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Non-Regulated Next Generation Networks**

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Incentives to Invest and to Give Access to Non-Regulated Next Generation Networks*

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Abstract

We analyze the incentives of a telecommunications incumbent to invest and give access to a downstream entrant to a next generation network, *NGN*. We model the industry as a duopoly, where a vertically integrated incumbent and a downstream entrant, that requires access to the incumbent's network, compete on Hotelling's line. The incumbent can invest in the deployment of a NGN that improves the quality of the retail services. Access to the old network is regulated, but access to the NGN is not. If the innovation is *drastic*, the incumbent always invests in the NGN, but does not give access to the entrant. If the innovation is *non-drastic* and if the access price to the old network is low, the incumbent voluntarily gives access to the NGN. If the innovation is *non-drastic*, there is no monotonic relation between the access price to the old network and the incumbent's incentives to invest. A regulatory *moratorium* emerges as socially optimal, if the innovation is large but *non-drastic*. We also analyze the case where both firms can invest in the deployment of a NGN.

Keywords: Next Generation Networks, Investment, Access, Regulation.

JEL Classification: L43, L51, L96, L98.

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1 Introduction

The deployment of next generation networks, *NGNs*, leading to a multi-service network for audio, video, and data services, sets the telecommunications sector on the verge of a new era.¹ In order to give firms the right incentives to invest, and to promote an efficient use of these infrastructures, sectorial regulators must set an adequate regulatory framework for these new telecommunications networks.

Sectoral regulators are considering three main regulatory approaches:

(i) The *Continuity approach*: This approach consists of maintaining the current regulatory system. Accordingly, the review of the market analysis follows the course already established, with the recourse to regulatory instruments in force and, presumably, with a substantial confirmation of the main regulatory measures in place.

(ii) The *Forbearance approach*: This approach consists of the abstention from regulatory intervention with regard to broadband networks. The absence of intervention may be permanent or temporary.

(iii) The *Equality of Access approach*: This approach consists on the enforcement of the principle of “equality of access” to NGNs, and the principle of “equivalence of input” to the incumbent’s wholesale offer, with a concurring progressive liberalization of the retail offers.

In this article, we analyze, in the context of the forbearance approach, the incentives of a telecommunications incumbent to: **(i)** invest in a NGN, and **(ii)** give access to the network to a downstream entrant.

We model the telecommunications industry as a differentiated products duopoly, where an incumbent and an entrant compete on Hotelling’s line (Hotelling, 1929). The incumbent is a vertically integrated firm that owns a network, and operates on the retail market. The entrant operates on the retail market, and requires access to the incumbent’s network. The incumbent can invest in the deployment of a NGN that improves the quality of the retail services. Access to the old network is regulated, but access to the new network is not.²

In this context, conceding access to the NGN amounts to allowing the entrant to increase the quality of its product, and has two opposing effects on the incumbent’s profit. First, it has the negative effect of reducing the retail profits of the incumbent: the *retail effect*.

¹A *Next Generation Network* is a "(...) packet-based network able to provide telecommunication services and able to make use of multiple broadband, QoS-enabled transport technologies and in which service-related functions are independent from underlying transport related technologies." See ITU (2001).

²Not regulating the new network corresponds, e.g., to the case of the US, where Verizon is deploying a next generation network, but is only obliged to offer to entrants wholesale services equivalent to those it already offered through the old network.

Second, it has the positive effect of increasing the wholesale profits of the incumbent: the *wholesale effect*. To understand this last effect note that, if the entrant uses the NGN, it produces a higher quality product, and thereby earns higher profits. This allows the incumbent to charge a higher access price, which increases wholesale profits.

We distinguish two cases: **(i)** the quality improvement generated by the investment is large – *drastic innovation*, and **(ii)** the quality improvement generated by the investment is small – *non-drastic innovation*. In the latter case, the entrant, using the old network, can compete against the incumbent, using the new network, if the access price is low enough. In the former case it cannot, even if the access price equals marginal cost.

If the innovation is *drastic*, the incumbent always invests in the NGN, but does not give access to the entrant, which is forced out of the market.

If the innovation is *non-drastic*, the regulator can control, through the regulation of the old network, whether the incumbent: **(i)** invests in a NGN, and **(ii)** gives access to the NGN.

If the access price to the old network is low, the wholesale profits on the old network are small, and therefore the wholesale effect dominates the retail effect. Thus, if the innovation is *non-drastic* and if the access price to the old network is low, the incumbent voluntarily gives access to the NGN. In this case, an increase in the access price to the old network decreases the incremental profit from investment, because it raises the incumbent's profit without investment. If the access price to the old network is high, the entrant exits the market after investment. Since the incumbent faces no competition, the incremental profit from the investment increases. Thus, if the innovation is *non-drastic*, there is no monotonic relation between the access price to the old network and the incumbent's incentives to invest.

Interestingly, a regulatory *moratorium* emerges as socially optimal, if the innovation is *non-drastic*, but large.

We also analyze the case where both firms can invest in the deployment of a NGN. If the investment cost is small, the possibility of both firms deploying a NGN may increase or decrease welfare, compared with the case where only the incumbent can invest. If the investment cost is large, the possibility of both firms investing never increases welfare, although this does not result from a duplication of the investment cost.

The academic literature on regulation only recently started to address the relation between access pricing and investment. Guthrie (2006) surveys the recent literature on the relationship between infrastructure investment and the different regulatory regimes. He concludes that much remains to be done. Valletti (2003) argues that it is important that

regulatory policies are designed in a way that enables regulators to commit to rules over a reasonable time period. Regulators should try to stabilize their policies to signal to firms that they can commit to their decisions.

Gans (2001) analyzes an investment timing game where two firms compete to invest in a new technology, but there is only one investment. In this case the regulator can induce the leader to invest at the socially optimal date through the use of the access price. This is a consequence of the preemption effect, typical of the investment race literature. Gans and King (2004) study the impact of access regulation on the timing of infrastructure investment, when there is uncertainty about the investment returns. This article suggests the use of a regulatory *moratorium* when the regulator has commitment problems. Vareda and Hoernig (2007) study the investment of two operators in new infrastructures, which allows them to offer new services, and show that a regulatory *moratorium* may be a necessary tool to give the leader the correct incentives to invest, at the same time that allows to charge a lower access price later on, in order to delay the follower's investment. Vareda (2007) studies the incumbent's incentives to invest in quality upgrades and cost reduction when the regulator forces it to unbundle its network. Foros (2004) shows that under some conditions the investment by an incumbent in the quality of its network is lower with price regulation since the access price is set equal to marginal cost. Kotakorpi (2006) considers a model with vertical differentiation, and obtains similar results. Brito et al. (2008) analyze if two-part access tariffs solve the dynamic consistency problem of the regulation of NGNs. Caillaud and Tirole (2004) analyze the funding of an infrastructure when an incumbent has private information about the profitability of the investment and the regulator does not have access to taxpayers' money.

The remainder of the article is organized as follows. We describe the model in Section 2. In Section 3, we characterize the equilibrium of the game. In Section 4, we discuss an extension. Finally, in Section 5, we conclude. All proofs are in the Appendix.

2 Model

2.1 Environment

Consider a telecommunications industry where two firms, the incumbent and the entrant, sell horizontally differentiated products. The *incumbent*, firm i , is a vertically integrated firm that owns a bottleneck input, to which we refer to as the old network. The *old network*, network o , is a telephone network with a local access network based on the twisted pair of

copper wire. The incumbent can make an investment to deploy a next generation network. The next generation network is also a bottleneck input that allows the supply of retail products of a higher quality than those supplied through the old network. We refer to the next generation network as the *new network*, or network n . The *entrant*, firm e , only operates in the retail market, and has to buy access to the network of the incumbent. We index firms with subscript $j = i, e$, and networks with subscript $v = o, n$. There is a third party in the industry, the *sectoral regulator*. Even if the new network is deployed, the incumbent must offer access to the old network at a regulated access price. Access to the new network is not mandatory. However, the incumbent may voluntarily sell access to the entrant. Costs and demand are common knowledge.

The game has five stages which unfold as follows. In stage 1, the sectoral regulator sets the access price to the old network. In stage 2, the incumbent makes an investment decision. In stage 3, if investment took place, the incumbent offers the entrant an access price to the new network. In stage 4, the entrant chooses which network it will use, if any. In stage 5, the incumbent and the entrant compete on retail tariffs.

2.2 Sectoral Regulator

The regulator sets the per unit access price of telecommunication services the entrant must pay to incumbent to have access to old network, the *access price*, denoted by α_o on $[0, +\infty)$.³

The regulator maximizes social welfare, i.e., the sum of the firms' profits and the consumer surplus, denoted by W .

2.3 Consumers

There is a large number of consumers, formally a continuum, whose measure we normalize to 1. Consumers are uniformly distributed along a Hotelling line segment of length 1, facing transportation costs tx to travel distance x , with t on $(0, +\infty)$. Consumers are otherwise homogeneous. As in Biglaiser and DeGraba (2001), we assume each consumer has a demand function for telecommunication services given by $y_j = (z + \Delta_v) - p_j$, where y_j on $(0, z + \Delta_v)$ is the number of units of telecommunication services purchased from firm j , p_j on $(0, z + \Delta_v)$ is the per unit price of telecommunication services of firm j , z is a parameter on $(\frac{4}{3}\sqrt{6t}, +\infty)$, and Δ_v is a parameter that takes value 0 for products supplied through the old network, i.e.,

³Regulating telecommunications markets by intervening at the wholesale level, namely by setting access prices, corresponds to the current EU and US practice.

for $v = o$, and takes value Δ_z on $(0, +\infty)$ for products supplied through the new network, i.e., for $v = n$.⁴ This means that consumers are willing to pay a premium for services delivered over the new network. The lower limit on z implies that all consumers have a positive surplus under the different market structures.

Let $\chi := \Delta_z(2z + \Delta_z)$. For $p_j = 0$, the incremental consumer surplus from the investment is $\frac{1}{2}\chi$. We take χ as a measure of quality improvement enabled by the next generation network.

2.4 Firms

The incumbent produces an input that: **(i)** uses in the production of a retail product, or **(ii)** sells to the entrant.

The incumbent is located at point 0 and the entrant at point 1 of the line segment where consumers are distributed.

Firms charge consumers two-part retail tariffs, denoted by $T_j(y_j) = F_j + p_j y_j$, $j = i, e$, where F_j on $[0, +\infty)$ is the fixed part of firm j .

The incumbent can invest in a new network at a fixed cost of I . We assume that I belongs to $(0, \frac{1}{2}\chi)$. The upper limit on I ensures that the investment on a new network is socially optimal. In Section 4 we allow both firms to invest.

Regarding the quality improvement enabled by the new network, we distinguish two cases: **(i)** if Δ_z is on $[\sqrt{z^2 + 6t} - z, +\infty)$, we say that the investment generates a *drastic innovation*; **(ii)** if Δ_z is on $[0, \sqrt{z^2 + 6t} - z)$, we say that the investment generates a *non-drastic innovation*. For notational convenience we use the equivalent condition that χ is on $[6t, +\infty)$ for *drastic innovation*, and that χ is on $[0, 6t)$ for *non-drastic innovation*.

All of the incumbent's marginal costs are constant and equal to zero. The entrant has marginal cost α_v on $\{\alpha_o, \alpha_n\}$, which corresponds to the unit price paid for access to the old or new network.

Given (α_o, α_n) , the entrant can either: **(i)** accept offer α_n , and use the new network, **(ii)** accept offer α_o , and use the old network, or **(iii)** reject α_n and α_o , and exit the market.

Denote by D_j , the demand, in terms of consumers, for firm $j = i, e$. The profits of firm

⁴Units of telecommunication services could be, e.g., minutes of communication or megabits.

$j = i, e$ for the whole game are:⁵

$$\pi_i = [p_i(z + \Delta_v - p_i) + F_i] D_i + \alpha_v(z + \Delta_v - p_e) D_e - \frac{\Delta_v}{\Delta_z} I, \quad (1)$$

$$\pi_e = [(p_e - \alpha_v)(z + \Delta_v - p_e) + F_e] D_e. \quad (2)$$

2.5 Equilibrium Concept

The sub-game perfect Nash equilibrium is: **(i)** a regulated access tariff for the old technology, **(ii)** an investment decision, **(iii)** an access price for the new network, **(iv)** a decision of which network to use, and **(v)** a set of retail tariffs, such that:

(E1) the retail tariffs maximize the firms' profits, given the access tariffs, the investment decision and the entry decision;

(E2) the entry decision maximizes the entrant's profits, given the access tariffs, the incumbent's investment decision, and the optimal retail tariffs function;

(E3) the access tariff for the new network maximizes the incumbent's profits, given the access tariff for the old network, the incumbent's decision to invest, the optimal entry decision and the optimal retail tariffs function;

(E4) the investment decision maximizes the incumbent's profits, given the access tariff for the old network, the optimal access tariff function for the new network, the optimal entry decision and the optimal retail tariffs function;

(E5) the access tariff for the old network maximizes social welfare, given the optimal investment decision, the optimal access tariff function for the new network, the optimal entry decision and the optimal retail tariffs function.

3 Equilibrium

In this Section, we characterize the equilibrium of the game, which we construct by working backwards.

3.1 Retail Price Stage

We characterize the equilibrium of the retail price game for five cases: **(i)** the incumbent does not invest in the new network, and the entrant exits the market, **(ii)** the incumbent

⁵For each consumer served by the entrant the incumbent earns $\alpha_v(z + \Delta_v - p_e)$, i.e., the wholesale markup times the number of minutes sold to each consumer. This represents the opportunity cost for the incumbent of serving directly each consumer.

invests in the new network, and the entrant exits the market **(iii)** the incumbent does not invest in the new network, and the entrant stays in the market, **(iv)** the incumbent invests in the new network, the entrant stays in the market, and selects the old network, and **(v)** the incumbent invests in the new network, the entrant stays in the market, and selects the new network. In cases (i)-(ii) the retail market is a monopoly. In cases (iii)-(v) the retail market is a duopoly. We use superscripts m_o, m_n, d_o, d_b, d_n to denote variables or functions associated with cases (i)-(v), respectively. In what follows we use the expression "net" as a shorthand for "net of the investment cost".

We start with the following Lemma.

Lemma 1: *In equilibrium, firms set the marginal price of the two-part retail tariff at marginal cost, i.e., $p_i = 0$ and $p_e = \alpha_v$, $v = o, n$.* ■

As usual with two-part tariffs, firms set the variable part of the retail tariff at marginal cost to maximize the gross consumer surplus, and then try to extract this surplus using the fixed part.

Given Lemma 1, from now on we only discuss the determination of the fixed fees.

3.1.1 Monopoly

Next, we characterize the equilibria of the retail game for the two cases where the retail market is a monopoly, which is given by the next Lemma.

Lemma 2: *If the retail market is a monopoly, in equilibrium, the incumbent charges the fixed fee, for $v = n, o$:*

$$F_i^{m_v}(z, \Delta_v) = \frac{(z + \Delta_v)^2}{2} - t. \quad (3)$$

■

The net profits of the incumbent for $v = n, o$, are:

$$\pi_i^{m_v}(z, \Delta_v) = \frac{(z + \Delta_v)^2}{2} - t. \quad (4)$$

3.1.2 Duopoly

Next, we characterize the equilibria of the retail price game for the three cases where the retail market is a duopoly, which is given by the next Lemma.

Let $\Delta_D := (\Delta_i - \Delta_e)(2z + \Delta_i + \Delta_e)$. Parameter Δ_D measures the incumbent's quality advantage with respect to the entrant.⁶

Lemma 3: *If the retail market is a duopoly, in equilibrium, the incumbent and the entrant charge fixed fees, for $j = o, b, n$:*

$$F_i^{d_j}(\Delta_i, \Delta_e, \alpha; z) = \begin{cases} t + \frac{1}{6}\alpha[6(z + \Delta_e) - 5\alpha] + \frac{1}{6}\Delta_D & \text{for } \alpha \text{ on } [0, \sqrt{6t - \Delta_D}) \\ \alpha(z + \Delta_e) - \frac{1}{2}\alpha^2 - t + \frac{1}{2}\Delta_D & \text{for } \alpha \text{ on } [\sqrt{6t - \Delta_D}, z + \Delta_e] \end{cases}$$

$$F_e^{d_j}(\Delta_i, \Delta_e, \alpha; z) = \begin{cases} t - \frac{1}{6}\alpha^2 - \frac{1}{6}D_j & \text{for } \alpha \text{ on } [0, \sqrt{6t - \Delta_D}) \\ 0 & \text{for } \alpha \text{ on } [\sqrt{6t - \Delta_D}, z + \Delta_e]. \end{cases}$$

■

The net profits of the incumbent and the entrant, gross of the fixed component of the access tariff, for $j = o, b, n$, are, respectively:

$$\pi_i^{d_j}(\Delta_i, \Delta_e, \alpha; z) = \begin{cases} \frac{(36t^2 + \alpha^4 - 60t\alpha^2) + 72\alpha t(z + \Delta_e) + \Delta_D(12t + \Delta_D + 2\alpha^2)}{72t} & \text{for } \alpha \text{ on } [0, \sqrt{6t - \Delta_D}) \\ \alpha(z + \Delta_e) - \frac{1}{2}\alpha^2 - t + \frac{1}{2}\Delta_D & \text{for } \alpha \text{ on } [\sqrt{6t - \Delta_D}, z + \Delta_e] \end{cases}$$

and

$$\pi_e^{d_j}(\Delta_i, \Delta_e, \alpha; z) = \begin{cases} \frac{1}{72t}[6t - (\Delta_D + \alpha^2)]^2 & \text{for } \alpha \text{ on } [0, \sqrt{6t - \Delta_D}) \\ 0 & \text{for } \alpha \text{ on } [\sqrt{6t - \Delta_D}, z + \Delta_e] \end{cases}$$

In a duopoly, the profit of the incumbent is non-decreasing in the marginal access price, while the profit of the entrant is non-increasing in the marginal access price.⁷ When the marginal access price increases, the marginal cost of the entrant increases relative to that of the incumbent. As a consequence, the market share, and thereby the profit of the incumbent increases, while the entrant's profit decreases.

3.2 Network Choice Stage

Next, we analyze the entrant's decision of which network to use.

⁶Clearly, in case $d_j = d_o$ we have $\Delta_i = \Delta_e = 0$, in case $d_j = d_n$ we have $\Delta_i = \Delta_e = \Delta_z$, and in case $d_j = d_b$ we have $\Delta_i = \Delta_z$ and $\Delta_e = 0$. Hence, in cases $d_j = d_o$ and $d_j = d_n$ we have $\Delta_D = 0$, and in case $d_j = d_b$ we have $\Delta_D = \chi$. For simplicity we omit the superscript d_j in Δ_D , Δ_i and Δ_e .

⁷The first part follows from the assumption that z belongs to $(\frac{4}{3}\sqrt{6t}, +\infty)$.

Lemma 4: (i) *Let there be no investment. The entrant:*

$$\begin{cases} \text{accepts } \alpha_o & \text{for } \alpha_o \text{ on } [0, \sqrt{6t}) \\ \text{exits} & \text{for } \alpha_o \text{ on } [\sqrt{6t}, +\infty). \end{cases}$$

(ii) *Let there be investment, and let the innovation be drastic, i.e., let χ be on $[6t, +\infty)$.*

The entrant:

$$\begin{cases} \text{accepts } \alpha_n & \text{for } (\alpha_o, \alpha_n) \text{ on } [0, +\infty) \times [0, \sqrt{6t}) \\ \text{exits} & \text{for } (\alpha_o, \alpha_n) \text{ on } [0, +\infty) \times [\sqrt{6t}, +\infty). \end{cases}$$

(iii) *Let there be investment, and let the innovation be non-drastic, i.e., let χ be on $[0, 6t)$. The entrant:*

$$\begin{cases} \text{accepts } \alpha_n & \text{for } (\alpha_o, \alpha_n) \text{ on } [0, \sqrt{6t - \chi}) \times [0, \sqrt{\alpha_o^2 + \chi}) \\ \text{accepts } \alpha_n & \text{for } (\alpha_o, \alpha_n) \text{ on } [\sqrt{6t - \chi}, +\infty) \times [0, \sqrt{6t}) \\ \text{accepts } \alpha_o & \text{for } (\alpha_o, \alpha_n) \text{ on } [0, \sqrt{6t - \chi}) \times [\sqrt{\alpha_o^2 + \chi}, +\infty) \\ \text{exits} & \text{for } (\alpha_o, \alpha_n) \text{ on } [\sqrt{6t - \chi}, +\infty) \times [\sqrt{6t}, +\infty). \end{cases}$$

■

If there is investment and the innovation is drastic, the entrant, when using the old network, cannot compete against the incumbent, using the new network. Thus, cases (i) and (ii) are similar in the sense that there are only two viable alternatives for the entrant: accepting access to the same network that the incumbent is using, or exiting the market. In case (i) the entrant's decision depends only on the regulator's choice of α_o , whereas in case (ii) it is the incumbent's choice of α_n that is relevant for the entrant's decision. The entrant either uses the same network as the incumbent, if the access price is not too high, i.e., if α_o is on $[0, \sqrt{6t})$, or exits the market.

Case (iii) presents a third alternative for the entrant if α_o is sufficiently low and α_n is sufficiently high: accepting access to the old network when the incumbent is using the new one.

3.3 Access Price Offer Stage

Next, we characterize the incumbent's equilibrium access price offer, which is presented in the next Lemma. Denote by $\alpha_n^*(\alpha_o)$, the optimal value of the access price to the new network, given α_o .

Lemma 5: (i) Let the innovation be drastic, i.e., let χ be on $[6t, +\infty)$. In equilibrium, the incumbent offers: $\alpha_n^*(\alpha_o) = \sqrt{6t}$, for all α_o on $[0, +\infty)$. (ii) Let the innovation be non-drastic, i.e., let χ be on $[0, 6t)$. In equilibrium, the incumbent offers:

$$\alpha_n^*(\alpha_o) = \begin{cases} \sqrt{\alpha_o^2 + \chi} & \text{for } \alpha_o \text{ on } [0, \sqrt{6t - \chi}) \\ \sqrt{6t} & \text{for } \alpha_o \text{ on } [\sqrt{6t - \chi}, +\infty). \end{cases}$$

■

If the innovation is drastic, the entrant, using the old network, cannot compete against the incumbent, using the new network. Thus, the incumbent offers a unacceptably high access price, i.e., $\alpha_n^*(\alpha_o) = \sqrt{6t}$, to induce the entrant to exit, and thereby become a monopolist. The same happens when the innovation is non-drastic, but the access price to the old network is high enough.

If the innovation is non-drastic and the access price to the old network is low, the entrant, using the old network, can compete against the incumbent, using the new network. Thus, since the incumbent cannot avoid competition from the entrant, it prefers to offer also a low access price to the new network. Conceding access to the new network allows the entrant to increase the quality of its product, and has two opposing effects on the incumbent's profit. First, it has the negative effect of reducing the retail profits of the incumbent: the *retail effect*. Second, it has the positive effect of increasing the wholesale profits of the incumbent: the *wholesale effect*. To understand this last effect note that, if the entrant uses the new network, it produces a higher quality product, and thereby earns higher profits. This allows the incumbent to charge a higher access price, which increases wholesale profits. The latter effect dominates.⁸

[Figure 1]

Figure 1 illustrates the entrant's equilibrium network choices as well as incumbent's access price offers.

3.4 Investment Stage

Next, we analyze the incumbent's decision to invest in the new network.

⁸This happens because the market is covered, and therefore, all consumers that the entrant captures are lost by the incumbent. If the market was partially covered, the incumbent would benefit additionally from the entrant's consumers that would otherwise be out of the market.

For $(\alpha_o, \Delta_z; z)$ on $[0, \sqrt{6t - \chi}] \times [0, \sqrt{z^2 + 6t} - z] \times (\frac{4}{3}\sqrt{6t}, +\infty)$, denote by

$$\Delta\Pi^i(\alpha_o, \Delta_z; z) := \pi_i^{d_n}(\Delta_z, \Delta_z, \alpha_n^*(\alpha_o); z) - \pi_i^{d_o}(0, 0, \alpha_o; z), \quad (5)$$

the incumbent's incremental profit from the investment, given that in stage 3 it offers an access price $\alpha_n^*(\alpha_o)$.⁹

The next auxiliary Remark states some useful properties of function $\Delta\Pi^i(\cdot)$.

Remark 1: Let values z_1 and z_2 be such that $\frac{4}{3}\sqrt{6t} < z_1 < z_2 < +\infty$. Function $\Delta\Pi^i(\cdot)$:¹⁰

(i) is quasi-convex with respect to α_o , and takes values below $\frac{1}{2}\chi$, if and only if, z is on $(\frac{4}{3}\sqrt{6t}, z_1)$,¹¹

(ii) is strictly decreasing with respect to α_o , and takes values below $\frac{1}{2}\chi$, if and only if, z is on (z_1, z_2) ;

(iii) is strictly decreasing with respect to α_o , and never takes values below $\frac{1}{2}\chi$, if and only if, z is on $(z_2, +\infty)$. ■

The following Lemma characterizes the optimal investment decision.

Lemma 6: (i) Let the innovation be drastic, i.e., let χ be on $[6t, +\infty)$. The incumbent invests for all α_o on $[0, +\infty)$. (ii) Let the innovation be non-drastic, i.e., let χ be on $[0, 6t)$.

The incumbent:

$$\begin{cases} \text{invests} & \text{for } (\alpha_o, I) \text{ on } [\sqrt{6t - \chi}, +\infty) \times [0, \frac{1}{2}\chi) \\ & \cup [0, \sqrt{6t - \chi}] \times [0, \Delta\Pi^i(\alpha_o, \Delta_z, z)) \\ \text{does not invest} & \text{for } (\alpha_o, I) \text{ on } [0, \sqrt{6t - \chi}] \times [\Delta\Pi^i(\alpha_o, \Delta_z, z), \frac{1}{2}\chi). \end{cases}$$

■

If the innovation is drastic, the incumbent always invests. This allows it to foreclose the market, and thus obtain a monopoly position.

Given our assumptions regarding I , the incumbent invests for any (I, α_o) , if z is sufficiently high, i.e., if z is on $(z_2, +\infty)$. If z is high, the investment option becomes more profitable, given that the increase in the access price received by the incumbent when it

⁹Recall that Δ_z on $[0, \sqrt{z^2 + 6t} - z]$ is equivalent to χ on $[0, 6t)$.

¹⁰Thresholds z_1 and z_2 are functions of (t, χ) . The functional forms of z_1 and z_2 are presented in the Appendix.

¹¹Note that this is impossible for $\chi > 0.428t$.

invests extends to a larger number of units. However, for lower values of z , if the innovation is non-drastic, and α_o is on $[0, \sqrt{6t - \chi})$, the incumbent may not always find it profitable to invest.

An interesting implication of the properties of $\Delta\Pi^i(\cdot)$ presented in Remark 1 is that the decision to invest may not be monotonic in α_o . Let z be on (z_1, z_2) . For α_o on $[0, \sqrt{6t - \chi})$, increasing α_o makes the no-investment decision more profitable, and hence discourages investment. For a sufficiently high α_o the incumbent will prefer not to invest for some values of I . However, for α_o on $[\sqrt{6t - \chi}, +\infty)$, the entrant's marginal cost is so large that it will not be able to compete with the incumbent, if the incumbent invests. This happens because in addition to the marginal cost disadvantage, the entrant will also sell an inferior service. For the same values of I , the incumbent will then invest and become a monopolist.

Furthermore, for z on $(\frac{4}{3}\sqrt{6t}, z_1)$, the decision to invest is not monotonic in α_o , even for values of the access price for which investing does not foreclose the market. Increasing α_o may make the incumbent switch from an investment decision to a no-investment decision, but the opposite may also occur. Take, for instance, the limit case of $\alpha_o = 0$. Then, as $\frac{\partial\alpha_n^*}{\partial\alpha_o}\Big|_{\alpha_o=0} = 0$, a small increase in α_o does not change the incumbent's profit when there is investment and access is granted at $\alpha_n^*(\alpha_o)$. However, the incumbent's profit when there is no-investment increases with α_o , given our assumption on z . Therefore, for small values of α_o , a small increase in the regulated access price decreases the incentives to invest.

As $\frac{\partial\alpha_n^*}{\partial\alpha_o}\Big|_{\alpha_o>0} < 1$, an increase in α_o always leads to a smaller increase in $\alpha_n^*(\alpha_o)$. But, since $\alpha_n^*(\cdot)$ is convex in α_o , the increase in $\alpha_n^*(\cdot)$ gets closer to the increase in α_o the higher α_o is, and thus the effect of a lower increase in α_n^* becomes weaker for higher values of α_o .¹² Additionally, as the investment increases the number of units purchased by each consumer, an increase in $\alpha_n^*(\cdot)$ affects a larger number of units than the increase in α_o . This may increase the incentives to invest.

3.5 Regulation of the Old Network Stage

Next, we discuss the regulator's choice of the access price to the old network.

The next Remark presents the regulator's objective function.

Remark 2:

¹²A lower value of z also makes $\frac{\partial\alpha_n^*}{\partial\alpha_o}\Big|_{\alpha_o>0}$ closer to 1.

(I) Welfare: (i) under monopoly is given by:

$$W^{m_v}(\Delta_v; z) = \frac{(z + \Delta_v)^2}{2} - \frac{1}{2}t - \frac{\Delta_v}{\Delta_z}I. \quad (6)$$

(ii) under duopoly with both firms using network $v = o, n$, is given by:

$$W^{d_v}(\Delta_v, \Delta_v, \alpha_v; z) = \begin{cases} \frac{72t(z+\Delta_v)^2 + 5\alpha_v^4 - 36t(t+\alpha_v^2)}{144t} - \frac{\Delta_v}{\Delta_z}I & \text{for } \alpha_v \text{ on } [0, \sqrt{6t}) \\ \frac{(z+\Delta_v)^2}{2} - \frac{1}{2}t - \frac{\Delta_v}{\Delta_z}I & \text{for } \alpha_v \text{ on } [\sqrt{6t}, z + \Delta_v]. \end{cases}$$

(II) Function $W^{d_v}(\cdot)$ is decreasing in α_v for α_v on $(0, \sqrt{\frac{18}{5}t})$, and increasing in α_v for α_v on $(\sqrt{\frac{18}{5}t}, \sqrt{6t})$. ■

[Figure 1]

Figure 1 illustrates the welfare function, $W^{d_v}(\cdot)$.¹³ Function $W^{d_v}(\cdot)$ is quasi-convex in α_v because increasing α_v has the following three effects. First, it has the negative effect of increasing transportation costs. Second, it has the negative effect of leading the entrant to set higher marginal retail price. Third, it has the positive effect of making some consumers shift from the entrant, where they have face a higher marginal retail price, to the incumbent, where they face a lower marginal retail price. If the access price is zero, the third effect is absent because the marginal price set by both firms is equal. Thus, increasing α_v unambiguously lowers welfare. If α_v is sufficiently high, the third effect may more than compensate the other two.

For a given α , investment may shift the welfare function upwards or downwards. This occurs because

$$W^{d_n}(\Delta_z, \Delta_z, \alpha; z) - W^{d_o}(0, 0, \alpha; z) = W^{m_n}(\Delta_z; z) - W^{m_o}(0; z) = \frac{\chi}{2} - I,$$

may be positive or negative. However, for investment to occur the regulator may need to change the value of α .

The regulator's only instrument, α_o , affects welfare in two different ways. If innovation is non-drastic and there is investment, the regulator's choice of α_o affects welfare, since $\alpha_n^*(\cdot)$ is increasing in α_o . Furthermore, if z is on $(\frac{4}{3}\sqrt{6t}, z_2)$, the decision to invest may depend on α_o , and thus the regulator can use α_o to affect the investment decision as well as the access price.

¹³The welfare function, $W^{d_v}(\cdot)$ is quasi-convex in α_v . Additionally, $W^{d_v}(\Delta_v, \sqrt{\frac{6t}{5}}; I) = W^{d_v}(\Delta_v, \sqrt{6t}; I) = W^{m_v}(\Delta_v, I)$, and $W^{d_v}(\Delta_v, 0; I) > W^{m_v}(\Delta_v, I)$.

If both firms use the same network and the entrant pays access price α_v , duopoly is socially preferable to monopoly, if and only if, α_v is on $\left[0, \sqrt{\frac{6}{5}t}\right)$ because:

$$W^{d_v}(\Delta_v, \Delta_v, \alpha_v; z) - W^{m_v}(\Delta_v; z) = \frac{(6t - \alpha_v^2)(6t - 5\alpha_v^2)}{144t}.$$

This occurs due to the following trade-off: monopoly leads to higher transportation costs, while, in duopoly, consumers served by the entrant purchase a suboptimal quantity if the marginal access price exceeds the marginal cost. Thus, if the network in use is the old network, the regulator prefers a duopoly if α_o is on $\left[0, \sqrt{\frac{6}{5}t}\right)$. If the network in use is the new network, the regulator prefers a duopoly if $\alpha_n^*(\alpha_o)$ is on $\left[0, \sqrt{\frac{6}{5}t}\right)$, or equivalently, if α_o is on $\left[0, \sqrt{\frac{6}{5}t - \chi}\right)$.

The next Lemma characterizes the socially optimal access price to the old network.

Lemma 7: *(i) Let the innovation be drastic, i.e., let χ be on $[6t, +\infty)$. In equilibrium, the regulator sets α_o on $[0, +\infty)$. (ii) Let the innovation be non-drastic, i.e., let χ be on $[0, 6t)$. In equilibrium, the regulator sets:*

$$\alpha_o = \begin{cases} 0 & \text{for } \chi \text{ on } \left[0, \frac{6}{5}t\right) \\ \sqrt{6t} & \text{for } \chi \text{ on } \left[\frac{6}{5}t, 6t\right). \end{cases}$$

■

If the innovation is drastic, the access price for the old network is irrelevant. From Lemma 6, the incumbent invests, for all α_o , and thereby the entrant exits the market, independently of the access price to the old network that the regulator sets.

If the innovation is non-drastic, for low values of χ , the regulator sets an α_o that leads to duopoly, while for high values of χ , the regulator sets an α_o that leads to a monopoly.

3.6 Equilibrium of the Whole Game

Having solved all the five stages of the commitment game, we can now summarize the equilibrium of the whole game, which we present in the next Proposition for further reference.

Proposition 1:

(I) If χ is on $\left[0, \frac{6}{5}t\right)$: (i) the regulator sets $\alpha_o = 0$, (ii) the incumbent invests, (iii) the incumbent offers $\alpha_n^ = \sqrt{\chi}$, (iv) the entrant accepts this offer, and (v) the incumbent and the entrant set $F_i^{d_n}(\Delta_v, \Delta_v, \sqrt{\chi}; z)$ and $F_e^{d_n}(\Delta_v, \Delta_v, \sqrt{\chi}; z)$, respectively.*

(II) If χ is on $[\frac{6}{5}t, 6t)$: (i) the regulator sets $\alpha_o = \sqrt{6t}$, (ii) the incumbent invests, (iii) the incumbent offers $\alpha_n^* = \sqrt{6t}$, (iv) the entrant exits the market, and (v) the incumbent sets $F_i^{mn}(z, \Delta_z) = \frac{(z+\Delta_z)^2}{2} - t$.

(III) If χ is on $[6t, +\infty)$, (i) the regulator sets any α_o on $[0, +\infty)$, (ii) the incumbent invests, (iii) the incumbent offers $\alpha_n^* = \sqrt{6t}$, (iv) the entrant exits the market, and (v) the incumbent sets $F_i^{mn}(z, \Delta_z) = \frac{(z+\Delta_z)^2}{2} - t$. ■

Case (III) is simple to discuss. With a drastic innovation, the entrant cannot compete with the incumbent even with access to the old network at $\alpha_o = 0$. As the incumbent is free to choose any access price to the new network, it will set a prohibitively high price, thus foreclosing the market. With access price as its only regulatory instrument, the regulator cannot change this outcome and any access price may be observed in equilibrium.

On the contrary, for a non-drastic innovation the regulator's choice of α_o will have an impact on the final market structure. The regulator may induce a duopoly by setting a low access price, or may induce a monopoly by setting a high access price. And both may be optimal.

On the one hand, a duopoly results in lower average transportation costs but, on the other hand, consumers served by the entrant will purchase less. This occurs because firms set two-part retail tariffs, with marginal price equal to marginal cost. If the access price is above marginal cost, those consumers served by the entrant will consume a suboptimal output. Whether duopoly fares better or worse than monopoly, depends on the magnitude of the price of access to the new network. If Δ_z is high – but still within the limits of a non-drastic innovation – the incumbent sets a high access price because $\alpha_n^*(\cdot)$ is increasing in Δ_z : the access price to the new network is higher, the higher the increase in quality enabled by the new network. Therefore, if Δ_z is low, a duopoly on the new network is socially preferable to a monopoly on the same network. If Δ_z is high, a monopoly on the new network is socially preferable to a duopoly on this network. This may be interpreted as a regulatory *moratorium*.

Interestingly, for a non-drastic innovation, we obtain the result that when the quality of the new network is high, the equilibrium access price to the old network is also high. One might expect precisely the opposite, i.e., expect that when the social value of the innovation is high, setting a low access price to the old network is required to give the incumbent incentive to invest. In our case, a high access price would work in the same way, allowing the incumbent to foreclose the market by investing, and thus giving incentive to invest.

However, this is not the justification for the regulatory *moratorium*, as the incumbent would also invest with $\alpha_o = 0$.

4 Extension

In this section, we consider the possibility of both firms investing in the new network.

Consider the model of Section 2, except that: in stage 2 both the incumbent and the entrant decide whether to invest, in stage 3, if only one firm invested it makes an access price offer to the rival, in stage 4, if one of the firms did not invest it chooses which network to use. We assume that the entrant and the incumbent have the same investment cost, and that the innovation is non-drastic.¹⁴

We start by solving the equilibrium of the game for a given α_o . Later, we discuss the impact on welfare of both firms investing.

If only the incumbent invests, the payoffs are the same as in Section 3.

If only the entrant invests, its net profit equals $\pi_i^{dn}(\Delta_z, \Delta_z, \alpha_n; z)$, while the incumbent's net profit equals $\pi_e^{dn}(\Delta_z, \Delta_z, \alpha_n; z)$. By the same reasoning as in Lemma 4, if only the entrant invests, it sets access price $\alpha_n^*(0) = \sqrt{\chi}$.

If both firms invest, the net profits are $\pi_i^{dn}(\Delta_z, \Delta_z, 0; z)$ for both firm, as this case is similar to a situation where a firm gives access to the other one at $\alpha_n = 0$.

For $(\alpha_o, \Delta_z; z)$ on $[0, \sqrt{6t - \chi}] \times [0, \sqrt{z^2 + 6t} - z] \times (\frac{4}{3}\sqrt{6t}, +\infty)$, denote by

$$\Delta\Pi_{e|I}(\Delta_z, \alpha_o; z) := \pi_i^{dn}(\Delta_z, \Delta_z, 0; z) - \pi_e^{dn}(\Delta_z, \Delta_z, \alpha_n^*(\alpha_o); z).$$

the entrant's incremental profit from the investment, given that the incumbent invested.

The following Lemma presents the equilibria of the investment game.¹⁵

Lemma 8: *In equilibrium:*

- (i) both firms invest, if and only if, I is on $[0, \Delta\Pi_{e|I}(\Delta_z, 0; z)]$;
- (ii) the entrant invests and the incumbent does not, if and only if, I is on $[\Delta\Pi_{e|I}(\Delta_z, 0; z), +\infty)$.
- (iii) the incumbent invests and the entrant does not, if and only if, I is on:

$$\begin{cases} [\Delta\Pi_{e|I}(\Delta_z, \alpha_o; z), \Delta\Pi_i(\Delta_z, \alpha_o; z)] & \text{for } \alpha_o \text{ on } [0, \sqrt{6t - \chi}] \\ [\pi_i^{dn}(\Delta_z, \Delta_z, 0; z), +\infty) & \text{for } \alpha_o \text{ on } [\sqrt{6t - \chi}, +\infty). \end{cases}$$

¹⁴If the innovation is drastic, the regulator cannot influence the outcome of the game.

¹⁵It can be showed that, for any α_o , we have $\Delta\Pi_{e|I}(\Delta_z, 0; z) < \Delta\Pi_i(\Delta_z, \alpha_o; z)$.

■

This Lemma suggests the following remarks. First, if the investment cost is low, both firms invest. Second, if both firms invest, the equilibrium is unique. Otherwise, it is possible to have two equilibria, one where only the entrant invests, and another where only the incumbent invests. Third, given that only one firm invests, it is more likely that it is the incumbent who invests. This happens because the incumbent pays a lower access price when it asks for access to the rival's network than the entrant, given that it has using the old network as an outside option. Hence, its incentives to invest, given that the entrant has invested, are relatively smaller than the entrant's incentives to invest, given that the incumbent has invested.

Figure 3 represents in the (α_o, I) -space all the relevant thresholds for I , in Lemma 8.¹⁶

[Figure 3]

Next we discuss how the possibility of investment by the entrant affects welfare when the regulator sets α_o optimally.

Suppose that I is on $[\Delta\Pi_{e|I}(\Delta_z, 0; z), +\infty)$. Since the investment cost is high, in equilibrium only one firm invests. There are two cases of interest, χ on $[0, \frac{6}{5}t]$ and χ on $(\frac{6}{5}t, +\infty)$. First, let χ be on $[0, \frac{6}{5}t]$. If only the incumbent is able to invest, then, from Proposition 1, it invests, offers $\alpha_n^* = \sqrt{\chi}$, and the entrant accepts. If both firms are able to invest and in equilibrium only the entrant invests, there will be a duopoly with $\alpha_n^*(0) = \sqrt{\chi}$. In addition, if both firms are able to invest and in equilibrium the incumbent invests, there will be either a duopoly with $\alpha_n^*(\alpha_o) = \sqrt{\chi + \alpha_o^2}$, if α_o is on $[0, \sqrt{6t - \chi})$; or a monopoly, if α_o is on $[\sqrt{6t - \chi}, +\infty)$. Given that χ is on $[0, \frac{6}{5}t]$, welfare under duopoly at access price $\alpha_n^* = \sqrt{\chi}$ is no smaller than at access price $\alpha_n^*(\alpha_o) = \sqrt{\chi + \alpha_o^2}$, or than under monopoly.¹⁷ Now let χ be on $(\frac{6}{5}t, +\infty)$. If only the incumbent is able to invest, it invests, offers $\alpha_n^* = \sqrt{6t}$, and becomes a monopolist. If both firms are able to invest, the possible equilibria are as above. With χ on $(\frac{6}{5}t, +\infty)$, welfare under monopoly with $\alpha_n^* = \sqrt{6t}$ exceeds welfare under duopoly at $\alpha_n^* = \sqrt{\chi}$ or $\alpha_n^*(\alpha_o) = \sqrt{\chi + \alpha_o^2}$.

Hence, independently of the regulator's choice of α_o , welfare cannot increase when both firms are able to invest if costs are so high that only one firm will invest in equilibrium. The

¹⁶Figure 3 refers to z on $(\frac{4}{3}\sqrt{6t}, z_2)$. For z on $(z_2, +\infty)$ the Figure is similar but without $\Delta\Pi^i(\alpha_o, \Delta_z, z)$.

¹⁷Setting $\alpha_o = 0$ is a sufficient condition for the expected welfare to be the same when only the incumbent is able to invest or when both firms are able to invest. Therefore, $\alpha_o = 0$ is the socially optimal access price if χ is on $[0, \frac{6}{5}t]$.

explanation is the following. For χ on $[0, \frac{6}{5}t]$ it follows from Proposition 1 that there will be a duopoly with a low access price when the incumbent is the only firm able to invest. When both firms are able to invest, access price $\alpha_o = 0$ leads to the same result, regardless of who invests. For χ on $(\frac{6}{5}t, +\infty)$, it follows from Proposition 1 that there will be a monopoly when the incumbent is the only firm able to invest. If the entrant invests it provides access to the incumbent, and the market structure is a duopoly, whereas if the incumbent invests there may be a monopoly. Interestingly, as the welfare function is quasi-convex, if χ is on $(\frac{6}{5}t, +\infty)$, a monopoly leads to a higher welfare than a duopoly on the new network with α_n^* on $(\sqrt{\chi}, +\infty)$. Therefore, investment by the entrant may reduce welfare.

Suppose that I is on $[0, \Delta\Pi_{e|I}(\Delta_z, 0; z))$. Since the investment cost is low, in equilibrium both firms will invest. Thus, welfare is independent of α_o . For χ on $[0, \frac{6}{5}t)$, welfare increases, if and only if, $W^{dn}(\Delta_z, \Delta_z, 0; z) - W^{dn}(\Delta_z, \Delta_z, \alpha_n^*(0); z) > I$, which is always true. For χ on $[\frac{6}{5}t, +\infty)$, welfare increases, if and only if, $W^{dn}(\Delta_z, \Delta_z, 0; z) - W^{m_n}(\Delta_z; z) > I$, or equivalently, $I < \frac{1}{4}t$. Thus, if both firms are able to invest, instead of just the incumbent, welfare may increase or decrease. This stems from the trade-off between access to the new network being priced at marginal cost for both firms, and the duplication of the investment cost.

We summarize the previous discussion in the next Remark.

Remark 3: *Suppose that both firms can invest. (i) Let I be on $[\Delta\Pi_{e|I}(\Delta_z, 0; z), +\infty)$, such that in equilibrium only one firm invests. Welfare does not increase compared with the case where only the incumbent invests. (ii) Let I be on $[0, \Delta\Pi_{e|I}(\Delta_z, 0; z), +\infty)$, such that in equilibrium both firms invest. Welfare may increase or decrease compared with the case where only the incumbent can invest. ■*

5 Conclusions

In this article, we analyzed the incentives of a telecommunications incumbent to invest and give access to a downstream entrant to a next generation network. We distinguished between the cases of a *drastic* and *non-drastic* innovation. If the innovation is *drastic*, the incumbent always invests, but does not give access to the entrant. If the innovation is *non-drastic*, the regulator can control, through the regulation of the old network, whether the incumbent **(i)** invests in a NGN, and **(ii)** gives access to the network. If the innovation is *non-drastic* and if the access price to the old network is low, the incumbent voluntarily gives

access to the new network. If the innovation is *non-drastic*, there is no monotonic relation between the access price to the old network and the incumbent's incentives to invest. A regulatory *moratorium* emerges as socially optimal, if innovation is large but *non-drastic*.

Appendix

Lemma 1: See DeGraba and Biglaiser. ■

Lemma 2: Consumers purchase from the monopolist if and only if

$$\frac{(z + \Delta_v)^2}{2} - tx - F_i > 0 \Leftrightarrow \chi < -\frac{1}{t} \left(F_i - \frac{1}{2} (z + \Delta_v)^2 \right)$$

Assuming an interior solution, the profit maximizing price is:

$$F_i = \frac{1}{4} (z + \Delta_v)^2.$$

However, we do not have an interior solution since, given our assumption on z ,

$$x = \frac{1}{4t} (z + \Delta_v)^2 > 1 \Leftrightarrow z + \Delta_v > 2\sqrt{t}.$$

In this case, the optimal fixed fee and profits are:

$$F_i^{m_v} = \pi_i^{m_v} = \frac{(z + \Delta_v)^2}{2} - t.$$
■

Lemma 3: To avoid a multiplicity of cases, we assume that firm $j = i, e$ faces demand $y_j = (z + \Delta_j) - p_j$ with Δ_j on $\{0, \Delta_z\}$. Additionally, the entrant has costs α_v on $\{\alpha_o, \alpha_n\}$, with $\max\{\alpha_o, \alpha_n\} < z + \Delta_e$. We start by finding the consumer who is indifferent between buying from the incumbent or from the entrant:

$$\begin{aligned} \frac{(z + \Delta_i)^2}{2} - tx - F_i &= \frac{(z + \Delta_e - \alpha)^2}{2} - t(1 - \chi) - F_e \Leftrightarrow \\ x &= \frac{1}{2} - \frac{F_i - F_e}{2t} - \frac{(z - \alpha + \Delta_e)^2 - (z + \Delta_i)^2}{4t}, \end{aligned}$$

with $\alpha < z$.

The demand function, in terms of consumers, facing the incumbent is

$$D_i = \begin{cases} 0 & F_i > F_e + \frac{\alpha(2(z+\Delta_e)-\alpha)+\Delta_D}{2} + t \\ 1 & F_i < F_e + \frac{\alpha(2(z+\Delta_e)-\alpha)+\Delta_D}{2} - t \\ x(F_i, F_e, \Delta_i, \Delta_e, \alpha; z) & \text{else} \end{cases}.$$

Clearly, $D_e = 1 - D_i$.

Given this indifferent consumer, and the fact that $p_i = 0$ and $p_e = \alpha$, profit functions, excluding investment costs, become:

$$\begin{aligned}\pi_i &= F_i x(F_i, F_e, \Delta_i, \Delta_e, \alpha; z) + \alpha(z + \Delta_e - \alpha)(1 - x(F_i, F_e, \Delta_i, \Delta_e, \alpha; z)) \\ \pi_e &= F_e(1 - x(F_i, F_e, \Delta_i, \Delta_e, \alpha; z)).\end{aligned}$$

Maximizing each profit function with respect to the fixed fee, we find:

$$\begin{aligned}F_i^{dv} &= t + \frac{1}{6}\alpha(6(z + \Delta_e) - 5\alpha) + \frac{1}{6}\Delta_D \\ F_e^{dv} &= t - \frac{1}{6}\alpha^2 - \frac{1}{6}\Delta_D.\end{aligned}$$

The indifferent consumer is given by

$$x^* = \frac{1}{2} + \frac{1}{12t}(\Delta_D + \alpha^2),$$

with $\alpha < \sqrt{6t - \Delta_D}$.

We now have to ensure that all consumers have a positive surplus, independently of the market structure considered:

$$\begin{aligned}\frac{(z + \Delta_i)^2}{2} - tx^*(\Delta_i, \Delta_e, \alpha; z) - F_i^*(\Delta_i, \Delta_e, \alpha; z) &> 0 \Leftrightarrow \\ 6t + \Delta_D + 4\alpha(z + \Delta_e) - 2(z + \Delta_i)^2 - 3\alpha^2 &< 0.\end{aligned}$$

When both firms use the same network it must be that $6t + 4(z + \Delta_i)\alpha - 2(z + \Delta_i)^2 - 3\alpha^2 < 0$ for all $\alpha < \sqrt{6t}$. This expression is maximized when $\alpha = \frac{2}{3}(z + \Delta_i)$ at $6t - \frac{2}{3}(z + \Delta_i)^2$ if $(z + \Delta_i) < \frac{3}{2}\sqrt{6t}$. If $(z + \Delta_i) > \frac{3}{2}\sqrt{6t}$ the maximum is $-2((z + \Delta_i) - \sqrt{6t})^2$ obtained at $\alpha = \sqrt{6t}$. Hence, we must have $6t - \frac{2}{3}(z + \Delta_i)^2 < 0 \Leftrightarrow (z + \Delta_i) > 3\sqrt{t}$ if $(z + \Delta_i) < \frac{3}{2}\sqrt{6t}$.

When the entrant uses the old and the incumbent uses the new network we must have $6t + 4z\alpha_o - 2z^2 - 3\alpha_o^2 - \Delta_z(2z + \Delta_z) < 0$ for all $\alpha_o < \sqrt{6t - \Delta_z(2z + \Delta_z)}$. This function takes maximum value, $6t - \frac{2}{3}z^2 - \Delta_z(2z + \Delta_z)$, at $\alpha_o = \frac{2}{3}z$ if $z < \frac{3}{2}\sqrt{6t}$. If $z > \frac{3}{2}\sqrt{6t}$, the maximum is $-2(z - \sqrt{6t})^2 - \Delta_z(2z + \Delta_z)$. Hence, it must be that $6t - \frac{2}{3}z^2 - \Delta_z(2z + \Delta_z) < 0 \Leftrightarrow z > \frac{1}{2}\sqrt{3}\sqrt{12t + \Delta_z^2} - \frac{3}{2}\Delta_z$ for $z < \frac{3}{2}\sqrt{6t}$. As $\sqrt{9t + \frac{3}{4}\Delta_z^2} - \frac{3}{2}\Delta_z < 3\sqrt{t} - \Delta_z < 3\sqrt{t}$ all restrictions are verified for $z > 3\sqrt{t}$.

We now show that the incumbent's profit function, $\pi_i(\Delta_i, \Delta_e, \alpha; z)$, increases in α for all $\alpha < \sqrt{6t - \Delta_D}$.

First note that $\frac{\partial(\pi_i^d(\Delta_i, \Delta_e, \alpha; z))}{\partial\alpha} = \frac{1}{18t}(18t(z + \Delta_e) + \alpha(\Delta_D - 30t) + \alpha^3)$, $\frac{\partial(\pi_i^d(\Delta_i, \Delta_e, \alpha; z))}{\partial\alpha} \Big|_{\alpha=0} = (z + \Delta_e) > 0$ and $\frac{\partial^2(\pi_i^d(\Delta_i, \Delta_e, \alpha; z))}{\partial\alpha^2} = \frac{1}{18t}(\Delta_D + 3\alpha^2 - 30t) < 0$. Thus, the incumbent's profit increases with α if $\frac{\partial(\pi_i^d(\Delta_i, \Delta_e, \alpha; z))}{\partial\alpha} \Big|_{\alpha=\sqrt{6t-\Delta_D}} > 0$.

Additionally, $\frac{\partial \left(\frac{\partial(\pi_i^d(\Delta_i, \Delta_e, \alpha; z))}{\partial \alpha} \Big|_{\alpha=\sqrt{6t-\Delta_D}} \right)}{\partial \Delta_D} = \frac{2}{3} \frac{1}{\sqrt{6t-\Delta_D}} > 0$. Hence, $\frac{\partial(\pi_i^d(\Delta_i, \Delta_e, \alpha; z))}{\partial \alpha} > 0$ for all $\alpha < \sqrt{6t-\Delta_D}$ if $\frac{1}{18t} \left(18t(z + \Delta_e) + \sqrt{6t-0}(0-30t) + (6t-0)^{\frac{3}{2}} \right) > 0 \Leftrightarrow z + \Delta_e > \frac{4}{3}\sqrt{6t}$, which is true given our assumption on z .

When $\alpha > \sqrt{6t-\Delta_D}$ the entrant will set $F_e = 0$. In this case, the incumbent's demand is

$$D_i = \begin{cases} 0 & F_i > \frac{\alpha(2(z+\Delta_e)-\alpha)+\Delta_D}{2} + t \\ 1 & F_i < \frac{\alpha(2(z+\Delta_e)-\alpha)+\Delta_D}{2} - t \\ x(F_i, F_e, \Delta_i, \Delta_e, \alpha; z) & \text{else} \end{cases} .$$

The best response is to set $F_i = \frac{1}{2}t + \frac{1}{4}\alpha(4(z + \Delta_e) - 3\alpha) + \frac{1}{4}\Delta_D$ if $x(\Delta_i, \Delta_e, \alpha; z) < 1$, which is impossible, or $F_i = \alpha(z + \Delta_e) - \frac{1}{2}\alpha^2 - t + \frac{1}{2}\Delta_D$, otherwise. This F_i is set in order to induce the consumer located at 1 to choose the incumbent. If $\alpha \geq z + \Delta_e$, no consumer will ever choose the entrant and the incumbent is effectively a monopolist. ■

Lemma 4: Consider initially that there is no investment in the new network: If $\alpha_o < \sqrt{6t}$, the entrant accepts α_o ; otherwise, it exits the market.

Consider now that there is investment in the new network: The entrant will choose a given network if it results in positive profits and if it is more profitable than choosing the other one. Hence, it will choose the new network if and only if $\alpha_n < \sqrt{6t}$ and

$$\begin{aligned} \pi_e^{dn}(\Delta_z, \Delta_z, \alpha_n; z) &\geq \pi_e^{db}(\Delta_z, 0, \alpha_o; z) \Leftrightarrow \\ \frac{1}{72t}(6t - \alpha_n^2)^2 &\geq \frac{1}{72t}(6t - \alpha_o^2 - \chi)^2 \Leftrightarrow \\ \alpha_n &\leq \sqrt{\alpha_o^2 + \chi}. \end{aligned}$$

(i) Assume $\alpha_o < \sqrt{6t - \chi}$. This means that accepting α_o results in a positive market share for the entrant. Then, if $\alpha_n < \sqrt{\alpha_o^2 + \chi}$, the entrant accepts α_n ; if $\alpha_n > \sqrt{\alpha_o^2 + \chi}$, it accepts α_o .

(ii) Assume that $\alpha_o > \sqrt{6t - \chi}$. This means that accepting α_o does not result in a positive market share for the entrant. Then, if $\alpha_n < \sqrt{6t}$, the entrant accepts α_n ; if $\alpha_n > \sqrt{6t}$, it exits the market.

If $6t - \chi < 0$ the entrant would have a non-positive market for any $\alpha_o \geq 0$. ■

Lemma 5: Assume initially that $\alpha_o < \sqrt{6t - \chi}$. We start by finding out the best offer for α_n in the incumbents perspective that is accepted by the entrant. This is the solution to

$$\max_{\alpha_n} \pi_i^{dn}(\Delta_z, \Delta_z, \alpha_n; z),$$

subject to $\alpha_n < \sqrt{\alpha_o^2 + \chi}$.

Therefore, the problem is to:

$$\max_{\alpha_n} (36t^2 + \alpha_n^4 - 60t\alpha_n^2) + 72\alpha_n t(z + \Delta_z),$$

with first-order conditions $\alpha_n^3 - 30t\alpha_n + 18t(z + \Delta_z) = 0$. Note that evaluated at $\alpha_n = 0$ the derivative is positive and that the second derivative $-12(\alpha_n^2 - 10t)$ is always negative, in the relevant range given that $\alpha_n < \sqrt{\alpha_o^2 + \chi} < \sqrt{6t}$. Thus, there are two candidates for optimum, depending on whether the constraint is binding or not.

The constraint is binding if

$$h(\alpha_o) := (\alpha_o^2 + \chi)^{\frac{3}{2}} - 30t(\alpha_o^2 + \chi)^{\frac{1}{2}} + 18t(z + \Delta_z) > 0. \quad (7)$$

As $\frac{\partial h(\alpha_o)}{\partial \alpha_o} = \frac{3(\alpha_o^2 + \chi - 10t)\alpha_o}{\sqrt{\chi + \alpha_o^2}} < 0$, a high α_o makes it more likely that the constraint is not binding. However, $h(\sqrt{6t - \chi}) = ((z + \Delta_z) - \frac{4}{3}\sqrt{6t})18t > 0$, given our assumption that $z > \frac{4}{3}\sqrt{6t}$. Thus, the constraint is always binding and the optimal access price is $\alpha_n^*(\alpha_o) = \sqrt{\chi + \alpha_o^2}$.

The incumbent will always give access to the new network to the entrant. This happens because for any $\alpha_o < z$ we have $f(\alpha_o) > 0$, where

$$\begin{aligned} f(\alpha_o) & : = \pi_i^{dn}(\Delta_z, \Delta_z, \sqrt{\chi + \alpha_o^2}; z) - \pi_i^{db}(\Delta_z, 0, \alpha_o; z) \\ & = -\left(z\alpha_o + \chi - \left(\sqrt{\chi + \alpha_o^2}\right)(z + \Delta_z)\right). \end{aligned}$$

Function $f(\alpha_o)$ is decreasing in α_o because $\frac{\partial f(\alpha_o)}{\partial \alpha_o} \Big|_{\Delta_z=0} = 0$ and $\frac{\partial^2 f(\alpha_o)}{\partial \alpha_o \partial \Delta_z} < 0$.

Additionally, $f(z) = 0$. Hence, for all $\alpha_o < z$ we have that $f(\alpha_o) > 0$.

Assume now that $\alpha_o > \sqrt{6t - \chi}$. Let us start by finding out the best offer α_n , in the incumbents perspective, that is accepted by the entrant. This is the solution to

$$\max_{\alpha_n} \pi_i^{dn}(\Delta_z, \Delta_z, \alpha_n; z),$$

subject to $\alpha_n < \sqrt{6t}$.

By the same reasons as above, there are two candidates for optimum, depending on whether the constraint is binding or not.

The constraint is binding if and only if $(6t)^{\frac{3}{2}} - 30t(6t)^{\frac{1}{2}} + 18t(z + \Delta_z) > 0$ which is equivalent to $(z + \Delta_z) > \frac{4}{3}\sqrt{6t}$. Given our assumption on z this always holds. The incumbent will then prefer that the entrant stays out of the market if and only if

$$\begin{aligned} \pi_i^{dn}(\Delta_z, \Delta_z, \sqrt{6t}; z) & < \pi_i^{mn}(z, \Delta_z) \Leftrightarrow \\ 6t + (z + \Delta_z)(z + \Delta_z - 2\sqrt{6t}) & > 0. \end{aligned}$$

But, as $6t + (z + \Delta_z)(z + \Delta_z - 2\sqrt{6t}) > 6t + \frac{4}{3}\sqrt{6t}(\frac{4}{3}\sqrt{6t} - 2\sqrt{6t}) = \frac{2}{3}t > 0$ this is always true. For the same reason, whenever $6t - \chi < 0$ the incumbent will prefer that the entrant stays out of the market. \blacksquare

Remark 1: With respect to $\Delta\Pi^i(\alpha_o, \Delta_z; z) = (z + \Delta_z) \sqrt{\alpha_o^2 + \chi} - z\alpha_o - \frac{\chi}{72t} (60t - \chi - 2\alpha_o^2)$, we have that:

- i) $\frac{\partial^2 \Delta\Pi^i(\alpha_o, \Delta_z; z)}{\partial \alpha_o^2} > 0$
- ii) $\left. \frac{\partial \Delta\Pi^i(\alpha_o, \Delta_z; z)}{\partial \alpha_o} \right|_{\alpha_o=0} = -z < 0$
- iii) $\left. \frac{\partial \Delta\Pi^i(\alpha_o, \Delta_z; z)}{\partial \alpha_o} \right|_{\alpha_o=\sqrt{6t-\chi}} = \frac{18t\sqrt{6t-\chi}\sqrt{\chi+z^2+\sqrt{6t}}(\chi\sqrt{6t-\chi}-18tz)}{18t\sqrt{6t}}$

This is positive if and only if $z < \frac{\sqrt{t}}{3} \left(\sqrt{6 - \chi/t} + \sqrt{(\chi/t)^2/6 - 11\chi/t + 60} \right)$.

Additionally, $\Delta\Pi^i(\sqrt{6t-\chi}, \Delta_z; z) < \frac{1}{2}\chi$ if and only if $z < \frac{\sqrt{t}}{72} \left(\left(\sqrt{6 - \chi/t} \right) (\chi/t + 84) + \sqrt{6(\chi/t + 156)(\chi/t + 12)} \right)$.

The two functions z_1 and z_2 that are helpful for the characterization of $\Delta\Pi^i(\alpha_o, \Delta_z; z)$ are then:

$$\begin{aligned} z_1(\chi, t) & : = \frac{\sqrt{t}}{3} \left(\sqrt{6 - \chi/t} + \sqrt{(\chi/t)^2/6 - 11\chi/t + 60} \right) \\ z_2(\chi, t) & : = \frac{\sqrt{t}}{72} \left(\left(\sqrt{6 - \chi/t} \right) (\chi/t + 84) + \sqrt{6(\chi/t + 156)(\chi/t + 12)} \right) \end{aligned}$$

with $z_2(\chi, t) > z_1(\chi, t)$ for all $\chi \in [0, 6t]$. \blacksquare

Lemma 6: (i) Assume that $\chi > 6t$. As seen in Lemma 4, the incumbent will not give access to the new network whatever the access price to the old network, and thus it obtains monopoly profit in case of investment.

If $\alpha_o < \sqrt{6t}$, in the absence of investment there will be entry. The incumbent will invest if and only if:

$$\pi_i^{mn}(z, \Delta_z) - I > \pi_i^{do}(0, 0, \alpha_o; z) \Leftrightarrow \pi_i^{mn}(z, \Delta_z) - \pi_i^{do}(0, 0, \alpha_o; z) > I.$$

As $\pi_i^{mn}(z, \Delta_z) - \pi_i^{do}(0, 0, \alpha_o; z)$ is decreasing in α_o , we have that $\pi_i^{mn}(z, \Delta_z) - \pi_i^{do}(0, 0, \alpha_o; z) > \pi_i^{mn}(z, \Delta_z) - \pi_i^{do}(0, 0, \sqrt{6t}; z) = \frac{\chi}{2} + \frac{(z-\sqrt{6t})^2}{2} > \frac{\chi}{2} > I$: the incumbent will always invest.

If $\alpha_o > \sqrt{6t}$, there will be no entry, independently of the investment decision. The incumbent will invest if and only if:

$$\pi_i^{mn}(z, \Delta_z) - I > \pi_i^{mo}(z, 0) \Leftrightarrow \frac{\chi}{2} > I.$$

which is always true by our assumption on I . Thus, the incumbent will always invest.

(ii) Assume that $\chi < 6t$. Assume that $\alpha_o < \sqrt{6t - \chi}$. As seen in Lemma 4, the incumbent will always give access to the new network to the entrant if it invests and it will give access to the old network if it does not invest. It will be profitable to invest if and only if:

$$\pi_i^{d_n}(\Delta_z, \Delta_z, \alpha_n^*; z) - I > \pi_i^{d_o}(0, 0, \alpha_o; z) \Leftrightarrow \Delta\Pi^i(\alpha_o, \Delta_z; z) > I.$$

Note that we have $\Delta\Pi^i(\alpha_o, 0; z) = 0$ and $\frac{\partial\Delta\Pi^i(\alpha_o, \Delta_z; z)}{\partial\Delta_z} = \frac{(z+\Delta_z)(18t(z+\Delta_z)+\alpha_n^{*3}-30t\alpha_n^*)}{18t\alpha_n^*} + \alpha_n^* > 0$.

Assume that $\alpha_o > \sqrt{6t - \chi}$. Then, the incumbent will prefer that the entrant exits if investment has taken place and this case is equal to (i). \blacksquare

Remark 2: Note that, $\frac{\partial W^d(\Delta_i, \Delta_i, \alpha; z)}{\partial\alpha} = \frac{1}{36t}(5\alpha^2 - 18t)\alpha = 0$ for $\alpha = 0$ or $\alpha = \sqrt{\frac{18}{5}t}$ and that $\frac{\partial^2 W^d(\Delta_i, \Delta_i, \alpha; z)}{\partial\alpha^2} = \frac{1}{12t}(5\alpha^2 - 6t)$. The second derivative at $\alpha = 0$ is $-\frac{1}{2} < 0$ and at $\alpha = \sqrt{\frac{18}{5}t} < \sqrt{6t}$ it is $1 > 0$. The third derivative is always non negative. \blacksquare

Lemma 7: We start by showing that (i) $\Delta\Pi^i(\alpha_o, \Delta_z; z)$ is decreasing until $\alpha_o = \sqrt{\frac{6t}{5}}$ and (ii) $\Delta\Pi^i(\sqrt{\frac{6t}{5}}, \Delta_z; z) > \frac{\chi}{2}$.

We have already showed that $\Delta\Pi^i(\alpha_o, \Delta_z; z)$ is convex, therefore we just need to show that the first derivative at $\sqrt{\frac{6t}{5}}$ is negative: $\frac{\partial\Delta\Pi^i(\alpha_o, \Delta_z; z)}{\partial\alpha_o} \Big|_{\alpha_o=\sqrt{\frac{6t}{5}}} = \frac{\chi}{18t}\sqrt{\frac{6t}{5}} - z + \sqrt{(\chi + z^2)} \frac{\sqrt{\frac{6t}{5}}}{\sqrt{(\chi + \frac{6}{5}t)}}$.

This is positive if $\sqrt{(\chi + z^2)} \frac{\sqrt{\frac{6t}{5}}}{\sqrt{(\chi + \frac{6}{5}t)}} > z - \frac{\chi}{18t}\sqrt{\frac{6t}{5}}$. As both terms in the inequality are positive, this implies that $\left(\sqrt{(\chi + z^2)} \frac{\sqrt{\frac{6t}{5}}}{\sqrt{(\chi + \frac{6}{5}t)}}\right)^2 > \left(z - \frac{\chi}{18t}\sqrt{\frac{6t}{5}}\right)^2$ which is equivalent to $\frac{6\sqrt{30}+5\frac{\chi}{t}\sqrt{30}-6\sqrt{5}\sqrt{5\frac{\chi}{t}+1356}}{450} < \frac{z}{\sqrt{t}} < \frac{6\sqrt{30}+5\frac{\chi}{t}\sqrt{30}+6\sqrt{5}\sqrt{5\frac{\chi}{t}+1356}}{450}$.

As $\frac{6\sqrt{30}+5\frac{\chi}{t}\sqrt{30}+6\sqrt{5}\sqrt{5\frac{\chi}{t}+1356}}{450} < \frac{6\sqrt{30}+30\sqrt{30}+6\sqrt{5}\sqrt{30+1356}}{450} < \frac{4}{3}\sqrt{6}$, it is impossible to have $\frac{\partial\Delta\Pi^i(\alpha_o, \Delta_z; z)}{\partial\alpha_o} \Big|_{\alpha_o=\sqrt{\frac{6t}{5}}} > 0$.

We now show that $\Delta\Pi^i(\sqrt{\frac{6t}{5}}, \Delta_z; z) > \frac{\chi}{2}$. Let

$$g(\chi) := \Delta\Pi^i(\sqrt{\frac{6t}{5}}, \Delta_z; z) - \frac{\chi}{2} = \frac{(5\chi - 468t)\chi}{360t} - z\sqrt{\frac{6}{5}t} + \left(\sqrt{\frac{6}{5}t} + \chi\right)(z + \Delta_z)$$

Assume that $g(\chi) < 0 \Leftrightarrow \sqrt{\left(\frac{6}{5} + \frac{\chi}{t}\right)\left(\frac{\chi}{t} + \left(\frac{z}{\sqrt{t}}\right)^2\right)} < \frac{(468-5\frac{\chi}{t})\frac{\chi}{t}}{360} + \frac{z}{\sqrt{t}}\sqrt{\frac{6}{5}}$. As both sides in the inequality are positive this implies that $\left(\frac{6}{5} + \frac{\chi}{t}\right)\left(\frac{\chi}{t} + \left(\frac{z}{\sqrt{t}}\right)^2\right) < \left(\frac{(468-5\frac{\chi}{t})\frac{\chi}{t}}{360} + \frac{z}{\sqrt{t}}\sqrt{\frac{6}{5}}\right)^2 \Leftrightarrow 180\left(\frac{z}{\sqrt{t}}\right)^2 + \left(\sqrt{30}\frac{\chi}{t} - \frac{468}{5}\sqrt{30}\right)\frac{z}{\sqrt{t}} + \frac{13\left(\frac{\chi}{t}\right)^2}{2} - \frac{621\frac{\chi}{t}}{5} - \frac{5\left(\frac{\chi}{t}\right)^3}{144} + 216 < 0$.

Therefore we should have $\frac{\sqrt{30}(468-5\frac{\chi}{t})}{1800} - \frac{\sqrt{5(5\frac{\chi}{t}-108)(5\frac{\chi}{t}+6)(5\frac{\chi}{t}-828)}}{1800} < \frac{z}{\sqrt{t}} < \frac{\sqrt{30}(468-5\frac{\chi}{t})}{1800} + \frac{\sqrt{5(5\frac{\chi}{t}-108)(5\frac{\chi}{t}+6)(5\frac{\chi}{t}-828)}}{1800}$.

By plotting these roots as a function of $\frac{\chi}{t} \in (0, 6)$, we observe that $\frac{\sqrt{30}(468-5\frac{\chi}{t})}{1800} + \frac{\sqrt{5(5\frac{\chi}{t}-108)(5\frac{\chi}{t}+6)(5\frac{\chi}{t}-828)}}{1800} < \frac{4}{3}\sqrt{6}$, meaning that it is impossible to have $g(\chi) < 0$.

This implies that $I < \frac{\chi}{2} < \Delta\Pi^i(\sqrt{\frac{6t}{5}}, \Delta_z; z)$, and thus, duopoly on the old network with $\alpha_o = (\Delta\Pi^i)^{-1}(I) > \sqrt{\frac{6}{5}t}$ is worst than monopoly on the old network, which is worst than a monopoly on the new network.

The regulator's choice is thus between duopoly and monopoly on the new network. To maximize welfare in the case of duopoly, the regulator will set $\alpha_o = 0$ leading to $\alpha_n^* = \sqrt{\chi}$. If $\chi < \frac{6}{5}t$ this results in higher welfare than the case of monopoly. However, when $\chi > \frac{6}{5}t$, monopoly on the new network is better than duopoly on the new network, and thus the regulator sets $\alpha_o^* = \sqrt{6t}$. ■

Proposition 1: This follows from the Lemmas above. ■

Lemma 8: We start by showing that (N, N) cannot be an equilibrium of this game because the entrant will always prefer to invest. A necessary condition for (N, N) to be an equilibrium is that the entrant prefers not to invest given that the incumbent does not invest. For the case of $\alpha_o < \sqrt{6t}$ this corresponds to:

$$\begin{aligned} \pi_e^{d_o}(0, 0, \alpha_o; z) &\geq \pi_i^{d_n}(\Delta_z, \Delta_z, \alpha_n^*(0); z) - I \Leftrightarrow \\ I &\geq \pi_i^{d_n}(\Delta_z, \Delta_z, \alpha_n^*(0); z) - \pi_e^{d_o}(0, 0, \alpha_o; z) \end{aligned}$$

Since $\frac{d\pi_e^{d_o}(0, 0, \alpha_o; z)}{d\alpha_o} < 0$, the minimum of $\pi_i^{d_n}(\Delta_z, \Delta_z, \alpha_n^*(0); z) - \pi_e^{d_o}(0, 0, \alpha_o; z)$ occurs when $\alpha_o = 0$ at $\pi_i^{d_n}(\Delta_z, \Delta_z, \alpha_n^*(0); z) - \pi_e^{d_o}(0, 0, 0; z) = \frac{(36t^2 + \chi^2 - 60t\chi) + 72\sqrt{\chi}t(z + \Delta_z)}{72t} - \frac{1}{2}t > \frac{\chi}{2}$. It can be showed that this is always true. Additionally, $I \geq \pi_i^{d_n}(\Delta_z, \Delta_z, \alpha_n^*(0); z) - \pi_e^{d_o}(0, 0, \alpha_o; z)$ implies that $I \geq \pi_i^{d_n}(\Delta_z, \Delta_z, \alpha_n^*(0); z)$. But this is the condition for the entrant to prefer to invest given that the incumbent does not invest for $\alpha_o > \sqrt{6t}$.

We will now analyze the other possibilities.

i) (Y, Y) is an equilibrium if and only if:

$$\pi_i^{d_n}(\Delta_z, \Delta_z, 0; z) - I \geq \pi_e^{d_n}(\Delta_z, \Delta_z, \alpha_n^*(0); z) \quad (8)$$

$$\pi_i^{d_n}(\Delta_z, \Delta_z, 0; z) - I \geq \pi_e^{d_n}(\Delta_z, \Delta_z, \alpha_n^*(\alpha_o); z) \quad (9)$$

when $\alpha_o < \sqrt{6t - \chi}$. As $\pi_e^{d_n}(\Delta_z, \Delta_z, \alpha_n^*(\alpha_o); z) \leq \pi_e^{d_n}(\Delta_z, \Delta_z, \alpha_n^*(0); z)$, the two inequalities hold if and only if:

$$\begin{aligned} I &\leq \Delta\Pi_{e|I}(\Delta_z, 0; z) := \pi_i^{d_n}(\Delta_z, \Delta_z, 0; z) - \pi_e^{d_n}(\Delta_z, \Delta_z, \alpha_n^*(0); z) \\ &= \frac{1}{2}t - \frac{(6t - \chi)^2}{72t} = \frac{(12t - \chi)\chi}{72t} < \frac{\chi}{2}. \end{aligned}$$

Note that, when $\alpha_o > \sqrt{6t - \chi}$, (9) should be replaced by $\pi_i^{dn}(\Delta_z, \Delta_z, 0; z) - I \geq 0$. But this is implied by (9). Hence, (Y, Y) is an equilibrium if and only if $I \leq \Delta\Pi_{e|I}(\Delta_z, 0; z)$.

ii) (N, Y) is an equilibrium if and only if $I \geq \Delta\Pi_{e|I}(\Delta_z, 0; z)$. We have already showed that when the incumbent does not invest the entrant will prefer to invest. Therefore, for (N, Y) to be an equilibrium we need only impose that $\pi_i^{dn}(\Delta_z, \Delta_z, 0; z) - I \leq \pi_e^{dn}(\Delta_z, \Delta_z, \alpha_n^*(0); z)$ which is equivalent to $I \geq \Delta\Pi_{e|I}(\Delta_z, 0; z)$.

iii) Consider that $\alpha_o < \sqrt{6t - \chi}$. (Y, N) is an equilibrium if and only if:

$$\begin{aligned}\pi_i^{dn}(\Delta_z, \Delta_z, \alpha_n^*(\alpha_o); z) - I &\geq \pi_i^{do}(0, 0, \alpha_o; z) \\ \pi_e^{dn}(\Delta_z, \Delta_z, \alpha_n^*(\alpha_o); z) &\geq \pi_i^{dn}(\Delta_z, \Delta_z, 0; z) - I\end{aligned}$$

which is equivalent to

$$I \leq \Delta\Pi^i(\alpha_o, \Delta_z; z) = (z + \Delta_z) \sqrt{\alpha_o^2 + \chi} - z\alpha_o - \frac{\chi}{72t} (60t - \chi - 2\alpha_o^2)$$

$$I \geq \Delta\Pi_{e|I}(\Delta_z, \alpha_o; z) := \pi_i^{dn}(\Delta_z, \Delta_z, 0; z) - \pi_e^{dn}(\Delta_z, \Delta_z, \alpha_n^*(\alpha_o); z) = \frac{1}{2}t - \frac{(6t - \chi - \alpha_o^2)^2}{72t}$$

Clearly, $\Delta\Pi_{e|I}(\Delta_z, 0; z) < \Delta\Pi_{e|I}(\Delta_z, \alpha_o; z)$ and $\Delta\Pi_{e|I}(\Delta_z, 0; z) < \Delta\Pi^i(\alpha_o, \Delta_z; z)$.¹⁸

Inspection of $\Delta\Pi_{e|I}(\Delta_z, \alpha_o; z)$ reveals that this is a continuous increasing function in α_o that for $\alpha_o \leq \sqrt{6t - \chi}$ takes values on the interval $[\Delta\Pi_{e|I}(\Delta_z, 0; z), \frac{1}{2}t]$.

Consider now that $\sqrt{6t - \chi} < \alpha_o < \sqrt{6t}$. (Y, N) is an equilibrium if

$$\begin{aligned}\pi_i^{mn}(\Delta_z; z) - I &\geq \pi_i^{do}(0, 0, \alpha_o; z) \\ 0 &\geq \pi_i^{dn}(\Delta_z, \Delta_z, 0; z) - I\end{aligned}$$

which is equivalent to

$$\begin{aligned}I &\leq \pi_i^{mn}(\Delta_z; z) - \pi_i^{do}(0, 0, \alpha_o; z) = \frac{1}{2}\chi + \frac{1}{2}z^2 - \frac{3}{2}t - z\alpha_o + \frac{5}{6}\alpha_o^2 - \frac{1}{72t}\alpha_o^4 \\ I &\geq \pi_i^{dn}(\Delta_z, \Delta_z, 0; z) = \frac{1}{2}t\end{aligned}$$

We have already shown that I is always lower than $\pi_i^{mn}(\Delta_z; z) - \pi_i^{do}(0, 0, \alpha_o; z)$. Thus, (Y, N) is an equilibrium if and only if $I \geq \frac{1}{2}t$.

Consider now that $\alpha_o > \sqrt{6t}$. (Y, N) is an equilibrium if

$$\begin{aligned}\pi_i^{mn}(\Delta_z; z) - I &\geq \pi_i^{mo}(0; z) \\ 0 &\geq \pi_i^{dn}(\Delta_z, \Delta_z, 0; z) - I\end{aligned}$$

¹⁸ $\Delta\Pi_{e|I}(\Delta_z, 0; z) - \Delta\Pi^i(\alpha_o, \Delta_z; z) = -\frac{\chi(\chi + \alpha_o^2)}{36t} + z\alpha_o + \chi - (\sqrt{\chi + \alpha_o^2})(z + \Delta_z)$. It is impossible that $z\alpha_o + \chi - (\sqrt{\chi + \alpha_o^2})(z + \Delta_z) > 0$. This is equivalent to $\frac{z\alpha_o + \chi}{(z + \Delta_z)} > \sqrt{\chi + \alpha_o^2}$. This implies, by squaring both sides of the inequality, that $\left(\frac{z\alpha_o + \chi}{(z + \Delta_z)}\right)^2 - (\chi + \alpha_o^2) > 0$. Simplifying we obtain $-(z + \Delta_z)^{-2}(2z + \Delta_z)(\alpha_o - z)^2\Delta_z > 0$ which is false. Hence, $\Delta\Pi_{e|I}(\Delta_z, 0; z) - \Delta\Pi^i(\alpha_o, \Delta_z; z) < 0$.

which is equivalent to

$$\begin{aligned} I &\leq \pi_i^{m_n}(\Delta_z; z) - \pi_i^{m_o}(0; z) = \frac{(z + \Delta_z)^2}{2} - t - \frac{(z + 0)^2}{2} + t = \frac{\chi}{2} \\ I &\geq \pi_i^{d_n}(\Delta_z, \Delta_z, 0; z) = \frac{1}{2}t \end{aligned}$$

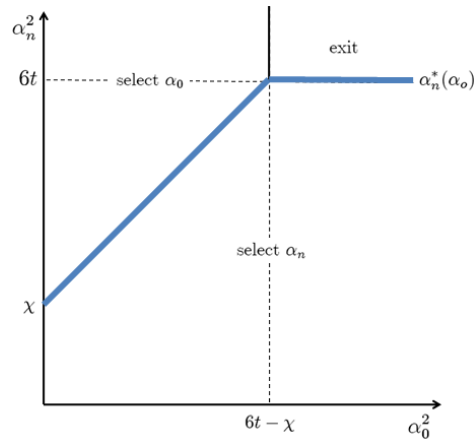
The first condition always holds. Thus, (Y, N) is an equilibrium if and only if $I \geq \frac{1}{2}t$. ■

Remark 3: This follows from Proposition 1 and Lemma 8. ■

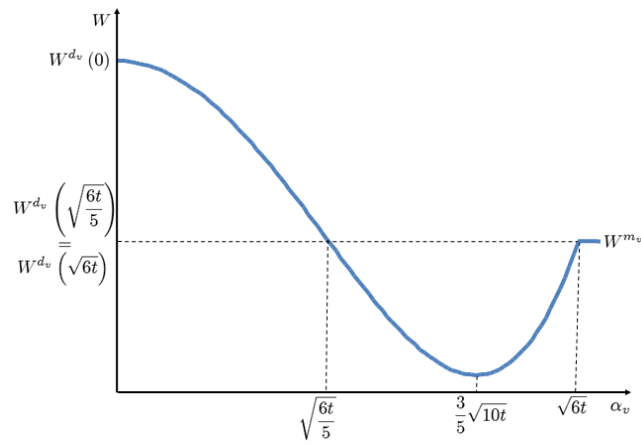
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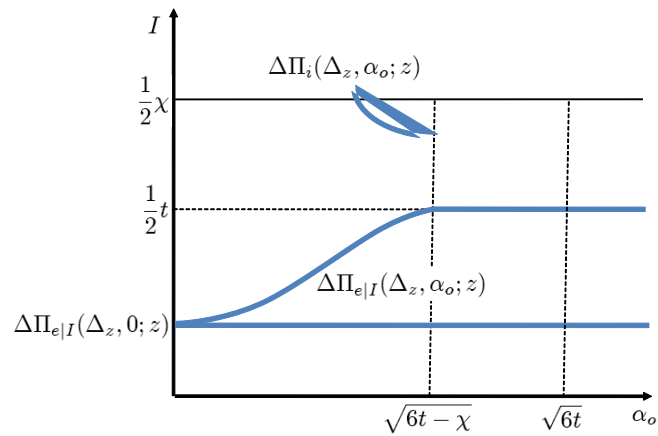
Figures



The entrant's decision to select the old or new network and the incumbent's choice of α_n .



Welfare as a function of the access price.



Equilibria when both firms may invest in the (α_o, I) space.