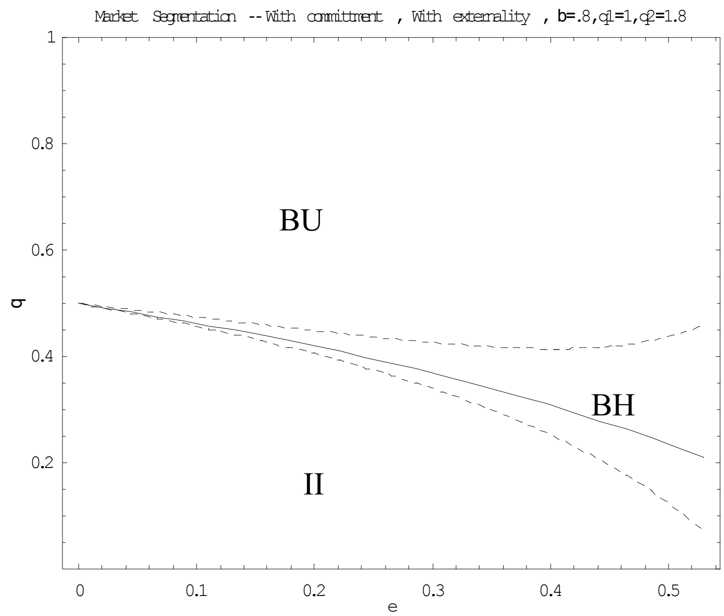

Figure 4. Market Segmentation of Selling v.s. Leasing with Externality (The solid curve: leasing; dotted curve: selling).

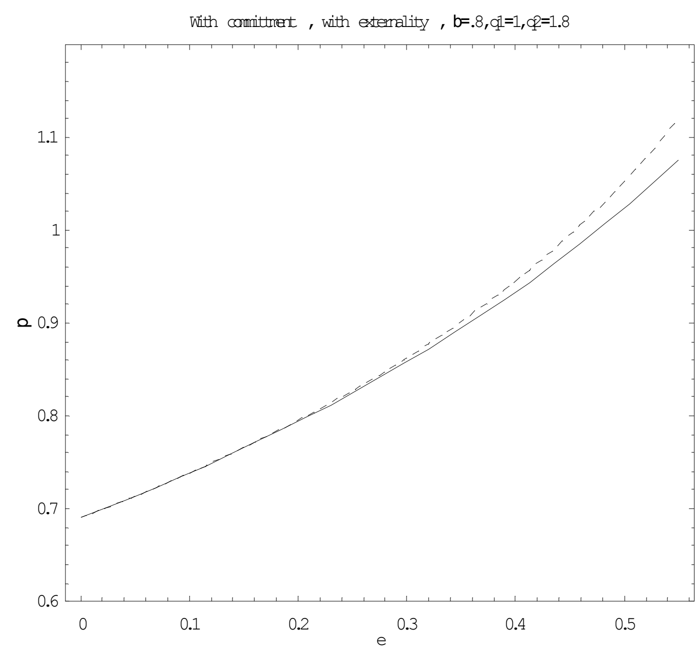

Figure 5. Profit comparison with externality (The solid curve: leasing; dotted curve: selling).

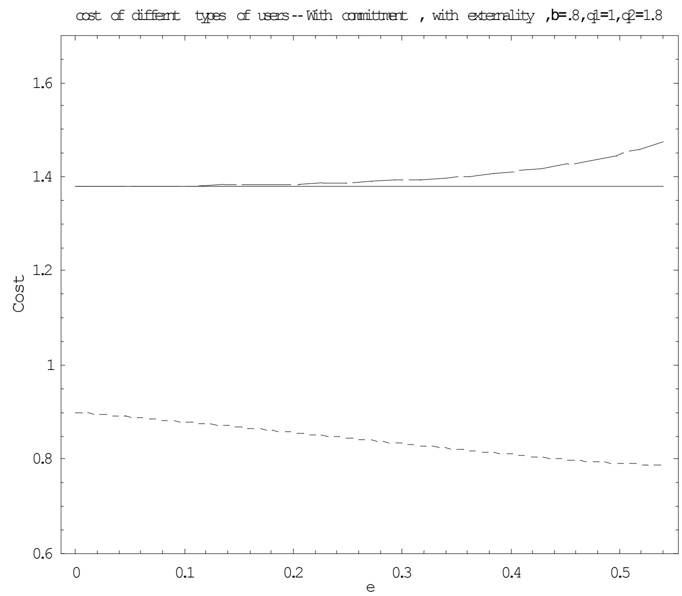

Figure 6. Cost of Using the Latest Version Software (The long dotted curve: hybrid; solid curve: pure leasing; short dotted curve: pure selling).

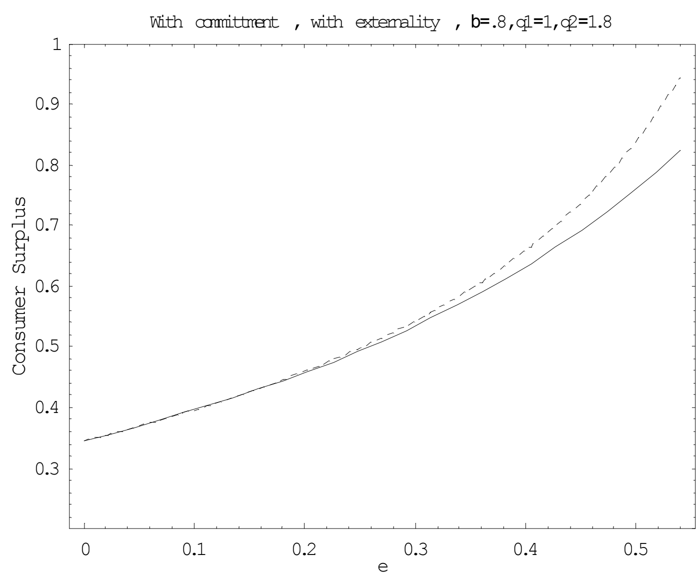

Figure 7. Consumer Surplus Comparison with Externality (The solid curve: leasing; dotted curve: selling).

This result is very interesting. The leasing policy has been suggested by Coase (1972), Bulow (1982) and other researchers as a way to effectively commit on second period prices to resolve the time-inconsistent problem. When we re-examine Coase Conjecture for durable good software which has a strong network externality effect, we find that leasing is a way of making commitment for a durable good monopoly, but its power is restricted when the durable good has a strong network effect. Leasing forces consumers to agree with the prices of both periods, which deprives some medium value consumers of the flexibility of buying but not upgrade. Therefore leasing strategy forgoes some market share for the ability of committing on second period price. Without considering network externality, previous studies about a monopoly durable seller claimed that leasing "can achieve all the standard results of a nondurable monopolist" (Bulow 1982). The above result shows that with the presence of network externality, that statement may not hold --- profit under pure leasing strategy may be lower than that of a nondurable monopolist. Therefore leasing is a *restricted* way of making commitment and it addresses the time-inconsistent problem at the cost of losing market share and reducing consumer surplus.

3.3.3 Hybrid

With network externality, the potential market segmentation for the hybrid case is the same as that in the benchmark case in Figure 2. Consumers can choose to enter the lease contract and automatic receive product upgrade in period 2 (type L), buy in period 1 and not upgrade in period 2 (BH), wait to buy in period 2 (IB) or remain inactive in either period (II). Those consumers who lease have a utility function $V_L(\theta) = \theta(q_1 + \beta q_2) + e(x_0 + \beta x_2) - (1+\beta)p_r$. The utility functions of the other three types of consumers are the same as in the pure selling case.

We solve the maximization problem to find the optimal prices and resulting firm profits.

$$\begin{aligned} &\max_{p_r, p_1, p_2} \Pi(p_r, p_1, p_2) = p_r x_L + p_1 x_{BH} + \beta(p_r x_L + p_2 x_{IB}) \\ &s.t. \quad x_L \geq 0, x_{BH} \geq 0, x_{IB} \geq 0, x_{II} \geq 0 \end{aligned}.$$

The equilibrium under the hybrid policy leads to the same consumer segmentation and vendor profit as in the pure selling equilibrium $\Pi^h = \Pi^s$. Because the quality is public knowledge to all the consumers, the total discounted price charged to lease and buy-and-upgrade customers should be the same in order to eliminate an arbitrage opportunity: $p_r^h = \dfrac{p_1^s + \beta p_u^s}{1+\beta}$. Therefore the above comparison and analysis between the selling and leasing policies also apply to this hybrid policy, with the replacement of pure selling policy to this hybrid policy.

3.4 Model 4– with externality, No Commitment

In the pervious model, we add network externality effect into the monopoly's consideration and show that leasing may not always achieve the monopoly equilibrium for a nondurable good seller. Therefore leasing is a restricted way of ameliorating the durable good time-inconsistent problem. This sub-section considers the case that when the monopoly cannot make a credible commitment on second period price, how the network externality effect can affect the licensing policy of the software vendor?

The equilibrium under the pure leasing policy is the same as in Section 3.3.1. We will next solve the equilibria under the pure selling and hybrid policies and compare them with that under leasing.

3.4.1 Pure selling

Since rational consumers will expect the upgrade price and the price of version II software to drop in the second period, the monopoly will have to decide the second period prices after consumers' first period decisions have been made, that is, based on the price and market share realized in period 1. We solve this two-stage Stackleberg game with backward induction.

Consumers' choices are the same as in Section 3.4.1, so is the potential market segmentation (II, IB, BH, BU).

$$\begin{aligned} &V_{II}(\theta_0) = V_{IB}(\theta_0 \mid q_2, x_2, p_2) \\ &V_{IB}(\theta_1 \mid q_2, x_2, p_2) = V_{BH}(\theta_1 \mid q_1, x_1, x_0, p_1) \\ &V_{BH}(\theta_2 \mid q_1, x_1, x_0, p_1) = V_{BU}(\theta_2 \mid q_1, q_2, x_1, x_2, p_1, p_u) \end{aligned} \tag{29}$$

where

$$\begin{aligned} &\theta_0 = 1 - x_2 \\ &\theta_1 = 1 - x_1 \\ &\theta_2 - \theta_1 = x_0 \end{aligned} \tag{30}$$

Taking consumers'' selections into account and given the price of version I software, the software vendor maximizes his second-period profit alone to decide on the second period upgrade price and the price of the version II software.

$$\max_{p_u, p_2} \Pi'(p_u, p_2 \mid p_1) = (x_1 - x_0)p_u + (x_2 - x_1)p_2$$

$$s.t. \quad \begin{cases} 0 \le x_0 \le x_1 \le x_2 \\ p_2 \ge p_u \end{cases}$$

After determining the optimal second period prices, the vendor continues to find the optimal first period selling price.

$$\begin{aligned} &\max_{p_1} \Pi(p_1) = x_1 p_1 + \beta \Pi' \\ &s.t. \quad x_1 \geq 0 \end{aligned}.$$

The prices directly affect consumers' adoption decision, and then affect market share. The market share of the software product increases network externality effect, which also affects consumers' utility. Therefore there exists a complicated relationship between the prices and market segmentations, which is described by the simultaneous equations (29) and (30). Since those market share vector (x_1, x_0, x_2) has a one-to-one mapping to the price vector (p_1, p_u, p_2), determining the market shares is equivalent to determining the prices. Hence we can convert the decision variables in the constrained optimization problem from price variables into the demand variables: the market shares of first period adopters, second period non-upgraders, and total market share in the second period.

When the vendor cannot commit on high prices in the second period, some medium and low value consumers will take the leapfrog strategy: they wait to buy in the second period when the installed base has been built up and they can benefit from both the network effect and the lower price. In market equilibrium, consumers strictly prefer leapfrogging to buying and holding. Therefore the market segmentation is (BU, IB, II).

When the intensity of the network effect e is small, the equilibrium price schedules and market shares are

$$x_1 = \frac{2q_1(q_2 - e)}{(4q_1 + \beta q_2)q_2 - 4e(q_1 + q_2) + 4e^2}$$

$$x_0 = 0$$

$$x_2 = \frac{\beta q_2^3 + 6q_1 q_2^2 - 2e(q_2(2q_1 - e) - (2q_2 - q_1))}{2(q_2 - e)((4q_1 + \beta q_2)q_2 - 4e(q_1 + q_2) + 4e^2)}$$

$$p_1 = \frac{q_2^2(4(1+\beta)q_1^2 + 2\beta^2 q_1 q_2 - \beta^2 q_2^2) - 4e^3(1+2\beta)q_1 + 2eq_2(2q_2^2 - (2+6\beta+\beta^2)q_1 q_2 - 4(1+\beta)q_1^2) + 4e^2(1+\beta)q_1^2 + (2+5)q_1 q_2 - \beta q_2^2)}{2(q_2 - e)((4q_1 + \beta q_2)q_2 - 4e(q_1 + q_2) + 4e^2)}$$

$$p_2 = \frac{q_2}{2}\left(1 - \frac{2q_1(q_2 - e)}{(4q_1 + \beta q_2)q_2 - 4e(q_1 + q_2) + 4e^2}\right)$$

$$p_u = \frac{2(q_2 - q_1)q_2^2(2q_1 + \beta q_2) + eq_2(8q_1^2 + 2(3+\beta)q_1 q_2 - (8+\beta)q_2^2) + 2e^2(6q_2^2 - 9q_1 q_2 - 2q_1^2) - 4e^3(q_2 - 2q_1)}{2(q_2 - e)(4q_1 + \beta q_2)q_2 - 4e(q_1 + q_2) + 4e^2}$$

When the intensity of externality e is large, the vendor has a greater incentive to charge a higher selling price towards the second version software. In that case, the condition that prevents consumers from hiding their previous purchasing history and buying second period software instead of upgrading --- $p_2 \geq p_u$ is binding. The equilibrium is characterized by

$$x_1 = \frac{q_1 q_2^2 + \beta(q_2 - q_1)((q_2 - 2q_1)q_2 + 2eq_1)}{2(\beta(q_2 - e)(q_2 - q_1)^2 + (q_1 - e)q_2^2)}$$

$$x_0 = 0$$

$$x_2 = \frac{q_1(\beta(q_2 - q_1)(q_2 - 2e) + (q_1 - 2e)q_2}{2(\beta(q_2 - e)(q_2 - q_1)^2 + (q_1 - e)q_2^2)} + x_1$$

$$p_1 = \frac{(q_1 - e)q_1 q_2^2 + \beta(q_2 - q_1)(2e^2 q_1 + (q_1 + e)q_2^2 - 4eq_1 q_2)}{2(\beta(q_2 - e)(q_2 - q_1)^2 + (q_1 - e)q_2^2)}$$

$$p_2 = p_u = \frac{q_2((q_2 - q_1)q_2(\beta q_2 + (1-\beta)q_1) + e((1-\beta)q_1^2 + (3+2\beta)q_1 q_2 - (2+\beta)q_2^2) - 2e^2 q_1)}{2(\beta(q_2 - e)(q_2 - q_1)^2 + (q_1 - e)q_2^2)}$$

Compared with the equilibrium under the leasing policy, the vendor gains a lower profit when he cannot commit on future prices but consumers are better off form the choices and lower prices.

3.4.2 Hybrid

If the vendor chooses to both sell and lease the software product, he can commit on the second period upgrade price but still leave the option of leapfrog to the lower value consumers. The equilibrium market segmentation is (L, IB, II). Similar to the game present in 3.4.1, the vendor decide the second period selling price taking the leasing price and first period selling price as given.

$$\max_{p_2} \Pi'(p_2 \mid p_r, p_1) = (x_1 - x_0)p_r + (x_2 - x_1)p_2$$
$$s.t. \quad 0 \leq x_0 \leq x_1 \leq x_2$$

After determining the optimal second period price, the vendor seeks the optimal first period selling price by maximizing the total discounted profit

$$\max_{p_1, p_r} \Pi(p_1, p_r) = (x_1 - x_0)p_r + x_0 p_1 + \beta\Pi'$$
$$s.t. \quad 0 \leq x_0 \leq x_1$$
.

To reduce the complexity of the problem, we transform the decision variables to market demand variables. The equilibrium market share and prices are:

$$x_1 = \frac{2(1+\beta)^2 q_1}{\beta q_2 + 4(1+\beta)^2 q_1 - (4+9\beta+4\beta^2)e}$$

$$x_0 = 0$$

$$x_2 = \frac{q_2(2(1+\beta)q_1 + \beta q_2) - e((4+9\beta+4\beta^2)q_2 - (2+6\beta+4\beta^2)q_1)}{2(q_2-e)(\beta q_2 + 4(1+\beta)^2 q_1 - (4+9\beta+4\beta^2)e)}$$

$$p_1 = \frac{4(1+\beta)^3 q_1^2 (q_2-e) - \beta^2 q_2^3 + e q_2((4+9\beta+4\beta^2)\beta q_2 - 2(2+11\beta+15\beta^2+6\beta^3)q_1) + 2e^2 q_1(2+9\beta+11\beta^2+4\beta^3)}{2(q_2-e)(\beta q_2 + 4(1+\beta)^2 q_1 - (4+9\beta+4\beta^2)e)}$$

$$p_2 = \frac{q_2(2(1+\beta)(2+\beta)q_1 + \beta q_2) - e((4+9\beta+4\beta^2)q_2 - 2(1+\beta)q_1)}{2(\beta q_2 + 4(1+\beta)^2 q_1 - (4+9\beta+4\beta^2)e)}$$

$$p_r = \frac{\beta^2 q_2^2 + 2\beta(2+3\beta+2\beta^2)q_1 q_2 + 4(1+\beta)^2 q_1^2) - e((4+9\beta+4\beta^2)\beta q_2 + 2(2+4\beta+\beta^2)q_1)}{2(1+\beta)(\beta q_2 + 4(1+\beta)^2 q_1 - (4+9\beta+4\beta^2)e)}$$

We use numerical examples to show the comparison among the three licensing policies (Figures 8-12). We can see leasing is still a dominating strategy among the three: the software vendor gains the highest profit under pure leasing and pure selling generates the lowest profits. Network externality increases consumer utility and thereby their willingness-to-pay. However, the software vendor needs to be able to commit on future prices in order to reap the benefit created by the network effect. When consumers expect the vendor to lower price in period 2, they will take advantage of the leapfrog choice available under the selling policy. This choice hurts the vendor's profit but benefit the consumers. Under the hybrid policy, the constraint that upgrade price cannot exceed price of second version software is eliminated, therefore the profit of hybrid equilibrium falls in-between profits of leasing and selling.

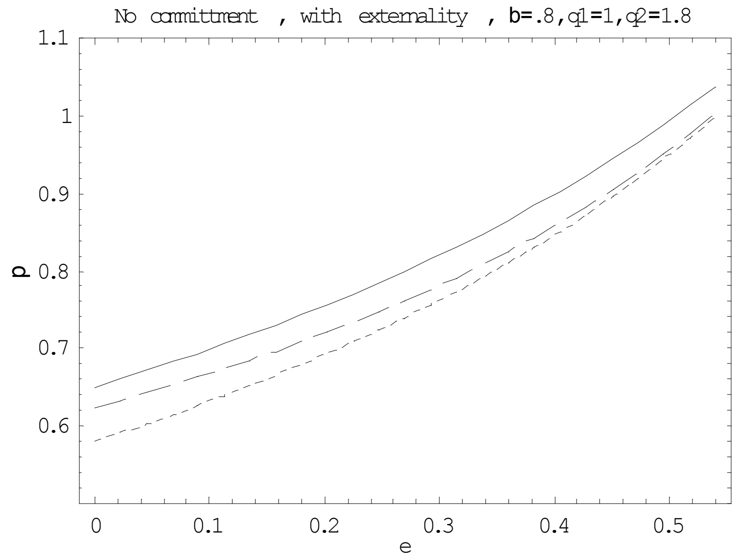

Figure 8. Profit comparison with externality and without commitment (The solid curve: pure leasing; long dotted curve: hybrid; short dotted curve: pure selling).

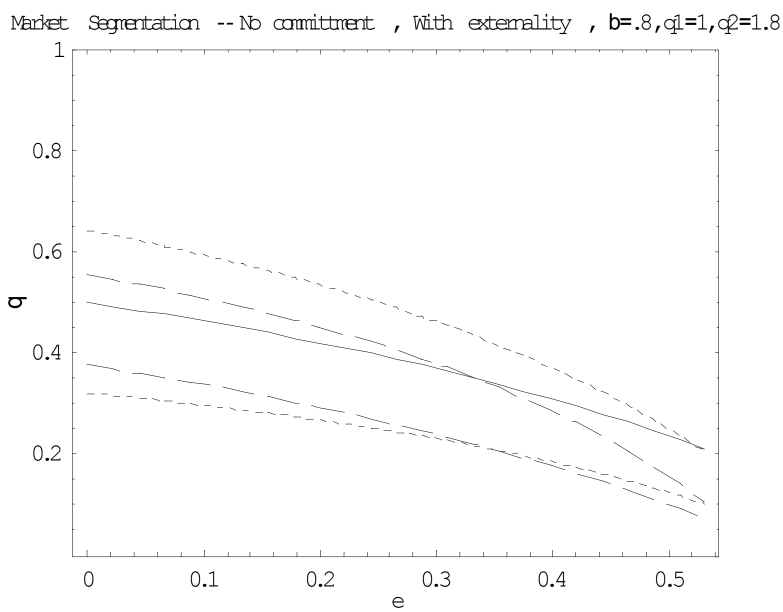

Figure 9. Market segmentations with externality and without commitment (The solid curve: pure leasing; long dotted curve: hybrid; short dotted curve: pure selling).

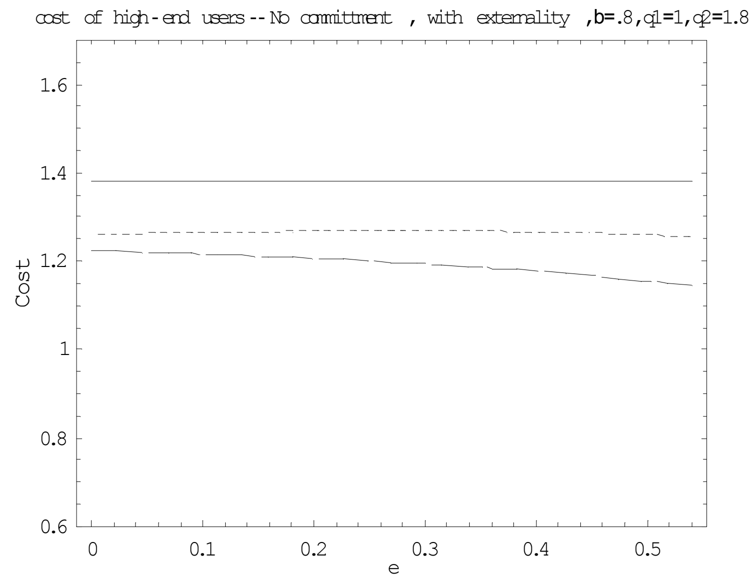

Figure 10. Cost of Using the Latest Version Software (The solid curve: pure leasing; short dotted curve: pure selling; long dotted curve: hybrid).

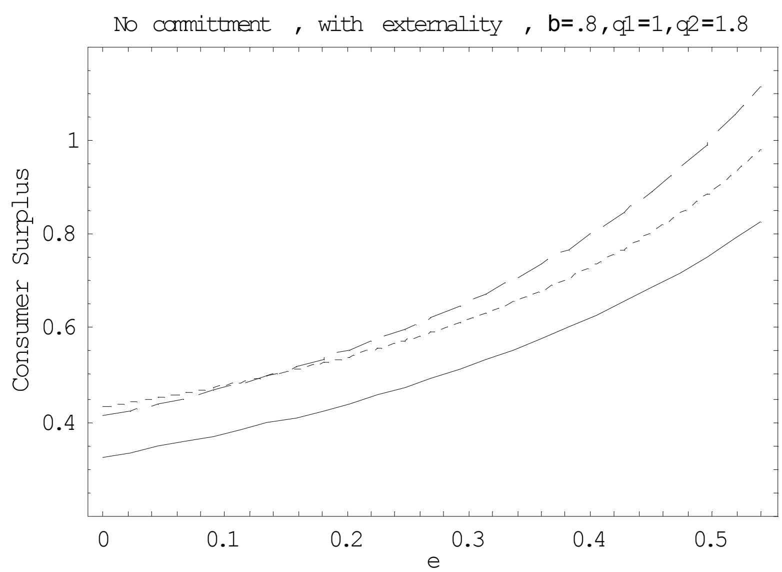

Figure 11. Consumer surplus comparison with externality and without commitment (The solid curve: pure leasing; long dotted curve: hybrid; short dotted curve: pure selling).

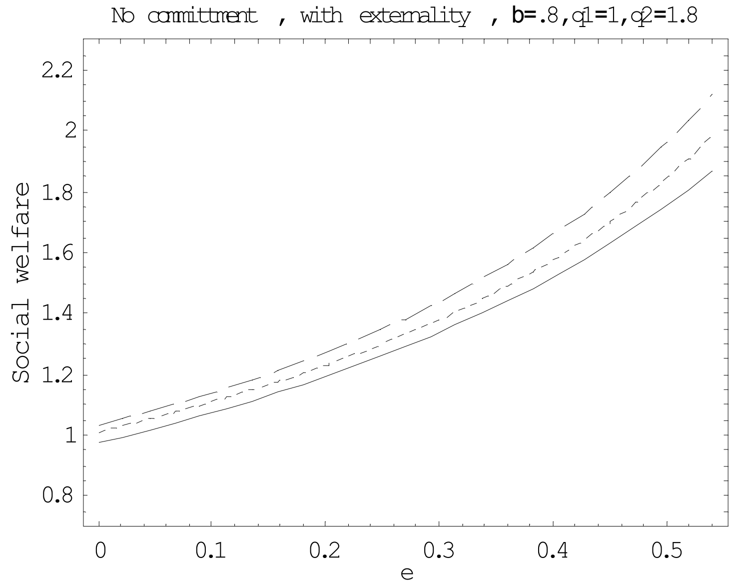

Figure 12. Social welfare comparison with externality and without commitment (The solid curve: pure leasing; long dotted curve: hybrid; short dotted curve: pure selling).

4 Conclusions

When software vendors can use the web to deliver software as a service based on a subscription model, we investigate whether web-based leases will become the dominant way of licensing software, as predicated by the “Coase conjecture” in traditional durable good market. We discuss leasing and selling software in a monopolist context, trying to explain the motivation for different strategies when there exists network externality effect. This paper identifies the costs and benefits of the software selling and leasing policies to both the vendors and the consumers considering the market dynamics, network externality and the ability of the monopolist’s ability to commit.

This project provides insights to them to facilitate their decision-making.

(1) Provide insights to Software Vendors (SVs);

Based on this study, we recommend the optimal strategy to software vendors to improve their products and profits. Whether to choose a pure selling or leasing policy or a hybrid one depends on a number of factors to be identified in this project:

- Degree and variance of quality improvement: high expectation or variance of quality improvement between software generations will increase the cannibalization of market shares of different generations of software. Therefore the time-inconsistent problem of selling a less appealing strategy to software vendors. On the other hand, a software vendor can control the quality improvement of new versions by controlling R&D investments and time-to-market of the new product.
- Intensity of network externality: a strong network effect will increase the consumers' value in using the product and therefore increase market share. As analyzed in the last section, when network externality is large enough, software vendors will be more willing to invest in market share and adopt the selling policy. As a result, contrary to the Coase conjecture, leasing may not always be a dominant licensing policy for software vendors. At the same time, software vendors can increase their long-term profit besides market share by building a strong network externality of their product.
- The trade-offs between ability to make commitment and the market share: we identify that leasing adds a restriction on the vendor's ability to enlarge market share in the first period for the benefit of committing on future prices. If the software vendor is able to commit on future prices with other strategies, then a selling or hybrid strategy may be preferred than leasing.

(2) Provide insights to consumers:

This paper helps consumers realize the benefits and costs of each of the licensing policies and help them make the best decision taking into account their own characteristics, product upgrades, and network effects. The analysis of consumer surplus and social welfare will also give implication to policy makers.

(3) Contribute to the academic research field:

Software is different from conventional durable goods because of the low marginal production cost, network externality in its distribution, easy to upgrade and strict Intellectual Property protection. Thus, the Coase Conjecture discussed in the durable goods literature does not apply to the software distribution strategy. Our paper fills in this gap by developing a model combining vertical differentiation and intertemporal price discrimination. Using the model, we find a market segmentation strategy for a software vendor to classify consumers by their quality preferences through different selling strategies.

By examining the differences in strategies regarding software and other durable products, this project can contribute to the literature of durable goods research. It extends the current models with new features of network externality and compatibility, which have not been fully considered into models due to technology unavailability or model intractability. We find that the leasing can help the monopoly selling commit on a high future price, but unlike proved in the previous studies for durable goods without network externality, leasing may not be able to achieve the monopoly profit as a nondurable seller can do over time. Therefore leasing is a restrict tool of making commitment at the cost of market share, consumer surplus and potentially firm profit. The results provide theoretical support for researchers in economics, marketing and technology management to better understand the licensing policies and impact on consumers and social welfare.

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