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Interconnecting Differentiated Networks

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Abstract

I examine interconnection decisions of differentiated firms. I find that previous results that firms never interconnect enough do not hold. In a Hotelling model consumers may suffer from interconnection, and firms may interconnect when it is not socially optimal. The firms interconnect too much when the network effects are steeper - this makes firms compete much less aggressively after interconnection, raising prices for consumers and profits for firms. Price and profit rise results holds under quality and installed base asymmetries, or only some firms in the industry interconnecting. More dimensions of differentiation make interconnection less attractive.

1 Introduction

Ever since Katz and Shapiro (1985) the prevalent opinion among economists was that private firms do not interconnect enough from the society's or the consumers' points of view. As a result, most of the public policy on network interconnection is aimed at encouraging firms to interconnect (see Gandal (2002) for a summary), with network effects-related issues becoming one of the poster children of market failure[1]. I show that consumers might suffer from interconnection of horizontally differentiated products with networks because of a subsequent price increase. Moreover, firms might interconnect when it is not socially optimal.

[1]Windows v. Apple OS, QWERTY v. Dvorak, Beta v VHS, the list goes on. See Liebowitz and Margolis (1994) for one of the few articles holding the opposite view.

A product is said to have a positive network effect when consumers purchasing it benefit from others purchasing the same product. Products which have positive effects range from phones (direct externality) to game consoles, see Gandal (2008) for empirical studies of network effects[2]. Interconnection between product A and product B means that the consumers who have purchased good A also enjoy the additional benefit when someone buys good B. For example, a person who has Yahoo Instant Messenger also enjoys the benefits of Google Talk's network after the two companies have interconnected their messengers. Similarly, people buying a Nissan sedan enjoy the indirect benefit of having all the Renault service facilities at their disposal, since the two car companies share platforms. Instead of interconnection the reader can think of interoperability or setting some common standard. I argue throughout the paper that such interconnection can be bad for consumers, because the demand becomes less elastic after interconnection, allowing firms to compete less aggressively and charge more. Similarily, firms want to interconnect for all the wrong reasons from the society welfare point of view. Firms sink interconnection costs too often because they pay interconnection costs to capture some more of the consumer surplus, without creating enough in return to justify the costs from the social welfare point of view.

Katz and Shapiro's conclusion is based on the assumption of Cournot competition between single-product firms with homogeneous products with positive network effects[3]. In their model consumers always benefit from interconnection. My model features horizontally differentiated firms which do not face any capacity constraints - standard differentiated Bertrand a la Hotelling (1929) setup. The only twists on Hotelling are the positive network effects, and the ability of the firms to interconnect. Sufficiently strong horizontal differentiation makes the common problem of the network effect literature - dealing with expectations - much easier. Even then, I impose assumptions on the model that do not let one of the firms take over the whole market. I am interested in examining the incentives to interconnect, which is not necessary once there is just one firm left.

Without interconnection firms compete for a consumer for two reasons. The first reason is increasing revenue. The second reason is that capturing this consumer makes the product more attractive to the all the other consumers. Interconnection gets rid of the second reason, effectively making the demand curve less elastic, and allowing firms to charge higher prices

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[2]In case of phones or email consumers get direct externality of more people being on the network being better. In case of the game consoles consumers derive benefits from other people having the same console indirectly by getting more software in a bigger market. Also there is the direct benefit of being able to interact with other users online, and the more users there are, the better off each user is. Platforms and two-sided markets are a related issue, and this paper could have been presented within the two-sided setup. For a recent paper examining compatibility in a two-sided setting see Miao (2008), in which the author examines the incentives of a monopolist to provide compatibility in a complementary market

[3]I refer to firms interconnecting their products as simply firms interconnecting from now on.

while maintaining the same customer base. The firms interconnect too much when the network effects are sufficiently steep, the opposite of what we would expect, as steep network effects imply the social need of having one big network.

The literature is generally concerned with two ways that firms can interconnect. The first way is mutually agreeing on a common standard, with each firm paying some fixed cost to interconnect. The second way is someone providing an adapter to let consumers (possibly imperfectly) enjoy the benefits of having the other network around, with each firm deciding how much (and to which network) interconnectivity to provide. I examine the first case of mutual agreement on a standard in this paper[4]. The underlying assumption throughout the paper is that the disutility from buying not an ideal good increases faster than the network effect[5]. If the assumption is not satisfied, then the network effects dominate, and we have to worry about one of the standards/platforms prevailing and capturing the whole market without interconnection. Since I am interested in interconnection, I do not examine these cases.

The literature on network effects has focused on the settings where the only differentiation between the products in the market is the number of consumers in each network. However, in the majority of the competitive network-effects settings (cell phone companies, instant messengers, social network sites), there is a horizontal brand (or technological standard) differentiation as well. For some of the examples generally given in the network externalities literature, a model with two-sided platforms might be a better fit. I consider network effects to be somewhat of a reduced approach, which lets the researcher focus on particular details. My goal in this paper is to point out the difference in the intuition and qualitative results between Cournot and Hotelling type network interconnection under different sets of circumstances.

The point that interconnection might hurt consumers in the Hotelling-type setup was first made by Spulber in his (2008a and 2008b) papers, and proposition 1 in the next section is a generalization of the arguments in the (2008b) paper. The fact that compatibility decreases elasticity of demand and lets the firms charge higher prices is also documented in the mix-and-match literature by Matutes and Regibeau (1988) and Economides (1989). The mix-and-match literature assumes that there are no explicit network effects for consumers or

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[4]For a paper on adapter interconnection with differentiation (vertical in their case), see Garcia and Vergari (2008). The authors find that with weak network effects market may achieve full compatibility (although with stronger network effects compatibility is underprovided). Moreover they find that a bigger firm might want to interconnect just as much as (or more than) a smaller firm, something that I show as well. Including adapters in my model would probably lead to something akin to endogenous travel cost models like Hendel and Neiva de Figueiredo (1997), at least in the linear version of my model.

[5]More rigorously, I require the derivative of the travel costs to be bigger than the derivative of the network effect function.

producers, however the "demand in mix-and-match models exhibits network externalities"[6]. Farrell and Saloner (1992) use the spatial setup to study a market where firms can produce converters for one-way compatibility with the other firm. They find that the availability of converters might be bad for society overall. While this also sounds like a too much interconnection can be bad result, this is not actually the case in their setup. The availability of converters is bad for the society only when without the converters consumers would have all joined the same network and achieved perfect standardization.

There are several related works in which interconnectionis not the main topic of the paper. Perhaps the most related one is Suleymanova and Wey (2008) in which the authors examine Bertrand competition in a market with network effects and heterogeneous switching costs. The heterogeneous switching costs behave a lot like the heterogeneous ideal points in Hotelling, and the results also have some similarity. For a small range of parameters, the authors find the result that with two active firms incompatibility might be better if the market share of one of them is really high, and so are the switching costs. For another range of parameters they find that having a monopoly without interconnection is better for social welfare than a duopoly with interconnection. Grilo et.al (2001) examine horizontally differentiated competition with network effects, and derive several benchmark results both for positive and negative network effects. Griva and Vettas (2004) examine both horizontally and vertically differentiated market with two firms, noting that, "full compatibility is equivalent to zero network effects. We find that, generally, the presence of network effects may lead firms to compete more intensively and their profits to be lower," which is a claim of this paper as well, and I show it in more detail in one of the subsections below.

I go on to the next step in the spatial setup - the incentives to relocate. Adopting the model of d'Aspremont et. al. (1979), with the addition of linear network externalities, I find that the effect of network externalities is non-monotonic in the position of the two firms. When the firms are moderate distance away, the network externalities decrease the incentive to move away, however if the firms are either very far or very close network externalities increase the incentive to move away.

In a latter section I examine a model with three firms on Salop's circle. Two of them are interconnected. I find that the not-interconnected firm is worse off in all the metrics - profits, market share, and price. This shows that even letting some of the firms in the market to interconnect might be bad for consumers and for the other firms in the market.

The next section focuses on asymmetric competition - first with one of the firms having a better product than another, and then with one of the firms getting to the market first,

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[6]See Economides (1996) for more on mix-and-match models and comparison with the network externality approach. The quote in the text is from that article.

and having some installed base of consumers by the time the second firm comes in. Both of the asymmetric cases show that prices go up after interconnection. More dimensions of differentiation make interconnection less valuable. Another result is that the market share of the bigger firm (in either context) is smaller with interconnection. If one naively uses Herfindahl index to measure competitiveness, the index is smaller if firms interconnect, yet the firms compete less aggressively and consumers pay higher prices.

The last section is a look at what happens in a not covered market. I use (a slightly modified) spokes model of Chen and Riordan (2007) to examine the uncovered market case. The main result of a post-interconnection decrease in consumer welfare still holds in this setup. However, the possibility of bringing in more consumers to the market mitigates the effect of the existing consumers becoming less elastic.

2 Interconnection in a Covered Market

On the demand side I follow the setup of Farrell and Saloner (1992), without adapters, or Spulber (2008b); but generalizing to N firms on a circle, with general externality function g, and a general travel cost function h. Consumers are distributed uniformly along a circle of circumference M and N firms are also located on the circle, at equal distances from each other. A consumer x away from a firm receives utility of

$$g(n^e) - h(x) - p, \tag{1}$$

where n^e is the expected number of consumers buying from this firm, and p is the price. Assume $g(0)$ is big enough so that every consumer buys a product, and that travel cost are steep enough, $h'(t) > g'(2t) > 0$. Firms may choose to interconnect before the market opens. This is a joint decision - each firms needs to prefer to interconnect for this to happen. Each firm has to pay $F > 0$ to interconnect. If the firms do interconnect, then consumers perceive all the goods as being part of the same network, and therefore instead of $g(n^e)$ all consumers receive $g(M)$ after purchasing the product.

Proposition 1 *In a symmetric equilibrium without interconnection firms charge*

$$p = \frac{M}{N} \left(h'(\frac{M}{2N}) - g'(\frac{M}{N}) \right). \tag{2}$$

Proof. I focus on the consumer indifferent between joining two firms given the prices. Fix the prices of all firms but one at $\bar{p}$. Then the shares of all the firms, except the two adjacent to the potential deviant firm, are $\frac{M}{N}$. The two firms bordering the deviant are going to have

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demand of $\frac{M}{2N}$ on the side not bordering the deviant's market. Then, the consumer at x who is indifferent between the deviant, whose price is p, and another firm, whose price is $\bar{p}$, is defined by[7]. The sufficiently high product differentiation makes the standard in the literature problem of expected equilibria much easier. If the consumer at .5 joins the firm at 0 as opposed to joining the firm at 1, then so does the consumer at .4. Therefore, just like in any Hotelling-based setup we just have to find the indifferent consumer at x. This point was made by Anderson et.al. (1992).

$$g(x + n^e) - h(x) - p = g(\frac{3M}{2N} - x) - h(\frac{M}{2N} - x) - \bar{p}, \tag{3}$$

where n^e is the expected number of consumers buying from the deviant firm on the other side (not x's side). The deviant firm's profit is $\Pi = 2xp$, and therefore the first order condition is

$$x + p\frac{dx}{dp} = 0 \tag{4}$$

Implicitly differentiating 3 with respect to p, we get

$$\frac{dx}{dp} = \frac{1}{g'(x + n^e) + g'(\frac{3M}{2N} - x) - h'(x) - h'(\frac{M}{2N} - x)} \tag{5}$$

Substituting this result into 4 and invoking the symmetry condition ($n^e = x = \frac{M}{2N}$ from $p = \bar{p}$), we get the result in the proposition. We also need to take care of the second order conditions. The second derivate of profit with respect to price is $2\frac{dx}{dp} + p\frac{d^2x}{dp^2}$. The first term is negative if $h'(t) > g'(2t)$ for $t > 0$, and the second term is negative if $h''(t) > g''(2t)$, and is zero in a symmetric equilibrium. ∎

The social welfare effects of interconnection are the increase in the network external- ity benefits to consumers and the sunk costs paid by the firms. The price difference does not matter for the social welfare, since this is simply a transfer from consumers to firms. However, firms interconnect only if the price difference is big enough to cover the (sunk) costs of interconnection. Therefore, the price difference might be sufficiently large for the interconnection to happen, but the network externalities might be too small to justify inter- connection from the social perspective. In this case, the consumers suffer as well, since they are the ones paying for interconnection costs.

If the firms choose to interconnect, then the sunk cost to interconnect, F, does not matter for pricing decision. The difference on the demand side is that it does not matter how many

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[7]Assuming that the inactive firm grabs $\frac{M}{2N}$ of the market from the other side. Different assumptions would lead to different SOCs (i.e. $h'(t) > g'(t)$ instead of $h'(t) > g'(2t)$)

other consumers buy the same product - in the end, everyone derives externalities from the joint network. Therefore, consumers' utility from network externalities does not enter firms' pricing decision either, and the firms price at $p = \frac{M}{N}h'(\frac{M}{2N})$ (Salop (1979) price with generalized travel cost function, can be derived using similar arguments as the proof for the previous proposition). Since the market share is the same in each symmetric equilibrium, $\frac{M}{N}$, we have the following corollary.

Corollary 1 *If the firms decide to interconnect, price increases by $\frac{M}{N}g'(\frac{M}{N})$. Firms interconnect (profits increase by more than the costs of interconnection) if and only if*

$$\left(\frac{M}{N}\right)^2 g'(\frac{M}{N}) > F. \tag{6}$$

Since the price increase depends on g' and consumer welfare depends on g, consumer welfare might decrease after interconnection, unlike in Katz and Shapiro(1985). This result immediately leads to re-examination of their central result that firms do not interconnect enough because consumer welfare effect is always positive[8]. Two following corollaries classify the parameter values and outcomes when firms interconnect.

Corollary 2 *Firms interconnect when it is socially optimal to, if and only if F is small enough that*

$$min\left((\frac{M}{N})^2 g'(\frac{M}{N}), \frac{M}{N}(g(M) - g(\frac{M}{N}))\right) > F. \tag{7}$$

Firms interconnect when it is NOT socially optimal to if

$$(\frac{M}{N})^2 g'(\frac{M}{N}) > F > \frac{M}{N}(g(M) - g(\frac{M}{N})). \tag{8}$$

With interconnection, consumer welfare increases if and only if

$$\frac{M}{N}g'(\frac{M}{N}) < g(M) - g(\frac{M}{N}). \tag{9}$$

In particular, it always decreases if the interconnection was not beneficial to the society.

For firms to interconnect when it is not socially optimal, the externality function g needs to be sufficiently steep. This means that firms interconnect too much exactly when consumers' enjoyment of the product rises too fast with the number of other consumers joining the network.

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[8]$\Delta SW = \Delta CW + \Delta\Pi$. In Katz and Shapiro $\Delta CW > 0$, and therefore $\Delta\Pi < \Delta SW$, and in particular it is possible that $\Delta\Pi < F < \Delta SW$, where F is the cost of interconnection, and in this case it it socially optimal to interconnect, but the firms do not want to.

Some (arguably most) products exhibit the properties that the network effects are increasing sharply in the number of users up to some point, and then they start to flatten out or decrease. The result of firms interconnecting too much is particularly relevant for these products. If the turning point is at a bigger number of consumers then any one one product enjoys in a symmetric equilibrium, then $g'(\frac{M}{N})$ is relatively large, so that the firms interconnect, however the difference in consumer utility between the big network and a small network $(g(M) - g(\frac{M}{N}))$ is much smaller since $g(M)$ is an overkill as far as the network size is concerned.

3   Product Positioning with Network Effects

Product positioning with networks had been neglected in the literature, at least partially stemming from the lack of models on horizontally differentiated networks. In this section I develop a simple model of product positioning with network effects, and examine what happens when firms have the ability to interconnect.

The model's only departure from Hotelling (1929) is the addition of network effects into the mix and quadratic travel cost to ensure an equilibrium (see d'Aspremont et. al. 1979). Consumers' preferred points are distributed uniformly on the unit interval. If the product is distance d away from a consumer's preferred point, consumer derives utility of $R - td^2 - p + jn^e$, where R is the reservation utility, t is the travel cost or differentiation parameter, p is the price of the product, j is the strength of network effects, and n^e is the expected number of consumers joining the network. The first(second) firm is located a(b) away 0(1). With $j = 0$ this is exactly the model in d'Aspremont et. al. (1979).

Proposition 2 *With network effects the firm closer to 0 charges*

$$p_0 = \frac{t}{3}((b-2)^2 - (a+1)^2) - j. \tag{10}$$

Proof. See Appendix. ■

As expected, the stronger the network effects are, the lower is the price. With interconnection, the price is the same as in d'Aspremont et.al. - the same as above with $j = 0$. The price increases both as the firm gets closer to its own end (i.e. as a gets smaller), and the farther away the competitor is (i.e. as b gets smaller).

Proposition 3 *The incentive to move farther from the competitor decreases in the strength of network effects $(\frac{\partial^2 \Pi_0}{\partial a \partial j} < 0)$ when the firms are moderate distance away, and increases $(\frac{\partial^2 \Pi_0}{\partial a \partial j} > 0)$ either when the firms are close or really far.*

Proof. See Appendix. ∎

The strength of network externalities effects demand elasticity in two ways. First, it effects the demand elasticity directly through the term related to the price difference between the two firms. Second, it enters indirectly because of the firms having the installed bases of a and b respectively, so the higher are a and b, the more effect the indirect term has. Hence, we have the non-monotonic behavior described above.

4 Partial and Sequential Interconnection

Some industries are fully interconnected, some are fully not interconnected and in some industries one company is interconnected with another, but not with a third one. An example would be the airline miles points networks - United is interconnected with Lufthansa, but not with American Airlines. Another one would be instant messengers, where Yahoo! Instant Messenger became interconnected with Google's after the recent deal. To focus on the interesting issues, and to keep the model as tractable as possible, there are three firms in the market. The market is a Salop's circle, where unit demand customers are distributed uniformly with mass 1, and have linear transportation cost t, and linear network benefit j. I continute to assume that $t > 2j$ and that the market is covered (each consumer buys a unit). The three firms are located at equal distances from each other, and two of them are interconnected. I drop the subscript for one of the interconnected firms, denote the other by subscript $_o$, and the stand-alone firm by $_s$. Since the two interconnected firms are symmetric, the proof of the following proposition only involves solving a system of two equations with two variables[9].

Fix an interconnected firm (The Firm from now on - the one without the subscript). The Firm charges one price, but has two different submarkets, on either side of The Firm. Both of the interconnected firms' networks include everything not in the stand-alone firm's network, but they still fight for consumers between them - with x consumers in the arc between the two interconnected firms buying from The Firm, and $\frac{1}{3} - x$ buying from the other firm. Similarly, x_s consumers buy from The Firm in the arc between The Firm and the stand-alone firm; and x_{so} consumers buying from the interconnected firm.

Proposition 4 *The interconnected firms charge*

$$p = \frac{t - j}{3(5t^2 - 9tj + 3j^2)} t(5t - 6j), \tag{11}$$

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[9]Equations are the first order conditions of an interconnected firm and the stand alone firm, and the variables are the respective prices.
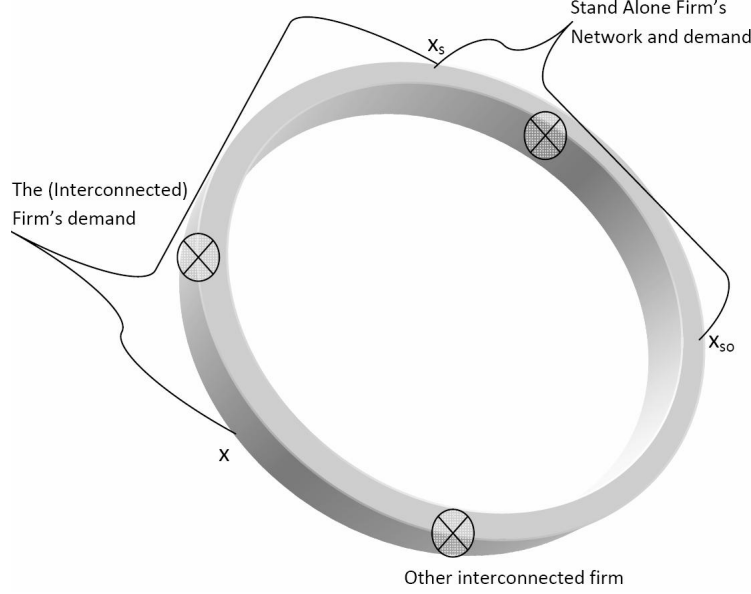
Figure 1: Partial Interconnection.

and the stand-alone firm charges

$$p_s = \frac{t - j}{3(5t^2 - 9tj + 3j^2)} \left(5t^2 - 10tj + 3j^2\right).$$ (12)

Proof. See Appendix. ∎

Since $t > j$, simple corollary follows.

Corollary 3 *In equilibrium, the stand-alone firm*

- *charges less than the interconnected firms ($p_s < p$),*

- *sells less quantity than either of the interconnected firms ($D_s < \frac{1}{3}$),*

- *derives less profits than either of the interconnected firms ($\Pi_s < \Pi$).*

In the standard Cournot model cooperation between two of the many firms leads to reduced output on the part of the cooperating firms, and therefore more profits for the not cooperating firms. The similar exercise is much harder to reproduce in the Salop's model, since one needs to take into account the fact that all the firms are not symmetric anymore - the firms closer to the cooperating firms are different than the firms farther away[10].

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[10]See Giraud-Heraud (2003) for an elegant solution of the case where cooperating firms are neighbors, with a system of N equations. Another way to model this would be to use the spokes model of Chen and Riordan (2007), as it would cut down on the number of equations in the system, since the interconnected firms would be symmetric, and so would be the not interconnected firms.

The results above show that similar logic holds for partial network interconnection. While it is not a merger, it is a form of cooperation, and one would expect cooperating firms to benefit at the expense of the other firm[11]. The results from Section 2 generalize to here - sometimes it is not optimal to let even *some* of the firms in the market interconnect, even if the ownership stays in different hands.

Corollary 4 *If all the firms interconnect, prices are even higher than with only two firms interconnecting.*

Proof. From the previous proposition we have to show that $p < \frac{t}{3}$, where the right hand side of the inequality is the familiar Salop price with three firms. The price of the two interconnected firms already contains $\frac{t}{3}$, and one can see that the prices with all the firms interconnecting are higher as long as $2t > 3j$, which is satisfied because of the $t > 2j$ assumption. ■

As expected, the prices in the market go up even more if the third firm joins the network. From the previous corollary one can easily see that if this happens the original members of the wide network lose market share. Therefore, for the first two firms to let the third firm join in, the price increase must be dramatic enough to cover not only for the costs of interconnection, but also for the market share decrease. Sequential (versus simultaneous) nature of interconnection does not influence the main result - consumers end up paying more, and firms might interconnect too much from the social and the consumer welfare points of view.

5   Effects of Asymmetry on Interconnection Decision

5.1   Different Quality - Vertical Differentiation Together With Horizontal

The ongoing assumption in the paper so far had been that the products are just horizontally differentiated. Another issue to examine is what happens if there are *also* quality differences between the products. There are two firms, located at 0 and M of the Hotelling interval [0,M], with consumers distributed uniformly. Assume that both h and g (the travel cost and the network externality functions) are linear, with coefficients of, respectively, t and j. Also, without loss of generality, assume that all the consumers derive an additional utility of $Q > 0$ from consuming firm 0's product.

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[11]Again, because we are dealing with the Hotelling setup here. In Cournot mergers the firms that do not merge benefit more.

Proposition 5 *With quality differences, both firms are less willing to interconnect. If the firms do interconnect, prices increase and market concentration decreases.*

Proof. The indifferent consumer is at x which is defined by

$$2x = \frac{p_2 - p_1}{t - j} + \frac{Q}{t - j} + M, \tag{13}$$

with subscript 2 denoting the firm at M, subscript 1 denoting the firm at 0 (the one with the superior product). The only difference from the familiar formula is the fraction with Q, and as one would expect, setting $Q = 0$ gives us back the standard formula. From 13 and the first order conditions for each firm we can get the following system describing the optimal prices:

$$p_1 = \frac{p_2}{2} + \frac{Q}{2} + (t - j)\frac{M}{2} \tag{14a}$$

$$p_2 = \frac{p_1}{2} - \frac{Q}{2} + (t - j)\frac{M}{2} \tag{14b}$$

The difference is the vertical Q term, which makes the firms asymmetrical, as expected. Solving the system, we get

$$p_1^* = M(t - j) + \frac{Q}{3} \tag{15a}$$

$$p_2^* = M(t - j) - \frac{Q}{3}, \tag{15b}$$

and from there, $x^* = \frac{M}{2} + \frac{Q}{6(t-j)}$. The prices and the indifferent consumer above give us the profits of the firms before interconnection. Firm 1's profit is xp_1 and firm 2's profit is $(M - x)p_2$. After interconnection the profits are the before interconnection profits with $j = 0$. The difference between pre- and post-interconnection profits for *both* firms is

$$\Pi_\Delta = \frac{M^2 j}{2} - \frac{j}{18t(t - j)} \times Q^2, \tag{16}$$

note that Q enters with a negative sign - for both firms the higher the quality difference is, the less they are willing to spend on interconnection. ∎

Quality differences introduce another dimension of differentiation into the setup, allowing the firms to derive higher profits than otherwise. Interconnection still means that firms do not compete for consumers as aggressively as before, however this is not as valuable since the firms are already differentiated in two dimensions, as firms do not compete on prices as hard to begin with. Both firms are less willing to interconnect in this setup. For both of

them the extra dimension of differentiation brings in more profits, and the interconnection is not as enticing. The result is likely to generalize to more dimensions than two - the more dimensions the firms are differentiated on, the less incentive they have to interconnect.

5.2   Previous Installed Base - Competing With Incumbent

We have looked at the quality asymmetry in the previous subsection. Now, assume that the quality of the products is the same, but one of the firms is a new entrant, with the other already having some consumers captured.

There are two firms, located at 0 and 1 of the Hotelling interval $[0, 1]$, with consumers distributed uniformly. Assume that both h and g (the travel cost and the network externality functions) are linear, with coefficients of, respectively, t and j, with $t > 2j$. Also, without loss of generality, assume that the firm at 0 already has A consumers captured from the previous periods. Suppose that these A consumers are going to stay with their firm no matter what, but the firm cannot charge different prices to the old and to the new consumers. I denote the firm with an installed base by subscript a, and the other firm by subscript b.

Proposition 6 *The bigger the installed base is, the more the firms want to interconnect. If the firms do interconnect, prices increase and market share of the bigger firm decreases.*

Proof. The proof is similar to the one for the previous proposition, and therefore relegated to the appendix. ■

Here we have the opposite result from the quality competition. The reason is that while the firms were differentiated in quality in the example before, there were no captured consumers. In this example, there is a captured installed base at the incumbent's disposal. This base is not under a threat from the entrant, and therefore the bigger the installed base is, the more the incumbent is interested in higher prices which come after interconnection. The entrant is not going to get the installed base one way or another, but with the incumbent becoming softer as the installed base grows, the entrant has more market share and a higher price charged to gain as the two firms interconnect.

6   Not Covered Market

In format (or standard) wars many consumers sit out and wait for either one of the standards to emerge as the winner, or for the firms to agree on a common standard. For example, some of the consumers would have been in the market for a high definition DVD equipment much sooner if there would have been an established standard from the beginning. From the

Firm 1

Firm 1's Monopoly
Market (10)

Spoke 1

Spoke (ray) 0

Competitive
Market (12)
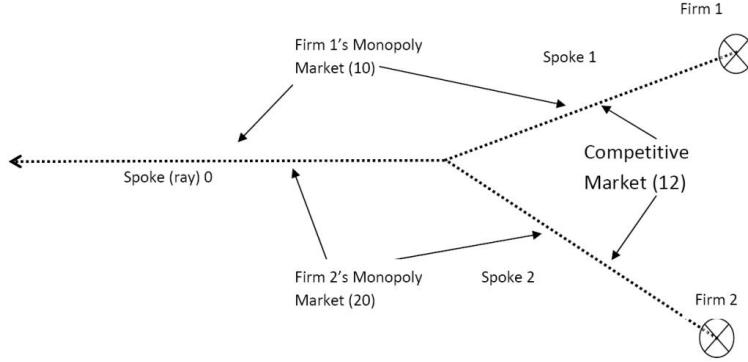
Firm 2's Monopoly
Market (20)

Spoke 2

Firm 2

Figure 2: Spokes and Network Effects.

theoretical perspective, one of the reasons that the consumer welfare always increases in Katz and Shapiro (1985) is that there are more consumers buying products after interconnection, which is not the case in any of the previous sections.

I tweak the model from above to address this issue. I use a slightly modified version of the spokes model of Chen and Riordan (2007) with two firms and three spokes[12]. The spoke setup is represented in Figure 2. I chose this set up since the spokes model is also non-local competition, just like the Cournot model, but unlike Hotelling or Salop. It does not matter for this special case, however extending to more firms than two should be easier from this groundwork.

The consumers are distributed uniformly along three spokes coming out of one hub, call them spoke 0, 1, and 2. Each spoke is of length 1. The two firms are located at the end of spokes 1 and 2, and I refer to them as, respectively, firm 1 and firm 2. Each consumer can only buy a product from two of the spokes - the spoke that the consumer is on, and one of the other ones with equal probability (in this case .5). If there is no product on one of those spokes (spoke 0 here), then the consumer is choosing between buying the product on the other spoke, or not buying the product at all.

This results in three submarkets. One of the submarkets is the standard Hotelling competition between firm 1 and firm 2, and this submarket consists of a (random) half of the consumers on spokes 1 and 2. I call this submarket 12 from now on. The other two submarkets are standard Hotelling monopoly setups, and consist of the other half of the consumers at spoke 1 and half of the consumers at spoke 0 for firm 1's monopoly submarket (call this submarket 10); and of the other half of the consumers at spoke 2 and the other half of the consumers at spoke 0 for firm 2's monopoly submarket (call this submarket 20). The transportation costs are linear with the parameter t, network effects are linear with the parameter

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[12]In what follows, the setup is equivalent to Hotelling with hinterlands - two firms located at 0 and 1, and consumers distributed along [a,b], with $[0, 1] \in [a, b]$.

j, as usual $t > 2j$, and each spoke has a weight of 1 of consumers. The modification of the spokes model is that I make spoke 0 into a ray. I am not interested in what happens if the market is covered, since I have examined this case in detail in the sections above, therefore with a ray instead of the 0 spoke there is always additional demand if the firms interconnect. I assume that the weight on each 1 unit of the ray is the same as the weight on either of the two spokes, so the total size of the market is infinity. Submarket 12 (the competition submarket) only has a size of 1 (half of spoke 1 and half of spoke 2). The consumer utilities are as in the sections before.

Note that the ray instead of a spoke assumption stacks the model in favor of the result of always too little interconnection. The firms have a much lower incentive of increasing prices, since there is a huge incentive of lowering prices to attract more consumers from the ray. Moreover, with interconnection the firms play a non-cooperative game in the small submarket 12 competing for consumers, however they are interested in cooperating on the (heavier weighted) ray, since more consumers for my rival means that the value of my product goes up for my monopoly market and I can get more revenue, all of which would imply strong incentives to keep prices down post interconnection.

Proposition 7 *With sufficiently high product differentiation or sufficiently low network effects, the prices go up with interconnection and consumer welfare suffers, resulting in the possibility of too much interconnection.*

Proof. While the logic of the proofs stays roughly the same, the algebra becomes much more involved. First, I find the equilibrium prices without interconnection. For that I need to find demand of the firms in each of the submarkets. The competitive submarket (12) can be characterized by the following, where x_{12} is how far away from firm 1 the consumer indifferent between buying from firm 1 and from firm 2 is located.

$$-p_1 - tx_{12} + j\frac{x_{12} + x_{10}}{2} = -p_2 - t(2 - x_{12}) + j\frac{2 - x_{12} + x_{20}}{2}, \qquad (17)$$

where p's are the prices, x_{10} and x_{20} are consumers on the ray (spoke 0) who, respectively, are indifferent between buying firm 1's product and not buying at all and buying firm 2's product and not buying at all. We have to divide the demand by 2 for network effects, since only half of the consumers before (for example) x_{12} are in the competitive submarket, the rest are in one of the monopoly submarkets. Similarly, we can characterize the other two

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x's:

$$R - p_1 - tx_{10} + j\frac{x_{12} + x_{10}}{2} = 0, \tag{18a}$$

$$R - p_2 - tx_{20} + j\frac{2 - x_{12} + x_{20}}{2} = 0. \tag{18b}$$

From the definitions of x's ((17),(18a), and (18b)), we have a system of three equations with three variables (the x's). Solving the system, we get:

$$x_{12} = 1 + \frac{p_2 - p_1}{2(t - j)}, \tag{19a}$$

$$x_{10} = \frac{2R + j}{2t - j} + \frac{3jp_1 + jp_2 - 4tp_1}{2(t - j)(2t - j)}. \tag{19b}$$

The profit function of firm 1 is (multiplied by 2 for convenience)

$$\Pi_1 = p_1(x_{10} + x_{12}), \tag{20}$$

differentiating with respect to p_1, we get:

$$\frac{\partial \Pi_1}{\partial p_1} = 1 + \frac{p_2 - 2p_1}{2(t - j)} + \frac{2R + j}{2t - j} + \frac{6jp_1 + jp_2 - 8tp_1}{2(t - j)(2t - j)}, \tag{21}$$

The second order condition holds, so setting the expression above to zero and invoking the symmetry conditions we get

$$p^* = \frac{2(t - j)(R + t)}{5t - 4j}. \tag{22}$$

Now we need to find the equilibrium prices with interconnection. It used to be that the intensity of network effects would not matter with interconnection. This is not the case now. While it will not matter (directly) for the competitive submarket, j directly effects the monopoly submarkets, since the higher the strength of the network effects, and the bigger the network, the more consumers prefer to purchase from their monopolist than not to purchase at all. Therefore the new equations which define the x's are going to be the same for the monopoly submarkets (x_{10} and x_{20}), and the previous equation (17) with j set to 0 for the competitive submarket (x_{12}). Solving this new system we get:

$$x_{12} = 1 + \frac{p_2 - p_1}{2t}, \tag{23a}$$

$$x_{10} = \frac{R + j}{t - j} - \frac{p_1}{t - j} + \frac{p_1 - p_2}{2t(t - j)}. \tag{23b}$$

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Given the new values, and differentiating the same profit function, we get

$$\frac{\partial \Pi_1}{\partial p_1} = 1 + \frac{p_2 - 2p_1}{2t} + \frac{R + j}{t - j} - \frac{2p_1}{t - j} + \frac{2p_1 - p_2}{2t(t - j)}, \tag{24a}$$

$$\frac{\partial^2 \Pi_1}{\partial p_1^2} = \frac{1}{t} - \frac{2}{t - j} + \frac{1}{t(t - j)}. \tag{24b}$$

The second order condition is satisfied iff $3t > j + 1$ and I assume it for the rest of the discussion. Involving the usual symmetry conditions the FOC gives us

$$p^* = \frac{2t(R + t)}{5t - j - 1}. \tag{25}$$

Prices after interconnection are higher for sufficiently high product differentiation (t). The change in consumer welfare (consumer welfare with interconnection less the consumer welfare without interconnection) is (with superscript i denoting interconnection, and superscript n denoting no interconnection):

$$CW^i - CW^n = \left[p^i - p^n + 2j(x_{10}^i - x_{10}^n)\right](1 + 2x_{10}^n) + (x_{10}^i - x_{10}^n)\left[R + j(1 + 2x_{10}^i) - p^i\right] - 2t \int_{x_{10}^n}^{x_{10}^i} z\,dz, \tag{26}$$

where all the x's and p's are the equilibrium values. The first term is how the welfare of existing consumers changed. The second term is the welfare of the new consumers, who did not buy before interconnection. The third term is the travel cost that the new consumers have to pay. The sum of the last two terms is positive, otherwise the new consumers would not buy. The first term is negative as long as the prices go up with interconnection. Overall, apriori it is not clear what happens. Note however that all the positive terms decrease with t, and the travel cost increases with t. Since all of our restrictions on the shapes of the travel cost and network effect functions involved t being sufficiently high, this means that a sufficiently large t still satisfies those restrictions, and makes the expression above negative. By previous arguments this implies that firms might interconnect too much. ∎

Even with the model stacked against a decrease in consumer welfare, I still get the result of too much interconnection with sufficiently high product differentiation. The possiblity of market expansion mitigates the results of an automatic post-interconnection price increase, but high enough product differentiation still delivers a result similar to the ones from previous sections.

7   Conclusion

I have examined the base aspects of differentiated Bertrand competition with the presence of network externalities and the option to interconnect. I have started with showing that the seminal result of not enough interconnection does not hold. When the network externality function is steep enough, then both consumer and social welfare suffer when firms interconnect, and therefore there might be too much interconnection, paradoxically, when the externality increases too fast. The effect comes from the fact that steep network externality function implies high demand elasticity, and the inability by firms to charge high prices. However, after interconnection the elasticity goes down, and the steeper the externality function, the less elastic demand becomes, since consumers do not care about which firm to join as far as the networks effects are concerned - they just care about the price.

I find that the results hold up with sequential interconnection in a three-firm market (first two firms interconnect between themselves, and then they might or might not let the third firm enter). The results also do not change qualitatively with either quality differences between the firms, or one of the firms having an installed base. Other comparative statics in these results are interesting in their own right.

The asymmetric cases show that the intuition from the symmetic case still holds. More interesting results are that more dimensions of differentiation make interconnection less valuable - firms are already not competing that hard on prices without interconnection. However, an incumbent with an installed base values interconnection more than firms in the symmetric case. Interconnection allows both the incumbent and the entrant not to compete as much on prices, and while incumbent might not gain as many new consumers as without interconnection, the increased price margin more than makes up for the loss. In both asymmetric cases (one product better than another, and one firm having captured consumers) with interconnection the Herfindahl index is lower, while the firms are competing less aggressively, indicating that with network interconnections naively using the index might lead to wrong conclusions.

The results from Bertrand differentiation are qualitatively different from the standard (Katz and Shapiro (1985)) Cournot-based approach to network externalities, and give different intuition to the effects of network externality. And as in general, one should think carefully whether the competition is Cournot or differentiated Bertrand, even in the case of network externalities.

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Appendix

Proof of Proposition 2

Proof. Call the firm at a's price p_0, and the other firm's price p_1. There is a marginal consumer at some x, who is indifferent between joining either of the networks. A nice feature of the Hotelling setup is there is a natural division of which consumer belongs to which network - all the consumers with preferred points to the left of the marginal consumer buy the product to the left, and the consumers to the right buy the other product[13]. This means that for the left firm $n^e = x$, where x is the marginal consumer. Therefore, the following equation characterizes x, with the left hand side the utility (without the reservation value) for the left firm's product, and the utility from the right firm's product on the right hand side.

$$jx - t(x-a)^2 - p_0 = j(1-x) - t(1-b-x)^2 - p_1 \tag{27a}$$

$$x = \frac{p_1 - p_0}{2(t(1-a-b)-j)} + \frac{t(1+b^2-a^2-2b)-j}{2(t(1-a-b)-j)} \tag{27b}$$

Profit of the firm on the left is $\Pi_0 = p_0 x$. Differentiating it with respect to p_0, and deriving a similar condition for the firm of the right, we get the following system of two equations with two variables (p_0 and p_1):

$$\frac{p_1 - 2p_0}{2(t(1-a-b)-j)} + \frac{t(1+b^2-a^2-2b)-j}{2(t(1-a-b)-j)} = 0 \tag{28a}$$

$$\frac{p_0 - 2p_1}{2(t(1-a-b)-j)} + 1 - \frac{t(1+b^2-a^2-2b)-j}{2(t(1-a-b)-j)} = 0 \tag{28b}$$

From the system, the price of the left firm is as stated in the proposition (switch a and b to get the right firm's price). ∎

Proof of Proposition 3

Proof. From the proof directly above, we know both the demand (x) and the optimal price (p_0) given a, b, and j. From d'Aspremont et. al., we know that $\frac{\partial \Pi_0}{\partial a} < 0$, or that profits increase as the firm moves away from the center. I am interested in the effects of j on both magnitude and sign of the incentives to agglomerate. Therefore, we need $\frac{\partial^2 \Pi_0}{\partial a \partial j}$. From symmetry, we know that $p_1 - p_0 = \frac{t}{3}(2a^2 - 2b^2 - 2a + 2b)$. Then after substituting the optimal

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[13] Noted by Anderson et. al. (1992), p.259

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prices:

$$x(a, b) = \frac{t(b^2 - a^2 - 2a - 4b + 3) - j}{6(t(1 - b - a) - j)}, \tag{29a}$$

$$\frac{dx}{da} = t\frac{t(1 - a - b)^2 + 2aj + j}{6(t(1 - b - a) - j)^2}, \tag{29b}$$

$$\frac{dx}{dj} = t\frac{b^2 - a^2 - a - 3b + 2}{6(t(1 - b - a) - j)^2}, \tag{29c}$$

$$\frac{d^2x}{dadj} = t\frac{t(3 - 2b)(1 - a - b) + 2aj + j}{6(t(1 - b - a) - j)^3}. \tag{29d}$$

It is possible (but tedious) to show then, that

$$\frac{\partial^2\Pi_0}{\partial a\partial j} = \frac{t^3(1 - a - b)\left[2a^3 - 2b^3 + 2a^2b - 2ab^2 + 6ab + 6b^2 - 3a^2 - 12a - 6b + 2\right]}{6(t(1 - b - a) - j)^2} \tag{30a}$$

$$+ \frac{jt^2\left[-4a^3 + 4ab^2 + a^2 - 8ab + 10a + 2b - 2\right]}{6(t(1 - b - a) - j)^2}. \tag{30b}$$

For simplicity, I examine the symmetric cases, where a=b. It is (relatively) easy to see that with $a, b \to 0$ the first term is positive, and the second one is negative, however since we assume that $t > j$ throughout the text, the whole expression is positive. As a and b move away from 0, the first term decreases, and the second term becomes positive, first moving into overall negative, and then becoming positive before $a = .5$. The less j is with respect to t, the bigger a it takes to go into the negative, and the bigger b it takes to become positive again. In the limit, as $j \to 0$, the derivative is positive when $a < 1 - \frac{\sqrt{7}}{3}$, which is approximately .12. As $j \to t$, the derivative is positive when $a < \frac{19 - \sqrt{181}}{18}$, which is approximately .31. ∎

Proof of Proposition 4

Proof. The demand in the first submarket, where the competition is the other interconnected firm, is the standard Salop/Hotelling demand, since from the previous sections we know that interconnected firms compete like standard product differentiated firms.

$$x = \frac{1}{6} + \frac{p_o - p}{2t}, \tag{31}$$

where p_o is the price of the other interconnected firm, and $\frac{1}{6}$ comes from the fact that there are three submarkets overall (each of size $\frac{1}{3}$), and if the firms are exactly the same, they split the submarket equally. The Firm competes with the stand-alone firm in the other submarket, and here we need to consider that the firms might potentially have different sized networks, and add the size of the other interconnected firm's network. Therefore, with utility from buying The Firm on the left hand side, and the utility from buying from the stand-alone

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firm on the right hand side, we get:

$$-p - tx_s + j(x_s + \frac{1}{3} + x_{so}) = -p_s - t(\frac{1}{3} - x_s) + j(\frac{2}{3} - x_s - x_{so}). \tag{32}$$

The term by j on the left hand side is the size of the interconnected network - what The Firm takes from the stand-alone firm, the arc between the interconnected firms, and what the other interconnected firm takes from the stand-alone firm. The similar expression on the other side is just the remainder (1 minus the expression on the left hand side). From this equation we get

$$x_s = \frac{1}{6} + \frac{p_s - p}{2(t - j)} + \frac{jx_{so}}{t - j}. \tag{33}$$

The demands for The Firm and the stand-alone firm respectively are $D = x + x_s$ and $D_s = \frac{2}{3} - x_s - x_{so}$. Differentiating profit of The Firm with respect to own price, and substituting from previous equations we get

$$\frac{\partial \Pi}{\partial p} = \frac{1}{3} + \frac{p_o - 2p}{2t} + \frac{p_s - 2p}{2(t - j)} + \frac{jx_{so}}{t - j}. \tag{34}$$

It is clear that the second order conditions are satisfied. We do not know x_{so}, however in equilibrium the two interconnected firms are symmetric, and therefore $x_{so} = x_s$, and $p_o = p$. Substituting that into 33 we get

$$x_{so} = \frac{p_s - p}{2(t - j)} + \frac{t - j}{6(t - 2j)}. \tag{35}$$

Then the first order condition for The Firm (plugging the right hand side above into 34 and making it equal to 0) gives us

$$t(t - j)(2t - 3j) - 3p(3t^2 - 6jt + 2j^2) + 3p_s t(t - j) = 0. \tag{36}$$

The FOC from the stand-alone firm gives

$$p_s = \frac{t - j}{3(2t - 3j)}(t - 3j + 3p). \tag{37}$$

This with equation 36 gives a system of two equations with two variables. The answers are in the proposition statement. ∎

Proof of Proposition 6

Proof. As before, we have to see where is the indifferent consumer located, and she is

located at x, where x satisfies the following equation

$$2x = \frac{p_b - p_a}{t - j} + \frac{Aj}{t - j} + 1. \tag{38}$$

Therefore, bigger firm's demand is $A + x$, and the smaller firm's demand is $1 - x$, where x is defined above. The profits are then, respectively, $(A + x)p_a$ and $(1 - x)p_b$. From this we get the following first order conditions:

$$2(A + x) - \frac{p_a}{t - j} = 0, \tag{39a}$$

$$2(1 - x) - \frac{p_b}{t - j} = 0. \tag{39b}$$

Since $t > 2j$, the second order conditions are satisfied for both firms, and we just have to solve a system of three equations and three variables, with the third one being 38. Skipping (quite a few) steps, we have the following results:

$$p_a^* = \frac{4}{3}At + (t - j) - Aj, \tag{40a}$$

$$p_b^* = \frac{2}{3}At + (t - j) - Aj, \tag{40b}$$

$$x^* = \frac{1}{2} - \frac{A}{3} + \frac{Aj}{6(t - j)}. \tag{40c}$$

Since x^* increases monotonically with j, and both prices decrease monotonically with j, we get the second part of the proposition (as before, the prices and market shares are going to be determined by the same equations with $j = 0$). The less trivial part are the incentives to interconnect. After plugging in profits after interconnection and subtracting the profits from before, we get the following differences in profits:

$$\Pi_a^\Delta = \Pi_b^\Delta = Aj + \frac{j}{2} + \frac{jA^2}{2} - \frac{tjA^2}{18(t - j)}. \tag{41}$$

Since $t > 2j$, the expression above increases in A - the bigger the installed base is, the more the firms want to interconnect, giving us the first part of the proposition. $\blacksquare$