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Competing with Menus of Tariff Options*

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Abstract

I study how firms actually compete in nonlinear tariffs by analyzing whether the incumbent and entrant's decisions to offer a given number of tariff options are interrelated. The goal is to shed some light on those dynamic and strategic aspects of tariff menus that are currently ignored by theoretical models of nonlinear pricing competition in order to highlight some basic features of the market that future theoretical work should address. This paper also introduces a generalized multivariate count data model that allows to account for the possibility of correlation of any sign among the pricing decisions of competing firms in a manner that is robust to the existence of over and underdispersion of counts. Pricing strategies appear to be strategic complements that respond positively to the existing heterogeneity of consumers' tastes. While this is a common source driving the number of tariff options offered, results also show that previous pricing decisions by the incumbent affect the entrant's current offering of tariff options, thus free riding on information about the market revealed by the likely better informed firm of the industry. The strategic complementarity result disappears when we only consider non-dominated tariffs.

Keywords: Nonlinear Pricing Competition; Tariff Menus; Strategic Complementarity; Bivariate Count Data Regression.

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1 Introduction

Firms engaging in nonlinear pricing rarely make use of fully nonlinear tariffs. Actually firms only offer few tariff options that approximate the fully nonlinear tariff. Thus, their pricing strategy consist of a bundle of countable features that characterizes each tariff option. A tariff option can therefore be interpreted as defined on a lattice of tariff features to accommodate the framework popularized by Milgrom and Roberts (1990a) and Topkis (1998) for environments where firms' strategies are discrete and their decision problems fail to be convex.

In addition, when firms compete, their pricing tactics may have important strategic effects: offering more or less tariff options may induce consumers to switch carriers or to induce different type of customers to subscribe each company. Thus, the offering of numerous tariff options by one carrier may trigger a similar response for the competitor if the number of tariff options are strategic complements,[1] or a smaller number of options if they are strategic substitutes. In this latter case, competing firms may differentiate their otherwise (almost) identical products through tariff design by appealing to customers that are significantly heterogeneous regarding the different features of the tariff options, *e.g.*, such as their complexity.

This is a rather unexplored area of nonlinear pricing that the present paper intends to document using the competitive telephone pricing of the early US cellular telephone industry between 1984 and 1992. This market was characterized by minimal product differentiation, lack of network externalities, and transition from monopoly to duopoly during the early years of the sample. The goal of the paper is not to develop a theory of how firms would incorporate different features of the tariff in competition but to document how they actually do so in a simplified setting so that this empirical evidence may serve as a reference for future research in this area of nonlinear pricing. Hence, this paper explores whether the existing theoretical models of nonlinear pricing competition can produce some sort of robust prediction to test *à la* Chiappori and Salanié (2000) if there is any strategic value attached to competing for customers through the design of tariff features, and in particular regarding the number of tariff options offered. I show that when we shift the attention from agents' choices to the principals' offering of contract options, such robust test does not exists. The reason is that there are few theoretical models advancing competing hypotheses, some of which support the idea of minimal tariff differentiation while others envision tariff options as a way to segment the market when other sources or differentiation are ineffective. Moreover, if firms face non-negligible commercialization costs per tariff option and if it is likely that these costs are different across firms, then any combination of number of tariff options can be rationalized as profit maximizing regardless of other elements of the model such as product differentiation or the distribution of consumer types. The reason is that these commercialization costs remain unobserved to the econometrician.

[1] See Milgrom and Roberts (1990b) and Vives (1990) for a theoretical treatment of strategic complementarities when firms' strategies are not continuous.

In this paper I only focus on the number of tariff options offered by the competing firms of the largest U.S. cellular telephone markets between 1984 and 1992. I ignore other tariff features such as length of the contract, bundling with particular telephone sets, or magnitude of the allowance of each tariff plan. These features certainly have an effect on the decision to subscribe to a particular plan and/or carrier, but the information on most of them is quite incomplete. Thus, I will focus on the number of tariff plans only and presume that there is significant unobserved heterogeneity that needs to be dealt with at the econometric stage. Miravete and Pernías (2006) empirically test for the existence of *complementarity* among the binary strategies of a firm. Alternatively, this paper focuses on the existence of *strategic complementarity* among the pricing tactics of competing firms.

In attempting to measure whether there is any strategic complementarity in offering more or less tariff options to compete for a common pool of potential customers, the econometric analysis needs to address the estimation of a multivariate count data regression model. The estimates reported here are the result of estimating a pseudo-maximum likelihood procedure based on a Gaussian copula function with double Poisson marginals that uniquely accommodates the observed underdispersion of the distribution of the number of tariffs and the possibility of negative correlation among the distribution of tariff options of the incumbent and entrant in the early U.S. cellular telephone industry. It should be noted that this estimation approach is the first one that allows to control for any combination of over and underdispersed distribution of counts together with the possibility of negative correlation among the number of tariff options offered by the competing firms. This is a sensible economic hypothesis that cannot be ruled out by imposing a positive correlation leading necessarily to conclude that the number of tariff options offered by competing firms are strategic complements. Furthermore, and contrary to any existing multivariate count data model, the consistent estimates are robust to the existence of unobserved heterogeneity leading to over and underdispersion of counts and the estimation approach, which avoids numerical integration, proves to be fast and easy to implement.

Results support the existence of strategic complementarity and thus, competition leads to minimal differences among the pricing tactics of competing cellular firms. The similar number of tariff options offered by incumbents and entrants may also indicate that there are no significant differences in commercialization costs across firms. More options are offered in more affluent markets and where there is evidence that consumers are more heterogeneous to better segment them and add to profits by reducing their informational rents.

The number of tariff options offered also increases over time although this is no longer true for all firms when we restrict our attention to non-dominated tariff options, *i.e.*, those that for some non-negligible number of consumption patters are at some point the least expensive tariff options to subscribe. When we consider these non-dominated tariff plans the strategic complementarity result disappears, indicating that firms simply optimize how to best implement the nonlinear tariff schedule by means of a menu of few tariff options. This points out an interesting result that complements those described by Miravete (2007): it is more likely that firms respond strategically to the number of tariff options offered by the competitor when these are mostly deceptive tariff options aimed to benefit from consumers mistakes.

Finally, the larger the number of effective tariff options offered by the incumbent during the monopoly phase in each market the more effective options are offered by the entrant. Firms imitate their competitors in an attempt to take advantage of their market research but this does not amount to the strategic complementarity of effective tariffs as a mechanism to segment the market and attract customers from the competitor's clientele. The data can only identify the effect of entrants learning out of the incumbent's supposedly better knowledge of the market.

The paper is organized as follows. Section 2 describes the data. Section 3 reviews different theoretical arguments that may help explaining the number of tariff plans that firms offer to their customers in competitive environments. Section 4 presents a feasible multivariate count data regression model based on the Gaussian copula and double Poisson distribution. Section 5 reports the results of this bivariate count data regression model in which the number of total and non-dominated tariff options offered by each firm is regressed against market and firm characteristics. This section also suggests interpretations that are consistent with the reported results and evaluates the existence of strategic complementarities among firms' offerings of tariff plans. Section 6 concludes.

2 Pricing in the Early U.S. Cellular Industry

This paper studies the pricing strategies of numerous cellular telephone carriers in the early U.S. cellular telephone industry. In particular, I focus on the number of tariff options that firms use to implement a fully nonlinear tariff. The data set, which has been described at length elsewhere,[2] contains a complete description of the tariff options offered by any of the two firms present in the largest markets of the U.S. between 1984 and 1988. The data, however, do not include subscription, neither individual consumption information, but it contains a large number of market and firm characteristics that can be used to control for the existence of firm and market specific heterogeneity. These data therefore do not allow me to analyze consumers' preferences for tariff options with different features such as in Economides, Seim, and Viard (2006) or Lambrecht, Seim, and Skiera (2006). The data are better suited to test for the existence of strategic complementarities among strategies that are countable and discrete in nature. Thus, I will focus on the contemporaneous interaction of firms' strategies regarding the number of tariff options offered.[3] As in

[2] See for instance Busse (2000), Miravete (2007), Miravete and Röller (2004), and Parker and Röller (1997).

[3] Busse (2000) focuses on exactly how similar are those other features of cellular tariffs when firms are present in several markets and Busse and Rysman (2005) study the different degree of concavity of nonlinear tariffs in yellow book advertising.

Table 1: Descriptive Statistics

	Incumbent		*Entrant*		*All Firms*	
Variables	Mean	Std.Dev.	Mean	Std.Dev.	Mean	Std.Dev.
PLANS	3.6970	1.2244	3.6465	1.2561	3.6717	1.2374
EFFPLANS	3.2020	1.2287	3.2020	1.2370	3.2020	1.2297
LASTMONO	1.5152	1.9025	1.5152	1.9025	1.5152	1.8977
LSTEFMON	1.1515	1.4240	1.1515	1.4240	1.1515	1.4204
YEAR88	0.3333	0.4738	0.3333	0.4738	0.3333	0.4726
YEAR92	0.3333	0.4738	0.3333	0.4738	0.3333	0.4726
FIRM-AGE	45.3030	31.5919	34.6432	30.5769	39.9731	31.4666
BELL	0.6667	0.4738	0.3030	0.4619	0.4848	0.5010
POVERTY	10.4505	2.7936	10.4505	2.7936	10.4505	2.7865
AP_{peak}	0.2288	0.2606	0.2210	0.2812	0.2249	0.2705
$AP_{off-peak}$	0.1526	9.5539	-1.4083	11.8900	-0.6279	10.7864
COVERAGE	0.0964	0.0789	0.0964	0.0789	0.0964	0.0787
Observations	99		99		198	

All variables are defined in main text.

Miravete (2007), I will distinguish however between actual and non-dominated tariffs to uncover whether the strategic interaction has anything to do with deceptive tactics.

Table 1 presents the descriptive statistics by type of firms.[4] The first variable, PLANS, denotes the number of actual tariff options offered by different firms at different points in time. This information was collected by *Economic and Management Consultants International, Inc.* and reported in *Cellular Price and Marketing Letter*, Information Enterprises, various issues, 1984–1988. For year 1992, Marciano (2000) combined information from *Cellular Directions, Inc.*, the *Cellular Telephone Industry Association*, and direct interviews with managers. Some of the tariff options offered were always more expensive than some others, or a combination of the other options offered by the same firm. Thus, EFFPLANS indicates the number of effective tariff options, *i.e.*, those that are the least expensive option for at least one out of the half a million usage profiles defined by the sum of peak and off-peak minutes adding up to a maximum of 1000 minutes of usage a month. LASTMONO and LSTEFMON are the number of actual and effective tariff options, respectively, offered by the incumbent during the last quarter where it enjoyed its monopoly position.

Every firm is observed three times in this data set: at the earliest quarter of the duoply phase of the market between 1984 and 1988, in the third quarter of 1988 (YEAR88), and in 1992 (YEAR92), respectively. Depending on the time of the award of the *wireline* license to the incumbent firm and the court resolution over the dispute for the awarding of the *non-wireline* license to an entrant, each firm accumulates a different experience, measured by FIRM-AGE, at the time of each observation. In addition to all these time related variables, BELL indicates whether the largest shareholder of each firm belongs to one of the "Baby Bells"

[4] There is more information available, but the limited size of the sample and the nonlinear nature of the estimation model prompted us to select a small number of regressors capable of producing meaningful results.

Table 2: Frequency Distributions of Number of Actual and Effective Tariff Options

Actual Options	*Incumbent* Cases	*Incumbent* Rel.Freq.	*Entrant* Cases	*Entrant* Rel.Freq.	*All Firms* Cases	*All Firms* Rel.Freq.
1	2	0.0202	6	0.0606	8	0.0404
2	18	0.1818	11	0.1111	29	0.1465
3	22	0.2222	26	0.2626	48	0.2424
4	28	0.2828	31	0.3131	59	0.2980
5	24	0.2424	19	0.1919	43	0.2172
6	5	0.0505	6	0.0606	11	0.0556
Mean, (Var.)	3.6969	(1.4991)	3.6464	(1.5778)	3.6717	(1.5113)

Effective Options	*Incumbent* Cases	*Incumbent* Rel.Freq.	*Entrant* Cases	*Entrant* Rel.Freq.	*All Firms* Cases	*All Firms* Rel.Freq.
1	8	0.0808	10	0.1010	18	0.0909
2	24	0.2424	17	0.1717	41	0.2071
3	25	0.2525	33	0.3333	58	0.2929
4	24	0.2424	23	0.2323	47	0.2374
5	18	0.1818	14	0.1414	32	0.1616
6	0	0.0000	2	0.0202	2	0.0101
Mean, (Var.)	3.2020	(1.5098)	3.2020	(1.5302)	3.2020	(1.5123)

Absolute and relative frequency distribution of the number of actual and non-dominated tariff options offered by each active firm.

(available from the *Federal Communications Commission*); and POVERTY reports the percentage of households with income below the poverty level according to the 1989 *Statistical Abstracts of the United States*.

The data include some potentially endogenous variables to describe some relevant features of the tariff and market where these firms operate. These variables are the curvature of the peak and off-peak tariff schedule as defined by AP_{peak} and $\text{AP}_{\text{off}-\text{peak}}$, and the ratio of total to potential subscribers, COVERAGE. Variable AP_{peak} is the equivalent of the Arrow-Pratt measure of risk aversion computed over a thousand minute interval of airtime usage by means of the quadratic polynomial that best fits the lower envelope of the peak component of the tariff. Variable $\text{AP}_{\text{off}-\text{peak}}$ is similarly computed using the off-peak component of the tariff instead. The idea behind these measures of risk aversion is that the more heterogeneous consumers are, in the sense the distribution of their taste has a higher hazard rate, the more concave the optimal tariff needs to be. If all consumers are alike, a single two-part tariff suffices to extract all the rents from consumers. Evidently, the degree of concavity computed makes use of the actual tariffs offered, and thus, these two variables may be endogenous. Finally, COVERAGE is defined as the maximum capacity installed (measured by the number of antennas $\times$ 1,300 customers) divided by the number of business in a particular city and the population divided by four (to approximate the number of households). This variable may also be endogenous as the participation decision of consumers may depend on tariff options tailored to their tastes. Thus, more numerous and less expensive options could lead to significantly higher participation rates.

Table 3: Contemporaneous Correlation Among Number of Tariff Options

Actual	1	2	3	4	5	6	Kendall's τ
1	1	0	1	0	0	0	0.3668
2	4	6	5	2	1	0	(5.38)
3	0	1	10	6	3	2	
4	0	3	6	11	6	2	
5	1	1	3	9	9	1	
6	0	0	1	3	0	1	
Effective	**1**	**2**	**3**	**4**	**5**	**6**	**Kendall's τ**
1	5	0	1	1	1	0	0.3280
2	1	10	8	4	1	0	(4.81)
3	0	4	12	5	3	1	
4	3	2	9	7	3	0	
5	1	1	3	6	6	1	
6	0	0	0	0	0	0	

Total cases for each combination of tariff options offered by the incumbent and entrant firm. Rows indicate the number of options of the entrant and columns those of the incumbent. Kendall's τ measures of the association among the number of tariff options. The corresponding absolute value t-statistics are shown in parentheses. There are 99 pairs of tariff strategies in the sample.

Table 2 presents the histogram of the actual and effective number of tariff options by type of cellular carrier. Firms in this early market offered fewer tariff options than what is thought of being customary today. It is noticeable though that while the number of effective tariff options is always smaller than the actual number of options, the difference is not excessive. Another remarkable feature of the data is that the distribution of plans is always underdispersed, *i.e.*, the variance of the distribution of number of plans never exceeds the mean, which is the opposite of what is normally encountered in dealing with count data.

Table 3 cross-tabulates the occurrences of the different strategies and is the first piece of evidence in favor of strategic complementarities in the number of tariff options. Regardless of whether we focus on the actual or on the effective number of tariff options, it is very infrequent to observe one firm offering only one or two options while the other attempts to segment the market by flooding customers with numerous choices. Most of the time, firms offer between 2 and 4 options while the competitor offers a similar number, differing at most in one option.

3 Competing through Tariff Design

Within the monopoly framework the actual implementation of nonlinear pricing through a menu of few tariff options can be rationalized quite easily:

- Discrete Types. One possibility is that the support of the distribution of types is discrete. Consumers purchase cellular, local, and long distance telephony, cable or satellite TV, and high speed internet access. In choosing which combination of these services to bundle what matters is the demand for access rather than usage and thus, in this example, any potential consumer type could be represented by one of the 2^5 nodes of a five dimensional lattice.[5]

- Commercialization Costs. The optimal screening of consumers with different willingness to pay involves quantity discounts when the distribution of types is well behaved and its support is compact.[6] Thus, the foregone profits of offering an additional tariff option (normally a two-part tariff or a "bucket" tariff that includes some free minutes associated to the fixed monthly fee) decreases rapidly with the number of tariff options as proven by Wilson (1993, §8.3). An equilibrium with a menu with few tariff options could therefore be easily justified as long as there is a sufficiently large fixed commercialization cost per tariff.

For the case of cellular telephone consumption it is appropriate to model consumers heterogeneity regarding cellular telephone usage as a continuous random variable with compact support. We do not expect that consumers only use 50, 110, or 137 minutes a month and not any other potential number of minutes. Thus, provided that the distribution of usage is well behaved, few tariff options are needed to implement any fully nonlinear tariff in the presence of commercialization costs.[7]

The commercialization costs argument shows that models of nonlinear pricing cannot provide with a robust empirical implication regarding how firms implement such contracts through simple menus of tariff options. This critique is valid both for monopoly and oligopoly nonlinear pricing models. Commercialization costs are generally unobservable to the econometrician, and thus, any combination of tariff options can be offered by competing firms if there is heterogeneity regarding unobserved commercialization costs across firms and markets.

Relative to the theoretical body dealing with monopoly nonlinear pricing, models of nonlinear pricing competition are still in its infancy. Perhaps the most interesting result of this limited literature is the fact that duopolists, once they serve the whole market, will offer a single two-part tariff to their customers if they are unable to differentiate their products at all. This result, simultaneously obtained by Armstrong and Vickers (2001) and Rochet and Stole (2002), suggests that firms should offer few rather

[5] Crawford and Shum (2006) make explicit use of the discrete support of types defined on bundles of channels to study price discounts in cable television.

[6] Besides the single crossing property, it is normally required that the single dimensional distribution of types fulfills the increasing hazard rate property. See for instance Maskin and Riley (1984).

[7] The fact that foregone profits decrease rapidly with the number of tariffs was first pointed out by Faulhaber and Panzar (1977) and formally proven by Wilson (1993, §8.3). Recently, Rogerson (2003) and Chu and Sappington (2007) have documented the large share of potential profits from fully nonlinear pricing that can be captured with just one simple two-part tariff.

than numerous tariff options when they compete. But this is true only when we restrict our attention to the symmetric equilibrium case without optimal exclusion of customers from the market. The assumption of symmetric equilibrium is very common but it necessarily implies complementarity among the firms' pricing strategies. The empirical rejection of a positive correlation among the number of effective tariff options offered by competing firms does not support the idea of symmetric equilibrium as the only likely outcome of nonlinear pricing games.

The early U.S. cellular telephone industry was far from becoming a mature industry over the time period covered by the data. Investment in the deployment of antennas, high charges relative to the inexpensive fixed line telephone, roaming, and a "consumer party pays" practice did not favor a rapid adoption of cellular phones by consumers. Thus, it is very unlikely that all potential consumers participate and nonlinear duopoly pricing does not collapses into a single two-part tariff per competing firm. The hypothesis of *minimal tariff differentiation* is consistent with the exclusive agency model of Rochet and Stole (2002). Firms face a similar underlying distribution of individual types and share not only the same marginal cost but also the same commercialization costs. Thus, the optimal nonlinear tariff becomes flatter as the possibility of extracting rents from consumers is reduced by the existence of competing firms. Therefore, the foregone profits of not offering an additional tariff in competition become smaller than in monopoly and fewer tariff options are needed to achieve certain percentage of the maximum profits from a fully nonlinear tariff. These models predict simultaneous uniform reduction of the markup for any consumption level as competition increases, something that the existing evidence does not support.[8] In fact, using the same data set of this paper, Miravete (2007) documents that entry of a second competitor triggers an increase in the number of tariff options available to consumers after controlling for all available firm and market specific characteristics. Similarly, Seim and Viard (2005) document that the number of tariff options per carrier increased in this same market at a later stage when additional competitors were allowed to enter as a result of the *1996 Telecommunications Act*.

Confronting this view leading necessarily to the strategic complementarity of the number of tariff options offered by competing cellular telephone carriers, Yang and Ye (2006) suggest that contract variety depends on two opposite effects. First, as the number of firms increases each firm becomes less differentiated and its residual demand becomes smaller. This *market size effect* is the only one considered in most models cited above. Second, the *market share effect* results from firms offering additional contracts when their market share becomes too small and differentiating themselves through the contract features is the only way to steal market share from competitors. The validity of this argument rests on the implicit assumption that consumers types are multidimensional —as in Rochet and Stole (2002)— so that contract features may affect the reservation utility of the potential clientele of these competing firms. Cellular telephone companies will

[8] Other papers that lead to the same symmetric equilibrium where the number of tariff options of each firm are necessarily strategic complements are those of Gal-Or (1988), Spulber (1989), Stole (1995), and Oren, Smith, and Wilson (1983).

offer more or less tariff options depending on how consumers value having to decide among numerous contract alternatives and how effective is the differentiation through contract variety relative to the product differentiation of the firm, *i.e.*, it ultimately depends on how each dimension of consumers' types enter their preferences. This environment, with the proper asymmetric distribution of types for usage and contract simplicity can explain situations where one firm offers many tariff options while competitors only offer a few. When the number of tariff options are strategic substitutes firms make use of menu options to segment the market and retain a profitable share of customers.

Therefore, firms may end up offering a similar number of options with almost identical features while competing through the design of tariffs, or alternatively, use their number and features to differentiate from each other. In all cases, features of tariffs unknown to the econometrician, the symmetry or asymmetry of the distribution of types, also unknown to the econometrician, and the unobserved firm specific commercialization costs drive the final decision on how many tariff options are offered. Thus, the number of tariff options offered can either be strategic complements or substitutes. Hence, the answer to this empirical question would depend on the specific study case. This paper reports the results for the early U.S. cellular telephone market.

4 A Bivariate Double Poisson Count Data Regression Model

Regardless of whether we focus on the total number of tariff options offered by the competing firms or on the number of effective (non-dominated) tariff options, Tables 2 and 3 present us with two features that none of the existing count data regression models can handle simultaneously: the distribution of counts appears to be underdispersed and positively correlated across firms. The bivariate Double Poisson count data regression model is a flexible approach based on a Gaussian copula function with assumed double Poisson marginal distributions and correlation of unrestricted sign that can be easily implemented an allows us to address simultaneously over or underdispersion and a potentially negative correlation of counts.

It has long been recognized that the Poisson model is in general too restrictive when estimating univariate count data regressions. Implicit to the Poisson model is the assumption of equidispersion of the distribution of counts, which is customarily rejected by the data. If the mean does not equal the variance of the distribution of counts the estimates are still consistent but inference is no longer robust, *e.g.*, Cameron and Trivedi (1998, 3.1-3.4). Many models, such as the Negative Binomial regression, have been suggested to address the existence of unobserved heterogeneity in the data that could explain the commonly observed overdispersion of the count distribution although underdispersion cannot be addressed with the same unobserved heterogeneity argument. However only Efron (1986) and Winkelmann (1995) suggests flexible enough model to address both over and underdispersion of the distribution of counts.

In presenting the econometric model, I will consider first the single equation count data regression model behind the assumed double Poisson marginal distributions of the bivariate model. Let $y_i = 0, 1, 2, \ldots$ be distributed according to a double Poisson distribution with parameters μ_i and ϕ_i, conditional on a set of regressors $\boldsymbol{x_i}$ in a sample with $j = 1, 2, \ldots, n$ observations. Efron (1986) shows that the probability frequency function of this double Poisson is:

$$f(y_i, \mu_i, \phi_i \mid \boldsymbol{x_i}) = K(\mu_i, \phi_i)\sqrt{\phi_i}\exp(-\phi_i\mu_i)\exp(-y_i)\frac{{y_i}^{y_i}}{y_i!}\left(\frac{e\mu_i}{y_i}\right)^{\phi_i y_i}, \tag{1a}$$

$$\mu_i = \exp(\boldsymbol{x_i}'\boldsymbol{\beta_i}), \tag{1b}$$

$$\frac{1}{K(\mu_i, \phi_i)} \simeq 1 + \frac{1-\phi_i}{12\phi_i\mu_i}\left(1 + \frac{1}{\phi_i\mu_i}\right). \tag{1c}$$

The advantage of this distribution over the standard Poisson is that the mean and variance do not depend on a single parameter, thus allowing for both over and underdispersion. Efron (1986) shows, among many other results, that:

$$\mathrm{E}[y_{i,j} \mid \boldsymbol{x_{i,j}}] \simeq \mu_{i,j}, \tag{2a}$$

$$\mathrm{V}[y_{i,j} \mid \boldsymbol{x_{i,j}}] \simeq \frac{\mu_{i,j}}{\phi_{i,j}}. \tag{2b}$$

Hence, the double Poisson includes the standard Poisson as a particular case, when $\phi_i = 1$, but it allows for overdispersion when $\phi_i < 1$ as well as for underdispersion when $\phi_i > 1$. Furthermore, the maximum likelihood first order conditions of the double Poison are:

$$\sum_{j=1}^{n} \frac{y_{i,j} - \mu_{i,j}}{(\mu_{i,j}/\phi_{i,j})}\frac{\partial\mu_{i,j}}{\partial\boldsymbol{\beta_i}} = \mathbf{0}, \tag{3}$$

which coincides with those of the standard Poisson regression model. The estimation of the parameters of interest $(\boldsymbol{\beta_i'}, \phi_i)'$ proceeds in two steps:[9] First, a simple and fast iteratively reweighted least squares procedure (Cameron and Trivedi (1998, §3.8)) is used to estimate the components of $\boldsymbol{\beta_i}$. After substituting into equation (2a) to obtain $\hat{\mu}_{i,j}$, the maximum likelihood estimate of ϕ_i coincides with the sample mean of the deviance measure (Cameron and Trivedi (1998, §5.3.2)):

$$\hat{\phi}_i = \sum_{j=1}^{n} y_{i,j}\ln\left(\frac{y_{i,j}}{\hat{\mu}_{i,j}}\right) - \left(y_{i,j} - \hat{\mu}_{i,j}\right). \tag{4}$$

[9]This is without loss of generality when, as in the case considered here, ϕ_i is assumed to be a constant because in such case the gradient characterizing the first order conditions of the maximum likelihood estimation is a block recursive system of equations where ϕ_i does not enter into the equations determining the components of $\boldsymbol{\beta_i}$.

Cameron and Trivedi (1998, §8) best summarize the difficulties of estimating a multivariate count data regression. Tractability has limited the estimation of these kind of models. Among the better known approaches Kocherlakota and Kocherlakota (1993) derive a bivariate Poisson distribution from two Poisson distributed variables resulting from the addition of a common Poisson variable to two independently distributed Poisson variables. The advantage of this approach —known as trivariate reduction— is that it allows for Poisson marginal distributions and a positive, although restricted, correlation coefficient that fully characterizes the dependence structure of variables. Similarly, Marshall and Olkin (1990) generate a multivariate count data distribution from mixtures and convolutions of distributions of count events. The advantage of this second approach is that it allows for the simultaneous existence of unobserved heterogeneity conducting to overdispersion and positive correlation of counts. Still, regardless of whether we focus on the bivariate Poisson distribution of Kocherlakota and Kocherlakota (1993), or the multivariate negative binomial distribution of Marshall and Olkin (1990), correlation is necessarily positive because there is a single source of heterogeneity to the distribution of the different counts that explains simultaneously overdispersion of the marginal distributions and their positive correlation. The moment approach of Gourieroux, Monfort, and Trognon (1984) suffers from the same shortcomings.

The bivariate double Poisson count data regression model can accommodate both over and underdispersion and allows for the possibility that counts are negatively correlated. Furthermore, these two features of the joint distribution of counts are not driven by a common unobserved factor to all univariate marginal distributions and the parameterization of the likelihood function allows for all possible combinations of over or unnderdispersed marginals and correlation of any sign. Allowing for the estimation to be flexible enough to accommodate the observed dispersion pattern in the data reduces the risk of misspecification bias. This is also accomplished by not excluding the possibility of negative correlation between the counts. Moreover, negative correlation is a sensible economic hypothesis for the present application that cannot be simply ruled out by assuming any of the other multivariate count data models currently available. A negative correlation estimate is consistent with the view that firms attempt to differentiate themselves through the design of the tariff options that they offer to, most likely, endogenously segmented and differentiated customers.

The present bivariate double Poisson model extends Efron (1986) approach to deal with multivariate counts making use of Gaussian copula function.[10] The clear advantage of this approach is that it is easily implementable, with first order conditions identical to those of the Poisson model. Estimation is fast and easy and only involves few computations beyond those of a pseudo-maximum likelihood count data estimation in order to account for over/underdispersion and correlation between counts. This compares

[10]For a recent and up to date introduction to copulas see Nelsen (2006). Copulas have been used before in econometric applications. Thus, Cameron, Li, Trivedi, and Zimmer (2004) apply the Frank (1979) copula to study the difference of positive count data variables and Park and Fader (2004) make use of a Gamma-Sarmanov copula to analyze the joint distribution of waiting times to repeated visits to a set of websites.

quite positively with the existing literature on multivariate count data regression which normally require heavy numerical integrations to evaluate multivariate count series with only restricted correlation of counts.

To ease the exposition I will focus, without loss of generality, on the bivariate case that is the objective of the present study. Let's assume that (φ_1, φ_2) are two continuous variables with joint probability density function $F(\varphi_1, \varphi_2)$ with continuous univariate marginal probability distribution functions given by $F_i(\varphi_i)$, $i = 1, 2$. The basic result on copulas, first proven by Sklar (1959), states that the following copula function C exists and is unique:

$$C : [0,1] \times [0,1] \Rightarrow [0,1], \quad \text{s.t.} \quad F(\varphi_1, \varphi_2) = C\left(F_1(\varphi_1), F_2(\varphi_2)\right) . \tag{5}$$

Then the joint probability density function is just the product of the marginal density functions and the copula density:

$$f(\varphi_1, \varphi_2) = f_1(\varphi_1) f_2(\varphi_2) \frac{\partial^2 C\left(F_1(\varphi_1), F_2(\varphi_2)\right)}{\partial F_1(\varphi_1) \partial F_2(\varphi_2)} . \tag{6}$$

Thus, a bivariate copula with standard uniform marginals can always be written as follows:

$$C(z_1, z_2) = F\left(F_1^{-1}(z_1), F_2^{-1}(z_2)\right) , \tag{7}$$

for $z_i = F_i(q_i)$, $i = 1, 2$, because $q_i = F_i^{-1}(z_i)$ is always uniformly distributed when the marginal distribution functions are continuous.[11] The Gaussian copula simply assumes that $F(\cdot)$ and $F_i(\cdot)$ are the bivariate and univariate standard normal distribution functions, respectively. This allows us to introduce the parameter ρ of correlation among counts:

$$C(z_1, z_2, \rho) = \Phi\left(\Phi_1^{-1}(z_1), \Phi_2^{-1}(z_2), \rho\right) . \tag{8}$$

In order to estimate this bivariate count data model with double Poisson marginals, we need to maximize a log-likelihood function where the contribution of each observation j is:

$$\log\left(f(y_1, y_2, \boldsymbol{\beta_1}, \boldsymbol{\beta_2}, \phi_1, \phi_2, \rho \mid \boldsymbol{x_1}, \boldsymbol{x_2}\right) = \sum_{i=1}^{2} \log\left(f(y_i, \mu_i, \phi_i \mid \boldsymbol{x_i})\right) + \log\left(c(q_1, q_2, \rho)\right) , \tag{9}$$

where $f_i(\cdot)$ is the univariate probability density function of the double Poisson given by equation (1a), $c(\cdot)$ is the probability density function of a standard bivariate normal distribution corresponding to (7), and:

$$q_i = \Phi_i^{-1}(z_i), \quad i = 1, 2. \tag{10}$$

[11] A proof of this widely used result, known as the "probability integral transformation", can be found in Casella and Berger (2001, pp.52-54).

Thus, the log-likelihood function of this model has two differentiated components consisting of the log-likelihood functions of two separate double Poissons plus the term involving the copula that exclusively determines ρ. Obtaining the value of parameters thus reduces to estimating two separate double Poisson count data regression models as explained before, and then, once we substitute the estimates $\hat{\boldsymbol{\beta}}_{\mathbf{1}}$, $\hat{\boldsymbol{\beta}}_{\mathbf{2}}$, $\hat{\phi}_1$, and $\hat{\phi}_2$ into $c(\cdot)$, obtain the estimate of ρ in a second stage simply as:

$$\hat{\rho} = \mathrm{CORR}(q_1, q_2)\,. \tag{11}$$

The only remaining difficulty of this approach is that $F_i(\varphi_i)$ is not continuous, but rather a discrete marginal distribution function defined on $\mathbb{N}$. Denuit and Lambert (2005) provide the solution to this problem by adding an independent random draw from a standard uniform distribution $u_{i,j}$ to each count $y_{i,j}$ so that the newly generated continuous distributions preserve the same association measure (Kendall's τ) than the original discrete distributions.[12] Fixing this problem only adds a simple step to properly defining z_i in equation (10) as the probability integral transformation of the continued variable associated to y_i. Let $F_i^\star(y_i^\star)$ denote the new continuous marginal distribution function of the continued variable:

$$y_{i,j}^\star = y_{i,j} + (u_{i,j} - 1)\,. \tag{12}$$

Then:

$$z_{i,j} = F_i^\star(y_{i,j}^\star) = F_i(y_{i,j} - 1) + f_i(y_{i,j}) \cdot u_{i,j}\,, \tag{13}$$

where $F_i(\cdot)$ and $f_i(\cdot)$ correspond, respectively, to the probability distribution and frequency functions of the double Poisson of equation (1a).

Thus, the combination of a Gaussian copula with double Poisson marginals reduces the estimation of a multivariate count data regression model to an easy sequence of univariate generalized linear regressions and the robust estimation of the correlation between the error terms of these equations. This procedure is easy to implement and produces consistent estimates of the parameters in a three sequential stages that allow to accommodate any combination of over and underdispersion of counts plus any correlation pattern between the counts. Inference will be based on bootstrapped standard errors.

5 Estimation Results

Table 4 reports the estimates of this model for the actual and effective number of tariff options, respectively. The estimates reflect that the distribution of actual tariff plans is underdispersed ($\phi_i > 1$ for both firms)

[12] This continuation procedure is applied by Heinen and Rengifo (2003) in the context of an autoregressive count data model.

Table 4: Number of Tariffs

	Actual		Effective	
Variables	Incumbent	Entrant	Incumbent	Entrant
CONSTANT	3.4170	2.8483	2.3359	1.2932
	(3.40)	(2.83)	(2.61)	(1.64)
FIRM-AGE	-0.0151	-0.0089	-0.0049	-0.0007
	(0.96)	(0.64)	(0.42)	(0.03)
BELL	0.5362	0.3045	-0.0180	0.3269
	(1.68)	(1.11)	(0.07)	(1.07)
POVERTY	-0.1292	-0.1608	-0.1385	-0.0889
	(2.02)	(3.02)	(2.38)	(1.87)
LASTMONO/ LSTEFMON	0.0525	0.0716	0.0343	0.1023
	(0.55)	(1.43)	(0.45)	(2.21)
YEAR88	0.8788	0.6395	0.4427	0.3974
	(1.58)	(1.58)	(1.11)	(0.79)
YEAR92	2.2780	2.0432	1.5640	1.4542
	(1.85)	(2.04)	(1.79)	(1.05)
AP_{peak}	0.9248	0.6255	1.2883	0.8658
	(1.72)	(1.74)	(2.82)	(2.16)
$\text{AP}_{\text{off-peak}}$	-0.0083	-0.0267	-0.0057	-0.0116
	(0.39)	(0.85)	(0.29)	(0.27)
COVERAGE	0.5946	-0.1875	0.1438	-0.0982
	(0.96)	(0.25)	(0.26)	(0.18)
ϕ_i	1.0946	1.0780	0.9528	0.9398
	(23.54)	(20.74)	(19.13)	(17.44)
ρ	0.3595		0.1285	
	(3.66)		(1.21)	
Observations	99	99	99	99
$-\ln L$	159.19	156.37	151.98	151.52
LM	1.6933	1.2914	3.6872	2.3916
$[p-value]$	[0.429]	[0.524]	[0.158]	[0.302]

Marginal effects evaluated at the sample mean of regressors. Absolute value, bootstrapped, t-statistics are reported between parentheses using 100 replications in each case. *LM* is the regression-based, heteroskedastic-robust, Lagrange multiplier test of endogeneity of Wooldridge (1997) for AP_{peak}, $\text{AP}_{\text{off-peak}}$, and COVERAGE. *LM* is asymptotically distributed as a χ^2_3 distribution under the null hypothesis of exogeneity and p-values are shown between brackets.

while the distribution of only effective tariff options is overdispersed ($\phi_i < 1$ also for both firms). Furthermore, the number of actual tariff options offered are strategic complements ($\rho > 0$) but the evidence does not support the existence of such strategic complementarity when we restrict our attention to non-dominated tariffs. Thus, once we account for relevant market and firm characteristics, these results support the interpretation that firms act independently of each other when implementing the optimal nonlinear tariff by means of a menu of few non-dominated tariff options. Still, they imitate each other if we consider the actual number of tariff options that include deceptive pricing. Thus, at this early stage of development of the cellular telephone industry, the evidence favors the *minimal tariff differentiation* of Rochet and Stole (2002) than the view of Yang and Ye (2006) when we focus on the total offering of tariff options. This is, on the other hand also consistent with the view of these latter authors. In this early industry the market is far from saturated and only two firms still can find a substantial share of customers to sign in without the need of having to steel customers from the competitor through other pricing tactics. In support of this view, notice that the effect of COVERAGEis never significant.

Table 4 also provides with other results that support the interpretation that pricing tactics, as defined by the actual number of options, are strategic complements. Thus, for instance, there is no effect of learning attributable to the experience accumulated by each firm in a particular market (non-significant FIRM-AGE). Similarly, the number of tariff options offered to consumers are independent of the ownership of the carrier. These two results favor the interpretation that firm differences associated to the cost of commercializing tariff options is negligible. This is true for the case of effective tariffs although BELL owned companies are marginally more likely to offer more tariff options, including deceptive ones.

The discrepancy on the complementarity result when we consider total and effective tariff options needs to be explored. An important result is the positive and significant effect of AP_{peak} on the effective number of tariff options, a result that is only marginally significant in the case of total tariff plans. This is important because the peak time band comprises at this early stage of the industry more than twelve hours a day and constitute the bulk of the market, clearly made mostly of business and high income individuals. The negative effect of POVERTY confirms that this is luxury good (handsets were initially priced at $3,000) and that screening is more intense in those markets with more affluent customers. Therefore, the more heterogeneous is the clientele base, the more effective tariff options are needed to successfully segment the relevant groups and extract most of the informational rents from individuals with different willingness to pay of the cellular service. This effect is common to both firms and the fact that it does not lead to complementarity between effective tariff plans may suggest that the distributions of consumer tastes that each firm confronts are not necessarily symmetric. Thus, the underlying distribution of consumer tastes drives the offering of effective tariff options while imitation of the competitor happens mostly to match the apparent offering of plans although some of them are actually always more expensive than other options offered by the same cellular carrier.

These results could be questioned because AP_{peak}, $\text{AP}_{\text{off-peak}}$, and COVERAGE may not be exogenous variables as the curvature measure depends on the tariff options actually offered and the decision to participate in the market is probably a function of the menu of tariff options available to different customers. Fortunately, the Lagrange multiplier test o Wooldridge (1997) concludes that these three variables can always be considered exogenous regressors. Thus, pricing has to be understood as the optimal strategy to discriminate consumers with a given distribution of tastes.

Finally, and perhaps one of the most interesting results of these regressions is that the entrant imitates selectively the pricing tactics of the incumbent, but only when referred to the non-dominated tariff options. The later pricing of the incumbent while still a monopolist conveys valuable information to the entrant regarding the distribution of consumer types. It is not important for the incumbent as this firm already knows the distribution of customer tastes. And it is not important for introducing deceptive (dominated) tariff options. Thus, free riding on the information about the market revealed by the pricing behavior of the incumbent and the heterogeneity of the entrant's customer's valuations explain the correlation in effective pricing, therefore ruling out strategic value to the number of effective tariff options as a way to segment the market and steal consumers. Results show that if such strategic value to the actual way in which pricing is implemented actually exists, it involves mostly dominated tariff options.

6 Concluding Remarks

This paper has described the pricing tactics of competing firms in order to determine whether they can be considered strategic complements or substitutes. Results support the idea of strategic complementarity only when considering the total tariff options offered but not if we focus on the non-dominated tariff options that approximate the optimal fully nonlinear tariff to screen among the heterogeneous consumers's valuations. This lack of complementarity among effective pricing strategies can only be explained by underlying asymmetries of the distributions of consumer valuations since results also rule out the existence of cost heterogeneity across firm types.

Thus, at least for an early industry, firms do not differentiate from each other through the design of tariffs. This may happen, as indicated by Yang and Ye (2006) once the market is more crowded with additional competitors. Some of the results reported by Economides et al. (2006) appear to indicate that this is the case, but it is unclear from their evidence that this differentiation suffices to turn pricing tactics into strategic substitutes when the industry matures.

The paper has also introduced an effective and easily implementable estimation method to obtain consistent estimates in multivariate count data settings. This approach solves a long standing problem in the multivariate count data regression literature and it allows to address series of counts that may be both positively and negatively correlated. In addition it can also accommodate the effect of unobservable heterogeneity leading not only to the common overdispersion of counts, but also to underdispersion.

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