NET Institute*

www.NETinst.org

Lapo Filistrucchi
Tilburg University & University of Siena

A SSNIP test for two-sided markets: the case of media

Lapo Filistrucchi[1]

Tilburg University & University of Siena

30 September 2008

Abstract

I discuss the design and implementation of a SSNIP test in order to identify the relevant market in a media market. I argue that in such a two-sided market the traditional SSNIP test cannot be applied as it is usually conceived but rather should be modified in order to take into account indirect network externalities. I discuss the issues of which price the hypothetical monopolist should be thought of as raising, of whether we should look at profits changes on only one side or on both sides of the market and of which feedback among the two sides of the market we should take into account.

I then derive the relevant formulas for Critical Loss Analysis. These look much uglier than in a single-sided market but in fact they are easy to calculate as they are still expressed in terms of elasticities and of current observed markups, prices and quantities. Data requirements are however higher as one needs to estimate the matrixes of the own and cross price elasticities of demand on the two-sides of the market and the matrixes of the network effects.

The paper fills a gap in the economic literature, so much more as market definition in media markets is at the centre of many recent competition policy and regulation cases around the world.

JEL codes: L40, L50, K20

Keywords: two-sided markets, SSNIP test, Hypothetical Monopolist test, critical loss analysis, critical elasticity analysis, market definition, media

1 – Introduction

In most economic models the relevant market is simply assumed. In practice however it is of great importance for any antitrust case. A wrong definition of the relevant market might for instance lead the antitrust authority or the courts to blocking a welfare enhancing merger or to allowing a welfare detrimental one. Also, in case of appeal, the recognition of a wrong market definition is sufficient for the courts to reject the whole analysis and rule in favour of the parties irrespective of any other argument brought up by the antitrust authority. Market definition is therefore the founding stone on which an antitrust case is built.

The "small significant non-transitory increase in price test" (SSNIP test), also known as "hypothetical monopolist test" (HM test), is a conceptual tool used to define the relevant market. In a standard market, starting from a set of candidate products for the relevant market, the SSNIP test is implemented by first simulating a price increase by a hypothetical monopolist who owns just one product and, as long as that leads to estimated losses in profits, progressively increasing the number of products owned by the monopolist. When profits are not estimated to decline following a small but significant increase in price by the hypothetical monopolist, the set of products owned by the monopolist in the last simulation constitutes the relevant market.

Many recent competition policy cases, such as the merger between Google and DoucleClick, call for both theoretical and empirical guidance on the design and implementation of a SSNIP test in such two-sided markets[2] as the media ones. A correct product market definition is also crucial in designing regulation in the media market, in light of the progress in information and communication technologies.

I discuss the design and implementation of a SSNIP test in order to identify the relevant market in a media market. In such a two-sided market the traditional SSNIP test cannot be applied as it is usually conceived. That is because a firm in a two-sided market sells two products or services to two distinct group of consumers and recognises that the demand from one type of consumers depends on the demand from the other type of consumers and vice versa, but consumers on the two-sides of the market do not internalise these indirect network effects. Since there is a link between demands on the two sides of the market, the profit function of a hypothetical monopolist who raises the price in a significant non-transitory way on one-side of the market is linked to the profit in the other market and the question arises of which feedbacks between the profits on the two-sides of the

---

[2] See also Argentesi & Ivaldi (2005).

market should be considered. Moreover, since in a two-sided market the profits of the hypothetical monopolist are determined by both the price level (roughly, the sum of the prices paid by the two sides) and the price structure (roughly, the ratio of the prices paid by the two-sides), it is not a priori clear whether the hypothetical monopolist should be thought of as raising a) the price level while optimally adjusting the price structure b) both prices together keeping fixed the price structure c) each of the two prices separately allowing the other price to be adjusted optimally d) each of the two prices while keeping the other price fixed.

In the present paper I therefore address these questions. I claim that profits on both sides of the market and all feedbacks should be taken into account and I suggest that in such two-sided markets as the media ones the test is run by raising each of the two prices separately allowing each time the other price to be adjusted.

The paper proceeds as follows. Section 2 presents the relevant economic literature. Section 3 discusses the rationale behind the SSNIP test in a single-sided market. Section 4 first proposes a SSNIP test for two-sided markets of the "media type" and derives the formulas necessary for the analysis. Section 5 concludes.

2 – The literature

Following the seminal works by Parker and Van Alstyne (2002), Rochet & Tirole (2002, 2003,2006) and Armstrong (2006), a growing number of papers have dealt with theoretical aspects of two-sided markets, e.g. Caillaud & Jullien(2003), Anderson & Gabszewicz (2005) and Guthrie & Wright(2007). Some of them, such as Evans(2003), Wright(2004) and Evans & Schmalensee (2005), have focused on competition policy in two-sided markets. They have pointed out for instance that, unlike the price level, due to the presence of indirect network externalities, the efficient price structure does not reflect the ratio of marginal costs on the two sides of the market and, more generally, that increased competition does not necessarily lead to a more balanced price structure nor to a more efficient one. Most policy contributions so far, except Emch & Thomson (2006), Evans & Noel (2005a, 2007), Argentesi & Filistrucchi (2007), Rochet & Tirole (2008) and Evans (2008), have mainly criticized the application of standard competition policy results to two-sided markets rather then suggesting alternative ones and, from the practical point of view, they argued against existing practice rather than providing new methods to practitioners.

More specifically, despite the rich literature on two-sided markets, only a few papers have dealt with market definition in two-sided markets, and in no way conclusively. Carlton(2007) simply claims that market definition in two-sided markets is more difficult than in the usual single-sided markets because of the uncertainty on the choice of the price the hypothetical monopolist should raise. Argentesi & Ivaldi (2005) discuss the issue in the context of the media market. Their paper however mainly argues for the need to take into account indirect network externalities in order to get unbiased estimates of the own and cross price elasticities of demand. They then present supporting results from a simple econometric exercise on a dataset of French newspapers in order to support their claim. Evans & Noel (2005a) argue for the need to take into account feedbacks between the two sides of the markets due to demand externalities and point to the difficulties arising in market definition when two-sided platforms compete with standard firms on one side of the market. Evans & Noel (2007) propose a way to extend the Critical Loss Analysis, an alternative to the SSNIP test, to two-sided platforms and derive formulas for its implementation. They propose to perform CLA by raising each price separately while keeping the other fixed and to take into account all feedbacks. They then illustrate the bias due to not taking into account feedbacks between the two-sides of the market. Evans (2008) discusses again market definition in two-sided markets, in a non technical way. Emch & Thomson (2006) discuss the design of a SSNIP test in the payment cards market and propose to apply the SSNIP test to the total price charged by the hypothetical monopolist while letting relative prices on the two sides of the market adjust optimally. Their paper however does not discuss the case of other markets and the proposed test is not directly applicable to most media markets, as I will argue below.

3 –Market Definition in a Two-Sided Market

3.1 – The SSNIP test for one-sided markets

In a potential market with many possible products, assuming an ideal situation where all relevant own and cross price elasticities of demand among these products have been estimated, the question arises of identifying a threshold on elasticities in order for substitution to be relevant enough to include two products in the same market.

The issue is solved applying the "small significant non-transitory increase in price" test (SSNIP test), also called the "hypothetical monopolist" test (HM test).

The SSNIP test or the HM test as a method for defining markets was first introduced in 1982 in the U.S. Department of Justice Merger Guidelines. In the EU it was used for the first time in the Nestlé/Perrier case in 1992 and has been officially recognised by the European Commission in its Commission's Notice for the Definition of the Relevant Market in 1997. There are however some differences in the way it is carried out by practitioners in the US and in the EU and more generally under different jurisdictions around the world[3].

In a standard market, the SSNIP test is, according to the EU merger guidelines, implemented by first simulating *a given price increase* above the current competitive level by an hypothetical monopolist which owns just one product and, as long as that leads to estimated losses in profits, progressively increasing the number of products owned by the monopolist. When profits are not estimated to decline following a small but significant increase in price by the hypothetical monopolist, the set of products owned by the monopolist in the last simulation constitutes the relevant market. According to the US guidelines, it is instead implemented by first simulating *the optimal price increase* above the current competitive level by a profit maximizing hypothetical monopolist and, as long as that is at least a small but significant non transitory increase, progressively raising the number of products owned by the monopolist. When the profit maximizing hypothetical monopolist will not raise prices by at least a small but significant non transitory increase, the set of products owned by the monopolist in the last simulation constitutes the relevant market.

Note that, although at first sight it might not seem so, there is indeed a difference between the test in the US and the test in the EU.

Both in the US and in the EU, there is also not unanimous consensus as to whether on all but the first step, one should raise just the price of the first product taken into consideration or should raise the prices of all products owned by the hypothetical monopolist at that step in the procedure.

In any case the basic idea behind the SSNIP test is that substitution among two products is enough to include them in the same market if an hypothetical monopolist would find it unprofitable to raise the price of one product in a small significant non-transitory way (EU merger guidelines) or if an hypothetical monopolist would find it profit maximizing to raise current competitive prices in a

---

[3] See Werden 2003 for a history of the Hypothetical Monopolist test and an explanation of the differences in the way it is implemented.

small significant non transitory way (US merger guidelines)[4]. In either case, in practice, the small significant change in price is usually 5% in the EU or 5-10% in the US and non-transitory is usually considered to mean 1 year.[5]

Using such a hypothetical monopolist as a benchmark reminds of issues concerning market power. This can however be misleading. The assessment of market power or the prediction of merger effects should be conducted after the relevant market has been defined.

Yet one of the rationales behind using a hypothetical monopolist as a benchmark that, by defining a market as the minimum set of products for which a small but significant increase in price by the hypothetical monopolist above the current (competitive) level is not unprofitable (EU version), or on which an hypothetical monopolist would maximise profits by increasing price in at least a small but significant way above the current (competitive) level (US version), one makes sure that the market is the smallest one that it is worth monopolising[6]. That monopolising the market must be worthwhile is clearly a requirement of economic theory, although the 5% and 10% values for the price increase are of course arbitrary[7].

Another rationale for having an hypothetical monopolist is to make sure strategic effects among competitors are ruled out when testing for substitution among products. For the same reason the prices of all products not owned by the hypothetical monopolist at a given stage of the procedure are kept constant.[8]

---

[4] See Werden 2002b for a rationalization of the different ways the SSNIP test is implemented according to different measures of substitutability between products.

[5] There is a well-known problem in the application of the SSNIP test, even to a single-sided market, when it comes to choosing the starting price level for the price increase, in that in the presence of perfect collusion or of monopoly any price increase above the current level is likely to be unprofitable, leading to a wider market definition and therefore to a finding of no (joint) dominance. It is the so-called "cellophane fallacy" from the Du Pont case in the US in the 1950s, , see Motta (2001) and Bishop & Walker (1999). I abstract here from this problem and assume the starting price level is a competitive one, though not necessarily the one obtained under the assumptions of the model of perfect competition.

[6] At the same time however, even absent strategic reactions from rivals, any firm owning a subset of the products in the market will not be able to profitably raise prices by 5% or 10%.

[7] One should note that in general choosing a 10% increase in prices implies a larger market definition than a 5% increase and therefore both a lower chance to find market power and abuses and a lower chance to identify concerns of unilateral effects in mergers.

[8] A debated issue is however whether one should take into account capacity constraints in the production of goods not owned by the hypothetical monopolist. I do not address this concern here.

In practice in order to implement the test assumptions are needed on the demand functions and the cost function. The most common assumptions are those of linear or constant elasticity demands and linear costs.

This is true also of Critical Loss Analysis (CLA) and Critical Elasticity Analysis (CEA), the usual ways a SSNIP test are implemented by practitioners.

Critical Loss Analysis in its EU version proceeds as follows: first, the critical loss in sales of an hypothetical monopolist owning just one product is calculated; this is the maximum loss in sales due to a price increase of 5% or 10% which would not make the price increase unprofitable; second, the actual loss in sales following a 5% or 10% increase in price is estimated; third, the actual loss in sales is compared to the critical loss in sales; if the actual loss in sales is smaller than then critical loss in sales, then a small significant price increase would be profitable and the market is defined; otherwise, the market is assumed to contain also another product and the analysis is repeated.

Critical Loss Analysis in its US version proceeds as follows: first, the critical loss in sales of an hypothetical profit-maximizing monopolist owning just one product is calculated; this is the maximum loss in sales the hypothetical monopolist would tolerate and still raise its price by 5% or 10% ; second, the actual loss in sales following a 5% or 10% increase in price is estimated; third, the actual loss in sales is compared to the critical loss in sales; if the actual loss in sales is smaller than then critical loss in sales, then a small significant price increase would be profit maximizing and the market is defined; otherwise, the market is assumed to contain also another product and the analysis is repeated.

Table 1 below reports the formulae that are commonly used to determine the critical loss in Critical Loss Analysis: m is the current (competitive) markup, t is the significance threshold for price increases t (i.e. usually 0.05 or 0.10).

Table 1: Critical Loss Formulae (constant marginal cost)

Demand Curve	US	EU
Linear	$\dfrac{t}{m+2t}$	$\dfrac{t}{m+t}$
Isoelastic	$1-(1+t)^{\frac{-1-t}{m+t}}$	$\dfrac{t}{m+t}$
Table 12: Critical Loss Formula (Werden, 2002)		

The actual loss formula is instead

$$\frac{\Delta q}{q} = \frac{\Delta p}{p}\left(\frac{\Delta q}{\Delta p}\frac{p}{q}\right)$$

and if demand is linear or for small changes in p

$$\frac{\Delta q}{q} = \frac{\Delta p}{p}\varepsilon_p^q.$$

Critical Elasticity Analysis in its EU version proceeds as follows: first, the critical elasticity faced by an hypothetical monopolist owning just one product is calculated; this is the maximum elasticity an hypothetical monopolist could face without a given price increase of 5% or 10% becoming unprofitable; second, the actual elasticity faced by an hypothetical monopolist is estimated; third, the actual elasticity is compared to the critical elasticity; if the elasticity is smaller than the critical elasticity, then a small significant price increase would be profitable and the market is defined; otherwise, the market is assumed to contain also another product and the analysis is repeated.

Critical Elasticity Analysis in its US version proceeds instead as follows: first, the critical elasticity faced by a hypothetical profit-maximizing monopolist owning just one product is calculated; this is the elasticity that would lead a profit maximizing hypothetical monopolist to increase prices of 5% or 10% ; second, the actual elasticity faced by the hypothetical monopolist is estimated; third, the actual elasticity is compared to the critical elasticity: if the actual elasticity is lower than then critical one, then a profit-maximising hypothetical monopolist would increase prices more than the given threshold and the market is defined; otherwise, the market is assumed to contain also another product and the analysis is repeated.

Table 2 below reports the formulae that are commonly used to determine the critical loss in Critical Elasticity Analysis.

Table 2: Critical Elasticities Formulae (constant marginal cost)

Demand	US	EU
Linear	$\dfrac{1}{m+2t}$	$\dfrac{1}{m+t}$
Isoelastic	$\dfrac{1+t}{m+t}$	$\dfrac{\log(m+t)-\log(m)}{\log(1+t)}$
Source: (Werden, 2002)		

Therefore, under the simplifying assumptions mentioned above, to check what happens to profits as prices rise, one should estimate the elasticities of demands with respect to prices and have a measure of the marginal cost (or of the mark-ups), in addition to observing current (competitive) quantities sold and prices (or revenues).

The same is not true of a two-sided market where estimation of indirect network effects between demands on the two-sides of the market is also necessary, as we will show in Section 4.

4 - Market Definition in a Two-Sided Markets

When discussing the extension of the SSNIP test to a two-sided market, some additional issues arise which make the analysis more complex than in the standard case discussed above:
- given that in a two-sided market there are indirect network externalities, should we take into account also (all?) feedbacks from one side of the market to the other? should we look at what happens to profits on only one side or on both sides of the market? how do we deal with products which are competing on one-side of the market but not on the other-side?
- given that in a two sided market the hypothetical monopolist sets (at least) two prices, which price should he be thought of as raising?

4.1 - Which price?

In a two-sided market one can distinguish:
a) the price level (roughly, the sum of the two prices)
b) the price structure (roughly, the ratio of the two prices)

Should then the hypothetical monopolist be thought of as raising:
i) the price level keeping fixed the price structure?
ii) the price level adjusting optimally the price structure?
iii) first one of the two prices keeping the other fixed and then the other price keeping the first fixed?
iv) first one of the two prices and then the other price, each time adjusting optimally the price structure?

Answering questions as the ones above requires first a distinction between two types of media market: those where the interaction between the two consumer groups is observable (e.g. an internet surfer clicks on an ad) and those where the interaction is not observable (e.g. a reader reads an ad or a reader is influenced by an ad). The presence of the interaction allows the two-sided media platform to charge one of the two sides (i.e. the advertisers) a per-interaction fee, possibly in addition to a subscription fee (i.e. in fact a two-part tariff) such as in the model of Rochet & Tirole (2006), while its absence only allows the platform to charge a subscription fee as in the model of Armstrong (2006). Clearly the presence of such an interaction is crucial for the practical implementation of the test in as much as such an interaction and the prices charged for it to the two-sides are observed.

Note however that in all media markets there is no real transaction between end-users, in the sense as there is for instance in a payment card market, as readers/viewers/listeners are not interested in seeing an ad, or their interest is only secondary with respect to that for content (news, films, music) provided by the platform.

Moreover, in most media markets the interaction is not observable. I will therefore mainly concentrate on this latter situation.

Figure 1

A Two-Sided Market: Media

Newspapers, TV, Radio, Internet…
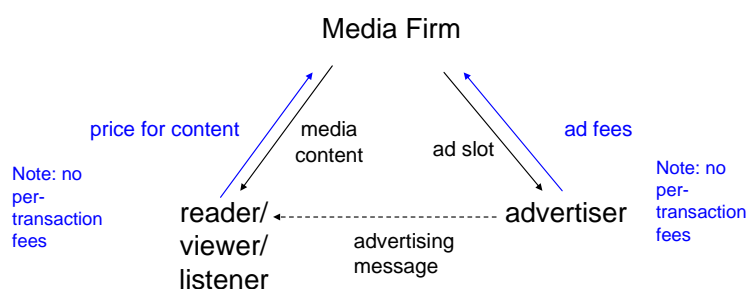
Note also that in a media market in the presence of multi-homing a platform can be in the relevant market on one-side but not on the other, e.g. TV might be a substitute for newspapers for an advertiser (as he just cares to reach his potential consumers) but not for a reader/viewer (as for instance a person can/likes to read his newspaper on the metro on his way to work and can/likes to watch TV at home in the evening). Moreover, as discussed in Evans & Noel (2005a), who bring the

example of bill-board advertising, in a market of the media type it might well be that a platform is competing with a one–sided firm on one of the two sides of the market. The same is not true in a payment card market, i.e. a card is either in the relevant market on both sides or not.

Given these different features, although one would ideally want an extension of the SSNIP test to two-sided markets which is the same for all types of two-sided markets, I believe the extension should be done differently according to the type of two-sided market.

In a market as the media one the hypothetical monopolist should be thought of as raising first one of the two prices and then the other price. Only in this way one could still interpret the test in terms of the original idea of a benchmark on the degree of substitution between products.

In addition I believe the hypothetical monopolist should be allowed to optimally adjust the price structure. First, the benchmark to decide when "substitution is enough" in a single-sided market is "what would happen to profits of an hypothetical monopolist" there is no reason why the latter should not be the benchmark also in a two-sided market. Second, more importantly, one still makes sure that the market is defined as the smallest one that it is worth monopolising (in the sense of where it is worth or optimal for a monopolist raising the price by 5% or 10% above the current competitive price). Finally, from a practical point of view, allowing the hypothetical monopolist to optimally adjust the price structure would have the positive side-effect to mitigate the otherwise unjustified worries of antitrust authorities that in a two-sided market with two positive indirect demand externalities considering all feedbacks would lead to a very wide market, since allowing the hypothetical monopolist to adjust optimally the price structure would, by definition, tend to increase the profitability of the price rise.

I therefore agree with Emch & Thomson (2006) in that in a two-sided market as the payment cards one the hypothetical monopolist should be thought of as raising the price level, adjusting optimally the price structure. I claim however that their extension of the traditional SSNIP test is not valid in a two-sided market of the media one. In such a market, as in Evans & Noel (2005b,2007), the hypothetical monopolist should be thought of as raising first the price on one side then that on the other side, but differently from Evans & Noel (2005b,2007) he should be allowed to adjust optimally the price structure.

4.2 - Which profits and which feedbacks ?

The issues of which feedbacks between the two-sides of the market should be taken into account and of whether we should look at what happens to profits on only one side or on both sides of the market are crucial.

For a two-sided market of the "media type", keeping in mind that the SSNIP test should aim at establishing whether there is "enough substitution", similarly, to Evans & Noel (2005b, 2007), I believe we should look at what happens to profits changes on all sides of the market and take into account all feedbacks (see figures 2 and 3). Many antitrust authorities seem to worry that in a two-sided market with two positive indirect demand externalities, considering all feedbacks would lead to a much wider market definition than single-sided market. This fear is unjustified as the point is exactly that the market is two-sided and should be treated as such. One should stress that there would be no feedback effect if there were no initial substitution effects in the market where the price has been increased (see figures 2 and 3). Moreover, if one allows the hypothetical monopolist to optimally adjust the price structure as claimed above, the feared enlargement of the market would be smaller, as the optimal adjustment of the price will lower the profit loss from the rise in price.

To this regard, assuming the relevant elasticities have been correctly estimated, in market where positive indirect network effects prevail (i.e. where accounting for feedbacks decreases the profitability of the rise in prices), it is true that if one applies the formula proposed by Evans&Noel (2005a,2007) the market is defined too widely (as it does not allow the hypothetical monopolist to optimally adjust the price on the other side), whereas if one applies the usual single-sided formulas, the market is defined too narrowly (as feedbacks are not accounted for). This relationship can sometimes provide sufficient indications in a competition policy cases[9].

With regard to the issue of how to deal with products which are competing on one-side of the market but not on the other-side, I believe one should take into account feedbacks from this products too in the analysis (see q^3_B in figures 2 and 3).

---

[9] For example, in a market where positive indirect network externalities prevail, if a given merger does not raise competitive concerns in a market defined according to the single-sided formula, a fortiori it wouldn't raise competitive concerns in the larger market defined according to the correct two-sided formula.
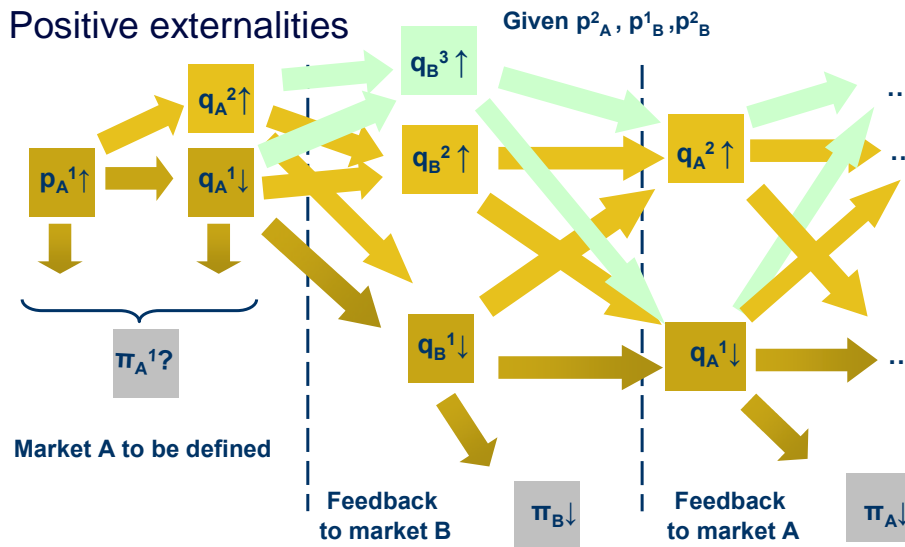
Figure 2

Media Market: Feedback & Profits - 1

Positive externalities

Given p^2_A , p^1_B , p^2_B

$q_A^2\uparrow$

$q_B^3\uparrow$

$p_A^1\uparrow$ → $q_A^1\downarrow$

$q_B^2\uparrow$

$q_A^2\uparrow$

$\pi_A^1?$

$q_B^1\downarrow$

$q_A^1\downarrow$

Market A to be defined

Feedback to market B

$\pi_B\downarrow$

Feedback to market A

$\pi_A\downarrow$

Figure 3

Media Market: Feedback & Profits - 2

One negative externality

For given p^2_A , p^1_B , p^2_B

and so on

$q_A^2\uparrow$

$q_B^3\uparrow$

$p_A^1\uparrow$ → $q_A^1\downarrow$

$q_B^2\uparrow$

$q_A^2\uparrow$

$\pi_A^1?$

$q_B^1\downarrow$

$q_A^1\uparrow$

Market A to be defined

Feedback to market B

$\pi_B\downarrow$

Feedback to market A

$\pi_A\uparrow$
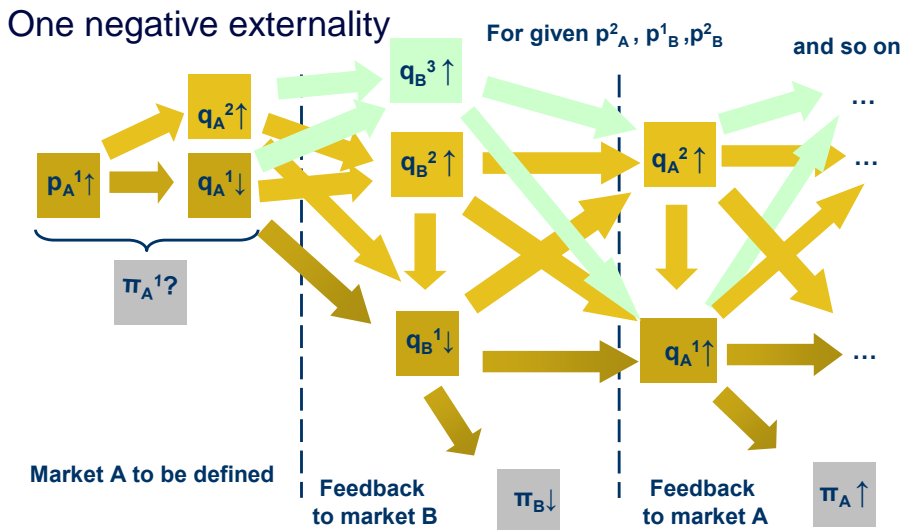
Finally, note that Evans & Noel (2007) distinguish between a short-run effect of the price increase, when the feedbacks have not taken place yet, and a long-run effect, when the feedback have all taken place. In fact, I believe such a distinction is in general useful from a theoretical point of view, as for sure there is a time dimension in the feedback story. However, the distinction is probably useless from the practical point of view in the case of the SSNIP test, as the price change is by definition "non transitory" and, as mentioned above, in a single-sided market "non transitory" is

usually taken to be a year. One could then claim that for most media markets one year is probably enough for all, or at least most, of the feedbacks to take place.

4.3 – The SSNIP test in a two-sided market

Taking into account the arguments above, I now proceed to develop the analytical formulas for the implementation of the test in a media market.

In a two-sided market with n candidate products j=1,2,…,n for the relevant market on each side m=A,B of the market, if the hypothetical firm is a monopolist over production of good 1, its profits in an adoption model are:

$$\Pi = p^A d^A(p^A, d^B) + p^B d^B(p^B, d^A) - C(d^A, d^B) \tag{1}$$

Then if it is possible to solve explicitly the system[10]
$$q^A = d^A(p^A, q^B)$$
$$q^B = d^B(q^A, p^B) \tag{2}$$

then
$$q^A = q^A(p^A, p^B)$$
$$q^B = q^B(p^A, p^B) \tag{3}$$

and

$$\Pi = p^A q^A(p^A, p^B) + p^B q^B(p^A, p^B) - C(q^A(p^A, p^B), q_B(p^A, p^B)) \tag{4}$$

If, in performing the SSNIP test, the hypothetical monopolist is allowed to adjust optimally the price of the good on the other side of the market, it will set

$$\frac{\partial \Pi}{\partial p^B} = q^B(p^A, p^B) + p^B \frac{\partial q^B(p^A, p^B)}{\partial p^B} + p^A \frac{\partial q^A(p^A, p^B)}{\partial p^B} - \frac{\partial C(q^A, q^B)}{\partial q^A} \frac{\partial q^A(p^A, p^B)}{\partial p^B} - \frac{\partial C(q^A, q^B)}{\partial q^B} \frac{\partial q^B(p^A, p^B)}{\partial p} = 0$$
$$\tag{5}$$

which identifies a relationship

$$f(p^A, \overset{*}{p^B}) = 0 \tag{6}$$

so that, if the equation can be solved explicitly for[11], it exists a function

---

[10] In fact, even if the system cannot be solved explicitly, one might be able to find the first derivatives (through the implicit function) and therefore to derive the first-order condition.

$$p^{B^{*}} = g^{*}(p^{A}) \tag{7}$$

which gives the optimal price on side B, $^{*}p^{B}$. for any price charged on side A, p^{A}.

<u>EU version</u>

Therefore, in the EU version of the SSNIP test, in the first step, to check what happens to Π as p^{A} increases, one should check the sign of

$$\Delta\Pi = \Pi(p^{A}(1+\frac{\Delta p^{A}}{p^{A}}), g^{B}(p^{A}(1+\frac{\Delta p^{A}}{p^{A}}))) - \Pi(p^{A}, p^{B})) \tag{8}$$

Note that, as for the SSNIP test in a single-sided market, assumptions are needed on the demand function and the cost function in order to calculate the first term on the right-hand side, which represents the counterfactual profits of the hypothetical monopolist and is therefore not observable by definition. One of the most common sets of assumptions are those of linear demand and linear cost.[12] Under these assumptions[13],

$$\Delta\Pi = t^{A}\Big(m^{A}R^{A}\varepsilon_{p^{A}}^{q^{A}} + m^{B}R^{B}\varepsilon_{p^{A}}^{q^{B}} + R^{A}\Big) + \big(t^{A}\big)^{2}R^{A}\varepsilon_{p^{A}}^{q^{A}} +$$

$$+ t^{A}\left[-t^{A}\frac{\varepsilon_{p^{B}}^{q^{A}}}{\varepsilon_{p^{B}}^{q^{B}}}\frac{R^{A}}{R^{B}} - t^{A}\frac{\varepsilon_{p^{A}}^{q^{B}}}{\varepsilon_{p^{B}}^{q^{B}}} - t^{A}\frac{1}{2}\left[\frac{\varepsilon_{p^{B}}^{q^{A}}}{\varepsilon_{p^{B}}^{q^{B}}}\right]\frac{R^{A}}{R^{B}} - m^{A}\frac{1}{2}\left[\frac{\varepsilon_{p^{B}}^{q^{A}}}{\varepsilon_{p^{B}}^{q^{B}}}\right]\frac{R^{A}}{R^{B}} - \frac{1}{2}\left[2+\frac{1}{\varepsilon_{p^{B}}^{q^{B}}}\right] - m^{B}\right]$$

$$\Big(R^{A}\varepsilon_{p^{B}}^{q^{A}} + R^{B}\varepsilon_{p^{A}}^{q^{B}}\Big) +$$

$$+ \left(-t^{A}\frac{\varepsilon_{p^{B}}^{q^{A}}}{\varepsilon_{p^{B}}^{q^{B}}}\frac{R^{A}}{R^{B}} - t^{A}\frac{\varepsilon_{p^{A}}^{q^{B}}}{\varepsilon_{p^{B}}^{q^{B}}} - t^{A}\frac{1}{2}\left[\frac{\varepsilon_{p^{B}}^{q^{A}}}{\varepsilon_{p^{B}}^{q^{B}}}\right]\frac{R^{A}}{R^{B}} - m^{A}\frac{1}{2}\left[\frac{\varepsilon_{p^{B}}^{q^{A}}}{\varepsilon_{p^{B}}^{q^{B}}}\right]\frac{R^{A}}{R^{B}} - \frac{1}{2}\left[2+\frac{1}{\varepsilon_{p^{B}}^{q^{B}}}\right] - m^{B}\right)$$

$$\Big(m^{A}R^{A}\varepsilon_{p^{B}}^{q^{A}} + m^{B}R^{B}\varepsilon_{p^{B}}^{q^{B}} + R^{B}\Big) +$$

$$+ \left(-t^{A}\frac{\varepsilon_{p^{B}}^{q^{A}}}{\varepsilon_{p^{B}}^{q^{B}}}\frac{R^{A}}{R^{B}} - t^{A}\frac{\varepsilon_{p^{A}}^{q^{B}}}{\varepsilon_{p^{B}}^{q^{B}}} - t^{A}\frac{1}{2}\left[\frac{\varepsilon_{p^{B}}^{q^{A}}}{\varepsilon_{p^{B}}^{q^{B}}}\right]\frac{R^{A}}{R^{B}} - m^{A}\frac{1}{2}\left[\frac{\varepsilon_{p^{B}}^{q^{A}}}{\varepsilon_{p^{B}}^{q^{B}}}\right]\frac{R^{A}}{R^{B}} - \frac{1}{2}\left[2+\frac{1}{\varepsilon_{p^{B}}^{q^{B}}}\right] - m^{B}\right)^{2} R^{B}\varepsilon_{p^{B}}^{q^{B}}$$

$$\tag{9}$$

---

[11] In fact, even if one cannot write the explicit function, one might be able to find the first derivatives (through the implicit function theorem) and therefore to derive $\dfrac{\partial g^{A}(p^{A})}{\partial p^{A}}$

[12] Alternative assumptions are those of constant elasticity demand functions and/or constant elasticity cost functions. These assumptions, as the linear ones, allow to derive the counterfactual from the observed data.
[13] See Appendix 1a

Instead, in the US version of the SSNIP test, in the first step, one should check whether an hypothetical monopolist, choosing p_1^A, p_1^B in order to maximize

$$\Pi = p^A q^A(p^A, p^B) + p^B q^B(p^A, p^B) - C(q^A(p^A, p^B), q_B(p^A, p^B)) \tag{10}$$

would raise the price p_1^A by more than the given threshold, that is whether

$$\frac{\Delta p^A}{p^A} = \frac{{}^*p^A - p^A}{p_1^A} > 5\% \text{ or } 10\% \tag{11}$$

where ${}^*p^A$ is the profit-maximizing price set by the hypothetical monopolist on side A.

From the first order conditions

$$\frac{\partial \Pi}{\partial p^A} = q^A(p^A, p^B) + p^A \frac{\partial q^A(p^A, p^B)}{\partial p^A} + p^B \frac{\partial q^B(p^A, p^B)}{\partial p^A} - \frac{\partial C(q^A, q^B)}{\partial q^A} \frac{\partial q^A(p^A, p^B)}{\partial p^B} - \frac{\partial C(q^A, q^B)}{\partial q^B} \frac{\partial q_1^B(p_1^A, p_1^B)}{\partial p_1^B} = 0$$

$$\frac{\partial \Pi}{\partial p^B} = q^B(p^A, p^B) + p^B \frac{\partial q^B(p^A, p^B)}{\partial p^B} + p^A \frac{\partial q^A(p^A, p^B)}{\partial p^B} - \frac{\partial C(q^A, q^B)}{\partial q^A} \frac{\partial q^A(p^A, p^B)}{\partial p^B} - \frac{\partial C(q^A, q^B)}{\partial q^B} \frac{\partial q^B(p^A, p^B)}{\partial p^B} = 0 \tag{12}$$

Again assumptions are needed on the demand function and the cost function. Under the usual assumptions of linear demand and linear costs[14]

$$\frac{\Delta p^A}{p^A} = -\frac{3}{4} m^B \frac{R^B}{R^A} \left(\frac{4\varepsilon_{p^B}^{q^B} \varepsilon_{p^A}^{q^B}}{4\varepsilon_{p^A}^{q^A} \varepsilon_{p^B}^{q^B} - \varepsilon_{p^A}^{q^B} \varepsilon_{p^B}^{q^A}} \right) - \frac{1}{2} m^A \left(\frac{4\varepsilon_{p^A}^{q^A} \varepsilon_{p^B}^{q^B}}{4\varepsilon_{p^A}^{q^A} \varepsilon_{p^B}^{q^B} - \varepsilon_{p^A}^{q^B} \varepsilon_{p^B}^{q^A}} \right) - \frac{1}{4} m^A \left(\frac{4\varepsilon_{p^A}^{q^B} \varepsilon_{p^B}^{q^A}}{4\varepsilon_{p^A}^{q^A} \varepsilon_{p^B}^{q^B} - \varepsilon_{p^A}^{q^B} \varepsilon_{p^B}^{q^A}} \right)$$
$$- \frac{1}{2} \left(\frac{4\varepsilon_{p^B}^{q^B}}{4\varepsilon_{p^A}^{q^A} \varepsilon_{p^B}^{q^B} - \varepsilon_{p^A}^{q^B} \varepsilon_{p^B}^{q^A}} \right) - \frac{1}{4} \frac{R^B}{R^A} \left(\frac{4\varepsilon_{p^B}^{q^A}}{4\varepsilon_{p^A}^{q^A} \varepsilon_{p^B}^{q^B} - \varepsilon_{p^A}^{q^B} \varepsilon_{p^B}^{q^A}} \right) \tag{13}$$

All in all, it is therefore clear that in order to perform the SSNIP test, in both the US and the EU version, one should not only estimate the own and cross elasticities of demand with respect to price and observe prices, sales and per unit costs on both sides of the market but also estimate the cross market network externalities.

4.3 – Critical Loss Analysis in a two-sided market

Critical Loss Analysis in its EU version in a two-sided market should proceed as follows: first, the critical loss(es) in sales on side A (and side B) of an hypothetical monopolist owning just one product is calculated; this is the maximum loss in sales due to a price increase of 5% or 10% on side A which would not make the price increase unprofitable; second, the actual loss(es) in sales on side

---

[14] See Appendix 1b

A (and side B) following a 5% or 10% increase in price on side A is (are) estimated; third, the actual loss(es) in sales is (are) compared to the critical loss in sales; if the actual loss(es) in sales is (are) smaller than then critical losses in sales, then a small significant price increase on side A would be profitable and the market is defined; otherwise, the market is assumed to contain also another product and the analysis is repeated.

Mathematically, assuming there n candidate products for the relevant market on side A of the market, in the first step, if the hypothetical firm is a monopolist over production of good 1, by increasing the price of its product 1 on side A by Δp^A and adjusting the price ratio $\dfrac{\Delta p^B}{\Delta p^A}$ optimally, the gains on side A will be $\Delta p^A (q^A + \Delta q^A)$ and those on side B will be $\dfrac{\Delta p^B}{\Delta p^A} \Delta p^A (q^B + \Delta q^B)$, while the losses will equal $-(p^A - \dfrac{\Delta C}{\Delta q^A})\Delta q^A$ and $-(p^B - \dfrac{\Delta C}{\Delta q^B})\Delta q^B$.

Equating the gains to the losses gives:

$$\Delta p^A (q^A + \Delta q^A) + \frac{\Delta p^B}{\Delta p^A} \Delta p^A (q^B + \Delta q^B) = -(p^A - \frac{\Delta C}{\Delta q^A})\Delta q^A - (p^B - \frac{\Delta C}{\Delta q^B})\Delta q^B \qquad (14)$$

This is the break-even condition. Clearly if the gains are higher than the losses (i.e. the left-hand side is higher than the right-hand side) then the price increase is profitable; vice versa the price increase is unprofitable.

As in a single-sided market I now proceed to manipulate the break-even condition in order to express it in terms of the given percentage price increase $\dfrac{\Delta p^A}{p^A}$, other observables (such as p^A, p^B, q^A, q^B) and of the percentage losses in sales, $\dfrac{\Delta q^A}{q^A}$ and $\dfrac{\Delta q^B}{q^B}$ the latter then being the critical losses, i.e. those that would guarantee a break-even following a given price increase.

After this manipulation[15] and assuming both linear demands and linear costs, the formula that defines implicitly the Critical Losses $\frac{\Delta q^A}{q^A}$ and $\frac{\Delta q^B}{q^B}$ on the two-sides of a two-sided market according to the EU version of the SSNIP test:

$$t^A R^A + R^A \left(t^A + m^A\right)\left(\frac{\Delta q^A}{q^A}\right) + m^B \left(\frac{\Delta q^B}{q^B}\right) R^B - \left[\left(\frac{\varepsilon_{p^B}^{q^A}}{\varepsilon_{p^B}^{q^B}}\right) + \left(\frac{\varepsilon_{p^A}^{q^B}}{\varepsilon_{p^B}^{q^B}}\right)\frac{R^B}{R^A}\right]t^A R^A \left(1 + \frac{\Delta q^B}{q^B}\right) +$$

$$- m^A R^A \left[\frac{\varepsilon_{p^B}^{q^A}}{\varepsilon_{p^B}^{q^B}}\right]\left(1 + \frac{\Delta q^B}{q^B}\right) - \frac{1}{\varepsilon_{p^B}^{q^B}} R^B \left(1 + \frac{\Delta q^B}{q^B}\right)^2 - m^B R^B \left(1 + \frac{\Delta q^B}{q^B}\right) = 0 \qquad (15)$$

where R^A are revenues from side A, $t^A = \frac{\Delta p^A}{p^A}$ is the percentage price increase on side A deemed relevant (5% or 10% usually) and $m^A = \frac{(p^A - \frac{\delta C^A}{\delta q^A})}{p^A}$ is the observed mark-up on side A . And similarly for side B.

Note that the part

$$t^A R^A + R^A \left(t^A + m^A\right)\left(\frac{\Delta q^A}{q^A}\right) + m^B \left(\frac{\Delta q^B}{q^B}\right) R^B$$

is the same as in Evans and Noel (2005a, 2007), while the rest is due to the hypothetical monopolist optimally adjusting the price structure.

The Actual Losses are instead:

$$\frac{\Delta q^A}{q^A} = \frac{\Delta p^A}{p^A}\frac{p^A}{\Delta p^A}\frac{\Delta q^A}{q^A} + \frac{\Delta p^B}{p^B}\frac{p^B}{\Delta p^B}\frac{\Delta q^A}{q^A}$$

$$\frac{\Delta q^B}{q^B} = \frac{\Delta p^A}{p^A}\frac{p^A}{\Delta p^A}\frac{\Delta q^B}{q^B} + \frac{\Delta p^B}{p^B}\frac{p^B}{\Delta p^B}\frac{\Delta q^B}{q^B} \qquad (16)$$

If demands are linear, one obtains

---

[15] See Appendix 2a

$$\frac{\Delta q^A}{q^A} = \frac{\Delta p^A}{p^A}\varepsilon_{P^A}^{q^A} + \frac{\Delta p^B}{p^B}\varepsilon_{P^B}^{q^A}$$

$$\frac{\Delta q^B}{q^B} = \frac{\Delta p^A}{p^A}\varepsilon_{P^A}^{q^B} + \frac{\Delta p^B}{p^B}\varepsilon_{P^B}^{q^B} \tag{17}$$

Note however once again that $\frac{\Delta p^B}{p^B}$ is not exogenous as in Evans&Noel (2005a, 2007) but is endogenously set by the hypothetical monopolist as a best reply to the exogenous change $\frac{\Delta p^A}{p^A}$.

Manipulating again the first order condition for profit maximization by the hypothetical monopolist, one can obtain [16] $\frac{\Delta p^B}{p^B}$ as a function of $\frac{\Delta p^A}{p^A}$:

$$\frac{\Delta p^B}{p^B} = -t^A \frac{1}{2}\left[\frac{\varepsilon_{p^B}^{q^A}}{\varepsilon_{p^B}^{q^B}}\right]\frac{R^A}{R^B} - m^A \frac{1}{2}\left[\frac{\varepsilon_{p^B}^{q^A}}{\varepsilon_{p^B}^{q^B}}\right]\frac{R^A}{R^B} - \frac{1}{2}\left[2 + \frac{1}{\varepsilon_{p^B}^{q^B}}\right] - m^B \tag{18}$$

So that substituting the latter into (17), the actual losses become[17]:

$$^{AL}\frac{\Delta q^A}{q^A} = \frac{\Delta p^A}{p^A}\left(\varepsilon_{P^A}^{q^A} - \frac{1}{2}\left[\frac{\varepsilon_{p^B}^{q^A}}{\varepsilon_{p^B}^{q^B}}\right]\frac{R^A}{R^B}\varepsilon_{P^B}^{q^A}\right) - m^A \frac{1}{2}\left(\frac{\varepsilon_{p^B}^{q^A}}{\varepsilon_{p^B}^{q^B}}\right)\frac{R^A}{R^B}\varepsilon_{P^B}^{q^A} - \frac{1}{2}\left(2 + \frac{1}{\varepsilon_{p^B}^{q^B}}\right)\varepsilon_{P^B}^{q^A} - m^B \varepsilon_{P^B}^{q^A}$$

$$^{AL}\frac{\Delta q^B}{q^B} = \frac{\Delta p^A}{p^A}\left(\varepsilon_{P^A}^{q^B} - \frac{1}{2}\left[\frac{\varepsilon_{p^B}^{q^A}}{\varepsilon_{p^B}^{q^B}}\right]\frac{R^A}{R^B}\varepsilon_{P^B}^{q^B}\right) - m^A \frac{1}{2}\left(\frac{\varepsilon_{p^B}^{q^A}}{\varepsilon_{p^B}^{q^B}}\right)\frac{R^A}{R^B}\varepsilon_{P^B}^{q^B} - \frac{1}{2}\left(2 + \frac{1}{\varepsilon_{p^B}^{q^B}}\right)\varepsilon_{P^B}^{q^B} - m^B \varepsilon_{P^B}^{q^B} \tag{19}$$

If, once the actual losses in (19) are substituted inside the break-even formula (15), the left-hand side is positive, then the price increase is profitable; vice versa the price increase is unprofitable.

Alternatively, one can solve the break-even condition (15) for the Critical Loss $^{CL}\frac{\Delta q^A}{q^A}$ and substitute on the right-hand side the expression for $^{AL}\frac{\Delta q^B}{q^B}$. One then obtains[18]:

---

[16] See Appendix 2b.
[17] For the critical loss on side A

$$\frac{\Delta q^A}{q^A} = \frac{\Delta p^A}{p^A}\varepsilon_{p^A}^{q^A} + \left[-\frac{\Delta p^A}{p^A}\frac{1}{2}\left[\frac{\varepsilon_{p^B}^{q^A}}{\varepsilon_{p^B}^{q^B}}\right]\frac{R^A}{R^B} - m^A \frac{1}{2}\left(\frac{\varepsilon_{p^B}^{q^A}}{\varepsilon_{p^B}^{q^B}}\right)\frac{R^A}{R^B} - \frac{1}{2}\left(2 + \frac{1}{\varepsilon_{p^B}^{q^B}}\right) - m^B\right]\varepsilon_{p^B}^{q^A}$$

$$\frac{\Delta q^A}{q^A} = \frac{\Delta p^A}{p^A}\varepsilon_{p^A}^{q^A} - \frac{\Delta p^A}{p^A}\frac{1}{2}\left[\frac{\varepsilon_{p^B}^{q^A}}{\varepsilon_{p^B}^{q^B}}\right]\frac{R^A}{R^B}\varepsilon_{p^B}^{q^A} - m^A \frac{1}{2}\left(\frac{\varepsilon_{p^B}^{q^A}}{\varepsilon_{p^B}^{q^B}}\right)\frac{R^A}{R^B}\varepsilon_{p^B}^{q^A} - \frac{1}{2}\left(2 + \frac{1}{\varepsilon_{p^B}^{q^B}}\right)\varepsilon_{p^B}^{q^A} - m^B \varepsilon_{p^B}^{q^A}$$

$$\frac{\Delta q^A}{q^A} = \frac{\Delta p^A}{p^A}\left(\varepsilon_{p^A}^{q^A} - \frac{1}{2}\left[\frac{\varepsilon_{p^B}^{q^A}}{\varepsilon_{p^B}^{q^B}}\right]\frac{R^A}{R^B}\varepsilon_{p^B}^{q^A}\right) - m^A \frac{1}{2}\left(\frac{\varepsilon_{p^B}^{q^A}}{\varepsilon_{p^B}^{q^B}}\right)\frac{R^A}{R^B}\varepsilon_{p^B}^{q^A} - \frac{1}{2}\left(2 + \frac{1}{\varepsilon_{p^B}^{q^B}}\right)\varepsilon_{p^B}^{q^A} - m^B \varepsilon_{p^B}^{q^A}$$

and similarly for the critical loss on side B.
[18] See Appendix 2c

$$^{CL}\frac{\Delta q^A}{q^A} = -\frac{t^A}{\left(t^A+m^A\right)} + \left[\left(\frac{\varepsilon_{p^B}^{q^A}}{\varepsilon_{p^B}^{q^B}}\right)+\left(\frac{\varepsilon_{p^A}^{q^B}}{\varepsilon_{p^B}^{q^B}}\right)\frac{R^B}{R^A}\right]\frac{t^A}{\left(t^A+m^A\right)}+\frac{m^A}{\left(t^A+m^A\right)}\left(\frac{\varepsilon_{p^B}^{q^A}}{\varepsilon_{p^B}^{q^B}}\right)+\frac{1}{\varepsilon_{p^B}^{q^B}}\frac{R^B}{R^A\left(t^A+m^A\right)}+\frac{m^B R^B}{R^A\left(t^A+m^A\right)}+$$

$$\left\{-\frac{m^B R^B}{R^A\left(t^A+m^A\right)}+\left[\left(\frac{\varepsilon_{p^B}^{q^A}}{\varepsilon_{p^B}^{q^B}}\right)+\left(\frac{\varepsilon_{p^A}^{q^B}}{\varepsilon_{p^B}^{q^B}}\right)\frac{R^B}{R^A}\right]\frac{t^A}{\left(t^A+m^A\right)}+\frac{m^A}{\left(t^A+m^A\right)}\left(\frac{\varepsilon_{p^B}^{q^A}}{\varepsilon_{p^B}^{q^B}}\right)+\frac{2}{\varepsilon_{p^B}^{q^B}}\frac{R^B}{R^A\left(t^A+m^A\right)}+\frac{m^B R^B}{R^A\left(t^A+m^A\right)}\right\}$$

$$\left\{t^A\varepsilon_{p^A}^{q^B}+\left[-m^A\frac{1}{2}\left(\frac{\varepsilon_{p^B}^{q^A}}{\varepsilon_{p^B}^{q^B}}\right)\frac{R^A}{R^B}-\frac{1}{2}\left(2+\frac{1}{\varepsilon_{p^B}^{q^B}}\right)-m^B\right]\varepsilon_{p^B}^{q^A}\right\}+$$

$$+\frac{1}{\varepsilon_{p^B}^{q^B}}\frac{R^B}{R^A\left(t^A+m^A\right)}\left\{t^A\varepsilon_{p^A}^{q^B}+\left[-m^A\frac{1}{2}\left(\frac{\varepsilon_{p^B}^{q^A}}{\varepsilon_{p^B}^{q^B}}\right)\frac{R^A}{R^B}-\frac{1}{2}\left(2+\frac{1}{\varepsilon_{p^B}^{q^B}}\right)-m^B\right]\varepsilon_{p^B}^{q^A}\right\}^2$$

<div align="right">(20)</div>

Then, as in the usual single-sided case, one only needs to check the actual loss $^{AL}\frac{\Delta q^A}{q^A}$ defined in

(19) against the critical loss $^{CL}\frac{\Delta q^A}{q^A}$ defined above in (20).

Note that both the Actual Loss formula (19) and the Critical Loss formula (20) look much more complex than in a single-sided market but in fact they are still expressed in terms of current elasticities and of current observed mark-ups, prices and quantities and rely on the same assumptions on demand and costs used to derive the formulas in a single-sided market .

US version

Critical Loss Analysis in its US version in a two-sided market should proceed as follows: first, the critical loss in sales on side A of an hypothetical monopolist owning just one product is calculated; this the maximum percentages of sales that a profit maximizing hypothetical monopolist would be willing to loose in order to increase price on side A of 5% or 10%; second, the actual loss in sales on side A following a 5% or 10% increase in price is estimated; third, the actual loss in sales is compared to the critical loss in sales; if the actual losses in sales is smaller than then critical loss in sales, then a small significant price increase would be profit maximizing and the market is defined; otherwise, the market is assumed to contain also another product and the analysis is repeated.

From the first order conditions for the hypothetical monopolist's profit maximization already reported in (12), one can obtain the equation for the optimal change in prices on one side A as a function of the optimal change in price on side B and of the optimal losses in sales on side A

$$\frac{\Delta p^B}{p^B} = -\frac{\Delta p^A}{p^A}\left[\frac{\varepsilon_{p^B}^{q^A}}{\varepsilon_{p^B}^{q^B}}\right]\frac{p^A q^A}{p^B q^B} + \frac{\left(p^A - \frac{\partial C}{\partial q^A}\right)}{p^A}\left[\frac{\varepsilon_{p^B}^{q^A}}{\varepsilon_{p^B}^{q^B}}\right]\frac{p^A q^A}{p^B q^B} - \frac{1}{\varepsilon_{p^B}^{q^B}}\left[1 + \frac{\Delta q^B}{q^B}\right] - \frac{\left(p^B - \frac{\partial C}{\partial q^B}\right)}{p^B} \qquad (21)$$

then, substituting inside (21) the expression for $\frac{\Delta p^B}{p^B}$ in (18), one obtains the expression for the Critical Loss on side A

$$^{CL}\frac{\Delta q^A}{q^A} = -\left(t^A - m^A\right)\left[1 - \frac{1}{2}\left(\frac{\varepsilon_{p^B}^{q^A}}{\varepsilon_{p^B}^{q^B}}\frac{\varepsilon_{p^A}^{q^B}}{\varepsilon_{p^A}^{q^A}}\right)\right]\varepsilon_{p^A}^{q^A} + \left[1 + \frac{1}{2\varepsilon_{p^B}^{q^B}} + 2m^B\right]\frac{R^B}{R^A}\varepsilon_{p^A}^{q^B} - 1 \qquad (22)$$

Then, as in the usual single-sided case, one only needs to check the actual loss $^{AL}\frac{\Delta q^A}{q^A}$ obtained in (19) against the critical loss $^{CL}\frac{\Delta q^A}{q^A}$ defined above in (22).

Note again that both the Actual Loss and the Critical Loss formulas look much more complex than in a single-sided market. This is due to the necessity to take into account the two-sidedness of the market. But in fact the formulas are still expressed in terms of current elasticities and of current observed mark-ups, prices and quantities and rely on the same assumptions on demand and costs used to derive the formulas in a single-sided market .

Yet, looking at the formulas, it is evident that data requirements for the implementation of the SSNIP test in two-sided markets, in both its US and EU versions, are higher than in a single –sided market as one needs to estimate not only the matrixes of the price elasticities of demand on each side of the market but also the matrixes of the price elasticities across sides of the market.

5 – Conclusions

I discussed the design and implementation of a SSNIP test for market definition in two-sided markets. I argued that in such a market the traditional SSNIP test cannot be applied as it is usually conceived. I proposed a SSNIP test for two-sided markets of the "media type".

The rationale behind the SSNIP test in a single-sided market is the following: it allows to set an (implicit) benchmark for the elasticities to be high enough for two candidate products to belong to the same relevant market; the benchmark is set in such a way that the market is defined as the smallest set of products on which a monopoly would find it profitable (or profit maximizing) to

exercise market power by raising non temporarily the price above the current (competitive) level (at least) by a small but significant percentage

In order to ensure the same rationale, the SSNIP test in a two-sided market should take into account changes in profits on both sides of the market and all feedbacks between profits on the two sides of the market following the hypothetical monopolist raise in prices. In addition it should be implemented by raising first the price on one side of the market then the price on the other side of the market, each time allowing the hypothetical monopolist to optimally adjust the price structure.

The antitrust authorities' worry that in a two-sided market with two positive indirect demand externalities, such a test would lead to a much wider market definition than single-sided market, is unjustified as the point is exactly that the market is two-sided market and should be treated as such. Yet, assuming a correct estimation of the relevant elasticities, in market where positive indirect network effects prevail, it is true that if one applies the formula proposed by Evans&Noel (2005a,2007) the market is defined too widely, whereas if one applies the usual single-sided formulas, the market is defined too narrowly. This relationship can sometimes provide sufficient indications in practice on the relevant market[19].

Finally I developed the corresponding formulas for critical loss analysis both for the EU and the US version of the test. These look much uglier than in a single-sided market but in fact they are easy to calculate as they are still expressed in terms of elasticities and of current observed markups, prices and quantities.

Data requirements for the implementation of the SSNIP test in two-sided markets are however higher than in a single –sided market as one needs to estimate not only the matrixes of the own and cross price elasticities of demand on the two-sides of the market but also the matrixes of the network effects.

References

- Anderson SP, Gabszewicz, JJ. 2005. The Media and Advertising: A Tale of Two-Sided Markets. CEPR Discussion Paper No. 5223
- Argentesi E, Filistrucchi L. 2007. Estimating Market Power in a Two-Sided Market: The Case of Newspapers. The Journal of Applied Econometrics, 22(7):1247 – 1266.

---

[19] For example, in a market where positive indirect network externalities prevail, if a given merger does not raise competitive concerns in a market defined according to the single-sided formula, a fortiori it wouldn't raise competitive concerns in the larger market defined according to the right two-sided formula.

- Argentesi E, Ivaldi M. 2005. Market Definition in Printed Media Industries: Theory, Practice, CEPR Discussion Paper n°5096.
- Armstrong M. 2006. Competition in Two-Sided Markets. Rand Journal of Economics,37(3):668-691.
- Bishop S, Walker M. (1999): The Economics of EC Competition Law. Sweet and Maxwell.
- Caillaud B, Jullien B. Chicken and Egg: Competition among Intermediation Service Providers. Rand Journal of Economics 34(2): 521-552.
- Carlton DW. 2007. Market Definition: Use and Abuse. Competition Policy International. Vol 3(1).
- Chandra A, Collard-Wexler A. Mergers in Two-Sided Markets: An Application to the Canadian Newspaper Industry. Stern School of Business Working Paper No. EC-07-03
- Emch E, Thomson T.S. 2006. Market Definition and Market Power in Payment Card Networks, The Review of Network Economics, 5 (1) , pp 45 – 60.
- Evans DS. 2008. Two-Sided Market Definition. Forthcoming as a chapter in ABA Section of Antitrust Law, Market Definition In Antitrust: Theory and Case Studies
- Evans DS, Noel MD. 2005a. Analyzing Market Definition and Power in Multi-sided Platform Markets, mimeo.
- Evans DS, Noel MD. 2005b. Defining Antitrust Markets When Firms Operate Two-Sided Platforms. Columbia Business Law Review, 667-702
- Evans DS, Noel MD. 2007. Noel, Michael D. and Evans, David S., Defining Markets that Involve Multi-Sided Platform Businesses: An Empirical Framework with an Application to Google's Purchase of DoubleClick, mimeo
- Evans DS, Schmalensee R. 2005a. The industrial Organization of Markets with Two-Sided Platforms, NBER Working Paper 11603.
- Evans DS. 2003. The Antitrust Economics of Multi-Sided Platform Markets. Yale Journal of Regulation 20: 325-381.
- Fletcher A, 2007, Competition Policy in Two-Sided Markets - Some brief remarks, Presentation at the 2nd LEAR Conference on the Economics of Competition Law
- Guthrie G, Wright J. 2007. Competing Payment Schemes, Journal of Industrial Economics, 2007, 55, 37-67.
- Harris BC, Simons JJ. 1989. Focusing Market Definition: How Much Substitution Is Enough, Research In Law And Economics 12, pp. 207-226.
- Holland M. 2007, Two-Sided Markets: A Challenge to Competition Policy?, mimeo.
- Ivaldi M, Lorincz S. 2005. A Full Equilibrium Relevant Market Test: Application to Computer Servers, IDEI Working Paper n. 341.
- Ivaldi M, Lorincz S. 2007. Implementing Relevant Market Tests in Antitrust Policy: Application to Computer Servers, mimeo.
- Kaiser U, Wright J. 2006. Price Structure in Two-Sided Markets: Evidence from the Magazine Industry. International Journal of Industrial Organization 24, 1– 28
- Parker GG, & Van Alstyne MV. 2002. Two-Sided Network Effects: A Theory of Information Product Design, Management Science, 51(10):1494–1504
- Rochet, JC, Tirole J. 2002. Cooperation among Competitors: The Economics of Payment Card Associations, Rand Journal of Economics, 33(4), 1-22.
- Rochet JC, Tirole J. 2003. Platform Competition in Two-Sided Markets. Journal of the European Economic Association 1(4): 990-1029.
- Rochet JC,Tirole J.2006. Two-Sided Markets: A Progress Report. Rand Journal of Economics, 37(3):645-667.
- Rochet JC, Tirole J. 2007. Must-Take Cards and the Tourist Test," DNB Working Papers 127.
- Rysman M. 2004. Competition Between Networks: A Study of the Market for Yellow Pages. Review of Economic Studies 71(2): 483-512.

- Rysman M. 2007. An Empirical Analysis of Payment Card Usage. Boston University, Journal of Industrial Economics, 55, 1-36.
- Sepinska E. 2008. The Merger between Google and DoubleClick: A Two-Sided Market Perspective, Master Thesis, Tilburg University.
- Werden GJ. 1998. Demand Elasticities in Antitrust Analysis, Antitrust Law Journal. 66, 387–96
- Werden GJ. 2002a. Beyond Critical Loss: Tailoring Applications of the Hypothetical Monopolist Paradigm, US DOJ Antitrust Division Economic Analysis Group Discussion Paper No. 02-9.
- Werden GJ. 2002b. Market Delineation Algorithms Based on the Hypothetical Monopolist Paradigm, US DOJ Antitrust Division Economic Analysis Group Discussion Paper No. 02-8.
- Werden GJ. 2003. The 1982 Merger Guidelines and the Ascent of the Hypothetical Monopolist Paradigm, Antitrust Law Journal 71, 253–76.
- Wright J. 2004. One-Sided Logic in Two-Sided Markets, Review of Network Economics, 1(3):42-63.

Appendix 1 – The SSNIP test formula

1a) In the EU

From

$$\Delta\Pi = \Pi(p^A(1+\frac{\Delta p^A}{p^A}), g^B(p^A(1+\frac{\Delta p^A}{p^A}))) - \Pi(p^A, p^B))$$

one gets

$$\Delta\Pi = p^A(1+\frac{\Delta p^A}{p^A})q^A(1+\frac{\Delta q^A}{q^A}) + p^B(1+\frac{\Delta p^B}{p^B})q^B(1+\frac{\Delta q^B}{q^B}) - p^A q^A - p^B q^B - \Delta C$$

or

$$\Delta\Pi = p^A q^A\left(1+\frac{\Delta q^A}{q^A}+\frac{\Delta p^A}{p^A}+\frac{\Delta p^A}{p^A}\frac{\Delta q^A}{q^A}\right) + p^B q^B\left(1+\frac{\Delta q^B}{q^B}+\frac{\Delta p^B}{p^B}+\frac{\Delta p^B}{p^B}\frac{\Delta q^B}{q^B}\right) - p^A q^A - p^B q^B - \Delta C$$

that is

$$\Delta\Pi = p^A q^A\left(\frac{\Delta q^A}{q^A}+\frac{\Delta p^A}{p^A}+\frac{\Delta p^A}{p^A}\frac{\Delta q^A}{q^A}\right) + p^B q^B\left(\frac{\Delta q^B}{q^B}+\frac{\Delta p^B}{p^B}+\frac{\Delta p^B}{p^B}\frac{\Delta q^B}{q^B}\right) - \Delta C$$

where

$\dfrac{\Delta p^A}{p^A}$ is exogenous

$$\Delta C = \frac{\Delta p^A}{p^A}\left(\frac{\Delta q^A}{q^A}\frac{p^A}{\Delta p^A}\frac{\Delta C}{\Delta q^A}q^A\right) + \frac{\Delta p^A}{p^A}\left(\frac{\Delta q^B}{q^B}\frac{p^A}{\Delta p^A}\frac{\Delta C}{\Delta q^B}q_1^B\right)$$

$$+ \frac{\Delta p^B}{p^B}\left(\frac{\Delta n^B}{n^B}\frac{p^B}{\Delta p^B}\frac{\Delta C}{\Delta n^B}n^B\right) + \frac{\Delta p^B}{p^B}\left(\frac{\Delta q^A}{q^A}\frac{p^B}{\Delta p^B}\frac{\Delta C}{\Delta q^A}n_1^A\right)$$

$$\frac{\Delta q^B}{q^B} = \frac{\Delta p^A}{p^A}\frac{\Delta q^B}{q^B}\frac{p^A}{\Delta p^A} + \frac{\Delta p^B}{p^B}\frac{\Delta q^B}{q^B}\frac{p^B}{\Delta p^B}$$

$$\frac{\Delta q^A}{q^A} = \frac{\Delta p^A}{p^A}\frac{\Delta q^A}{q^A}\frac{p^A}{\Delta p^A} + \frac{\Delta p^B}{p^B}\frac{\Delta q^A}{q^A}\frac{p^B}{\Delta p^B}$$

and

$$\frac{\Delta p^B}{p^B} = \frac{\Delta^* p^B}{p^B} + \frac{{}^* p^B - p^B}{p^B} = \frac{\Delta p^A}{p^A}\frac{\Delta^* p^B}{\Delta p^A}\frac{p^A}{p^B} + \frac{{}^* p^B - p^B}{p^B}$$

Under the assumptions of linear demand and linear cost,

$$\Delta C = \frac{\Delta p^A}{p^A}\varepsilon_{p^A}^{q^A}c^A q^A + \frac{\Delta p^A}{p^A}\varepsilon_{p^A}^{q^B}c^B q^B + \frac{\Delta p^B}{p^B}\varepsilon_{p^B}^{q^B}c^B q^B + \frac{\Delta p^B}{p^B}\varepsilon_{p^B}^{q^A}c^A q^A$$

$$\frac{\Delta q^A}{q^A} = \frac{\Delta p^A}{p^A}\varepsilon_{p^A}^{q^A} + \frac{\Delta p^B}{p^B}\varepsilon_{p^B}^{q^A}$$

$$\frac{\Delta q^B}{q^B} = \frac{\Delta p^A}{p^A}\varepsilon_{p^A}^{q^B} + \frac{\Delta p^B}{p^B}\varepsilon_{p^B}^{q^B}$$

and, since the best reply function is linear because demands are linear , its slope is (see 2a below)

$$\frac{\Delta^* p^B}{\Delta p^A} = \frac{\partial^* p^B}{\partial p^A} = -\left[\frac{\partial q^A}{\partial p^B}\bigg/\frac{\partial q^B}{\partial p^B}\right] - \left[\frac{\partial q^B}{\partial p^A}\bigg/\frac{\partial q^B}{\partial p^B}\right]$$

while (see 2a below)

$$\frac{{}^* p^B - p^B}{p^B} = -\frac{\Delta p^A}{p^A}\frac{1}{2}\left[\frac{\varepsilon_{p^B}^{q^A}}{\varepsilon_{p^B}^{q^B}}\right]\frac{R^A}{R^B} - m^A\frac{1}{2}\left[\frac{\varepsilon_{p^B}^{q^A}}{\varepsilon_{p^B}^{q^B}}\right]\frac{R^A}{R^B} - \frac{1}{2}\left[2+\frac{1}{\varepsilon_{p^B}^{q^B}}\right] - m^B$$

so that

$$\frac{\Delta p^B}{p^B} = -\frac{\Delta p^A}{p^A}\left[\frac{\partial q^A}{\partial p^B}\bigg/\frac{\partial q^B}{\partial p^B}\right]\frac{p^A}{p^B} - \frac{\Delta p^A}{p^A}\left[\frac{\partial q^B}{\partial p^A}\bigg/\frac{\partial q^B}{\partial p^B}\right]\frac{p^A}{p^B} - \frac{\Delta p^A}{p^A}\frac{1}{2}\left[\frac{\varepsilon_{p^B}^{q^A}}{\varepsilon_{p^B}^{q^B}}\right]\frac{R^A}{R^B} - m^A\frac{1}{2}\left[\frac{\varepsilon_{p^B}^{q^A}}{\varepsilon_{p^B}^{q^B}}\right]\frac{R^A}{R^B} - \frac{1}{2}\left[2+\frac{1}{\varepsilon_{p^B}^{q^B}}\right] - m^B$$

which is

$$\frac{\Delta p^B}{p^B} = -\frac{\Delta p^A}{p^A}\left[\frac{\partial q^A}{\partial p^B}\frac{p^B}{q^A}\bigg/\frac{\partial q^B}{\partial p^B}\frac{p^B}{q^B}\right]\frac{p^A q^A}{p^B q^B} - \frac{\Delta p^A}{p^A}\left[\frac{\partial q^B}{\partial p^A}\frac{p^A}{q^B}\bigg/\frac{\partial q^B}{\partial p^B}\frac{p^B}{q^B}\right]\frac{p^B}{q^B} - \frac{\Delta p^A}{p^A}\frac{1}{2}\left[\frac{\varepsilon_{p^B}^{q^A}}{\varepsilon_{p^B}^{q^B}}\right]\frac{R^A}{R^B} - m^A\frac{1}{2}\left[\frac{\varepsilon_{p^B}^{q^A}}{\varepsilon_{p^B}^{q^B}}\right]\frac{R^A}{R^B} - \frac{1}{2}\left[2+\frac{1}{\varepsilon_{p^B}^{q^B}}\right] - m^B$$

so that

$$\frac{\Delta p^B}{p^B} = -\frac{\Delta p^A}{p^A}\frac{\varepsilon_{p^B}^{q^A}}{\varepsilon_{p^B}^{q^B}}\frac{R^A}{R^B} - \frac{\Delta p^A}{p^A}\frac{\varepsilon_{p^A}^{q^B}}{\varepsilon_{p^B}^{q^B}} - \frac{\Delta p^A}{p^A}\frac{1}{2}\left[\frac{\varepsilon_{p^B}^{q^A}}{\varepsilon_{p^B}^{q^B}}\right]\frac{R^A}{R^B} - m^A\frac{1}{2}\left[\frac{\varepsilon_{p^B}^{q^A}}{\varepsilon_{p^B}^{q^B}}\right]\frac{R^A}{R^B} - \frac{1}{2}\left[2+\frac{1}{\varepsilon_{p^B}^{q^B}}\right] - m^B$$

So that, substituting for $\dfrac{\Delta q^A}{q^A}, \dfrac{\Delta q^B}{q^B}$ and ΔC ,

$$\Delta\Pi = p^A q^A\left(\frac{\Delta q^A}{q^A} + \frac{\Delta p^A}{p^A} + \frac{\Delta p^A}{p^A}\frac{\Delta q^A}{q^A}\right) + p^B q^B\left(\frac{\Delta q^B}{q^B} + \frac{\Delta p^B}{p^B} + \frac{\Delta p^B}{p^B}\frac{\Delta q^B}{q^B}\right) - \Delta C$$

$$\Delta\Pi = p^A q^A\left(\frac{\Delta p^A}{p}\varepsilon_{p^A}^{q_1^A} + \frac{\Delta p^B}{p^B}\varepsilon_{p^B}^{q^A} + \frac{\Delta p^A}{p^A} + \frac{\Delta p^A}{p^A}\frac{\Delta p^A}{p^A}\varepsilon_{p^A}^{q^A} + \frac{\Delta p^A}{p^A}\frac{\Delta p^B}{p^B}\varepsilon_{p^B}^{q^A}\right)$$

$$+ p^B q^B\left(\frac{\Delta p^A}{p^A}\varepsilon_{p^A}^{q^B} + \frac{\Delta p^B}{p^B}\varepsilon_{p^B}^{q^B} + \frac{\Delta p^B}{p^B} + \frac{\Delta p^B}{p^B}\frac{\Delta p^A}{p^A}\varepsilon_{p^A}^{q^B} + \frac{\Delta p^B}{p^B}\frac{\Delta p^B}{p^B}\varepsilon_{p^B}^{q^B}\right)$$

$$- \frac{\Delta p^A}{p^A}\varepsilon_{p^A}^{q^A}c^A q^A - \frac{\Delta p^A}{p^A}\varepsilon_{p^A}^{q^B}c^B q^B - \frac{\Delta p^B}{p^B}\varepsilon_{p^B}^{q^B}c^B q^B - \frac{\Delta p^B}{p^B}\varepsilon_{p^B}^{q^A}c^A q^A$$

which is equivalent to

$$\Delta\Pi = \frac{\Delta p^A}{p^A} p^A q^A \varepsilon_{p^A}^{q^A} + \frac{\Delta p^B}{p^B} p^A q^A \varepsilon_{p^B}^{q^A} + \frac{\Delta p^A}{p^A} p^A q^A + \frac{\Delta p^A}{p^A}\frac{\Delta p^A}{p^A} p^A q^A \varepsilon_{p^A}^{q^A} + \frac{\Delta p^A}{p^A}\frac{\Delta p^B}{p^B} p^A q^A \varepsilon_{p^B}^{q^A} +$$

$$+ \frac{\Delta p^A}{p^A} p^B q^B \varepsilon_{p^A}^{q^B} + \frac{\Delta p^B}{p^B} p^B q^B \varepsilon_{p^B}^{q^B} + \frac{\Delta p^B}{p^B} p^B q^B + \frac{\Delta p^B}{p^B}\frac{\Delta p^A}{p^A} p^B q^B \varepsilon_{p^A}^{q^B} + \frac{\Delta p^B}{p^B}\frac{\Delta p^B}{p^B} p^B q^B \varepsilon_{p^B}^{q^B} +$$

$$- \frac{\Delta p^A}{p^A}\varepsilon_{p^A}^{q^A} c^A q^A - \frac{\Delta p^A}{p^A}\varepsilon_{p^A}^{q^B} c^B q^B - \frac{\Delta p^B}{p^B}\varepsilon_{p^B}^{q^B} c^B q^B - \frac{\Delta p^B}{p_1^B}\varepsilon_{p^B}^{q^A} c^A q^A$$

that is

$$\Delta\Pi = \frac{\Delta p^A}{p^A} p^A q^A \varepsilon_{p^A}^{q^A} + \frac{\Delta p^A}{p^A} p^B q^B \varepsilon_{p^A}^{q^B} + \frac{\Delta p^A}{p^A} p^A q^A - \frac{\Delta p^A}{p^A}\varepsilon_{p^A}^{q^A} c^A q^A - \frac{\Delta p^A}{p^A}\varepsilon_{p^A}^{q^B} c^B q^B +$$

$$\left(\frac{\Delta p^A}{p^A}\right)^2 p^A q^A \varepsilon_{p^A}^{q^A} +$$

$$+ \frac{\Delta p^A}{p^A}\frac{\Delta p^B}{p^B} p^A q^A \varepsilon_{p^B}^{q^A} + \frac{\Delta p^B}{p^B}\frac{\Delta p^A}{p^A} p^B q^B \varepsilon_{p^A}^{q^B}$$

$$+ \frac{\Delta p^B}{p^B} p^A q^A \varepsilon_{p^B}^{q^A} + \frac{\Delta p^B}{p^B} p^B q^B \varepsilon_{p^B}^{q^B} + \frac{\Delta p^B}{p^B} p^B q^B - \frac{\Delta p^B}{p^B}\varepsilon_{p^B}^{q^B} c^B q^B - \frac{\Delta p^B}{p^B}\varepsilon_{p^B}^{q^A} c^A q^A$$

$$+ \left(\frac{\Delta p^B}{p^B}\right)^2 p^B q^B \varepsilon_{p^B}^{q^B}$$

or

$$\Delta\Pi = \frac{\Delta p^A}{p^A}\left(p^A q^A \varepsilon_{p^A}^{q^A} + p^B q^B \varepsilon_{p^A}^{q^B} + p^A q^A - \varepsilon_{p^A}^{q^A} c^A q^A - \varepsilon_{p^A}^{q^B} c^B q^B \right) +$$

$$+ \left(\frac{\Delta p^A}{p^A}\right)^2 p^A q^A \varepsilon_{p^A}^{q^A} +$$

$$+ \frac{\Delta p^A}{p^A}\frac{\Delta p^B}{p^B}\left(p^A q^A \varepsilon_{p^B}^{q^A} + p^B q^B \varepsilon_{p^A}^{q^B} \right) +$$

$$+ \frac{\Delta p^B}{p^B}\left(p^A q^A \varepsilon_{p^B}^{q^A} + p^B q^B \varepsilon_{p^B}^{q^B} + p^B q^B - \varepsilon_{p^B}^{q^B} c^B q^B - \varepsilon_{p^B}^{q^A} c^A q^A \right) +$$

$$+ \left(\frac{\Delta p^B}{p^B}\right)^2 p^B q^B \varepsilon_{p^B}^{q^B}$$

and substituting also for $\dfrac{\Delta p^B}{p^B}$

$$\Delta\Pi = \frac{\Delta p^A}{p^A}\left(p^A q^A \varepsilon_{p^A}^{q^A} + p^B q^B \varepsilon_{p^A}^{q^B} + p^A q^A - \varepsilon_{p^A}^{q^A} c^A q^A - \varepsilon_{p^A}^{q^B} c^B q^B\right) + \left(\frac{\Delta p^A}{p^A}\right)^2 p^A q^A \varepsilon_{p^A}^{q^A} +$$

$$+ \frac{\Delta p^A}{p^A}\left(-\frac{\Delta p_1^A}{p_1^A}\frac{\varepsilon_{p^B}^{q^A}}{\varepsilon_{p^B}^{q^B}}\frac{R^A}{R^B} - \frac{\Delta p_1^A}{p_1^A}\frac{\varepsilon_{p^A}^{q^B}}{\varepsilon_{p^B}^{q^B}} - \frac{\Delta p^A}{p^A}\frac{1}{2}\left[\frac{\varepsilon_{p^B}^{q^A}}{\varepsilon_{p^B}^{q^B}}\right]\frac{R^A}{R^B} - m^A\frac{1}{2}\left[\frac{\varepsilon_{p^B}^{q^A}}{\varepsilon_{p^B}^{q^B}}\right]\frac{R^A}{R^B} - \frac{1}{2}\left[2 + \frac{1}{\varepsilon_{p^B}^{q^B}}\right] - m^B\right)$$

$$\left(p^A q^A \varepsilon_{p^B}^{q^A} + p^B q^B \varepsilon_{p^A}^{q^B}\right) +$$

$$+\left(-\frac{\Delta p^A}{p^A}\frac{\varepsilon_{p^B}^{q^A}}{\varepsilon_{p^B}^{q^B}}\frac{R^A}{R^B} - \frac{\Delta p^A}{p^A}\frac{\varepsilon_{p^A}^{q^B}}{\varepsilon_{p^B}^{q^B}} - \frac{\Delta p^A}{p^A}\frac{1}{2}\left[\frac{\varepsilon_{p^B}^{q^A}}{\varepsilon_{p^B}^{q^B}}\right]\frac{R^A}{R^B} - m^A\frac{1}{2}\left[\frac{\varepsilon_{p^B}^{q^A}}{\varepsilon_{p^B}^{q^B}}\right]\frac{R^A}{R^B} - \frac{1}{2}\left[2 + \frac{1}{\varepsilon_{p^B}^{q^B}}\right] - m^B\right)$$

$$\left(p^A q^A \varepsilon_{p^B}^{q^A} + p^B q^B \varepsilon_{p^B}^{q^B} + p^B q^B - \varepsilon_{p^B}^{q^B} c^B q^B - \varepsilon_{p^B}^{q^A} c^A q^A\right) +$$

$$+\left(-\frac{\Delta p^A}{p^A}\frac{\varepsilon_{p^B}^{q^A}}{\varepsilon_{p^B}^{q^B}}\frac{R^A}{R^B} - \frac{\Delta p^A}{p^A}\frac{\varepsilon_{p^A}^{q^B}}{\varepsilon_{p^B}^{q^B}} - \frac{\Delta p^A}{p^A}\frac{1}{2}\left[\frac{\varepsilon_{p^B}^{q^A}}{\varepsilon_{p^B}^{q^B}}\right]\frac{R^A}{R^B} - m^A\frac{1}{2}\left[\frac{\varepsilon_{p^B}^{q^A}}{\varepsilon_{p^B}^{q^B}}\right]\frac{R^A}{R^B} - \frac{1}{2}\left[2 + \frac{1}{\varepsilon_{p^B}^{q^B}}\right] - m^B\right)^2 p^B q^B \varepsilon_{p^B}^{q^B}$$

so that

$$\Delta\Pi = t^A\left(R^A\varepsilon_{p^A}^{q^A} + R^B\varepsilon_{p^A}^{q^B} + R^A - \varepsilon_{p^A}^{q^A}c^A q^A - \varepsilon_{p^A}^{q^B}c^B q^B\right) + \left(t^A\right)^2 R^A\varepsilon_{p^A}^{q^A} +$$

$$+ t^A\left(-t^A\frac{\varepsilon_{p^B}^{q^A}}{\varepsilon_{p^B}^{q^B}}\frac{R^A}{R^B} - t^A\frac{\varepsilon_{p^A}^{q^B}}{\varepsilon_{p^B}^{q^B}} - t^A\frac{1}{2}\left[\frac{\varepsilon_{p^B}^{q^A}}{\varepsilon_{p^B}^{q^B}}\right]\frac{R^A}{R^B} - m^A\frac{1}{2}\left[\frac{\varepsilon_{p^B}^{q^A}}{\varepsilon_{p^B}^{q^B}}\right]\frac{R^A}{R^B} - \frac{1}{2}\left[2 + \frac{1}{\varepsilon_{p^B}^{q^B}}\right] - m^B\right)$$

$$\left(R^A\varepsilon_{p^B}^{q^A} + R^B\varepsilon_{p^A}^{q^B}\right) +$$

$$+\left(-t^A\frac{\varepsilon_{p^B}^{q^A}}{\varepsilon_{p^B}^{q^B}}\frac{R^A}{R^B} - t^A\frac{\varepsilon_{p^A}^{q^B}}{\varepsilon_{p^B}^{q^B}} - t^A\frac{1}{2}\left[\frac{\varepsilon_{p^B}^{q^A}}{\varepsilon_{p^B}^{q^B}}\right]\frac{R^A}{R^B} - m^A\frac{1}{2}\left[\frac{\varepsilon_{p^B}^{q^A}}{\varepsilon_{p^B}^{q^B}}\right]\frac{R^A}{R^B} - \frac{1}{2}\left[2 + \frac{1}{\varepsilon_{p^B}^{q^B}}\right] - m^B\right)$$

$$\left(R^A\varepsilon_{p^B}^{q^A} + R^B\varepsilon_{p^B}^{q^B} + R^B - \varepsilon_{p^B}^{q^B}c^B q^B - \varepsilon_{p^B}^{q^A}c^A q^A\right) +$$

$$+\left(-t^A\frac{\varepsilon_{p^B}^{q^A}}{\varepsilon_{p^B}^{q^B}}\frac{R^A}{R^B} - t^A\frac{\varepsilon_{p^A}^{q^B}}{\varepsilon_{p^B}^{q^B}} - t^A\frac{1}{2}\left[\frac{\varepsilon_{p^B}^{q^A}}{\varepsilon_{p^B}^{q^B}}\right]\frac{R^A}{R^B} - m^A\frac{1}{2}\left[\frac{\varepsilon_{p^B}^{q^A}}{\varepsilon_{p^B}^{q^B}}\right]\frac{R^A}{R^B} - \frac{1}{2}\left[2 + \frac{1}{\varepsilon_{p^B}^{q^B}}\right] - m^B\right)^2 R^B\varepsilon_{p^B}^{q^B}$$

or equivalently

$$\Delta\Pi = t^A\left(m^A R^A \varepsilon_{p^A}^{q^A} + m^B R^B \varepsilon_{p^A}^{q^B} + R^A\right) + \left(t^A\right)^2 R^A \varepsilon_{p^A}^{q^A} +$$

$$+ t^A\left[-t^A \frac{\varepsilon_{p^B}^{q^A}}{\varepsilon_{p^B}^{q^B}}\frac{R^A}{R^B} - t^A \frac{\varepsilon_{p^A}^{q^B}}{\varepsilon_{p^B}^{q^B}} - t^A \frac{1}{2}\left[\frac{\varepsilon_{p^B}^{q^A}}{\varepsilon_{p^B}^{q^B}}\right]\frac{R^A}{R^B} - m^A \frac{1}{2}\left[\frac{\varepsilon_{p^B}^{q^A}}{\varepsilon_{p^B}^{q^B}}\right]\frac{R^A}{R^B} - \frac{1}{2}\left[2 + \frac{1}{\varepsilon_{p^B}^{q^B}}\right] - m^B\right]$$

$$\left(R^A \varepsilon_{p^B}^{q^A} + R^B \varepsilon_{p^A}^{q^B}\right) +$$

$$+\left(-t^A \frac{\varepsilon_{p^B}^{q^A}}{\varepsilon_{p^B}^{q^B}}\frac{R^A}{R^B} - t^A \frac{\varepsilon_{p^A}^{q^B}}{\varepsilon_{p^B}^{q^B}} - t^A \frac{1}{2}\left[\frac{\varepsilon_{p^B}^{q^A}}{\varepsilon_{p^B}^{q^B}}\right]\frac{R^A}{R^B} - m^A \frac{1}{2}\left[\frac{\varepsilon_{p^B}^{q^A}}{\varepsilon_{p^B}^{q^B}}\right]\frac{R^A}{R^B} - \frac{1}{2}\left[2 + \frac{1}{\varepsilon_{p^B}^{q^B}}\right] - m^B\right)$$

$$\left(m^A R^A \varepsilon_{p^B}^{q^A} + m^B R^B \varepsilon_{p^B}^{q^B} + R^B\right) +$$

$$+\left(-t^A \frac{\varepsilon_{p^B}^{q^A}}{\varepsilon_{p^B}^{q^B}}\frac{R^A}{R^B} - t^A \frac{\varepsilon_{p^A}^{q^B}}{\varepsilon_{p^B}^{q^B}} - t^A \frac{1}{2}\left[\frac{\varepsilon_{p^B}^{q^A}}{\varepsilon_{p^B}^{q^B}}\right]\frac{R^A}{R^B} - m^A \frac{1}{2}\left[\frac{\varepsilon_{p^B}^{q^A}}{\varepsilon_{p^B}^{q^B}}\right]\frac{R^A}{R^B} - \frac{1}{2}\left[2 + \frac{1}{\varepsilon_{p^B}^{q^B}}\right] - m^B\right)^2 R^B \varepsilon_{p^B}^{q^B}$$

$$(9)$$

1b) In the US

From the first order conditions for profit maximization by the hypothetical monopolist

$$\frac{\partial\Pi}{\partial p^A} = q^A(p^A, p^B) + p^A \frac{\partial q^A(p^A,p^B)}{\partial p^A} + p^B \frac{\partial q^B(p^A,p^B)}{\partial p^A} - \frac{\partial C(q^A,q^B)}{\partial q^A}\frac{\partial q^A(p^A,p^B)}{\partial p^B} - \frac{\partial C(q^A,q^B)}{\partial q^B}\frac{\partial q_1^B(p_1^A,p_1^B)}{\partial p_1^B} = 0$$

$$\frac{\partial\Pi}{\partial p^B} = q^B(p^A, p^B) + p^B \frac{\partial q^B(p^A,p^B)}{\partial p^B} + p^A \frac{\partial q^A(p^A,p^B)}{\partial p^B} - \frac{\partial C(q^A,q^B)}{\partial q^A}\frac{\partial q^A(p^A,p^B)}{\partial p^B} - \frac{\partial C(q^A,q^B)}{\partial q^B}\frac{\partial q^B(p^A,p^B)}{\partial p^B} = 0$$

$$(12)$$

one can obtain

$${}^*p^A = -\left({}^*p^B - \frac{\partial C}{\partial q^B}\right)\left[\frac{\partial q^B}{\partial p^A}\Big/\frac{\partial q^A}{\partial p^A}\right] - \left[{}^*q^A\Big/\frac{\partial q^A}{\partial p^A}\right] + \frac{\partial C}{\partial q^A}$$

$${}^*p^B = -\left({}^*p^A - \frac{\partial C}{\partial q^A}\right)\left[\frac{\partial q^A}{\partial p^B}\Big/\frac{\partial q^B}{\partial p^B}\right] - \left[{}^*q^B\Big/\frac{\partial q^B}{\partial p^B}\right] + \frac{\partial C}{\partial q^B}$$

since ${}^*q^A = q^A + \Delta q^A$ and ${}^*q^B = q^B + \Delta q^B$

$${}^*p^A = -{}^*p^B\left[\frac{\partial q^B}{\partial p^A}\Big/\frac{\partial q^A}{\partial p^A}\right] + \frac{\partial C}{\partial q^B}\left[\frac{\partial q^B}{\partial p^A}\Big/\frac{\partial q^A}{\partial p^A}\right] - \left[\Delta q^A\Big/\frac{\partial q^A}{\partial p^A}\right] - \left[q^A\Big/\frac{\partial q^A}{\partial p^A}\right] + \frac{\partial C}{\partial q^A}$$

$${}^*p^B = -{}^*p^A\left[\frac{\partial q^A}{\partial p^B}\Big/\frac{\partial q^B}{\partial p^B}\right] + \frac{\partial C}{\partial q^A}\left[\frac{\partial q^A}{\partial p^B}\Big/\frac{\partial q^B}{\partial p^B}\right] - \left[q^B\Big/\frac{\partial q^B}{\partial p^B}\right] - \left[\Delta q^B\Big/\frac{\partial q^B}{\partial p^B}\right] + \frac{\partial C}{\partial q^B}$$

multiplying and dividing the third term on the right-hand side in the first equation by Δp^A and in the

second equation by Δp^B

$$^*p^A = -^*p^B \left[\frac{\partial q^B}{\partial p^A} \middle/ \frac{\partial q^A}{\partial p^A} \right] + \frac{\partial C}{\partial q^B} \left[\frac{\partial q^B}{\partial p^A} \middle/ \frac{\partial q^A}{\partial p^A} \right] - \Delta p^A \left[\frac{\Delta q^A}{\Delta p^A} \middle/ \frac{\partial q^A}{\partial p^A} \right] - \left[q^A \middle/ \frac{\partial q^A}{\partial p^A} \right] + \frac{\partial C}{\partial q^A}$$

$$^*p^B = -^*p^A \left[\frac{\partial q^A}{\partial p^B} \middle/ \frac{\partial q^B}{\partial p^B} \right] + \frac{\partial C}{\partial q^A} \left[\frac{\partial q^A}{\partial p^B} \middle/ \frac{\partial q^B}{\partial p^B} \right] - \left[q^B \middle/ \frac{\partial q^B}{\partial p^B} \right] - \Delta p^B \left[\frac{\Delta q^B}{\Delta p^B} \middle/ \frac{\partial q^B}{\partial p^B} \right] + \frac{\partial C}{\partial q^B}$$

and, since $\dfrac{\Delta q^A}{\Delta p^A} = \dfrac{\partial q^A}{\partial p^A}$ and $\dfrac{\Delta q^B}{\Delta p^B} = \dfrac{\partial q^B}{\partial p^B}$, because demands are linear

$$^*p^A = -^*p^B \left[\frac{\partial q^B}{\partial p^A} \middle/ \frac{\partial q^A}{\partial p^A} \right] + \frac{\partial C}{\partial q^B} \left[\frac{\partial q^B}{\partial p^A} \middle/ \frac{\partial q^A}{\partial p^A} \right] - \Delta p^A - \left[q^A \middle/ \frac{\partial q^A}{\partial p^A} \right] + \frac{\partial C}{\partial q^A}$$

$$^*p^B = -^*p^A \left[\frac{\partial q^A}{\partial p^B} \middle/ \frac{\partial q^B}{\partial p^B} \right] + \frac{\partial C}{\partial q^A} \left[\frac{\partial q^A}{\partial p^B} \middle/ \frac{\partial q^B}{\partial p^B} \right] - \left[q^B \middle/ \frac{\partial q^B}{\partial p^B} \right] - \Delta p^B + \frac{\partial C}{\partial q^B}$$

since $\Delta p^A = {}^*p^A - p^A$ and $\Delta p^B = {}^*p^B - p^B$

$$^*p^A = -^*p^B \left[\frac{\partial q^B}{\partial p^A} \middle/ \frac{\partial q^A}{\partial p^A} \right] + \frac{\partial C}{\partial q^B} \left[\frac{\partial q^B}{\partial p^A} \middle/ \frac{\partial q^A}{\partial p^A} \right] - {}^*p^A + p^A - \left[q^A \middle/ \frac{\partial q^A}{\partial p^A} \right] + \frac{\partial C}{\partial q^A}$$

$$^*p^B = -^*p^A \left[\frac{\partial q^A}{\partial p^B} \middle/ \frac{\partial q^B}{\partial p^B} \right] + \frac{\partial C}{\partial q^A} \left[\frac{\partial q^A}{\partial p^B} \middle/ \frac{\partial q^B}{\partial p^B} \right] - \left[q^B \middle/ \frac{\partial q^B}{\partial p^B} \right] - {}^*p^B + p^B + \frac{\partial C}{\partial q^B}$$

bringing to the left-hand side all terms with $^*p^A$ in the first equation and all terms with $^*p^B$ in the second equation

$$2^*p^A = -^*p^B \left[\frac{\partial q^B}{\partial p^A} \middle/ \frac{\partial q^A}{\partial p^A} \right] + \frac{\partial C}{\partial q^B} \left[\frac{\partial q^B}{\partial p^A} \middle/ \frac{\partial q^A}{\partial p^A} \right] + p^A - \left[q^A \middle/ \frac{\partial q^A}{\partial p^A} \right] + \frac{\partial C}{\partial q^A}$$

$$2^*p^B = -^*p^A \left[\frac{\partial q^A}{\partial p^B} \middle/ \frac{\partial q^B}{\partial p^B} \right] + \frac{\partial C}{\partial q^A} \left[\frac{\partial q^A}{\partial p^B} \middle/ \frac{\partial q^B}{\partial p^B} \right] - \left[q^B \middle/ \frac{\partial q^B}{\partial p^B} \right] + p^B + \frac{\partial C}{\partial q^B}$$

dividing both sides in both equations by 2

$$^*p^A = -^*p^B \frac{1}{2}\left[\frac{\partial q^B}{\partial p^A} \middle/ \frac{\partial q^A}{\partial p^A} \right] + \frac{\partial C}{\partial q^B} \frac{1}{2}\left[\frac{\partial q^B}{\partial p^A} \middle/ \frac{\partial q^A}{\partial p^A} \right] + \frac{1}{2} p^A - \frac{1}{2}\left[q^A \middle/ \frac{\partial q^A}{\partial p^A} \right] + \frac{1}{2}\frac{\partial C}{\partial q^A}$$

$$^*p^B = -^*p^A \frac{1}{2}\left[\frac{\partial q^A}{\partial p^B} \middle/ \frac{\partial q^B}{\partial p^B} \right] + \frac{\partial C}{\partial q^A} \frac{1}{2}\left[\frac{\partial q^A}{\partial p^B} \middle/ \frac{\partial q^B}{\partial p^B} \right] - \frac{1}{2}\left[q^B \middle/ \frac{\partial q^B}{\partial p^B} \right] + \frac{1}{2} p^B + \frac{1}{2}\frac{\partial C}{\partial q^B}$$

since $^*p^A = \Delta p^A + p^A$ and $^*p^B = \Delta p^B + p^B$

$$p^A + \Delta p^A = -\left(p^B + \Delta p^B\right)\frac{1}{2}\left[\frac{\partial q^B}{\partial p^A} \middle/ \frac{\partial q^A}{\partial p^A} \right] + \frac{\partial C}{\partial q^B} \frac{1}{2}\left[\frac{\partial q^B}{\partial p^A} \middle/ \frac{\partial q^A}{\partial p^A} \right] + \frac{1}{2} p^A - \frac{1}{2}\left[q^A \middle/ \frac{\partial q^A}{\partial p^A} \right] + \frac{1}{2}\frac{\partial C}{\partial q^A}$$

$$p^B + \Delta p^B = -\left(p^A + \Delta p^A\right)\frac{1}{2}\left[\frac{\partial q^A}{\partial p^B} \middle/ \frac{\partial q^B}{\partial p^B} \right] + \frac{\partial C}{\partial q^A} \frac{1}{2}\left[\frac{\partial q^A}{\partial p^B} \middle/ \frac{\partial q^B}{\partial p^B} \right] - \frac{1}{2}\left[q^B \middle/ \frac{\partial q^B}{\partial p^B} \right] + \frac{1}{2} p^B + \frac{1}{2}\frac{\partial C}{\partial q^B}$$

bringing p^A and p^b on the right-hand side

$$\Delta p^A = -\left(p^B + \Delta p^B\right)\frac{1}{2}\left[\frac{\partial q^B}{\partial p^A}\Big/\frac{\partial q^A}{\partial p^A}\right] + \frac{\partial C}{\partial q^B}\frac{1}{2}\left[\frac{\partial q^B}{\partial p^A}\Big/\frac{\partial q^A}{\partial p^A}\right] - \frac{1}{2}p^A - \frac{1}{2}\left[q^A\Big/\frac{\partial q^A}{\partial p^A}\right] + \frac{1}{2}\frac{\partial C}{\partial q^A}$$

$$\Delta p^B = -\left(p^A + \Delta p^A\right)\frac{1}{2}\left[\frac{\partial q^A}{\partial p^B}\Big/\frac{\partial q^B}{\partial p^B}\right] + \frac{\partial C}{\partial q^A}\frac{1}{2}\left[\frac{\partial q^A}{\partial p^B}\Big/\frac{\partial q^B}{\partial p^B}\right] - \frac{1}{2}\left[q^B\Big/\frac{\partial q^B}{\partial p^B}\right] - \frac{1}{2}p^B + \frac{1}{2}\frac{\partial C}{\partial q^B}$$

rearranging terms

$$\Delta p^A = -\left(p^B - \frac{\partial C}{\partial q^B}\right)\frac{1}{2}\left[\frac{\partial q^B}{\partial p^A}\Big/\frac{\partial q^A}{\partial p^A}\right] + \Delta p^B\frac{1}{2}\left[\frac{\partial q^B}{\partial p^A}\Big/\frac{\partial q^A}{\partial p^A}\right] - \frac{1}{2}\left(p^A - \frac{\partial C}{\partial q^A}\right) - \frac{1}{2}\left[q^A\Big/\frac{\partial q^A}{\partial p^A}\right]$$

$$\Delta p^B = -\left(p^A - \frac{\partial C}{\partial q^A}\right)\frac{1}{2}\left[\frac{\partial q^A}{\partial p^B}\Big/\frac{\partial q^B}{\partial p^B}\right] + \Delta p^A\frac{1}{2}\left[\frac{\partial q^A}{\partial p^B}\Big/\frac{\partial q^B}{\partial p^B}\right] - \frac{1}{2}\left[q^B\Big/\frac{\partial q^B}{\partial p^B}\right] - \frac{1}{2}\left(p^B - \frac{\partial C}{\partial q^B}\right)$$

dividing both sides in the first equation by p^A and both sides in the second by p^b

$$\frac{\Delta p^A}{p^A} = -\left(p^B - \frac{\partial C}{\partial q^B}\right)\frac{1}{2}\frac{1}{p^A}\frac{\frac{\partial q^B}{\partial p^A}}{\frac{\partial q^A}{\partial p^A}} + \Delta p^B\frac{1}{2}\frac{\frac{\partial q^B}{\partial p^A}}{\frac{\partial q^A}{\partial p^A}}\frac{1}{p^A} - \frac{1}{2}\frac{\left(p^A - \frac{\partial C}{\partial q^A}\right)}{p^A} - \frac{1}{2}\frac{\frac{q^A}{p^A}}{\frac{\partial q^A}{\partial p^A}}$$

$$\frac{\Delta p^B}{p^B} = -\left(p^A - \frac{\partial C}{\partial q^A}\right)\frac{1}{2}\frac{1}{p^B}\frac{\frac{\partial q^A}{\partial p^B}}{\frac{\partial q^B}{\partial p^B}} + \Delta p^A\frac{1}{2}\frac{1}{p^B}\frac{\frac{\partial q^A}{\partial p^B}}{\frac{\partial q^B}{\partial p^B}} - \frac{1}{2}\frac{\frac{q^B}{p^B}}{\frac{\partial q^B}{\partial p^B}} - \frac{1}{2}\frac{\left(p^B - \frac{\partial C}{\partial q^B}\right)}{p^B}$$

then multiplying and dividing the first two terms on the right-hand side in the first equation by

$p^A\ q^A\ q^b$ and in the second equation by $p^B\ q^A\ q^b$

$$\frac{\Delta p^A}{p^A} = -\left(p^B - \frac{\partial C}{\partial q^B}\right)\frac{1}{2}\frac{q^B}{p^A q^A}\frac{\frac{\partial q^B}{\partial p^A}\frac{p^A}{q^B}}{\frac{\partial q^A}{\partial p^A}\frac{p^A}{q^A}} + \Delta p^B\frac{1}{2}\frac{q^B}{p^A q^A}\frac{\frac{\partial q^B}{\partial p^A}\frac{p^A}{q^B}}{\frac{\partial q^A}{\partial p^A}\frac{p^A}{q^A}} - \frac{1}{2}\frac{\left(p^A - \frac{\partial C}{\partial q^A}\right)}{p^A} - \frac{1}{2}\frac{\frac{q^A}{p^A}}{\frac{\partial q^A}{\partial p^A}}$$

$$\frac{\Delta p^B}{p^B} = -\left(p^A - \frac{\partial C}{\partial q^A}\right)\frac{1}{2}\frac{q^A}{p^B q^B}\frac{\frac{\partial q^A}{\partial p^B}\frac{p^B}{q^A}}{\frac{\partial q^B}{\partial p^B}\frac{p^B}{q^B}} + \Delta p^A\frac{1}{2}\frac{q^A}{p^B q^B}\frac{\frac{\partial q^A}{\partial p^B}\frac{p^B}{q^A}}{\frac{\partial q^B}{\partial p^B}\frac{p^B}{q^B}} - \frac{1}{2}\frac{\frac{q^B}{p^B}}{\frac{\partial q^B}{\partial p^B}} - \frac{1}{2}\frac{\left(p^B - \frac{\partial C}{\partial q^B}\right)}{p^B}$$

further multiplying and dividing the first two terms on the right-hand side in the first equation by

p^B and in the second equation by p^A

$$\frac{\Delta p^A}{p^A} = -\frac{\left(p^B - \frac{\partial C}{\partial q^B}\right)}{p^B}\frac{1}{2}\frac{p^B q^B}{p^A q^A}\frac{\frac{\partial q^B}{\partial p^A}\frac{p^A}{q^B}}{\frac{\partial q^A}{\partial p^A}\frac{p^A}{q^A}} + \frac{\Delta p^B}{p^B}\frac{1}{2}\frac{p^B q^B}{p^A q^A}\frac{\frac{\partial q^B}{\partial p^A}\frac{p^A}{q^B}}{\frac{\partial q^A}{\partial p^A}\frac{p^A}{q^A}} - \frac{1}{2}\frac{\left(p^A - \frac{\partial C}{\partial q^A}\right)}{p^A} - \frac{1}{2}\frac{\frac{q^A}{p^A}}{\frac{\partial q^A}{\partial p^A}}$$

$$\frac{\Delta p^B}{p^B} = -\frac{\left(p^A - \frac{\partial C}{\partial q^A}\right)}{p^A}\frac{1}{2}\frac{p^A q^A}{p^B q^B}\frac{\frac{\partial q^A}{\partial p^B}\frac{p^B}{q^A}}{\frac{\partial q^B}{\partial p^B}\frac{p^B}{q^B}} + \frac{\Delta p^A}{p^A}\frac{1}{2}\frac{p^A q^A}{p^B q^B}\frac{\frac{\partial q^A}{\partial p^B}\frac{p^B}{q^A}}{\frac{\partial q^B}{\partial p^B}\frac{p^B}{q^B}} - \frac{1}{2}\frac{\frac{q^B}{p^B}}{\frac{\partial q^B}{\partial p^B}} - \frac{1}{2}\frac{\left(p^B - \frac{\partial C}{\partial q^B}\right)}{p^B}$$

and substituting for the elasticities

$$\frac{\Delta p^A}{p^A} = -\frac{\left(p^B - \frac{\partial C}{\partial q^B}\right)}{p^B}\frac{1}{2}\frac{p^B q^B}{p^A q^A}\frac{\varepsilon^{q^B}_{p^A}}{\varepsilon^{q^A}_{p^A}} + \frac{\Delta p^B}{p^B}\frac{1}{2}\frac{p^B q^B}{p^A q^A}\frac{\varepsilon^{q^B}_{p^A}}{\varepsilon^{q^A}_{p^A}} - \frac{1}{2}\frac{\left(p^A - \frac{\partial C}{\partial q^A}\right)}{p^A} - \frac{1}{2}\frac{1}{\varepsilon^{q^A}_{p^A}}$$

$$\frac{\Delta p^B}{p^B} = -\frac{\left(p^A - \frac{\partial C}{\partial q^A}\right)}{p^A}\frac{1}{2}\frac{p^A q^A}{p^B q^B}\frac{\varepsilon^{q^A}_{p^B}}{\varepsilon^{q^B}_{p^B}} + \frac{\Delta p^A}{p^A}\frac{1}{2}\frac{p^A q^A}{p^B q^B}\frac{\varepsilon^{q^A}_{p^B}}{\varepsilon^{q^B}_{p^B}} - \frac{1}{2}\frac{1}{\varepsilon^{q^B}_{p^B}} - \frac{1}{2}\frac{\left(p^B - \frac{\partial C}{\partial q^B}\right)}{p^B}$$

substituting for revenues

$$\frac{\Delta p^A}{p^A} = -\frac{\left(p^B - \frac{\partial C}{\partial q^B}\right)}{p^B}\frac{1}{2}\frac{R^B}{R^A}\frac{\varepsilon^{q^B}_{p^A}}{\varepsilon^{q^A}_{p^A}} + \frac{\Delta p^B}{p^B}\frac{1}{2}\frac{R^B}{R^A}\frac{\varepsilon^{q^B}_{p^A}}{\varepsilon^{q^A}_{p^A}} - \frac{1}{2}\frac{\left(p^A - \frac{\partial C}{\partial q^A}\right)}{p^A} - \frac{1}{2}\frac{1}{\varepsilon^{q^A}_{p^A}}$$

$$\frac{\Delta p^B}{p^B} = -\frac{\left(p^A - \frac{\partial C}{\partial q^A}\right)}{p^A}\frac{1}{2}\frac{R^A}{R^B}\frac{\varepsilon^{q^A}_{p^B}}{\varepsilon^{q^B}_{p^B}} + \frac{\Delta p^A}{p^A}\frac{1}{2}\frac{R^A}{R^B}\frac{\varepsilon^{q^A}_{p^B}}{\varepsilon^{q^B}_{p^B}} - \frac{1}{2}\frac{1}{\varepsilon^{q^B}_{p^B}} - \frac{1}{2}\frac{\left(p^B - \frac{\partial C}{\partial q^B}\right)}{p^B}$$

and, since costs are linear, one can also substitute also for mark-ups

$$\frac{\Delta p^A}{p^A} = -m^B\frac{1}{2}\frac{R^B}{R^A}\frac{\varepsilon^{q^B}_{p^A}}{\varepsilon^{q^A}_{p^A}} + \frac{\Delta p^B}{p^B}\frac{1}{2}\frac{R^B}{R^A}\frac{\varepsilon^{q^B}_{p^A}}{\varepsilon^{q^A}_{p^A}} - \frac{1}{2}m^A - \frac{1}{2}\frac{1}{\varepsilon^{q^A}_{p^A}}$$

$$\frac{\Delta p^B}{p^B} = -m^A\frac{1}{2}\frac{R^A}{R^B}\frac{\varepsilon^{q^A}_{p^B}}{\varepsilon^{q^B}_{p^B}} + \frac{\Delta p^A}{p^A}\frac{1}{2}\frac{R^A}{R^B}\frac{\varepsilon^{q^A}_{p^B}}{\varepsilon^{q^B}_{p^B}} - \frac{1}{2}\frac{1}{\varepsilon^{q^B}_{p^B}} - \frac{1}{2}m^B$$

then substituting the second into the first one

$$\frac{\Delta p^A}{p^A} = -m^B\frac{1}{2}\frac{R^B}{R^A}\frac{\varepsilon^{q^B}_{p^A}}{\varepsilon^{q^A}_{p^A}} - \frac{1}{2}m^A - \frac{1}{2}\frac{1}{\varepsilon^{q^A}_{p^A}} + \frac{1}{2}\frac{R^B}{R^A}\frac{\varepsilon^{q^B}_{p^A}}{\varepsilon^{q^A}_{p^A}}\left(-m^A\frac{1}{2}\frac{R^A}{R^B}\frac{\varepsilon^{q^A}_{p^B}}{\varepsilon^{q^B}_{p^B}} + \frac{\Delta p^A}{p^A}\frac{1}{2}\frac{R^A}{R^B}\frac{\varepsilon^{q^A}_{p^B}}{\varepsilon^{q^B}_{p^B}} - \frac{1}{2}\frac{1}{\varepsilon^{q^B}_{p^B}} - \frac{1}{2}m^B\right)$$

so that

$$\frac{\Delta p^A}{p^A} = -m^B\frac{1}{2}\frac{R^B}{R^A}\frac{\varepsilon^{q^B}_{p^A}}{\varepsilon^{q^A}_{p^A}} - \frac{1}{2}m^A - \frac{1}{2}\frac{1}{\varepsilon^{q^A}_{p^A}} - \frac{1}{2}\frac{R^B}{R^A}\frac{\varepsilon^{q^B}_{p^A}}{\varepsilon^{q^A}_{p^A}}m^A\frac{1}{2}\frac{R^A}{R^B}\frac{\varepsilon^{q^A}_{p^B}}{\varepsilon^{q^B}_{p^B}} + \frac{1}{2}\frac{R^B}{R^A}\frac{\varepsilon^{q^B}_{p^A}}{\varepsilon^{q^A}_{p^A}}\frac{\Delta p^A}{p^A}\frac{1}{2}\frac{R^A}{R^B}\frac{\varepsilon^{q^A}_{p^B}}{\varepsilon^{q^B}_{p^B}} +$$

$$- \frac{1}{2}\frac{1}{\varepsilon^{q^B}_{p^B}}\frac{1}{2}\frac{R^B}{R^A}\frac{\varepsilon^{q^B}_{p^A}}{\varepsilon^{q^A}_{p^A}} - \frac{1}{2}m^B\frac{1}{2}\frac{R^B}{R^A}\frac{\varepsilon^{q^B}_{p^A}}{\varepsilon^{q^A}_{p^A}}$$

and simplifying

$$\frac{\Delta p^A}{p^A} = -\frac{1}{2}m^B\frac{R^B}{R^A}\frac{\varepsilon_{p^A}^{q^B}}{\varepsilon_{p^A}^{q^A}} - \frac{1}{2}m^A - \frac{1}{2}\frac{1}{\varepsilon_{p^A}^{q^A}} - \frac{1}{4}m^A\frac{\varepsilon_{p^A}^{q^B}}{\varepsilon_{p^A}^{q^A}}\frac{\varepsilon_{p^B}^{q^A}}{\varepsilon_{p^B}^{q^B}} + \frac{1}{4}\frac{\Delta p^A}{p^A}\frac{\varepsilon_{p^A}^{q^B}}{\varepsilon_{p^A}^{q^A}}\frac{\varepsilon_{p^B}^{q^A}}{\varepsilon_{p^B}^{q^B}}$$

$$-\frac{1}{4}\frac{R^B}{R^A}\frac{\varepsilon_{p^A}^{q^B}}{\varepsilon_{p^B}^{q^B}\varepsilon_{p^A}^{q^A}} - \frac{1}{4}m^B\frac{R^B}{R^A}\frac{\varepsilon_{p^A}^{q^B}}{\varepsilon_{p^A}^{q^A}}$$

bringing all terms with $\dfrac{\Delta p^A}{p^A}$ to the left-hand side

$$\frac{\Delta p^A}{p^A}\left(1 - \frac{1}{4}\frac{\varepsilon_{p^A}^{q^B}}{\varepsilon_{p^A}^{q^A}}\frac{\varepsilon_{p^B}^{q^A}}{\varepsilon_{p^B}^{q^B}}\right) = -\frac{3}{4}m^B\frac{R^B}{R^A}\frac{\varepsilon_{p^A}^{q^B}}{\varepsilon_{p^A}^{q^A}} - \frac{1}{2}m^A - \frac{1}{2}\frac{1}{\varepsilon_{p^A}^{q^A}} - \frac{1}{4}m^A\frac{\varepsilon_{p^A}^{q^B}}{\varepsilon_{p^A}^{q^A}}\frac{\varepsilon_{p^B}^{q^A}}{\varepsilon_{p^B}^{q^B}} +$$

$$-\frac{1}{4}\frac{R^B}{R^A}\frac{\varepsilon_{p^A}^{q^B}}{\varepsilon_{p^B}^{q^B}\varepsilon_{p^A}^{q^A}}$$

and dividing both sides by $\dfrac{4\varepsilon_{p^A}^{q^A}\varepsilon_{p^B}^{q^B} - \varepsilon_{p^A}^{q^B}\varepsilon_{p^B}^{q^A}}{4\varepsilon_{p^A}^{q^A}\varepsilon_{p^B}^{q^B}}$ one obtains

$$\frac{\Delta p^A}{p^A} = -\frac{3}{4}m^B\frac{R^B}{R^A}\frac{\varepsilon_{p^A}^{q^B}}{\varepsilon_{p^A}^{q^A}}\frac{4\varepsilon_{p^A}^{q^A}\varepsilon_{p^B}^{q^B}}{\left(4\varepsilon_{p^A}^{q^A}\varepsilon_{p^B}^{q^B} - \varepsilon_{p^A}^{q^B}\varepsilon_{p^B}^{q^A}\right)} - \frac{1}{2}m^A\frac{4\varepsilon_{p^A}^{q^A}\varepsilon_{p^B}^{q^B}}{\left(4\varepsilon_{p^A}^{q^A}\varepsilon_{p^B}^{q^B} - \varepsilon_{p^A}^{q^B}\varepsilon_{p^B}^{q^A}\right)} - \frac{1}{2}\frac{1}{\varepsilon_{p^A}^{q^A}}\frac{4\varepsilon_{p^A}^{q^A}\varepsilon_{p^B}^{q^B}}{\left(4\varepsilon_{p^A}^{q^A}\varepsilon_{p^B}^{q^B} - \varepsilon_{p^A}^{q^B}\varepsilon_{p^B}^{q^A}\right)} +$$

$$-\frac{1}{4}m^A\frac{\varepsilon_{p^A}^{q^B}}{\varepsilon_{p^A}^{q^A}}\frac{\varepsilon_{p^B}^{q^A}}{\varepsilon_{p^B}^{q^B}}\frac{4\varepsilon_{p^A}^{q^A}\varepsilon_{p^B}^{q^B}}{\left(4\varepsilon_{p^A}^{q^A}\varepsilon_{p^B}^{q^B} - \varepsilon_{p^A}^{q^B}\varepsilon_{p^B}^{q^A}\right)} - \frac{1}{4}\frac{R^B}{R^A}\frac{\varepsilon_{p^A}^{q^B}}{\varepsilon_{p^B}^{q^B}\varepsilon_{p^A}^{q^A}}\frac{4\varepsilon_{p^A}^{q^A}\varepsilon_{p^B}^{q^B}}{\left(4\varepsilon_{p^A}^{q^A}\varepsilon_{p^B}^{q^B} - \varepsilon_{p^A}^{q^B}\varepsilon_{p^B}^{q^A}\right)}$$

finally, simplifying,

$$\frac{\Delta p^A}{p^A} = -\frac{3}{4}m^B\frac{R^B}{R^A}\frac{4\varepsilon_{p^B}^{q^B}\varepsilon_{p^A}^{q^A}}{\left(4\varepsilon_{p^A}^{q^A}\varepsilon_{p^B}^{q^B} - \varepsilon_{p^A}^{q^B}\varepsilon_{p^B}^{q^A}\right)} - \frac{1}{2}m^A\frac{4\varepsilon_{p^A}^{q^A}\varepsilon_{p^B}^{q^B}}{\left(4\varepsilon_{p^A}^{q^A}\varepsilon_{p^B}^{q^B} - \varepsilon_{p^A}^{q^B}\varepsilon_{p^B}^{q^A}\right)} - \frac{1}{4}m^A\frac{4\varepsilon_{p^A}^{q^B}\varepsilon_{p^B}^{q^A}}{\left(4\varepsilon_{p^A}^{q^A}\varepsilon_{p^B}^{q^B} - \varepsilon_{p^A}^{q^B}\varepsilon_{p^B}^{q^A}\right)}$$

$$-\frac{1}{2}\frac{4\varepsilon_{p^B}^{q^B}}{\left(4\varepsilon_{p^A}^{q^A}\varepsilon_{p^B}^{q^B} - \varepsilon_{p^A}^{q^B}\varepsilon_{p^B}^{q^A}\right)} - \frac{1}{4}\frac{R^B}{R^A}\frac{4\varepsilon_{p^A}^{q^B}}{\left(4\varepsilon_{p^A}^{q^A}\varepsilon_{p^B}^{q^B} - \varepsilon_{p^A}^{q^B}\varepsilon_{p^B}^{q^A}\right)}$$

$$(13)$$

Appendix 2 – Critical Loss Formula in the EU

a) Critical Losses

From the break-even condition

$$\Delta p^A(q^A + \Delta q^A) + \frac{\Delta p^B}{\Delta p^A}\Delta p^A(q^B + \Delta q^B) = -(p^A - \frac{\Delta C}{\Delta q^A})\Delta q^A - (p^B - \frac{\Delta C}{\Delta q^B})\Delta q^B \tag{14}$$

rearranging and breaking the equation into three parts to simplify the calculations gives:

$$\left[\Delta p^A(q^A + \Delta q^A) + (p^A - \frac{\Delta C}{\Delta q^A})\Delta q^A\right] + \frac{\Delta p^B}{\Delta p^A}\Delta p^A(q^B + \Delta q^B) + (p^B - \frac{\Delta C}{\Delta q^A})\Delta q^B = 0$$

$$\underbrace{\qquad\qquad\qquad\qquad\qquad\qquad}_{[I]}\quad\underbrace{\qquad\qquad}_{[II]}\quad\underbrace{\qquad\qquad}_{[III]}$$

where [I] and [III] are the same as in Evans and Noel (2005a, 2007), while [II] is due to the

hypothetical monopolist optimally adjusting the price structure.

With simple manipulation [I] can be rewritten as[20]:

$$\left(\frac{\Delta p^A}{p^A}\right)q^A p^A + p^A q^A \left(\frac{\Delta p^A}{p^A} + \frac{(p^A - \frac{\Delta C}{\Delta q^A})}{P^A}\right)\left(\frac{\Delta q^A}{q^A}\right)$$

while [III] can be rewritten as[21]

$$(p^B - \frac{\Delta C}{\Delta q^B})\Delta q^B = \frac{(p^B - \frac{\Delta C}{\Delta q^B})}{p^B}\frac{\Delta p^A}{p^A}\left(\frac{\Delta q^B}{q^B}\frac{p^A}{\Delta p^A}\right)q^B p^B$$

---

[20] $\Delta p^A(q^A + \Delta q^A) + (p^A - \frac{\Delta C}{\Delta q^A})\Delta q^A = \Delta p^A q^A + (\Delta p^A + p^A - \frac{\Delta C}{\Delta q})\Delta q^A = \frac{\Delta p^A q^A p^A}{p^A} + \frac{(\Delta p^A + p^A - \frac{\Delta C}{\Delta q^A})\Delta q^A p^A q^A}{p^A q^A} =$

$= \left(\frac{\Delta p^A}{p^A}\right)q^A p^A + \left(p^A q^A\right)\left(\frac{\Delta p^A}{p^A} + \frac{(p^A - \frac{\Delta C}{\Delta q^A})}{P^A}\right)\left(\frac{\Delta q^A}{q^A}\right)$

[21] $(p^B - \frac{\Delta C}{\Delta q^B})\Delta q^B = \frac{(p^B - \frac{\Delta C}{\Delta q^B})\Delta q^B p^B q^B}{p^B q^B} = \frac{(p^B - \frac{\Delta C}{\Delta q^B})}{p^B}\frac{\Delta q^B}{q^B}q^B p^B = \frac{(p^B - \frac{\Delta C}{\Delta q^B})}{p^B}\frac{\Delta p^A}{p^A}\left(\frac{\Delta q^B}{q^B}\frac{p^A}{\Delta p^A}\right)q^B p^B =$

$= \frac{(p^B - \frac{\Delta C}{\Delta q^B})}{p^B}\frac{\Delta p^A}{p^A}\left(\frac{\Delta q^B}{q^B}\frac{p^A}{\Delta p^A}\right)q^B p^B$

If costs are linear [I] becomes

$$t^A R^A + R^A \left(t^A + m^A \left(\frac{\Delta q^A}{q^A} \right) \right)$$

where $t^A = \frac{\Delta p^A}{p^A}$ is the percentage price increase on side A deemed relevant (5% or 10% usually)

and $m^A = \frac{(p^A - c^A)}{p^A}$ is the observed mark-up on side A .

If also demands are linear [III] is equal to

$$m^B t^A \frac{\Delta q^B}{q^B} R^B .$$

Then note that [II] can be decomposed[22] as

$$\frac{\Delta p^B}{\Delta p^A} \Delta p^A (q^B + \Delta q^B) = \frac{\Delta^* p^B}{\Delta p^A} \Delta p^A (q^B + \Delta q^B) + \frac{^*p^B - p^B}{\Delta p^A} \Delta p^A (q^B + \Delta q^B)$$

$$\underbrace{\hphantom{\frac{\Delta^* p^B}{\Delta p^A} \Delta p^A (q^B + \Delta q^B)}}_{\text{[IIa]}} \quad \underbrace{\hphantom{\frac{^*p^B - p^B}{\Delta p^A} \Delta p^A (q^B + \Delta q^B)}}_{\text{[IIb]}}$$

where p^B is the observed market price on side B while $^*p^B$ is the optimal price the hypothetical monopolist would have set in correspondence with the observed price p^A and $^*\Delta p^B$ is the change in the hypothetical monopolist's optimal price $^*p^B$ following the change in p^A .

In order to obtain [IIa] note that, as discussed above, from the first order condition for profit maximization with respect to p^B in (5), one can obtain the best reply function in (6). The slope of the best reply function is given by differentiating the expression for $^*P^B$ with respect to P^A :

$$\frac{\partial^* P^B}{\partial P^A} = \frac{\partial^* g(P^A)}{\partial P^A} .$$

---

[22] $\frac{\Delta P^B}{\Delta P^A} \Delta P^A (Q^B + \Delta Q^B) = \frac{\Delta^* P^B + {}^*P^B - {}^0 P^B}{\Delta P^A} \Delta P^A (Q^B + \Delta Q^B) = \left(\frac{\Delta^* P^B}{\Delta P^A} + \frac{{}^*P^B - {}^0 P^B}{\Delta P^A} \right) \Delta P^A (Q^B + \Delta Q^B) = = \frac{\Delta^* P^B}{\Delta P^A} \Delta P^A (Q^B + \Delta Q^B) + \frac{{}^*P^B - {}^0 P^B}{\Delta P^A} \Delta P^A (Q^B + \Delta Q^B)$

If demands and costs are linear, the best reply function is also linear and equal

$$\text{to}^*p^B = -(p^A - \frac{\partial C}{\partial q^A})\left[\frac{\partial q^A}{\partial p^B}\Big/\frac{\partial q^B}{\partial p^B}\right] - \left[^*q^B\Big/\frac{\partial q^B}{\partial p^B}\right] + \frac{\partial C}{\partial q^B}$$

Since demands are linear, $\dfrac{\partial q^B}{\partial p^A}$ does not change with p^A (or p^B), so that

$$\frac{\partial^* q^B}{\partial p^A} = \frac{\partial q^B}{\partial p^A}$$

Then the slope of the best reply function is

$$\frac{\partial^* p^B}{\partial p^A} = -\left[\frac{\partial q^A}{\partial p^B}\Big/\frac{\partial q^B}{\partial p^B}\right] - \left[\frac{\partial q^B}{\partial p^A}\Big/\frac{\partial q^B}{\partial p^B}\right]$$

and

$$\frac{\Delta^* p^B}{\Delta p^A} = \frac{\partial^* p^B}{\partial p^A}.$$

Then

$$\frac{\Delta^* p^B}{\Delta p^A}\Delta p^A(q^B + \Delta q^B) = \frac{\partial^* p^B}{\partial p^A}\Delta p^A(q^B + \Delta q^B)$$

and substituting for

$$\frac{\partial^* p^B}{\partial p^A} = -\left(\frac{\partial q^A}{\partial p^B}\Big/\frac{\partial q^B}{\partial p^B}\right) - \left(\frac{\partial q^B}{\partial p^A}\Big/\frac{\partial q^B}{\partial p^B}\right)$$

one gets

$$\frac{\Delta^* p^B}{\Delta p^A}\Delta p^A(q^B + \Delta q^B) = -\left[\left(\frac{\partial q^A}{\partial p^B}\Big/\frac{\partial q^B}{\partial p^B}\right) + \left(\frac{\partial q^B}{\partial p^A}\Big/\frac{\partial q^B}{\partial p^B}\right)\right]\frac{\Delta p^A}{p^A}q^B p^A\left(\frac{q^B + \Delta q^B}{q^B}\right)$$

and multiplying and dividing the terms in square brackets

$$\frac{\Delta^* p^B}{\Delta p^A}\Delta p^A(q^B + \Delta q^B) = -\left[\left(\frac{\dfrac{\partial q^A}{\partial p^B}\dfrac{p^B}{q^A}}{\dfrac{\partial q^B}{\partial p^B}\dfrac{p^B}{q^B}}\right)\frac{q^A}{q^B} + \left(\frac{\dfrac{\partial q^B}{\partial p^A}\dfrac{p^A}{q^B}}{\dfrac{\partial q^B}{\partial p^B}\dfrac{p^B}{q^B}}\right)\frac{p^B}{p^A}\right]\frac{\Delta p^A}{p^A}q^B p^A\left(\frac{q^B + \Delta q^B}{q^B}\right)$$

then multiplying and dividing by $\dfrac{q^B}{q^A}$, one finds

$$\frac{\Delta^* p^B}{\Delta p^A}\Delta p^A(q^B+\Delta q^B)=-\left[\left(\frac{\dfrac{\partial q^A}{\partial p^B}\dfrac{p^B}{q^A}}{\dfrac{\partial q^B}{\partial p^B}\dfrac{p^B}{q^B}}\right)+\left(\frac{\dfrac{\partial q^B}{\partial p^A}\dfrac{p^A}{q^B}}{\dfrac{\partial q^B}{\partial p^B}\dfrac{p^B}{q^B}}\right)\frac{p^B}{p^A}\frac{q^B}{q^A}\right]\frac{q^A}{q^B}\frac{\Delta p^A}{p^A}q^B p^A\left(\frac{q^B+\Delta q^B}{q^B}\right)$$

and simplifying

$$\frac{\Delta^* p^B}{\Delta p^A}\Delta p^A(q^B+\Delta q^B)=-\left[\left(\frac{\dfrac{\partial q^A}{\partial p^B}\dfrac{p^B}{q^A}}{\dfrac{\partial q^B}{\partial p^B}\dfrac{p^B}{q^B}}\right)+\left(\frac{\dfrac{\partial q^B}{\partial p^A}\dfrac{p^A}{q^B}}{\dfrac{\partial q^B}{\partial p^B}\dfrac{p^B}{q^B}}\right)\frac{p^B}{p^A}\frac{q^B}{q^A}\right]\frac{\Delta p^A}{p^A}q^A p^A\left(\frac{q^B+\Delta q^B}{q^B}\right)$$

and substituting the elasticities

$$\frac{\Delta^* p^B}{\Delta p^A}\Delta p^A(q^B+\Delta q^B)=-\left[\left(\frac{\varepsilon_{p^B}^{q^A}}{\varepsilon_{p^B}^{q^B}}\right)+\left(\frac{\varepsilon_{p^A}^{q^B}}{\varepsilon_{p^B}^{q^B}}\right)\frac{R^B}{R^A}\right]t^A R^A\left(1+\frac{\Delta q^B}{q^B}\right)$$

which is the expression for [IIa].

In order to obtain [IIb], note first that price $^*P^B$ is the profit-maximizing one the hypothetical monopolist would have set in correspondence to the observed price P^A.

Then, from the first order condition with respect to P^B for the hypothetical monopolist's profit maximization:

$$^*p^B=-(p^A-\frac{\partial C}{\partial q^A})\left[\frac{\partial q^A}{\partial p^B}\bigg/\frac{\partial q^B}{\partial p^B}\right]-\left[^*q^B\bigg/\frac{\partial q^B}{\partial p^B}\right]+\frac{\partial C}{\partial q^B}$$

Since $^*q^B=q^B+\Delta q^B$,

$$^*p^B=-(p^A-\frac{\partial C}{\partial q^A})\left[\frac{\partial q^A}{\partial p^B}\bigg/\frac{\partial q^B}{\partial p^B}\right]-\left[(q^B+\Delta q^B)\bigg/\frac{\partial q^B}{\partial p^B}\right]+\frac{\partial C}{\partial q^B}$$

so that

$$^*p^B=-(p^A-\frac{\partial C}{\partial q^A})\left[\frac{\partial q^A}{\partial p^B}\bigg/\frac{\partial q^B}{\partial p^B}\right]-\left[q^B\bigg/\frac{\partial q^B}{\partial p^B}\right]-\left[\Delta q^B\bigg/\frac{\partial q^B}{\partial p^B}\right]+\frac{\partial C}{\partial q^B}$$

Then, multiplying and dividing by p^B all the terms on the right-hand side, by q^A and q^B the first term and by q^B third term only, one gets

$$^*p^B = -(p^A - \frac{\partial C}{\partial q^A})\left[\frac{\partial q^A}{\partial p^B}\frac{p^B}{q^A} \Big/ \frac{\partial q^B}{\partial p^B}\frac{p^B}{q^B}\right]\frac{q^A}{q^B} - p^B\left[\frac{q^B}{p^B} \Big/ \frac{\partial q^B}{\partial p^B}\right] - p^B\left[\frac{\Delta q^B}{q^B}\frac{q^B}{p^B} \Big/ \frac{\partial q^B}{\partial p^B}\right] + \frac{\partial C}{\partial q^B}$$

and substituting for the elasticities

$$^*p^B = -(p^A - \frac{\partial C}{\partial q^A})\left[\frac{\varepsilon^{q^A}_{p^B}}{\varepsilon^{q^B}_{p^B}}\right]\frac{q^A}{q^B} - p^B\left[\frac{1}{\varepsilon^{q^B}_{p^B}}\right] - p^B\left[\frac{\Delta q^B}{q^B}\frac{1}{\varepsilon^{q^B}_{p^B}}\right] + \frac{\partial C}{\partial q^B}$$

then grouping the second and third term

$$^*p^B = -(p^A - \frac{\partial C}{\partial q^A})\left[\frac{\varepsilon^{q^A}_{p^B}}{\varepsilon^{q^B}_{p^B}}\right]\frac{q^A}{q^B} - p^B\left[\frac{1}{\varepsilon^{q^B}_{p^B}} + \frac{\Delta q^B}{q^B}\frac{1}{\varepsilon^{q^B}_{p^B}}\right] + \frac{\partial C}{\partial q^B}$$

subtracting p^B from both sides

$$^*p^B - p^B = -(p^A - \frac{\partial C}{\partial q^A})\left[\frac{\varepsilon^{q^A}_{p^B}}{\varepsilon^{q^B}_{p^B}}\right]\frac{q^A}{q^B} - p^B\frac{1}{\varepsilon^{q^B}_{p^B}}\left[1 + \frac{\Delta q^B}{q^B}\right] - \left(p^B - \frac{\partial C}{\partial q^B}\right)$$

and dividing both sides by p^B

$$\frac{^*p^B - p^B}{p^B} = -(p^A - \frac{\partial C}{\partial q^A})\left[\frac{\varepsilon^{q^A}_{p^B}}{\varepsilon^{q^B}_{p^B}}\right]\frac{q^A}{p^B q^B} - \frac{1}{\varepsilon^{q^B}_{p^B}}\left[1 + \frac{\Delta q^B}{q^B}\right] - \frac{\left(p^B - \frac{\partial C}{\partial q^B}\right)}{p^B}$$

then both multiplying and dividing the first term on the right-hand side by p^A

$$\frac{^*p^B - p^B}{p^B} = -\frac{\left(p^A - \frac{\partial C}{\partial q^A}\right)}{p^A}\left[\frac{\varepsilon^{q^A}_{p^B}}{\varepsilon^{q^B}_{p^B}}\right]\frac{p^A q^A}{p^B q^B} - \frac{1}{\varepsilon^{q^B}_{p^B}}\left[1 + \frac{\Delta q^B}{q^B}\right] - \frac{\left(p^B - \frac{\partial C}{\partial q^B}\right)}{p^B}$$

or equivalently

$$\frac{^*p^B - p^B}{p^B} = -m^A\left[\frac{\varepsilon^{q^A}_{p^B}}{\varepsilon^{q^B}_{p^B}}\right]\frac{R^A}{R^B} - \frac{1}{\varepsilon^{q^B}_{p^B}}\left[1 + \frac{\Delta q^B}{q^B}\right] - m^B$$

Substituting this into

$$\frac{^*p^B - p^B}{\Delta p^A}\Delta p^A(q^B + \Delta q^B) = \left(\frac{^*p^B - p^B}{p^B}\right)p^B q^B\left(\frac{q^B + \Delta q^B}{q^B}\right)$$

one obtains the expression for [IIb]

$$\frac{^*p^B - p^B}{\Delta p^A}\Delta p^A(q^B + \Delta q^B) = -m^A R^A\left[\frac{\varepsilon^{q^A}_{p^B}}{\varepsilon^{q^B}_{p^B}}\right]\left(1 + \frac{\Delta q^B}{q^B}\right) - \frac{1}{\varepsilon^{q^B}_{p^B}}R^B\left(1 + \frac{\Delta q^B}{q^B}\right)^2 - m^B R^B\left(1 + \frac{\Delta q^B}{q^B}\right)$$

.

Finally, summing up the four parts $[I]+[II]+[IIa]+[IIb]$ of the equation gives the formula that

defines implicitly the Critical Loss $\dfrac{\Delta q^A}{q^A}$ in a two-sided market according to the EU version of the

SSNIP test:

$$
t^A R^A + R^A\left(t^A + m^A\right)\left(\frac{\Delta q^A}{q^A}\right) + m^B\left(\frac{\Delta q^B}{q^B}\right)R^B - \left[\left(\frac{\varepsilon_{p^B}^{q^A}}{\varepsilon_{p^B}^{q^B}}\right) + \left(\frac{\varepsilon_{p^B}^{q^B}}{\varepsilon_{p^B}^{q^B}}\right)\frac{R^B}{R^A}\right] t^A R^A\left(1 + \frac{\Delta q^B}{q^B}\right) +
$$

$$
- m^A R^A\left[\frac{\varepsilon_{p^B}^{q^A}}{\varepsilon_{p^B}^{q^B}}\right]\left(1 + \frac{\Delta q^B}{q^B}\right) - \frac{1}{\varepsilon_{p^B}^{q^B}} R^B\left(1 + \frac{\Delta q^B}{q^B}\right)^2 - m^B R^B\left(1 + \frac{\Delta q^B}{q^B}\right) = 0 \tag{15}
$$

Again the first three terms are the same as in Evans & Noel (2005b,2007).

2b) Optimal price change on side B following a given price change on side A
From the first-order condition for profit maximization by the hypothetical monopolist in (12) one obtains,

$$
{}^*p^B = -(p^A + \Delta p^A - \frac{\partial C}{\partial q^A})\left[\frac{\partial q^A}{\partial p^B} \Big/ \frac{\partial q^B}{\partial p^B}\right] - \left[{}^*q^B \Big/ \frac{\partial q^B}{\partial p^B}\right] + \frac{\partial C}{\partial q^B},
$$

and, since ${}^*q^B = q^B + \Delta q^B$,

$$
{}^*p^B = -(p^A + \Delta p^A - \frac{\partial C}{\partial q^A})\left[\frac{\partial q^A}{\partial p^B} \Big/ \frac{\partial q^B}{\partial p^B}\right] - \left[\left(q^B + \Delta q^B\right)\Big/ \frac{\partial q^B}{\partial p^B}\right] + \frac{\partial C}{\partial q^B}
$$

Then, multiplying and dividing by p^B the first and the second term on the right-hand side, by q^A and q^B the first term and by Δp^B the third term only, one gets

$$
{}^*p^B = -(p^A + \Delta p^A - \frac{\partial C}{\partial q^A})\left[\frac{\partial q^A}{\partial p^B}\frac{p^B}{q^A} \Big/ \frac{\partial q^B}{\partial p^B}\frac{p^B}{q^B}\right]\frac{q^A}{q^B} - p^B\left[\frac{q^B}{p^B} \Big/ \frac{\partial q^B}{\partial p^B}\right] - \Delta p^B\left[\frac{\Delta q^B}{\Delta p^B} \Big/ \frac{\partial q^B}{\partial p^B}\right] + \frac{\partial C}{\partial q^B}
$$

then, given $\dfrac{\Delta q^B}{\Delta p^B} = \dfrac{\partial q^B}{\partial p^B}$,

$$
{}^*p^B = -(p^A + \Delta p^A - \frac{\partial C}{\partial q^A})\left[\frac{\partial q^A}{\partial p^B}\frac{p^B}{q^A} \Big/ \frac{\partial q^B}{\partial p^B}\frac{p^B}{q^B}\right]\frac{q^A}{q^B} - p^B\left[\frac{q^B}{p^B} \Big/ \frac{\partial q^B}{\partial p^B}\right] - \Delta p^B + \frac{\partial C}{\partial q^B}
$$

and substituting for the elasticities

$$
{}^*p^B = -(p^A + \Delta p^A - \frac{\partial C}{\partial q^A})\left[\frac{\varepsilon_{p^B}^{q^A}}{\varepsilon_{p^B}^{q^B}}\right]\frac{q^A}{q^B} - p^B\left[\frac{1}{\varepsilon_{p^B}^{q^B}}\right] - \Delta p^B + \frac{\partial C}{\partial q^B}
$$

then

$$
{}^*p^B = -(p^A + \Delta p^A - \frac{\partial C}{\partial q^A})\left[\frac{\varepsilon_{p^B}^{q^A}}{\varepsilon_{p^B}^{q^B}}\right]\frac{q^A}{q^B} - p^B\left[\frac{1}{\varepsilon_{p^B}^{q^B}}\right] - {}^*p^B - p^B + \frac{\partial C}{\partial q^B}
$$

so that

$$2^{*}p^{B} = -(p^{A} + \Delta p^{A} - \frac{\partial C}{\partial q^{A}})\left[\frac{\varepsilon_{p^{B}}^{q^{A}}}{\varepsilon_{p^{B}}^{q^{B}}}\right]\frac{q^{A}}{q^{B}} - p^{B}\left[\frac{1}{\varepsilon_{p^{B}}^{q^{B}}}\right] - \left(p^{B} - \frac{\partial C}{\partial q^{B}}\right)$$

and, dividing both sides by 2,

$$^{*}p^{B} = -(p^{A} + \Delta p^{A} - \frac{\partial C}{\partial q^{A}})\frac{1}{2}\left[\frac{\varepsilon_{p^{B}}^{q^{A}}}{\varepsilon_{p^{B}}^{q^{B}}}\right]\frac{q^{A}}{q^{B}} - p^{B}\frac{1}{2}\left[\frac{1}{\varepsilon_{p^{B}}^{q^{B}}}\right] - \frac{1}{2}\left(p^{B} - \frac{\partial C}{\partial q^{B}}\right)$$

subtracting p^{B} from both sides

$$^{*}p^{B} - p^{B} = -(p^{A} + \Delta p^{A} - \frac{\partial C}{\partial q^{A}})\frac{1}{2}\left[\frac{\varepsilon_{p^{B}}^{q^{A}}}{\varepsilon_{p^{B}}^{q^{B}}}\right]\frac{q^{A}}{q^{B}} - p^{B}\frac{1}{2}\left[\frac{1}{\varepsilon_{p^{B}}^{q^{B}}}\right] - \frac{1}{2}\left(p^{B} - \frac{\partial C}{\partial q^{B}}\right) - p^{B}$$

grouping the second and fourth term on the right-hand side

$$^{*}p^{B} - p^{B} = -(p^{A} + \Delta p^{A} - \frac{\partial C}{\partial q^{A}})\frac{1}{2}\left[\frac{\varepsilon_{p^{B}}^{q^{A}}}{\varepsilon_{p^{B}}^{q^{B}}}\right]\frac{q^{A}}{q^{B}} - p^{B}\frac{1}{2}\left[2 + \frac{1}{\varepsilon_{p^{B}}^{q^{B}}}\right] - \frac{1}{2}\left(p^{B} - \frac{\partial C}{\partial q^{B}}\right)$$

and dividing both sides by p^{B}

$$\frac{^{*}p^{B} - p^{B}}{p^{B}} = -(p^{A} + \Delta p^{A} - \frac{\partial C}{\partial q^{A}})\frac{1}{2}\left[\frac{\varepsilon_{p^{B}}^{q^{A}}}{\varepsilon_{p^{B}}^{q^{B}}}\right]\frac{q^{A}}{p^{B}q^{B}} - \frac{1}{2}\left[2 + \frac{1}{\varepsilon_{p^{B}}^{q^{B}}}\right] - \frac{\left(p^{B} - \frac{\partial C}{\partial q^{B}}\right)}{p^{B}}$$

then both multiplying and dividing the first term on the right-hand side by p^{A}

$$\frac{^{*}p^{B} - p^{B}}{p^{B}} = -\frac{\Delta p^{A}}{p^{A}}\frac{1}{2}\left[\frac{\varepsilon_{p^{B}}^{q^{A}}}{\varepsilon_{p^{B}}^{q^{B}}}\right]\frac{p^{A}q^{A}}{p^{B}q^{B}} - \frac{\left(p^{A} - \frac{\partial C}{\partial q^{A}}\right)}{p^{A}}\frac{1}{2}\left[\frac{\varepsilon_{p^{B}}^{q^{A}}}{\varepsilon_{p^{B}}^{q^{B}}}\right]\frac{p^{A}q^{A}}{p^{B}q^{B}} - \frac{1}{2}\left[2 + \frac{1}{\varepsilon_{p^{B}}^{q^{B}}}\right] - \frac{\left(p^{B} - \frac{\partial C}{\partial q^{B}}\right)}{p^{B}}$$

or equivalently

$$\frac{\Delta p^{B}}{p^{B}} = -t^{A}\frac{1}{2}\left[\frac{\varepsilon_{p^{B}}^{q^{A}}}{\varepsilon_{p^{B}}^{q^{B}}}\right]\frac{R^{A}}{R^{B}} - m^{A}\frac{1}{2}\left[\frac{\varepsilon_{p^{B}}^{q^{A}}}{\varepsilon_{p^{B}}^{q^{B}}}\right]\frac{R^{A}}{R^{B}} - \frac{1}{2}\left[2 + \frac{1}{\varepsilon_{p^{B}}^{q^{B}}}\right] - m^{B}$$

(18)

2c) Critical Loss on side A
From the break-even condition (15)

$$R^{A}\left(t^{A} + m^{A}\right)\left(\frac{\Delta q^{A}}{q^{A}}\right) = -t^{A}R^{A} - m^{B}\left(\frac{\Delta q^{B}}{q^{B}}\right)R^{B} + \left[\left(\frac{\varepsilon_{p^{B}}^{q^{A}}}{\varepsilon_{p^{B}}^{q^{B}}}\right) + \left(\frac{\varepsilon_{p^{A}}^{q^{B}}}{\varepsilon_{p^{B}}^{q^{B}}}\right)\frac{R^{B}}{R^{A}}\right]t^{A}R^{A}\left(1 + \frac{\Delta q^{B}}{q^{B}}\right) +$$

$$+ m^{A}R^{A}\left[\frac{\varepsilon_{p^{B}}^{q^{A}}}{\varepsilon_{p^{B}}^{q^{B}}}\right]\left(1 + \frac{\Delta q^{B}}{q^{B}}\right) + \frac{1}{\varepsilon_{p^{B}}^{q^{B}}}R^{B}\left(1 + \frac{\Delta q^{B}}{q^{B}}\right)^{2} + m^{B}R^{B}\left(1 + \frac{\Delta q^{B}}{q^{B}}\right)$$

then

$$\frac{\Delta q^A}{q^A} = -\frac{t^A}{\left(t^A+m^A\right)} - \frac{m^B R^B}{R^A\left(t^A+m^A\right)}\frac{\Delta q^B}{q^B} + \left[\left(\frac{\varepsilon^{q^A}_{p^B}}{\varepsilon^{q^B}_{p^B}}\right)+\left(\frac{\varepsilon^{q^B}_{p^A}}{\varepsilon^{q^B}_{p^B}}\right)\frac{R^B}{R^A}\right]\frac{t^A R^A}{R^A\left(t^A+m^A\right)}\left(1+\frac{\Delta q^B}{q^B}\right)+$$

$$+\frac{m^A}{\left(t^A+m^A\right)}\left[\frac{\varepsilon^{q^A}_{p^B}}{\varepsilon^{q^B}_{p^B}}\right]\left(1+\frac{\Delta q^B}{q^B}\right)+\frac{1}{\varepsilon^{q^B}_{p^B}}\frac{R^B}{R^A\left(t^A+m^A\right)}\left(1+\frac{\Delta q^B}{q^B}\right)^2+\frac{m^B R^B}{R^A\left(t^A+m^A\right)}\left(1+\frac{\Delta q^B}{q^B}\right)$$

so that

$$\frac{\Delta q^A}{q^A} = -\frac{t^A}{\left(t^A+m^A\right)} - \frac{m^B R^B}{R^A\left(t^A+m^A\right)}\frac{\Delta q^B}{q^B} + \left[\left(\frac{\varepsilon^{q^A}_{p^B}}{\varepsilon^{q^B}_{p^B}}\right)+\left(\frac{\varepsilon^{q^B}_{p^A}}{\varepsilon^{q^B}_{p^B}}\right)\frac{R^B}{R^A}\right]\frac{t^A R^A}{R^A\left(t^A+m^A\right)}\left(1+\frac{\Delta q^B}{q^B}\right)+$$

$$+\frac{m^A}{\left(t^A+m^A\right)}\left[\frac{\varepsilon^{q^A}_{p^B}}{\varepsilon^{q^B}_{p^B}}\right]\left(1+\frac{\Delta q^B}{q^B}\right)+\frac{1}{\varepsilon^{q^B}_{p^B}}\frac{R^B}{R^A\left(t^A+m^A\right)}\left[1+2\frac{\Delta q^B}{q^B}+\left(\frac{\Delta q^B}{q^B}\right)^2\right]+\frac{m^B R^B}{R^A\left(t^A+m^A\right)}\left(1+\frac{\Delta q^B}{q^B}\right)$$

or

$$\frac{\Delta q^A}{q^A} = -\frac{t^A}{\left(t^A+m^A\right)} - \frac{m^B R^B}{R^A\left(t^A+m^A\right)}\frac{\Delta q^B}{q^B} +$$

$$+\left[\left(\frac{\varepsilon^{q^A}_{p^B}}{\varepsilon^{q^B}_{p^B}}\right)+\left(\frac{\varepsilon^{q^B}_{p^A}}{\varepsilon^{q^B}_{p^B}}\right)\frac{R^B}{R^A}\right]\frac{t^A R^A}{\left(t^A+m^A\right)} + \left[\left(\frac{\varepsilon^{q^A}_{p^B}}{\varepsilon^{q^B}_{p^B}}\right)+\left(\frac{\varepsilon^{q^B}_{p^A}}{\varepsilon^{q^B}_{p^B}}\right)\frac{R^B}{R^A}\right]\frac{t^A}{\left(t^A+m^A\right)}\frac{\Delta q^B}{q^B} +$$

$$+\frac{m^A}{\left(t^A+m^A\right)}\left[\frac{\varepsilon^{q^A}_{p^B}}{\varepsilon^{q^B}_{p^B}}\right] + \frac{m^A R^A}{R^A\left(t^A+m^A\right)}\left[\frac{\varepsilon^{q^A}_{p^B}}{\varepsilon^{q^B}_{p^B}}\right]\frac{\Delta q^B}{q^B} +$$

$$+\frac{1}{\varepsilon^{q^B}_{p^B}}\frac{R^B}{R^A\left(t^A+m^A\right)} + \frac{1}{\varepsilon^{q^B}_{p^B}}\frac{R^B}{R^A\left(t^A+m^A\right)}\left(2\frac{\Delta q^B}{q^B}\right) + \frac{1}{\varepsilon^{q^B}_{p^B}}\frac{R^B}{R^A\left(t^A+m^A\right)}\left(\frac{\Delta q^B}{q^B}\right)^2 +$$

$$+\frac{m^B R^B}{R^A\left(t^A+m^A\right)}\left(1+\frac{\Delta q^B}{q^B}\right)$$

which is

$$\frac{\Delta q^A}{q^A} = -\frac{t^A}{\left(t^A+m^A\right)} + \left[\left(\frac{\varepsilon^{q^A}_{p^B}}{\varepsilon^{q^B}_{p^B}}\right)+\left(\frac{\varepsilon^{q^B}_{p^A}}{\varepsilon^{q^B}_{p^B}}\right)\frac{R^B}{R^A}\right]\frac{t^A}{\left(t^A+m^A\right)} + \frac{m^A}{\left(t^A+m^A\right)}\left[\frac{\varepsilon^{q^A}_{p^B}}{\varepsilon^{q^B}_{p^B}}\right] + \frac{1}{\varepsilon^{q^B}_{p^B}}\frac{R^B}{R^A\left(t^A+m^A\right)} + \frac{m^B R^B}{R^A\left(t^A+m^A\right)} +$$

$$-\frac{m^B R^B}{R^A\left(t^A+m^A\right)}\frac{\Delta q^B}{q^B} + \left[\left(\frac{\varepsilon^{q^A}_{p^B}}{\varepsilon^{q^B}_{p^B}}\right)+\left(\frac{\varepsilon^{q^B}_{p^A}}{\varepsilon^{q^B}_{p^B}}\right)\frac{R^B}{R^A}\right]\frac{t^A}{\left(t^A+m^A\right)}\frac{\Delta q^B}{q^B} + \frac{m^A}{\left(t^A+m^A\right)}\left[\frac{\varepsilon^{q^A}_{p^B}}{\varepsilon^{q^B}_{p^B}}\right]\frac{\Delta q^B}{q^B} +$$

$$+\frac{1}{\varepsilon^{q^B}_{p^B}}\frac{R^B}{R^A\left(t^A+m^A\right)}\left(2\frac{\Delta q^B}{q^B}\right) + \frac{m^B R^B}{R^A\left(t^A+m^A\right)}\frac{\Delta q^B}{q^B} +$$

$$+\frac{1}{\varepsilon^{q^B}_{p^B}}\frac{R^B}{R^A\left(t^A+m^A\right)}\left(\frac{\Delta q^B}{q^B}\right)^2$$

or

$$\frac{\Delta q^A}{q^A} = -\frac{t^A}{\left(t^A+m^A\right)} + \left[\left(\frac{\varepsilon_{p^B}^{q^A}}{\varepsilon_{p^B}^{q^B}}\right) + \left(\frac{\varepsilon_{p^A}^{q^B}}{\varepsilon_{p^B}^{q^B}}\right)\frac{R^B}{R^A}\right]\frac{t^A}{\left(t^A+m^A\right)} + \frac{m^A}{\left(t^A+m^A\right)}\left[\frac{\varepsilon_{p^B}^{q^A}}{\varepsilon_{p^B}^{q^B}}\right] + \frac{1}{\varepsilon_{p^B}^{q^B}}\frac{R^B}{R^A\left(t^A+m^A\right)} + \frac{m^B R^B}{R^A\left(t^A+m^A\right)} +$$

$$-\frac{m^B R^B}{R^A\left(t^A+m^A\right)}\frac{\Delta q^B}{q^B} + \left[\left(\frac{\varepsilon_{p^B}^{q^A}}{\varepsilon_{p^B}^{q^B}}\right) + \left(\frac{\varepsilon_{p^A}^{q^B}}{\varepsilon_{p^B}^{q^B}}\right)\frac{R^B}{R^A}\right]\frac{t^A}{\left(t^A+m^A\right)}\frac{\Delta q^B}{q^B} + \frac{m^A}{\left(t^A+m^A\right)}\left[\frac{\varepsilon_{p^B}^{q^A}}{\varepsilon_{p^B}^{q^B}}\right]\frac{\Delta q^B}{q^B}$$

$$+\frac{1}{\varepsilon_{p^B}^{q^B}}\frac{R^B}{R^A\left(t^A+m^A\right)}\left(2\frac{\Delta q^B}{q^B}\right) + \frac{m^B R^B}{R^A\left(t^A+m^A\right)}\frac{\Delta q^B}{q^B} +$$

$$+\frac{1}{\varepsilon_{p^B}^{q^B}}\frac{R^B}{R^A\left(t^A+m^A\right)}\left(\frac{\Delta q^B}{q^B}\right)^2$$

so that one obtains the Critical Loss $^{CL}\dfrac{\Delta q^A}{q^A}$ as a function of the Critical Loss $^{CL}\dfrac{\Delta q^B}{q^B}$

$$^{CL}\frac{\Delta q^A}{q^A} = -\frac{t^A}{\left(t^A+m^A\right)} + \left[\left(\frac{\varepsilon_{p^B}^{q^A}}{\varepsilon_{p^B}^{q^B}}\right) + \left(\frac{\varepsilon_{p^A}^{q^B}}{\varepsilon_{p^B}^{q^B}}\right)\frac{R^B}{R^A}\right]\frac{t^A}{\left(t^A+m^A\right)} + \frac{m^A}{\left(t^A+m^A\right)}\left(\frac{\varepsilon_{p^B}^{q^A}}{\varepsilon_{p^B}^{q^B}}\right) + \frac{1}{\varepsilon_{p^B}^{q^B}}\frac{R^B}{R^A\left(t^A+m^A\right)} + \frac{m^B R^B}{R^A\left(t^A+m^A\right)} +$$

$$\left\{-\frac{m^B R^B}{R^A\left(t^A+m^A\right)} + \left[\left(\frac{\varepsilon_{p^B}^{q^A}}{\varepsilon_{p^B}^{q^B}}\right) + \left(\frac{\varepsilon_{p^A}^{q^B}}{\varepsilon_{p^B}^{q^B}}\right)\frac{R^B}{R^A}\right]\frac{t^A}{\left(t^A+m^A\right)} + \frac{m^A}{\left(t^A+m^A\right)}\left(\frac{\varepsilon_{p^B}^{q^A}}{\varepsilon_{p^B}^{q^B}}\right) + \frac{2}{\varepsilon_{p^B}^{q^B}}\frac{R^B}{R^A\left(t^A+m^A\right)} + \frac{m^B R^B}{R^A\left(t^A+m^A\right)}\right\}\left(^{CL}\frac{\Delta q^B}{q^B}\right) +$$

$$+\frac{1}{\varepsilon_{p^B}^{q^B}}\frac{R^B}{R^A\left(t^A+m^A\right)}\left(^{CL}\frac{\Delta q^B}{q^B}\right)^2$$

and substituting for $^{CL}\dfrac{\Delta q^B}{q^B} = {}^{AL}\dfrac{\Delta q^B}{q^B}$ from (19), one gets

$$^{CL}\frac{\Delta q^A}{q^A} = -\frac{t^A}{\left(t^A+m^A\right)} + \left[\left(\frac{\varepsilon_{p^B}^{q^A}}{\varepsilon_{p^B}^{q^B}}\right) + \left(\frac{\varepsilon_{p^A}^{q^B}}{\varepsilon_{p^B}^{q^B}}\right)\frac{R^B}{R^A}\right]\frac{t^A}{\left(t^A+m^A\right)} + \frac{m^A}{\left(t^A+m^A\right)}\left(\frac{\varepsilon_{p^B}^{q^A}}{\varepsilon_{p^B}^{q^B}}\right) + \frac{1}{\varepsilon_{p^B}^{q^B}}\frac{R^B}{R^A\left(t^A+m^A\right)} + \frac{m^B R^B}{R^A\left(t^A+m^A\right)} +$$

$$\left\{-\frac{m^B R^B}{R^A\left(t^A+m^A\right)} + \left[\left(\frac{\varepsilon_{p^B}^{q^A}}{\varepsilon_{p^B}^{q^B}}\right) + \left(\frac{\varepsilon_{p^A}^{q^B}}{\varepsilon_{p^B}^{q^B}}\right)\frac{R^B}{R^A}\right]\frac{t^A}{\left(t^A+m^A\right)} + \frac{m^A}{\left(t^A+m^A\right)}\left(\frac{\varepsilon_{p^B}^{q^A}}{\varepsilon_{p^B}^{q^B}}\right) + \frac{2}{\varepsilon_{p^B}^{q^B}}\frac{R^B}{R^A\left(t^A+m^A\right)} + \frac{m^B R^B}{R^A\left(t^A+m^A\right)}\right\}$$

$$\left\{t^A \varepsilon_{p^A}^{q^B} + \left[-m^A\frac{1}{2}\left(\frac{\varepsilon_{p^B}^{q^A}}{\varepsilon_{p^B}^{q^B}}\right)\frac{R^A}{R^B} - \frac{1}{2}\left(2+\frac{1}{\varepsilon_{p^B}^{q^B}}\right) - m^B\right]\varepsilon_{p^B}^{q^A}\right\} +$$

$$+\frac{1}{\varepsilon_{p^B}^{q^B}}\frac{R^B}{R^A\left(t^A+m^A\right)}\left\{t^A \varepsilon_{p^A}^{q^B} + \left[-m^A\frac{1}{2}\left(\frac{\varepsilon_{p^B}^{q^A}}{\varepsilon_{p^B}^{q^B}}\right)\frac{R^A}{R^B} - \frac{1}{2}\left(2+\frac{1}{\varepsilon_{p^B}^{q^B}}\right) - m^B\right]\varepsilon_{p^B}^{q^A}\right\}^2$$

$$(20)$$

Appendix 3 –The Critical Loss Formula in the US

a) Critical Loss on Side A

A hypothetical monopolist that wants to maximise profits will set prices to satisfy:

$$\frac{\partial \Pi}{\partial p_1^A} = q_1^A(p_1^A, p_1^B) + p_1^A \frac{\partial q_1^A(p_1^A, p_1^B)}{\partial p_1^A} + p_1^B \frac{\partial q_1^B(p_1^A, p_1^B)}{\partial p_1^A} - \frac{\partial C(q_1^A, q_1^B)}{\partial q_1^A} \frac{\partial q_1^A(p_1^A, p_1^B)}{\partial p_1^B} - \frac{\partial C(q_1^A, q_1^B)}{\partial q_1^B} \frac{\partial q_1^B(p_1^A, p_1^B)}{\partial p_1^B} = 0$$

$$\frac{\partial \Pi}{\partial p_1^B} = q_1^B(p_1^A, p_1^B) + p_1^B \frac{\partial q_1^B(p_1^A, p_1^B)}{\partial p_1^B} + p_1^A \frac{\partial q_1^A(p_1^A, p_1^B)}{\partial p_1^B} - \frac{\partial C(q_1^A, q_1^B)}{\partial q_1^A} \frac{\partial q_1^A(p_1^A, p_1^B)}{\partial p_1^B} - \frac{\partial C(q_1^A, q_1^B)}{\partial q_1^B} \frac{\partial q_1^B(p_1^A, p_1^B)}{\partial p_1^B} = 0$$

$$(12)$$

From these first order condition for the hypothetical monopolist's profit maximization, one obtains:

$$^*p^A = -(^*p^B - \frac{\partial C}{\partial q^B})\left[\frac{\partial q^B}{\partial p^A} \Big/ \frac{\partial q^A}{\partial p^A}\right] - \left[^*q^A \Big/ \frac{\partial q^A}{\partial p^A}\right] + \frac{\partial C}{\partial q^A}$$

$$^*p^B = -(^*p^A - \frac{\partial C}{\partial q^A})\left[\frac{\partial q^A}{\partial p^B} \Big/ \frac{\partial q^B}{\partial p^B}\right] - \left[^*q^B \Big/ \frac{\partial q^B}{\partial p^B}\right] + \frac{\partial C}{\partial q^B}$$

Since $^*q^B = q^B + \Delta q^B$, $^*q^A = q^A + \Delta q^A$

$$^*p^A = -(^*p^B - \frac{\partial C}{\partial q^B})\left[\frac{\partial q^B}{\partial p^A} \Big/ \frac{\partial q^A}{\partial p^A}\right] - \left[(q^A + \Delta q^A) \Big/ \frac{\partial q^A}{\partial p^A}\right] + \frac{\partial C}{\partial q^A}$$

$$^*p^B = -(^*p^A - \frac{\partial C}{\partial q^A})\left[\frac{\partial q^A}{\partial p^B} \Big/ \frac{\partial q^B}{\partial p^B}\right] - \left[(q^B + \Delta q^B) \Big/ \frac{\partial q^B}{\partial p^B}\right] + \frac{\partial C}{\partial q^B}$$

so that from the second

$$^*p^B = -(^*p^A - \frac{\partial C}{\partial q^A})\left[\frac{\partial q^A}{\partial p^B} \Big/ \frac{\partial q^B}{\partial p^B}\right] - \left[q^B \Big/ \frac{\partial q^B}{\partial p^B}\right] - \left[\Delta q^B \Big/ \frac{\partial q^B}{\partial p^B}\right] + \frac{\partial C}{\partial q^B}$$

then, multiplying and dividing by p^B all the terms on the right-hand side, by q^A and q^B the first term and by q^B third term only, one gets

$$^*p^B = -(^*p^A - \frac{\partial C}{\partial q^A})\left[\frac{\partial q^A}{\partial p^B} \frac{p^B}{q^A} \Big/ \frac{\partial q^B}{\partial p^B} \frac{p^B}{q^B}\right]\frac{q^A}{q^B} - p^B\left[\frac{q^B}{p^B} \Big/ \frac{\partial q^B}{\partial p^B}\right] - p^B\left[\frac{\Delta q^B}{q^B} \frac{q^B}{p^B} \Big/ \frac{\partial q^B}{\partial p^B}\right] + \frac{\partial C}{\partial q^B}$$

and substituting for the elasticities

$$^*p^B = -(^*p^A - \frac{\partial C}{\partial q^A})\left[\frac{\varepsilon_{p^B}^{q^A}}{\varepsilon_{p^B}^{q^B}}\right]\frac{q^A}{q^B} - p^B\left[\frac{1}{\varepsilon_{p^B}^{q^B}}\right] - p^B\left[\frac{\Delta q^B}{q^B} \frac{1}{\varepsilon_{p^B}^{q^B}}\right] + \frac{\partial C}{\partial q^B}$$

then grouping the second and third term

$$^*p^B = -(^*p^A - \frac{\partial C}{\partial q^A})\left[\frac{\varepsilon_{p^B}^{q^A}}{\varepsilon_{p^B}^{q^B}}\right]\frac{q^A}{q^B} - p^B\left[\frac{1}{\varepsilon_{p^B}^{q^B}} + \frac{\Delta q^B}{q^B} \frac{1}{\varepsilon_{p^B}^{q^B}}\right] + \frac{\partial C}{\partial q^B}$$

subtracting p^B from both sides

$$^*p^B - p^B = -(^*p^A - \frac{\partial C}{\partial q^A})\left[\frac{\varepsilon_{p^B}^{q^A}}{\varepsilon_{p^B}^{q^B}}\right]\frac{q^A}{q^B} - p^B \frac{1}{\varepsilon_{p^B}^{q^B}}\left[1 + \frac{\Delta q^B}{q^B}\right] - \left(p^B - \frac{\partial C}{\partial q^B}\right)$$

and dividing both sides by p^B

$$\frac{^*p^B - p^B}{p^B} = -(^*p^A - \frac{\partial C}{\partial q^A})\left[\frac{\varepsilon_{p^B}^{q^A}}{\varepsilon_{p^B}^{q^B}}\right]\frac{q^A}{p^B q^B} - \frac{1}{\varepsilon_{p^B}^{q^B}}\left[1 + \frac{\Delta q^B}{q^B}\right] - \frac{\left(p^B - \frac{\partial C}{\partial q^B}\right)}{p^B}$$

then both multiplying and dividing the first term on the right-hand side by p^A

$$\frac{^*p^B - p^B}{p^B} = -\frac{\left(^*p^A - \frac{\partial C}{\partial q^A}\right)}{p^A}\left[\frac{\varepsilon_{p^B}^{q^A}}{\varepsilon_{p^B}^{q^B}}\right]\frac{p^A q^A}{p^B q^B} - \frac{1}{\varepsilon_{p^B}^{q^B}}\left[1 + \frac{\Delta q^B}{q^B}\right] - \frac{\left(p^B - \frac{\partial C}{\partial q^B}\right)}{p^B}$$

since $^*p^A = p^A + \Delta p^A$

$$\frac{^*p^B - p^B}{p^B} = -\frac{\left(\Delta p^A + p^A - \frac{\partial C}{\partial q^A}\right)}{p^A}\left[\frac{\varepsilon_{p^B}^{q^A}}{\varepsilon_{p^B}^{q^B}}\right]\frac{p^A q^A}{p^B q^B} - \frac{1}{\varepsilon_{p^B}^{q^B}}\left[1 + \frac{\Delta q^B}{q^B}\right] - \frac{\left(p^B - \frac{\partial C}{\partial q^B}\right)}{p^B}$$

then

$$\frac{\Delta p^B}{p^B} = -\frac{\Delta p^A}{p^A}\left[\frac{\varepsilon_{p^B}^{q^A}}{\varepsilon_{p^B}^{q^B}}\right]\frac{p^A q^A}{p^B q^B} + \frac{\left(p^A - \frac{\partial C}{\partial q^A}\right)}{p^A}\left[\frac{\varepsilon_{p^B}^{q^A}}{\varepsilon_{p^B}^{q^B}}\right]\frac{p^A q^A}{p^B q^B} - \frac{1}{\varepsilon_{p^B}^{q^B}}\left[1 + \frac{\Delta q^B}{q^B}\right] - \frac{\left(p^B - \frac{\partial C}{\partial q^B}\right)}{p^B}$$

Similarly

$$\frac{\Delta p^A}{p^A} = -\frac{\Delta p^B}{p^B}\left[\frac{\varepsilon_{p^A}^{q^B}}{\varepsilon_{p^A}^{q^A}}\right]\frac{p^B q^B}{p^A q^A} + \frac{\left(p^B - \frac{\partial C}{\partial q^B}\right)}{p^B}\left[\frac{\varepsilon_{p^A}^{q^B}}{\varepsilon_{p^A}^{q^A}}\right]\frac{p^B q^B}{p^A q^A} - \frac{1}{\varepsilon_{p^A}^{q^A}}\left[1 + \frac{\Delta q^A}{q^A}\right] - \frac{\left(p^A - \frac{\partial C}{\partial q^A}\right)}{p^A} \tag{21}$$

And, substituting in the latter, the expression (18) for $\frac{\Delta p^B}{p^B}$ derived in Appendix 2b, one obtains

$$\frac{\Delta p^A}{p^A} = -\left[\frac{\Delta p^A}{p^A}\frac{1}{2}\left[\frac{\varepsilon_{p^B}^{q^A}}{\varepsilon_{p^B}^{q^B}}\right]\frac{R^A}{R^B} - m^A\frac{1}{2}\left[\frac{\varepsilon_{p^B}^{q^A}}{\varepsilon_{p^B}^{q^B}}\right]\frac{R^A}{R^B} - \frac{1}{2}\left[2 + \frac{1}{\varepsilon_{p^B}^{q^B}}\right] - m^B\right]*$$

$$*\left[\frac{\varepsilon_{p^A}^{q^B}}{\varepsilon_{p^A}^{q^A}}\right]\frac{p^B q^B}{p^A q^A} + m^B\left[\frac{\varepsilon_{p^A}^{q^B}}{\varepsilon_{p^A}^{q^A}}\right]\frac{p^B q^B}{p^A q^A} - \frac{1}{\varepsilon_{p^A}^{q^A}}\left[1 + \frac{\Delta q^A}{q^A}\right] - m^A$$

so that

$$\frac{\Delta p^A}{p^A} = -\frac{\Delta p^A}{p^A}\frac{1}{2}\left[\frac{\varepsilon_{p^B}^{q^A}}{\varepsilon_{p^B}^{q^B}}\frac{\varepsilon_{p^A}^{q^B}}{\varepsilon_{p^A}^{q^A}}\right] + m^A\frac{1}{2}\left[\frac{\varepsilon_{p^B}^{q^A}}{\varepsilon_{p^B}^{q^B}}\frac{\varepsilon_{p^A}^{q^B}}{\varepsilon_{p^A}^{q^A}}\right] + \frac{1}{2}\left[2 + \frac{1}{\varepsilon_{p^B}^{q^B}}\right]\left[\frac{\varepsilon_{p^A}^{q^B}}{\varepsilon_{p^A}^{q^A}}\right]\frac{R^B}{R^A} +$$

$$+ m^B\left[\frac{\varepsilon_{p^A}^{q^B}}{\varepsilon_{p^A}^{q^A}}\right]\frac{R^B}{R^A} + m^B\left[\frac{\varepsilon_{p^A}^{q^B}}{\varepsilon_{p^A}^{q^A}}\right]\frac{R^B}{R^A} - \frac{1}{\varepsilon_{p^A}^{q^A}}\left[1 + \frac{\Delta q^A}{q^A}\right] - m^A$$

and bringing all terms in $\dfrac{\Delta p^A}{p^A}$ on the right-hand side

$$0 = -\frac{\Delta p^A}{p^A}\left[1 - \frac{1}{2}\left(\frac{\varepsilon_{p^B}^{q^A}}{\varepsilon_{p^B}^{q^B}}\frac{\varepsilon_{p^A}^{q^B}}{\varepsilon_{p^A}^{q^A}}\right)\right] + m^A\left[1 - \frac{1}{2}\left(\frac{\varepsilon_{p^B}^{q^A}}{\varepsilon_{p^B}^{q^B}}\frac{\varepsilon_{p^A}^{q^B}}{\varepsilon_{p^A}^{q^A}}\right)\right] + \left[1 + \frac{1}{2\varepsilon_{p^B}^{q^B}}\right]\left[\frac{\varepsilon_{p^A}^{q^B}}{\varepsilon_{p^A}^{q^A}}\right]\frac{R^B}{R^A} +$$

$$+ 2m^B\left[\frac{\varepsilon_{p^A}^{q^B}}{\varepsilon_{p^A}^{q^A}}\right]\frac{R^B}{R^A} - \frac{1}{\varepsilon_{p^A}^{q^A}} - \frac{1}{\varepsilon_{p^A}^{q^A}}\frac{\Delta q^A}{q^A}$$

then bringing the term in $\dfrac{\Delta q^A}{q^A}$ on the left-hand side

$$\frac{1}{\varepsilon_{p^A}^{q^A}}\frac{\Delta q^A}{q^A} = -\frac{\Delta p^A}{p^A}\left[1 - \frac{1}{2}\left(\frac{\varepsilon_{p^B}^{q^A}}{\varepsilon_{p^B}^{q^B}}\frac{\varepsilon_{p^A}^{q^B}}{\varepsilon_{p^A}^{q^A}}\right)\right] + m^A\left[1 - \frac{1}{2}\left(\frac{\varepsilon_{p^B}^{q^A}}{\varepsilon_{p^B}^{q^B}}\frac{\varepsilon_{p^A}^{q^B}}{\varepsilon_{p^A}^{q^A}}\right)\right] + \left[1 + \frac{1}{2\varepsilon_{p^B}^{q^B}} + 2m^B\right]\left[\frac{\varepsilon_{p^A}^{q^B}}{\varepsilon_{p^A}^{q^A}}\right]\frac{R^B}{R^A} - \frac{1}{\varepsilon_{p^A}^{q^A}}$$

and solving for $\dfrac{\Delta q^A}{q^A}$

$$\frac{\Delta q^A}{q^A} = -\left(\frac{\Delta p^A}{p^A} - m^A\right)\left[1 - \frac{1}{2}\left(\frac{\varepsilon_{p^B}^{q^A}}{\varepsilon_{p^B}^{q^B}}\frac{\varepsilon_{p^A}^{q^B}}{\varepsilon_{p^A}^{q^A}}\right)\right]\varepsilon_{p^A}^{q^A} + \left[1 + \frac{1}{2\varepsilon_{p^B}^{q^B}} + 2m^B\right]\left[\frac{\varepsilon_{p^A}^{q^B}}{\varepsilon_{p^A}^{q^A}}\right]\frac{R^B}{R^A}\varepsilon_{p^A}^{q^A} - \frac{1}{\varepsilon_{p^A}^{q^A}}\varepsilon_{p^A}^{q^A}$$

or equivalently

$$\frac{\Delta q^A}{q^A} = -\left(t^A - m^A\right)\left[1 - \frac{1}{2}\left(\frac{\varepsilon_{p^B}^{q^A}}{\varepsilon_{p^B}^{q^B}}\frac{\varepsilon_{p^A}^{q^B}}{\varepsilon_{p^A}^{q^A}}\right)\right]\varepsilon_{p^A}^{q^A} + \left[1 + \frac{1}{2\varepsilon_{p^B}^{q^B}} + 2m^B\right]\frac{R^B}{R^A}\varepsilon_{p^A}^{q^B} - 1$$

$$\tag{22}$$