Working Paper #08-09

October 2008

Competition vs. Regulation in Mobile Telecommunications

Johan Stennek
Gothenburg University and CEPR

Thomas P. Tangerås
Research Institute of Industrial Economics

Competition vs. Regulation in Mobile Telecommunications*

Johan Stennek†and Thomas P. Tangerås‡

October 20, 2008

Abstract

This paper questions whether competition can replace sector-specific regulation of mobile telecommunications. We show that the monopolistic outcome may prevail independently of market concentration when access prices are determined in bilateral negotiations. A light-handed regulatory policy can induce effective competition. Call prices are close to the marginal cost if the networks are sufficiently close substitutes. Neither demand nor cost information is required. A unique and symmetric call price equilibrium exists under symmetric access prices, provided that call demand is sufficiently inelastic. Existence encompasses the case of many networks and high network substitutability.

Key Words: network competition; two-way access; mobile termination rates; entry; collusion

JEL classification: L12, L14, L51, L96

*We are grateful for comments from Mark Armstrong, Matthew Bennet, Mats Bergman, Jonas Björnerstedt, Luke Froeb, Martin Perry and seminar participants at EARIE '06 in Amsterdam, IFN, Gothenburg University, Swedish Ministry of Industry, Employment and Communications, Swedish Regulatory Authority on Post and Telecommunications, Swedish Competition Authority, IIOC'07 in Savannah, SNEE'07 in Mölle, Norio VI in Stockholm, Tilburg University, EEA-ESEM'07 in Budapest, University of Bergen and the 2007 Breugel/IDEI conference on networks in Brussels and at the Royal Institute of Technology. We gratefully acknowledge financial support from the NET Institute (www.netinst.org), the Kauffman Foundation, the Swedish Competition Authority and Jan Wallander's Research Foundation.

†Gothenburg University and CEPR, Box 640, SE-405 30 Gothenburg, Sweden, E-mail: johan.stennek@economics.gu.se,

‡Research Institute of Industrial Economics (IFN), Box 55665, SE-102 15 Stockholm, Sweden, E-mail: thomas.tangeras@ifn.se

1 Introduction

The mobile telecommunications industry has changed dramatically during the last fifteen years, in terms of technology as well as market organization. While most national markets in the OECD area were monopolies in the late 1980:s, most of them now have three or more competing mobile networks. This development is summarized in Table 1.

1

Table 1: Competition in mobile phone infrastructure in 30 OECD countries 1989 - 2004.

	89	90	91	92	93	94	95	96	97	98	99	00	01	02	03	04
Mono-poly	24	23	23	18	15	11	11	6	3	0	0	0	0	0	0	0
Duo-poly	6	7	7	11	12	14	13	16	18	14	9	5	4	4	4	4
Trio-poly	0	0	0	1	3	4	4	5	4	8	13	15	14	10	12	13
4 or more	0	0	0	0	0	1	2	3	5	8	8	10	12	16	14	13

Source: OECD Communications Outlook 2005.

1.pdf

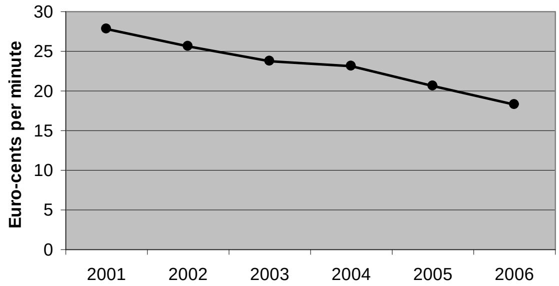
Also competition between existing wireless networks appears to have increased. Consumers can easily compare the networks' prices on special websites on the Internet. Consumers who want to switch networks may retain their phone number, so-called number-portability. Since the networks are required to interconnect, everybody can reach everybody else independently of to which network they subscribe. Universal service obligations reduce the differentiation across mobile networks.

The customers in Western Europe pay much lower prices today than five years ago. Figure 1 shows that the networks' mobile revenues, per originated minute, has fallen by a third between 2001 and 2006.

1

Figure 1: Mobile revenues per originated minute in 9 European countries 2001 - 2006

Adapted from *International Telecommunications Market* , Ofcom 2007.



2.pdf

With apparently fiercer competition today than ever before, we ask whether now is the time to deregulate mobile telecommunications.

Call price competition and its limits There used to be good reasons for regulation. While telecom networks compete for customers in the retail market, they cooperate in the wholesale market by providing each other call termination. Telecom companies have both a horizontal and a vertical relation, which distinguishes telecommunications from most other markets. By compensating a rival by means of an *access price* for every call terminated in the rival's network, the telecom company transfers some of the revenues it collects from its customers to the rival. Therefore, each network may have weak incentives to capture market shares by offering the customers lower call prices. In fact, competition can be so weak that the monopoly outcome prevails.

In essence, the competitive conditions are determined in the termination market. By agreeing to charge high (per-minute) access prices, the networks commit to artificially high marginal costs for calls. Therefore, they indirectly commit to charge high (per-minute) call prices from their customers. Since interconnection agreements are legally enforceable, there is no need for complicated punishment mechanisms to sustain the collusive price. Reduced network differentiation does not help since the networks can raise their access prices to offset the increase in competition.

This phenomenon – often called access price collusion – was described in the seminal papers by Armstrong (1998) and Laffont, Rey and Tirole (1998a and b), henceforth A-LRT. An important short-coming of their analysis is the assumption that there are only two networks in the market. While duopolies were common in the past, they have almost disappeared today. Since there is a crucial difference between duopolies and less concentrated markets, the old insights must be reexamined.

Access price competition and its limits With more than two networks in the market, one or more networks are left out of any bilateral access price negotiation.[1] The access prices might be expected to take the effects on both involved parties into account (as argued by Cave, 2004), but being largely negligent about the effects on the profits of third-party rivals. This makes a crucial difference because of the competitive externalities caused by access prices.

[1] Bilateral negotitations were common, for instance, in the Nordic countries prior to the NRAs' decisions of industry wide access price regulations.

We show that entry beyond duopoly stimulates a new form of competition – which we call access price competition. Reducing their common access price commits two networks to more aggressive call pricing in order to poach customers from third-party rivals. Since subscription demand is more elastic the more competitors there are, the networks have the incentive to reduce access prices with every new network entering the market.

However, we also show that the access charges paid to third-party rivals soften this access price competition considerably. With a larger number of networks, the fraction of calls terminated by third parties will be larger and more income will be drained from the two partners. Expressed differently, when the market is more fragmented, the share of calls subject to double margins is higher.

In the present model, both the competitive effect and the cost effect are proportional to the size of the customer base of the two networks and exactly offset each other at the monopoly call price. Therefore, our first main result is that market concentration has no effect on call prices, if few restrictions are placed on the networks' price setting strategies. The collusive outcome prevails independent of the market concentration.

Our analysis therefore suggests that the observed decline in call prices in Europe (see Figure 1, above) may partly result from the sector-specific regulation that these countries have put in place to support the increase in competition. It is not clear that the increase in competition would have led to lower prices absent regulation. But regulation comes with substantial costs.

Regulation and its limits The European framework for electronic communications is "market based," i.e. the obligations should be imposed only to the extent they are deemed essential for competition to work. The National Regulatory Authorities (NRAs) in the Member States therefore need to collect a great deal of information to implement the regulation. As a first step, they must assess the competitive situation in the relevant markets. This is an exercise full of conceptual and practical problems. For instance, it is not clear how to measure market power in the wholesale (i.e. termination) market where there are only a few companies acting as both sellers and buyers at the same time. At the next stage, if competition is deemed inadequate, appropriate remedies must be found. If the operators are required to charge access prices based on costs, a cost model must be constructed. This raises several questions. Should the networks be valued according to the historical cost or the replacement value? How should joint costs be assigned to call origination and call termination? Next, the NRAs must collect the relevant cost data and the networks will also be burdened with compiling this information.

A recent Swedish official report (SOU, 2006) describes excessive bureaucratic complications. Companies and regulators often have divergent views of most issues, from the competitive situation to the level of cost, and the decisions are usually appealed. Legal proceedings are known to drag on for years. In addition, companies have great difficulties in predicting the eventual decisions by regulatory agencies and courts. The official report speculated that this legal uncertainty reduces investments and limits the supply of services to consumers. This conclusion is supported in a report to the European Commission (London Economics, 2006) comparing investment in electronic communications throughout the EU. It concludes that regulatory uncertainty is sometimes detrimental to higher levels of investment. These conclusions should be read in the light of the empirical studies suggesting that telecom investments have a surprisingly large impact on economic growth. A third of the growth in the OECD area over a 20-year period can be attributed to the direct and indirect impact of telecommunications (Röller and Waverman, 2001; Waverman, Meschi and Fuss, 2005). Thus, the burden of telecom regulation may have significant effects on the economy in general.

In view of these problems, it is not surprising that sector-specific rules are typically regarded as intermediary solutions.[2] In the words of the European Commission (2005):

> "*Regulation is seen as essentially a temporary phenomenon, required to make the transition from the formerly monopolistic telecommunications industry to a fully functioning market system. ... [A]s the sector evolves, operators will increasingly build their own infrastructures and compete more effectively. ... [R]egulation can be rolled back, and competition law ... will replace sector-specific intervention.*"

Despite this long-term objective, the European Commission (2007) recently recommended continued regulation of mobile termination charges. This decision is consistent with our previous argument that competition and regulation should also play complementary roles in the long run. But the decision also raises new questions. Does there exist an alternative to the failures of the unconstrained market and the burdens of detailed regulation? Is it possible to devise *structural rules* for the networks' market behavior – rules that are simple and informationally undemanding, yet effective in preventing monopolization?

[2] See OECD (2006) and Kerf, Neto and Geradin (2005).

How to reduce prices with limited information We study a regulation – called *STR-regulation* – combining four well-known structural rules: (*i*) interconnection is mandatory; (*ii*) networks are not allowed to charge different prices for calls within the same network (on-net) and calls to other networks (off-net); (*iii*) access prices must be the same in both directions (reciprocal) and (*iv*) below a ceiling, which is independent of the cost level. The first three rules are primarily designed to prevent the exploitation of network externalities and to prevent that one network monopolizes the market (see Stennek and Tangerås, 2008). The fourth rule is designed to prevent access price collusion.

Our second main result is that increased competition (more firms and easier switching) leads to lower prices when the market is governed by *STR*-regulation. Most interestingly, the equilibrium call prices even decline towards the marginal cost as networks become increasingly closer substitutes. The networks are simply unable to offset the increased competition by jacking up the access price beyond a certain level. And when network differentiation is reduced even further, call prices must decline as well.

Remarkably, the exact level of the access price ceiling set by the regulator may not be very important. In fact, *any* access price ceiling is sufficient to push call prices down to the marginal cost when the networks are near-perfect substitutes. The access price ceiling may be set very high, but the ceiling may also be set low and even below cost. A special case of *STR*-regulation is the Bill-and-Keep regime, in which access price are zero.

More generally, our results show that the equilibrium call price becomes less responsive to changes in the access price ceiling as the networks become closer substitutes. This result has an important implication for regulatory policy. In particular, the regulator does not require detailed information about costs if the networks are less differentiated and consumer switching is easy.

Contributions From a more theoretical point of view, a first main contribution is the extension of the bilateral access pricing model from the duopoly case to a general n-network oligopoly. Independently of whether negotiations are bilateral or multilateral, the two-way structure of mobile telecommunications networks renders the condition that negotiated outcomes be immune to (possibly secret) bilateral deviations a natural requirement. The extension to general oligopoly is non-trivial because there will now be competition also in the access price negotiations. This analysis is complicated by the fact that the networks cannot observe the access prices negotiated between two other networks.

A second contribution is the derivation of a sufficient condition for the existence of a pure strategy call price equilibrium in a market with *STR*-regulation. Existence of a pure strategy equilibrium is far from trivial in two-way access situations because the profit functions of the networks are not necessarily concave in call prices. A network who finds a small price reduction unprofitable may still benefit from a large price cut that monopolizes the market and avoids all access prices. A-LRT's solutions, an access price close to the termination cost or highly differentiated networks are not useful for our purposes. We want to avoid the case in which regulators can tie access prices to cost or demand data, and we want to evaluate the performance of the regulation also when networks are close substitutes. Our analysis shows that an equilibrium exists, provided that call demand is sufficiently inelastic.

Discussion of the assumptions The main assumption of the model is that access prices are set in bilateral negotiations, as was e.g. the case in the Nordic countries before industry-wide regulation was introduced. It is the bilateral negotiations that open for access price competition. Much to our own surprise, our results demonstrate that the monopoly outcome nevertheless prevails. One may also show that multilateral negotiations result in the same outcome. Monopoly pricing is thus relatively robust and does not depend on the exact institutional framework.

We assume access prices to be secret. Commercial contracts are usually private information, even so in telecom. With public access prices, and bilateral negotiations, our conjecture is that the outcome of an unregulated market would be even worse. Firms would then have an incentive to increase access prices even above the monopoly level to induce a soft call price response from their competitors.

In other respects, however, the market for telecommunications is clearly much more complicated than the model we consider. To begin with, we have focused on mobile telephony and neglected fixed telephony. Mobile penetration exceeds 100% in many developed countries, the UK and Sweden being two examples. Mobile telephony is superior to fixed telephony in many dimensions, not only regarding mobility, but also in terms of available services, such as text and multimedia messaging. With falling prices we expect mobile telephony to become the dominating medium for voice telephony.

Many operators offer calling plans in which calls within the network (on-net calls) are cheaper than calls terminated in a competitor's network (off-net calls). We show in a companion paper (Stennek and Tangerås, 2008) that the network externalities arising from such price discrimination are strong enough to allow monopolization, even when networks are highly substitutable. A prerequisite for a competitive market with substitutable networks, therefore, is that price discrimination between off-net and on-net calls is banned. Such a ban is in place, for instance, in Estonia where the incumbent is prohibited from discriminating between fixed on-net and off-net calls. The present paper takes off from there.

In an extension of our analysis, we consider two-part tariffs. Access price collusion is still a problem, provided the networks are close substitutes and the fixed fee is constrained to be non-negative, e.g., due to arbitrage conditions. Then, the fixed fee is zero in equilibrium and the networks compete only in call prices. It is not uncommon for a network to be competing entirely in call prices despite not being compelled to do so. This is the case, for example, with prepaid cards. Jeon and Hurkens (2008) contain additional examples of networks competing in linear prices.

In our model all consumers have the same call demand and, therefore, the networks offer only one contract each. In reality most networks offer a menu of contracts, presumably to price discriminate between consumers with different call patterns. Dessein (2003) analyzes non-linear pricing in duopoly. We expect entry to both increase competition and to raise the perceived cost of calls even when consumers are heterogenous. Access price competition with heterogenous consumers is left for future research.

Many of our propositions are valid if call demand is sufficiently inelastic. We simulate the model to gauge the significance of the price elasticity of demand. Fortunately, the simulation shows that the elasticities can be set quite generously without jeopardizing the results.

Telecom operators have large fixed costs. Ramsey pricing is the appropriate welfare benchmark, not marginal cost pricing. One consequence is that it is not socially optimal to strive for "perfect competition", for example by removing all network differentiation. We leave the issue of optimal network differentiation for future research.

A main contribution of the paper is to show that market concentration has two opposing effects on prices. The reason is that firms in this type of market have both a horizontal and a vertical relation. A double-margins effect tends to increase prices, a competitive effect pulls in the other direction. We have shown that the double margins effect could be substantial; the two effects exactly cancel out in our model. In a more general, asymmetric model with vertical differentiation, imbalanced call pattern, price discrimination and so on, the two opposing effects would still be there. It is unclear which of the two would dominate. It is entirely possible that the cost effect would be most important; entry would then lead to higher prices.

Related Literature There is a small literature on two-way access with more than two networks. Calzada and Valletti (2008) focus on how access prices can be used to deter entry. Our focus is on post-entry access pricing. Their model is one of *multilateral* negotiations, whereas our emphasis is on competitive access prices and *bilateral* negotiations. Finally, Calzada and Valletti restrict their attention to the case when networks are highly differentiated. Our results hold for any degree of network substitutability (provided call demand is not too elastic). Gilo and Spiegel (2004) analyze the implications of transit when a third party seeks access to two interconnected networks, but they abstract from competition.

Doganoglu and Tauman (2002) contain results on network substitutability related to our results. They prove the existence of a unique symmetric call price equilibrium in a model with two networks for any degree of network substitutability, but under two restrictive assumptions: (i) the access price is above the termination cost but below the demand intercept, or (ii) a network's access price is a linear function of the competitor's call price. We extend their analysis by eliminating the restrictions on the access price and by allowing for general n-network oligopoly. Our analysis also brings out the crucial role played by the price elasticity of call demand. Jeon and Hurkens (2008) extend the analysis of call price contingent access price regulation to allow for general n-firm oligopoly.[3]

2 Access Price Competition and its Limits

There are two potential problems in an unregulated market. The first problem is that the networks may compete too much as a result of network externalities, especially when the networks are close substitutes. In our companion paper (Stennek and Tangerås, 2008) we show that only one network may then be viable in equilibrium absent regulation.[4] We also show that this problem

[3]Related work emphasizing the role of inelastic call demand includes Armstrong (2004). He studies competition between two networks for heterogeneous subscribers under the assumption of *perfectly* inelastic call demand.

Additional results on two-part tariffs are contained in Gans and King (2001), Dessein (2003), Jeon et al. (2004), Valletti and Cambini (2005), Berger (2005) and Calzada and Valletti (2008).

The basic duopoly framework of A-LRT has also been extended in other directions, such as the gradual evolution of market shares following entry from monopoly to duopoloy (De Bijl and Peitz, 2002), asymmetric networks (Carter and Wright, 2003) and investments (Valletti and Cambini, 2005).

[4]Calzada and Valletti (2008) consider limit access pricing with low network substitutability and a single access price governing all relations. Their conclusions depend crucially on whether the networks are assumed to compete in utilities or prices for subscribers.

can be avoided by simple regulations requiring that (*i*) all pairs of networks must interconnect, that (*ii*) all pairs of networks must set reciprocal access prices and that (*iii*) a network must charge the same price for off-net and on-net calls.

The second problem is that the networks may compete too little as a result of access price collusion. To focus the present paper on access price collusion, we assume that the above three regulations are in place. The question is whether access price collusion will occur in equilibrium and, if so, what additional regulations are necessary to eliminate such collusion.

The model is a four-stage game. The $n \geq 2$ networks first set access prices in pair-wise negotiations. The networks then set call prices simultaneously and independently. In the basic model each network $i, j, k \in N$ sets a call price, $p_i \geq 0$. Our analysis of subscription fees is discussed in section 4.1. The consumers observe the call prices and subscribe to a network. Finally, the consumers decide how many calls to make.

Call Demand In the fourth stage, consumers make their calls. In addition to their mobile phones, the consumers have access to an alternative, but inferior means of communication such as a public phone which does not require a subscription ("the outside option"). The price of alternative calls is exogenously set at v, where v includes both the price and any additional disutility from using the outside option. The outside option guarantees that call prices are bounded above when demand is very inelastic.

Every consumer in network i makes q_i mobile calls at (the non-discriminatory) price p_i or q_0 calls with the outside option at price v to every other consumer. It is thus assumed that the consumers value calls the same way and that they are equally good friends with everybody else, giving rise to a so-called balanced call pattern. Utility is quadratic in the number of calls, i.e.

$$U(q_i, q_0) = \left(q_i + q_0 - \frac{1}{2}(q_i + q_0)^2\right)\frac{1}{\varepsilon} - p_i q_i - v q_0.$$

Consequently, the demand for mobile calls is linear and equal to $D(p_i) = 1 - \varepsilon p_i$ for $p_i \leq v$ and zero for $p_i > v$. The price-elasticity of demand is $\eta(p_i) = \varepsilon p_i / (1 - \varepsilon p_i) \geq 0$. Since the elasticity is increasing in ε, we will refer to a low ε as a low elasticity of demand.

Subscription Demand In stage three, consumers subscribe to one of the mobile networks. The outside option does not require a subscription. But consumers base their choice of mobile network on the net benefits of the networks over the outside option. The indirect utility of the outside option is $U(0, D(v)) = (1-\varepsilon v)^2 / 2\varepsilon$, and the net benefit of network i is

$$V(p_i) = U(D(p_i), 0) - U(0, D(v)) = (v - p_i)\left(1 - \varepsilon \tfrac{v+p_i}{2}\right), \tag{1}$$

for $p_i \leq v$ and zero for all prices above v. The price elasticity of the net benefit function is $\sigma_{vp}(p_i) = -(\partial V(p_i)/\partial p_i)(p_i/V(p_i)) = D(p_i)\, p_i/V(p_i)$.

To describe network choice, we employ a random utility model. The basic assumption is that the networks are horizontally differentiated, e.g. in terms of customer management services, but that these differences does not affect the quality of calls. Network i's market share is equal to

$$S_i = \frac{V(p_i)^{\frac{1}{\gamma}}}{\sum_{j=1}^{n} V(p_j)^{\frac{1}{\gamma}}}, \tag{2}$$

when at least one network charges a price strictly below v. To derive equation (2), assume that a subscriber selects network i over j only if $V(p_i)\exp\{\delta_i\} \geq V(p_j)\exp\{\delta_j\}$, where δ_i and δ_j are two double exponentially distributed utility terms, independent across subscribers and networks.[5]

The price-elasticity of the demand for subscriptions is

$$\sigma(p, \gamma, n) = -\left.\frac{\partial S_i}{\partial p_i}\frac{p_i}{S_i}\right|_{p_i = p_j = p \forall p_j \in N \backslash i} = \frac{1}{\gamma}\frac{n-1}{n}\sigma_{vp}(p) \tag{3}$$

when all networks charge the same price p. If γ is close to zero, subscription demand is very elastic and the network with the lowest price captures most of the subscribers. If γ is very large, subscription demand is very inelastic and the networks divide the market approximately equally, independent of their prices. The network substitutability parameter γ may capture many different factors such as customer heterogeneity in combination with product differentiation, switching costs, and bounded rationality.

The advantage of the random utility model over the commonly used Hotelling model is that the market is fully covered at all prices with positive demand, independently of the degree of network substitutability. The advantage of using the net benefit rather than the indirect utility function is that a network failing to provide subscribers with any net benefit over the outside option ($p_i \geq v$) will have a zero market share, if any competitor offers a positive net benefit ($p_j < v$). This implies that no network will raise its price above v to rely solely on incoming calls for its profit.

Call prices In stage two, the networks set (non-negative) call prices. With mandatory interconnection, reciprocal access prices and a ban on price dis-

[5] See, e.g., Doganoglu and Tauman (2002) and Anderson, de Palma and Thisse (1992).

crimination, the profit of network i is given by

$$\pi_i = S_i \left[S_i (p_i - c) D(p_i) + \sum_{k \neq i} S_k (p_i - a_{ik} - c_o) D(p_i) + \sum_{k \neq i} S_k (a_{ik} - c_t) D(p_k) \right], \tag{4}$$

where c_t is the marginal cost of call termination, c_o is the marginal cost of call origination, c is the total marginal cost of a completed call $c_t + c_o$, and a_{ik} is the reciprocal access price between networks i and k. We assume that the marginal cost is lower than the willingness to pay for the first unit, i.e. $c < v < \varepsilon^{-1}$. The term in brackets is the profit per subscriber. The first term is the profit from on-net calls and the second term the profit from outgoing off-net calls. The third term is the profit on incoming off-net calls.

Consistent with actual practice, we assume that networks can only observe their own access prices when setting the call price. Since each network cannot observe the access prices between other networks, the game is one of imperfect information. Consequently, perfect Bayesian equilibrium is an appropriate solution concept. We assume that the networks take all unobserved access prices to be at their equilibrium levels (that is, they have passive beliefs, see Rubinstein and Wolinsky, 1990, and McAfee and Schwartz, 1994). Of course, in a repeated game setting, call prices may be taken as signals of access prices, but we leave this signalling issue for future research.

Access price In the first stage, the networks negotiate access prices. For simplicity, we assume that each negotiation is efficient from the point of view of the two networks. Formally, every pair of networks has delegated its choice of the access price to a separate agent. Thus, an agent called A_{ij} sets the reciprocal access price $a_{ij} = a_{ji}$ for traffic between networks i and j. The objective of agent A_{ij} is to maximize the sum of expected profits of networks i and j. There are $n(n-1)/2$ agents and they set the access prices simultaneously and independently of each other.[6]

To guarantee that a network will not have an incentive to make phony calls to the other network, the marginal cost of off-net calls must be non-negative, i.e. $a_{ij} \geq -c_o$ for all pair of networks.

Definition of Equilibrium A perfect Bayesian equilibrium consists of $n(n-1)/2$ access prices a^*_{ij}, n call price mappings $p^*_i(a_i)$ where a_i is the vector of access prices network i has agreed with other networks, and n

[6] This is essentially the Nash-equilibrium-in-Nash-bargaining-solutions approach, introduced by Davidson (1988) and Horn and Wolinsky (1988). See Björnerstedt and Stennek (2007) for a non-cooperative foundation for this approach.

belief mappings $\mu_i^*(a_{-i}|a_i)$ where a_{-i} are all the access prices not known by network i. An equilibrium has three defining characteristics:

First, network i sets the call price $p_i^*(a_i)$, which maximizes the expected profit

$$\pi_i^e(p_i, a_i) = \int \pi_i\left(p_i, p_{-i}^*(a), a_i\right) \mu_i^*(a_{-i}|a_i)\, da_{-i}$$

where $p_{-i}^*(a) = (p_1^*(a_1), .., p_{i-1}^*(a_{i-1}), p_{i+1}^*(a_{i+1}), .., p_n^*(a_n))$ are the equilibrium call price mappings of i's competitors, and where the ex post profit of network i is given by equation (4).

Second, agent A_{ij} sets the reciprocal access price $a_{ij}^* = a_{ji}^*$, which maximizes the sum of the expected profits of networks i and j,

$$\pi_i^e\left(p_i^*\left(a_{ij}, a_{i-j}^*\right), \left(a_{ij}, a_{i-j}^*\right)\right) + \pi_j^e\left(p_j^*\left(a_{ji}, a_{j-i}^*\right), \left(a_{ji}, a_{j-i}^*\right)\right),$$

taking all other access prices $a_{i-j}^* = a_i^* \backslash a_{ij}^*$ and $a_{j-i}^* = a_j^* \backslash a_{ji}^*$ as given.

It is easy to demonstrate that the access price only indirectly affects the sum of profits via its effect on equilibrium call prices. One might say that the access price constitutes an instrument of collusion. If possible, the access price will be set to implement the monopoly price, that is $p_i = p^m$. The monopoly price p^m is characterized by the Lerner formula $(p^m - c)/p^m = 1/\eta(p^m)$ if $p^m \leq v$ and $p^m = v$ otherwise.

Third, there is Bayesian updating. If a network observes all its access prices to be at their equilibrium levels, the network will believe that also the access prices it cannot observe are at their equilibrium levels, i.e., $\mu_i^*\left(a_{-i}^*|a_i^*\right) = \infty$. We invoke passive beliefs off the equilibrium path – i.e., a network continues to believe all other access prices to be at their equilibrium levels, also following a deviation; that is, $\mu_i^*\left(a_{-i}^*|a_i\right) = \infty$ also if $a_i \neq a_i^*$.

We also restrict the attention to semi-symmetric equilibria, i.e. equilibria prescribing all access prices to be the same and equal to a^* (which is now a scalar).

2.1 Call Prices

This section derives the equilibrium in call prices, following universal agreement on the same access price $a \geq -c_o$. It is instructive to study the derivative of network i's profit with respect to its own call price, for simplicity assuming all other networks to charge the same price p_j, i.e.

$$\begin{aligned} \frac{\partial \pi_i}{\partial p_i} &= S_i D(p_i) \\ &+ \frac{\partial S_i}{\partial p_i}\left[(p_i - c)D(p_i) + (1 - S_i)(a - c_t)(D(p_j) - D(p_i))\right] \\ &+ S_i (p_i - c - (1 - S_i)(a - c_t)) D'(p_i) \\ &+ S_i \frac{\partial S_i}{\partial p_i}(a - c_t)\left[D(p_i) - D(p_j)\right]. \end{aligned} \tag{5}$$

The first three lines represent the standard trade-off between price and sales; an increased price leads to a higher mark-up on every call, but reduces the customer base and reduces call demand. The fourth line represents two "composition effects" resulting from the subscribers switching to the competing network as a result of a price increase: access costs are increased, but so are access revenues. The composition effect may be positive or negative, and will be discussed more below.

Lemma 1 *Following universal agreement on the same access price $a \geq -c_o$, there exists at most one equilibrium. If it exists, it is characterized by the call price $p^*(\gamma, a, n) \in [c, v)$, implicitly given by*

$$\frac{p^* - c}{p^*} = \frac{1}{\eta(p^*) + \sigma(p^*, \gamma, n)}\left[1 + \frac{n-1}{n}\frac{a - c_t}{p^*}\eta(p^*)\right]. \tag{6}$$

The price p^* is increasing in the access price and in the degree of network differentiation. A larger number of networks leads to reduced call prices if, and only if, the access price is sufficiently low relative to network substitutability to ensure an equilibrium price below the monopoly level.

The proof is in Appendix A.1.

The interesting thing to note here is that market concentration has an ambiguous effect on call prices. Upon inspection of (6), we see that market concentration affects the call price through two channels. The elasticity of subscriber demand $\sigma(p^*, \gamma, n)$ increases the more fragmented (i.e., less concentrated) the market is, which tends to push down the call price. This is the standard *competition effect* of concentration, affecting prices in most industries. In telecom, the networks' effective marginal cost also increases with fragmentation. The effective marginal cost is defined as $C(a, n) = c + \frac{n-1}{n}(a - c_t)$, taking into account that a share of calls are terminated off-net and are therefore subject to an access price mark-up. This *double-margins effect* pulls the call price in the opposite direction. Either effect may dominate, but if the access price is sufficiently low the competition effect is stronger.

Even if the number of firms grows without any bound, the price will not necessarily be pushed down to the marginal cost. This is true also when networks are close substitutes and the access price is low. The reason is that the access price markup will increase in importance, the more fragmented the market becomes. (Another reason is that every single network has a fraction of loyal customers since every network offers its own variety to the market. The second mechanism is a well-known effect of the random utility model, cf. Anderson, de Palma and Thisse, 1992.)

2.2 Access prices

In the first stage all network pairs bargain over access prices. It is instructive, however, to think about networks i and j's (formally, agent A_{ij}) choice of the access price a_{ij} as a choice of their call prices. The only effect of a deviation in the access price from a is to induce the two call prices p_i and p_j to deviate from p^*. In equilibrium all other network-pairs stick to the recommended access price a and call price p^* even if i and j would deviate, since the deviation is unobservable to outsiders. Moreover, it is easy to see that $\pi_i^e + \pi_j^e$ only depends on their common access price a_{ij} indirectly, through the effect on p_i and p_j.

We do not need to define threat points for the negotiations. Since the access price is reciprocal and the two firms are symmetric, there is no room for conflicts over the acces price within any negotiation.

A marginal deviation in the access price to induce a marginal deviation in i's call price from p^* has the following effect on the expected joint profit

$$\begin{array}{rcl} \frac{\partial(\pi_i^e+\pi_j^e)}{\partial p_i} & = & \frac{1}{n} D\left(p^*\right) \\ & + & \frac{1}{n}\left(p^* - \left[C\left(a,n\right) - \frac{1}{n}\left(a - c_t\right)\right]\right) D'\left(p^*\right) \\ & + & \left(\frac{\partial S_i}{\partial p_i} + \frac{\partial S_j}{\partial p_i}\right)\left(p^* - c\right) D\left(p^*\right), \end{array} \tag{7}$$

where $C\left(a,n\right) - \frac{1}{n}\left(a - c_t\right)$ is the joint marginal cost of the two networks. Note that the maximization problem facing the agent is similar to that facing an individual network. The optimal price is a trade-off between a higher mark-up (first row) and lower call demand (second row) and smaller customer bases (third row). The difference is that the network pair internalizes the effects of a change in p_i on the rival-cum-partner's profit π_j^e. A higher p_i increases network j's profit as its customer base increases by $\partial S_j / \partial p_i$. This is the standard cartel motive and tends to raise the agreed price above the individually optimal level p^*. On the other hand, a higher p_i reduces network j's access revenues by $(a - c_t) D'(p^*) n^{-2}$ as i's customers make fewer calls. Internalization of downstream effects is the standard motive for vertical integration and pulls in favor of lower call prices.

To determine the equilibrium access and call price, use full market coverage ($\partial S_j / \partial p_i = -(n-1)^{-1} \partial S_i / \partial p_i$) and the equilibrium relation (6) to get

$$\frac{\partial(\pi_i^e + \pi_j^e)}{\partial p_i} = -\left(\frac{p^* - c}{p^*} - \frac{1}{\eta(p^*)}\right) \frac{\eta(p^*) D(p^*)}{n(n-1)}.$$

Clearly, only $p^* = p^m$ could be a symmetric equilibrium. At any other call price, any two networks would deviate by altering their joint access price. But if the access price induces the monopoly call price, i.e.

$$a^m(n) = c_t + \frac{n}{n-1} \frac{p^m \sigma(p^m, \gamma, n)}{\eta(p^m)^2} = c_t + \frac{1}{\gamma} \frac{p^m \sigma_{vp}(p^m)}{\eta(p^m)^2} = \alpha^m, \tag{8}$$

no pair has any local incentive to deviate. To see that $a^m(n)$ implements the monopoly call price, substitute the Lerner rule into the equilibrium relation (6), and use the expression for $\sigma(p^m, \gamma, n)$ in (3).[7]

Proposition 1 *No single network can profit from a unilateral deviation from the monopoly price p^m, and no pair of networks can jointly profit from a bilateral deviation from α^m, to induce a deviation in their call prices from the monopoly level, provided that the networks are sufficiently differentiated.*

The proof is in Appendix A.2.

The monopoly outcome prevails independent of market concentration. The proposition extends A-LRT's monopolization result to markets with more than two networks. Why does not competition work, despite decentralized negotiations? On the one hand, fragmentation strengthens access price competition, since a smaller share of the (horizontal) competitive externalities between networks can be internalized in any bilateral negotiation. However, also the share of the (vertical) double-margins externality that can be internalized in any bilateral negotiation is smaller. In general, either effect may dominate, but the two effects will cancel in any symmetric and fully covered market with a balanced call pattern.[8]

[7] Note that the collusive access price is independent of the market structure. This result is particular to our specification of subscription demand.

[8] A proof of the general claim is available on request.

3 Regulation

In the present section we study the effect of imposing a price cap $\overline{a} \geq -c_o$, in addition to the first three STR rules. The question is if competition will have an effect on call prices under STR-regulation. By increased competition we mean lower market concentration or reduced network differentiation. First, we must discuss the existence of equilibrium, however.

3.1 A New Condition for the Existence of Equilibrium

A pure strategy equilibrium may fail to exist when networks are close substitutes and when the access price cap is not necessarily close to the marginal cost of call termination. When networks are close substitutes, they will have to set a high access price so as to deter marginal price cuts. However, a large price cut may still be profitable as it allows the deviating network to seize nearly all consumers, thereby avoiding the access costs. A price equal to the marginal cost cannot be an equilibrium either, because a network can change its price a little and earn a positive profit. Equation (5) shows that a small increase in the price above the marginal cost would be profitable if call demand were inelastic (since $a \geq -c_o$).

The root of the existence problem is the traffic flowing between networks, represented by the final term in (5), i.e.

$$S_i \tfrac{\partial S_i}{\partial p_i} \left(a - c_t\right) \left[D\left(p_i\right) - D\left(p_j\right)\right].$$

When network i increases the call price, its remaining consumers will make more off-net calls than before since a larger fraction of subscribers now belong to other networks, a cost effect. On the other hand, the access revenues also go up since more calls now flow into i's network from the outside. This composition effect can be positive or negative, depending on the call price differences across networks. If i charges a comparatively low (high) call price the cost (access revenue) effect dominates.

Existence is restored by minimizing the composition effect. The two solutions devised by A-LRT are not useful in our context. Their first solution is to assume that the networks are poor substitutes, i.e. $\partial S_i/\partial p_i \approx 0$. Then, the market shares are insensitive to price changes and the composition effect vanishes. The networks are essentially local monopolists, and set prices with an effective marginal cost equal to $c + S_j\left(a - c_t\right)$. Their second solution is to assume that the access price is close to the termination cost, i.e. $a \approx c_t$. Then, the traffic between the networks is of minor importance to profits and the composition effect vanishes.

Our solution to the existence problem is instead to require call demand to be inelastic.

Lemma 2 *Consider a market under* STR*, including a cap on access prices. There exists a unique and symmetric pure strategy equilibrium in call prices if demand is sufficiently inelastic (that is, if ε sufficiently low). The equilibrium call price is characterized in equation (6). The equilibrium access price is the monopoly access price or, if lower, the access price ceiling, that is, $a^* = \min\{\alpha^m, \overline{a}\}$.*

The proof is in Appendix A.3.

A low elasticity of demand has a similar effect on the profit function as an access price close to the termination cost. The idea is that the traffic between the networks is of minor importance to profits when the difference in demand is small. The difference in equilibrium demand is indeed small when demand is inelastic since the equilibrium prices are contained in $[c, v]$. As $|D(p_i) - D(p_j)| \leq \varepsilon (v - c)$, the difference drops towards zero and the composition effect vanishes as demand becomes more inelastic.

3.2 Effect of Competition under Regulation

The rest of this section describes the effect of increased competition when *STR* (including the price cap) is in place. First we discuss network differentiation, then market concentration.

Network Differentiation The closer substitutes are the networks, the higher is the monopoly access price α^m, defined in equation (8). The necessary access price even increases without any bound (i.e. $\alpha^m \to \infty$), as the networks become closer to being perfect substitutes ($\gamma \to 0$). When networks are close substitutes, any cap on access prices must consequently be binding. In fact, by limiting the networks' ability to offset the competitive pressure by charging high access prices, the networks are forced to marginal cost pricing when networks are close substitutes, i.e. $\lim_{\gamma \to 0} p^*(\gamma, \overline{a}, n) \to c$.

Proposition 2 STR *induces the networks to charge call prices as close to the marginal cost as is desired, independent of the access price ceiling, provided that the networks are sufficiently close substitutes, and assuming that demand is sufficiently inelastic.*

The proof is in Appendix A.4.

Note that it is the combination of *STR* and competition (i.e. having more than one network and high network substitutability) that drives down call prices. Network substitutability is crucial for efficiency. If the networks are poor substitutes ($\gamma \to \infty$) and the access price ceiling is above marginal cost ($\overline{a} > c_t$), then *STR* induces the networks to set the access price close to the marginal cost of termination ($a \to c_t$) and charge the monopoly price. The crucial role of having a second network is evident from inspecting the construction of the *STR*-rules. *STR* is a complement to but cannot replace competition. Reversely, increased network substitutability and a second network will not have any effect on call prices, unless the access price ceiling $\overline{a}$ is binding. That is, competition cannot replace regulation.

As the access price ceiling can be set arbitrarily high, the informational requirement is minimal. The only restriction is that $\overline{a} \geq -c_o$. In particular, *STR* combined with a Bill-and-Keep regime ($\overline{a} = 0$) is a possible solution. The level of the access price should have no direct bearing on investment incentives because equilibrium access revenues are always zero under a balanced call pattern. Investment costs must necessarily be covered in the retail market.

Market Concentration Market concentration has an effect on call prices if, and only if, the access price ceiling is binding, $\alpha^m > \overline{a}$. There are two ways of ensuring a binding access price ceiling. The first is to set the ceiling sufficiently low, the second is to reduce network differentiation to increase α^m. In sum:

Proposition 3 *Market fragmentation leads to (weakly) lower call prices under* STR. *If the access price ceiling is too generous and the networks too differentiated, the monopoly price prevails independently of the number of networks. If the access price ceiling is sufficiently low or networks are sufficiently close substitutes, call prices go down when the number of networks goes up.*

This result demonstrates that competition is a complement to regulation and not a substitute, since market concentration has an effect only when the access price ceiling is binding.

It is possible that our model underestimates the effect of market concentration on call price competition, however. A new network is like a new variety in this model, and the product space is never overcrowded. The presence of loyal subscribers tends to limit the intensity of call price competition as new networks enter the market. The effect of a crowded product space can be incorporated into the model by considering a more general network differentiation parameter $\gamma(n, \theta)$, where θ now signifies switching costs etc., and where $\partial\gamma/\partial n < 0$. In this case, competition dominates double margins even at the monopoly price. Hence, unilateral deviations from the monopoly price become increasingly profitable as the market becomes less concentrated, which tends to drive the equilibrium access price $a^m(n)$ up to the ceiling. If $\lim_{n\to\infty}\gamma(n) = 0$, additional reductions in market concentration would eventually drive call prices down to the marginal cost.

However, the substantial costs of building new networks, the technical limitations to unbounded entry and the anti-competitive effects of access pricing, lead us to question whether reduced network differentiation is not a more fruitful approach than entry in achieving a competitive environment in telecommunications.

Taken together, Propositions 1 - 3 show that the observed increase in competition, both in terms of entry and easier switching, may be part of the explanation for the observed fall in call prices in western Europe, but it may be mistake to remove regulation altogether.

3.3 Structural versus Cost-Based Regulation

A possible cost-based regulation (henceforth "*CBR*") is to peg access prices down to the marginal cost of call termination, i.e. $\overline{a}^{CBR} = c_t$. When the networks have the same costs, *CBR* leads to reciprocal access prices. The networks will also face the same marginal costs for on-net and off-net calls, and Ramsey pricing prescribes the same prices for off- and on-net calls. There is no price discrimination and no tariff-mediated network externalities. In equilibrium, all networks would set the same call price, characterized by

$$\frac{p^{CBR}-c}{p^{CBR}} = \frac{1}{\eta\left(p^{CBR}\right)+\sigma\left(p^{CBR},\gamma,n\right)}.$$

Since p^* is increasing in the access price, it is clear that STR induces a higher price than CBR, whenever $\overline{a} > \overline{a}^{CBR} = c_t$.

One way of viewing STR is as a slight weakening of CBR, preserving the reciprocity of access prices and the absence of call price discrimination, but disconnecting the access price ceiling from the production cost. The advantage of STR is that it does not require any cost information, and the advantage of *CBR* appears to be a lower call price. Note, however, that STR provides stronger incentives for cost containment than *CBR* by making the networks residual claimants on any efforts to reduce the termination costs. Taking these incentives into account, it may well be the case that $p^{CBR} > p^{STR}$.[9]

[9] A more detailed comparison of STR and *CBR* in terms of dynamic efficiency is left to future research.

The relative efficiency of CBR is small in situations of high and low network substitutability. If networks are very poor substitutes, both policies perform equally poorly, thereby inducing the monopoly price ($\lim_{\gamma\to\infty} p^{CBR} = \lim_{\gamma\to\infty} p^* = p^m$). If networks are very close substitutes, both policies perform equally well, inducing marginal cost pricing. To conclude, STR is a substitute for cost-based regulation, and STR is likely to perform better whenever information is sparse or investment incentives important.

4 Extensions

4.1 Two-Part Tariffs

Telecom networks typically use two-part tariffs, with a call price p_i and a subscription fee F_i. It has been argued that access price collusion may then not be a problem. This conclusion is based on Laffont, Rey and Tirole's (1998a-b) result that networks using two-part tariffs do not have any incentive to raise their access prices above the cost of termination. Assuming the networks to be poor substitutes, the networks set the call price equal to the effective marginal cost and use the subscription fee to extract the resulting consumer surplus. They set the access price equal to the marginal cost, so as to avoid distortions in the call price, since the maximization of the industry profits is the same as the maximization of the social surplus (when $n = 2$).

We show that the effect of two-part tariffs to a large extent depends on network differentiation. (The formal analysis is relegated to Appendix A.5.) For instance, if the networks are nearly perfect substitutes, the subscription fee is competed down to zero and the networks barely break even. If arbitrage possibilities prevent the networks from setting negative subscription fees, they can profit from setting an access price above the termination cost, since they would then have a positive margin on calls.

In fact, the possibility of two-part tariffs does not affect the equilibrium, provided that the subscription fees must be non-negative and the networks are sufficiently close substitutes. In any symmetric equilibrium, with two networks, for example, the access price then is equal to $\max\{\overline{a},\ a^m\}$, the call price is equal to p^*, and the subscription fee is set to zero.

In reality, the true arbitrage condition may be somewhat below zero in case the networks can frame a negative fee as a partial subsidy of handsets, but this is of no consequence for our results as long as the subscription subsidy cannot be too large.

4.2 Anti-competitive Arbitrage

At the turn of the millennium, the main Swedish mobile carrier Telia launched a campaign offering late night calls at SEK .75 per minute. As termination charges were well above that level, an arbitrage opportunity on off-net calls arose. For example, the access price charged by the main competitor Comviq at the time was SEK 1.60, which opened a per minute arbitrage window of SEK .85=1.60-.75 less the marginal cost of termination. A small company called Faxback identified the arbitrage opportunity and struck a deal with Comviq. Comviq agreed to pay Faxback SEK 1.20 per minute for all calls made by Faxback's Telia subscriptions to a certain phone number in Comviq's network. Soon thereafter, Faxback connected a large set of Telia mobile phones to its computers and started making eight-hour nightly nonsense calls.[10] After a while, Telia's computerized intelligence system discovered the plot. The campaign was eventually withdrawn, and Faxback was sued for fraudulent behavior. In a recent verdict, Faxback was freed by the Stockholm City court which deemed the arbitrage legal.

The interesting point is that arbitrage of the Faxback type is anti-competi tive. Arbitrage effectively eliminates the incentive to undercut the competitor by establishing a call price floor. In the notation of the present paper, arbitrage arises whenever $a-p > c_t$. The no-arbitrage condition is $p \geq a-c_t$. Assume that the access price ceiling is generous and call demand not too inelastic, i.e. $p^m \leq \overline{a} - c_t$, but subscription demand is very elastic so that $\alpha^m > \overline{a}$. If arbitrage were infeasible, the equilibrium access price would be $a^* = \overline{a}$ and the call price $p^*(\gamma, \overline{a}, n) < p^m$. However, with arbitrage, the access price $a = p^m + c_t \leq \overline{a}$ is sufficient to sustain p^m as the equilibrium.

These results suggest that policy makers should take steps to prevent arbitrage. Arbitrage would be eliminated under a Bill-and-Keep regime, since arbitrage would then imply a negative call price, $p < -c_t$. Second, Faxback-arbitrage is only feasible if agreements of the Comviq-type are legal. Third, direct arbitrage is feasible only if networks are allowed to operate affiliates with a significant amount of subscriptions in a competitor's network.

4.3 More on Call Demand Elasticity

The practical relevance of our proposed policy hinges on the sensitivity of our results to the price elasticity of demand. If it were the case that equilibrium could only be guaranteed for unrealistically low demand elasticities, *STR* would not produce call prices close to the marginal cost, but most likely a

[10] The only sound heard during the calls was the whistling rooster from the Robin Hood movies.

situation with fluctuating prices. Our proof of existence suggests that the upper bound on ε is reduced as γ is pushed towards zero.

To gauge the significance of the price elasticity of demand, we use a numerical simulation. Fortunately, the simulation indicates that the elasticities can be set quite generously. To calibrate the model, we look at the situation in the Swedish market prior to the imposition of access price caps in the late 1990's. At the time, the call price was approximately $p = 5$ SEK per minute (divide by 10 to translate into Euro). Estimates of the short-term marginal cost per minute were not too far from $c = 0.1$, and it may be assumed that call termination and call origination were equally expensive, i.e. $c_o = c_t = 0.05$. Absent regulation, the observed price was probably close to its monopoly level, and the negotiated access price, which was around $a = 3$ per minute at the time, was sufficient to sustain collusion.

Using the Lerner rule, $(5 - 0.1)/5 = \eta(5)^{-1}$, we get an equilibrium elasticity of call demand around $\eta^m = 1$. This elasticity was elevated by the lack of competition, and the deep elasticity parameter can be calibrated to be around $\varepsilon = 0.1$. Substituting the prices, estimated costs and $\eta^m = 1$ into the pricing equation (6), i.e.

$$\frac{5 - 0.1}{5} = \frac{1}{1 + \sigma^m}\left(1 + \frac{1}{2}\frac{3 - 0.05}{5}1\right),$$

we may infer the approximate subscriber elasticity to be $\sigma^m = 0.3$. Assuming the price of a pay-phone call (including the disutility of using such a device) to be $v = 9$, the deep network substitutability parameter can be calibrated to be around $\gamma = 3.5$, recalling $\sigma^m = D(p^m)/2\gamma V(p^m)$.

The question is now whether this situation can be construed as an equilibrium of the model. The answer is yes: substituting the observed and inferred numerical values into the profit function, the profit function is nicely concave whenever the competitor charges the monopoly price.

The next issue is to study the effect of a policy shift in line with *STR*. Consider first a reduction in γ, but without any access price ceiling. For instance, increasing network substitutability to $\sigma' = 1$ (approximately corresponding to $\gamma' = 1$) would imply that the networks have to set an access price of more than $a' = 10$ to induce the monopoly price.

Consider a further increase of the network substitutability, say to $\gamma'' = 0.5$ (which would correspond to $\sigma = 2$ at a symmetric monopoly price), but now assume that the regulator imposes a price ceiling of $\overline{a} = 15$. At the maximal access price, the monopoly price can no longer be sustained as an equilibrium and the call price falls to $p'' = 4.4$. Successive increases in network substitutability, first to $\gamma''' = 0.1$ and then to $\gamma'''' = 0.05$, lead to successive falls in the equilibrium price, first to $p''' = 1.5$ and then to $p'''' = 0.9$. All cases are summarized in the following table.

Case	$\overline{a}$	a	γ	p	$\sigma(p)$	$\eta(p)$
Original situation	-	3	3.5	5	0.3	1
Experiment 1	-	10	1	5	1	1
Experiment 2	15	15	0.5	4.4	1.6	0.8
Experiment 3	15	15	0.1	1.5	1.8	0.2
Experiment 4	15	15	0.05	0.9	2	0.1

Note that the equilibrium prevails under the successive reductions of γ, despite keeping $\varepsilon = 0.1$ fixed. The reduction of the price elasticity of call demand, η, is due to the reduced price. These demand elasticities are quite low, but broadly consistent with econometric evidence (approximately 0.5 on US data, see e.g. Hausman, 2002).

As is evident from Figure 2, the profit function becomes increasingly peaked around the equilibrium price as γ decreases. We anticipate further reductions in γ to have no effect on the existence of equilibrium. Hence, for plausible initial values of γ and ε, and with a very generous access price ceiling $\overline{a} = 15$, changes in the degree of network substitutability have no effect on the existence of equilibrium.

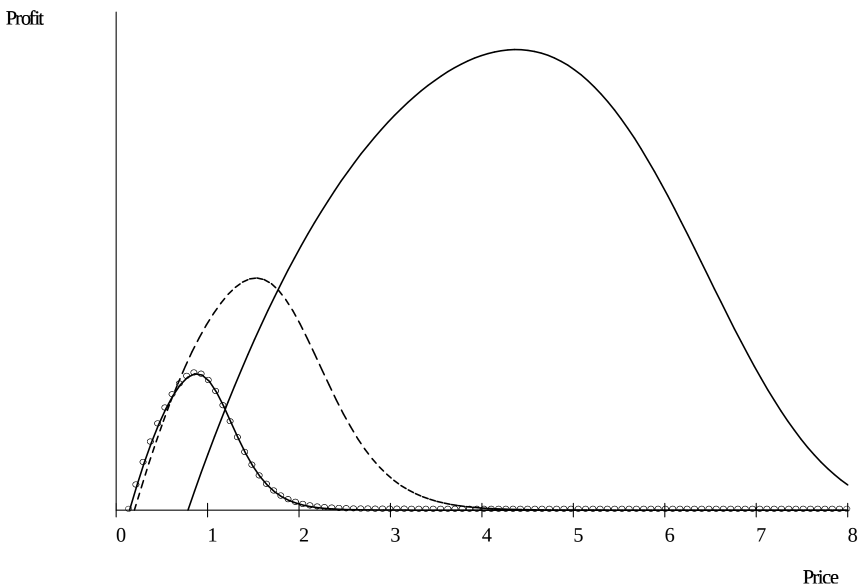


Figure 2: Experiments 2-4

The source of the existence problem is that a network may find it profitable to significantly cut its price to corner the market. However, at high degrees of network substitutability, the price charged by the competitor will be close to the marginal cost; thus, the possibility of undercutting the rival diminishes as networks become highly substitutable. Our simulations indicate that at realistic values, the strategic effect working through the competitor's price is sufficient to render price cuts unprofitable, even with highly substitutable networks.

5 Concluding Remarks

The analysis brings out a complementarity between regulation and competition. Without regulation, access price collusion leads to monopolization, independent of the number of networks. While most people seem to agree that a sector-specific regulation is necessary during the transition from monopoly to competition, our work shows that regulation may be required also in the long run, when there are several competing networks in the market, they have all built up sizable customer bases, and when access price collusion may be a more acute problem than price squeezes.[11] This stands in contrast to the commonly held view (cf. Armstrong and Sappington, 2006; Vogelsang, 2006) that competition and regulation are substitutes in the long run.[12]

Since competitive problems are ubiquitous in the mobile market, independent of market concentration and network differentiation, it appears reasonable to base the regulation on the legal presumption that competition does not work. The regulatory obligations may be universal. We also show that the required obligations may be limited to defining structural rules (STR-regulation) for the networks' pricing rather than to setting access price ceilings close to some measure of cost. The necessary information may therefore be minimal and the problems of cost-containment avoided. Most importantly, STR-regulation is transparent to the industry.

Finally, we show that the call price competition and access price competition induced by entry are partly offset by an increase in the networks' effective cost, as a larger share of calls are terminated in the rivals' networks and therefore subject to the access price markup. Considering the substantial costs of

[11] For more on price squeezes, see Bouckaert and Verboven (2004) and Valletti (2003).

[12] There is also some anecdotal evidence that sector-specific regulation may contribute to lower prices. In 2001, mobile call charges were much higher (more than twice as high) in New Zealand which had until then almost exclusively relied on antitrust rules, than in the UK and in the US which relied on sector-specific rules. Australia and Chile with models somewhere between the two extremes also had an intermediate price level (Kerf et al., 2005).

building new networks, our results suggest that other methods for reducing call prices may be preferred. We have demonstrated that efforts to reduce network differentiation may be one such alternative. Number-portability has been one remedy; reducing the duration of subscriptions and reducing inertia in switching between networks have been suggested as additional remedies.[13]

References

Anderson, Simon, André de Palma and Jacques-Francois Thisse (1992): *Discrete Choice Theory of Product Differentiation*, Cambridge, Massachusetts, MIT Press.

Armstrong, Mark (1998): Network Interconnection in Telecommunications, *Economic Journal*, vol.108, pp. 545-564.

Armstrong, Mark (2004): Network Interconnection with Asymmetric Networks and Heterogeneous Calling Patterns, *Information Economics and Policy*, vol. 16, pp. 375-390.

Armstrong, Mark and David E. M. Sappington (2006): Regulation, Competition, and Liberalization, *Journal of Economic Literature*, vol. 44, pp. 325-366.

Berger, Ulrich (2005): Bill-and-Keep vs. Cost-Based Access Pricing Revisited, *Economics Letters* vol. 86, pp. 107-112.

Bouckaert, Jan and Frank Verboven (2004): Price Squeezes in a Regulatory Environment, *Journal of Regulatory Economics*, vol. 26, pp. 321-351.

Björnerstedt, Jonas and Johan Stennek (2007): Bilateral Oligopoly and the Efficiency of Intermediate Goods Markets, *International Journal of Industrial Organization*, vol. 25, pp. 884-907.

Calzada, Joan and Tommaso M. Valletti (2008): Network Competition and Entry Deterrence, *Economic Journal*, vol. 118, pp. 1223-1244.

Carter, Michael and Julian Wright (2003): Asymmetric Network Interconnection, *Review of Industrial Organization*, vol. 22, pp. 27-46.

Cave, Martin (2004): Regulatory Policy towards Mobile Termination, Report prepared for TelstraClear, July 2004, Mec 1126.

[13] For an extended discussion of the policy implications, see Stennek and Tangerås (2007).

Davidson, Carl (1988): Multiunit Bargaining in Oligopolistic Industries, *Journal of Labor Economics*, vol. 6, pp. 397-422.

De Bijl, Paul and Martin Peitz (2002): *Regulation and Entry into Telecommunications Markets*, New York: Cambridge University Press.

Dessein, Wouter (2003): Network Competition in Nonlinear Pricing, *RAND Journal of Economics*, vol. 34, pp. 593-611.

Doganoglu, Toker and Yair Tauman (2002): Network Competition and Access Charge Rules, *Manchester School*, vol. 70, pp. 16-35.

European Commission (2005): Electronic Communications: New Regulatory Framework - Principles, Fact Sheet 13, http://europa.eu.int/information_society/doc/factsheets/013-regulatory_framework.pdf, November 9, 2006.

Gans, Joshua S. and Stephen P. King (2001): Using 'Bill and Keep' Interconnect Arrangements to Soften Network Competition, *Economics Letters*, vol. 71, pp. 413-420.

Gilo, David and Yossi Spiegel (2004): Network Interconnection with Competitive Transit, *Information Economics and Policy*, vol. 16, pp. 439-458.

Hausman, Jerry (2002): Mobile Telephone, in: Cave, Martin E., Sumit K. Majumdar and Ingo Vogelsang (eds.): *Handbook of Telecommunications Economics, Volume 1.* Amsterdam: North-Holland, pp. 563-604.

Horn, Henrik and Asher Wolinsky (1988): Bilateral Monopolies and Incentives for Merger, *RAND Journal of Economics*, vol. 19, pp. 408–419.

Jeon, Doh-Shin, Jean-Jacques Laffont and Jean Tirole (2004): On the "Receiver-Pays" Principle, *RAND Journal of Economics*, vol. 35, pp. 85-110.

Jeon, Doh-Shin and Sjaak Hurkens (2008): A Retail Benchmarking Approach to Efficient Two-Way Access Pricing: No Termination-Based Price Discrimination, *RAND Journal of Economics*, forthcoming.

Kerf, Michel, Isabel Neto and Damien Géradin (2005): *How Antitrust and Sector Regulation Affect Telecom Competition*, Public Policy for the Private Sector, Note Number 296, The World Bank, Washington.

Laffont, Jean-Jacques, Patrick Rey and Jean Tirole (1998a): Network Competition: Overview and Nondiscriminatory Pricing, *RAND Journal of Economics*, vol. 29, pp. 1-37.

Laffont, Jean-Jacques, Patrick Rey and Jean Tirole (1998b): Network Competition: Price Discrimination, *RAND Journal of Economics*, vol. 29, pp. 38-56.

OECD (2005): *Communications Outlook*, OECD, Paris.

OECD (2006): *Telecommunication Regulatory Institutional Structures and Responsibilities*, OECD, Paris.

Röller, Lars-Hendrik & Waverman, Leonard. (2001): Telecommunications Infrastructure and Economic Development: A Simultaneous Approach, *American Economic Review*, vol. 91, pp. 909-923.

Stennek, Johan and Thomas P. Tangerås (2008): Intense Network Competition, manuscript, Gothenburg University and IFN.

Stennek, Johan and Thomas P. Tangerås (2007): Better Regulation of Mobile Telecommunications, *European Policy Analysis* 2007:1 (http://www.sieps.se/epa/2007/ EPA_1_2007.pdf).

SOU (2006): *Effektivare LEK*, Statens offentliga utredningar, Stockholm, 2006:88.

Valletti, Tommaso M. (2003): *Access Services to Public Mobile Networks*, prepared for the European Commission.

Valletti, Tommaso M. and Carlo Cambini (2005): Investments and Network Competition, *RAND Journal of Economics*, vol. 36, pp. 446-467.

Vogelsang, Ingo (2006): Access and Interconnection Pricing Issues in Telecommunications, CESifo DICE Report, *Journal for Institutional Comparisons*, vol.4, pp. 3-9.

Waverman, Leonard, Meloria Meschi and Melvyn Fuss (2005): The Impact of Telecoms on Economic Growth in Developing Countries. *Vodafone Policy Paper Series*, No. 2, pp. 10-23.

Wright, Julian (2002): Access Pricing under Competition: an Application to Cellular Networks. *Journal of Industrial Economics*, vol. 50, pp. 317-335.

A Appendix

We maintain the following assumptions throughout: (i) all pairs of networks are interconnected, (ii) networks do not price discriminate between off-net and on-net calls and (iii) all access prices are reciprocal, but not necessarily the same for all pairs of networks.

A.1 Proof of Lemma 1

Assume that all access prices faced by network i are identical and equal to a, and that i expects all other networks to charge the same call price $p^* \in [c, v)$. Then, network i's expected profit is

$$\pi_i(p) = S_i(p)\left((p-c)\,D(p) + (1 - S_i(p))\,(a - c_t)\,(D(p^*) - D(p))\right),$$

as a function of its call price p. The first-order condition for a symmetric equilibrium ($p = p^*$) is given by the first-order condition

$$\tfrac{\partial \pi_i(p)}{\partial p} = \tfrac{\partial S_i}{\partial p_i}(p^* - c)\,D(p^*) + \tfrac{1}{n}\left(D(p^*) + \left(p^* - c - (1 - \tfrac{1}{n})(a - c_t)\right) D'(p^*)\right) = 0.$$

Using $\frac{\partial S_i}{\partial p} = -(n-1)\,D(p^*)\,/V(p^*)\,\gamma n^2$ and $D(p^*) = 1 - \varepsilon p^*$, we find that any symmetric equilibrium $p^*(\gamma, a, n)$ must be a solution to $g(p^*, \gamma, a, n) = 0$, where

$$g(p, \gamma, a, n) = 1 - \varepsilon\left(2p - c - 1 - \tfrac{n-1}{n}(a - c_t)\right) - \tfrac{1}{\gamma}\tfrac{n-1}{n}(p - c)\tfrac{D^2(p)}{V(p)} \qquad (9)$$

is a third-degree polynomial. As we will show that $g(p, \gamma, a, n)$ is strictly decreasing in p for all prices in the interval $[c, v)$, there is at most one solution to $g(p^*, \gamma, a, n) = 0$ in this interval. It is easy to rewrite $g(p^*, \gamma, a, n) = 0$ as (6). To prove the claimed monotonicity of g, note that the derivative

$$\tfrac{\partial g}{\partial p} = -2\varepsilon - \tfrac{1}{\gamma}\tfrac{n-1}{n}\left(\tfrac{D^2(p)}{V(p)} + (p - c)\tfrac{d}{dp}\left[\tfrac{D^2(p)}{V(p)}\right]\right),$$

is negative for all $p \in [c, v)$ since

$$\tfrac{d}{dp}\left[\tfrac{D^2(p)}{V(p)}\right] = \tfrac{D(p)}{V^2(p)}\left(2D'(p)\,V(p) + D^2(p)\right) = \tfrac{D(p)D^2(v)}{V^2(p)} > 0.$$

Comparative statics To demonstrate how the equilibrium price varies with the access price, market concentration and the substitutability of the networks, note that $dp^*/d\gamma = -(\partial g/\partial\gamma)\,/\,(\partial g/\partial p)$, $dp^*/da = -(\partial g/\partial a)\,/\,(\partial g/\partial p)$ and $dp^*/dn = -(\partial g/\partial n)\,/\,(\partial g/\partial p)$. Since $\partial g/\partial p < 0$, $\partial g/\partial a = (n-1)\,\varepsilon/n >$

0 and $\partial g/\partial\gamma = (n-1)(p^* - c) D^2(p^*) / (V(p^*) n\gamma^2) \geq 0$, the first two results follow. Using eq. (6), it can easily be verified that

$$\frac{\partial g}{\partial n} = \left(\frac{p^*-c}{p^*} - \frac{1}{\eta(p^*)}\right)\frac{\eta(p^*)D(p^*)}{n(n-1)},$$

which completes the comparative statics exercise of the proof. ■

A.2 Proof of Proposition 1

In a sense, Proposition 1 is stronger than a usual existence result. If the networks i and j (formally, agent A_{ij}) set the access price $a_{ij} = a^m$ there exists an equilibrium of the continuation game with all networks charging the monopoly price. If a pair of networks would set a very high access price, an equilibrium in call prices may possibly fail to exist, however, as one firm might profit from cornering the market. But we show that *any* pair of prices p_i and p_j would lead to a lower joint profit than both networks charging the monopoly price p^m.

In the proofs that follow we shall make repeated use of the following result, which is an extension of a Lemma in Armstrong (1998):

Lemma 3 *Let $f : P \to \mathbb{R}$ and $z : P \to \mathbb{R}$ be twice continuously differentiable functions defined on a compact set $P \in \mathbb{R}^m$, $m \in \{1,2\}$. Assume that (i) f has a unique maximand p^*; (ii) the matrix of second partial derivatives of f is negative definite at p^*; (iii) $z(p^*) = 0$ and (iv) $\partial z(p)/\partial p_i|_{p=p^*} = 0$ for all i. Then, p^* is the unique maximand of $f(p) + \tau z(p)$ for all sufficiently small (but positive) τ.*

Proof. The first leading principal minor $\partial^2 f/\partial p_1^2 + \tau\partial^2 z/\partial p_1^2$ of the matrix of second partial derivatives of $f + \tau z$ is negative at p^* for all sufficiently small (but positive) τ since $\partial^2 f/\partial p_1|_{p=p^*} < 0$. The determinant

$$\begin{aligned}
&\left(\frac{\partial^2 f}{\partial p_1^2} + \tau\frac{\partial^2 z}{\partial p_1^2}\right)\left(\frac{\partial^2 f}{\partial p_2^2} + \tau\frac{\partial^2 z}{\partial p_2^2}\right) - \left(\frac{\partial^2 f}{\partial p_1 \partial p_2} + \tau\frac{\partial^2 z}{\partial p_1 \partial p_2}\right)\left(\frac{\partial^2 f}{\partial p_2 \partial p_1} + \tau\frac{\partial^2 z}{\partial p_2 \partial p_1}\right) \\
&= \frac{\partial^2 f}{\partial p_1^2}\frac{\partial^2 f}{\partial p_2^2} - \frac{\partial^2 f}{\partial p_1 \partial p_2}\frac{\partial^2 f}{\partial p_2 \partial p_1} + \tau\frac{\partial^2 f}{\partial p_1^2}\frac{\partial^2 z}{\partial p_2^2} + \tau\frac{\partial^2 z}{\partial p_1^2}\left(\frac{\partial^2 f}{\partial p_2^2} + \tau\frac{\partial^2 z}{\partial p_2^2}\right) \\
&-\tau\frac{\partial^2 f}{\partial p_1 \partial p_2}\frac{\partial^2 z}{\partial p_2 \partial p_1} - \tau\frac{\partial^2 z}{\partial p_1 \partial p_2}\left(\frac{\partial^2 f}{\partial p_2 \partial p_1} + \tau\frac{\partial^2 z}{\partial p_2 \partial p_1}\right)
\end{aligned}$$

of that same matrix (for the case with $n = 2$) is positive at p^* for all sufficiently small (but positive) τ since $\frac{\partial^2 f}{\partial p_1^2}\frac{\partial^2 f}{\partial p_2^2}|_{p=p^*} > \frac{\partial^2 f}{\partial p_1 \partial p_2}|_{p=p^*}$. As $\partial f/\partial p_i|_{p=p^*} + \tau\partial z/\partial p_i|_{p=p^*} = 0$ for all i, there exists a $\tau_1 > 0$ such that p^* is a strict local maximum of $f + \tau z$ for all $\tau < \tau_1$. Hence, there exists a neighborhood B around p^* with the property $f(p) + \tau z(p) < f(p^*) + \tau z(p^*)$

for all $p \in B - p^*$ and for all $\tau < \tau_1$. Since B is open, its complement $P - B$ is closed. Being a closed subset of the compact set P, $P - B$ is itself compact. Compactness of $P - B$ along with continuity of f and z imply the existence of a p^f (p^z) which maximizes f (z) over $P - B$. $f(p) + \tau z(p) \leq f(p^f) + \tau z(p^z) < f(p^*) = f(p^*) + \tau z(p^*)$ for all $p \in P - B$ and $\tau < \tau_2 = (f(p^*) - f(p^f))/z(p^z) > 0$. $\tau_2 > 0$ because p^* is the unique maximand of f and $p^* \notin P - B$. Setting $\overline{\tau} = \min\{\tau_1, \tau_2\} > 0$, we conclude that p^* is the unique maximand of $f + \tau z$ in P for all $\tau < \overline{\tau}$. ■

No individual firm has an incentive to deviate Assume that all access prices are identical and equal to α^m, and that network i expects all other networks to charge the monopoly call price p^m. In this case, i's expected profit is

$$\pi_i(p) = S_i(p)[(p - c)D(p) + (1 - S_i(p))(\alpha^m - c_t)(D(p^m) - D(p))],$$

when its call price is $p \in [0, v]$. There is no point in considering prices above v since the profit is zero for all $p \geq v$. To rewrite network i's profit on the form $\pi_i(p) = x(p) + \frac{\varepsilon}{\gamma} y(p)$, we first add and subtract

$$\left(1 - \tfrac{1}{n}\right)(\alpha^m - c_t)(S_i(p)(D(p^*) - D(p)))$$

to π_i, then substitute in α^m defined in (8), and finally define

$$x(p) = S_i(p)[(p_i - c)D(p) + \left(1 - \tfrac{1}{n}\right)(\alpha^m - c_t)(D(p^m) - D(p))]$$

and

$$y(p) = \tfrac{p^m \sigma_{vp}(p^m)}{\eta(p^m)^2} S_i(p)(\tfrac{1}{n} - S_i(p))(p - p^m).$$

The decomposition of the profit into the two $x(p)$ and $y(p)$ is done to separate out the "composition effect" from the more regular effects of a change in price on profit. We will refer to $x(p)$ as the *normalized* profit. The first claim demonstrates that the monopoly price maximizes the normalized profit function.

Claim 1 *p^m is the unique maximand of x. Moreover, $\partial^2 x/\partial p^2|_{p=p^m} < 0$.*

Proof. We first show that x has a unique maximand. The set $\Omega \subset [0, v]$ of p's for which $x \geq 0$ is non-empty due to the fact that $x(p^m) > 0$. Obviously, every maximand of x must be contained in Ω. We now demonstrate that x has a unique maximand in Ω. By continuity of x, the set of p's for which $x < 0$ is open. Hence, Ω is closed. Being a closed subset of the compact set $[0, v]$, Ω is itself compact, and thus admits a maximum. The normalized profit per subscriber, i.e. $x(p)/S_i(p)$, is strictly concave in p, which renders Ω convex since $S_i \geq 0$. Differentiate:

$$\frac{\partial x}{\partial p} = \frac{S_i'(p)}{S_i(p)} x\left(p\right) + S_i\left(p\right) \left(1 + \varepsilon c + \varepsilon \frac{(n-1)}{n}\left(\alpha^m - c_t\right) - 2\varepsilon p\right)$$

and

$$\frac{\partial^2 x}{\partial p^2} = 2\frac{S_i'(p)}{S_i(p)}\frac{\partial x}{\partial p} + \left[\frac{d}{dp}\left[\frac{S_i'(p)}{S_i(p)}\right] - \left(\frac{S_i'(p)}{S_i(p)}\right)^2\right] x\left(p\right) - 2\varepsilon S_i\left(p\right).$$

Note that

$$\frac{d}{dp}\left[\frac{S_i'(p)}{S_i(p)}\right] = \frac{1}{\gamma} S_i'\left(p\right)\frac{D(p)}{V(p)} - \frac{1}{\gamma}\left(1 - S_i\left(p\right)\right)\frac{d}{dp}\left[\frac{D(p)}{V(p)}\right] < 0,$$

since $S_i'\left(p\right)/S_i\left(p\right) = -\frac{1}{\gamma}(1 - S_i\left(p\right))\frac{D(p)}{V(p)} < 0$ and

$$\frac{d}{dp}\left[\frac{D(p)}{V(p)}\right] = \frac{D^2(p) - \varepsilon V(p)}{V^2(p)} = \frac{(1-\varepsilon p)^2 + (1-\varepsilon v)^2}{2V^2(p)} > 0.$$

Hence, $\partial^2 x/\partial p^2 < 0$ for all $p \in \Omega$ satisfying $\partial x/\partial p \geq 0$. In particular every interior solution $\partial x/\partial p = 0$ is a strict local optimum. Hence, x is strictly quasi-concave on Ω and therefore has a unique maximand on $[0, v]$.

By strict quasi-concavity, any solution $\partial x/\partial p = 0$ in Ω is an optimum. Using $\alpha^m = c_t + p^m \sigma_{vp}\left(p^m\right)/\gamma\eta\left(p^m\right)^2$, it is easy to verify that $\partial x/\partial p|_{p=p^m} = 0$, hence p^m is the unique maximand of x. $p^m \in \Omega$ and $\partial x/\partial p|_{p=p^m} = 0$ imply $\partial^2 x/\partial p^2|_{p=p^m} < 0$, which completes the proof. ■

Finally note that the normalized profit x satisfies assumptions (i) and (ii) of Lemma 3. Even assumptions (iii) and (iv) of the Lemma are fulfilled, since $y\left(p^m\right) = 0$ and $\partial y/\partial p|_{p=p^m} = 0$. Consequently, p^m is the unique maximand of $\pi_i\left(p\right) = x(p) + \frac{\varepsilon}{\gamma} y(p)$ for $\frac{\varepsilon}{\gamma}$ sufficiently low, but positive.

No pair of firms can increase their joint profit by deviating To prove that no pair of firms can increase their joint profit by deviating from α^m we will follow a very similar procedure. If there are only two firms in the industry, they cannot profit from a joint deviation from monopoly prices since their joint profit is just the industry profit, which is maximized precisely at monopoly prices. Assume therefore that $n \geq 3$. The joint expected profit of firms i and j is

$$\begin{aligned} \pi_{ij}\left(\mathbf{p}\right) = \; & S_i\left(p_i - c\right) D\left(p_i\right) + S_j\left(p_j - c\right) D\left(p_j\right) \\ + \; & \left(1 - S_i - S_j\right)\left(\alpha^m - c_t\right)\left(S_i\left(D\left(p^m\right) - D\left(p_i\right)\right) + S_j\left(D\left(p^m\right) - D\left(p_j\right)\right)\right). \end{aligned} \tag{10}$$

when they charge call prices $\mathbf{p} = (p_i, p_j)$, all other firms charge the monopoly call price p^m, and all access prices except possibly a_{ij} are equal to α^m.

Again, we need to decompose the joint profit into a normalized profit and a part catching the composition effect,

$$\pi_{ij}(\mathbf{p}) = X(\mathbf{p}) + \tfrac{\varepsilon}{\gamma}Y(\mathbf{p}) \text{ where } X(\mathbf{p}) = X_i(\mathbf{p}) + X_j(\mathbf{p}).$$

To do this, add and subtract

$$\left(1-\tfrac{2}{n}\right)(\alpha^m - c_t)\left(S_i\left(D\left(p^m\right)-D\left(p_i\right)\right) + S_j\left(D\left(p^m\right)-D\left(p_j\right)\right)\right)$$

from π_{ij} and define

$$X_i(\mathbf{p}) = S_i(\mathbf{p})\left(\left(p_i - c\right)D\left(p_i\right) + \left(1-\frac{2}{n}\right)(\alpha^m - c_t)\left(D\left(p^m\right)-D\left(p_i\right)\right)\right),$$

and

$$Y(\mathbf{p}) = \tfrac{p^m\sigma_{vp}(p^m)}{\eta(p^m)^2}\left(\frac{2}{n} - S_i(\mathbf{p}) - S_j(\mathbf{p})\right)\left(S_i(\mathbf{p})\left(p_i - p^m\right) + S_j(\mathbf{p})\left(p_j - p^m\right)\right).$$

Let $\widehat{p}^m$ be the monopoly price at marginal cost $\widehat{c} = c + \frac{n-2}{n}(\alpha^m - c_t) > c$, which is the perceived marginal cost of any pair of firms maximizing their joint profit, i.e. $\widehat{p} = \min\{v, \frac{1+\varepsilon\widehat{c}}{2\varepsilon}\}$. Consider first the problem of maximizing the normalized profit X. Let $\widetilde{\mathbf{p}} = (\widetilde{p}_i, \widetilde{p}_j)$ be a pair of prices that maximize X, and let the optimal normalized profit be denoted by $\widetilde{X} = X(\widetilde{\mathbf{p}})$ and the optimal market share by $\widetilde{S}_i = S_i(\widetilde{\mathbf{p}})$. We first show that X is maximized only if both firms charge the same call price:

Claim 2 *The policy $\widetilde{\mathbf{p}} = (\widetilde{p}_i, \widetilde{p}_j)$ maximizes X only if $\widetilde{p}_i = \widetilde{p}_j \in (0, \widehat{p}^m)$.*

Proof. We establish the result via four intermediary steps.

Step 1 demonstrates that both prices must be set below the price of calls from public phones, i.e. $\widetilde{p}_i < v$ *and* $\widetilde{p}_j < v$. It would clearly not be optimal to set $\widetilde{p}_j \geq v$ and $\widetilde{p}_i \geq v$ since that would result in a zero normalized profit, $\widetilde{X} = 0$, while setting one price equal to the monopoly price would result in a larger profit $X(p^m, v) = \frac{1}{n-1}(p^m - c) > 0$. Note that this also implies that the optimal normalized profit must be strictly positive, $\widetilde{X} > 0$. It could also not be optimal to set only one price below, e.g. $\widetilde{p}_i < v$ and $\widetilde{p}_j \geq v$. The alternative pricing policy $p_i = p_j = \widetilde{p}_i$ would lead to a higher profit, i.e.

$$X(\widetilde{p}_i, \widetilde{p}_i) - \widetilde{X} = \left(2S_i(\widetilde{p}_i, \widetilde{p}_i) - \widetilde{S}_i\right)\tfrac{\widetilde{X}_i}{\widetilde{S}_i} > 0$$

since

$$2S_i - \widetilde{S}_i = \frac{V(\widetilde{p}_i)^{\frac{1}{\gamma}}V(p^*)^{\frac{1}{\gamma}}(n-2)}{\left(2V(\widetilde{p}_i)^{\frac{1}{\gamma}} + (n-2)V(p^*)^{\frac{1}{\gamma}}\right)\left(V(\widetilde{p}_i)^{\frac{1}{\gamma}} + (n-2)V(p^*)^{\frac{1}{\gamma}}\right)} > 0.$$

Thus we conclude that both prices must be (strictly) below v.

Step 2 demonstrates that both prices must be set above marginal cost, i.e. $\widetilde{p}_i > c$ *and* $\widetilde{p}_j > c$. Maximization of the normalized profit X implies $\widetilde{X}_i > 0$ or $\widetilde{X}_j > 0$ or both since $\widetilde{X} > 0$. Assume wlog that $\widetilde{X}_j > 0$. First, note that $\widetilde{X}_j > 0$ implies $\widetilde{p}_j > c$. Second, note that $\partial X/\partial p_i > 0$ for all $\widetilde{p}_i \leq c$. This can be inferred from

$$\frac{V(p_i)}{S_i D(p_i)}\frac{\partial X}{\partial p_i} = -\frac{1}{\gamma}\frac{X_i}{S_i} + \left(D\left(p_i\right) + \left(p_i - \widehat{c}\right) D'\left(p_i\right)\right)\frac{V(p_i)}{D(p_i)} + \frac{1}{\gamma}X, \tag{11}$$

since $\widetilde{X} > 0$, $\widetilde{X}_i \leq 0$ and $p_i \leq c \leq \widehat{c}$. Hence, both prices must be above marginal cost.

Step 3 demonstrates that both prices must be set below the optimal price of a high cost monopolist, in particular $\widetilde{p}_i < \widehat{p}^m$ *and* $\widetilde{p}_j < \widehat{p}^m$. If $\widehat{p}^m = v$, the result is a direct implication of Step 1. Consider the other possibility, $\widehat{p}^m < v$. Note that

$$\begin{aligned}\frac{V(p_i)}{D(p_i)}\frac{\partial X}{\partial p_i} + \frac{V(p_j)}{S_j D(p_j)}\frac{\partial X}{\partial p_j} = &\ \left(D\left(p_i\right) + \left(p_i - \widehat{c}\right) D'\left(p_i\right)\right)\frac{S_i V(p_i)}{D(p_i)} + \left(D\left(p_j\right) + \left(p_j - \widehat{c}\right) D'\left(p_j\right)\right)\frac{S_j V(p_j)}{D(p_j)} \\ - &\ \frac{1}{\gamma}(1 - S_i - S_j)X < 0\end{aligned}$$

if $\widetilde{p}_i \in [\widehat{p}^m, v)$ and $\widetilde{p}_j \in [\widehat{p}^m, v)$ both hold. In this case, the two first-order conditions $\frac{\partial X}{\partial p_i} = \frac{\partial X}{\partial p_j} = 0$ cannot simultaneously be met. Therefore, $\widetilde{p}_i < \widehat{p}^m$ or $\widetilde{p}_j < \widehat{p}^m$, or both.

Assume wlog that $\widetilde{p}_i < \widehat{p}^m \leq \widetilde{p}_j$. There are two possibilities to consider. In the first case the low price firm has a higher normalized profit per subscriber, i.e. $\widetilde{X}_i/\widetilde{S}_i > \widetilde{X}_j/\widetilde{S}_j$. Lower the high price p_j until normalized profits per subscribers are the same, i.e. $X_j/S_j = \widetilde{X}_i/\widetilde{S}_i$. By a comparison of profits,

$$X - \widetilde{X} = X_i + X_j - \widetilde{X} = \left(\frac{S_i}{\widetilde{S}_i} + \frac{S_j}{\widetilde{S}_i}\right)\widetilde{X}_i - \widetilde{X}_i - \widetilde{X}_j = \frac{S_i + S_j - \widetilde{S}_i - \widetilde{S}_j}{\widetilde{S}_i}\widetilde{X}_i + \widetilde{S}_j\left(\frac{\widetilde{X}_i}{\widetilde{S}_i} - \frac{\widetilde{X}_j}{\widetilde{S}_j}\right)$$

Now, the total market share of the two networks is increased by the price reduction, i.e. $\widetilde{S}_i + \widetilde{S}_j < S_i + S_j$, since $\partial(S_i + S_j)/\partial p_j = -\frac{1}{\gamma}S_j(1 - S_i - S_j)D(p_j)/V(p_j) < 0$. As $\widetilde{X}_i/\widetilde{S}_i > \widetilde{X}_j/\widetilde{S}_j$ by assumption, it follows that $X > \widetilde{X}$, which contradicts optimality of $\widetilde{X}$. Consider now the possibility that the high price firm earns the higher profit per subscriber, i.e. $\widetilde{X}_i/\widetilde{S}_i \leq \widetilde{X}_j/\widetilde{S}_j$. Subtract the two first-order conditions

$$\begin{aligned}\frac{V(p_i)}{S_i D(p_i)}\frac{\partial X}{\partial p_i} - \frac{V(p_j)}{S_j D(p_j)}\frac{\partial X}{\partial p_j} = &\ \frac{1}{\gamma}\left(\frac{\widetilde{X}_j}{\widetilde{S}_j} - \frac{\widetilde{X}_i}{\widetilde{S}_i}\right) + \left(D\left(\widetilde{p}_i\right) + \left(\widetilde{p}_i - \widehat{c}\right) D'\left(\widetilde{p}_i\right)\right)\frac{V(\widetilde{p}_i)}{D(\widetilde{p}_i)} \\ - &\ \left(D\left(\widetilde{p}_j\right) + (\widetilde{p}_j - \widehat{c})D'\left(\widetilde{p}_j\right)\right)\frac{V(\widetilde{p}_j)}{D(\widetilde{p}_j)}.\end{aligned}$$

The first term on the RHS is non-negative by assumption. The second term is positive since $\widetilde{p}_i < \widehat{p}^m$, and the third term is non-negative since $\widetilde{p}_j \geq \widehat{p}^m$.

The RHS is positive, and so the two first-order conditions $\frac{\partial X}{\partial p_i} = \frac{\partial X}{\partial p_j} = 0$ cannot be satisfied simultaneously even in this remaining case. By necessity, therefore, $\widetilde{p}_i < \widehat{p}^m$ and $\widetilde{p}_j < \widehat{p}^m$.

Step 4 demonstrates that $\widetilde{p}_i = \widetilde{p}_j \in (c, \widehat{p}^m)$. Steps 1 - 3 hold the joint implication that $\widetilde{p}_i \in (c, \widehat{p}^m)$ and $\widetilde{p}_j \in (c, \widehat{p}^m)$. Using (11), note that

$$\begin{aligned} \frac{\partial X}{\partial p_i} = \frac{\partial X}{\partial p_j} \Rightarrow & \quad \gamma \left(D\left(\widetilde{p}_i\right) + \left(\widetilde{p}_i - \widehat{c}\right) D'\left(\widetilde{p}_i\right)\right) \frac{V(\widetilde{p}_i)}{D(\widetilde{p}_i)} - \frac{\widetilde{X}_i}{\widetilde{S}_i} \\ = & \quad \gamma(D(\widetilde{p}_j) + (\widetilde{p}_j - \widehat{c})D'(\widetilde{p}_j)) \frac{V(\widetilde{p}_j)}{D(\widetilde{p}_j)} - \frac{\widetilde{X}_j}{\widetilde{S}_j}. \end{aligned}$$

Define

$$\begin{aligned} h(p) = & \quad \gamma \left(D\left(p\right) + \left(p - \widehat{c}\right) D'\left(p\right)\right) \frac{V(p)}{D(p)} \\ - & \quad \underbrace{\left(p - c\right) D\left(p\right) - \left(1 - \frac{2}{n}\right) \left(\alpha^m - c_t\right) \left(D\left(p^m\right) - D\left(p\right)\right)}_{X_i/S_i} \end{aligned}$$

With this definition, $\frac{\partial X}{\partial p_i} = \frac{\partial X}{\partial p_j}$ implies $h(\widetilde{p}_i) = h(\widetilde{p}_j)$.

$$h'(p) = 2\varepsilon \left(\tfrac{1+\varepsilon\widehat{c}}{2\varepsilon} - p\right) \left[\gamma \tfrac{d}{dp} \left[\tfrac{V(p)}{D(p)}\right] - 1\right] - 2\varepsilon\gamma \tfrac{V(p)}{D(p)} < 0$$

for all $p \in (c, \widehat{p}^m)$, where the inequality follows from $\frac{d}{dp}\left[\frac{V(p)}{D(p)}\right] < 0$, see the proof of Claim 1, and $p < \widehat{p}^m = \min\{v, \frac{1+\varepsilon\widehat{c}}{2\varepsilon}\}$. Since $h(\widetilde{p}_i) \neq h(\widetilde{p}_j)$ for all $\widetilde{p}_i \neq \widetilde{p}_j$ meeting $\widetilde{p}_i \in (c, \widehat{p}^m)$ and $\widetilde{p}_j \in (c, \widehat{p}^m)$, by necessity $\widetilde{p}_i = \widetilde{p}_j \in (c, \widehat{p}^m)$. ■

Next, let us find the symmetric price $p_i = p_j = p$ that maximizes X. With symmetric prices, X simplifies to

$$X(p,p) = S(p) \left(\left(p - c\right) D\left(p\right) + \left(1 - \tfrac{2}{n}\right) \left(\alpha^m - c_t\right) \left(D\left(p^m\right) - D\left(p\right)\right)\right),$$

where $S\left(p\right) = 2V\left(p\right)^{\frac{1}{\gamma}} / (2V\left(p\right)^{\frac{1}{\gamma}} + (n-2) V\left(p^m\right)^{\frac{1}{\gamma}})$ is the total market share of networks i and j.

Claim 3 *p^m is the unique maximand of $X(p,p)$. The matrix of second partial derivatives of $X(\mathbf{p})$ is negative definite at (p^m, p^m).*

Proof. We omit the proof that $X(p,p)$ has a unique maximand on $[0,v]$, since this is analogous to the proof in Claim 1 that $x(p)$ has a unique maximand on $[0,v]$. It is easy to verify that $\partial X(p,p)/\partial p|_{p=p^m} = 0$, hence p^m is the unique maximand of $X(p,p)$. Using (11), the cross-partial derivatives of $X(\mathbf{p})$ are $\frac{\partial^2 X}{\partial p_i^2} = h'(p^m)\frac{nD(p^m)}{V(p^m)} < 0$ and $\frac{\partial^2 X}{\partial p_i \partial p_j} = 0$ evaluated at $p_i = p_j = p^m$. The matrix of cross-partial derivatives therefore is negative definite at monopoly prices. ■

By virtue of Claim 2 and Claim 3 above, we conclude that (i) $X(p^m, p^m) > X(\mathbf{p})$ for all $\mathbf{p} \in [0, v]^2 - (p^m, p^m)$; (ii) the matrix of second partial derivatives of $X(\mathbf{p})$ is negative definite at (p^m, p^m). Even assumptions (iii) and (iv) of Lemma 3 are fulfilled, since $Y(p^m, p^m) = 0$ and $\partial Y / \partial p_i|_{\mathbf{p}=(p^m,p^m)} = \partial Y / \partial p_j|_{\mathbf{p}=(p^m,p^m)} = 0$. Consequently, $p_i = p_j = p^m$ is the unique maximand of $\pi_{ij}(\mathbf{p}) = X(\mathbf{p}) + \frac{\varepsilon}{\gamma} Y(\mathbf{p})$ for $\frac{\varepsilon}{\gamma}$ sufficiently low, but positive. This completes the proof of Proposition A.2. ■

A.3 Proof of Lemma 2

In the proof to follow we shall make use of the following general lemma:

Lemma 4 *Consider a market with n networks under* STR, *with an access price ceiling $\overline{a}$. Assume that all access prices are common knowledge, but not necessarily the same. There exists a pure strategy equilibrium in call prices if demand is sufficiently inelastic (that is, if ε sufficiently low). If, in addition, all access prices are identical and equal to $a \in [-c_o, \overline{a}]$, the equilibrium is unique and symmetric and equal to $p^*(\gamma, a, n) \in [c, v)$ implicitly defined in eq. (6) in Section 2.*

Proof. We will show that there exists a unique and symmetric pure strategy equilibrium for all $\varepsilon < \overline{\varepsilon}(\gamma, \overline{a})$ and some $\overline{\varepsilon}(\gamma, \overline{a}) > 0$. The equilibrium price is above the marginal cost, c, but below a certain highest price $\mathcal{P}(\gamma, \varepsilon, \overline{a}) \in (c, v)$.

First, some preliminary observations. Let $\mathbf{p}_{-i} = (p_1, .., p_{i-1}, p_{i+1}, p_n)$, so that $\mathbf{p} = (p_i, \mathbf{p}_{-i})$, and let a_{ik} be the reciprocal access price between networks i and k. With n networks, the profit of network $i \in N = \{1, 2, .., n\}$ is

$$\pi_i(\mathbf{p}) = S_i \left((p_i - c) D(p_i) + \textstyle\sum_{k \neq i} S_k (a_{ik} - c_t) (D(p_k) - D(p_i)) \right),$$

The marginal profit is equal to

$$\begin{aligned} \tfrac{\partial \pi_i}{\partial p_i} &= \tfrac{\partial S_i}{\partial p_i} (p_i - c) D(p_i) + S_i (D(p_i) + (p_i - c) D'(p_i)) \\ &\quad + \textstyle\sum_{k \neq i} (a_{ik} - c_t) S_k \left(\left(\tfrac{\partial S_i}{\partial p_i} + \tfrac{S_i}{S_k} \tfrac{\partial S_k}{\partial p_i} \right) (D(p_k) - D(p_i)) - S_i D'(p_i) \right). \end{aligned} \tag{12}$$

Substitute $\frac{\partial S_i}{\partial p_i} = -\gamma^{-1} S_i (1 - S_i) \frac{D(p_i)}{V(p_i)}$ and $\frac{\partial S_k}{\partial p_i} = \gamma^{-1} S_i S_k \frac{D(p_i)}{V(p_i)}$ into the marginal profit and rewrite to obtain

$$\begin{aligned} R_i(\mathbf{p}) = \tfrac{\partial \pi_i}{\partial p_i} \tfrac{\gamma V(p_i)}{S_i D(p_i)} &= -(1 - S_i)(p_i - c) D(p_i) + \gamma \left(1 + (p_i - c) \tfrac{D'(p_i)}{D(p_i)} \right) V(p_i) \\ &\quad - \textstyle\sum_{k \neq i} (a_{ik} - c_t) S_k \left((1 - 2S_i)(D(p_k) - D(p_i)) + \gamma \tfrac{D'(p_i)}{D(p_i)} V(p_i) \right) \end{aligned} \tag{13}$$

Since $\gamma V(p_i)/S_i D(p_i) > 0$ for all $p_i < v$, $sgn\{\partial\pi_i/\partial p_i\} = sgn\{R_i(\mathbf{p})\}$ for all $p_i < v$.

Existence The existence proof proceeds in four claims. The first two claims establish that a network will never set a price above $P(\gamma,\varepsilon,\overline{a}) \in (c,v)$ nor below c, given that everybody else charges a call price at or above c, at least one competitor sets a price in $[c,v)$ and ε is sufficiently small.

Claim 4 *There exists an $\varepsilon_1(\gamma,\overline{a}) > 0$ such that for all $\varepsilon < \varepsilon_1(\gamma,\overline{a})$, $p_i < c$ is a strictly dominated strategy.*

Proof. Note that $R_i(\mathbf{p})$ can be rewritten as

$$\begin{aligned} R_i(\mathbf{p}) = &-(1-S_i)(p_i-c)D(p_i) - \gamma\left(S_i c + \textstyle\sum_{k\neq i}(a_{ik}+c_o)S_k\right)\frac{D'(p_i)}{D(p_i)}V(p_i) \\ &+\gamma\left(1-\frac{\varepsilon p}{1-\varepsilon p}\right)V(p_i) - \varepsilon\textstyle\sum_{k\neq i}S_k(1-2S_i)(a_{ik}-c_t)(p_i-p_k), \end{aligned}$$

where we have used $D(p) = 1-\varepsilon p$ in the last line. The sum of the terms on the first line is strictly positive for $p_i \leq c$. The expression on the second line is strictly positive for ε sufficiently small. Hence, $\pi_i(\mathbf{p}) < \pi_i(c,\mathbf{p}_{-i})$ for all $p_i < c$ and for all $\mathbf{p}_{-i}$, provided that ε is sufficiently small. ■

Let M_{-i} be the set of networks not including i that charge a call price strictly below v. Let $\overline{p}(\mathbf{p}_{-i})$ be the maximal of these prices for the case that $M_{-i} \neq \emptyset$.

Claim 5 *Assume that $\varepsilon < (v+c)^{-1}$. If $p_k \geq c$ for all $k \neq i$ and $p_j < v$ for some $j \neq i$, then there exists a $\mathcal{P}(\gamma,\varepsilon,\overline{a}) \in (c,v)$ such that $\partial\pi_i/\partial p_i < 0$ for all $p_i \in [\max\{\mathcal{P};\overline{p}(\mathbf{p}_{-i})\},v)$. $\mathcal{P}$ is increasing in γ, ε and $\overline{a}$.*

Proof. By manipulating terms and using linearity of demand, we get

$$\begin{aligned} R_i(\mathbf{p}) = &-\left(\tfrac{1}{2}(1-\varepsilon(v+c))(p_i-c) - \gamma V(p_i) - \varepsilon\gamma\max\{0;\overline{a}\}\frac{V(p_i)}{D(p_i)}\right) \\ &-\varepsilon\gamma\left(p_i - c + \max\{0;\overline{a}\} - \textstyle\sum_{k\neq i}S_k a_{ik} + (1-S_i)c_t\right)\frac{V(p_i)}{D(p_i)} \\ &-\left(\left(\tfrac{1}{2}-S_i\right)(1-\varepsilon(v+c)) + \varepsilon(1-S_i)(v-p_i+2cS_i)\right)(p_i-c) \\ &-\varepsilon\textstyle\sum_{k\neq i}S_k(1-2S_i)(c(p_k-c)+(a_{ik}+c_o)(p_i-p_k)). \end{aligned}$$

Define the term in parenthesis in the first line as $H(p_i) = H(p_i,\gamma,\varepsilon,\overline{a})$. Note that H is strictly increasing in p_i since $\varepsilon < (v+c)^{-1}$, $-V'(p_i) = D(p_i)$ and $-\frac{d}{dp_i}\left[\frac{V(p_i)}{D(p_i)}\right] = \frac{(1-\varepsilon p_i)^2+(1-\varepsilon v)^2}{2(1-\varepsilon p_i)^2} > 0$. Moreover, $H(c) < 0$ and $H(v) > 0$ since $V(v) = 0$. Hence, there exists a unique $\mathcal{P}(\gamma,\varepsilon,\overline{a}) \in (c,v)$ implicitly defined by $H(\mathcal{P}) = 0$, with the property that $H(p_i) > 0$ for all $p_i > \mathcal{P}$. Note also that the second line is strictly negative for all $p_i \in [\mathcal{P}, v)$, whereas the two final lines are non-positive for all $v > p_i \geq \overline{p}(M_{-i})$ and $p_k \geq c$ for all $k \neq i$. Hence, $R_i(\mathbf{p}) < 0$ for all $p_i \in [\max\{\mathcal{P}(\gamma, \varepsilon, \overline{a}); \overline{p}(M_{-i})\}, v)$. The properties of $\mathcal{P}(\gamma, \varepsilon, \overline{a})$ follow from implicit differentiation of $H(\mathcal{P}, \gamma, \varepsilon, \overline{a}) = 0$. ∎

Claim 6 *There exists a pure strategy Nash equilibrium, $\widetilde{\mathbf{p}} \in [c, \mathcal{P}(\gamma, \varepsilon, \overline{a})]^n$, for every $\varepsilon < \varepsilon_2(\gamma, \overline{a})$, and for some $\overline{\varepsilon}_2(\gamma, \overline{a}) > 0$.*

Proof. First, note that claims 4 and 5 guarantee that every network will set a price in $[c, \mathcal{P}]$ given that all other networks set a price in $[c, \mathcal{P}]$, provided that ε is sufficiently small. Second, note that π_i is continuous in p on the domain $p \in [c, \mathcal{P}]^n$. The existence proof amounts to verifying quasi-concavity of π_i in p_i on $[c, \mathcal{P}]$ for all $\varepsilon < \varepsilon_2(\gamma, \overline{a})$ and some $\varepsilon_2(\gamma, \overline{a}) > 0$. Claim 4 implies $R_i(c, \mathbf{p}_{-i}) > 0$, and Claim 5 implies $R_i(\mathcal{P}, \mathbf{p}_{-i}) < 0$ for all $p_{-i} \in [c, \mathcal{P}]^{n-1}$. Hence, there exists a $\widehat{p}_i(\mathbf{p}_{-i}) \in (c, \mathcal{P})$ which satisfies $R_i(\widehat{p}_i, \mathbf{p}_{-i}) = 0$. If $\widehat{p}_i$ is uniquely defined, π_i is single-peaked and therefore strictly quasi-concave in p_i on $[c, \mathcal{P}]$. We now demonstrate that $R_i(\mathbf{p})$ is strictly decreasing in p_i in the interval $[c, \mathcal{P}]$, provided that ε is sufficiently small. Note that

$$\begin{aligned}
\frac{\partial R_i(\mathbf{p})}{\partial p_i} &= -\left(\gamma + (1 - S_i)\left(1 + \tfrac{1}{\gamma} S_i \tfrac{D(p_i)}{V(p_i)}(p_i - c)\right)\right) D(p_i) \\
&\quad -\varepsilon\left(\gamma \tfrac{V(p_i)}{D(p_i)} - \sum_{k \neq i} S_i S_k (a_{ik} - c_t)\right) \\
&\quad -\varepsilon\gamma\left(p_i - c - \sum_{k \neq i} S_k (a_{ik} - c_t)\right) \tfrac{d}{dp_i}\left[\tfrac{V(p_i)}{D(p_i)}\right] \\
&\quad -\tfrac{\varepsilon}{\gamma} \sum_{k \neq i} (a_{ik} - c_t) S_i S_k (3 - 4S_i) \tfrac{D(p_i)}{V(p_i)} (p_i - p_k) \\
&\quad -\varepsilon\left(\sum_{k \neq i} (a_{ik} - c_t) S_k (1 - 2S_i) - (1 - S_i)(p_i - c)\right),
\end{aligned}$$

where we have used Roy's identity $V'(p_i) = -D(p_i)$, the explicit expressions for $\partial S_i / \partial p_i$ and $\partial S_j / \partial p_i$ as well as linear demand $D(p) = 1 - \varepsilon p$ and $D'(p) = -\varepsilon$. Since $\lim_{\varepsilon \to 0} V(p)/D(p) = v - p$,

$$\lim_{\varepsilon \to 0} \frac{\partial R_i(\mathbf{p})}{\partial p_i} = -\left(\gamma + (1 - S_i)\left(1 + \tfrac{1}{\gamma} S_i \frac{p_i - c}{v - p_i}\right)\right) < 0 \;\forall p_i \leq \mathcal{P}(\gamma, \varepsilon, \overline{a}) < v. \tag{14}$$

Hence, $R_i(\mathbf{p})$ is strictly decreasing in p_i in the interval $[c, \mathcal{P}]$ provided that ε is sufficiently small. ∎

Existence is ensured for all ε smaller than the minimum of $\varepsilon_1(\gamma, \overline{a})$ used in Claim 4, $(v+c)^{-1}$ used in Claim 5 and $\varepsilon_2(\gamma, \overline{a})$, used in Claim 6.

Uniqueness Claim 4 establishes that all equilibrium prices must be at or above the marginal cost provided that ε is sufficiently small. The uniqueness proof proceeds in two claims. Claim 7 establishes that all equilibrium prices are contained in $[c, \mathcal{P})$ provided that ε is sufficiently small. Claim 8 establishes that any equilibrium in which two networks charge symmetric access prices forces them to charge identical call prices, provided that ε is sufficiently small. This holds the implication that all networks charge the same call price if all access prices are the same. In the proof of Lemma 1, we showed that there can be at most one symmetric equilibrium when all firms charge the same access price a. This symmetric equilibrium call price p^* is implicitly defined in (6) in Section 2. Let $\widetilde{\mathbf{p}}_{-i}$ be the equilibrium call prices of all networks except i, $\widetilde{p}_i$ the equilibrium call price of network i and $\widetilde{\mathbf{p}}_i = (\widetilde{p}_i, \widetilde{\mathbf{p}}_{-i})$.

Claim 7 *Assume that $\varepsilon < (v+c)^{-1}$ and $\varepsilon < \varepsilon_1(\gamma, \overline{a})$ defined in Claim 4 are both satisfied. In any equilibrium $\widetilde{\mathbf{p}}$, $\widetilde{p}_i \in [c, \mathcal{P})$ for all $i \in N$ with $\mathcal{P}(\gamma, \varepsilon, \overline{a})$ defined in Claim 5.*

Proof. We first demonstrate that at least one network charges a call price strictly below v in equilibrium. Suppose, on the contrary, that $\widetilde{p}_i \geq v \; \forall i \in N$. The industry profit is $\sum_{k \in M} S_k (v-c) D(v) \leq (v-c) D(v)$ in this case, where M is the (possibly empty) set of networks which charge a call price exactly equal to v. It follows that at least one network, say network j, earns a profit strictly below $(v-c) D(v)$. Any deviation by j to $p_j = v - \delta$, $\delta > 0$, would render j the monopoly status and profit $(v - \delta - c) D(v - \delta)$. By setting δ arbitrarily close to but below v, j could strictly increase its profit. Having established that at least one network charges a call price below v in equilibrium, we now show that all networks set a price strictly below v in equilibrium. Suppose wlog that $\widetilde{p}_i \geq v$ in equilibrium. Since, in this case, $M_{-i} \neq \emptyset$ and $\widetilde{p}_k \geq c$ for all $k \neq i$, we know from Claim 5 that $\pi_i(\max\{\mathcal{P}; \overline{p}(\widetilde{\mathbf{p}}_{-i})\}, \widetilde{\mathbf{p}}_{-i}) > \pi_i(\widetilde{\mathbf{p}}) = 0$ for all $\widetilde{p}_i \geq v$, and so $\widetilde{p}_i \geq v$ cannot be an equilibrium. Having established that all networks charge a call price strictly below v in equilibrium, we now show that all networks set a price strictly below $\mathcal{P}$ in equilibrium. Assume wlog that network i charges the maximal price, i.e. $\widetilde{p}_i \geq \overline{p}(\widetilde{\mathbf{p}}_{-i})$. For any $\widetilde{p}_i \in [\mathcal{P}, v)$, i will strictly profit by lowering its call price, see Claim 5; hence, the maximal equilibrium price must necessarily be strictly below $\mathcal{P}$. ■

Claim 8 *Assume that networks i and j set symmetric access prices, $a_{ik} = a_{jk}$ $\forall k \neq i, j$. There exists an $\varepsilon_3(\gamma, \overline{a}) > 0$ such that for all $\varepsilon < \varepsilon_3(\gamma, \overline{a})$ any call price equilibrium $\widetilde{\mathbf{p}} \in [c, \mathcal{P}]$, satisfies $\widetilde{p}_i = \widetilde{p}_j$.*

Proof. Any interior equilibrium $\widetilde{p}_k \in [c, v)$ $\forall k \in N$ must satisfy the two first-order conditions $R_i(\widetilde{\mathbf{p}}) = 0$ and $R_j(\widetilde{\mathbf{p}}) = 0$, where $R_i(\mathbf{p})$ was defined in equation (13) and $R_j(\mathbf{p})$ can be equivalently defined. It follows that every interior equilibrium must satisfy $R_i(\widetilde{\mathbf{p}}) - R_j(\widetilde{\mathbf{p}}) = 0$. $R_i(\mathbf{p})$ is strictly decreasing in p_i in the interval $[c, \mathcal{P}]$ provided that ε is sufficiently small; see (14). Note that

$$\begin{aligned}\frac{\partial R_j(\mathbf{p})}{\partial p_i} &= \tfrac{1}{\gamma} S_i S_j \tfrac{D(p_i)}{V(p_i)} (p_j - c) D(p_j) + \varepsilon (a_{ij} - c_t) S_i (1 - 2S_j) \\ &\quad + \varepsilon \textstyle\sum_{k \neq j} S_i S_k (a_{jk} - c_t) \left(1 - \tfrac{1}{\gamma} \tfrac{D(p_i)}{V(p_i)} (1 - 4S_j)(p_j - p_k)\right) \\ &\quad - \varepsilon (a_{ij} - c_t) S_i (1 - S_i) \left(1 - \tfrac{1}{\gamma}(1 - 2S_j) \tfrac{D(p_i)}{V(p_i)} (p_j - p_i)\right),\end{aligned}$$

where we have used the explicit expressions for $\partial S_i / \partial p_i$ and $\partial S_j / \partial p_i$ and $\partial S_k / \partial p_i$ as well as linear demand $D(p) = 1 - \varepsilon p$ and $D'(p) = -\varepsilon$. Since $\lim_{\varepsilon \to 0} V(p) / D(p) = v - p$,

$$\lim_{\varepsilon \to 0} \frac{\partial R_j(\mathbf{p})}{\partial p_i} = \tfrac{1}{\gamma} S_i S_j \tfrac{p_j - c}{v - p_i} \geq 0 \;\forall p_i < v,\; p_j \geq c. \tag{15}$$

To summarize, $R_i(\mathbf{p}) - R_j(\mathbf{p})$ is strictly decreasing in p_i in the interval $[c, \mathcal{P}]$, provided that ε is sufficiently small. For every $\widetilde{p}_j \in [c, \mathcal{P}]$, therefore, there can be at most one solution $\widetilde{p}_i \in [c, \mathcal{P}]$ to $R_i(\widetilde{\mathbf{p}}) - R_j(\widetilde{\mathbf{p}}) = 0$, provided that ε is sufficiently small. Since i and j charge symmetric access prices, $R_i(\widetilde{\mathbf{p}}) = R_j(\widetilde{\mathbf{p}})$ is satisfied for $\widetilde{p}_i = \widetilde{p}_j$. Thus, $R_i(\widetilde{\mathbf{p}}) \neq R_j(\widetilde{\mathbf{p}})$ for all $\widetilde{p}_i \neq \widetilde{p}_j$, which excludes the possibility of an asymmetric equilibrium. ■

Defining $\overline{\varepsilon}(\gamma, \overline{a})$ as the minimum of $\varepsilon_1(\gamma)$, $(v + c)^{-1}$, $\varepsilon_2(\gamma, \overline{a})$ and $\varepsilon_3(\gamma, \overline{a})$ completes the existence and uniqueness proof. ■

Existence and uniqueness of the continuation game Consider now the continuation game following universal agreement on a. Assume that all access prices except possibly $a_{ij} = a_{ji} = \widehat{a}$ are equal to a. Assume that $\varepsilon < \overline{\varepsilon}(\gamma, \overline{a})$ defined in Lemma 4. Applying passive beliefs, network $k \neq i, j$'s expected profit in the continuation game is

$$\pi_k(p_k, \widetilde{\mathbf{p}}_{-k}) = S_k(p_k, \widetilde{\mathbf{p}}_{-k}) \left((p_k - c) D(p_k) + \sum_{l \neq k} S_l(p_k, \widetilde{\mathbf{p}}_{-k})(a - c_t)(D(\widetilde{p}_l) - D(p_k))\right)$$

Note that the maximization problem facing network k is the same as in Lemma 4, with the exception that here, all access prices are equal to a. All networks except i and j face the same maximization problem. By virtue of Lemma 4, they all charge the same call price $\widetilde{p}_k = p^*(\gamma, a, n)$ implicitly defined in eq. (6) in Section 2. Network i's expected profit is

$$\begin{aligned}\pi_i(p_i, \widetilde{\mathbf{p}}_{-i}) &= S_i(p_i, \widetilde{\mathbf{p}}_{-i})\left[(p_i - c)D(p_i) + S_j(p_i, \widetilde{\mathbf{p}}_{-i})(\widehat{a} - c_t)(D(\widetilde{p}_j) - D(p_i))\right. \\ &\quad \left. + (1 - S_i(p_i, \widetilde{\mathbf{p}}_{-i}) - S_j(p_i, \widetilde{\mathbf{p}}_{-i}))(a - c_t)(D(p^*(\gamma, a, n)) - D(p_i))\right],\end{aligned}$$

where we have applied passive beliefs on i's expectations about the actions of the networks other than j. Note that even the maximization problem facing network i and j are identical and the same as in Lemma 4, with the exception that here $a_{ij} = a_{ji} = \widehat{a}$ and all other access prices are equal to a. Since i and j face symmetric access prices, they charge the same unique equilibrium call price $\widetilde{p}_i = \widetilde{p}_j = \widetilde{p}(\gamma, \widehat{a}, a, n)$ implicitly defined by the solution to $\partial\pi_i(p_i, \widetilde{\mathbf{p}}_{-i})/\partial p_i|_{p_i = \widetilde{p}_i} = 0$:

$$\begin{aligned}\tfrac{\widetilde{p}-c}{\widetilde{p}} = \tfrac{1}{\eta(\widetilde{p})+\sigma_i(\widetilde{\mathbf{p}})}&\Big[1 + \Big(S_i(\widetilde{\mathbf{p}})\tfrac{(\widehat{a}-c_t)}{\widetilde{p}} + (1 - 2S_i(\widetilde{\mathbf{p}}))\tfrac{(a-c_t)}{\widetilde{p}}\Big)\eta(\widetilde{p}) \\ &- [\sigma_{ji}(\widetilde{\mathbf{p}})S_i(\widetilde{\mathbf{p}}) + (1 - 3S_i(\widetilde{\mathbf{p}}))\sigma_i(\widetilde{\mathbf{p}})]\tfrac{(a-c_t)}{\widetilde{p}}\tfrac{(D(p^*)-D(\widetilde{p}))}{D(\widetilde{p})}\Big]\end{aligned}, \tag{16}$$

where $\sigma_{ji} = (\partial S_j/\partial p_i)(p_j/S_j)$ is the cross-price subscriber elasticity. As is easily verified, $\widetilde{p}(\gamma, a, a, n) = p^*(\gamma, a, n)$.

Comparative statics of the continuation game We show that $\widetilde{p}$ is non-decreasing in $\widehat{a}$. In equilibrium, $R_i(\widetilde{\mathbf{p}}) = R_j(\widetilde{\mathbf{p}}) = 0$, where R_i was defined in (13). The numerator of

$$\frac{\partial\widetilde{p}_i}{\partial\widehat{a}} = \frac{\frac{\partial R_i}{\partial p_j}\frac{\partial R_j}{\partial\widehat{a}} - \frac{\partial R_j}{\partial p_j}\frac{\partial R_i}{\partial\widehat{a}}}{\frac{\partial R_i}{\partial p_i}\frac{\partial R_j}{\partial p_j} - \frac{\partial R_j}{\partial p_i}\frac{\partial R_i}{\partial p_j}}$$

is positive for ε sufficiently low, since in that case, $\partial R_j/\partial p_j < 0$ and $\partial R_i/\partial p_j \geq 0$, whereas $\partial R_i/\partial\widehat{a} = \partial R_j/\partial\widehat{a} = -\gamma S_j\frac{D'(\widetilde{p})}{D(\widetilde{p})}V(\widetilde{p}) > 0$ always holds. Using (14) and (15),

$$\begin{aligned}\lim_{\varepsilon\to 0}\left(\tfrac{\partial R_i}{\partial p_i}\tfrac{\partial R_j}{\partial p_j} - \tfrac{\partial R_j}{\partial p_i}\tfrac{\partial R_i}{\partial p_j}\right) &= (\gamma + 1 - S_j)\left(\gamma + (1 - S_i)\left(1 + \tfrac{1}{\gamma}S_i\tfrac{p_i-c}{v-p_i}\right)\right) \\ &\quad + S_j(1 - S_j)\left(1 + \tfrac{1}{\gamma}(1 - S_i)\right)\tfrac{p_j-c}{v-p_j} \\ &\quad + \tfrac{1}{\gamma^2}S_iS_j(1 - S_j - S_i)\tfrac{p_i-c}{v-p_i}\tfrac{p_j-c}{v-p_j},\end{aligned}$$

which is strictly positive for all $p_i \in [c, \mathcal{P}]$ and $p_j \in [c, \mathcal{P}]$. Consequently, $\widetilde{p}$ is non-decreasing in $\widehat{a}$ for ε sufficiently low, but positive.

Existence of an access price equilibrium To summarize so far, for ε sufficiently low, but positive, there exists a unique equilibrium in the continuation game following universal agreement on a, but where i and j possible deviate to an alternative access price $a_{ij} = a_{ji} = \widehat{a}$. All networks except i and j charge the same call price $p^*(\gamma, a, n)$ implicitly defined in eq. (6). Networks i and j charge the same call price $\widetilde{p}(\gamma, \widehat{a}, a, n)$ implicitly defined in eq. (16). $\widetilde{p}(\gamma, \widehat{a}, a, n)$ is non-decreasing in $\widehat{a}$, with $\widetilde{p}(\gamma, a, a, n) = p^*(\gamma, a, n)$. ■

Consider the incentives of agent A_{ij} who maximizes joint profit of i and j. Substitute p^m for p^*, α^m for a and set $p_i = p_j = p$ in (10) to get joint profit

$$\pi_{ij}(p) = S(p)((p-c)D(p) + (1 - S(p))(a - c_t)(D(p^*(\gamma, a, n)) - D(p)))$$

where $S(p) = 2V(p)^{\frac{1}{\gamma}} / (2V(p)^{\frac{1}{\gamma}} + (n-2)V(p^*(\gamma, a, n))^{\frac{1}{\gamma}})$ is the total market share of networks i and j. Since $\widetilde{p}$ is non-decreasing in $\widehat{a}$, the set of prices call prices that A_{ij} can implement is restricted by $P(a) = [\widetilde{p}(\gamma, -c_o, a, n), \widetilde{p}(\gamma, \overline{a}, a, n)]$. A_{ij} thus maximizes $\pi_{ij}(p)$ over $P(a)$. Recall that a is the proposed equilibrium access price.

As in the proof of Proposition 1, joint profit can be decomposed in two parts $\pi_{ij}(p) = \widetilde{x}(p) + \varepsilon(a - c_t)\widetilde{y}(p)$ where

$$\widetilde{x}(p) = S(p)\left((p-c)D(p) + \tfrac{n-2}{n}(a - c_t)(D(p^*(\gamma, a, n)) - D(p))\right),$$

and

$$\widetilde{y}(p) = S(p)\left(\tfrac{2}{n} - S(p)\right)(p - p^*(\gamma, a, n)).$$

Claim 9 *$\widetilde{x}(p)$ has a unique maximand for every $P(a)$. The maximand is equal to p^m and $\partial^2\widetilde{x}/\partial p^2|_{p=p^m} < 0$ when $\alpha^m \leq \overline{a}$ and the price range is $P(\alpha^m)$. The maximand is equal to $p^*(\gamma, \overline{a}, n)$ and $\partial^2\widetilde{x}/\partial p^2|_{p=p^*(\gamma,\overline{a},n)} < 0$ when the price range is $P(\overline{a})$ and $\overline{a} < \alpha^m$.*

Proof. We omit the proof that $\widetilde{x}(p)$ has a unique maximand on $P(a)$, since it is analogous to the proof in Claim 1 that $x(p)$ has a unique maximand on $[0, v]$. It is easy to verify that $\partial\widetilde{x}/\partial p|_{p=p^m} = 0$, hence p^m is the unique maximand of $\widetilde{x}(p)$ on $P(\alpha^m)$. Also, $\partial^2\widetilde{x}/\partial p^2|_{p=p^m} < 0$ is straightforward in view of the algebra in the proof of Claim 1. Consider next the case when the price range is $P(\overline{a})$. After a few straightforward manipulations:

$$\frac{\partial\widetilde{x}(p)}{\partial p}|_{p=p^*(\gamma,\overline{a},n)} = -\frac{2}{n(n-1)}\left[\frac{p^*-c}{p^*} - \frac{1}{\eta(p^*)}\right]\eta(p^*)D(p^*).$$

For $\overline{a} < \alpha^m$, $p^*(\gamma, \overline{a}, n) < p^*(\gamma, \alpha^m, n) = p^m$, since p^* is increasing in a, see Proposition 1. In this case $\frac{\partial\widetilde{x}(p)}{\partial p}|_{p=p^*(\gamma,\overline{a},n)} > 0$ and i and j would benefit from a higher call price. However, this is impossible to implement because it would involve setting an access price above $\overline{a}$, which is disallowed. Finally, $\partial^2\widetilde{x}/\partial p^2|_{p=p^*(\gamma,\overline{a},n)} < 0$ is obvious in light of Claim 1. ■

$\widetilde{x}$ satisfies assumptions (i) and (ii) of Lemma 3 both for $P(\alpha^m)$ and $P(\overline{a})$. Even assumptions (iii) and (iv) of the Lemma are fulfilled, since $\widetilde{y}(p^*(\gamma, a, n)) = 0$ and $\partial\widetilde{y}/\partial p|_{p=p^*(\gamma,a,n)} = 0$ for all a. Consequently, p^m is the unique maximand of $\pi_{ij}(p) = \widetilde{x}(p) + \varepsilon(\alpha^m - c_t)\widetilde{y}(p)$ when $\alpha^m \leq \overline{a}$ and $\frac{\varepsilon}{\gamma}$ is sufficiently low, but positive. Second, $p^*(\gamma, \overline{a}, n)$ is the unique maximand of $\pi_{ij}(p) = \widetilde{x}(p) + \varepsilon(\overline{a} - c_t)\widetilde{y}(p)$ when $\alpha^m > \overline{a}$ and ε is sufficiently low, but positive. By construction $p^*(\gamma, \alpha^m, n) = p^m$. Therefore, α^m is an access price equilibrium when $\alpha^m \leq \overline{a}$, whereas $\overline{a}$ is an access price equilibrium when $\alpha^m > \overline{a}$, provided ε is sufficiently low.

Uniqueness of the access price equilibrium It cannot be the case that $a < \alpha^m < \overline{a}$ in symmetric equilibrium. For in this case, $p^*(\gamma, a, n) < p^m$, $\partial\widetilde{x}/\partial p|_{p=p^*} > 0$ and A_{ij} would benefit from setting $\widehat{a} > a$ to induce a call price $\widetilde{p} > p^*$. It cannot be the case that $a \in (\alpha^m, \overline{a}]$ in symmetric equilibrium, either. For in this case, $p^*(\gamma, a, n) > p^m$, $\partial x/\partial p|_{p=p^*} < 0$ and A_{ij} would benefit from setting $\widehat{a} < a$ to induce a call price $\widetilde{p} < p^*$. Finally, it cannot be the case that $a < \overline{a} \leq \alpha^m$ in symmetric equilibrium. For in this case, $p^*(\gamma, a, n) < p^m$ and A_{ij} would benefit from setting $\widehat{a} > a$ to induce a call price $\widetilde{p} > p^*$. ■

A.4 Proof of Proposition 2

When the price elasticity of call demand is sufficiently low, the unique equilibrium price $p^* \in [c, v)$ is given by (6). Using $D(p^*) = 1 - \varepsilon p^*$, $\eta(p^*) = \varepsilon p^*/(1 - \varepsilon p^*)$, and $\sigma(p^*) = (n-1)D(p^*)p^*/n\gamma V(p^*)$, we get the following bounds on $p^*(\gamma, a, n)$:

$$0 \leq p^* - c = \frac{\gamma\left(D(p^*) + \frac{n-1}{n}(a - c_t)\varepsilon\right)}{\gamma\varepsilon + \frac{n-1}{n}\frac{D^2(p^*)}{V(p^*)}} < \frac{\gamma\left(D(v) + (\overline{a} - c_t)\varepsilon\right)}{\gamma\varepsilon + \frac{D^2(c)}{2V(c)}} \tag{17}$$

where the second inequality follows from the fact that p^* is increasing in $\overline{a}$, $D(p^*) < D(v)$, $n \geq 2$ and $\frac{d}{dp}\left[\frac{D^2(p)}{V(p)}\right] = \frac{D(p)D^2(v)}{V^2(p)} > 0$. Since ε is bounded from above, the last term in the above equation goes to zero as γ goes to zero. Clearly, $\lim_{\gamma\downarrow 0} p^*(\gamma, a, n) = c$. ■

A.5 Two-part Tariffs

This appendix derives the access prices that maximise industry profits under two-part tariffs. The purpose is to show that introducing two-part tariffs does not add much to the analysis if the fixed fee must be non-negative and the networks are sufficiently close substitutes. Consider the case where $n \geq 2$ networks each charge a subscription fee $F_i \geq 0$ in addition to the non-discriminatory call price $p_i \geq 0$. The subscriber's indirect utility is $V(p_i) - F_i$ and i's customer base $S_i = \left((V(p_i) - F_i)^{\frac{1}{\gamma}}\right)(\sum_{j \in N} (V(p_j) - F_j)^{\frac{1}{\gamma}})^{-1}$. Each network maximizes the Lagrangian $L_i = \pi_i + \lambda_i F_i$, where

$$\pi_i = S_i \left[(p_i - c) D(p_i) + \sum_{j \neq i} S_j (a - c_t)(D(p_j) - D(p_i)) + F_i \right]$$

is the network's profit. Any symmetric equilibrium $p_i = p$ for all $i \in N$, $F_i = F$ for all $i \in N$, is given by the solutions to

$$\frac{\partial L_i}{\partial p_i} = \frac{\partial S_i}{\partial p_i} [(p - c) D(p) + F] + \frac{1}{n} \left(D(p) + \left(p - c - \frac{n-1}{n}(a - c_t) \right) D'(p) \right) = 0,$$

$$\frac{\partial L_i}{\partial F_i} = \frac{\partial S_i}{\partial F_i} [(p - c) D(p) + F] + \frac{1}{n} + \lambda = 0$$

and $\lambda F = 0$, $\lambda \geq 0$. Subtract $\partial L_i / \partial F_i$ from $\partial L_i / \partial p_i$ and use $D(p_i) \frac{\partial S_i}{\partial F_i} = \frac{\partial S_i}{\partial p_i}$ to get

$$\lambda F = -\frac{1}{n} \left(p - c - \frac{n-1}{n}(a - c_t) \right) \frac{\eta(p)}{p} F = 0. \tag{18}$$

There are two types of equilibria, First, there is the standard solution $p(a) = c + \frac{n-1}{n}(a - c_t)$, with fixed fee

$$F(\gamma, a, n) = \frac{n-1}{n-1+\gamma n} \left[\frac{n}{n-1} \gamma V \left(c + \frac{n-1}{n}(a - c_t) \right) - \frac{n-1}{n}(a - c_t) D \left(c + \frac{n-1}{n}(a - c_t) \right) \right], \tag{19}$$

where F was obtained by substituting $p = c + \frac{n-1}{n}(a - c_t)$ into $\partial L_i / \partial p_i$ and using the symmetric relation $\frac{\partial S_i}{\partial p_i} = -\frac{n-1}{\gamma n^2} \frac{D(p)}{V(p) - F}$. Second, there is a corner solution $F = 0$ and p given by $\partial \pi_i / \partial p_i = 0$, i.e. $p = p^*(\gamma, a, n)$. By using (19), we see that $F(\gamma, a) \geq 0$ if and only if

$$\gamma > \overline{\gamma}(a, n) = \frac{(n-1)^2}{n^2}(a - c_t) \frac{D\left(c + \frac{n-1}{n}(a - c_t)\right)}{V\left(c + \frac{n-1}{n}(a - c_t)\right)}.$$

The inequality is violated for all γ sufficiently low (but positive) provided $a > c_t$. Note also that $\partial \overline{\gamma} / \partial a > 0$ for all $a > c_t$ by the fact that $\frac{d}{dp}\left[\frac{D(p)}{V(p)}\right] > 0$, see the proof of Claim 5. Hence, there exists an $A(\gamma, n) = \overline{\gamma}^{-1}(\gamma)$, such that the standard solution applies if and only if $a \leq A(\gamma, n)$, and the corner solution applies if and only if $a \geq A(\gamma)$.

Consider next the profit maximizing choice of a. Since we are interested in the case with high network substitutability and a generous access price ceiling $\overline{a} > c_t$, assume γ to be sufficiently low to ensure $\alpha^m > A(\gamma, n)$ and $\overline{a} > A(\gamma, n)$ (recall that α^m is decreasing in γ, whereas $\partial A / \partial \gamma > 0$ and $A(0, n) = c_t$). For all $a \in [-c_o, A]$, the symmetric equilibrium profit is

$$\pi(a) = \tfrac{\gamma}{n-1+\gamma n} \left[\tfrac{n-1}{n} (a - c_t) D\left(c + \tfrac{n-1}{n}(a - c_t)\right) + V\left(c + \tfrac{n-1}{n}(a - c_t)\right) \right].$$

If the access charge is set collectively, i.e. if, for instance $n = 2$, the marginal profit is

$$\pi'(a) = \tfrac{\gamma}{n-1+\gamma n} \tfrac{(n-1)^2}{n^2} (a - c_t) D'\left(c + \tfrac{n-1}{n}(a - c_t)\right)$$

which implies that c_t is the collectively optimal choice of a in $[-c_o, A]$. For all $a \in [A, \overline{a}]$, the symmetric equilibrium profit is

$$\pi(a) = \tfrac{1}{n} \left(p^*(\gamma, a, n) - c\right) D\left(p^*(\gamma, a, n)\right),$$

which we know reaches its maximum at $\min\{\alpha^m, \overline{a}\}$. The profit function is non-monotonic in a with two local maximands c_t and $\min\{\alpha^m, \overline{a}\}$. Which of these is the global maximand depends on γ. Note that $\pi(\alpha^m) = \frac{1}{n}(p^m - c) D(p^m)$ is independent of γ and $\pi(c_t) = \frac{\gamma}{n-1+\gamma n} V(c)$ vanishes in the limit as $\gamma \to 0$. Hence, $\pi(\alpha^m) > \pi(c_t)$ for γ sufficiently low. Comparing $\pi(c_t)$ and $\pi(\overline{a})$ is more difficult since also $\lim_{\gamma \to 0} \pi(\overline{a}) = 0$. Note, however, that

$$\begin{aligned} \frac{\pi(\overline{a})}{\pi(c_t)} = \frac{\frac{1}{n}(p^*-c)D(p^*)}{\frac{\gamma}{n-1+\gamma n}V(c)} &= \frac{n-1+\gamma n}{n} \frac{D(p^*)}{V(c)} \frac{(p^*-c)}{\gamma} \\ &= \frac{n-1+\gamma n}{n} \frac{D(p^*(\gamma,a,n))}{V(c)} \frac{D(p^*) + \frac{n-1}{n}(a-c_t)\varepsilon}{\gamma\varepsilon + \frac{n-1}{n}\frac{D^2(p^*)}{V(p^*)}} \end{aligned}$$

where the second equality follows from substituting in (17).

$$\lim_{\gamma \to 0} \frac{\pi(\overline{a})}{\pi(c_t)} = 1 + \frac{(n-1)(\overline{a} - c_t)\varepsilon}{nD(c)} > 1$$

implies that even $\pi(\overline{a}) > \pi(c_t)$ for γ sufficiently low. It follows that the access charge that maximises industry profit is $a^* = \min\{\alpha^m, \overline{a}\}$, the subscription fee is 0 and the call price $p^*(\gamma, a^*, n)$ in symmetric equilibrium, provided that $\overline{a} > c_t$ and networks are sufficiently close substitutes.